Correspondence in the Generalized Theory of Gravitation*

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I. INTRODUCTION

THE success of general relativity as a physical theory has made it necessary to investigate further possibilities of taking over more of the geometrical concepts for the description of natural phenomena. In its present form (Einstein's 1915 theory) general relativity is an expression of field concepts in an incomplete sense. It describes (i) gravitational phenomena in the large, and (ii) it gives the equations of motion of singularities in an external field; it does not say anything about (i) gravitational phenomena in the small, and (ii) electric charges and their interaction with fields.

The most important consequence of the nonlinearity of the gravitational field equations is the fact that they contain the law of motion. In this sense general relativity together with Maxwell's field constitutes a well-defined but an incomplete theory. In this paper the incompleteness of the theory is not taken in the sense of its as yet unquantized state. We believe the nonlinearity of the theory is more important than its possible quantized forms. It is somewhat difficult to understand the principle of equivalence in the quantized version of general relativity. The quantum nature of all atomic phenomena may be expected to result from an unquantized nonlinear theory obtained by a consistent and well-defined generalization of general relativity.

In order to test the internal consistency of a generalized theory of gravitation one can advance a correspondence argument like in quantum theory. One anticipates the reduction of the total field to general relativistic field equations on which the generalized theory is to be based. The trivial alternative in the correspondence argument is to abolish the electromagnetic field in the equations of the generalized theory that results in the free field equations of general relativity without Maxwell's field. The second alternative is obtained by means of a fundamental length in the generalized theory, the vanishing of which results in the field equations of general relativity in the presence of electromagnetic field. Of the two alternatives the latter is certainly more satisfactory in that it does not end up with a pure gravitational field which it wanted to unify with electricity from the beginning. In terms of universal constants the special relativity is based on the constancy of the velocity of light c and quantum theory on the finite value of Planck's constant h. By making c tend to infinity one obtains the nonrelativistic laws and making h tend to zero one gets classical description. In addition to the universal constants c and γ (gravitational constant) in the generalized theory a third constant is needed to formulate a correspondence argument for the theory. The latter role is played by a fundamental constant r_0 in the author's[1] version of the generalized theory of gravitation, which in the following will be referred to as (I).

In the limit of $r_0=0$ we obtain the equations of general relativity with the electromagnetic field as the source of gravitation plus Maxwell's equations for charge free fields. The existence of charges is linked up with a nonvanishing fundamental length just as the existence of spin arises from a finite h.

II. DISCUSSION OF THE FIELD EQUATIONS

The total field is described by a nonsymmetric tensor $g=a_{\alpha\beta}+\varphi_{\alpha\beta}$. The physical interpretation of symmetric and antisymmetric parts will result from a correspondence argument. The metric of the total field contains both symmetric and antisymmetric variables and it can be measured by using standard measuring rods and clocks. In (I) the metric was defined by

$$\overset{\alpha\beta}{b}=\overset{\alpha\beta}{\mathfrak{g}^-}/(-\mathrm{Det}\,\overset{\alpha\beta}{\mathfrak{g}^-})^{-\frac{1}{2}}, \tag{II.1}$$

where

$$\overset{\alpha\beta}{\mathfrak{g}^-}=\sqrt{(-g)}\,a^{\alpha\beta}\left[\frac{\overset{\alpha}{\delta}_\mu(1+\Omega)-\varphi^{\alpha\nu}\varphi_{\mu\nu}}{1+\Omega-\Lambda^2}\right]$$

$$=\sqrt{-g}\,a^{\alpha\beta}[\lambda\overset{\alpha}{\delta}_\mu-\rho\varphi^{\alpha\nu}\varphi_{\mu\nu}]. \tag{II.2}$$

