7. H. Weyl, *Raum. Zeit. Materie* (Verlag Julius Springer, Berlin, 1923), fifth edition.
8. P. Jordan, *Schwerkraft und Weltall* (Vieweg, Braunschweig, 1955), second edition.
9. E. Schrödinger, *Space-Time Structure* (Cambridge University Press, Cambridge, 1950).
10. M. A. Tonnelat, *La thèorie du champ unifiê d'Einstein* (Gauthier-Villars, Paris, 1955).
11. Proceedings of the Conference on Relativity at Berne, 1955. To appear shortly as a supplement to Helv. Phys. Acta.

II. Papers

12. H. P. Robertson, Revs. Modern Phys. **21**, 378 (1949).
13. Durbin, Loar, and Havens, Phys. Rev. **88**, 179 (1952).
14. W. Pauli, Phys. Rev. **58**, 716 (1940). Also Progr. Theoret. Phys. **5**, 527 (1950).
15. E. P. Wigner, Ann. Math. **40**, 149 (1939).
16. (a) V. Bargmann and E. P. Wigner, Proc. Natl. Acad. Sci. **34**, 211 (1948); (b) T. D. Newton and E. P. Wigner, Revs. Modern Phys. **21**, 400 (1949).
17. C. Møller, Dan. Mat. Fys. Medd. **30**, No. 10 (1955).
18. Ph. Freud, Ann. Math. **40**, 417 (1939).
19. A. Einstein, Can. J. Math. **2**, 120 (1950).
20. E. Schrödinger, Proc. Irish Acad. **54A**, 79 (1951).
21. L. Infeld and A. Schild, Revs. Modern Phys. **21**, 408 (1949).
22. (a) Einstein, Infeld, and Hoffmann, Ann. Math. **39**, 66 (1938); (b) A. Einstein and L. Infeld, Ann. Math. **41**, 455 (1940); (c) A. Einstein and L. Infeld, Can. J. Math. **1**, 209 (1949); (d) L. Infeld, Acta Phys. Polonica **XIII**, 187 (1954).
23. J. A. Wheeler, Phys. Rev. **97**, 511 (1955).
24. V. L. Ginsburg, Uspekhi fisz. Nauk, **LIX**, 11–49 (May, 1956).
25. S. F. Singer, Phys. Rev. **104**, 11 (1956).
26. L. D. Landau, "On the quantum theory of fields," in *Niels Bohr and the Development of Physics* (New York and London, 1955).
27. Th. Kaluza, Sitz. Preuss. Akad. Wiss. 966, 1921.
28. O. Klein, Z. Physik **37**, 895 (1926).
29. Y. Thiry, Compt. rend. acad. sci. (Paris) **226**, 216 (1948).
30. P. A. M. Dirac, Proc. Roy. Soc. (London) **A165**, 199 (1938).
31. M. Fierz, Helv. Phys. Acta **XXIX**, 128 (1956).
32. O. Klein, Arkiv Mat. Astron. Fysik B **34A**, No. 1 (1946).
33. A. Einstein and B. Kaufman, Ann. Math. **62**, 128 (1955).
34. E. Schrödinger, Proc. Roy. Irish Acad. **51**, 163 (1947); **51**, 205 (1948); **52**, 1 (1948).
35. (a) P. G. Bergmann, Phys. Rev. **75**, 680 (1949); (b) P. G. Bergmann and J. H. M. Brunings, Revs. Modern Phys. **21**, 480 (1949); (c) J. L. Anderson and P. G. Bergmann, Phys. Rev. **83**, 1018 (1951); (d) P. G. Bergmann and I. Goldberg, Phys. Rev. **98**, 531 (1955).
36. L. Rosenfeld, Ann. inst. Henri Poincaré **2**, 25 (1932).

REVIEWS OF MODERN PHYSICS VOLUME 29, NUMBER 2 APRIL, 1957

Nuclear Physics

VICTOR F. WEISSKOPF

Department of Physics and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts

I

WITHIN the framework of a short review it is impossible to discuss all outstanding problems in nuclear physics. I therefore restrict myself to the problem of nuclear structure and stability. The main problem can be formulated as follows: Given A nucleons, what can we conclude from our present incomplete knowledge of nucleon interactions, in regard to the structure and properties of the nucleus which they form?

