Nuclear Shell Models

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BEFORE talking about recent progress of the nuclear shell model, it might be worthwhile to give a very brief outline of its history. The shell model is about as old as nuclear physics, or at least dates from the discovery that the neutron was a constituent of the nucleus. By the shell model I mean the assumption that it is a good approximation to describe each nucleon as moving in an average field of force produced by all the other nucleons. This system is so analogous to the Bohr atomic model that it is not necessary to go into details. One obtains a series of degenerate levels which are to be filled according to Pauli's principle. Whenever one encounters two levels which are widely separated, the filling of the lower one means something like a shell closure.

Much has been said about the average potential in Dr. Weisskopf's talk. For the moment, let us assume that it is spherically symmetric; this assumption is not as well founded as in the atomic case. In Dr. Mottelson's talk it will be seen what a great variety of models is accessible when this simplifying assumption is discarded. The other assumption was (at least in the early stages of the shell model, that is, in the early 30's) that the average potential should be a function of the coordinates only. We now know that it might be more reasonable to include a velocity dependence of the potential, say with a term which is proportional to the square of the momentum, but it should still be scalar. This new idea about the best average potential does not affect the general arguments seriously.

I should now like to say a few words about the rather great success which the old shell model attained through the efforts of many theoreticians, especially Wigner and his co-workers, in explaining many facts about the light nuclei which were generally known with rather poor accuracy at that time. I mention as an example, that the shell closures of He^4 and O^{16}, and to a certain extent of Ca^{40}, were the first theoretical predictions about the particular stability of certain nuclei. The shell model provided a very neat explanation of the surprising fact that until far up in the atomic table there was only one element (argon) which had no odd mass isotope: One knew that the only odd mass argon isotope which might be expected to be stable was ${}_{18}A_{21}{}^{39}$, this would lie on the slope of increasing neutron excess with increasing charge, but the shell model explained rather quantitatively the instability of this isotope just by the fact that the twenty-first neutron would have to be brought into a new shell and would be very loosely bound. Hence the isobar ${}_{19}K_{20}{}^{39}$ has a lower binding energy in spite of the greater Coulomb energy, and therefore there is no odd mass isotope of argon.

These are only a few examples for the general success of the early shell model. Yet the model somehow got out of vogue. This was partly for the reason that there were other nucleon numbers which were distinguished in the same way as the numbers 2 in helium, 8 in oxygen, and 20 in calcium, from the natural radioactive decay one knew that there were similar phenomena at the numbers $Z=82$ and $N=126$, and from the table of stable nuclei together with the abundance chart it could be concluded that $Z=50$, and $N=50$ and also $N=82$ played a similar role. However, these numbers never arose as shell closures from a reasonable assumption about the average potential even if one included velocity dependence. The main reason, however, was the tremendous success of Niels Bohr's compound nucleus model in explaining the data on nuclear reactions. The underlying idea of the compound nucleus is that all nucleons somehow equally share the responsibility for the properties of the nuclear states involved, and Bohr rather convincingly suggested that this were a necessary consequence of the close packing and the short range of the nuclear forces. It was generally believed that this feature not only held for the excited levels in nuclear reactions, but also for the lowest levels, and I think that mainly in consequence of this generalization the shell model was not considered seriously for about ten years.

This morining we learned about Dr. Brueckner's confidence that these arguments may be overcome, and that the independent particle model can be justified in spite of close packing, and the short range forces, and even for forces with a repulsive core. The principal objections may therefore no longer be serious, and I hope that in the near future this point can be completely cleared up. As long as it is not convincingly shown that the Hartree model is a good approach even for these extreme forms of nuclear interactions, the success of the shell model is like explaining magic by miracles.

