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Nuclear Reactions

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1. INTRODUCTION

THE richness of phenomena encountered in the study of nuclear reactions is revealed by the colorful terminology in current use: cloudy crystal ball model, knock-out processes, boil-off processes, stripping and pickup reactions, and so on. In a brief survey with any pretensions to coherence, it is impossible to make detailed mention of all phenomena. Consequently, the present review will be purposely restricted to the central features of nuclear reactions. Coulomb excitation, heavy ion reactions, fission, and other such specialized subjects will not be discussed. Furthermore, little will be said about the older and well-known subjects such as the Breit-Wigner formula and the statistical theory of the compound nucleus. Rather we will stress the developments of recent years, with particular emphasis on the cloudy crystal ball (or complex potential) model and direct reactions.

Ideally, the task of a comprehensive theory of nuclear reactions is to predict the cross sections for all the individual energetically-allowed processes that can be initiated by the bombardment of a given target nucleus with a given projectile. In the search for such a theory, it is convenient to begin by limiting oneself to a more modest goal, namely the prediction of the cross sections for scattering and absorption.

When the target nucleus and projectile come together in a bombardment, there are two basic types of events that can occur as a result of their interaction. Either the nuclei are scattered by each other without change in relative energy or internal structures (scattering event), or else they exchange energy and change their internal structures (absorption event).* In the latter case, the event implies the formation of a "compound system" of all the particles.[1] This may decay quickly (direct process) or after a long time interval (compound nucleus process). Evidently, any given individual cross section is determined by the relative probability for the decay of the compound system into the appropriate final nuclei. A theory giving all these relative probabilities can be called a theory of "individual cross sections" as opposed to one which only gives only the two basic cross sections for scattering and absorption. Section 2 is devoted to discussion of the cross sections for diffraction and absorption. This is followed, in Sec. 3, by a review of the present theory of individual cross sections. Finally, in Sec. 4, some remarks are made on the general contemporary situation in the study of nuclear reactions.

2. CROSS SECTIONS FOR SCATTERING AND ABSORPTION

The sum of the scattering and absorption cross sections defined in the Introduction must evidently constitute what is normally understood to be the "total" cross section. However, it is not true[2] that the two cross sections separately can be exactly identified with what one normally calls the "elastic" and "nonelastic" (or "reaction") cross sections. The reason for this comes from the possibility that, of the totality of absorption events which constitute the absorption cross section, some of them may ultimately lead to the production of the original bombarding pair of nuclei. In the scheme which divides the total cross section into elastic and nonelastic, such events would be included in the former, not the latter. These special events are said to account for the "compound-elastic" part of the elastic cross section. This distinguishes them from the "scattering" or "shape-elastic" events which constitute the rest of the elastic cross section. Fortunately, except at energies of less than a few Mev, the compound-elastic cross section is quite negligible. Thus in the following, we will identify the usual quantitative definitions of elastic and nonelastic cross sections with the shape-elastic and absorption cross sections, respectively. Later on [Sec. 2(d)] we will describe the special situation arising at low energies.

At this stage we make an important specialization in the discussion, namely it will be assumed that the projectile in the bombardment is a nucleon. Clearly, a necessary preliminary to the basic understanding of reactions initiated by deuterons, alphas and other composite particles is the understanding of reactions initiated by simple single particles.

2(a) Parametric Analysis of Cross Sections

Let us, for simplicity, ignore the presence of the intrinsic nucleon spin. The shape-elastic and absorption

* Strictly speaking, there may also be the third possibility of a charge in relative energy without a change in internal structure. This will be so if energy can be stored in collective motion which preserves internal structure. For instance, if the target nucleus has a nonspherical shape, it may be set into simple rotational motion. We ignore this possibility until we make special mention of it in Sec. 4(b).

[1] V. F. Weisskopf, Address at Amsterdam Conference on Nuclear Reactions (July, 1956).

[2] Feshbach, Porter, and Weisskopf, Phys. Rev. 96, 448 (1954).

cross sections then have the well-known forms

$$\sigma_{\mathrm{sh.el}}(\theta)d\Omega=\sigma_{\mathrm{el}}(\theta)d\Omega=\frac{\pi}{k^2}\left|\sum_l (2l+1)(1-S_l)\frac{P_l(\cos\theta)}{(4\pi)^{\frac{1}{2}}}\right|^2 d\Omega$$

$$\sigma_{\mathrm{abs}}=\sigma_{\mathrm{nonel}}=\frac{\pi}{k^2}\sum_l (2l+1)(1-|S_l|^2),$$

where k is the wave number of the relative motion of the colliding nuclei, l labels the orbital angular momenta of relative motion, and S_l is the scattering amplitude for the lth partial wave defined by the following asymptotic form of the wave function of relative motion at large distances of separation

$$\frac{1}{r}\{e^{-i[kr-(l\pi/2)]}-S_l e^{+i[kr-(l\pi/2)]}\}P_l(\cos\theta).$$

When the nucleon spin is taken into account, these formulas become somewhat more complicated. For instance, for a zero spin target nucleus, each S_l is replaced by two scattering amplitudes $S_{l+\frac{1}{2}}$ and $S_{l-\frac{1}{2}}$ for two possible values, $J=l\pm\frac{1}{2}$. In addition, there is a new cross section to be considered, the polarization cross section, which depends on the differences $(S_{l+\frac{1}{2}}-S_{l-\frac{1}{2}})$. The presence of nucleon spin is only significant when there is some force coupling the spin to the orbital motion, so that $S_{l+\frac{1}{2}}$ and $S_{l-\frac{1}{2}}$ are not equal. Experimentally, it appears that this coupling is not very large, although it is certainly necessary to take it into account if the observed differential cross sections are to be fitted in better than a qualitative manner. (Also, evidently, it is necessary to consider it if one wishes to fit the observed polarization cross sections.) For the present discussion, it is convenient to ignore the spin-orbit coupling.