Our knowledge of interactions is, in fact, extremely limited. The properties of the deuteron and the nucleon-nucleon scattering experiments contain much information about the interaction between nuclear pairs. We have deduced phase shifts as functions of energy from many scattering experiments, but so far it was not possible to explain these phase shifts as arising from a well-defined potential. Hence, it is hard to apply this knowledge to the situation within nuclei which is so different from the scattering of free and isolated nucleons. Because of the proximity of many scattering centers within one wavelength inside the nucleus, the conditions of scattering are fundamentally altered; any definite conclusions from the two-body scattering can be drawn only with a knowledge of the scattering potential and not from the mere phase shifts. As an example, let us look at a slow nucleon whose wavelength $\lambda\!\!\!^{-}$ is large compared to the radius r^* of the scatterer. The scattered wave ψ_{sc} will then be an S wave ($\psi_{sc}=[\sin(kr-\delta)]/r$). If it hits a second scatterer at a long distance $d\gg\lambda\!\!\!^{-}$ from the first scatterer, the wave will be essentially constant over the second scattering region, and there will again be mainly S scattering. However if the second scatterer is near the first ($d\ll\lambda\!\!\!^{-}$), the scattered wave ψ_{sc} is far from constant over the scattering region because of the preponderance of the $1/r$ dependence, if $d\ll\lambda\!\!\!^{-}$. In this case, p scattering will be quite important in spite of the low energy. In the usual mathematical terminology, this condition is expressed by saying that the elements of the scattering matrix between states of different energy are also important in our problem. This makes our problem rather difficult since the scattering phase shifts only determine the scattering matrix elements between states of equal energy.

Apart from the incomplete knowledge of the interaction between two free nucleons, our problem is complicated by the fact that we do not know whether this interaction is changed considerably when the two nucleons are surrounded by other nucleons. The existence of such a many-body effect is neither proved nor disproved. The average distance d of nucleons in nuclear matter, $d \approx 1.8 \times 10^{-13}$ cm, is not very much larger than twice the root mean square r^* of the spacial extension of the proton as measured by the Stanford group, $r^* \sim 0.8 \times 10^{-13}$ cm. Hence, the "meson clouds" of the nucleons almost touch one another in nuclear matter. It is a matter of personal inclination whether the fact $d > 2r^*$ can be used as an argument in favor of a nondense packing in which the forces are expected to be similar to the ones between two isolated particles or whether $d \sim 2r^*$ is an argument in favor of dense packing with appreciable changes in the meson clouds and therefore in the nuclear forces within nuclear matter. At present no direct experimental decision of this alternative has yet been devised.

Since it is not possible to derive the properties of nuclei deductively from the nucleon interactions, we must start at the other end and study the observed properties and try to find some explanations of the salient features of the observed facts. They are:

(A) *The existence of "nuclear matter."* Nuclear shape measurements have shown that all nuclei except the very lightest reach a universal density near the center of about $\rho_0 = 1.7 \times 10^{38}$ particles/cm^3. This seems to be the density at which nucleons reach a certain equilibrium. We are allowed to extrapolate and to assume that, were it not for the Coulomb repulsion, nuclear matter would be thinkable in a stable state of infinite extension and of the density ρ_0. It would consist of an equal mixture of protons and neutrons, having a binding energy of roughly 15 Mev per particle. Here the "saturation" of nuclear forces enters, which prevents nuclear matter both from collapsing to a density less than ρ_0 and from flying apart.

(B) *Independent particle aspect.* The dynamics of nuclear matter exhibits certain unexpectedly simple properties at low excitation energies: They can be fairly well reproduced by assuming that the nucleons within nuclear matter move almost independently of each other in a constant potential. Considering a nucleus as a finite spherical chunk of nuclear matter, we obtain a model of the nucleus consisting of independent nucleons moving within a spherical potential well in the lowest quantum states permitted by the exclusion principle. With the additional assumption of a strong spin-orbit coupling, this model reproduces a surprisingly large amount of experimental facts concerning the properties of nuclei at low excitation. On that basis we can understand the systematics of nuclear binding energies, most of the structure of nuclear spectra, and a good part of the electric and magnetic properties. We learn from this that nuclear matter at low excitation behaves somewhat like a degenerate gas, a fact which poses a grave problem of understanding at the present stage of our knowledge. The source of the spin-orbit coupling is also a problem which has not been satisfactorily explained.

(C) *The third group of important nuclear properties waiting for explanation contains features of less general character.* We quote here the deviation from sphericity found in many nuclei, the problems connected with the interpretation of the ensuing rotational spectra, and the vast amount of material accumulated in the study of nuclear reactions.

As mentioned earlier, there are two approaches to the problem of understanding the principles underlying the nuclear structure. We characterize them by the assumptions $r^* \ll d/2$ and $r^* \gg d/2$, where r^* is the radius of the meson cloud around a nucleon and d is the average distance apart of the nucleons. Actually, of course, $r^* \sim d/2$. In the first approach one endeavors to understand the phenomena on the basis of two-body forces between pairs of particles. Examples of this approach are the papers by Brueckner, Eden, Levinson, Mahmoud, Bethe, Goldstone and others.[1] We will call it the Brueckner approach. In the other approach one assumes that the meson clouds of all nucleons merge completely into a one-meson field within the nucleus.

