Collective Motion in the Nucleus

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FOR many purposes it is possible to consider each nucleon as moving, almost independently, in an average nuclear field generated by all the other nucleons. The existence of such independent particle motion in the nucleus gives rise to a nuclear shell structure which is similar in many respects to the familiar atomic shell structure. There is, however, at least one very important difference between the field in which the electrons move and that in the nucleus. In the atom the field is dominated by the attraction of the heavy central nucleus. The fact that the nuclear field is generated entirely by the nucleons themselves implies that the nucleus will be much less stable against oscillations in shape. Already in the early considerations about nuclear structure, it was pointed out by Bohr and Kalckar[1] that this relative instability in shape should imply the existence of low-frequency shape oscillations as an important mode in the nuclear dynamics. The understanding of most of the low energy nuclear properties requires an analysis of the interplay between the collective shape oscillations and the independent particle degrees of freedom in the nucleus.[2]

In general, the period for the shape oscillations of lowest frequency is considerably longer than the period for independent particle motion in the nucleus. One may thus employ an adiabatic approximation in solving the equations of motion. That is, we can proceed in two steps: first we solve for the intrinsic motion subject to a constraint which specifies a particular shape and orientation. In practice we may attempt to satisfy this constraint approximately by solving for the motion of nucleons in a potential which possesses the required shape and orientation. The eigenvalues, $E_i(\alpha)$, of this first part of the problem are functions of the parameters, α, which define the nuclear shape. These functions are called the potential energy surfaces. They play a role very similar to the potential energy surfaces in molecules. Next, we consider the additional energy which the system acquires when we let α vary slowly with time. Expanding the energy in a power series in $\dot{\alpha}$ we obtain the collective Hamiltonian

$$H=E_i(\alpha)+\tfrac{1}{2}B_i(\alpha)\dot{\alpha}^2. \quad (1)$$

The neglect of the nondiagonal couplings proportional to $\dot{\alpha}$ is justified only if there are no degeneracies or near-degeneracies in the spectrum of the intrinsic motion. In general it is only the even-even nuclei that are free from such degeneracies. I shall mainly confine myself in this talk to these even-even nuclei for which the above especially simple collective Hamiltonian is adequate. Unfortunately, I shall not have time, within the scope of this talk, to discuss the many very interesting effects which arise in the odd-A nuclei.

In the early discussions of the nuclear collective properties, an attempt was made to estimate the functions $E(\alpha)$ and $B(\alpha)$ by employing a liquid drop model. Thus, $E(\alpha)$ would depend on the nuclear "surface tension" which in turn could be related to the observed nuclear binding energies, and B might be calculated from the kinetic energy associated with surface oscillations of an irrotational fluid.

We now know, however, that the functions $E(\alpha)$ and $B(\alpha)$are influenced in an essential manner by the nuclear shell structure. Let us consider, for example, the function $E_0(\alpha)$ which represents the lowest potential energy surface in an even-even nucleus. Beginning with a closed shell nucleus, we know that the system possesses an especially great binding energy which is associated with the degeneracies of the nucleon orbits in a spherical potential. Any attempt to distort the system from the spherical shape is very costly in energy, and the potential energy curve for this configuration is thus stable at the spherical shape and rises very steeply as we go away from this shape (curve a, Fig. 1).

Adding additional nucleons beyond the closed shell, the potential energy curve will be a result of the competition between the particles outside closed shells, which exert a polarizing effect on the nuclear shape, and the particles in closed shells which prefer the spherical shape. The existence of this important polarizing effect of the individual nucleons outside of closed shells was first recognized by Rainwater.[3] He pointed out that since the orbital motion of a single nucleon is mainly confined to a plane perpendicular to the direction of its angular momentum vector, the particle will exert a centrifugal force in this plane tending to distort the nucleus.

The polarizing tendency of the particles outside of closed shells is modified in an important way by the residual forces which are not included in the average nuclear field. These tend to couple the nucleons into a state of definite total angular momentum J. Mayer,

[1] N. Bohr and F. Kalckar, Mat.Fys.Medd.Dan.Vid.Selsk. **14**, no. 10 (1937).

