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Structure of the Nucleon*

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IT is generally believed that nucleons have some sort of internal structure. By this we mean that if one attempts to discuss their interactions with other particles and, in particular, with the electromagnetic field, one finds that they cannot be described in terms of kinematical quantities and static properties such as charge, spin, and magnetic moment. It is, of course, possible that in fact these should suffice and that our theoretical ideas about the fundamental interactions are so entirely wrong as to have not even phenomenological, qualitative validity. For example, one could conjecture that apparent deviations from a description in terms of point particles in electromagnetic interactions means a breakdown of quantum electrodynamics. Such arguments are difficult to combat except with successful, less radical explanations. In view of the perhaps qualified success of meson theory to correlate, via dispersion relations, much of the data in the field of π-meson physics, we adopt the attitude that quantum field theory indeed has some finite domain of applicability and attempt a description of nucleon structure based on it.

So long as we confine attention to phenomena which involve energies very small compared to the mass of the lightest particles that are strongly coupled to nucleons we expect that structure will play no important role. Thus, the scattering of zero frequency light can do no more than measure the total electric charge. If the frequency is raised slightly, the scattering amplitude can be completely characterized by giving beside the charge the anomalous magnetic moment. It can be shown quite generally that the next term in the frequency expansion involves the detailed dynamical structure of the nucleon. For example, one finds for the coherent forward scattering amplitude the expression

$$f(\omega)=-\frac{e^2}{m}+\frac{\omega^2}{2\pi^2}\int_0^\infty d\omega'\frac{\sigma(\omega')}{\omega'^2-\omega^2-i\epsilon},$$
$$\simeq-\frac{e^2}{m}+\frac{\omega^2}{2\pi^2}\int_0^\infty d\omega'\frac{\sigma(\omega')}{\omega'^2}, \qquad (1)$$

where σ is the total cross section for the scattering of unpolarized light. For small frequencies the ω under the integral may be neglected. From a theoretical standpoint we must know all of the channels into which the initial system of photon and nucleon may lead and be able to compute the various cross sections. If the nucleon were a point particle σ would be composed entirely of electromagnetic cross sections—the total light scattering cross section, electron pair production in the field of the nucleon, production of nucleon-antinucleon pairs, etc. Thus, σ would be proportional at least to e^4 and thus one would expect only minute deviations from the Thomson amplitude $(-e^2/m)$.

Experimentally this is not at all the case. We expect and indeed find strong deviations from point particle behavior as the frequency ω approaches the threshold for real meson production (~150 Mev). Once we admit the possibility of coupling to the meson field, large cross sections such as that for photomeson production enter the picture. We do not discuss the light scattering data in detail but merely state that a rather satisfactory understanding of the problem is obtained from dispersion relations used in conjunction with experimental and theoretical knowledge of photomeson production. Although this process depends very much on nucleon structure, it is rather difficult to draw detailed conclusions from it. The reason is basically that there are two classes of diagrams which contribute; those in which the initial and final photons are separated somewhere by a bare nucleon line and those for which this is not the case. Whereas the processes contained in diagram (a) involve in a loose sense the nucleon structure via the Γ's, those in diagram (b) are hopelessly far removed from any such simple connection (see Fig. 1).

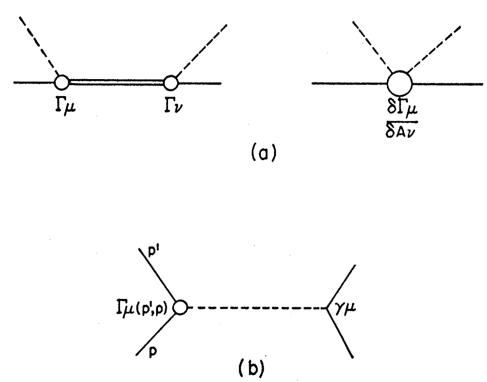

FIG. 1. Feynman diagrams for (a) Compton scattering and (b) electron scattering.

* This paper is based largely on theoretical work carried out in collaboration with P. Federbush and S. B. Treiman, which will be published shortly elsewhere. Some of the ideas have been independently discussed by G. F. Chew, R. Karplus, S. G. Gasiorowicz, and F. Zachariasen, and we are indebted to Professor Chew for very helpful correspondence on this work which will also be published soon.

