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# Nuclear Radii as Determined by Scattering of Neutrons

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## I. ENERGIES OF 40 MEV AND ABOVE

FROM the time the neutron was first discovered, many analyses were made to determine the radius of heavy nuclei as a function of atomic weight by means of scattering experiments. Total cross sections of neutrons on complex nuclei are equated to $2\pi R^2$ and assuming

$$R=r_0A^{\frac{1}{3}}+C,$$

a least-squares fit to the data gives values for $r_0$ and $C$.[1–3] At the experimental energies then used (7 to 25 Mev) nuclear matter was assumed to be completely absorbing. After the 184-in. cyclotron at Berkeley was put into operation in 1947, further analyses of data[4] then obtained were made on the basis that the nucleus became somewhat transparent[5] at the high incident neutron energies (90 Mev) available. The nucleus was represented by a complex square well potential, the real part of which was related to the depth of the Fermi sphere, and the imaginary part to the neutron-nucleon interaction within the nucleus. The Schroedinger equation with this complex potential was solved by the WKB approximation, this being equivalent to ray tracing through the nuclear matter, neglecting refraction at the surfaces. The "optical model"[6,7] analysis resulted in a fairly good representation of nuclear matter with parameters that seemed reasonable. The values are shown in Table I in the column headed 90 Mev. The radius of $1.37A^{\frac{1}{3}}\times10^{-13}$ cm seemed to be in the "popular" range at that time.

Other experiments were soon performed on the 184-in. cyclotron at higher energies, in the range 270 to 300 Mev. Since the optical model was a high-energy approximation, it was thought that better results would be obtained at these energies than at 90 Mev. This was not the case, especially if one searched for "reasonable" potentials. The best results[8] were obtained for a radius close to that used at the lower energy ($1.39A^{\frac{1}{3}}\times10^{-13}$ cm), but the real potential had to be made extremely small, effectively zero. (See Table I under 270 Mev.) A further experimental study of total cross section as a function of energy by DeJuren and Moyer[9] showed a rather sharp drop in cross section near 100 Mev. At approximately 160 Mev the cross section leveled out and continued at a fairly constant value up to the maximum energy then obtainable, namely 300 Mev. The optical model hence would require the same parameters at all energies greater than 160 Mev to fit the data. Jastrow[10] showed how the sharp break might come about as a consequence of nucleon-nucleon scattering considerations. More recent work of Riesenfeld and Watson[11] provides a scheme for computing the optical model potentials from nucleon-nucleon phase shifts. These calculations show that the real potential does drop off with energy, but may not become zero by 300 Mev. Some analyses show an attractive well of as much as 10 Mev depth.

TABLE. I.

| $E$ (Mev) | 90 | 270 |
|---|---|---|
| $r_0$ | 1.37 | 1.39 |
| $-V_0$ (Mev) | 30 | 0 |
| $-W_0$ (Mev) | 15 | 17 |

These analyses were based on both total and nonelastic cross sections. Some angular distributions of scattered neutrons were measured, but only at relatively small angles (to the first minimum). At 90 Mev, the measured forward scattering was higher than the predicted values, but the relative shapes were correct. At 300 Mev no fit could be made.

Experimental data at other energies, both lower[12–15] than 90 Mev and higher[16,17] than 300 Mev later became available, but the former did not readily lend itself to analysis because of the high-energy approximation involved and the latter showed the same cross sections as at 300 Mev. Taylor[18] did carry out the necessary calculation for all data available in 1954 and showed that a radius of approximately $(1.23A^{\frac{1}{3}}+0.8)\times10^{-13}$ cm

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[1] D. C. Grahame and G. T. Seaborg, Phys. Rev. **53**, 795 (1938).

[2] Amaldi, Bocciarelli, Gacciapuoti, and Trabacchi, Nuovo cimento **3**, 203 (1946).

[3] R. Sherr, Phys. Rev. **68**, 240 (1945).

[4] Cook, McMillan, Peterson, and Sewell, Phys. Rev. **75**, 7 (1949).

[5] R. Serber, Phys. Rev. **72**, 1114 (1947).

[6] Fernbach, Serber, and Taylor, Phys. Rev. **78**, 1352 (1949).

[7] Bratenahl, Fernbach, Hildebrand, Leith, and Moyer, Phys. Rev. **77**, 597 (1950).