The consequences of the latter approach are difficult to argue in view of our lack of any specific meson theory. It is plausible, however, that such a model would lead to a lack or to a strong reduction of all interactions in nuclear matter at its normal density at which the meson field probably reaches a certain saturation value. This picture has been discussed by Duerr and by Johnson and Teller[2] and contains some interesting speculations as to the nuclear interactions of antinucleons. The "Teller approach" contains the lack of interactions as a basic assumption and explains automatically our group B of facts. There exist certain observations, however, which seem to indicate that the lack of interaction between nucleons in nuclear matter is restricted only to low excitations. If this is correct, this lack is only apparent and is caused by the special properties of nuclear matter in its lowest states, as it would be from the point of view of the Brueckner approach. Some of these observations are: The analysis of nuclear reactions has shown that the mean free path of a nucleon entering nuclear matter is large only for relatively low energy of the particle. It can only be considered as moving independently of the others when the incident energy is less than, say, 10 Mev. At higher energies, but not at too high ones, say from 10 to 50 Mev, the mean free path is quite short compared to the nuclear radius, i.e., it is subject to strong interactions. Other indications of strong forces within the nucleus come from photoproduction of π mesons, from p-d processes, and other pickup reactions. In all these cases, the results indicate that the nuclear wave functions contain momentum components which are much higher than the ones one would get in the case of independent free motion. Hence, strong forces must be present which cause them. How-

[1] For references see H. A. Bethe, Phys. Rev. **103**, 1353 (1956).
[2] H. Duerr, Phys. Rev. **103**, 469 (1956); M. H. Johnson and E. Teller, Phys. Rev. **98**, 783 (1955).

ever the arguments presented here against the Teller approach are not quite convincing. After all, it is only assumed that the interactions disappear in the center of the nucleus where the density reaches the maximum value. The nuclear forces would be functions of the density and assume their known strength at zero density. Hence there would be increasingly stronger interaction towards the surface of the nucleus where the density drops to zero. It is perfectly possible that the effects mentioned before can all be explained by these surface effects. In particular, the π-meson production and the pickup reactions are typical surface phenomena.

The Teller approach will not be further discussed in this review. We would like to face the question whether one can understand the nuclear properties if the forces between the constituents are roughly what we know from the two-nucleon interactions. A large amount of work has been spent recently on this problem by Brueckner and his colleagues, and a formalism has been developed which seems to be adequate to describe the situation. It is a generalization of a formalism originally invented to deal with multiple-scattering phenomena.

There certainly is a good deal of similarity in the problem of a wave scattered by a dense array of centers and the problem of the motion of a nucleon in a nucleus. The path of a nucleon should suffer many scatterings in traversing nuclear matter. The aforementioned group B of facts, however, indicates that there must be a fundamental difference in the two problems which prevents those scatterings from being effective in the nucleus. It was pointed out some time ago[3] that the Pauli exclusion principle plays an important role in this since, from a naive point of view, all end states into which a scattering of two nucleons can lead are already occupied at low excitations. It is not clear, in the first place, why one can use free particle states as actually existing in nuclear matter. However, the present theoretical treatments of the situation seem to bear out this idea and point towards the decisive role of the exclusion principle. Unfortunately, the mathematical difficulties are still very great, and it is not clearly understood what properties of the nuclear force make the independent particle approach a good approximation and lead to the observed density in nuclear matter.

II

Let us now approach the problem in a less deductive way. We learn from experience that the independent particle model is a good description of the facts and we ask what consequences we can draw from this circumstance. It means that the wave function Ψ of the nucleus is approximately a product of one-particle wave functions φ_i, duly antisymmetrized so as to fulfill the Pauli principle:

$$\Psi = A \prod_\alpha \varphi_\alpha(r_i). \tag{1}$$

Here r_i is the coordinate of the ith particle, α is the quantum state in which it is located, and "A" symbolizes the antisymmetrization which makes all α's different for particles of the same kind. The wave functions φ_α are given by a one-particle Hamiltonian

$$H_i\varphi_\alpha = [T_i + U_\alpha(r_i)]\varphi_\alpha = E_\alpha\varphi_\alpha, \tag{2}$$

where the first term is the kinetic energy $T_i = p_i^2/2m$, and the second term is the potential energy, which is assumed to be a potential well of the size of the nucleus. It also contains a spin-orbit term which we will neglect in the following rough estimates. We put approximately

$$\begin{aligned} U_\alpha(r) &= -V_\alpha \quad \text{for} \quad r<R \\ U_\alpha(r) &= 0 \qquad\ \ \text{for} \quad r>R, \end{aligned} \tag{3}$$

where R is the nuclear radius and V_α is a positive energy (the well depth). We do not assume that all particles have the same well depth; hence $U_\alpha(r)$ and V_α may depend upon the state α in which the particle is found. The actual exact Hamiltonian can be written in the form

$$H = \sum_{i=1}^{A} T_i + \tfrac{1}{2}\sum_{i,k=1}^{A} V_{ik}(\mathbf{r}_i - \mathbf{r}_k), \tag{4}$$

where V_{ik} is the interaction potential between the pair of particles i and k. We assume that only pair forces exist. If (1) is an approximate solution of (4), then the one-particle potential energy $U_\alpha(r_i)$ is the average effect of all other particles on the ith particle.