Hence,

$$b=\frac{g^2}{a}\mathrm{Det}(\lambda\overset{\alpha}{\delta}_\mu-\rho\varphi^{\alpha\nu}\varphi_{\mu\nu})$$

$$=\frac{g^2}{a}(\lambda^4-\rho\lambda^3P_1+\lambda^2\rho^2P_2-\lambda\rho^3P_3+\rho^4P_4), \tag{II.3}$$

where

$$P_1=\overset{\alpha}{c}_\alpha=2\Omega,$$

$$P_2=\frac{1}{2!}\overset{\alpha\beta}{\delta}_{\mu\nu}\overset{\mu}{c}_\alpha\overset{\nu}{c}_\beta=\Omega^2-2\Lambda^2,$$

$$P_3=\frac{1}{3!}\overset{\alpha\beta\gamma}{\delta}_{\mu\nu\sigma}\overset{\mu}{c}_\alpha\overset{\nu}{c}_\beta\overset{\sigma}{c}_\gamma=-2\Omega\Lambda^2,$$

$$P_4=\frac{1}{4!}\overset{\alpha\beta\gamma\delta}{\delta}_{\mu\nu\rho\sigma}\overset{\mu}{c}_\alpha\overset{\nu}{c}_\beta\overset{\rho}{c}_\gamma\overset{\sigma}{c}_\delta=\mathrm{Det}(\varphi^{\alpha\nu}\varphi_{\beta\nu})=\Lambda^4,$$

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[1] B. Kurşunoğlu, Phys. Rev. 88, 1369 (1952).

and

$$\overset{\alpha}{c}_{\beta}=\varphi^{\alpha\mu}\varphi_{\beta\mu}.$$

In deriving these relations we use the identities

$$a^{\mu\nu}\varphi_{\alpha\nu}\varphi_{\beta\mu}\varphi^{\beta\gamma}=\varphi_{\alpha\cdot}^{\ \mu}\varphi_{\beta\mu}\varphi^{\beta\gamma}=\Omega\varphi_{\alpha\cdot}^{\ \gamma}+\Lambda f_{\alpha\cdot}^{\ \gamma},$$

$$\varphi^{\mu\rho}\varphi_{\nu\rho}\varphi^{\nu\sigma}\varphi_{\mu\sigma}=2\Omega^2+4\Lambda^2,$$

$$\varphi^{\mu\rho}\varphi_{\nu\rho}\varphi^{\nu\sigma}\varphi_{\lambda\sigma}\varphi^{\sigma\gamma}\varphi_{\mu\gamma}=2\Omega^3+6\Omega\Lambda^2,$$

$$\varphi^{\mu\alpha}\varphi_{\sigma\alpha}\varphi^{\sigma\beta}\varphi_{\rho\beta}\varphi^{\rho\gamma}\varphi_{\nu\gamma}\varphi^{\nu\delta}\varphi_{\mu\delta}=2\Omega^4+4\Lambda^4+8\Omega^2\Lambda^2,$$

$$\varphi^{\alpha\mu}\varphi_{\beta\mu}=\overset{\alpha}{\delta}_{\beta}\Omega+f^{\alpha\mu}f_{\beta\mu},$$

$$f^{\alpha\mu}\varphi_{\beta\mu}=\overset{\alpha}{\delta}_{\beta}\Lambda$$

and

$$\varphi_{\alpha\beta}=\tfrac{1}{2}\sqrt{(-a)}\ \epsilon_{\alpha\beta\mu\nu}f^{\mu\nu},$$

$$\varphi^{\alpha\beta}=-\frac{1}{2}\frac{1}{\sqrt{(-a)}}\epsilon^{\alpha\beta\mu\nu}f_{\mu\nu}.$$

Hence (II.3) leads to

$$\frac{ab}{g^2}=\frac{1}{(1+\Omega-\Lambda^2)^2}$$

or

$$b=a. \tag{II.4}$$

The contravariant metric thus given by

$$b^{\alpha\beta}=\frac{a^{\alpha\beta}(1+\Omega)-\varphi^{\alpha\mu}\varphi^{\beta}{}_{\cdot\mu}}{(1+\Omega-\Lambda^2)^{\frac{1}{2}}}. \tag{II.5}$$

The covariant metric tensor can be obtained from

$$b_{\alpha\mu}=\frac{a}{3!}\epsilon_{\alpha\beta\gamma\delta}\epsilon_{\mu\nu\rho\sigma}b^{\beta\nu}b^{\gamma\rho}b^{\delta\sigma}$$

as

$$b_{\alpha\beta}=\frac{a_{\alpha\beta}+\varphi_{\alpha\cdot}^{\ \mu}\varphi_{\beta\mu}}{(1+\Omega-\Lambda^2)^{\frac{1}{2}}},\quad b^{\alpha\mu}b_{\beta\mu}=\overset{\alpha}{\delta}_{\beta}. \tag{II.6}$$