What we would like to have is a probe which interacts just once with the nucleon (rather than twice as in the above mentioned process). The most natural probes are the electromagnetic and mesonic fields. Of the two, only the electromagnetic field can be utilized directly, because of our unfortunate inability to fabricate classical meson fields. Thus, just as light scattering depends on nucleon structure, so does meson scattering and for much the same reasons the scattering of mesons is difficult to interpret directly in terms of the structure. One may, however, ask theoretically for the scattering of a nucleon by a meson field but we shall not discuss this matter here. We are left, then, with the electromagnetic field which proves to be a very useful tool.

During the past few years Hofstadter and his collaborators have carried out very precise experiments on the scattering of electrons by protons, deuterons and heavier nuclei. The process of electron scattering may be described to lowest order in the electric charge by the Feynman diagram shown in Fig. 1. We see that there is a single interaction between the nucleon and the virtual electromagnetic field caused by the deflection of the electron. Our interest is in the nucleon vertex function $\Gamma_\mu(p',p)$ which enters the matrix element for the scattering amplitude in the form $\bar{u}(p')\Gamma_\mu(p'p)u(p)$ with $u(p)$ a Dirac spinor for a nucleon with four momentum $p=[p, i(m^2+\mathbf{p}^2)]^{\frac{1}{2}}$. In general, the relation between the above matrix element of $\Gamma_\mu(p'p)$, the vertex operator, and the current density operator of the system $j_\mu=-\Box A_\mu$ is (disregarding for a moment the isotopic spin of the nucleons)

$$J_\mu=\left(\frac{p_0p_0'}{m^2}\right)^{\frac{1}{2}}\langle p'|j_\mu|p\rangle$$
$$=ie(p'-p)^2D_{Fc}[(p'-p)^2]\bar{u}(p')\Gamma_\mu(p',p)u(p),$$

where D_{Fc} is the complete renormalized photon propagation function and e is the renormalized electric charge. In our approximation, i.e., lowest order in e, $(p'-p)^2D_{Fc}$ is to be replaced by unity. A failure of electrodynamics as well as radiative corrections to photon propagation would appear in this formalism by having $(p'-p)^2D_{Fc}$ different from unity and as such would multiply into any structure effects.

It is conventional to write J_μ in terms of two invariant functions F_1 and F_2 called the charge and magnetization density, respectively. These are defined as follows:

$$J_\mu=i\bar{u}(p')\{\gamma_\mu F_1[(p'-p)^2]$$
$$-i\sigma_{\mu\nu}(p'-p)_\nu F_2[(p'-p)^2]\}u(p),$$

where now taking isotopic spin into account, F_1 and F_2 contain an isotopic scalar and an isotopic vector part, e.g., $F_1=F_1^S+\tau_3F_1^V$. The experimental data on electron scattering are usually expressed in terms of these functions. There are certain things known about all four functions for small momentum transfers $q^2\equiv(p'-p)^2$. In particular for $q^2=0$, we have

$$F_1^S(0)=\frac{F_1^P+F_1^N}{2}=\frac{e}{2},\quad F_2^S(0)=\frac{\mu_P+\mu_N}{2},$$
$$F_1^V(0)=\frac{F_1^P-F_1^N}{2}=\frac{e}{2},\quad F_1^V(0)=\frac{\mu_P-\mu_N}{2},$$

where the μ's are the *anomalous* static moments. The frequently referred to mean square radii are defined from the power series expansion in q^2 of the F's, e.g.,

$$\langle(r_1^S)^2\rangle_{\text{Av}}=-\frac{6}{F_1^S(0)}\frac{d}{dq^2}F_1^S(q^2)\Big|_{q^2=0}$$

