Neutron-Electron Interaction*

LESLIE L. FOLDY
Case Institute of Technology, Cleveland, Ohio

INTRODUCTION

THE *neutron-electron interaction* is the term used to describe a weak, short range, spin-, and velocity-independent interaction between the neutron and the electron.

Since the neutron has a vanishing total charge there is no Coulomb interaction between a neutron and an electron. Both particles have magnetic moments and hence there exists the familiar spin-dependent magnetic dipole-dipole interaction between them and also a velocity-dependent interaction between the magnetic moment of the neutron and the magnetic field associated with the convection current of an electron in motion. These interactions have been studied quite extensively by scattering slow neutrons from paramagnetic ions and are not of interest here.[1] A further electromagnetic interaction between the two particles which is both spin- and velocity-*independent* is expected to exist if there is a separation of electrical charges in the neutron so that while neutral as a whole, it contains regions of nonvanishing charge density. There will then exist electric fields inside the neutron and an electron (or for that matter, any charged particle) penetrating this extended charge distribution will be subjected to electrostatic forces. This spin- and velocity-independent interaction is the subject of this review article. At least some charge separation of the type described above is to be expected simply on the basis that the free neutron satisfies the Dirac equation and possesses an anomalous magnetic moment. The contribution of this term (discussed in more detail below) is referred to as the magnetic contribution or Foldy term.[2-4] If there is a further *intrinsic* separation of charge in the neutron, there will be an additional contribution to the neutron-electron interaction which is referred to as the *intrinsic* neutron-electron interaction. On the basis of current meson-theoretical ideas concerning nucleon structure, such a further contribution is to be expected from the fact that a neutron is part of the time dissociated into a negative (pi) meson and a proton.[5]

It is clear that the interaction is not a specific interaction between neutron and electron, but between the neutron and any charged particle, and that the interaction stems from an internal *electromagnetic* structure of the neutron. There is not precluded the existence of a specific interaction between electron and neutron of a nonelectromagnetic nature, but in the interests of economy the postulation of such an interaction is unattractive until it is clear that a purely electromagnetic interpretation of the experiments is untenable. In this light, the term "neutron-electron interaction" is something of a misnomer, but in view of its extensive use in the literature, this terminology is used here.

ELEMENTARY THEORY

For the charge density at the point $\mathbf{r}+\mathbf{b}$ associated with a neutron with center located at the point $\mathbf{r}$, let us write $\rho(\mathbf{b})$. From the fact that this function must satisfy certain elementary invariance properties it follows that ρ is a spherically symmetric function of its argument. The electrostatic interaction energy of this charge distribution in an electrostatic potential $\phi(\mathbf{r})$ is given by

$$V(\mathbf{r})=\int\rho(\mathbf{b})\phi(\mathbf{r}+\mathbf{b})d\mathbf{b}. \quad (1)$$

If the potential is slowly varying over the region occupied by the neutron's charge distribution, we may make a moment expansion:

$$V(\mathbf{r})=\int\rho(\mathbf{b})\left[\phi(\mathbf{r})+\sum_i b_i\frac{\partial\phi(\mathbf{r})}{\partial r_i}+\tfrac{1}{2}\sum_{i,j}b_ib_j\frac{\partial^2\phi(\mathbf{r})}{\partial r_i\partial r_j}+\cdots\right]d\mathbf{b}. \quad (2)$$

Carrying out the integrations, in view of the spherical symmetry of the charge density of total charge zero, this reduces to

$$V(\mathbf{r})=\frac{1}{6}\int b^2\rho(b)d\mathbf{b}\cdot\nabla^2\phi(\mathbf{r}). \quad (3)$$

In the experiments to be described what is measured is the scattering amplitude a_e of the neutron by a bound electron. In Born approximation for neutrons of very long wavelength this is related to the volume integral of the potential $V(\mathbf{r})$:

$$\int V(\mathbf{r})d\mathbf{r}=-\frac{2\pi\hbar^2}{M}a_e=\frac{1}{6}\int b^2\rho(\mathbf{b})d\mathbf{b}\cdot\int\nabla^2\phi(\mathbf{r})d\mathbf{r}$$
$$=-\frac{2\pi Q}{3}\int b^2\rho(\mathbf{b})d\mathbf{b}, \quad (4)$$

* The preparation of this paper has been assisted by the U. S. Atomic Energy Commission.

[1] See, for example, D. J. Hughes, *Pile Neutron Research* (Addison-Wesley Publishing Company, Cambridge, 1953).
[2] L. L. Foldy, Phys. Rev. **83**, 688L (1951); G. Breit, Proc. Natl. Acad. Sci. U. S. **37**, 837 (1951).
[3] L. L. Foldy, Phys. Rev. **87**, 688 (1952).
[4] L. L. Foldy, Phys. Rev. **87**, 693 (1952).
[5] E. Fermi and L. Marshall, Phys. Rev. **72**, 1139 (1947).

where M is the neutron mass and Q is the total of the charge giving rise to the potential $\phi(\mathbf{r})$. The magnitude of the electron-neutron interaction is usually expressed by giving the equivalent constant potential V_0 over a sphere whose radius is the classical electron radius, e^2/mc^2, in order to have the same volume integral as that observed in the experiment. Substituting the electronic charge $-e$ for Q, we then have

$$V_0=\frac{e}{2}\left(\frac{2mc}{e^2}\right)^3\int\rho(b)b^2d\mathbf{b}, \tag{5}$$

so that the neutron-electron interaction measures the second radial moment of the charge density associated with a neutron.

The potential V_0 is purely conventional, since the electron classical radius plays no fundamental role in the problem. In discussing the experimental results, we follow current practice and give the values of V_0 so obtained. These values are related to the actual quantities of intrinsic interest later.

The elementary theoretical analysis, while demonstrating in a satisfactory way the origin of the physical effect, is inadequate for a complete understanding of the relation of the experimental results to the intrinsic charge distribution in the neutron. This is a consequence of certain relativistic effects which lead to a difference between the intrinsic charge distribution and that ostensibly exhibited in the experiments described. The relativistic analysis is presented later.

HISTORY

The first attempt to discover an interaction between electrons and neutrons was made by P. I. Dee[6] in the same year, 1932, in which the neutron was discovered by Chadwick. Dee searched for recoil electrons and for ion pairs produced by (fast) neutrons in a cloud chamber and was able to conclude only that such an interaction had a cross section no greater than 1% of the cross section for interaction of a neutron with a nitrogen nucleus. In 1936, Condon[7] pointed out that by studying the scattering of slow neutrons by electrons bound in atoms, in which case the reduced mass of the system was much increased, one could reduce the upper limit found by Dee by a factor of a thousand. Dee's results correspond to a value for V_0 satisfying the inequality

$$|V_0|<3\times10^9\text{ ev}, \tag{6}$$

while Condon could conclude

$$|V_0|<3\times10^6\text{ ev}. \tag{7}$$

Condon also pointed out that the existence of a neutron-electron interaction would give rise to an isotope shift in spectral lines. Previously Breit had suggested that the isotope shift of spectral lines in heavy atoms was due to the difference in radii of isotopic nuclei, a suggestion which has been substantiated by independent determinations of nuclear sizes. Thus isotope shift data can be employed to place an upper limit on V_0 which is not much smaller than that given in (7).