The determinant of $\underset{\smile}{\mathfrak{g}}^{\alpha\beta}$ is

$$\mathrm{Det}(\underset{\smile}{\mathfrak{g}}^{\alpha\beta})=-a\Lambda^2=\mathrm{Det}(f_{\mu\nu}).$$

We may interpret the geometrical entity $g_{\alpha\beta}$ as a physical tensor by introducing a critical field q (see I) given by

$$q=pc^2/(2\gamma)^{\frac{1}{2}}=c^2/r_0(2\gamma)^{\frac{1}{2}} \tag{II.7}$$

and write

$$g_{\alpha\beta}=a_{\alpha\beta}+r_0(2\gamma)^{\frac{1}{2}}\varphi_{\alpha\beta}/c^3. \tag{II.8}$$

The theory is invariant with respect to the sign of r_0. Both $g_{\alpha\beta}$ and $g_{\beta\alpha}=a_{\alpha\beta}-r_0(2\gamma)^{\frac{1}{2}}/c^2\varphi_{\alpha\beta}$ satisfy the same set of equations. Another important invariance of the theory refers to the λ invariance defined by

$$\overset{*}{\Gamma}{}^{\gamma}_{\alpha\beta}=\Gamma^{\gamma}_{\alpha\beta}+\overset{\gamma}{\delta}_{\alpha}\lambda_{,\beta}, \tag{II.9}$$

where λ is a scalar. The most important significance of the λ invariance lies in its exclusion of complex field variables $g_{\alpha\beta}$ and complex $\Gamma^{\gamma}_{\alpha\beta}$. As we shall see (in the next section) the complex $g_{\alpha\beta}$ would lead to a velocity of light exceeding the constant c.

The field equations are

$$\underset{-}{R}_{\alpha\beta}=-p^2\left[a_{\alpha\beta}-\frac{a_{\alpha\beta}+(2\gamma/p^2c^4)\varphi_{\alpha\cdot}^{\ \mu}\varphi_{\beta\mu}}{[(1+(2\gamma/p^2c^4)\Omega-(4\gamma^2/p^4c^8)\Lambda^2]^{\frac{1}{2}}}\right], \tag{II.10}$$

$$\underset{\smile}{R}_{\alpha\beta,\gamma}+\underset{\smile}{R}_{\beta\gamma,\alpha}+\underset{\smile}{R}_{\gamma\alpha,\beta}=-p^2I_{\alpha\beta\gamma}, \tag{II.11}$$

$$\psi_{\alpha\beta,\gamma}+\psi_{\beta\gamma,\alpha}+\psi_{\gamma\alpha,\beta}=0, \tag{II.12}$$

where the negative sign before p^2 is taken in anticipation of a comparison of the theory with general relativity and where

$$\psi_{\alpha\beta}=-\tfrac{1}{2}\epsilon_{\alpha\beta\mu\nu}\underset{\smile}{\mathfrak{g}}^{\mu\nu}=\frac{f_{\alpha\beta}+\dfrac{2\gamma}{p^2c^4}\varphi_{\alpha\beta}\Lambda}{\left(1+\dfrac{2\gamma}{p^2c^4}\Omega-\dfrac{4\gamma^2}{p^4c^8}\Lambda^2\right)^{\frac{1}{2}}}. \tag{II.13}$$

The electric current is defined by

$$\mathcal{J}^{\alpha}=\frac{1}{3!}\epsilon^{\mu\nu\rho\alpha}I_{\mu\nu\rho}$$

$$I_{\mu\nu\rho}=\varphi_{\mu\nu,\rho}+\varphi_{\nu\rho,\mu}+\varphi_{\rho\mu,\nu}. \tag{II.14}$$

If we use the algebraic equations

$$g_{\alpha\beta,\gamma}=g_{\mu\beta}\Gamma^{\mu}_{\alpha\gamma}+g_{\alpha\mu}\Gamma^{\mu}_{\gamma\beta}$$

defining the displacement field $\Gamma^{\gamma}_{\alpha\beta}$, and make p tend to infinity, the field equations reduces to

$$G_{\alpha\beta}=(2\gamma/c^4)T_{\alpha\beta}, \tag{II.15}$$

$$f_{\alpha\beta,\gamma}+f_{\beta\gamma,\alpha}+f_{\gamma\alpha,\beta}=0, \tag{II.16}$$

where

$$T_{\alpha\beta}=f_{\alpha\cdot}^{\ \mu}f_{\beta\mu}-\tfrac{1}{4}a_{\alpha\beta}f^{\mu\nu}f_{\mu\nu}. \tag{II.17}$$