In terms of these "isotopic" radii, one may write for the charge radii $\langle r_1^2\rangle_{\text{Av}}=\frac{1}{2}[\langle(r_1^S)^2\rangle_{\text{Av}}\pm\langle(r_1^V)^2\rangle_{\text{Av}}]$ where the upper sign refers to the proton and the lower to the neutron. The magnetization radii are similarly defined

$$\langle(r_2^P)^2\rangle_{\text{Av}}=\frac{\mu_P+\mu_N}{\mu_P}\langle(r_2^S)^2\rangle_{\text{Av}}+\frac{\mu_P-\mu_N}{\mu_P}\langle(r_2^V)^2\rangle_{\text{Av}},$$
$$\langle(r_2^N)^2\rangle_{\text{Av}}=\frac{\mu_P+\mu_N}{\mu_N}(r_2^S)^2-\frac{\mu_P-\mu_N}{\mu_N}\langle(r_2^V)^2\rangle_{\text{Av}}.$$

It has been known for a long time that the mean square radius associated with the neutron charge density function, F_1^N is almost zero and this poses one of the most difficult theoretical problems in understanding nucleon structure. The reason is that the analysis of the Stanford electron scattering data seems to indicate that $\langle(r_1^P)^2\rangle_{\text{Av}}$ is very large, not less than $0.18/\mu^2$ and perhaps more likely to be $0.32/\mu^2$. In order to reconcile these facts, one must conjecture that there exists a large isotopic scalar contribution to $\langle r_1^2\rangle_{\text{Av}}$ so that the proton is sizable and the neutron cancels. In view of the fact that the contributions to this scalar part, as we shall see, would appear to be of rather short range, it is difficult to see, in detail, a way out. The other facts which are also obtained from the electron scattering experiments are that for large momentum transfer ($q^2>4\mu^2$) for protons $F_1/e\approx F_2/\mu_p$ and there is no evidence for any large difference between neutron and proton F_2's. The proton data has been generally analyzed in terms of the assumption $F_1/e=F_2/\mu_p$ and F_2 taken to be the Fourier transform of an exponential $\sim(\alpha^2+q^2)^{-2}$. There were many tries made, but this form fits the data admirably and leads to

$$\langle r^2\rangle_{\text{Av}}=(0.32\pm0.032)/\mu^2.$$

Such distributions are very unnatural from a theoretical standpoint and it would be very nice if the mean square radius could be measured in a truly model-independent fashion. This may in fact be done for F_1 by doing electron-proton scattering experiments in which the momentum transfer is very small; it is not so easy to get $\langle r_2^2 \rangle_{\text{Av}}$ directly.

In spite of the fact that there are some theoretical difficulties in giving a rigorous proof, it is probably true that the F's can be represented in the following dispersion form:

$$F_1{}^S(q^2) = \frac{e}{2} - \frac{q^2}{\pi} \int_{(3\mu)^2}^{\infty} d\xi^2 \frac{\rho_1{}^S(\xi^2)}{\xi^2(\xi^2+q^2)}$$

$$F_1{}^V(q^2) = \frac{e}{2} - \frac{q^2}{\pi} \int_{(2\mu)^2}^{\infty} d\xi^2 \frac{\rho_1{}^V(\xi^2)}{\xi^2(\xi^2+q^2)}$$

$$F_2{}^S(q^2) = \frac{1}{\pi} \int_{(3\mu)^2}^{\infty} d\xi^2 \frac{\rho_2{}^S(\xi^2)}{\xi^2+q^2}$$

$$F_2{}^V(q^2) = \frac{1}{\pi} \int_{(2\mu)^2}^{\infty} d\xi^2 \frac{\rho_2{}^V(\xi^2)}{\xi^2+q^2}.$$

In the dispersion relation for F_1 we have been conservative and have subtracted a constant in order to minimize the contribution from very large mass (i.e., ξ^2) values. From these formulas alone it is very difficult to come to any conclusions about mean square radii or asymptotic behavior unless definite assumptions are made about the various ρ's. The ρ's stand for the absorptive parts of the F's in the sense of dispersion theory and the parameter ξ^2 represents the total (mass)^2 of the intermediate states contributing to the absorptive parts. The fact that the scalar and vector parts have different lower limits comes from simple isotopic spin considerations; one finds that the ρ's have one contribution proportional to the matrix element $\langle 0 | j_\mu | n \rangle$, where $|n\rangle$ is a state with n π-mesons, and it is easy to show that under the symmetry operation which carries each component of the meson field ϕ_α into $-\phi_\alpha$ the scalar part of j_μ changes sign whereas the vector part does not. Thus, the vector part of j_μ is connected to states with 2, 4, $\cdots$ pions and the scalar part to states with 3, 5, $\cdots$ pions. It is the least massive state in each case which sets the lower limits.