The total energy E of the system is given by the expectation value of H in the state Ψ:

$$E = \int \Psi^* H \Psi dr = \sum_\alpha \langle T_\alpha\rangle + \tfrac{1}{2}\sum_\alpha \langle U_\alpha\rangle,$$

where T_α is the expectation value of the kinetic energy in the state α, and $\langle U_\alpha\rangle$ is the average value of U_α in that state α. The sums are extended over all occupied states α. Here the energy appears as a sum of contributions from each occupied state. The factor $\frac{1}{2}$ in the second term is of great importance. It comes from the fact that the potential energy of the particle in the state α is the sum of all pair interactions with the other particles. Hence, if no factor $\frac{1}{2}$ were present, the interaction between a given pair i-k of particles would have been counted twice, once in $\langle U_\alpha\rangle$ and then again in $\langle U_\beta\rangle$, when α is the state of the ith particle and β the state of the kth one. This factor $\frac{1}{2}$ also makes it impossible to define a "model Hamiltonian," i.e., a Hamiltonian for our wave function (1). The Hamiltonian $H = \sum_i H_i$, whose eigenfunctions are the ones defined by (1), has eigenvalues different from (4); namely, the values one would get without the factor $\frac{1}{2}$.

We have assumed that the potentials U_α are square wells with a depth V_α. Hence, neglecting surface effects, each wave function φ_α has a well-defined momentum p_α and a kinetic energy T_α:

$$T_\alpha = p_\alpha^2/2m, \quad \langle U_\alpha\rangle = -V_\alpha.$$

[3] V. Weisskopf, Science 113, 1 (1951).

Also, in the lowest energy state, all levels up to the Fermi kinetic energy T_F are occupied. Then we get

$$\sum_{\alpha} \langle T_\alpha \rangle = \tfrac{3}{5} A T_F, \tag{5}$$

where the sum is taken over all occupied levels. The packing fraction P is given by

$$P = -\frac{E}{A} = -\tfrac{3}{5} T_F + \tfrac{1}{2} V_{\text{Av}}, \quad V_{\text{Av}} = \frac{1}{A} \sum_{\alpha} V_\alpha, \tag{6}$$

where V_{Av} is the average value of the well depth over all occupied states.

We now determine the separation energy S, which is the minimum energy necessary to remove one particle from the nucleus. The easiest one to remove is the one on the top of the Fermi distribution. We get

$$S = -T_F + V_F, \tag{7}$$

where V_F is the well depth for the top particle.[4] Now, S and P are closely related; in fact, in the limit of large nuclei they become equal if surface effects are neglected. The equality of these two magnitudes is an expression of the fact that the total energy is proportional to the number of constituents. By equating P and S we get from (6) and (7)

$$S = -\tfrac{1}{5} T_F + V_{\text{Av}} - V_F. \tag{8}$$

This equation demonstrates the necessity of assuming well depths depending on the momentum p_i. If all depths were equal, we would have $V_{\text{Av}} = V_F$, and we would get the nonsensical result of a negative separation energy.

Equation (8) does not tell us in what way V_i depends on the state i. Let us make the simplest assumption of a quadratic dependence upon the mementum p_i:

$$V_i = V_0 - \frac{p_i^2}{p_F^2} V_1, \tag{9}$$

where V_0 and V_1 are constants and p_F is the Fermi momentum. Then, of course, $V_{\text{Av}} = V_0 - \tfrac{3}{5} V_1$, $V_F = V_0 - V_1$, and we calculate V_0 and V_1 from (7) and (8):

$$V_1 = \tfrac{1}{2}(5S + T_F), \quad V_0 = \tfrac{1}{2}(7S + 3T_F). \tag{10}$$

A quadratic dependence of the well depth can always be expressed in terms of an effective mass m^*. The energy of the ith particle is then written in the form

$$E_i = \frac{p_i^2}{2m} - V_i = \frac{p_i^2}{2m^*} - V_0$$

with

$$\frac{m}{m^*} = 1 + \frac{V_1}{T_F} = \tfrac{3}{2} + \tfrac{5}{2}\frac{S}{T_F}. \tag{11}$$

These relations should only serve as a first orientation and must not be regarded as quantitative. Nevertheless, it is interesting to put in some numbers. The Fermi energy T_F depends only upon the nuclear density and is (with $R = 1.2 \times A^{\frac{1}{3}} \times 10^{-13}$ cm) $T_F = 33$ Mev. Hence, with $S = 8$ Mev we get $m/m^* = 2.1$, which is near enough to the value $m/2$ commonly used for the effective mass.

Evidently the assumption (9) of a quadratic dependence of V_i on p_i is arbitrary. All we can conclude is the fact that the average of V_i is larger than V_F. The main point is the recognition that a decrease of the well depth with increasing momentum of the particle follows directly from two facts: One is the independent-particle motion, the other is the approximate equality of P and S, which is a consequence of the saturation of nuclear forces.