The equations (II.11) lead to

$$\varphi_{\alpha\beta,\gamma}+\varphi_{\beta\gamma,\alpha}+\varphi_{\gamma\alpha,\beta}=0 \tag{II.18}$$

which expresses the vanishing of free electric charges in the limit of $r_0=0$. Thus the existence of free charges is linked up with a finite fundamental length r_0. We can apply the correspondence argument to the conservation laws of the total field

$$\mathfrak{T}^{\rho}_{\alpha,\rho}=0, \tag{II.19}$$

where

$$-4\pi p^2 q^{-2}\overset{\alpha}{\mathfrak{T}}_{\beta}=\tfrac{1}{2}(\mathfrak{g}^{\alpha\mu}R_{\beta\mu}+\mathfrak{g}^{\mu\alpha}R_{\mu\beta}-\overset{\alpha}{\delta}_{\beta}\mathfrak{g}^{\mu\nu}R_{\mu\nu})$$
$$+\tfrac{1}{2}(\overset{\mu\nu}{\mathfrak{g}}_{,\beta}\overset{\alpha}{\mathfrak{B}}_{\mu\nu}-\overset{\alpha}{\delta}_{\beta}\overset{\alpha}{\mathfrak{B}})$$
$$=(\overset{\alpha\mu}{\underline{\mathfrak{g}}}R_{\underline{\beta\mu}}-\tfrac{1}{2}\overset{\alpha}{\delta}_{\beta}\overset{\mu\nu}{\underline{\mathfrak{g}}}R_{\underline{\mu\nu}})$$
$$+q^{-2}(\overset{\alpha\mu}{\underset{\smile}{\mathfrak{g}}}R_{\underset{\smile}{\beta\mu}}-\tfrac{1}{2}\overset{\alpha}{\delta}_{\beta}\overset{\mu\nu}{\underset{\smile}{\mathfrak{g}}}R_{\underset{\smile}{\mu\nu}})$$
$$+\tfrac{1}{2}(\overset{\mu\nu}{\mathfrak{g}}_{,\beta}\overset{\alpha}{\mathfrak{B}}_{\mu\nu}-\overset{\alpha}{\delta}_{\beta}\overset{\alpha}{\mathfrak{B}}). \quad \text{(II.20)}$$

If we make p tend to infinity

$$-\overset{\alpha}{\mathfrak{T}}_{\beta}=\frac{c^4}{8\pi\gamma}(\mathfrak{G}^{\alpha\mu}G_{\beta\mu}-\tfrac{1}{2}\overset{\alpha}{\delta}_{\beta}\mathfrak{G}^{\mu\nu}G_{\mu\nu})+\frac{c^4}{16\pi\gamma}\overset{\alpha}{t}_{\beta}$$
$$=\frac{1}{4\pi}(-\alpha)^{\frac{1}{2}}\overset{\alpha}{T}_{\beta}+\frac{c^4}{16\pi\gamma}\overset{\alpha}{t}_{\beta}, \quad \text{(II.21)}$$

where

$$\mathfrak{G}^{\alpha\beta}=(-\alpha)^{\frac{1}{2}}a^{\alpha\beta}$$

and

$$\overset{\alpha}{t}_{\beta}=\overset{\mu\nu}{\mathfrak{G}}_{,\beta}\overset{\alpha}{\mathfrak{L}}_{\mu\nu}-\overset{\alpha}{\delta}_{\beta}\mathfrak{L}$$

which is the energy momentum tensor of general relativity in the presence of electromagnetic and gravitational fields.

There is no way of obtaining Einstein's version of the theory from the present one without loss of internal consistency and the physical interpretation that follows from the correspondence argument. To make $p=0$ is as meaningless as making $c=0$ in special relativity or $h=\infty$ in quantum theory. However, one can also introduce a critical field strength q in Einstein's theory and consider the limit of q tending to infinity in which case the resulting equations are two sets of disconnected equations for gravitational field in empty space and an unfamiliar set for the antisymmetric field. One does not get Maxwell's equations.