Before analyzing the structure of these representations further it should be noted that the experimentally favored exponential, for $F_2{}^V$, say, corresponds to a $\rho_2{}^V(\xi^2) = \delta'(\xi^2 - \alpha^2)$ which is fantastically unlikely from a theoretical standpoint. One may go further and ask that making, say, the superficially unwarranted assumption that for the proton $F_1(q^2)/F_1(0) = F_2(q^2)/F_2(0)$ one represent the data with a positive definite $\rho(\xi^2)$. If one looks at $1/F_2(q^2)$ it is easy to show that it should be concave downward, but experimentally one finds it to be concave upward (see Fig. 2). The fact that the ρ's may not be positive precludes any elementary statements about the various F's.

It is our feeling that it would be very worthwhile to re-examine the experimental data from the standpoint of our dispersion representations. In particular the

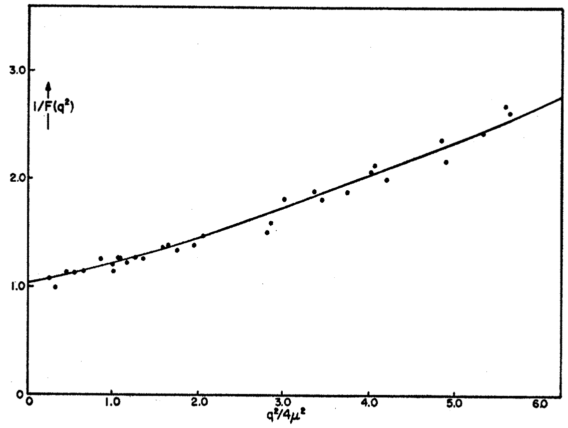

FIG. 2. Form factor for the proton obtained by assuming $F_1/F_1(0) = F_2/F_2(0)$. The points are the experimental data and the curve corresponds to an exponential distribution with $r^2 = 0.32/\mu^2$.

simplifying assumption that

$$F_1(q^2)/F_1(0) = F_2(q^2)/F_2(0)$$

should be studied with different classes of form factors than have been heretofore used. The most natural form of trial function would be obtained by choosing $\rho(\xi^2) = \sum a_i \delta(\xi^2 - \alpha_i^2)$ which would simulate the contributions from various classes of intermediate states of various masses of the variety to be discussed below. We have explored briefly one very simple example, namely with two terms only and $a_1 = -a_2 = (\alpha_1^2\alpha_2^2/\alpha_2^2 - \alpha_1^2)$. One finds then, say, $F_2(q^2)/F_2(0) = \alpha_1^2\alpha_2^2/(\alpha_1^2+q^2)(\alpha_2^2+q^2)$. The case of $\alpha_1^2 = \alpha_2^2 \approx 37\mu^2$ corresponds to the favorite Stanford exponential with a mean square radius of $0.32/\mu^2$. If, however, one takes $\alpha_1^2 = 100\mu^2$, $\alpha_2^2 = 20.5\mu^2$ one obtains a function which is essentially indistinguishable for most of the range from the first and which has only a slightly different mean square radius, $0.35/\mu^2$. It is obvious that with more parameters one could get agreement with these for large momentum transfers and get a radically different mean square radius. It may be, incidentally, that the approximate and unexpected equality of F_1 and F_2 for large momentum transfers comes about in some manner as indicated by our example.