It is not hard to understand in a qualitative way why a momentum-dependent well depth may arise from a nuclear force which has saturation effects. A simple example is the case of repulsive forces at close distances; if a particle moves slowly, it will follow a path avoiding near collisions. When it goes fast, it will penetrate somewhat into the region of repulsion, thus increasing the average potential energy. Another example is an exchange force in which antisymmetric pairs lead to repulsion. The nearest distance of approach between antisymmetric pairs is of the order of the reciprocal relative momentum. Hence, the repulsion will be more effective for a fast-moving particle. This latter example was already treated in 1935 by VanVleck.[5] At that time the independent particle model was in fashion and he has shown that the exchange terms in the average potential energy go to zero with increasing energy of the particle. This gives rise to a momentum dependence of the potential which is quadratic only for momenta small compared to p_F. It reaches a constant value for $p \gg p_F$.

The question arises as to whether the momentum dependence of the well depth can be found directly from experiments. The most direct way would be a study of the nuclear energy levels. The excitation energies are altogether higher if the well depth decreases with increasing momentum. In fact, they should be twice as high as calculated with the normal mass when m^*/m is about $\tfrac{1}{2}$. Unfortunately, however, nuclear spectroscopy is not as direct a way as it seems to find

[4] Note must be taken here of the fact that the nucleus decreases its size by a factor $(A-1/A)^{\frac{1}{3}}$ when one particle is removed. In order to see clearly the effects of this, let us divide the separation of a particle in two steps. First we remove the particle without changing the wave functions of any of the other particles. This requires the energy (7). We then have a nucleus with $A-1$ particles, but with slightly too large a radius. As a second step we compress this nucleus to its normal density. This will raise all kinetic energies slightly, but it also will decrease the potential energy because of the increased density. These two effects must cancel each other almost completely since we know that the actual nuclear density ρ_0 is a stable equilibrium value and hence $\partial E/\partial \rho = 0$ for $\rho = \rho_0$. Hence, (7) is the actual separation energy apart from very small corrections. As to details of the corrections, see R. A. Berg and L. Wilets, Phys. Rev. **101**, 201 (1956).

[5] J. H. VanVleck, Phys. Rev. **48**, 367 (1935).

this effect since one rarely observes the actual energy differences between shells. What one measures are mostly energy differences within one shell, which depend on the finer details of the well shape and are therefore not useful for our problem. The studies of Ross, Mark, and Lawson[6] have shown that the essential features of the low-lying nuclear spectra are not very sensitive to the effective mass.

Levels of higher excitations are difficult to observe and to identify. Once the excitation energy becomes comparable to the distance between shells, the number of levels is so great that a comparison with theoretical predictions becomes hardly feasible. In this connection it is worth mentioning Wilkinson's[7] idea that the giant resonance in photonuclear process might be explained by one-particle transitions, but only if the effective mass is taken into account.

Recently Rand[8] investigated this suggestion quantitatively and has calculated the cross section for nuclear excitation by gamma rays on the basis of an independent particle model in which the particles move in a square well and have an effective mass of one-half. The dependence of these cross sections upon the energy of the gamma rays (he considered only electric dipole transitions) shows the characteristic maxima, usually called "giant resonances." They are almost at the correct energies, although perhaps 10 to 20% lower than the experimental maxima. Without the reduced effective mass, however, the independent particle model gives absorption maxima at energies less than half of the experimental value.[9]

It has been often suggested that the effective mass enters into the contribution of the orbital motion to the magnetic moment. One might suspect that the orbital g factor $(e/2mc)$ is changed by replacing m by m^*. The experimental evidence does *not* bear this out. The slope of the so-called Schmidt lines directly gives the orbital g value, and it seems that the experimental magnetic moments reproduce the slope fairly well. It is true that the values do not lie very close to the lines because of configuration interaction, but a change of the g factor to twice its value seems out of the question. The problem of the effect of the momentum dependent potential on the orbital g factor is difficult and has not been completely cleared up. It is certainly incorrect, however, merely to change m into m^* in the expression for g. One must keep in mind that there is *no* Hamiltonian in which the m^* appears. The mass m^* only enters into the calculation of the expectation value of the energy in the independent particle model state Ψ. The exact Hamiltonian H contains the mass m and not m^*, and the corresponding operator of the orbital magnetic moment is $(e/2mc)L_p$, where L_p is the orbital angular momentum of the protons and m is the actual mass. Therefore, if Ψ is a good wave function, one should use the actual mass in calculating the magnetic moment, and the original Schmidt lines are correct. However, Ψ might be a good wave function for energy computations, but a bad one for magnetic properties.