[8] S. Fernbach, UCRL-1382, Berkeley (1951).

[9] J. DeJuren and B. J. Moyer, Phys. Rev. **81**, 919 (1951).

[10] Robert Jastrow, Phys. Rev. **82**, 261 (1951).

[11] W. B. Riesenfeld and K. M. Watson, Phys. Rev. **102**, 1157 (1956).

[12] A. E. Taylor and E. Wood, Phil. Mag. **44**, 95 (1953).

[13] W. I. Linlor and B. Ragent, Phys. Rev. **92**, 835 (1953).

[14] B. Ragent, UCRL-2337, Berkeley (1953).

[15] R. H. Hildebrand and C. E. Leith, Phys. Rev. **80**, 842 (1950).

[16] V. A. Nedzel, Phys. Rev. **94**, 174 (1954).

[17] Snow, Coor, Hill, Hornyak, Smith, and Snow, Phys. Rev. **98**, 1369 (1955).

[18] T. B. Taylor, thesis, Cornell University (1954).

held for energies between 40 Mev and 1 Bev, but the real potential behaved strangely, having a resonance-like peak in the vicinity of 60 Mev. The measured total cross sections show a sharp dip in this energy region. With the smaller radius, Taylor found it unnecessary to drop the real potential to zero near 300 Mev. Instead it has a minimum of the order of 10 Mev, and then rises to approximately 26 Mev for incident neutrons of 1 Bev. However the approximation used in this calculation does not indicate the sign of the potential. It could be positive or negative, that is, the potential could go through zero near 300 Mev and continue to become more negative approaching −26 Mev at this 1 Bev point. (The negative sign here indicates a repulsive potential.)

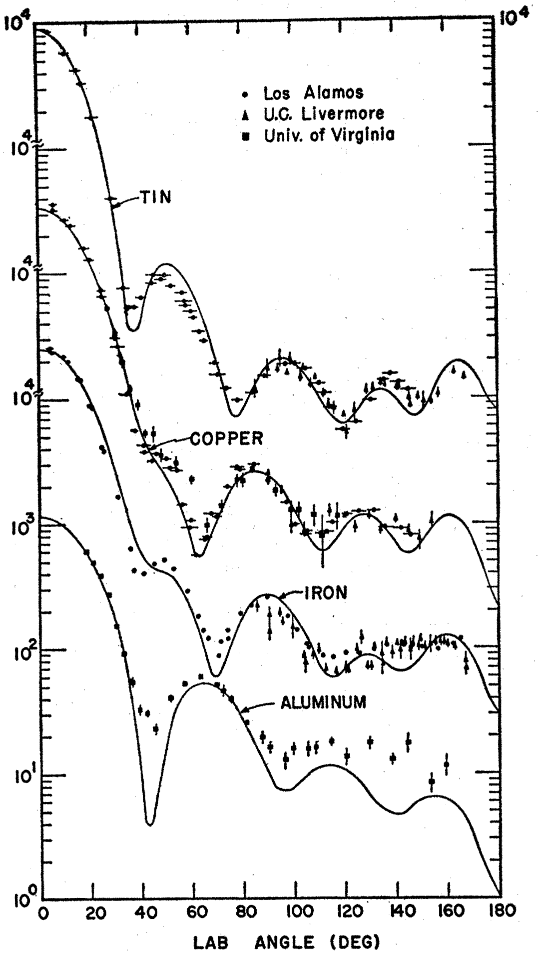

FIG. 1. Experimental and theoretical differential cross sections for 14-Mev neutrons scattered from Sn, Cu, Fe, and Al. The experimental data presented are not completely corrected for multiple scattering nor have angular and energy resolution been taken into account. The data of Berko *et al.*, University of Virginia, have been normalized to the point at 18° for the heavy elements and 30° for the lighter elements.

Thus the same total neutron cross sections can be computed from several sets of parameters. For a given radius, a potential can be found which makes the computed value agree with the experimental. Experimental differential cross-section data would be of great importance in better defining the parameters. From the meager data available at 90 Mev, it seems that the Taylor analysis provides the better agreement with experiment.