Let us look now more closely into the structure of the vertex operator and its relation to the representations written. For the sake of definiteness let us describe in diagrams the structure of the vector part of the vertex. These are shown in Fig. 3; there are, of course, an infinite number of graphs which have been left out. They are mostly too horrible to describe, undoubtedly uncomputable, and we hope they are numerically unimportant. We see by referring to Fig. 3 that we are instructed to add the amplitudes for processes in which the (virtual) photon interacts with a "bare" nucleon, in which a virtual pair of pions is produced at a complicated vertex which then scatter

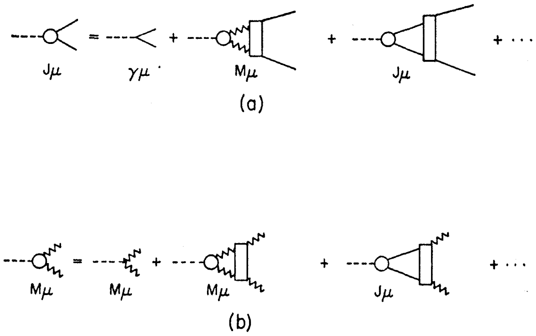

FIG. 3. Dispersion theory diagrams showing the structure of the contributions to the form factor for the proton J_μ and the meson M_μ.

on the nucleon and finally a process in which the photon produces a nucleon-antinucleon pair (through the full vertex we are trying to compute) which then interacts inside a "black box" with the nucleon. There are a few other diagrams which are simple enough to describe: We could have produced a pair of K mesons instead of π mesons or a hyperon-antihyperon pair rather than nucleon-antinucleon. We comment on these later.

We are inclined to think that the contribution from the two-meson intermediate state could be the most important insofar as the low momentum transfer phenomena were concerned. This follows from our dispersion representations simply from the fact that only the less massive intermediate states can feel small changes in q^2. This is not a very sharp conclusion since the weight functions are not necessarily smooth or positive definite. To evaluate this contribution, however, requires a knowledge of the continuation into the unphysical world—a sort of never-never land where momenta become imaginary—of pion nucleon scattering amplitudes, or if you prefer, the amplitude for the annihilation of a nucleon-antinucleon pair into two pions, the total energy of which is less than that of the pair. Neither of these processes can be calculated theoretically with much confidence for the relatively extensive excursions into "never-never land" which are required. It has been argued by Chew *et al.*, that one may be able to treat the processes in perturbation theory in which case the continuations are easy.† If that be done, then except for the appearance of the meson vertex this would be old-fashioned perturbation theory.

As we can see from Fig. 3, the structure of the meson vertex can be analyzed in a similar fashion to that of the nucleon vertex. (This quantity, call it $M\mu$, is essentially the form factor for electron meson scattering which is unfortunately not very accessible experimentally. One can, of course, scatter pions from electron targets, but the mass ratio makes it impossible to transfer sufficient momentum to make such an experiment worthwhile. It is possible that the meson form factor plays a role in pion production in electron-proton collisions.) The first thing we encounter aside from the production of a pair of bare pions is the vertex $M\mu$ itself producing a pair of pions which undergo "black box" scattering before emerging. If all higher states (including that involving the nucleon vertex shown in the figure, the neglect of which can be rigorously justified) are neglected this problem can be very completely analyzed. It is easy to show from gauge invariance considerations that only p-wave pions are involved (in the center-of-mass system, of course) so that we need know only the p wave, isotopic spin one, phase shift for pion-pion scattering as a function of energy. This is not a very well-known quantity but it is reasonable to expect that the interaction has a very short range so that for moderate energies a presentation like $\tan\delta = k^3a^3$ with k the center-of-mass wave number and a the scattering length seems reasonable. Perturbation theoretic estimates (with $g^2/4\pi\sim 15$) indicate $a\gtrsim 1/m$ with m the nucleon mass. The integral equation implied by our diagrams may actually be solved for any (in general complex) phase shift with the result (given for real δ)

$$M_\mu = \langle 0|j_\mu|q_i k_j\rangle (4k_0q_0)^{\frac{1}{2}} = i\epsilon_{3ij}(q-k)_\mu \frac{e}{\sqrt{2}} M[(q+k)^2],$$

where

$$M[(q+k)^2] = \exp\left[-\frac{(q+k)^2}{\pi}\int_{4\mu^2}^{\infty} d\xi^2 \frac{\delta[(\xi^2/4-\mu^2)^{\frac{1}{2}}]}{\xi^2\{\xi^2+(q+k)^2-i\epsilon\}}\right].$$

With our particular model this leads to a pion mean square radius given by

$$\langle r^2\rangle_{\text{Av}} = \frac{3}{4\mu^2}\alpha^2 \frac{2-\alpha+\alpha^2+\alpha^3}{(1+\alpha)(1+\alpha^2+\alpha^4)},$$

where $\alpha = \mu a$. For $a = 2/m$, $(r^2)^{\frac{1}{2}} = 0.4\times 10^{-13}$ cm which is quite large.