At any given energy the set of complex S_l form a doubly infinite number of parameters that are, in principle, deducible from experiment and which are only restricted by the conditions $|S_l|\leqslant 1$. We might guess that any theory of reactions containing a similar number of parameters is always capable of fitting the data.[3] In situations of physical interest, only a restricted number of l waves are involved in nuclear collisions because of the finite size of nuclei. In a given collision, there are only $\sim 2l_{\max}$ parameters to be considered, where $l_{\max}$ is the l value of the highest partial wave involved in the collision. If a theory contains a similar number of parameters, then it can always be made to fit the data. Such a theory gives no physical insight into the mechanism of reactions. It only gives a "representation" of the data, but does not provide evidence for or against a particular model, although it may be inspired by a model.

[3] J. A. Wheeler, Phys. Rev. 99, 630(A) (1955).

2(b) Complex Potential Representation

Many successful attempts have been made to fit observed cross sections by representing the interaction of incident nucleons with target nuclei by a spherical single particle potential well $-V(r)-iW(r)$, where $V(r)$ is the real, refracting part and $iW(r)$ is the imaginary, absorbing part. For any such potential, one solves the Schrodinger equation for each l wave, thereby determining the amplitudes S_l. In the high energy limit, where many l waves are involved in the collision, the formulas for $\sigma_{\mathrm{el}}(\theta)$ and σ_{abs} naturally take on the forms characteristic of the "optical model" that can be used in this limit.[4] S_l is evaluated as $\exp(ik\int nds)$ where the integral is taken along the optical path corresponding to impact parameter $b=l/k$, and where n is a complex refractive index defined by

$$n^2-1=(V+iW)/E,$$

E being the relative energy of collision. It is usual to write

$$n=1+\frac{1}{k}\left(k_1+\frac{iK}{2}\right),$$

where k_1 and K are real quantities and K is the "absorption coefficient" equal to the reciprocal of the mean free path against collision. If W is small compared to $V+E$, then

$$\frac{k_1}{k}\approx\left(1+\frac{V}{E}\right)^{\frac{1}{2}}-1$$

$$\frac{K}{k}\approx\frac{W}{E\left(1+\frac{V}{E}\right)^{\frac{1}{2}}}.$$

These relations are correct to a sufficient approximation in cases of physical interest.

In order to fit the data with the complex potential model, it is evidently necessary to restrict the potential to approximately the size of the target nucleus if the theoretical value of $l_{\max}$ is to be made to coincide with the experimental one. This still leaves one with freedom to adjust the shapes of $V(r)$ and $W(r)$, and this freedom surely means that, at any given energy, the data can always be fitted by a suitably chosen complex potential.† At very high energies, where the elastic scattering closes up into the forward direction, there is effectively only one experimental quantity, *viz.*, σ_{abs}, and the

[4] Fernbach, Serber, and Taylor, Phys. Rev. 75, 1352 (1949).

† For instance, the quantity $1-|S_l|^2$ giving the absorption in the lth wave is essentially $\sim\int W(r)|\psi_l(r)|^2d\tau$; i.e., the volume integral of $W(r)$ weighted by the square modulus of the wave function $\psi_l(r)$ of the lth wave. Thus one can give σ_{abs} any desired value by adjusting the value of $W(r)$ near the impact distance for the lth wave where $|\psi_l|^2$ is peaked. However it must be stressed that no *rigorous* proof has been given of the conjecture that the set of S_l can always be fitted by a suitable choice of potential $V(r)+iW(r)$. Such a proof has been reported for only the special case of $W(r)=0$, i.e., no absorption.[3]

corresponding theoretical parameter is an integral over $K(r)$. Thus, at any energy, there seems to be ample freedom in the model to guarantee a fit of the data.

2(c) Evidence for the Optical Properties of Nuclear Matter

The very important question that now arises is: how can one establish whether the complex potential corresponds to physical reality or not? In other words, we want to know if nuclear matter really has optical properties so that the complex potential is a genuine physical model, or whether the complex potential is just to be regarded as no more than a representation of data. Merely to fit the data at a given energy with an adjustable complex potential proves nothing. To confirm that the model has a physical basis, one must go further and compare the experimental values of V and W as functions of radial distance and energy with values computed theoretically using a calculation based on optical model ideas. Generally speaking, such ideas enable one to express V and W in terms of the scattering amplitudes of individual collision processes between the incident nucleon and the nucleons in the target. Since the observed amplitudes for nucleon-nucleon scattering are smooth functions of energy, it follows that a precondition for the physical reality of the complex potential is that V and W are smooth functions of energy. Furthermore, since the radial dependences of V and W are expected to be simply related to that of the nuclear density, the known smoothness of the latter leads to the requirement that V and W be also smooth functions of radial distance.

Experimentally elastic and absorption cross sections have been measured for neutrons and protons incident on many nuclei with bombarding energies up to 1.4 Bev. These cross sections have been analyzed using the complex potential with the following results.