Discovery of the magnetic moment of the neutron gave the first indication that *some* interaction exists between the electron and neutron, though not the interaction of interest here. However, the rise of Yukawa's meson theory of nuclear forces made it apparent that a suitable theoretical basis for understanding the internal structure of nucleons was established. Of particular interest was the possibility of explaining the anomalous magnetic moment of both proton and neutron, but it was also recognized that the fact that the theory predicts a virtual dissociation of a neutron into proton and negative meson for some fraction of the time would lead one to expect a spin-independent interaction. Simple arguments, to the effect that in the dissociation the meson could reach to a distance of about its own Compton wavelength $\hbar/\mu c$ from the proton, then allow one to estimate an upper limit to the magnitude of the second radial moment of the charge distribution and hence for V_0, namely,

$$|V_0|<\frac{e^2}{2}\left(\frac{\hbar}{\mu c}\right)^2\left(\frac{mc^2}{e^2}\right)^3=5\times10^4\text{ ev}, \tag{8}$$

and further to establish that the sign of V_0 should be negative, corresponding to an attractive force between neutron and electron.

The first attempts to determine an interaction of the magnitude given by (8) were made in 1947 by Havens, Rabi, and Rainwater,[8,9] and by Fermi and Marshall[10] by methods described later. While the latter were only able to place an upper limit of about 5000 ev on the magnitude of V_0, the former were able to show that there probably exists an attractive interaction with a magnitude for V_0 of the order of a few thousand volts. The subsequent refinement of these experiments, together with a measurement of a different type, established that an attractive interaction does indeed exist with $V_0=-4050$ ev, to an accuracy of 5%.

The developments in the theory of the interaction were not equally satisfactory. While the essential assumptions of Yukawa's theory were bolstered by the discovery of the π meson, the detailed working out of the consequences of the theory encountered severe problems. Only one method was available for treating the problems in a rigorous relativistic fashion, namely, the perturbation theory (with renormalization) which assumes a weak coupling between mesons and nucleons. Results of such calculations, while substantiating in general the qualitative phenomena which had been anticipated, were disappointing quantitatively. In the

[6] P. I. Dee, Proc. Roy. Soc. (London) **A136**, 727 (1932).
[7] E. U. Condon, Phys. Rev. **49**, 459 (1936).
[8] Havens, Rabi, and Rainwater, Phys. Rev. **72**, 634 (1947).
[9] Havens, Rabi, and Rainwater, Phys. Rev. **82**, 345 (1951).
[10] E. Fermi and L. Marshall, Phys. Rev. **72**, 1139 (1947).

most favored form of the theory, the calculated ratio of the anomalous magnetic dipole moments of neutron and proton differs by a factor of about 8 from the experimental ratio which is approximately -1. Calculation of the anomalous moments is intimately connected with the calculation of the neutron-electron interaction, so it was clear that the theory for the latter could not be considered trustworthy.

The theoretical situation was upset even more when in 1951 Foldy[2] showed that the observed interaction is the sum of the two terms mentioned in the introduction, a magnetic term whose value can be calculated with knowledge only of the anomalous moment of the neutron, and the intrinsic term which depends on meson-theoretic details and is directly connected with the intrinsic charge separation arising from the virtual dissociation of the neutron into proton and negative meson. Since the magnetic term amounts to -4080 ev in its contribution to V_0, the experiments indicate that the intrinsic contribution can be no more than a few hundred volts, a value much too small to be expected from almost any reasonable form of meson theory. This last puzzle is still unresolved and is discussed later.

EXPERIMENTAL DETERMINATIONS OF THE NEUTRON-ELECTRON INTERACTION

To emphasize the problems involved in a precise experimental determination of the neutron-electron interaction, figures for the order of magnitude of the scattering length and associated cross section to be measured may be reviewed. If one scatters fast neutrons from electrons, in which case the electrons may be regarded as free, and the reduced mass of the neutron-electron system is virtually the electron mass, the associated scattering length is about 10^{-19} cm corresponding to a total cross section of about 5×10^{-37} cm^2. By scattering slow neutrons from electrons bound in atoms or molecules, the reduced mass of the system becomes of the order of the neutron mass and the corresponding scattering length increases to about 1.5×10^{-16} cm and the total cross section of a single electron (which would show up in a measurement of incoherent scattering of the neutron) would be about 3×10^{-31} cm^2. These cross sections are too small to be measured directly.

The experimental techniques which have been used have therefore concentrated on measurement of the scattering amplitude. They thus take advantage of the coherence of the scattering from many electrons and from the atomic nucleus. This can be done in several ways. If one measures the coherent scattering of neutrons by an atom containing a nucleus and Z electrons, then the coherent scattering amplitude for neutrons whose wavelength is long compared to the size of the atom will be the algebraic sum of the nuclear scattering amplitude a_n and the scattering amplitude a_e caused by each of the electrons

$$a_T=a_n+Za_e. \tag{9}$$

The scattering will be isotropic in the center-of-mass system of atom and neutron and the total scattering cross section in this case will be

$$\sigma_T=4\pi a_T^2=4\pi(a_n^2+2Za_na_e+Z^2a_e^2). \tag{10}$$

Since $a_n\sim10^{-12}$ cm and $a_e\sim1.5\times10^{-16}$ cm, for an atom of high atomic number, the total cross section will be a few percent different from that of a bare nucleus. A single measurement of the total cross section of an atom for neutrons of such long wavelength gives no information since one would have to know the coherent scattering length for a bare nucleus. The latter cannot be measured directly, but it can be inferred from a measurement of the *variation* of the total cross section of the atom with neutron wavelength. This is possible since as the wavelength decreases and becomes comparable with the size of the electronic cloud in the atom, destructive interference of the scattered waves from the various electrons sets in for scattering in all but the forward direction. At sufficiently short wavelengths this destructive interference becomes virtually complete and the total coherent scattering cross section approaches that of the nucleus alone. Provided that over this range of wavelength the nuclear scattering length does not change appreciably (which will be true provided there are no neutron resonances in the neighborhood of the wavelength region of interest) one then has a measurement of the coherent scattering length of the nucleus. It is not necessary to extend the measurements over the full energy range indicated. If one knows the scattering form factor of the electron cloud as a function of neutron wavelength, then one need only measure the variation of the atomic cross section over a limited energy range. In essence, this is the basis of the original measurement of Havens, Rabi, and Rainwater[8,9] who used first liquid lead and then liquid bismuth as the scattering material and measured the total cross section as a function of neutron wavelength. Corrections then had to be made for the energy variation of the neutron capture cross section of these materials, for relative velocity effects caused by thermal motion of the target atoms, and for liquid diffraction effects. After application of these corrections, these workers obtained a value,

$$V_0=-(5300\pm1000) \text{ ev},$$

where the negative sign indicates that the interaction between neutron and electron is attractive. A repetition of this experiment carried out more recently by Melkonian, Rustad, and Havens[11] yielded the more precise value

$$V_0=-(4165\pm265) \text{ ev}.$$

The principle of employing interference between the scattering amplitudes from the different electrons in an atom is also the basis for the method originally employed by Fermi and Marshall,[10] and subsequently