At any rate, some years ago Dr. Goeppert-Mayer and, independently, my friends Haxel and Suess in Germany dug up the old shell model. At that time we were not very hopeful that we could learn very much about elementary nuclear interactions in this way, but we thought that even a poor model might be useful in finding some correlations and orientations in the rapidly increasing wealth of isolated nuclear data. We thought the situation might be rather similar to that in organic chemistry before quantum mechanics was developed. At that time, the experimental chemists could not wait until the physicists had developed an entirely consistent and fundamental theory of matter, and they had to use concepts of their own, like valence or the Kekulé model. We investigated the shell model in this spirit and we were quite surprised that it appears to be possible to obtain some fundamental physical information from it.

The first reason why we reconsidered the shell model was the rapidly increasing evidence for the particular behavior of the nucleon numbers mentioned above; since they were not explained at that time, the name "magic numbers" was used in the literature. Beyond that, I should like to mention two other aspects. First of all there was the different behavior of the magnetic moments of odd proton and odd neutron nuclei, namely that nuclear moments of odd neutron nuclei did not show any dependence on the angular momentum (they are by no means greater for large angular momenta than for small angular momenta) whereas there is definitely an increase of the odd proton moments with increasing angular momentum. This suggests that only the orbital momentum of the odd charged particle, contributes to the magnetic moment of odd mass nuclei. I will not go into details, but in addition there is a grouping which even suggested a difference in orientation of the spin of the odd particle with respect to its orbital momentum. Thirdly, there was another point which Niels Bohr emphasized in my discussions with him, namely the absence of rotational levels. At that time none were known, and there are still no rotational levels with moments of inertia of the whole nucleus. Now in a model in which each particle is moving in an average field, there is no place for a rotation of the nucleus as a whole because any angular momentum must be given by the orbital momenta of the single particles. Of course, there might be superpositions, but the shell model was the simplest way to get rid of the surprising fact that there were no rigid nuclear rotations.

We next tried to find the proper "magic numbers" and, as is well known, they could be obtained uniquely by one additional assumption about the spherically symmetric potential. To the purely radial potential one has to add a strong spin-orbit coupling of the form

$$f(r)\boldsymbol{\sigma}\cdot\mathbf{r}\times\mathbf{p}.$$

This additional strong spin-orbit coupling gives the right ordering of levels.

At that time, the existence of a potential of this type was an arbitrary assumption, although such a term does arise as a first order relativistic effect if one considers the motion of a spin $\frac{1}{2}$ nucleon to be described by a Dirac-like equation with a spherically symmetric scalar field $V(r)$. In the Pauli approximation this leads to a term of the type

$$\left(1-A(\boldsymbol{\sigma}\cdot\mathbf{L})\frac{1}{r}\frac{d}{dr}\right)V(r),$$

where

$$A=\left(\frac{h}{2mc}\right)^2.$$

The sign of the term with $(\boldsymbol{\sigma}\cdot\mathbf{L})$ is negative, otherwise the term is just the same as the Thomas term. In the electromagnetic case, where the potential is the fourth component of a four-vector, it has a positive sign. The negative sign is just what is necessary to account for the experimental data, but the factor A is too small by at least one order of magnitude, and therefore the spin-orbit potential was originally an *ad hoc* hypothesis.

I do not know to what extent the Brueckner theory is now able to give a spin-orbit coupling of this type from conventional nuclear forces as they are suggested by meson theories (for example, from the tensor forces). At the moment one can hardly give a simple reason for the occurrence of this peculiar potential. Of course, the spin-orbit splitting need not necessarily be described by such a term, but it is certainly the simplest one.

I should like to emphasize the experimental evidence for this spin-orbit coupling from the scattering of neutrons or protons by helium. Helium can be considered as a very spherically symmetric nucleus, it has no excited states up to its dissociation energy of about 20 Mev, and therefore I think that if the shell model should work at all, it would be most usefully applied to the interaction of the helium nucleus with an incoming proton or neutron. The analysis of the proton scattering showed very different behavior of the phase shifts for the $p_{\frac{1}{2}}$ and $p_{\frac{3}{2}}$ states. There is a rather sharp resonance which means the $p_{\frac{3}{2}}$ phase shift goes through $\pi/2$ at about 2.5 Mev (laboratory system) whereas the $p_{\frac{1}{2}}$ phase shift shows a very gradual slope. This obviously indicates a very strong spin-orbit interaction. These phase shifts were confirmed at Minnesota by the experimental observation of the polarization of proton beams, through double scattering in the classical arrangement by which Barkla showed polarization of x-rays.