The quantity of relevance for the nucleon vertex is the real part of M which is shown in Fig. 4, again for the choice $a = 2/m$. This shows that $\text{Re}M(\xi^2)$ falls off rather rapidly for negative argument. The way that M enters the dispersion relations is as follows: if the contribution to $\rho(\xi^2)$ from the two pion intermediate state for point pions is called ρ_p then the correct contribution including meson structure is obtained by multiplying ρ_p essentially by $\text{Re}M(-\xi^2)$. Setting $\text{Re}M(-\xi^2) = 1$, i.e., neglecting the meson structure, gives entirely undeserved weight to large values of ξ^2. It may even be true that the pessimism about the high mass (large ξ^2) values which led to using the conservative forms for $F_1{}^S$ and $F_1{}^V$ is unwarranted. Since we cannot reli-

† *Note added in proof.*—A more careful investigation of the two meson contribution has shown that this argument is probably not accurate. The corrections to perturbation theory are in fact so large that they cannot be reliably estimated. The direction of the effects is such as to maintain the fairly quantitative agreement with the anomalous moment, increase the magnetization mean square radius over the perturbation theory value, and to drastically reduce the charge density radius. These results will be discussed in detail elsewhere.

ably calculate ρ_p for these values it is hard to test this point.

If a calculation of the two meson state is made on the basis of perturbation theory and the meson vertex function given in Fig. 4 is used, we get a rough idea about the importance of this particular intermediate state and of the effect of the meson vertex. One finds for the vector magnetic moment and magnetization mean square radius the values $1.65e/2m$ and $0.125/\mu^2$, respectively, for point pions. Taking account of the vertex we find $1.8e/2m$ and $0.16/\mu^2$. The charge density function is in principle much more susceptible to influence by the meson form factor by virtue of the fact that large intermediate mass contributions are expected to be rather more important than for the magnetization density. Again using perturbation theory, we find $\langle (r_1^V)^2 \rangle_{\text{Av}} = 0.24/\mu^2$ for point pions and $0.18/\mu^2$ with the pion vertex. These numbers are to be regarded as only illustrative and have only semiquantitative significance.

The comparison with experiment at this point is as follows: The vector magnetic moment is about $1.8e/2m$ which agrees exactly and no doubt fortuitously with our theoretical value. The magnetization mean square radius for the proton (and neutron also if only this two pion state is included) on the other hand is only about $\frac{1}{2}$ of the apparent experimental value. It should be emphasized, however, that the "experimental" value is obtained very indirectly. The magnetization form factor plays an important role in electron scattering only when the momentum transfers are so large that one is well beyond the range where the characterization of the form factor by a mean square radius is adequate. That is, one cannot write in the relevant region $F_2(q^2) = F_2(0)[1-(q^2r^2/6)]$ and determine $\langle r^2 \rangle_{\text{Av}}$ from experiment. What has been done is to attempt to fit an analytic curve to $F(q^2)$ for large q^2 and to calculate from this r^2. Unless one has enormous confidence in one's ability to guess really the correct curve it is hard to see how one can be very sure of the behavior of F for the small q^2 values which define the mean square radius.

As to the proton charge density mean square radius, there may soon be a reliable, model independent determination from experiments done at rather small momentum transfers. The first results from these experiments seem to indicate that this is quite large, perhaps as much as $0.32/\mu^2$. The two pion state predicts only about $0.09/\mu^2$ and even if one uses the empirical fact that the neutron radius is zero, i.e., assume $r_S^2 = r_V^2$, one finds only $0.18/\mu^2$. If the meson vertex were less important than we have estimated $0.09/\mu^2$ would be increased to $0.12/\mu^2$ and again assuming zero neutron radius to a value of $0.24/\mu^2$. Life would be considerably simpler if it were to turn out that the proton charge radius were actually much smaller and it would be worth the effort to do the difficult low momentum transfer experiments with high accuracy. If the proton radius were smaller there would be less burden on the