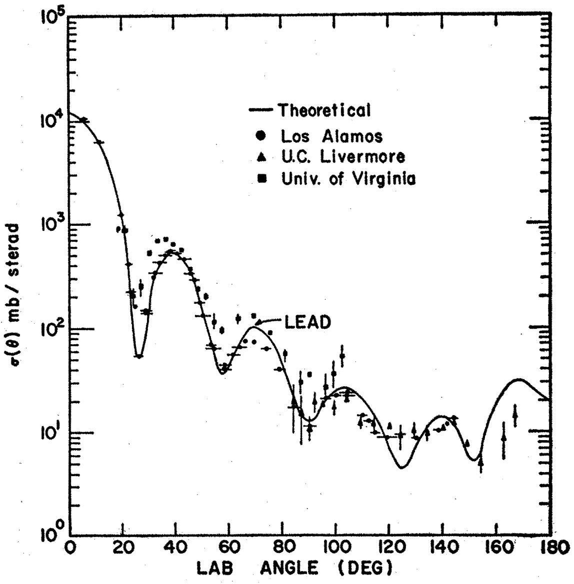

FIG. 2. Experimental and theoretical cross sections for 14-Mev neutrons scattered from Pb. The data of Berko *et al.* have been normalized to the point at 18°.

## II. ENERGIES BELOW 40 MEV

In 1952, Feshbach, Porter, and Weisskopf[19] did an exact phase-shift calculation for the scattering of low-energy neutrons (1 to 3 Mev) from complex nuclei. The results for the diffraction scattering showed remarkable

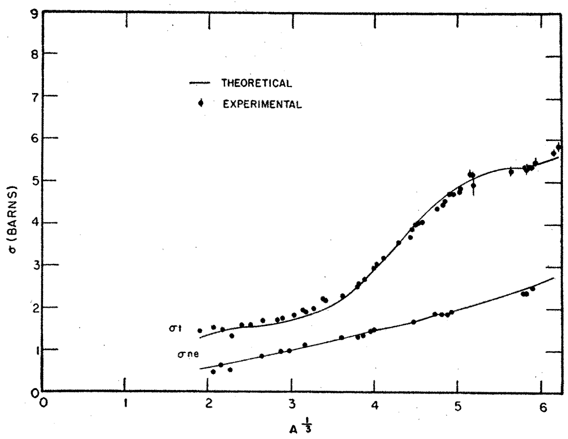

FIG. 3. Experimental and theoretical total and nonelastic cross sections of 14-Mev neutrons as a function of $A^{\frac{1}{3}}$.

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[19] Feshbach, Porter, and Weisskopf, Phys. Rev. **90**, 166 (1953).

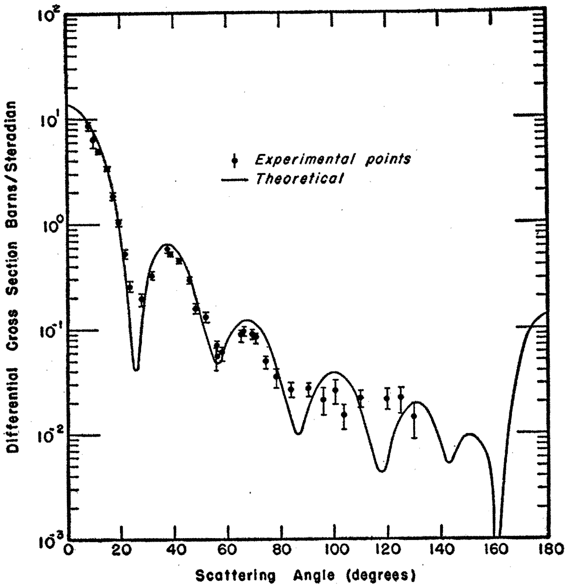

FIG. 4. Experimental and theoretical cross sections for 15-Mev neutrons scattered from Bi. Spin-orbit potential not included.