The first scattering on helium produces the polarization and the next scattering provides the analysis of the polarization. The amount of polarization predicted by the phase shifts (as obtained from the angular dependence of the single scattering at different energies) was quantitatively confirmed.

This experiment has been extended, and the whole phase shift analysis confirmed by experiments at Illinois. Jentschke and co-workers let the first scattering occur at rather high energy (5.5 Mev) and before the second scattering the protons are slowed down to 2.5 Mev (this does not affect the polarization because the slowing down is caused by ionization). Because of the different effects of the phase shifts at the different energies, the theory predicts a reversal of the intensities in two symmetric directions, as compared with the Minnesota experiment. Complete agreement with the theoretical prediction was found. I think that the spin-orbit interaction for a nucleon moving in nuclear matter is now an established fact.

Breit and his co-workers tried to obtain these phase shifts through the whole region from zero to 15 Mev under the assumption that the proton or neutron moves in a velocity independent, spherically symmetrical potential with a spin-orbit term of the Thomas type superimposed. For a Gaussian well one had to take the spin-orbit coupling 30 times the Thomas value, and for a square well potential, one had to take even 50 times the Thomas value. The phase shifts were described over the whole energy interval with velocity independent central forces. Perhaps this is an accident but we cannot draw conclusions until these calculations are repeated with velocity dependent potentials.

I shall not discuss how these shell model predictions were useful in giving the spins and parities, etc., not only of stable nuclei, but also of radioactive nuclei, in the classification of β decays, in the explanation of the phenomenon of isomerism, etc. But I will discuss some recent more detailed calculations which were made with the aim of getting more quantitative agreement with experimental facts, and of getting information about elementary nuclear forces.

The closed shell nucleus is very trivial, and equally simple are nuclei with one neutron or one proton missing from a closed shell, but as soon as there are several particles outside closed shells the situation is very peculiar. If the shell model were really a very good approximation the particles in an incompletely filled shell could couple their spins in various ways, even if one takes the requirements of all the symmetry principles: isotopic spin, and Pauli's exclusion principle into account. The prediction would be that very close to the ground state there should be other levels with different spins, so that such nuclei should have a very confusing spectrum. From the fact that this is not the case, one sees that simple shell model functions are a crude approximation and that there are certainly correlations between the nucleons. The adopted procedure is quite similar to what was done in the theory of atomic shells and the early nuclear shell model: One takes the shell model functions as first approximation, describes the interactions by pairs of particle forces, and then one calculates what mixtures of different shell model configurations gives the lowest energy. One hoped that at least the bulk of the eigenfunction could be represented by only one configuration. That is actually not true, and to solve this problem there have been carried out a great deal of numerical calculations by very many theoreticians.

It is impossible to talk about all the details which have been obtained. However, I should like to mention two which I happen by accident to know best and which I think are rather consistent. One is the work by Kurath on the whole $p_{\frac{3}{2}}$ and $p_{\frac{1}{2}}$ shell, that is for all nuclei between helium and oxygen. In pure jj coupling first the $p_{\frac{3}{2}}$ shell would be filled, and after that one had to fill the $p_{\frac{1}{2}}$ shell. But all other forces besides the spin-orbit force (which we have not really understood as yet) tend to produce a mixture of these configurations so that in the simplest case the Li^6 ground state should be a linear combination of $(p_{\frac{3}{2}})^2$, $(p_{\frac{3}{2}},p_{\frac{1}{2}})$, and $(p_{\frac{1}{2}})^2$.

Calculations show that, with reasonable forces between pairs of particles and a single particle spin-orbit coupling superimposed, one arrives at a very considerable mixing of configurations. The situation is intermediate between a pure LS configuration and a pure jj configuration. Kurath showed that with rather good over-all agreement not only the ground states but also the excited states can be obtained.