[11] Melkonian, Rustad, and Havens, Bull. Am. Phys. Ser. II, **1**, 62 (1956).

refined by Hammermesh, Ringo, and Wattenberg[12] to obtain precision results for the neutron-electron interaction. In this case use is made of the fact that when the atomic size is not very small compared to the neutron wavelength, the interference between the waves scattered by different electrons can be described by a form factor $f(\theta)$, where θ is the angle of scattering of the neutron, such that the total scattering amplitude at an angle θ is given by

$$a_T(\theta)=a_n+Za_e f(\theta). \tag{11}$$

Here $f(0)=1$, and for sufficiently long neutron wavelength, $f(\theta)$ is a monotonically decreasing function of θ as θ varies from 0 to π. Thus the coherent scattering cross section is not isotropic in the center-of-mass system but is peaked forwards or backwards depending on the relative sign of the amplitudes a_n and a_e. Measurement of this asymmetry, combined with knowledge of the coherent nuclear scattering amplitude a_n and of the form factor $f(\theta)$, yields a value for a_e. In the measurements mentioned, krypton and xenon gases were used as the scattering materials. Unfortunately, thermal motion of the gas atoms contributes to the asymmetry to an appreciable degree, and rather elaborate corrections for this relative motion effect, dependent on the energy spectrum of neutrons used and the energy sensitivity of the neutron detectors must be applied. Until recently the coherent nuclear scattering amplitudes of krypton and xenon had not been measured, so estimates of these were used. From the Argonne Laboratory data of Hammermesh, Ringo, and Wattenberg, when combined with the coherent cross sections measured by Crouch, Krohn, and Ringo,[13] the following values were obtained:

krypton: $V_0=-4500$ ev
xenon: $V_0=-3000$ ev
mean $V_0=-(3900\pm800)$ ev.

The spread between the two values is somewhat large, but is consistent with the assigned errors.[14]

The third method for precision measurement of the neutron-electron interaction is that employed by Hughes, Harvey, Goldberg, and Stafne[15] at Brookhaven National Laboratory. It is based on the observation that the index of refraction of a material for neutron waves of wavelength λ is given by the expression,

$$n^2=1+\frac{\lambda^2}{\pi}\sum_i n_i a_i, \tag{12}$$

where a_i is the coherent scattering length for forward scattering of neutrons by particles of type i and n_i is the number of such particles per unit volume of the material. For a material with N atoms per unit volume, this becomes

$$n^2=1+\frac{\lambda^2}{\pi}N(a_n+Za_e), \tag{13}$$

where a_n is again the nuclear scattering length and a_e that of an electron. If the index of refraction can be measured and if a_n can be determined independently, a_e can be determined. One method for precisely measuring the relative refractive index of two materials is to measure the critical angle, θ_c, for total reflection given by

$$\theta_c^2=n_A-n_B \quad (\text{for } \theta_c\ll 1), \tag{14}$$

where n_A and n_B are the refractive indices of the two materials. Hughes and his co-workers selected as the materials bismuth and liquid oxygen, since for this combination, the nuclear scattering amplitudes contribute about equal amounts to the respective refractive indices, and hence the scattering of the neutrons by the electrons is largely responsible for the refractive index difference which enters the critical angle formula. One has

$$\frac{\pi}{\lambda^2}\theta_c^2=N_{\mathrm{Bi}}a_{\mathrm{Bi}}\left\{\frac{N_{\mathrm{O}}a_{\mathrm{O}}}{N_{\mathrm{Bi}}a_{\mathrm{Bi}}}-1\right\}+\{N_{\mathrm{Bi}}Z_{\mathrm{Bi}}-N_{\mathrm{O}}a_{\mathrm{O}}\}a_e, \tag{15}$$

where the subscripts Bi and O refer to bismuth and oxygen, respectively. Since $N_{\mathrm{Bi}}a_{\mathrm{Bi}}\simeq N_{\mathrm{O}}a_{\mathrm{O}}$, one does not require inordinate precision in the measurement of a_{Bi}, and furthermore, the ratio $N_{\mathrm{O}}a_{\mathrm{O}}/N_{\mathrm{Bi}}a_{\mathrm{Bi}}$ can be measured more accurately than the individual absolute scattering lengths. The scattering length a_{Bi} was measured by the method indicated earlier, namely by measuring the total scattering cross sections at neutron energies where the electronic scattering effects are virtually annulled by interference. Corrections for relative motion or knowledge of the atomic form factors are not required. The final result was

$$V_0=-(3860\pm370) \text{ ev}.$$

The consistency of the results obtained by three such different techniques encourages confidence that no irrelevant side effects are appreciably affecting the measurements, and represents a tribute to the experimental skill of those who carried out measurements of this tiny effect.

A grand average of the three precision results, according to the assigned probable errors, gives $V_0=-(4050\pm200)$ ev, which is in very close agreement with the value predicted by the magnetic term alone (-4080 ev), thus leaving no evidence for the existence of an intrinsic interaction.

[12] Hammermesh, Ringo, and Wattenberg, Phys. Rev. **85**, 483(L) (1952).
[13] Crouch, Krohn, and Ringo, Phys. Rev. **102**, 1321 (1956).
[14] In a private communication, Dr. G. R. Ringo has suggested that the discrepancy between the two values may be caused by a p-wave resonance in one of the isotopes of Xe.
[15] Harvey, Hughes, and Goldberg, Phys. Rev. **87**, 220(A) (1952); Phys. Rev. **88**, 163(A) (1952); Hughes, Harvey, Goldberg, and Stafne, Phys. Rev. 90, 497(L) (1953).

PHENOMENOLOGICAL ANALYSIS OF THE NEUTRON-ELECTRON INTERACTION

The accepted interpretation of the observed neutron-electron interaction is that it is primarily, at least, an electromagnetic interaction of the neutron. Thus a corresponding interaction would be expected between the neutron and any charged particle. There exists no substantiation of this conjecture, however. The only other charged particle by which this hypothesis could be easily tested is the proton, but the nuclear interaction between neutron and proton is not sufficiently well understood to allow the identification of a small electromagnetic contribution. More favorable in this respect would be the measurement of the interaction between a neutron and a μ meson, but experimental techniques have not reached the point where this is feasible. In favor of the stated hypothesis is the fact that the observed interaction is of the proper order of magnitude to be expected for an electromagnetic interaction so that any specific interaction of a nonelectromagnetic character between neutron and electron cannot be very large.

Accepting the electromagnetic basis of the neutron-electron interaction and assuming that the *free* neutron satisfies the Dirac equation, a general phenomenological analysis of the interaction is easily constructed. One need only consider the scattering of a Dirac particle by a weak but otherwise arbitrary electromagnetic field. Before making such an analysis, the difference between the *intrinsic* electromagnetic properties of a Dirac particle and the properties which it ostensibly exhibits in an experimental situation will be discussed.