Recently Blin-Stoyle[10] has suggested the possibility that the effective mass might play a role in determining the moment of inertia of a deformed nucleus. According to Inglis[11] this moment is given by

$$J=\sum_k \frac{|(0|L_z|k)|^2}{E_k-E_0},$$

where L is the operator of the angular momentum around the axis of rotation, and the indices 0 and k denote the ground state and an excited state of the deformed nonrotating nucleus. E_0 and E_k are the respective energies. If this formula is applied to a system of independent particles of mass m moving in an ellipsoidal potential well, one gets for J the so-called "rigid" moment of inertia, which is the one corresponding to the rigid rotation of the mass distribution. This moment is larger than the observed ones. Blin-Stoyle points out that an increase of the excitation energies of the system (and that is just what $m^*/m<1$ means in effect) would decrease the resulting value of J. Figure 1 shows that the observed values of nuclear moments of inertia seem to reach just one-half of the rigid moment for large deformations. This is just the value expected for $m^*/m=\frac{1}{2}$. If this argument is correct, it is an example showing that interactions between particles can reduce the moment of inertia. Bohr and Mottelson have mentioned that a similar effect takes place when one introduces explicitly attractive interactions between particles outside closed shells. They have shown that

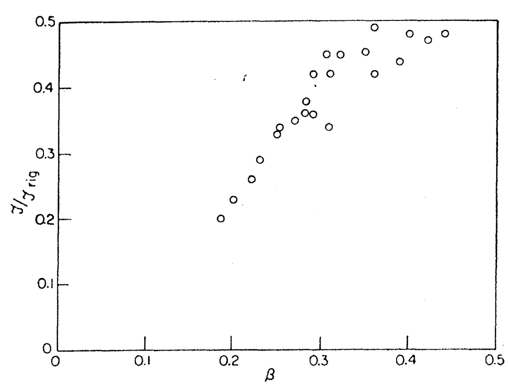

FIG. 1. The ratio of the observed values of the moments of inertia to the value for rigid rotation are plotted against the deformation parameter β. Values taken from Bohr and Mottelson, Kgl. Danske Vid. Medd. **30**, No. 1 (1955).

[6] Ross, Mark, and Lawson, Phys. Rev. **102**, 1613 (1956) and a subsequent paper to be published soon.

[7] D. Wilkinson, Proceedings of the Amsterdam Conference on Nuclear Reactions, 1956, to be published in *Physica. Proceedings of the Glasgow Conference on Nuclear Physics* (Pergamon Press, London, 1955), p. 161.

[8] S. Rand, Phys. Rev. (to be published).

[9] J. L. Burkhardt, Phys. Rev. **91**, 420 (1953).

[10] R. Blin-Stoyle, Nuclear Phys. **2**, 169 (1956).

[11] D. R. Inglis, Phys. Rev. **96**, 1059 (1954).

the small values of J for small deformations are very probably caused by such effects.

III

We now turn to the discussion of nuclear reactions. The recognition of the relatively independent motion of nucleons in nuclear matter has had its effect upon the interpretation of nuclear reactions. Under the impact of the successes of the shell model it seems questionable to assume that a nucleon, which enters the nucleus from outside, will share its energy immediately with all other constituents as assumed previously. The old description of a nuclear reaction as proceeding in two stages, the formation of a compound nucleus and the subsequent independent decay, is an idealization which cannot be considered to be valid in all cases.

We would like to propose a more general scheme for the description of nuclear reactions that allows for phenomena which do not fall within the framework of the old two-stage description; Fig. 2 presents a graphic representation of such a scheme. We divide the nuclear reaction in three successive stages, the independent-particle stage (I.P. stage), the compound-system stage (C.S. stage), and the final stage. In the first stage we find the incident particle interacting with the target nucleus, but in this stage the nucleus acts upon the particle as a potential well. The particle enters this well without losing its distinct individuality; it is refracted, and partially reflected, at the surface because of the change of potential. In order to provide for the subsequent events in the next stages, the potential must have a real part, V_1, and also an imaginary part, V_2. Then the first stage is described by an incident wave which is not only scattered by the nucleus, but also partially absorbed. "Absorption" in this picture means that the particle disappears from the entrance channel such that it can no longer be considered as existing as an independent particle distinct from the target nucleus. It is just this absorption which leads to the actual nuclear reaction. The scattering which takes place in this stage is called "shape-elastic" scattering. Both scattering and absorption in this stage are given directly by the scattering and absorption of a wave in a complex potential V_1+iV_2. Hence, the events in the

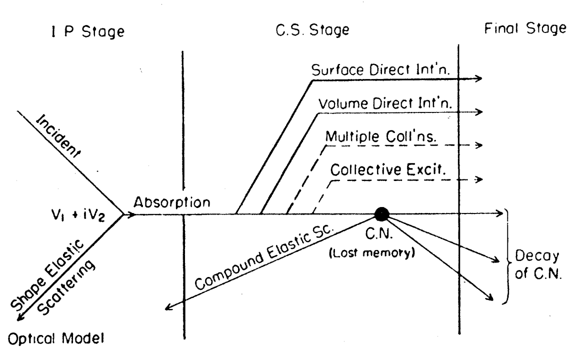

FIG. 2. Nuclear reaction scheme.

first stage are described by the optical model of the nucleus. It is a description of what happens in the entrance channel only.