Observed Radial Dependence of V and W

As far as the radial dependences are concerned, all the data are consistent with a shape much like that of the nuclear density distribution with its flat central part and an appreciable surface fall-off distance. These two qualitative features are essential for fitting the data,[5,6] at least in the case of $V(r)$. The radial distribution of W is less well determined and some success has been achieved with a distribution of W concentrated at the nuclear surface.[5] For both V and W there is uncertainty about the details of the shapes deduced from experiment. This arises partly from the neglect of spin-orbit coupling in most attempts to fit the data. To carry out a full analysis of the polarization, differential and absorption cross sections with a complex potential model including a spin-orbit term $U(r)\mathbf{l}\cdot\mathbf{s}$ would be very laborious. Unfortunately it seems that such an analysis must be done if the complex potential model is to be exhaustively tested.

Observed Energy Dependences of the Central Depths of V and W

The values of central depths of V and W that are extracted from the data depend on the radii of the radial distributions of V and W. (In principle, such radii should be determinable uniquely from a careful analysis of the data and so V and W could be assigned unique values. In reality, most analyses have not approached this ideal stage.) To a first approximation, the cross sections are determined by the combination $(k+k_1+iK/2)R$, where R is the mean radius and k_1 and K are the "bulk" values of our previously defined quantities. It follows that V and W depend on the choice of R as $(V+E)\sim R^{-2}$, $W\sim R^{-2}$. The values of V and W that we now quote, have been obtained from the reported values by normalizing to the choice $R=1.33A^{\frac{1}{3}}\times 10^{-13}$ cm.

For protons,[6] the central depth of V after correction for the mean Coulomb potential is observed to decrease from 55 Mev at zero bombarding energy roughly according to the law $V=55-0.5E$. Above $E=40$ Mev, the values flatten off to $V\sim 20$ Mev at $E=100$ Mev and appear to stay at this value up to $E\gtrsim 400$ Mev.[7] At higher energies, the values of V are not obtained very precisely from the data.

For neutrons,[2,7] the same remarks apply except that the depth of V at zero bombarding energy[2] is about 10 Mev less than for protons. Such a difference is to be generally expected from the velocity dependence of V. It is easy‡ to see that, if the mean potentials felt by a neutron and a proton are assumed to be the same as functions of *kinetic* energy except for the Coulomb potential V_c, then the two potentials as functions of *total* or *bombarding* energy differ not only by V_c but also by a term which is opposite in sign to V_c and proportional to V_c.

For protons and neutrons the value of the central depth of W increases from $W\sim 3$ Mev at zero bombarding energy to $W\sim 15$ Mev at $E=32$ Mev.[6] There is a broad maximum with $W\sim 20$ Mev centered at $E\sim 70$ Mev, followed by a shallow minimum of $W\sim 12$ Mev at $E\sim 200$ Mev. From then on W increases monotonically passing through $W\sim 17$ Mev at $E=350$ Mev[7] and $W\sim 70$ Mev at $E\sim 1400$ Mev.[8]

[5] Bjorklund, Fernbach, and Sherman, Phys. Rev. **101**, 1832 (1956).

[6] Melkanoff, Moszkowski, Nodvik, and Saxon, Phys. Rev. **101**, 507 (1956).

[7] T. B. Taylor, Phys. Rev. **92**, 831 (1953); R. Jastrow and R. M. Sternheimer (to be published); F. Mandl and T. H. R. Skyrme, Phil. Mag. **44**, 1028 (1953).

‡ If, for a neutron, $V=-\alpha+\beta T$, where T is the kinetic energy, the corresponding formula for V as a function of *total* energy $E=T+V$ is $V=(-\alpha+\beta E)/(1+\beta)$. For a proton, the formula $V=-\alpha+\beta T+V_c$ leads to $V=(-\alpha+\beta E+V_c)/(1+\beta)$ $=(-\alpha+\beta E)/(1+\beta)+V_c-\beta/(1+\beta)V_c$. This differs from the neutron potential not only in V_c but also in the term $-\beta/(1+\beta)V_c$ which tends to cancel V_c.

[8] R. W. Williams, Phys. Rev. **98**, 1387 (1955).

To appreciate the physical implications of these magnitudes of W, it is convenient to consider the ratio of the corresponding mean free paths $1/K$ to a typical nuclear radius which we will choose to be 7×10^{-13} cm. This ratio exceeds unity only below a few Mev. Above these energies it falls steadily to about 0.25 around $E=40$ Mev, then rises to a maximum of about 0.7 at $E\sim350$ Mev and falls again to 0.3 at 1.4 Bev. Thus the mean free path of a nucleon against collision in nuclear matter is always of the order of the nuclear radius, but only actually exceeds it at low energies.

On the simplest classical particle picture, one expects the mean free path $1/K$ to be simply $(\rho\langle\sigma\rangle)^{-1}$ where ρ is the nucleon density in nuclear matter and $\langle\sigma\rangle$ is a suitable average of the nucleon-nucleon total cross section, taken over the various velocities of neutrons and protons in nuclear matter. This relation follows from the more general one of the classical optical wave model[9]:

$$n-1=\rho k^{-1}\langle 2\pi k_n{}^{-1}f_n(0)\rangle.$$

Here $f_n(0)$ is the forward part of the nucleon-nucleon scattering amplitude and the preceding expression for K follows on taking the imaginary part of this equation. It is unfortunate that one cannot proceed to calculate k_1 directly from the observed nucleon-nucleon data. The reason is essentially that the singlet and triplet scattering amplitudes are not determined separately by the data as analyzed at present.§ No such difficulty is present in the case of K however and this quantity is straightforwardly computed. The observed values of K above $E\sim100$ Mev are immediately found to be predicted correctly. From the simple formula for K, its energy dependence for $E>100$ Mev ought to be that of the nucleon-nucleon cross sections. The fall-off in the n-p cross section from 100 to 300 Mev and the subsequent rise (due to meson processes) are directly reflected in the observed values of K.