The critical field strength is responsible for the nonlinearity of the theory. The constant r_0 implies an extended charge distribution of elementary particles and nonexistence of point charges. That the electron has an extended charge distribution can be observed experimentally with electron scattering of wave length r_0. For $r_0=10^{-13}$ cm the corresponding electron energy is about 600 Mev. Some experiments on electron scattering do actually imply an extended charge distribution.

Invariance of the theory with respect to the sign of r_0 introduces the concept of negative length. Measurement of negative length may be compared to saying that there is a fictitious space where moving clocks go faster and moving measuring rods get longer! In actual case the measured length is still positive just as an electron moving in the direction of negative time arrow has a positive energy. Under these circumstances if we contemplate the origin of mass as being from the electromagnetic field then the field must create positive and negative charges simultaneously. In this sense the electromagnetic field creates pairs of particles and does not allow for single acts of creation of particles. This interpretation makes it possible to think of the existence of negative mass along with the positive mass. The vanishing of r_0 is one reason for general relativity leaving the sign of the mass quite arbitrary.

III. LIGHT CONE

The signature of the total field as in the theory of gravitation is defined by the symmetric part of $g_{\alpha\beta}$ so that the presence of the electromagnetic field does not spoil the invariance of the propagation of light but introduces an anisotropy in the form of the light cone. Because of the result $\mathrm{Det}(b_{\alpha\beta})=a$ the requirement that $\mathrm{Det}(b_{\alpha\beta})$ is everywhere different from zero and b be negative is satisfied. The propagation of light, i.e., the interval dx^α for two neighboring events shall satisfy

$$ds^2=b_{\alpha\beta}dx^\alpha dx^\beta=0, \quad \text{(III.1)}$$

which proposition is compatible with the correspondence argument since for vanishing fundamental length it reduces to the law of propagation of light in the presence of a purely gravitational field. The motivation behind choosing $b_{\alpha\beta}$ as the metric of the total field can also be understood from the Bianchi identities

$$(\overset{\alpha\mu}{\underline{\mathfrak{g}}}R_{\underline{\beta\mu}}-\tfrac{1}{2}\overset{\alpha}{\delta}_{\beta}\overset{\mu\nu}{\underline{\mathfrak{g}}}R_{\underline{\mu\nu}})_{||\alpha}=\tfrac{1}{2}\overset{\mu\nu}{\underset{\smile}{\mathfrak{g}}}(R_{\underset{\smile}{\mu\nu},\beta}+R_{\underset{\smile}{\nu\beta},\mu}+R_{\underset{\smile}{\beta\mu},\nu}), \quad \text{(III.2)}$$

which is the limit of $r_0=0$ reduce to

$$(\mathfrak{A}^{\alpha\mu}G_{\beta\mu}-\tfrac{1}{2}\overset{\alpha}{\delta}_{\beta}\mathfrak{A}^{\mu\nu}G_{\mu\nu})_{|\alpha}=0 \quad \text{(III.3)}$$

while in the absence of charges they become

$$(\overset{\alpha\mu}{\underline{\mathfrak{g}}}R_{\underline{\beta\mu}}-\tfrac{1}{2}\overset{\alpha}{\delta}_{\beta}\overset{\mu\nu}{\underline{\mathfrak{g}}}R_{\underline{\mu\nu}})_{||\alpha}=0. \quad \text{(III.4)}$$

Thus in the absence of charges the tensor

$$\overset{\alpha\mu}{\underline{\mathfrak{g}}}R_{\underline{\beta\mu}}-\tfrac{1}{2}\overset{\alpha}{\delta}_{\beta}\overset{\mu\nu}{\underline{\mathfrak{g}}}R_{\underline{\mu\nu}}$$

or if the field equations are satisfied the tensor

$$\overset{\alpha\mu}{\underline{\mathfrak{g}}}a_{\underline{\beta\mu}}-\tfrac{1}{2}\overset{\alpha}{\delta}_{\beta}\overset{\mu\nu}{\underline{\mathfrak{g}}}a_{\underline{\mu\nu}}$$