If one allowed for admixtures of further configurations which arise from higher single particle states, the agreement could certainly be improved, because this procedure is a legitimate and systematic way of taking all correlations between the particle positions into account. But it is reassuring that with p configurations alone the experimental data can be reproduced so well. To give this agreement the single particle spin-orbit force has to increase systematically from the beginning of the shell (He^5) towards the end of the shell (N^{15}). Such a behavior of the phenomenological $f(r)\boldsymbol{\sigma}\cdot\mathbf{L}$-force follows from the assumption that it arises from spin-orbit interactions between pairs of particles. The beginning of the shell (He^6) is therefore better represented by a pure LS configuration, whereas the end of the shell (N^{14}) resembles more closely a pure jj configuration.

Not only the energies and quantum numbers of the nuclear states are well accounted for, but also the matrix elements for radiative and β transitions agree fairly well with the observed data. This is a more sensitive test for the quality of the eigenfunctions, because it is well known that the energies (which can be obtained from a minimum principle) usually come out quite well even with poor eigenfunctions, whereas the transition matrix elements are much more sensitive. The long lifetime of C^{14}, i.e., the small matrix element for its β transition to N^{14}, which was a worrying puzzle to the theoreticians for many years, is now well understood. The matrix element becomes accidentally so small through a destructive interference between the contributions of the various configuration which represents the ground states of C^{14}, respectively N^{14}. From central forces between pairs of particles and $\boldsymbol{\sigma}\cdot\mathbf{L}$ forces alone one cannot obtain a configuration mixture with such amplitudes and phases that the matrix-element is smaller by a factor 10^{-3} than the value calculated from a simple shell model configuration. However, it was shown by the Harwell group that the addition of a small tensor interaction between the two particles, of sign and magnitude as in the deuteron, is sufficient to yield precisely such a configuration mixture that the calculated β-transition matrix element vanishes, whereas the ordering and spacing of the excited levels is practically not disturbed.

Kurath's p-shell results agree well with those obtained by the Harwell group, and this group achieved equally satisfactory results for the nuclei with one to three nucleons beyond the O^{16} shell. In this case they find that an appreciable mixing of the $d_{\frac{5}{2}}$, $s_{\frac{1}{2}}$, and $d_{\frac{3}{2}}$ configurations results from the same type of internucleon forces. Again good agreement with observed data not only of the level schemes but also of electromagnetic moments and of the transition probabilities is obtained without additional f-state configurations. All these calculations were carried through with shell model eigenfunctions obtained from a velocity independent potential and a $f(r)\boldsymbol{\sigma}\cdot\mathbf{L}$ term.

In these calculations one difficulty in the concept of the shell model has also been eliminated, namely, the additional degrees of freedom of the "center" around which the auxiliary single particle potential is assumed to be rotationally invariant, which has no physical significance. The total nuclear wave function should be invariant under translation, whereas the determinant of the A single particle wave functions gives an oscillation of the center of gravity around the nonphysical "center." For the nuclear ground states this unrealistic feature can be removed. However, if one considers all possible states of all shell-model configurations one would obtain states which have no physical meaning. The simplest example is an "excited state" of He^4. The ground state is an $\{s^4\}$ configuration. For excited states a configuration $\{s^3p^1\}$ could be considered. Among the properly antisymmetrized functions there appears one which can be written as $\{s^2p^1\}_{J=1}=\mathbf{R}\cdot\{s^4\}$, where $\mathbf{R}$ is the center of gravity vector of the four nucleons. This would describe an unphysical state. Only after elimination of all such spurious shell model states does one obtain level schemes which agree with the observed ones.

I could give only a brief summary of a few of the numerous calculations which have shown that the shell model can account for a great number of quantitative nuclear data as well as for general features of nuclear structure if correlation effects are properly accounted for by the introduction of mixed configurations. This practical success must however be solidified by a general proof that the Hartree method is self-consistent for the nucleus.