The distinction between these two sets of properties arises[16] from the relativistic phenomenon of "Zitterbewegung." Even for a free Dirac particle, the position of the particle does not move along a straight line with constant velocity but instead carries out a dancing motion (Zitterbewegung) with the speed of light centered on a point which does move uniformly. The physical extent of this dancing motion is of the order of the Compton wavelength of the particle. In the case of the Dirac electron (neglecting radiative corrections, which are small), the electron is presumed to carry a point electric charge $-e$ located at its position. As the electron moves through an electric field, however, the dancing motion causes this charge to explore the field over a region whose extent is the electron Compton wavelength, whence the motion of the charge is not that which would be expected of a point charge, but instead like that of a charge spatially extended over a finite volume. The effect of this is described in the theory of the Dirac electron[17] by the so-called "Darwin term" and is responsible for a shift in the energy levels of an s electron in the hydrogen atom, for example, over and above what would be expected from the change of mass with velocity.

Furthermore, the dancing motion is such that it has a net circulation around the spin direction of the electron. This is equivalent to a small current loop, and hence in the presence of magnetic fields the electron behaves as if it has a magnetic moment (the *normal* magnetic moment) $e\hbar/2mc$, although intrinsically no such moment is possessed by the particle. The apparent finite extent of the charge distribution of an electron and its magnetic moment arise from the purely point charge character of the intrinsic electron when compounded with the effect of Zitterbewegung.

It is, of course, possible to associate with a Dirac particle an intrinsic extended charge and current distribution of finite extent centered on its instantaneous position. The electromagnetic structure ostensibly exhibited by the particle will, however, again be modified by the effects of Zitterbewegung as this intrinsic charge and current density distribution is carried about in the dancing motion of the instantaneous position. The apparent spatial extent of the charge distribution is compounded from the intrinsic extension and the additional "spatial smearing" caused by the Zitterbewegung. The apparent current distribution is modified by the convective transport of the intrinsic charge distribution in the Zitterbewegung.

Of primary theoretical interest are the *intrinsic* electromagnetic properties of a Dirac particle. Thus one is faced with the problem of disentangling from the direct experimental result the separate contributions arising from intrinsic structure and from Zitterbewegung. The relativistically covariant phenomenological analysis which we now present makes this possible.

We now consider the general description of the scattering of a Dirac particle by a weak but otherwise arbitrary electromagnetic field, described by a four-vector potential $A_\mu(x)\equiv(\mathbf{A}(\mathbf{r},t),i\phi(\mathbf{r},t))$ with $x\equiv(\mathbf{r},it)$. For the present calculation we use units in which $\hbar$ and c are unity. We write the general form for the S-matrix element for scattering of the Dirac particle from a momentum-energy state $p_\mu\equiv[\mathbf{p},i(M^2+\mathbf{p}^2)^{\frac{1}{2}}]$ and spinor state u_p to a momentum-energy state $p_\mu'\equiv[\mathbf{p}',i(M^2+\mathbf{p}'^2)^{\frac{1}{2}}]$ and spinor state $u_{p'}$, by this electromagnetic field, retaining only terms linear in the potentials in view of the assumed weakness of the field[18]:

$$-\int d^4x\left\{\bar{u}_{p'}e^{-ip_\mu'x_\mu}\sum_{n=0}^{\infty}\left[\epsilon_n\gamma_\mu\Box^n A_\mu+\tfrac{1}{2}\mu_n\gamma_\mu\gamma_\nu\Box^n\left(\frac{\partial A_\mu}{\partial x_\nu}-\frac{\partial A_\nu}{\partial x_\mu}\right)\right]e^{ip_\mu x_\mu}u_p\right\}. \quad (16)$$

[16] P. A. M. Dirac, *Quantum Mechanics* (Oxford University Press, New York, 1947), third edition, Chap. XI; L. L. Foldy and S. A. Wouthuysen, Phys. Rev. **78**, 29 (1950); S. Tani, Progr. Theoret. Phys. (Kyoto) **6**, 267 (1951); K. Huang, Am. J. Phys. **20**, 479 (1952); H. Feshbach and F. Villars, Revs. Modern Phys. **30**, 24 (1958).

[17] C. G. Darwin, Proc. Roy. Soc. (London) **A118**, 654 (1928). See also L. L. Foldy and S. A. Wouthuysen, reference 16.

[18] G. Salzman, Phys. Rev. **99**, 973 (1955). A. C. Zemach, Phys. Rev. **104**, 1771 (1957); L. L. Foldy, Phys. Rev. **87**, 688 (1952). Such a representation of the S-matrix element is already implied in the early papers of J. Schwinger on quantum electrodynamics: Phys. Rev. **73**, 416 (1948); **74**, 1439 (1948); **75**, 651 (1949).

Here the summation convention on repeated indices is used, the γ_μ are the Dirac matrices, $\Box$ is the d'Alembertian operator, and the coefficients ϵ_n and μ_n characterize the intrinsic electromagnetic structure of the particle. The form of the S-matrix element is completely determined by conditions of Lorentz invariance and gauge invariance; the only arbitrariness lies in the values of the ϵ_n and μ_n coefficients. If A_μ is expanded as a linear combination of plane waves of momentum-energy four-vector k_μ, then for each such plane wave the series of terms with coefficients ϵ_n can be summed to yield a *form factor* $F_1(k_\mu k_\mu)$ and the series of terms with coefficients μ_n can be summed to yield a second *form factor* $F_2(k_\mu k_\mu)$; the first describes the intrinsic charge density, the second the intrinsic magnetization density, of the Dirac particle in a transition corresponding to the momentum-energy transfer $k_\mu = p_\mu' - p_\mu$.[19]

The same S-matrix element would be obtained in first Born approximation from the following extension of the ordinary Dirac equation[18]:

$$\gamma_\mu \frac{\partial \psi}{\partial x_\mu} + \frac{Mc}{\hbar}\psi - \frac{i}{\hbar c}\sum_{n=0}^{\infty}\left[\epsilon_n \gamma_\mu \Box^n A_\mu + \tfrac{1}{2}\mu_n \gamma_\mu \gamma_\nu \Box^n\left(\frac{\partial A_\mu}{\partial x_\nu} - \frac{\partial A_\nu}{\partial x_\mu}\right)\right]\psi = 0. \quad (17)$$

In this equation the coefficient ϵ_0 is the total charge on the Dirac particle. The term prefixed by the coefficient μ_0 is the representation of an anomalous or intrinsic magnetic dipole moment for the Dirac particle of the form introduced by Pauli.[20] The remaining terms in these series, which are less familiar, represent successively higher radial moments of the intrinsic charge and current distribution associated with the particle.