[2] A. Bohr, Mat.Fys.Medd.Dan.Vid.Selsk. **26**, No. 14 (1952). D. L. Hill and J. A. Wheeler, Phys. Rev. **89**, 1102 (1953). A. Bohr and B. R. Mottelson, Mat. Fys.Medd.Dan.Vid.Selsk. **27**, No. 16 (1953). The present talk is largely based on Chapter V of the review article by Alder, Bohr, Huus, Mottelson, and Winther, Revs. Modern Phys. **28**, 432 (1956). This last reference also contains a more complete bibliography and compilation of relevant experimental data.

[3] J. Rainwater, Phys. Rev. **79**, 432 (1950).

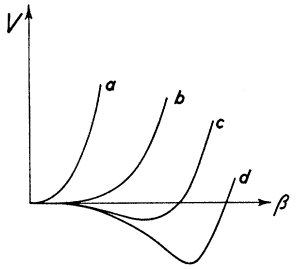

FIG. 1. Potential energy curves for the lowest configurations in even-even nuclei.[2]

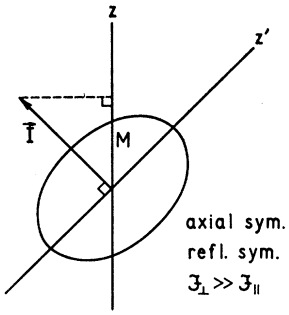

FIG. 2. Coupling scheme for the lowest rotational band in an even-even nucleus.

Flowers, Racah, and others have shown that the short range attractive forces, which act in the nucleus, favor a state of $J=0$ for an even-even nucleus. Thus, in a spherical potential ($\alpha=0$), the particles in an even-even nucleus will have $J=0$, a spherical density distribution, and no net polarizing effect on the nuclear shape. However, when we consider intrinsic motion in a nonspherical field, J is no longer a constant of the motion, the particles become decoupled from each other and exert their polarizing effect.

Thus if we add a few particles beyond a closed shell we obtain a potential energy curve which remains stable at the spherical shape, as a consequence of the residual interactions, but which is much softer against distortion away from the spherical shape as a consequence of the polarizing tendencies of the added nucleons (curve b, Fig. 1). Continuing to add still more nucleons beyond the closed shell, the spherical shape continues to become less and less stable, until, if there are enough nucleons outside of closed shells, the spherical shape may become unstable and the lowest energy of the nucleus is then associated with a nonspherical shape (curve c, Fig. 1). With still more nucleons added, the magnitude of the equilibrium deformation increases and the stability of the system with respect to displacements from the deformed equilibrium also increases (curve d, Fig. 1.).

I would like now to discuss the characteristic collective excitation spectra which are associated with these different types of potential energy surfaces.

The situation is most simple for nuclei with nonspherical equilibrium shapes, so I shall start by discussing the spectra of the nuclei in the regions farthest from closed shells. The regions in which nonspherical nuclei have been observed are roughly defined by $A>24$, $150<A<190$, and $A>220$; these are just the regions farthest from the closed shell configurations. Such systems, possessing a nonspherical shape, exhibit a very low energy collective motion which simply corresponds to the reorientation of the nucleus in space with preservation of the shape and intrinsic structure. Indeed the nuclei in the above regions do exhibit rotational spectra which for the lowest intrinsic state in the even-even nuclei have the form

$$E_{\text{rot}}=\frac{\hbar^2}{2\mathfrak{I}}I(I+1) \quad I=0,2,4,6,\cdots \text{even parity.} \quad (2)$$