Below $E=100$ Mev the decrease in K can be attributed to the effects of the Pauli principle which forbids collisions that would otherwise take place. A simple calculation[10] with a degenerate Fermi gas at zero temperature reproduces the observed values rather well if one ignores the velocity dependence of the nucleon potential and takes $V=T_F+8$ Mev, i.e. $E=T-(T_F+8$ Mev$)$ where T is the kinetic energy of the bombarding particle and T_F is the maximum energy in the Fermi distribution. A more consistent calculation which takes the velocity dependence into account reduces the predicted values by nearly an order of magnitude, thereby destroying the agreement with experiment. However the agreement can again be restored by allowing the Fermi gas to have a suitable temperature.[11] The effect of doing this just about cancels the effects of the velocity dependence.

In spite of the absence of an optical calculation of V, the agreement obtained in the case of W between optical theory and experiment seems sufficient to establish the physical reality of the complex potential. The final task is now to explain *why* nuclear matter has apparent optical properties. This is really a problem of nuclear structure rather than nuclear reactions, so this difficult question will not be discussed here. However it seems appropriate to draw attention to the fact that the model's success is somewhat surprising. The model is essentially a classical one, based on the concept of two-particle collisions with energy conservation in each collision. On the other hand, the nucleus is a quantum system of closely packed particles and there are usually particles present in the wave zone for a given collision between two particles, and these would normally lead to nonconservation of energy in the collision. Perhaps the present situation can be optimistically summarized by the observation that the optical model picture is at least internally consistent in the following sense: the model certainly demands as a necessary condition that the localization length (i.e., the wavelength) of the incident particle be much less than the distance between collisions. Assuming the validity of the model, one computes this latter distance (i.e., the mean free path $1/K$) and finds it to be large and to satisfy the necessary condition.

2(d) Special Situation at Low Energies

In the beginning of this section, we mentioned that the absorption and shape-elastic cross sections differ from the usual nonelastic and elastic cross sections by an amount equal to the compound-elastic cross section, thus[2]

$$\sigma_{\rm el}=\sigma_{\rm sh.el}+\sigma_{\rm comp.el}$$

$$\sigma_{\rm nonel}=\sigma_{\rm abs}-\sigma_{\rm comp.el}$$

At energies above a few Mev, $\sigma_{\rm comp.el}$ is reduced to a negligible amount because of the competition of this process with all the many possible decay modes of the compound system. In the case of low energies, when this is not the case, the first thing to notice is that the complex potential model is to be associated with the prediction of $\sigma_{\rm sh.el}$ and $\sigma_{\rm abs}$ and *not* with $\sigma_{\rm el}$ and $\sigma_{\rm nonel}$.‖ (This follows from the physical meaning of the complex potential in which "absorption" signifies an energy-exchanging collision of the incident nucleon with a target nucleon.) Thus, it might be thought that, in order to compare the observed $\sigma_{\rm el}$ and $\sigma_{\rm nonel}$ with the

[9] R. Jastrow, Phys. Rev. **82**, 261 (1951); N. C. Francis and K. M. Watson, Phys. Rev. **92**, 291 (1953).

§ Attempts have been made to compute k_1 by hypothesizing a nucleon-nucleon potential and evaluating the forward scattering amplitude in Born approximation. For example, A. Kind and L. Jess (to be published). See also reference 15.

[10] A. M. Lane and C. F. Wandel, Phys. Rev. **98**, 1524 (1955); E. Clementel and C. Villi, Nuovo cimento **10**, 176 (1955).

[11] K. A. Brueckner, Phys. Rev. **103**, 172 (1956).

‖ The complex potential model makes no specification at all about $\sigma_{\rm comp.el}$, and so does not predict $\sigma_{\rm el}$ and $\sigma_{\rm nonel}$. To determine $\sigma_{\rm comp.el}$, one has to introduce a complete detailed theory of reactions as in Sec. 3 (see reference 13).

complex potential model, one must subtract and add $\sigma_{\text{comp.el}}$ to these quantities. This is correct, but this is not the only modification to be made at low energies. There is a second one, which appears at first sight to be quite different, but really is also due to the non-negligible value of $\sigma_{\text{comp.el}}$. This second modification is that cross sections must always be averaged over the fine-structure resonances before comparing with the complex potential. The complex potential model cannot retain its usual physical meaning when applied to sharply defined energies, if only because the values of V and W must be allowed to fluctuate to follow the resonances. There is a good physical reason[1] why the complex potential cannot be applied to vanishingly small energy intervals, but is restricted to energy intervals including many levels. If the split of the total cross section into $\sigma_{\text{sh.el}}$ and σ_{abs}, as given by the complex potential, is to be a valid physical one, there must be no interference between the compound-elastic scattering and the diffraction-elastic scattering, i.e. the two must be completely incoherent. On a time scale, this means that the incident wave packet must be sufficiently short in time that the shape-scattered wave is well clear of the nucleus before the compound-elastic wave appears at the nuclear surface. The time for the latter is $2\pi\hbar/D$, where D is the mean fine-structure level spacing. Application of the uncertainty relation $\Delta t\Delta E \gtrsim h$ shows that, for no interference, the incident energy must be spread over many resonances.