In particular the term prefixed by ϵ_1 gives a partial description of a radial extension of the intrinsic charge distribution with ϵ_1 itself related to the second radial moment[21]:

$$\epsilon_1 \sim \frac{1}{6}\int r^2 \rho(r) d\mathbf{r}, \quad (18)$$

where $\rho(\mathbf{r})$ represents the intrinsic charge density. Since the effect of this term is similar to that of the "Darwin term" in the theory of the Dirac electron, it may be called the *intrinsic* Darwin term. The term in μ_1 arises from a radial extension of the intrinsic magnetization of the Dirac particle, the coefficient μ_1 measuring the second radial moment of this magnetization.

From either the S-matrix element or from the equivalent Dirac equation, it is a simple matter to calculate the scattering amplitude for a Dirac particle when scattered by a weak, slowly varying, purely electrostatic potential $\phi(\mathbf{r})$. This is given by (with restoration of $\hbar$, c):

$$a(\mathbf{k}) = -\frac{M}{2\pi\hbar^2}\int e^{-i\mathbf{k}\cdot\mathbf{r}}\sum_{n=0}^{\infty}\left\{\epsilon_n + \frac{\hbar}{2Mc}\mu_{n-1} + \frac{1}{2}\left(\frac{\hbar}{2Mc}\right)^2 \epsilon_{n-1} + \cdots\right\}\Delta^n \phi(\mathbf{r}) d\mathbf{r}, \quad (19)$$

where $\hbar\mathbf{k}$ represents the momentum transfer in the scattering, and $\Delta = \nabla^2$ is the Laplacian operator. When the momentum transfer is small only the terms $n=0$ and $n=1$ are important. The term for $n=0$ contributes simply

$$a_0(\mathbf{k}) = -\frac{M\epsilon_0}{2\pi\hbar^2}\int e^{-i\mathbf{k}\cdot\mathbf{r}}\phi(\mathbf{r}) d\mathbf{r}, \quad (20)$$

to the scattering amplitude and represents the ordinary electrostatic scattering of a particle bearing a point charge ϵ_0. The contribution of the term $n=1$ can be written

$$a_1(\mathbf{k}) = -\frac{M}{2\pi\hbar^2}\left[\epsilon_1 + \frac{\hbar}{2Mc}\mu_0 + \frac{1}{2}\left(\frac{\hbar}{2Mc}\right)^2\epsilon_0\right] \times \int e^{-i\mathbf{k}\cdot\mathbf{r}}\nabla^2\phi(\mathbf{r}) d\mathbf{r}, \quad (21)$$

and in the limit $\mathbf{k} \to 0$ becomes

$$a_1 = \frac{2MQ}{\hbar^2}\left[\epsilon_1 + \frac{\hbar}{2Mc}\mu_0 + \frac{1}{2}\left(\frac{\hbar}{2Mc}\right)^2\epsilon_0\right], \quad (22)$$

where Q is the total charge producing the potential $\phi(\mathbf{r})$.

In the case of the neutron, for which ϵ_0 and hence a_0 is zero, the term a_1 is responsible for the coherent scattering by the electronic charge in an atom and hence gives rise to the observed neutron-electron interaction. With $Q = -e$, the electronic charge, the scattering amplitude a_1 is to be identified with the scattering amplitude a_e introduced in our discussion of the experiments; hence

$$a_e = -\frac{2Me}{\hbar^2}\left[\epsilon_1 + \frac{\hbar}{2Mc}\mu_0\right]. \quad (23)$$

This result can easily be translated into the purely conventional potential V_0 by noting that the scattering amplitude of a constant potential V_0 over a sphere of radius e^2/mc^2 is given by

$$a_e = -\frac{2}{3}\frac{MV_0}{\hbar^2}\left(\frac{e^2}{mc^2}\right)^3, \quad (24)$$

[19] These form factors are essentially the same as those employed by other speakers at this conference. See Yennie, Levy, and Ravenhall, Revs. Modern Phys. **29**, 144 (1957).

[20] W. Pauli, Revs. Modern Phys. **13**, 203 (1941).

[21] We employ a correspondence symbol rather than an equal sign here since there is some ambiguity in relating the relativistic coefficients to the physical extension of a static charge distribution. The indicated correspondence is perhaps the most reasonable one.

whence

$$V_0=3e\left(\frac{mc^2}{e^2}\right)^3\left[\epsilon_1+\frac{\hbar}{2Mc}\mu_0\right]. \tag{25}$$

Before discussing this particular result, let us return to the expression for the scattering length a_1 in the more general case given in Eq. (21), and discuss the significance of the terms included therein. The propriety of the appearance of ϵ_1, since it represents the second radial moment of the intrinsic charge distribution is obvious. The term in ϵ_0 is the Darwin term described as arising from the Zitterbewegung of the intrinsic charge. The term in μ_0 is new, however, and requires explanation. This term for the neutron is the magnetic term referred to in the introduction. It is also a consequence of Zitterbewegung through the following mechanism: When a Dirac particle possesses an intrinsic or anomalous moment μ_0, this moment is carried about by the particle in its Zitterbewegung. From the theory of relativity it is known that when a magnetic moment is set into motion, an electric moment is developed with direction perpendicular both to the direction of motion and to the direction of the magnetic moment. In the Zitterbewegung of the Dirac particle, the radial component of this electric moment does not average to zero. Hence there is a radial separation of charge with a spherical layer of charge of one sign lying outside a spherical layer of equal but opposite charge. Such a spherical double layer of charge has a finite second radial moment for its charge distribution: it is this which contributes the term in μ_0 to the scattering amplitude.

In the case of the neutron, the Darwin term is absent, but since the neutron possesses an anomalous moment of -1.91 nuclear magnetons, the magnetic term contributes to the observed interaction. Its contribution can be calculated on the basis of knowledge only of the moment without reference to its origin. Substitution of numerical values yields

$$V_{0,\mathrm{mag.}}=-4080 \text{ ev.}$$

Comparing with the observed interaction V_0, the intrinsic interaction is only

$$V_{0,\mathrm{intrinsic}}=(30\pm 200) \text{ ev.}$$

The unexpected smallness of this result compared to meson theory expectations represents the most puzzling feature of the neutron-electron interaction.

From the measured V_0 and the known magnetic moment of the neutron, we have then for the initial coefficients characterizing the intrinsic electromagnetic structure of the neutron:

$$\begin{aligned}\epsilon_0{}^N&=0,\\ \mu_0{}^N&=-1.91(e\hbar/2Mc),\\ \epsilon_1{}^N&=(0.03\pm0.2)\cdot(e\hbar^2/8M^2c^2).\end{aligned} \tag{26}$$

A similar analysis is possible for the proton, where the experiments on high-energy electron scattering by protons by Hofstadter and his collaborators[22] yield a value for the second radial moment of the proton charge distribution. The results are

$$\begin{aligned}\epsilon_0{}^P&=e,\\ \mu_0{}^P&=1.79(e\hbar/2Mc),\\ \epsilon_1{}^P&=20(e\hbar^2/8M^2c^2).\end{aligned} \tag{27}$$

High-energy electron scattering experiments on deuterons compared with protons also confirm the fact that $\epsilon_1{}^N\ll\epsilon_1{}^P$.