The nature of this rotational spectrum tells us quite a bit about the symmetry properties of the nuclear shape. Thus the dependence of the nuclear energy on $I(I+1)$ implies that we have to do with the rotations of a symmetrical top; the fact that we most often observe only the even spin and even parity states in these spectra[4] implies that usually the nuclear shape is symmetric with respect to reflection in a plane perpendicular to the symmetry axis and passing through the center of the nucleus; and finally, the absence of other states in the rotational band which would correspond to rotation about the symmetry axis implies that the effective moment of inertia for such rotations is at least an order of magnitude smaller than that for rotations about an axis perpendicular to the nuclear symmetry axis. The nuclear coupling scheme is sketched in Fig. 2. These symmetry properties of the nuclear shape are related to the nuclear potential energy surfaces. It has indeed been shown that the nuclear shell structure will almost always prefer such axially symmetric shapes. The smallness of $\mathfrak{I}_\parallel$ is a special case of a more general result which we shall discuss further a little later.

The experimental evidence on the rotational spectra of the even-even nuclei is shown in Fig. 3. This gives the ratio of the measured excitation energies of the excited states to the energy observed for the first excited, 2+, states in these nuclei. According to the above expression we would expect the second excited state to have $I=4+$ and an energy $3\frac{1}{3}$ times that of the 2+ state, the third and fourth states should have $I=6+$ and 8+, and energy respectively, 7 and 12 times the 2+ energy. The experimental data agree quite well with this interpretation; in the regions of the largest deformations the energy ratio of the 4+ to the 2+ states agree with the above expression within a few tenths of a percent. The deviations are always negative, increase with increasing I, and with the approach to the transition regions where the deformed shape is no longer a stable equilibrium. Such deviations can be interpreted in terms of centrifugal distortion, which slightly changes the nuclear shape as a function of the rotational frequency.

[4] In some cases, especially around $A=224$, odd-spin odd-parity rotational bands have been observed. It has been suggested by R. Christy that these be interpreted in terms of a deviation of the nuclear shape from reflection symmetry.

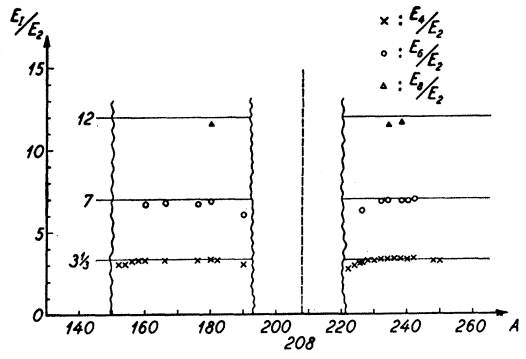

FIG. 3. Experimentally observed energy ratios for excited states in rotational bands in even-even nuclei.[2]

Besides the simple energy expression governing the rotational spectrum, there are also intensity rules that govern the relative strength of β or γ transitions to different members of a rotational band. These intensity rules follow directly from the geometric nature of the rotation, in the same way that one obtains intensity rules for the relative strength of the transitions in a fine structure or a hyperfine structure multiplet. The experimental evidence supports these intensity rules within the experimental accuracy, which is about 10% in the best studied cases.

While the relative energies and intensities in a rotational band are geometrical quantities which are independent of the detailed nature of the intrinsic motion, the effective moment of inertia, $\Im$, appearing in the rotational energy, is a dynamical quantity which depends in an essential way on the intrinsic structure of the system. We may attempt to estimate $\Im$ by employing a method first suggested in this connection by Inglis.[5] He pointed out that, since $\Im$ represents the additional kinetic energy which the nucleons must have in order to follow the rotation of the nucleus, we can estimate $\Im$ by considering the motion of particles in a rotating nuclear field. Due to the equivalence of a rotation to an external magnetic field, this problem is

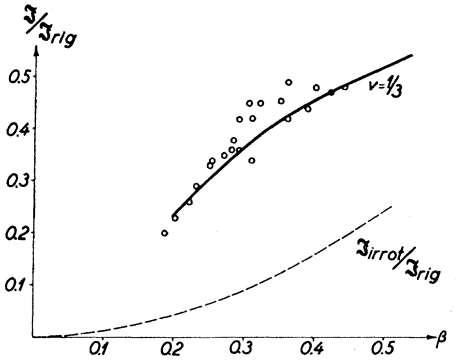

FIG. 4. The magnitudes of the observed moments of inertia[6] are plotted as a function of the deformation parameter β.