2(e) "Strength Function" Representation

For some years there has been available a proper general theory of reactions formulated in terms of fine structure resonance levels.[12] This theory is a quantum mechanical theory from the beginning, and so the use of it avoids the awkward questions of interpretation that arise with the use of the complex potential [see Sec. 2(c)]. It is to be expected that this theory would be more formal and forbidding. Nevertheless, it is possible to use the fine structure theory to define absorption and shape-elastic cross sections which are identical in physical meaning to those we have already introduced.[2,13] (The essential point of comparison is that the absorption cross section includes that part of the elastic cross section that is incoherent with the shape-elastic cross section.) These cross sections are given by the formulas of Sec. 2(a) with S_l replaced by its average $\langle S_l\rangle$ taken over many resonances. S_l is related in a simple way to the logarithmic derivative f_l of the wave function[14] evaluated at the nuclear surface, i.e., at the interaction radius $r=a$. It can be shown[13,15] that $\langle S_l\rangle$ is given by the same relation with f_l replaced by a quantity $(R_\infty^{(l)}+i\pi s_R^{(l)})^{-1}$ where $R_\infty^{(l)}$ and $s_R^{(l)}$ are two parameters defined statistically in terms of the widths and energies of the fine structure resonances.¶ s_R is the "strength function," defined as $\langle\gamma_\lambda\rangle^2/D$; i.e., the mean reduced with γ_λ^2 of resonance levels λ, divided by the mean level spacing of these levels. R_∞ is essentially the deviation of the potential scattering phase angle from that due to a hard sphere of the size of the target nucleus of interaction radius $r=a$.

Since the complex potential and the strength function theories both specify the same cross sections, the condition that they give the same cross sections gives relations between the parameters of the two theories. Thus, since we know that the complex potential fits the data with certain values of V and W, we can avoid reanalyzing the data with the strength function theory by using these relations. The condition that the two theories give the same cross sections is equivalent to the condition that they give the same effective value of f. This quantity is $(R_\infty+i\pi s_R)^{-1}$ in the case of the strength function theory. In the case of a square complex potential $V+iW$, with single particle energies E_p (say), f may be written as[13,15]

$$\frac{1}{f}=\sum_p \frac{\hbar^2/Ma^2}{E_p-E-iW}$$

and so, taking the imaginary part

$$s_R=\frac{1}{\pi}\sum_p \frac{(\hbar^2/Ma^2)W}{(E_p-E)^2+W^2}.$$

Thus the form of s_R needed to fit the observed data is a sum of Cauchy terms, each one centered on a single particle energy E_p. It is of considerable interest to try to predict this form of s_R from the theory of nuclear structure. (In particular, the numerical prediction of the width $2W$ of the peaks is especially important.[14]) The starting point for such an attempt is the relation[13]:

$$R_\infty+i\pi s_R=R(E+i\delta),$$

where $R(E)=\sum_\lambda \gamma_\lambda^2/E_\lambda-E$ is the "R function" of the fine structure theory[12] and where δ is an energy much larger than the mean level spacing. The presence of δ removes the singularities in R and makes it smooth. Eventually δ must be allowed to go to zero. Now we may write[15]

$$\gamma_\lambda^2\left(\frac{\hbar^2}{Ma^2}\right)^{-1}=(\sum_p \langle\lambda|p0\rangle)^2\approx\sum_p |\langle\lambda|p0\rangle|^2$$

[12] E. P. Wigner and L. Eisenbud, Phys. Rev. **72**, 29 (1947); P. L. Kapur and R. E. Peierls, Proc. Roy. Soc. (London) **A166**, 277 (1937); G. Breit, Phys. Rev. **69**, 472 (1946).

[13] E. P. Wigner, Proc. Cambridge Phil. Soc. **47**, 790 (1951); Ann. Math. **53**, 36 (1951); **55**, 7 (1952); R. G. Thomas, Phys. Rev. **97**, 224 (1955).

[14] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952).

[15] Lane, Thomas, and Wigner, Phys. Rev. **98**, 693 (1955).

¶ Since the fine-structure theory is general, there must be as many parameters in it as there are in the set of S_l. For each (complex) S_l, there correspond the two parameters $R_\infty^{(l)}$ and $S_R^{(l)}$. From this point in the discussion, we drop the label l.

where $\langle\lambda|$ is the resonance state λ and $|p0\rangle$ is the product state of the single particle state p and the ground state $|0\rangle$ of the target nucleus. The last inequality follows on assuming random signs for the amplitudes $\langle\lambda|p0\rangle$. Thus

$$R(E+i\delta)=\left(\frac{\hbar^2}{Ma^2}\right)\sum_p\sum_\lambda\frac{\langle\lambda|p0\rangle^2}{E_\lambda-E-i\delta}$$
$$=\left(\frac{\hbar^2}{Ma^2}\right)\sum_p\left\langle p0\left|\frac{1}{H-E-i\delta}\right|p0\right\rangle.$$