The phenomenological analysis just described contains no information in itself as to the expected magnitude for the ϵ_n and μ_n coefficients. For this a fundamental theory which yields a microscopic description of the intrinsic electromagnetic structure of nucleons is needed. Even without such a theory, the disparately small value of $\epsilon_1{}^N$ is greatly in evidence from the following argument: From the quantities ϵ_0, μ_0, and ϵ_1, one can form ratios which have the dimensions of a length, namely μ_0/ϵ_0 and ϵ_1/μ_0. For the proton these two lengths are of comparable order of magnitude and also of the order of the "mechanical size" of the proton as given by its Compton wavelength. On the other hand, for the neutron the ratio $\epsilon_1{}^N/\mu_0{}^N$ is much smaller.

An almost obvious fact is worth noting, namely that a *relativistic* phenomenological analysis, such as has been here presented, is essential for understanding the experimental results. This follows from the fact that the magnetic term, which is of relativistic origin, dominates the observed effect. A nonrelativistic treatment would have completely missed this term and given a misleading result for the second radial moment of the intrinsic charge distribution. The situation is somewhat analogous to that encountered in understanding the fine structure of the energy levels of the hydrogen atom. An attempt to describe this nonrelativistically in terms of the Pauli spin theory would not only be doomed to failure, but would give a completely misleading picture of a phenomenon which is essentially relativistic in all its aspects. Three relativistic phenomena are responsible for this fine structure: the relativistic variation of mass with velocity, the normal moment of the electron which has a relativistic origin in Zitterbewegung, and the Darwin term arising also from relativistic Zitterbewegung.

For emphasis, the point may be put still another way. The above phenomenological analysis can be applied to discussing, in principle at least, some of the electromagnetic properties of an atom whose total angular momentum is $\hbar/2$, such as the deuterium atom in its ground state with electron and nuclear spin antiparallel. In this case, the intrinsic coefficient ϵ_1 is of the order of the electron charge times the square of the

[22] See Hofstadter, Bumiller, and Yearian, Revs. Modern Phys. **30**, 482 (1958).

Bohr radius, while the "anomalous" magnetic moment of the atom is of the order of a Bohr magneton. Thus the ratio of the magnetic term to the intrinsic coefficient ϵ_1 is here of the order of 10^{-9}, and hence the magnetic term could be disregarded in discussing the charge distribution in such an atom, as is the case in practice. A nonrelativistic analysis of this problem would be perfectly satisfactory. Had this ratio been correspondingly small in the case of a neutron, a nonrelativistic analysis would have been adequate in that case as well. Instead, the experimental ratio is of the order of 20!

The phenomenological analysis has omitted the possible contribution of terms of second or higher order in the electromagnetic field. Such terms should exist theoretically as a consequence of the polarizability of the charge and current distribution associated with a nucleon. Their contribution can be calculated from any given meson theory,[23] or they can be estimated from the cross section for photoproduction of mesons on nucleons.[24] Such calculations indicate that in general the polarization effects should be small compared to the observed effects in the experiments so far performed on both protons and neutrons. However, the intrinsic neutron-electron interaction is so anomalously small as to suggest that such a polarization correction may nevertheless be necessary when experiments improve to a degree where a precision result for ϵ_1 can be obtained.

MESON THEORY

Up to this point the discussion has centered on the experimental results and their appropriate phenomenological analysis with no attempt to understand them in terms of a fundamental theory of nucleon structure. Any review of the neutron-electron interaction would be incomplete without some description of these attempts. In this connection the neutron-electron interaction can not be logically isolated from the other electromagnetic properties of nucleons, and hence it is necessary to include in this discussion the intrinsic Darwin coefficient ϵ_1 for the proton and the anomalous magnetic moments of neutron and proton as well.

The meson theory gave promise of a fundamental basis in terms of which the electromagnetic structure of nucleons (that is, the deviations in their electromagnetic behavior from that of simple Dirac particles) might be rationalized. In this theory nucleons are considered the sources of a *mesonic* field in the same manner in which charges are the sources of an electromagnetic field. The quanta of the meson field differ from those of the electromagnetic field in that they have a finite rest and may themselves carry an electric charge. The virtual emission and reabsorption of mesons by a nucleon then modifies the electromagnetic properties of a nucleon in a manner in some ways analogous to that in which the virtual emission and reabsorption of photons modifies the electromagnetic properties of an electron or other charged particle.

The coupling of the electromagnetic field to its sources is measured by the fine-structure constant $e^2/\hbar c = 1/137$. Smallness of this number permits treatment of this interaction as a small perturbation (weak coupling theory) and allows the solution of electrodynamic problems by considering the expansion of the associated S-matrix element as a relatively rapidly converging power series in the fine structure constant. In this treatment it is essential to recognize that even the mass and charge of the sources of the electromagnetic field are modified through the interaction. Consistent recognition of this fact throughout the calculations constitutes the so-called renormalization program. Only after the discovery of how this could be done in a relativistically consistent manner was quantum electrodynamics able to describe with precision and without ambiguity the electromagnetic behavior of electrons and positrons.

The success of the perturbation method combined with the renormalization program in electrodynamics gave hope that similar success might be achieved in treatment of the meson field. This hope has not been realized, presumably because the coupling in this case is not sufficiently weak. In spite of this, the covariant weak coupling theory has still some virtue in that it is the only current theory which is relativistically covariant and in that it gives a qualitative insight, at least, into the nature of the phenomena arising from the existence of the meson field. Quantitatively, however, it has failed completely in describing the important phenomena of presumably mesonic origin: nucleon-nucleon forces, meson scattering, the production of mesons by collisions between nucleons, or between photons and nucleons, and the electromagnetic properties of nucleons.

The failures of weak coupling meson theory do not necessarily imply a fundamental weakness in meson theory itself, but only an inadequacy of the mathematical methods for dealing with it. If the covariance of the theory is sacrificed to the end of avoiding the weak coupling approximation, then in this mutilated form, the theory has been extremely valuable in making explicable many of the features of the experimental phenomena listed above in a semiquantitative, or, in some cases, in a quantitative way. What is not clear is the precise relationship between the mutilated theory and its relativistically covariant primitive, but even here, recent developments, such as the derivation of the dispersion relations, have made the understanding of these relations much clearer, if not entirely transparent.[25]

The experimental study of mesons has led to the conclusion that the π-meson field transforms relativistically as a pseudoscalar field and that positive, negative, and neutral mesons are coupled in a symmetrical manner

[23] J. M. Jauch and K. M. Watson, Phys. Rev. **72**, 1254 (1947); **73**, 268 (1948).

[24] S. D. Drell and M. A. Rudermann, Phys. Rev. **106**, 561 (1957).