[5] D. R. Inglis, Phys. Rev. **96**, 1059 (1954).

very similar to the calculation of the diamagnetism of a gas of charged particles.

If we first consider this problem in the limit where we may use classical mechanics we find that the moment of inertia must have the value $\Im_{rig}$ corresponding to a rigid rotation of the nuclear density distribution. This result corresponds with the statement that a classical electron gas has no diamagnetism, as was first shown by N. Bohr and later independently by van Leeuwen. We can see this result most easily by looking at the system in the rotating coordinate system in which the potential remains constant in time, but where we must add the appropriate Coriolis forces to the Hamiltonian. However, the Coriolis forces do not change the isotropy of the velocity distribution. There is thus no net current in the rotating coordinate system; in the space fixed system the whole nucleus is rotating as a rigid body. This result is independent of any details of the intrinsic structure such as the mean free path of the particles in the potential.

When we consider the actual nucleus we find that there is a very important quantum-mechanical correction to the above result, which is a consequence of the tendency of the intrinsic structure to go into a state of $J=0$ as the nuclear deformation, α, goes to zero. As mentioned previously, this tendency is a consequence of the residual forces in the nucleus. Since a state with $J=0$ has a spherical density distribution, its energy is independent of the orientation of the nuclear field, and thus it does not contribute to the moment of inertia. Even for the appreciable eccentricities encountered in the most deformed nuclei, this single quantum state with $J=0$ constitutes a significant part of the intrinsic state of motion and thus the nuclear moments of inertia are appreciable below the value $\Im_{rig}$ and become smaller as the eccentricity becomes smaller.[6] The experimentally measured moments of inertia are shown in Fig. 4 in units of the moment $\Im_{rig}$. The abscissa is the nuclear eccentricity, β, deduced from the nuclear quadrupole moment determinations. The solid curve is obtained from an extremely simplified model which provides an interpolation between the small and great deformation limits. This interpolation expression depends on the strength of the residual interactions as measured by the parameter v.

As we approach the closed shell configurations the deformations get smaller, the moments of inertia get smaller, and the accuracy of the simple rotational energy spectrum gets poorer. Finally, we reach a point where the nuclear potential energy surface no longer has a nonspherical equilibrium and then the nuclear coupling scheme changes in an essential manner. The nucleus now has a spherical equilibrium shape and the system cannot exhibit simple rotational motion; the low-frequency collective degrees of freedom now correspond to

[6] For a more detailed discussion of the nuclear moments of inertia, see A. Bohr and B. R. Mottelson, Mat.Fys.Medd.Dan. Vid.Selsk. **30**, No. 1 (1955).

TABLE I. Data on nuclear vibrations around a spherical equilibrium shape. The table[2] lists all cases in which the second excited state is known to have spin and parity 2+ (the first excited states are all also known to have 2+ character). The second, third, and fourth columns list, respectively, the energies of the first excited state, the second excited state, and the ratio of these two energies. The fifth column gives the ratio of the observed cross section for Coulomb excitation to that expected for a single proton transition. The sixth column gives the ratio of $M1$ to $E2$ radiation in the cascade transition between the second excited state and the first. The last column gives the ratio of the reduced transition probabilities for $E2$ radiation for the crossover decay to ground and the cascade decay of the second excited state.