The matrix elements can now be developed in a perturbation series[16] in terms of $H-H_0$ where H_0 is the Hamiltonian for the state $\langle p0|$. Each term in the series is a sum over processes in which the incident particle is scattered through several states before returning to its original state p. One can argue that, if the matrix elements of $H-H_0$ connect states separated by energies up to some energy $\mathcal{E}$, then under reasonable conditions on $\mathcal{E}$, one process contributes more than all the others.[16] This is the process in which the incident particle is returned to its original state by each alternate scattering. The contributions of such processes to all terms in the perturbation expansion can be summed in closed form to give just the required Cauchy form. In this form,

$$W(E)=\pi\frac{\mathrm{Av}|\langle p0|(H-H_0)|p't\rangle|^2}{D(E)},$$

where $|p't\rangle$ labels those product states with energies near E, and the average is taken over these states. $D(E)$ is the mean level spacing at energy E.

This calculation seems to be a more precise derivation of the same result obtained from more intuitive considerations[17] in which one calculates the energy shift in the state $|p0\rangle$ due to the interaction $(H-H_0)$. The previous approximation in which the incident particle is returned to its original state by each alternate scattering corresponds to the Tamm-Dancoff method of radiation theory in which single photons are allowed to be created and destroyed. The energy shift from this theory contains an imaginary part just like the W given by the above formula. Although quantitative numbers are unreliable, the qualitative values of W from this formula are in agreement with experiment.[17] In particular, the Pauli principle is decisive in reducing the value of W at low energies.

The strength function theory does not depend in any way on the complex potential model and the calculation of cross sections just described may be said to give a direct description of these cross sections without any reference to the complex potential. However the latter model is a very simple physical model that is known to fit the data. Therefore a better description of the strength function theory would perhaps be that it gives a proper quantum mechanical basis for the complex potential model. In this connection we note that the foregoing expression for $W(E)$ can be reduced to the form

$$W(E)=\frac{\hbar^2}{2M}\rho\langle\sigma_{\mathrm{Born}}\rangle(k+k_1)$$

which is very similar to that implied by the optical theory in Sec. 2(c). The only difference between the two expressions is that, where the optical formula contains the actual nucleon-nucleon total cross sections, the other contains Born approximation expressions for these quantities. It is not clear at present whether this difference is a result of approximations made in the use of the strength function theory (the selection of leading terms) or whether it implies that the classical optical model ought to be modified to the extent of this difference. At high energies the Born approximation reproduces the observed total cross sections and the two expressions become the same.

3. INDIVIDUAL CROSS SECTIONS

So far, this review has only mentioned the prediction of the cross sections for shape-elastic scattering and absorption of incident particles. Now the attempts to decompose the absorption cross section into its component parts (the individual cross sections) will be described.

3(a) Statistical Theories of Reactions

In general, a given bombardment may result in the production of very many different products. In the course of producing these, a very large number of nucleon-nucleon collisions occur and one must resort to some type of statistical treatment. Until a few years ago, it was assumed that, for all but high energies (say > 50 Mev), the incident nucleon is absorbed into the target nucleus as soon as it crosses the surface and forms a long-lived compound state. The qualitative features of this state have been described so many times[14] that there is no need to repeat them here. The prediction of any individual cross section on this model has the form of a product of (i.e. the geometrical cross section πa^2 at sufficiently high energies) and the probability of decay into the particular final products specified by the cross section. This probability is computed on a statistical basis assuming that the intrinsic probabilities for decomposition into all individual final products are the same. One prediction of this theory is that the spectrum of emitted particles of a given type (say protons) should be essentially Maxwellian ($e^{-E/T}$) except for a low energy cutoff due to barrier penetration.

An essential condition for the validity of this "strong absorption" picture is that the mean free path of the incident nucleon against collision be much smaller than

[16] C. Bloch, Nuclear Phys. (to be published).
[17] M. Cini and S. Fubini, Nuovo cimento 10, 75 (1955).

the dimensions of the target nucleus. As we have already seen, the use of the complex potential model in analyzing observed cross sections shows that this condition is *not* satisfied. The empirical mean free paths obtained in this way are of the order of the nuclear radius with the consequence that the incident particle, after its first collision, still has an appreciable probability of escaping from the nucleus. It was fortunate that this information from the complex potential model appeared when it did, because there was growing experimental evidence against certain aspects of the "strong absorption" picture,[14] and it was not known at what point the theory was at fault. It is fairly straightforward to modify the "strong absorption" model to allow for the long mean free paths. Using a classical treatment of particle-particle collisions, one imagines[18] the incident particle to collide with a target nucleon, then the two nucleons collide with other nucleons and so on, so that a cascade develops. After each collision there is a certain small probability that a nucleon will escape directly. For a reasonably high energy, say 50 Mev, there are a very large number of collisions, so that it is almost certain that one or two nucleons will escape directly. The end of what might be called the "cascade stage" of the reaction comes when all individual nucleons have less than the requisite energy for escaping out of their mean potential well. At this point we can imagine that a true compound nucleus is formed and that this lives for a relatively long time before eventually decaying.** ††