[25] Chew, Goldberger, Low, and Nambu, Phys. Rev. **106**, 1337, 1345 (1957).

to nucleon sources to give such systems the property known as charge independence or invariance under rotations in isotopic spin space. Thus consideration need be given only to the symmetrical pseudoscalar meson theory. Less is known concerning the coupling of the field to the nucleon sources. The simplest coupling (pseudoscalar coupling) is known to lead to a renormalizable theory, and has current theoretical preference. The so-called pseudovector coupling theory is not renormalizable, but shares certain properties with the pseudoscalar coupling theory through well-known equivalence theorems. The lack of renormalizability does not necessarily exclude it from consideration, but has as a consequence the fact that it cannot by itself be handled covariantly in a manner that gives meaningful results. There can exist further types of coupling which are nonlinear in the meson field, but these have not been explored in any systematic way. Our discussion of the covariant weak coupling theories is limited to the pseudoscalar coupling theory.

Before proceeding to details, it is appropriate to discuss the consequences, such as they are, of charge independence for the electromagnetic properties of nucleons. In a theory which involves only nucleons and mesons,[26] the charge and current density of the meson field transform as the z component of a vector in isotopic spin space and hence give equal but opposite-signed contributions to electromagnetic properties for neutron and proton. On the other hand, the nucleon contribution to the charge and current density transforms as a linear combination of isotopic vector and isotopic scalar and hence there is no simple relation between its contribution to the properties of the neutron as compared with its contribution to those of the proton. Consequently, charge independence in itself imposes no relationship between the electromagnetic coefficients ϵ_n and μ_n for neutrons and those for protons. It is nevertheless useful for some purposes to consider the isotopic scalar and isotopic vector parts of the coefficients ϵ_n and μ_n, defined to be

$$\begin{aligned} \epsilon_n{}^V&=\tfrac{1}{2}(\epsilon_n{}^P-\epsilon_n{}^N), & \epsilon_n{}^S&=\tfrac{1}{2}(\epsilon_n{}^P+\epsilon_n{}^N), \\ \mu_n{}^V&=\tfrac{1}{2}(\mu_n{}^P-\mu_n{}^N), & \mu_n{}^S&=\tfrac{1}{2}(\mu_n{}^P+\mu_n{}^N), \end{aligned} \tag{28}$$

since the meson charge current contributes only to the isotopic vector part, and hence this form of expression isolates, to some degree, certain of the theoretical contributions to these coefficients. From these definitions, it is clear that the experimental values indicate that for the anomalous magnetic moment of a nucleon, the isotopic vector part is much larger than the isotopic scalar part (by a factor of about 30). On the other hand, the isotopic scalar and isotopic vector parts of ϵ_1 are very closely equal in magnitude.

[26] These remarks have reference to the "bare" meson and nucleon charge current in a theory in which there is no "interaction" charge-current density, such as the pseudoscalar coupling theory. In the more general case, the meson and nucleon contributions cannot be separated.

TABLE I. Comparison of weak coupling pseudoscalar meson theory (pseudoscalar coupling) results with experiment.

Quantity	Experiment	Theory
$\mu_0{}^N/\mu_0{}^P$	-1.07	-7.8
$\epsilon_1{}^N/\epsilon_1{}^P$	0.0015 ± 0.01	-0.28
$\epsilon_1{}^N/(\mu_0{}^N\hbar/2Mc)$	-0.007 ± 0.05	0.32
$\epsilon_1{}^P/(\mu_0{}^P\hbar/2Mc)$	5.1	9.0

A. Weak Coupling Pseudoscalar Theory

Calculations with the weak coupling approximation to the pseudoscalar coupling theory have been carried out by a large number of authors.[27] After correction of some minor errors in some of the earlier papers and a clarification of some confusion concerning the magnetic term, all of the calculations agree. Since the values of the coefficients ϵ_n and μ_n are proportional to the square of the coupling constant, comparison of the results with experiment is facilitated by quoting ratios, which are independent of the coupling constant. The results are presented in Table I which bespeaks a sad commentary on and writes an epitaph to the unmodified weak coupling theory. Several points are, however, of interest.

(a) The theory obviously predicts much too large an isotopic scalar part for the anomalous magnetic moment. Since the isotopic scalar part arises from the nucleon current only, the experimental values indicate that either the nucleon contribution to the current is strongly suppressed or that its isotopic scalar part is strongly suppressed relative to its isotopic vector part. Neither of these alternatives occur in the weak coupling theory.

(b) The discrepancies between theory and experiment for the ratios which involve $\epsilon_1{}^N$ are also large, but not so large as they might easily have been. The theoretical value of $\epsilon_1{}^N$ is smaller than in general might be expected because of a moderate cancellation between meson and nucleon contributions to it, while such a cancellation does not occur in $\epsilon_1{}^P$. In fact, if the π-meson mass were approximately double its actual value, this cancellation would have been complete and would lead to $\epsilon_1{}^N=0$, without appreciably changing $\epsilon_1{}^P$. Thus a small value for the neutron-electron interaction can be achieved by cancellation of mesonic and nucleonic contributions, but such a cancellation would be a purely fortuitous phenomenon.

It is interesting, and at the same time puzzling, that agreement with experiment on the magnetic moments requires suppression of the nucleon contribution to the current density, while agreement with experiment on the coefficients ϵ_1 requires that the nucleon contribution

[27] J. M. Luttinger, Phys. Rev. **74**, 893 (1948); M. Slotnick and W. Heitler, Phys. Rev. **75**, 1645 (1949); K. M. Case, Phys. Rev. **76**, 1 (1949); S. M. Dancoff and S. D. Drell, Phys. Rev. **76**, 205 (1949); S. Borowitz and W. Kohn, Phys. Rev. **76**, 818 (1949); B. D. Fried, Phys. Rev. **88**, 1142 (1952). See also: M. Rosenbluth, Phys. Rev. **79**, 615 (1950); K. Nakabayashi and I. Sato, Progr. Theoret. Phys. **6**, 252 (1951); S. Goto, Progr. Theoret. Phys. **12**, 699 (1954); K. Ishida, Progr. Theoret. Phys. **18**, 493 (1957).

to the charge density be large and comparable with the meson contribution. It is not easy to see how these conditions can be achieved by any simple modification of the weak coupling theory.

B. Chew-Low Static Theory

Direct attempts to take into account higher order terms in the weak coupling theory in a covariant way have not been particularly successful in ameliorating the discrepancies of the weak coupling theory with experiment. However, if covariance is sacrificed it is possible to obtain results from the theory which are noteworthy in their success in correlating experimental results associated with meson scattering, meson photoproduction, and internucleonic forces. In particular the so-called "static model" which has been analyzed in great detail by Chew and Low[28] is worthy of mention. This is a mutilated form of the pseudovector coupling theory in which the nucleon is represented as a static extended source of the meson field. It is, however, connected with the pseudoscalar coupling theory through the equivalence theorems, and corresponds, at least roughly, to the pseudoscalar theory with nucleon recoil and pair production suppressed and with the matrix element for emission of mesons of high momentum reduced virtually to zero by the introduction of a momentum cutoff, corresponding to the extended source. In this theory, not only is the application of weak-coupling methods justified, but higher order corrections can be calculated in a consistent manner.