is conserved covariantly with respect to $b_{\alpha\beta}$. The metric $b_{\alpha\beta}$ is not independent of the fundamental length r_0 and in this sense the zero interval in (III.1) is one between two "extended events" and not between two point events. This can best be seen by choosing a local coordinate system in which the mixed tensor $\mathfrak{A}^{\alpha\mu}b_{\beta\mu}$ is represented by a diagonal matrix $\rho_\alpha\delta^\alpha_\beta$. From the transformation law

$$(a^{\alpha\mu}b_{\beta\mu})^* = \overset{*\alpha}{b}_\beta = \frac{\partial \overset{*}{x}^\alpha}{\partial x^\mu}\frac{\partial x^\nu}{\partial \overset{*}{x}^\beta}\overset{\mu}{b}_\nu, \qquad \text{(III.5)}$$

we would like to find a transformation which will result in

$$\overset{*\alpha}{b}_\beta = \rho_\alpha \overset{\alpha}{\delta}_\beta. \qquad \text{(III.6)}$$

From (III.5) and (III.6) we get

$$\frac{\partial \overset{*}{x}^\alpha}{\partial x^\mu}\rho_\alpha = \frac{\partial \overset{*}{x}^\alpha}{\partial x^\nu}\overset{\nu}{b}_\mu$$

or

$$\frac{\partial \overset{*}{x}^\alpha}{\partial x^\mu}(\overset{\mu}{b}_\beta - \overset{\mu}{\delta}_\beta \rho_\alpha) = 0. \qquad \text{(III.7)}$$

These 16 equations will determine the four eigenvalues ρ_α and the transformation $\partial \overset{*}{x}^\alpha/\partial x^\beta$. The eigenvalues are found from the condition

$$\mathrm{Det}(\overset{\alpha}{b}_\beta - \overset{\alpha}{\delta}_\beta \rho_\gamma) = 0. \qquad \text{(III.8)}$$

By applying the methods of Sec. II, we obtain the equation

$$\rho^4 - I_1\rho^3 + I_2\rho^2 - I_1\rho + 1 = 0, \qquad \text{(III.9)}$$

where

$$I_1 = \frac{4+2\Omega}{(1+\Omega-\Lambda^2)^{\frac{1}{2}}},$$

$$I_2 = 6 + \frac{\Omega^2+4\Lambda^2}{1+\Omega-\Lambda^2} = \tfrac{1}{4}I_1^2 + 2.$$

The equation (III.9) can be written as

$$\rho^2 + \frac{1}{\rho^2} - I_1\left(\rho + \frac{1}{\rho}\right) + I_2 = 0$$

so by putting $x = \rho + 1/\rho$ it reduces to

$$x^2 - I_1 x + \tfrac{1}{4}I_1^2 = 0$$

with the roots

$$\rho = \tfrac{1}{4}I_1 \pm \tfrac{1}{2}(I_2-6)^{\frac{1}{2}}.$$

The other two roots are obtained by noting that the equation (III.9) is also satisfied by $1/\rho$. Thus the eigenvalues are

$$\rho_1 = \tfrac{1}{4}I_1 + \tfrac{1}{2}\sqrt{(I_2-6)} = \rho_4,$$
$$\rho_2 = \tfrac{1}{4}I_1 - \tfrac{1}{2}\sqrt{(I_2-6)} = \rho_3. \qquad \text{(III.10)}$$

In the local frame as chosen, the space-time interval is

$$ds^2 = a_{\alpha\beta}\Delta x^\alpha \Delta x^\beta, \qquad \text{(III.11)}$$

where

$$\Delta x^\alpha = (\rho_\alpha)^{\frac{1}{2}} dx^\alpha, \qquad \text{(III.12)}$$

so there will be an electromagnetic extension of the two neighboirng point events. Because of the algebraic invariants involved in the definition of the metric it is not possible to choose a local frame in which the metric tensor will take the form of the metric of the flat space time. This is the same as saying that the principle of equivalence would not hold in the presence of an electromagnetic field.