Nucleus	E_2 (Mev)	E'_2 (Mev)	E'_2/E_2	$\frac{B(E2;0\to2)}{B_{sp}(E2)}$	$(M1/E2)_{2'\to2}$	$\frac{B(E2;2'\to0)}{B(E2;2'\to2)}$
$_{26}Fe^{58}$	0.81	1.62	2.00		0.2	0.01
$_{28}Ni^{60}$	1.33	2.18	1.64	17		(3×10^{-3})
$_{30}Zn^{64}$	1.00	2.27	2.27	15		(0.1)
Zn^{66}	1.05	2.40	2.29	11		(0.05)
$_{34}Se^{76}$	0.55	1.19	2.17	44	~1	0.1
$_{36}Kr^{82}$	0.77	1.45	1.88			(0.01)
Kr^{84}	0.9	1.9	2.1			>0.1
$_{40}Zr^{92}$	0.93	1.83	1.97			(0.05)
$_{44}Ru^{100}$	0.54	1.36	2.52	22		(0.05)
Ru^{102}	0.47	1.10	2.34	45		(0.15)
$_{52}Te^{122}$	0.57	1.26	2.21	26	0.1	0.01
Te^{126}	0.65	1.40	2.16	17		(0.004)
$_{54}Xe^{126}$	0.39	0.86	2.20			(0.01)
Xe^{128}	0.46	0.99	2.15			(0.01)
$_{78}Pt^{192}$	0.32	0.61	1.90		0.025	0.004
Pt^{194}	0.33	0.62	1.88	50	small	0.01
Pt^{196}	0.35	0.69	1.97	38	0.05	$<4\times10^{-4}$
$_{80}Hg^{198}$	0.41	1.09	2.66	29	0.7	0.04
$_{84}Po^{214}$	0.61	1.38	2.26	13	>2	

quadrupole vibrations around the spherical equilibrium shape. Each quantum of vibration carries two units of angular momentum, since we are dealing with quadrupole oscillations. If we assume small amplitude oscillation, the motion will be approximately harmonic and the excitation spectrum of an even-even nucleus would have the form indicated in Fig. 5. Of course, higher order terms in the collective Hamiltonian will modify the exact equality of the energy spacings and remove the degeneracies.

As was first pointed out by Scharff-Goldhaber and Weneser,[7] the low-lying excitation spectra of the even-even nuclei, outside of the above regions where rotations are observed and excluding the few nuclei immediately adjacent to closed shells, agree very well with this picture. Thus the first excited states all have spin 2 and even parity. The energy of this state varies in a regular manner with A and in the sense suggested by the qualitative considerations about the potential energy curves. The frequency decreases regularly as we go away from closed shells, corresponding to the decreasing stability of the spherical shape caused by the polarizing effect of the nucleons outside of closed shells.

Coulomb excitation of these first excited states has been observed in about fifty nuclei, and in every case the cross section has been appreciably greater than would be expected for the excitation of a single proton; the enhancement ranges between about a factor of ten to fifty. This provides very direct evidence that we are dealing here with an excitation mode that involves the motion of an appreciable number of nucleons. The Coulomb excitation cross sections also increase as we go away from closed shells, reflecting with the expected increase in the amplitude of the oscillations of shape.

The second excited vibrational states have been observed in many cases and their energy is found to vary between 2 and 2.5 times that of the first excited state (with two or three unusual exceptions lying outside this interval). The observed second excited states have in all cases $I=0+$, $2+$, or $4+$. The whole triplet is not usually observed because the states are populated in β or γ decays which obey very strict selection rules on the allowed spin changes.

The vibrational character of the second excited state is especially shown by the γ decay of the second excited $2+$ states. These states may decay by $M1$ or $E2$ radiation to the first excited $2+$ state or by $E2$ radiation to the $0+$ ground state. If all matrix elements were of the order of single particle values, the $M1$ cascade decay would be the strongest by factors of the order of 100. However, the observed $M1$ cascade radiation is usually no stronger than the $E2$ cascade and is sometimes appreciably weaker. This smallness of the $M1$ transition matrix elements follows at once from the fact that the fundamental excitation mode involved is a quad-

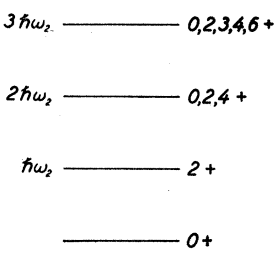

FIG. 5. Excitation spectrum corresponding to quadrupole oscillation of small amplitude. Energies are shown on the left, and spins and parities on the right.

[7] G. Scharff-Goldhaber and J. Weneser, Phys. Rev. 98, 212 (1955).