The second stage contains the events which cause the absorption in the first stage. This "absorption" comprises any effect in which the particle leaves the entrance channel and therefore has undergone an interaction with the target nucleus which cannot be described by a potential only. The state of the system after the particle has been removed from the entrance channel will be called a compound system (C.S.). It describes the situation which exists when the particle interacts with the target to a more intimate extent than can be described by a potential in the entrance channel. This can happen in many ways. The incident particle can collide with a nucleon in the target (direct interactions); it can set up some collective motion as surface vibrations or nuclear rotations. The concept of C.S. is somewhat more general than the concept of "compound-nucleus" (C.N.) as used before in the two-stage Bohr description of a nuclear reaction. The C.N. is characterized by the fact that the state of the incident particle is indistinguishable from the state of any other nucleon; it has completely "coalesced" with the target. The C.S., however, is a state in which some energy exchange between target and projectile has taken place, regardless of the role of the incident particle. Hence the C.S. includes the C.N., but contains also other forms of interaction, namely, all those in which the incident particle is removed from the entrance channel.

There remains a question of what part of the interaction between the incoming particle and the target nucleus can be described by a potential in the entrance channel and what part leads to an absorption in respect to this channel and therefore to a C.S. This problem is not yet clearly understood. After all, the phenomenon described by a potential well is also an interaction, but the target acts in this case only as a whole. Although it might be partially excited when the particle is within the nucleus, it remains in its original state after the particle has left.

The third and final stage contains the processes in which the reaction products separate from each other. In some respects it is similar to the first stage, since the emitted particle can be considered as an outgoing wave from a potential well representing the residual nucleus.

The first stage is the one we know most about. As long as one restricts oneself to the first stage only, all details of the nuclear reaction are buried in what appears as an absorption from the entrance channel. Hence an understanding of this stage provides only a very rough picture of the reaction. Nevertheless, it gives information about the total cross sections, about the elastic scattering, although only about the shape-elastic part, and about the reaction cross sections. Another limitation comes from the short duration of this stage. The time the particle spends within the nucleus before being "absorbed" is only of the order of a few

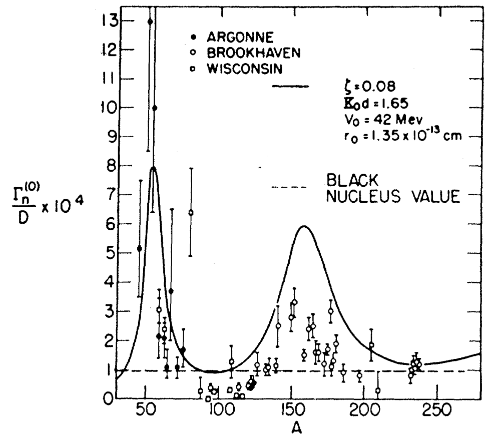

FIG. 3. The average value of Γ_n/D (neutron width) divided by level distance as a function of A. The theoretical curve is calculated with a potential as given by (12) with $R=r_0A^{\frac{1}{3}}$ and $K_0=(2mV_0/\hbar^2)^{\frac{1}{2}}$.

traversals of the nuclear well or even less. It then is impossible within the first stage to ascribe an energy to the particle which is better defined than by the margin given by the uncertainty principle, and this margin is about 0.1 Mev or more. Hence the optical model can only give information which is an average over a relatively large energy region; it therefore cannot account for the closely-spaced resonances which are observed in so many nuclear reactions.

Within these limitations the optical model description of the first stage has been very successful. It can reproduce the main features of the total neutron cross section as a function of energy and nuclear radius by means of a simple potential square well with rounded edges. It can predict reaction cross sections; for example, the theory predicted before the experiment that the average neutron width Γ_n divided by the level distance D for low energies, when plotted against the nuclear radius, should have maxima at certain radii. These are the ones at which a standing wave can be set up within the nucleus [$R=(n+\frac{1}{2})\Lambda$, where Λ is the wavelength inside, and n is an integer]. Figure 3 shows that the experiments have borne out this prediction. It also can predict the size and the angular dependence of the elastic scattering of neutrons and protons.[12]

Actually all one can calculate by this method is the shape elastic scattering which differs from the observed one by the compound elastic scattering. However, this difficulty is not important for protons, in which case the compound elastic part is negligible. In the case of neutrons, the compound elastic cross section, which is important only at lower energies, can be estimated from the difference between the observed reaction cross section and the calculated absorption cross section. The angular dependence of the compound scattering is nearly isotropic. It is gratifying that the difference between the observed elastic scattering and the calculated shape-elastic one can be explained completely by the experimentally determined compound-elastic part.[13]

The best potential for the description of the first stage of a nuclear reaction is at present

$$\begin{aligned}
&V=V_1+iV_2, \quad V_2=\zeta V_1\\
&V_1=-V_0(1+\exp[(r-R)/d])^{-1}\\
&V_0=43\pm3\text{ Mev}, \quad d=0.5\pm0.1\times10^{-13}\text{ cm} \qquad (12)\\
&R=(1.27A^{\frac{1}{3}}+0.6)0.10^{-13}.
\end{aligned}$$