The method of calculation for the "cascade stage" of the reaction depends on what one wishes to predict. For instance, it is clear that the products of the first collision will have a strong memory of the incident direction and energy. The products of second collisions will have less memory i.e. they tend to appear at wider angles and lower energies. With each collision, memory is progressively lost so that particles from the sixth or seventh collisions emerge nearly isotropically with low energies. If one observes the high-energy particles in the forward direction, it follows that these are mostly due to first collisions. It is quite easy to make an analytical calculation[19] of the energy and angular spectra of these nucleons (using the "impulse approximation"). This can be extended to the second collision,[19] but the method becomes tedious for higher order collisions. This means, in particular, that it is not useful for predicting the excitation energy left in the compound nucleus. For following these higher order collisions, one usually takes resort to the type of mathematics known as Monte Carlo calculations, in which one follows many cascades through the nucleus and builds up statistical results.[18] Such calculations have been done for an incident nucleon energy of 100 Mev on a nucleus of mass 64, under the assumption that a nucleon cannot escape after its energy falls below the maximum Fermi gas kinetic energy in the target nucleus plus 8 Mev.[20] Out of 200 particles, 30 pass through the nucleus without collision. 4 lead to the ejection of 4 particles, 42 to the ejection of 3 particles, 82 to the ejection of 2 particles, 43 to the ejection of one particle, and 1 to capture with no ejected particles. In all cases when particles are ejected, the consequent compound nucleus has a mean excitation energy of about 25 to 40 Mev.

Calculations at lower incident energies[21] indicate a transition between 30 Mev and 20 Mev from a situation in which each capture usually leads to a direct ejection to a situation where each capture does not usually lead to a direct ejection. Such a transition is expected from the following considerations. At lower energies, there are three tendencies. One of these is the increase in the nucleon mean free path, which tends to increase the direct ejection process. On the other hand, the number of collisions in which direct ejection is energetically possible falls sharply with decreasing energy. Finally surface reflection effects become important[21,22] so that, even if a particle reaches the surface with enough energy to escape, it is probable that it will be reflected back into the nucleus. The latter two effects easily compensate the first one, so that only a few direct ejection processes occur below 20 Mev or so incident energy on a medium sized nucleus.

3(b) Surface Reactions at Energies $\lesssim$20 Mev

Certain low-energy ($E<25$ Mev) reactions have been given an attention quite out of proportion to their small cross sections. These reactions can be generally called "surface reactions" because they only occur when the bombarding particles pass through the edge of the target nucleus. The usual direct ejection cross sections fall to negligible values at low bombarding energies, largely because of surface reflection. If the incident particle interacts with a target particle while it is actually in the surface, then this reflection effect is not so important. On the other hand, the proba-

[18] R. Serber, Phys. Rev. **72**, 1114 (1947); M. L. Goldberger, Phys. Rev. **74**, 1269 (1948).

** It may happen that so much energy is left in restricted regions of the compound nucleus that anomalous effects such as "spot boiling" occur. Theoretical work on this phenomenon indicates that it should become important when local nuclear temperatures exceed 7 Mev [see A. Kind and G. Patergnani, Nuovo cimento **11**, 106 (1954)].

†† The decay of the compound nucleus is expected to be essentially that pictured by the earlier "strong absorption" theory with particle spectra of Maxwellian type. The fact that the nucleon mean free path is longer than implied by this theory is only expected to affect the spectra to the extent of imposing certain mild modulations on them. These come from the fact that the transmission probability of a nucleon escaping across the nuclear surface is modulated because of its motion in the average nuclear potential.

[19] F. Mandl and T. H. R. Skyrme, Proc. Phys. Soc. (London) **A65**, 107 (1952).

[20] J. W. Meadows, Phys. Rev. **98**, 744 (1955).

[21] Hayakawa, Kawai, and Kikuchi, Progr. Theoret. Phys. **13**, 415 (1955).

[22] L. R. B. Elton and L. C. Gomes, Phys. Rev. **105**, 1027 (1957); G. Brown and H. Muirhead, Phil. Mag. (to be published).

bility of finding a particle in the surface relative to finding it in the volume is small so the cross sections for these processes are small. The reason why they have excited so much attention is that they can be made to show up very dramatically under suitable experimental conditions. Because of the direct nature of these processes, they tend to produce particles of high energies‡‡ directed in the forward hemisphere. This is in contrast to the compound nucleus process which tends to produce particles of low energy distributed over all angles.

The two main types of surface reaction initiated by incident nucleons are the "surface knock-out"[23] (in which inelastic nucleons appear) and the "surface pickup" (in which deuterons appear). In other words, the interaction of the incident nucleon with a target nucleon may either lead to the escape of one of these two nucleons, or to the escape of both joined together as a deuteron. The so-called "stripping" processes[24] are simply the time inverses of the pickup processes. It is to be expected that the only significant contributions to surface reactions come from the first collisions so that, in calculations, one does not have to follow cascades using the classical statistical method. Instead it is simple to make a quantum mechanical calculation in which the initial and final nuclei are represented as proper quantum states. For a first orientation one can carry out a Born approximation calculation in which the initial and final particle states are represented by plane waves. However this leads to an unwanted and spurious volume contribution[24] unless one simply ignores the part of the Born approximation matrix element from inside the nuclear volume. A better and more consistent treatment consists in representing the initial and final particle states by distorted waves corresponding to some complex potential.[25] §§ In this way, the volume contribution is naturally made small as a result of surface reflection and the absorption inside the potential.

The main qualitative features of surface reactions leading to definite final states are that the details of the angular distribution are sensitive to the amount of angular momentum transferred in the reaction, while the absolute cross section depends on the probability of finding nucleons in the surface of the target nucleus. Consequently, a proper analysis of the data enables one to derive useful information about the spins of states and their reduced widths[26] for break-up into a nucleon and a daughter state.