It is convenient to express the metric $b_{\alpha\beta}$ in terms of $\psi_{\alpha\beta}$. We use the antisymmetric tensors $\psi_{\alpha\beta}$ and $\phi_{\alpha\beta}$ to record the invariants

$$\Lambda = \tfrac{1}{4}\phi^{\alpha\beta}\psi_{\alpha\beta},$$

$$Q = \tfrac{1}{2}\psi^{\alpha\beta}\psi_{\alpha\beta} = \frac{\Omega\Lambda^2 - \Omega + 4\Lambda^2}{1+\Omega-\Lambda^2}$$

$$= -\tfrac{1}{2}\phi^{\alpha\beta}\phi_{\alpha\beta}(= -\tfrac{1}{2}b^{\alpha\mu}b^{\beta\nu}\varphi_{\mu\nu}\varphi_{\alpha\beta}),$$

where

$$\phi_{\alpha\beta} = \frac{1}{\sqrt{-a}}a_{\alpha\mu}a_{\beta\nu}\overset{\mu\nu}{\underset{\smile}{\mathfrak{g}}} = \frac{\varphi_{\alpha\beta} - \Lambda f_{\alpha\beta}}{(1+\Omega-\Lambda^2)^{\frac{1}{2}}}.$$

Hence

$$(1+Q-\Lambda^2)^{\frac{1}{2}} = \frac{1+\Lambda^2}{(1+\Omega-\Lambda^2)^{\frac{1}{2}}}$$

and

$$\phi^{\alpha\mu}\phi_{\beta\mu} = -\overset{\alpha}{\delta}_\beta Q + \psi^{\alpha\mu}\psi_{\beta\mu},$$

$$\varphi_{\alpha\beta} = \frac{\phi_{\alpha\beta} + \Lambda\psi_{\alpha\beta}}{(1+Q-\Lambda^2)^{\frac{1}{2}}} \qquad \text{(III.13)}$$

$$f_{\alpha\beta} = \frac{\psi_{\alpha\beta} - \Lambda\phi_{\alpha\beta}}{(1+Q-\Lambda^2)^{\frac{1}{2}}}. \qquad \text{(III.14)}$$

The metric takes the form

$$b_{\alpha\beta} = \frac{a_{\alpha\beta} + \psi_{\alpha\cdot}^{\ \mu}\psi_{\beta\mu}}{\sqrt{(1+Q-\Lambda^2)}} \qquad \text{(III.15)}$$

which shows that the metrical properties of the total field is the same with respect to $\varphi_{\alpha\beta}$ and $\psi_{\alpha\beta}$ fields. The electromagnetic field is described by four vectors

$$\mathbf{D} = (\varphi_{23}, \varphi_{31}, \varphi_{12}), \quad \mathfrak{H} = (\varphi_{41}, \varphi_{42}, \varphi_{43}),$$
$$\mathbf{B} = (\psi_{23}, \psi_{31}, \psi_{12}), \quad \mathcal{E} = (\psi_{14}, \psi_{24}, \psi_{34})$$

that reduce to two vectors in the correspondence limit.†

The metric can be written as

$$b_{\alpha\beta} = a_{\alpha\mu}c_\beta^{\ \mu} \qquad \text{(III.16)}$$

where

$$\overset{\mu}{c}_\beta = \frac{\delta_\beta^{\ \mu} + \psi^{\mu\rho}\psi_{\beta\rho}}{(1+Q-\Lambda^2)^{\frac{1}{2}}}$$

† In connection with the recent observation on parity nonconservation, it is interesting to note that the existence of the Λ term (where $\Lambda = \mathcal{E}\cdot\mathbf{B}$) in (III.13) and (III.14) implies different field strength in the reflected coordinate system. The parity conservation must also be violated for very strong fields such as at a charged particle itself.

and

$$\mathrm{Det}(c_\beta{}^\mu)=1.$$

Because of the form (III.16) the signature of the metric will remain unchanged by coordinate transformations. But there exists no local frame in which the measuring rods and clocks will operate without being contracted and retarded. These facts about $b_{\alpha\beta}$ make it clear that we can take over the concept of light cone from general relativity to study the propagation of $\varphi_{\alpha\beta}$ waves. By using the form (III.15) of the metric we find that the distortion of the light cone due to the finite value of the critical field is given by

$$a_{\alpha\beta}dx^\alpha dx^\beta=-q^{-2}\psi_{\alpha\cdot}^{\;\;\mu}\psi_{\beta\mu}dx^\alpha dx^\beta.$$