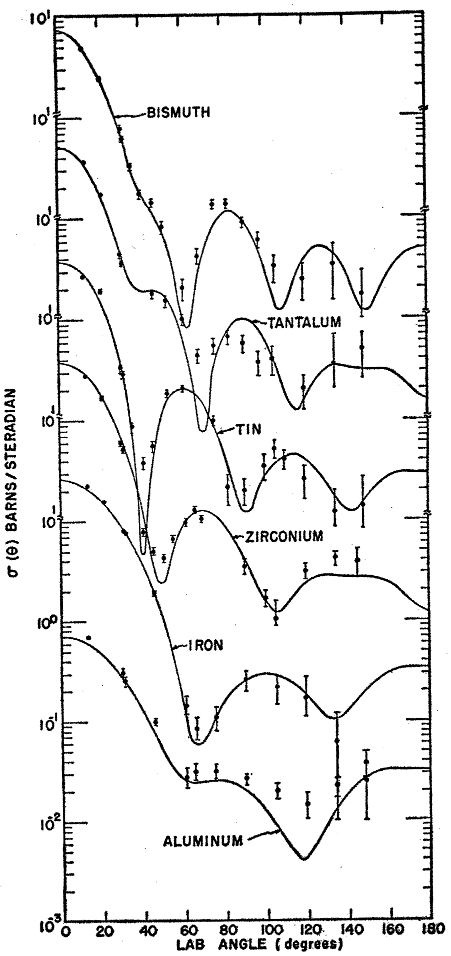

FIG. 5. Experimental and theoretical differential cross sections for 7 Mev neutrons scattered from Al, Fe, Sn, Ta, Zr, and Bi.

agreement with the experimental data of Barschall.[20] Here again the nucleus was characterized by a complex square well potential, the parameters being

$$R_0 = 1.4A^{\frac{1}{3}} \times 10^{-13}\text{ cm},$$
$$-V_0 = 19\text{ Mev},$$
$$-W_0 \cong 1\text{ Mev}.$$

The success of the "cloudy crystal ball" led to exact phase shift calculation at other fairly low energies, for both the scattering of neutrons[21,22] and protons.[23,24] In the case of protons the experimental data[25] at 18 Mev was extremely good. With it Chase and Rohrlich showed that a square well does not provide a good fit. Saxon and Woods arrived at the same conclusion and were led to use a potential of the form

$$V = -(V_0 + iW_0)/[1 + \exp(R - R_0)/a].$$

They found a marked improvement in the computed fit to the data with the addition of the diffuse edge. Unfortunately, the parameters required varied from nucleus to nucleus.

A similar potential was applied to calculations involving neutron scattering at energies between 2.5 and 14 Mev.[26] Small angle diffraction scattering can be predicted fairly well, but at large angles the calculated values oscillate more violently than the experimental data. At energies lower than 7 Mev corrections had to be made for the compound elastic scattering (assuming it to be isotropic). As with proton scattering, parameters were varied from element to element to obtain best agreement between the experimental and theoretical

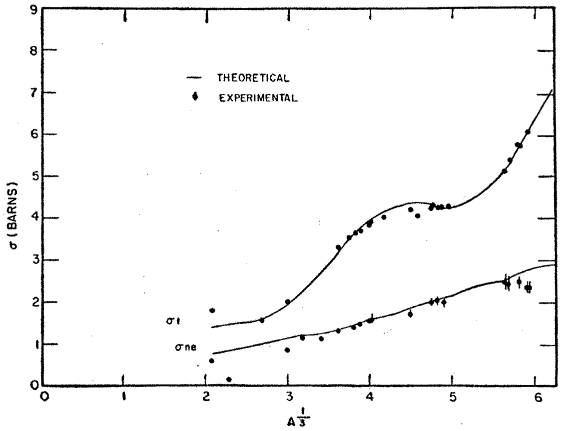

FIG. 6. Experimental and theoretical total and nonelastic cross sections of 7-Mev neutrons as a function of $A^{\frac{1}{3}}$.

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[20] H. H. Barschall, Phys. Rev. **86**, 431 (1952).

[21] Bjorklund, Fernbach, and Sherman, Phys. Rev. **101**, 1832 (1956).

[22] F. E. Bjorklund and S. Fernbach, Phys. Rev. **109**, 1295 (1958).

[23] F. Rohrlich and D. M. Chase, Phys. Rev. **94**, 81 (1954).

[24] R. D. Woods and D. S. Saxon, Phys. Rev. **95**, 577 (1954).

[25] B. Dayton and G. Schrank, Phys. Rev. **101**, 1358 (1956).

[26] Beyster, Walt, and Salmi, Phys. Rev. **104**, 1319 (1956).