In general, the cross section for the production of any particular final state at energies $\lesssim 20$ Mev contains two contributions, one from the (delayed) compound nucleus process and one from the (direct) surface reaction. If the incident energy is spread over a range larger than the resonance widths, it can be argued that the two contributions are incoherent. For more sharply defined energies, the two contributions may interfere coherently. As yet, no satisfactory theory that includes both processes in a natural way has been formulated, although some progress has been made towards such a theory.[27]

4. GENERALIZATIONS AND CONCLUDING REMARKS

Hitherto we have consistently retained two restrictions specified in the opening paragraphs, *viz.*, we have taken the bombarding particle to be a nucleon, and assumed the target nucleus to be spherical. We end by removing these restrictions to some extent.

Bombardment with Composite Particles

The scattering and absorption cross sections for bombardments with deuterons, alphas, and heavy ions have been analyzed with a complex potential just like those for nucleon bombardment.[28] One can find complex potentials that fit the data rather well, but the physical interpretation of these potentials is not obvious. One may guess that the absorption coefficient K should be considerably larger than that for nucleon bombardment at corresponding energies since the interaction of any single one of the incoming nucleons with the target will be enough to cause loss of energy. This speculation is confirmed by observation. The experimental values of the mean free paths for alphas and deuterons[28] are much less than nuclear radii and the mean free path for a nitrogen ion is only a few percent of a nuclear radius.[29]

Nonspherical Target Nuclei

It is now known that many target nuclei have a spheroidal deformation in their shape. Although these deformations are not usually large enough to invalidate our discussion of reactions with spherical nuclei, they do give rise to an important new type of process. This is a special and interesting process which cannot be labeled as diffraction or absorption in the senses in

‡‡ The energy spectrum actually depends on a second factor besides the kinematical, momentum conserving, factor which favors high energies. This second factor is the "parentage overlap" between the initial and final nuclear states. Generally speaking, the shell-model predicts that this overlap should decrease with increasing energy separation between the initial and final states. [See A. M. Lane and D. H. Wilkinson, Phys. Rev. **97**, 1199 (1955)]. This factor is somewhat compensated by the fact that the exponential tails of the nucleon wave function are more extended for higher excited states, but, on the whole, it appears that this factor also favors high energies.

[23] Austern, Butler, and McManus, Phys. Rev. **92**, 350 (1953).

[24] S. T. Butler, Proc. Roy. Soc. (London) **A208**, 559 (1951); P. B. Daitch and J. B. French, Phys. Rev. **87**, 900 (1951); Bhatia, Huang, Huby, and Newns, Phil. Mag. **43**, 485 (1952).

[25] J. Horowitz and A. M. L. Messiah, J. phys. radium **14**, 695 (1953); W. Tobocman and M. M. Kalos, Phys. Rev. **97**, 132 (1955).

§§ This procedure is only appropriate if the incident energy covers several states of the compound nucleus. Otherwise the complex potential model loses its meaning [see Sec. 2(d)].

[26] J. E. Bowcock, Proc. Phys. Soc. (London) **A68**, 512 (1955).

[27] R. G. Thomas, Phys. Rev. **100**, 25 (1955); C. Bloch (to be published).

[28] C. E. Porter, Phys. Rev. **99**, 1400 (1955).

[29] C. E. Porter, Phys. Rev. **103**, 674 (1956).

which we have used the terms. It consists in the target nucleus being set into rotational motion as a result of the interaction of the incident nucleon with the deformed potential field of the nucleus. This implies the excitation of rotational states and a consequent loss of energy by the nucleon. In principle, the problem can be handled quantitatively by solving the one-body Schrödinger equation for a nucleon interacting with a deformed one-body potential associated with a body possessing a certain moment of inertia. In practice however there are analytical problems in solving this equation, and, in the reported treatments,[30] various approximations have been made in order to estimate cross sections for the processes.

4(c) Concluding Remarks

During the last ten years, several new and important nuclear reaction phenomena have been established experimentally. These include the mountain resonances in total nucleon cross sections, surface reactions, and knock-out processes. In the present review, we have discussed how the earlier models of nuclear reactions have been revised to give some account of these phenomena. The basic essential revision has been to allow the nucleon mean free path to be comparable with nuclear dimensions, instead of assuming it to be very small as in the earlier "strong absorption" model. The result of this revision is a very satisfying synthesis between the ideas of the Bohr model of the compound nucleus and those of the nuclear shell model. The revised model retains many useful features of the "strong absorption" model. For instance, the theory of the compound nucleus that was part of the older picture is still widely applicable within the framework of the revised model.

There is such a wide measure of agreement between the present model and experiment that it seems unlikely that any further substantial revision will be necessary in the future. In such circumstances, it is natural that considerable theoretical attention[16,17] should have been given to the problem of trying to understand the validity of the model from a fundamental point of view. There are many interesting but difficult questions which arise in such attempts. Most of these however belong to the study of nuclear structure rather than of nuclear reactions.

[30] C. F. Wandel, thesis, Copenhagen (1953) (unpublished); D. M. Brink, Proc. Phys. Soc. (London) **A68**, 994 (1955); S. Hayahawa and S. Yoshida, Proc. Phys. Soc. (London) **A68**, 656 (1955); Progr. Theoret. Phys. **14**, 1 (1955).