This theory considers the physical nucleon to be constituted of a "core" and a "meson field" or "meson cloud." The latter describes at least the outlying part of the proper meson field. The core, represented by the extended meson source function, presumably represents in an overly simplified way the complicated effects of nucleon recoil, nucleon-antinucleon pairs, heavier particles which may be coupled to nucleons, and the inner part of the proper meson field. Its specification is much less complete than that of the outlying meson field, and this is particularly true of its electromagnetic properties. Although there is no difficulty in computing the contribution of the outlying meson field to electromagnetic properties of the physical nucleon, the contribution of the core is completely ambiguous.

The outlying meson field contributes only to the isotopic vector part of the anomalous magnetic moment of the nucleon, which is the dominant part of the moment. Its contribution can be calculated[29] in terms of the fundamental parameters (the coupling constant and the cutoff momentum) which are already fixed by fitting the experimental data on meson scattering, and is in quite reasonable agreement with the experimental isotopic vector part of the moment, considering the various uncertainties. The contribution of the core, on the other hand, depends very much on the electromagnetic properties which are ascribed to it. If it is assigned a normal Dirac moment in the proton state and no moment in the neutron state, then its isotopic vector part is such as not seriously to disturb the agreement with experiment obtained from the meson contribution. Its isotopic scalar part is much too large when compared with the experimental values, however. There is no real justification for this treatment of the core in view of its complex nature, and the ambiguity concerning its proper treatment is so great as to make it difficult to decide whether the reasonable value obtained for the contribution of the meson cloud should be given any significance.

The situation with respect to the coefficient ϵ_1 measuring the neutron-electron interaction and the mean-square charge radius of the proton is also unsatisfactory.[30] The outlying meson field contribution, if considered by itself yields much too large a value for the neutron, and somewhat too small a value for the proton. Since experimentally the isotopic scalar and isotopic vector parts of ϵ_1 are nearly equal, the meson cloud alone, which contributes an isotopic vector part only, cannot give agreement with experiment. Again one is at a loss concerning the proper treatment of the core. In order to obtain the small observed intrinsic neutron-electron interaction it would be necessary to assume that the core charge is spread out over a region comparable in size with the meson cloud itself. This is a very unattractive hypothesis since it implies that electromagnetically, the core is as large as the entire nucleon and hence the physical significance of the separation between core and meson cloud becomes obscure.

From this discussion it is clear that the interpretation of the results for electromagnetic properties of nucleons on the static model is highly ambiguous because of the uncertainty in the proper treatment of the core. There exists, however, a further ambiguity (which may not be entirely separated from the first) as a consequence of the fact that the theory is basically nonrelativistic. As emphasized earlier, a nonrelativistic treatment of the neutron-electron interaction, and for that matter, the magnetic moment as well, is highly suspect since the proper disposition of relativistic corrections which are substantially of the same order as the effects considered is very much in question.

It has been the general practice to identify the second radial moment of the charge density in the static model directly with the experimental *intrinsic* neutron-electron

[28] G. F. Chew, Phys. Rev. **94**, 1748, 1755 (1954); **95**, 1669 (1954); F. E. Low, Phys. Rev. **97**, 1392 (1955); Lehmann, Symanzik, and Zimmerman, Nuovo cimento **1**, 1 (1955); G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570 (1956). See also reference 24. The following papers also deal with related nucleonic models: R. G. Sachs, Phys. Rev. **87**, 1100 (1952); Phys. Rev. **95**, 1065 (1954); B. T. Feld, Ann. phys. **1**, 58 (1957).

[29] M. Friedman, Phys. Rev. **97**, 1123 (1955); H. Myazawa, Phys. Rev. **101**, 1564 (1956).

[30] G. Salzman, Phys. Rev. **99**, 973 (1955); **105**, 1076 (1957); S. B. Treiman and R. G. Sachs, Phys. Rev. **103**, 435 (1956); P. R. Suura, Phys. Rev. **108**, 470 (1957).

interaction rather than with the ostensible interaction as directly observed in the experiments, which contains the magnetic term as well. We do not feel that it is definitely settled that this is the proper treatment and give here an argument indicating that the proper treatment is very much in question until one knows the precise relationship of a static theory to its presumed relativistic primitive.

The physical origin of the magnetic term can be understood in terms of the transport of the intrinsic magnetic moment of a neutron in its Zitterbewegung. This means that the current distribution which gives rise to the anomalous moment must be transported. In the meson theory the current distribution arises in part at least from the virtual meson cloud about the nucleon. One might argue that if the "natural frequencies" of the meson cloud are appreciably lower than the Zitterbewegung frequency $Mc^2/\hbar$, then the meson field would be unable to follow the Zitterbewegung but would instead tend to center on the average position of the nucleon and would not partake in the Zitterbewegung. In such a contingency, the magnetic term would be expected to be absent, or at least reduced by a factor depending on the degree of "slip" of the meson cloud relative to the nucleon source. If this were the case, this could be interpreted in the specific context of the static model as meaning that the "core" in the static theory receives part of its extension from the Zitterbewegung of the nucleon source, and hence that the magnetic term should not be added to the result from this theory in order to compare with the direct experimental value. This does not mean that our relativistic phenomenological analysis is incorrect, but that there can exist a dynamic relationship between μ_0 and ϵ_1 such that the magnetic term is in part cancelled by effects contained in the intrinsic term. In any case, so long as one has a completely relativistic theory there is not the least ambiguity in the proper treatment of the magnetic term.

We hold no particular brief for the correctness of this argument but consider it only as an example of the problems which can arise in attempting to draw firm conclusions from a nonrelativistic theory and hence as an admonition for caution. On this basis we are tempted to conclude that the static theory is simply inadequate to deal properly with the problem of the electromagnetic structure of nucleons.

It should be mentioned that the π-mesonic field is not the only field responsible for electromagnetic structure for the nucleon. The coupling of nucleons to the K-meson field will also make its contribution to this structure, and as Sandri[31] has pointed out, its effect can be of the form to reduce the intrinsic neutron-electron interaction.

FINAL REMARKS

The outstanding characteristic of the intrinsic neutron-electron interaction, as has been repeatedly emphasized, is its unexpected smallness. From the point of view of current theories of nucleon structure it should be compounded from a number of contributions—the π-meson cloud, nucleon recoil, K-meson effects, etc.—each of which individually is much larger than the observed effect. Assuming the correctness of the present viewpoint, one is forced to the conclusion that the smallness of $\epsilon_1{}^N$ must be the result of fortuitous cancellation of relatively large contributions—a conclusion which is not particularly attractive but also one which cannot be rejected on any *a priori* grounds.

The only other simple manner of explaining its smallness is to assume that there exists a basic symmetry principle or selection rule which rigorously enforces the charge *density* of the neutron to be identically zero. This would imply a basic asymmetry between the neutron and proton which would be puzzling in view of the many similarities of the two particles. Clearly, new experiments which give detailed information about the charge density function for the neutron and not simply its second radial moment would be most helpful in this situation.

ACKNOWLEDGMENT

The author wishes to express his indebtedness to his colleague, Dr. Smio Tani, for valuable discussions.

[31] G. Sandri, Phys. Rev. **101**, 1616 (1956); also K. Ishida, reference 26.