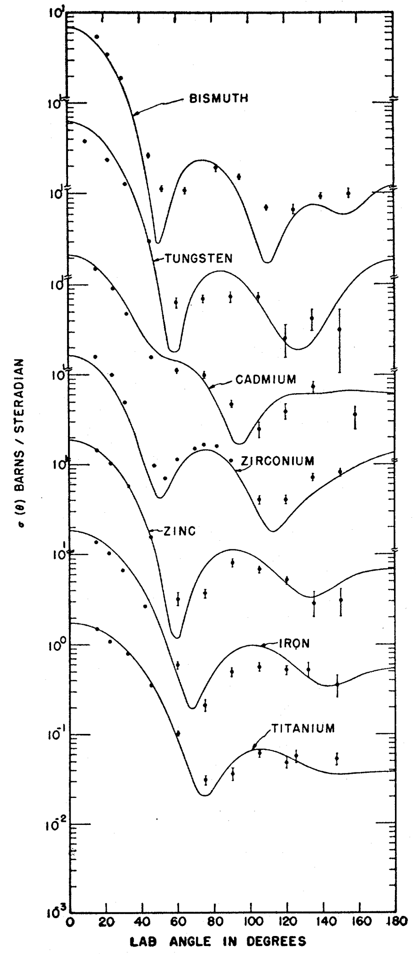

Fig. 7. Experimental and theoretical differential cross sections for 4.1-Mev neutrons scattered from Bi, W, Cd, Zr, Zn, Fe, and Ti.

data. The imaginary potential, in particular changed by as much as a factor of 2 over the range of $A$.

Another attempt to fit the neutron scattering data at these energies was made with a slightly different potential.[21] The real part was unchanged, of the Saxon-Woods form, but the imaginary potential was assumed to be a gaussian centered at $R_0$, of the form

$$-W_0\{\exp[-(R-R_0)^2/b^2]\}.$$

It was assumed that most nonelastic events take place at the nuclear surface, the Pauli principle prohibiting such collisions inside the nucleus. With this potential much better agreement was obtained between computed and experimental nonelastic cross sections than had been obtained with a centered potential falling off at the edge. Even the angular distributions seem somewhat improved. Large angle scattering still seems to require improvement. The most interesting aspect of using this potential was that the same parameters could be used for all elements.

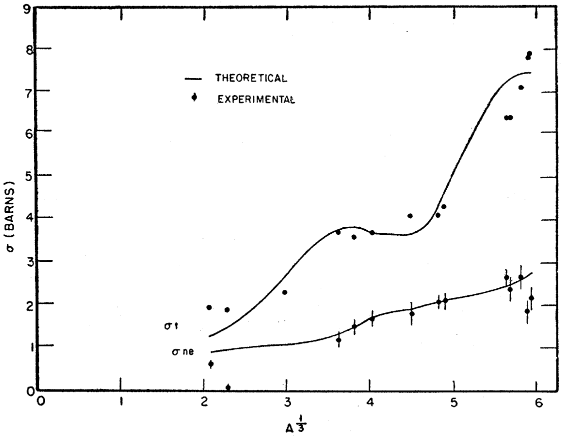

Fig. 8. Experimental and theoretical total and nonelastic cross sections of 4.1-Mev neutrons as a function of $A^{\frac{1}{3}}$.

At this time high-energy polarization experiments were being performed, and it was obvious that in addition to the central potential a spin-orbit potential was required at these energies to fit high-energy proton scattering data. The addition of such a term led to less violent oscillation of the computed diffraction pattern. For this reason it was added to the calculation at 14 Mev in the form of a Thomas-type term. Its magnitude was taken to be similar to that used in shell-model calculation,[27] namely, ~35 to 40 times the usual Thomas term. The results of these calculations are shown in Figs. 1–3 where a comparison is made with the experimental data. Figure 4 shows the result of a calculation with the best parameters, neglecting the spin-orbit potential.

Similar attempts to fit data at 7 Mev and 4.1 Mev were made using the same radius, but slightly different values for the potentials. These results are shown in Figs. 5–8. The parameters used are given in Table II.