This can be written

$$\left(1+\frac{q^{-2}}{2}Q\right)a_{\alpha\beta}dx^\alpha dx^\beta=-q^{-2}T_{\alpha\beta}dx^\alpha dx^\beta \quad \text{(III.17)}$$

or

$$\left(1+\frac{q^{-2}}{2}Q\right)=-q^{-2}T_{\alpha\beta}\frac{dx^\alpha}{du}\frac{dx^\beta}{du}, \quad \text{(III.18)}$$

where

$$du^2=a_{\alpha\beta}dx^\alpha dx^\beta. \quad \text{(III.19)}$$

The result (III.18) is in complete agreement with Schrödinger's[2] calculation obtained under certain assumptions about the ψ field from the equations (II.12) and (II.16). Schrödinger's calculation is based on a special frame where $a_{\alpha\beta}$ assumes a Galilean form. He chooses the principal directions of the three-dimensional part of $T_j{}^i$ of $T_\beta{}^\alpha$ as the coordinate axes and takes $T_i{}^4$ parallel to the x_1 direction, in which case (III.18) assumes a simpler form. In (III.18) we can regard the ψ field consisting of two parts: an infinitely weak, rapidly oscillating part that represents the light wave whose propagation is to be investigated, and a beckground field with no restriction on its magnitude. For the special case of Galilean a the equation (III.18) is

$$\frac{p^2c^4}{2\gamma}(c^2-v^2)=-(\mathbf{v}\cdot\mathbf{B})^2+c^2B^2+(\boldsymbol{\mathcal{E}}\times\mathbf{v})^2-2c\mathbf{v}\cdot(\boldsymbol{\mathcal{E}}\times\mathbf{B}). \quad \text{(III.20)}$$

In the case of a purely electric field $\mathcal{E}$ the D field is given by

$$\mathbf{D}=\kappa_0\boldsymbol{\mathcal{E}}, \quad \text{(III.21)}$$

where

$$\kappa_0=\left(1-\frac{\mathcal{E}^2}{q^2}\right)^{-\frac{1}{2}}$$

can be regarded as a dielectric constant for the background field. The velocity of light v in the direction of the wave normal in the background field, independently of its polarization and frequency, is

$$v^2=\frac{c^2}{1+(\mathcal{E}^2/q^2)\sin^2\theta}. \quad \text{(III.22)}$$

If the background field $\mathcal{E}$ is very small compared to the critical field q, we have

$$v^2=c^2(\kappa_0{}^2\sin^2\theta+\cos^2\theta), \quad \text{(III.23)}$$

where θ is the angle between the wave normal and $\boldsymbol{\mathcal{E}}$. Another simple case is obtained for a background field with parallel $\mathcal{E}$ and B fields. Both results agree with Schrödinger's calculations. For complex field variables $g_{\alpha\beta}$ the result (III.22) would have a denominator smaller than 1, so the velocity of light would exceed the constant c.

IV. CONCLUSION

With the physical interpretation proposed in this paper, the concept of generalized theory of gravitation may be taken more seriously than it has been looked on in the past. The correspondence argument used in this theory puts great emphasis on the validity of the general relativity as a physical theory. The latter is either a virtue or an undesirable basis for a theory aiming at a unified description of the natural phenomena. However, one thing is quite clear now, that general theory of relativity with all its tested consequences, is the only genuine field theory in physics. It is a field theory that is independent of any mechanical constants like mass and charge, where masses are singularities in the field. The generalized theory is expected to account for electric charge and mass. One of the necessary conditions for the generalized field to represent mass and charge must come from the regularity of the field everywhere. Particles may be represented as stationary solutions of the field equations with large fluctuations confined to regions of the dimensions of r_0. Unfortunately, we have not yet been able to get exact solutions of the field equations to verify the above conjectures. One of the most important reasons for requiring exact solutions lies in the fact that the critical field may be approached at places where particles are situated. In this case we cannot disregard the nonlinearity of the theory. Strong gravitational and electromagnetic interactions may be expected to take place in regions of high fluctuations. The concept of negative length may be linked up with negative mass (or antimatter). The shape of the universe would be like a dipole with repelling poles consisting of matter and antimatter. The poles of the dipole should be rotating around the center of the dipole to maintain a steady state universe against gravitational repulsion of the poles.

It still remains to be seen in what way Planck's constant will appear in the theory, without which it would be empty of any physical content.

[2] O. Hittmair and E. Schrödinger, Commun. Dublin Inst. Advanced Studies, Series A, No. 8 (1951).