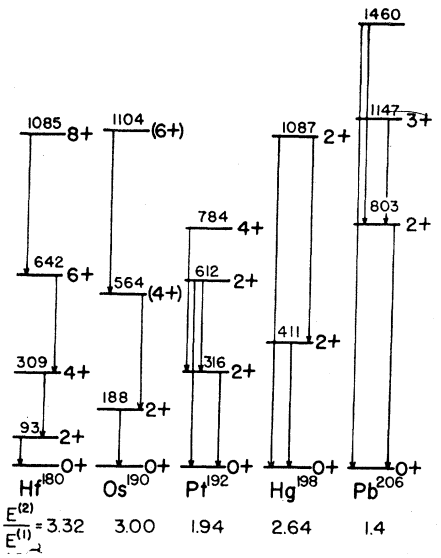

FIG. 6. Representative low-energy excitation spectra of even-even nuclei in the region $170<A<208$. The ratio of the energy of the second excited state, $E^{(2)}$, to that of the first excited state, $E^{(1)}$, is shown under each level scheme. In the region of non-spherical nuclei the limiting expression (2) predicts that this ratio should be $3\frac{1}{3}$; in the region where the spherical shape is stable, the assumption of approximately harmonic vibration implies that this ratio should be close to 2 (see Fig. 5).

rupole motion, even though $\Delta I=0$, and thus in this description the $M1$ matrix element vanishes.

The reduced $E2$ transition probabilities for the cascade and crossover decay of the second excited 2+ states are observed to differ by an appreciable factor. The reduced transition probability for the cascade decay is observed to range from 10 to 1000 times greater than that for the crossover transition. The existence of this selection rule also follows as a simple consequence of the above description of these states. Since the electric quadrupole transition operator is linear in the quadrupole shape parameter, α, it cannot change the number of vibrational quanta by more than one.

The empirical evidence on the vibrational spectra of even-even nuclei is summarized in Table I. This contains all the cases in which the second excited states are known to have spin and parity 2+.

To summarize, the general picture of the lowest excited states of the even-even nuclei is as follows:

In the regions farthest from closed shells the nuclei possess a nonspherical shape as a consequence of the polarizing effect of the nucleons outside of closed shells. The lowest excitations are then rotations with the spectrum (2). As we approach towards a closed shell the magnitude of the nuclear eccentricity decreases and the moment of inertia also decreases. At a point which may be still quite distant from the closed shell configuration, the spherical shape becomes a stable equilibrium as a consequence of the residual forces between the nucleons and the whole nuclear coupling scheme changes. The lowest excitations now correspond to approximately harmonic shape oscillation of quadrupole type. With the further approach to a closed shell configuration the frequency of such shape oscillations increases. Finally, in the immediate neighborhood of a closed shell configuration the collective vibrational frequencies may become comparable to those of independent particle motion. It is then no longer possible to employ an adiabatic approximation as in the derivation of (1). In such cases it is most appropriate to treat completely all the degrees of freedom of the particles outside of the closed shells, as has been done by Inglis, Elliott, and Flowers, Redlich, Ford, Levinson, Pryce, and others.

The general sequence of spectra is illustrated in Fig. 6. The levels of Hf^{180} follow the rotational spectrum quite accurately, as is the case for all the nuclei in this region (Fig. 3). With Os^{190} deviations from the rotational spectrum (2) are about ten percent. The transition to vibrational spectra occurs at Pt^{192}. All the observed spectra from this point to the immediate neighborhood of the closed shell configuration of Pb^{208} correspond to approximately harmonic vibrations.

Finally, by the time we come to Pb^{206} the collective frequencies have become higher than those of individual particle motion. The observed low-lying excited states of this nucleus have been shown[8] to correspond well with the two-neutron excitations expected from the known single particle levels available in this region.

[8] M. H. L. Pryce, Proc. Phys. Soc. (London) **A65**, 773 (1952); D. E. Alburger and M. H. L. Pryce, Phys. Rev. **95**, 1482 (1954).