Total and nonelastic cross sections at 26 Mev recently measured[28] but not completely corrected are

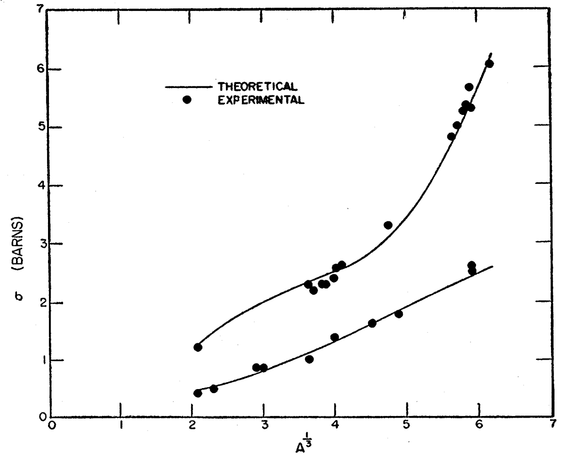

Fig. 9. Experimental and theoretical total and nonelastic cross sections of 26-Mev neutrons as a function of $A^{\frac{1}{3}}$.

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[27] Ross, Mark, and Lawson, Phys. Rev. **102**, 1613 (1956).

[28] Private communication. Data obtained by Bratenahl, Peterson, and Stoering, Livermore, California.

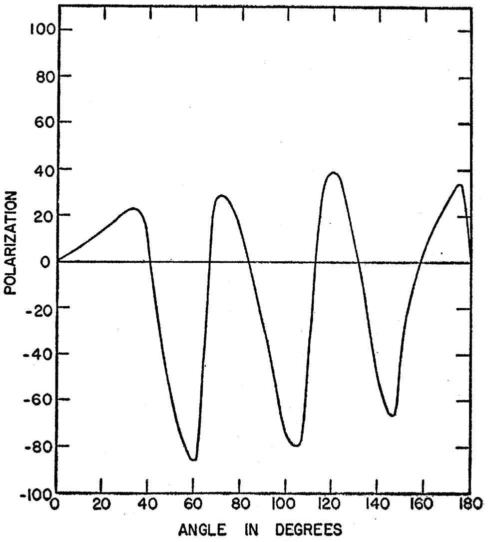

FIG. 10. Percent polarization (theoretical) of 14-Mev neutrons by Cu.

shown in Fig. 9 along with computed values using the parameter shown in Table II.

Figure 10 shows the computed polarization for 14 Mev neutrons by copper. No experimental polarization data is available as yet.

## III. CONCLUSIONS

From the low-energy data, one is led to a nuclear radius of the order of $(1.25\pm0.05)A^{\frac{1}{3}}\times10^{-13}$ cm. The deviation from 1.25 is assigned to account for the fact that this figure could be changed within these limits for some elements without seriously affecting the results. The nuclear potential at these energies can be best represented by a potential of the form

TABLE II.

| $F$ (Mev) | $V_0$ (Mev) | $W_0$ (Mev) | $\lambda$ | $a$ ($10^{-13}$ cm) | $b$ ($10^{-13}$ cm) | $r_0$ ($10^{-13}$ cm) |
|---|---|---|---|---|---|---|
| 4.1 | 50 | 7 | 35 | 0.65 | 0.98 | 1.25 |
| 7 | 45.5 | 9.5 | 35 | 0.65 | 0.98 | 1.25 |
| 14 | 44 | 11 | 35 | 0.65 | 0.98 | 1.25 |

$$-V(R)=V_0/[1+\exp(R-R_0)/a]$$
$$+iW_0\{\exp[-(R-R_0)^2/b^2]\}+\lambda V_0\frac{1}{R}\frac{dp}{dR}\boldsymbol{\sigma}\cdot\mathbf{l},$$

where $p=[1+\exp(R-R_0)/a]^{-1}$. $V_0$ may be as high as 50 Mev at extremely low incident neutron energies, decreasing slowly to 37 Mev at neutron energies of 26 Mev; $W_0$ increases from zero to approximately 10 Mev at an incident neutron energy of approximately 7 Mev; and $a$ and $b$ are constant at $0.65\times10^{-13}$ cm and $1\times10^{-13}$ cm, respectively. The spin-orbit potential is real only and 30 times the Thomas term.

At higher neutron energies, the only available analyses are for square wells using approximate methods. However, the radius seems to be within the limits assigned at the lower energies. The real central potential continues to fall and possibly goes through zero at approximately 300 Mev and becomes larger in a negative direction. The imaginary potential remains more or less constant at 15 Mev. The data presently available are not good enough to determine the need for a spin-orbit potential at the higher energies. Presumably much of the parameter data obtained from proton scattering studies could be assumed to apply to neutron scattering as well.