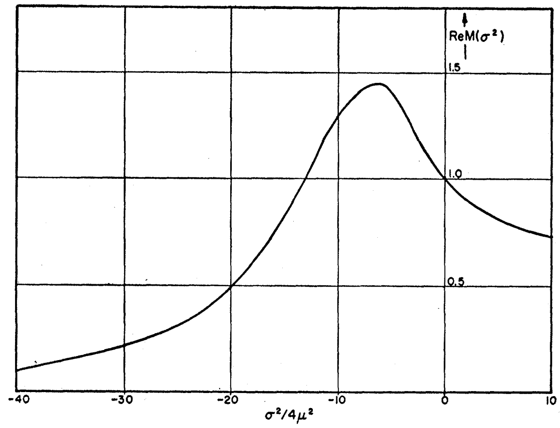

FIG. 4. The real part of the meson form factor evaluated under the assumption that the pion-pion p-wave phase shift is given by $\tan\delta = k^3a^3$ with $a = 2/m$.

isotopic scalar part of the vertex to yield the small neutron radius. It is very difficult, incidentally, to see how the latter experimental determination could be wrong since the same result has been found in many ways.

Nothing we have described as yet makes any contribution to the isotopic scalar part of the vertex. The problem here is to understand the very small ($\sim -0.06e/2m$) scalar magnetic moment and the apparently large mean square radius for the charge density. Unless the magnetization density were anomalously large, if the magnetic moment comes out right the magnetization mean square radii for neutron and proton would be relatively unaffected because of the weighting factor $\mu_S/\mu_{N,P}$.

The most conspicuous contributor, historically, to the scalar part is the intermediate state involving a nucleon-antinucleon pair. This state enters in perturbation theory on an entirely equal footing with the two pion state, and if it is included we obtain ridiculous answers to all questions. Whereas it may be possible to justify the use of perturbation theory for the two pion state, it cannot be done for the pair state. We have been able to show, however, using unitarity limitations on the nucleon-nucleon scattering amplitudes, that is, what goes on inside the last black box in Fig. 4, that this state actually makes entirely negligible contributions to both the magnetic moment and mean square radius:

$$\mu_A \lesssim \frac{1}{10}\frac{e}{2m}, \quad \langle r_1^2 \rangle_{\text{Av}} \lesssim \frac{3}{100}\frac{1}{\mu^2}.$$

This conclusion is not based on a particular theory but follows from general quantum mechanical principles and our dispersion formalism.

Where then does the isotopic scalar vertex come from? The most natural sources from states of finite degree of complexity are those involving two K mesons or three pions. We have calculated the two K contribution using perturbation theory and find that even if the K-baryon coupling constants were as large as the pion-nucleon constant, they would make a negligible contribution to the moments and radii. We include here in the language of perturbation theory only the K-current contribution. The usual simultaneous inclusion of the baryon current would yield large and erroneous effects but we can again kill this contribution by a unitarity argument so it should not be included. Unless perturbation theory for the K current is wildly off, we must look elsewhere.

This leaves us with the three pion state. Our analysis of this contribution is not very conclusive since its evaluation requires a knowledge of matrix elements for rather scary processes like nucleon-antinucleon annihilation into three pions as well as a vertex corresponding to the production of three pions by a virtual photon. From general principles such as angular momentum, conservation and charge conjugation invariance a few things can be said but scarcely enough to make any quantitative statements at present. Barring weird behavior in the high mass region which isn't impossible, this state is the only one which has a reasonable chance of explaining the facts. Arguing dimensionally it might be expected to contribute about 4/9 the mean square radius of the two pion state or roughly speaking about the same order of magnitude (the two pion contribution is rather smaller than one would intuitively expect, that is $1/m\mu$ rather than $1/\mu^2$). Unfortunately, it will probably be rather hard to make a sharp statement about this contribution.

In summary, then, the situation seems to be as follows: we seem to be able to understand in a semi-quantitative way the isotopic vector contributions to both the charge and magnetization density form factors for small momentum transfers. The old bugaboo of the nucleon pair contributions has been laid to rest. The most likely candidate for the isotopic scalar contribution is the at present uncomputable 3 pion state.

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