The value of ζ which determines the imaginary part depends strongly on the energy. For low energies up to a few Mev one finds $\zeta\approx0.08$. At higher energies it increases sharply and reaches about 0.2 around 10 Mev and perhaps 0.4 to 0.5 at still higher energies. These values and the sharp rise with energy can be qualitatively understood on the basis of the independent particle model and the Pauli principle. The latter prevents the exchange of energy and momentum between the incoming particle and another nucleon if the resulting states are occupied by other particles. With increasing energy of the incoming particle, this will happen less frequently, and therefore more opportunities exist for the incoming particle to interact.[14]

Although ζ is the most energy-dependent variable in the potential (12), the other values are not completely energy independent either. In particular, the well depth V_0 has a tendency of decreasing with increasing energy. Some proton measurements indicate that $V_0\sim36$ Mev at an incident energy of 30 Mev, and an analysis by Taylor[15] has shown that it reaches a value of about 15 Mev at very high energies. This is just what one would expect from previous considerations as to the momentum dependence of the well depth.

The second stage of nuclear reactions is much less well understood. Here we face a varied range of phenomena, which can be grouped between the two following extremes: One is the direct interaction in which the incoming particles hit one single nucleon in the nucleus, and one of the two partners leaves the nucleus without interfering at all with the other nucleons; the other extreme is a formation of a real compound nucleus in which the energy of the incoming particle is divided among all (or very many) partners before the final stage of the reaction.

In recent years many reactions have been observed which lie between these extremes and some which definitely are almost pure direct interactions. Most important among the latter ones are the stripping and pickup reactions. In spite of this fact it is still probable

[12] R. D. Woods and D. S. Saxon, Phys. Rev. **95**, 577 (1954); **101**, 506 (1956); Fujimoto and Hussain, Phil. Mag. **46**, 542 (1955); Burge, Fujimoto, and Hussain, Phil. Mag. **1**, 19 (1956).

[13] L. Rosen and L. Stewart, Phys. Rev. **99**, 1052 (1955); Beyster, Walt, and Salmi, Phys. Rev. **104**, 1319 (1956).

[14] V. F. Weisskopf, Helv. Phys. Acta **23**, 187 (1950); Science **113**, 1 (1951). This idea was worked out more quantitatively by A. M. Lane and C. F. Wandel, Phys. Rev. **98**, 1524 (1955).

[15] T. B. Taylor, Phys. Rev. **92**, 831 (1953).

that compound nucleus formation plays an important part in most reactions. There is good experimental evidence[16] that the low-energy neutrons emerging from nuclear reactions have a "Maxwellian" distribution in energy (if the energy is high enough to allow for many excited states of the residual nucleus) which is isotropic in space. Hence the statistical description of the second stage as a "heated" compound nucleus is a good approximation. One can conclude from these observations that, for incident energies up to 25 Mev (neutrons or protons), a C.N. is formed with at least 80 to 90% probability.

In recent years attention was mostly concentrated upon those reactions which do not go via C.N. Especially in the reactions in which charged particles are emitted, the C.N. necessarily plays a minor role, since the potential barrier discriminates against charged particle emission after the C.N. is formed. Most of the emissions of charged particles must come from the nuclear surface, because it is a region in which the barrier is somewhat lower. Figure 4 shows a plot by N. Rosen, comparing the observed proton spectrum in a (*n*-*p*) reaction on Zn with the one calculated from the statistical theory. The maximum is much lower than the barrier at the nuclear radius (6 to 7 Mev) would admit. Hence, most of these protons must have been produced in an outer surface region.

This example should serve to show that the problems

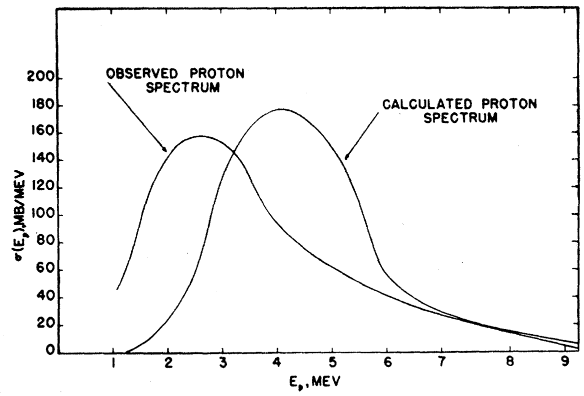

FIG. 4. Comparison of calculated and observed proton spectra from $Zn^{64}+n+14.1$ Mev.

of nuclear reactions are far from being understood. Many more similar examples could be quoted. The situation is rendered difficult because of the likelihood that the properties of the nuclear surface enter essentially into those reactions which do not go via C.N. What happens in the surface is even less understood than what happens in the interior of the nucleus.

The increasing complexity of experimental facts from nuclear reactions shows how far we are yet from a systematic understanding of what is going on in a nucleus during a reaction. It is to be expected that the picture will be clearer when some of the fundamental questions of the structure of nuclear matter are answered.

[16] E. R. Graves and N. Rosen, Phys. Rev. **94**, 224 (1953); N. Rosen and Stewart, Phys. Rev. **97**, 224 (1955); E. R. Graves and R. W. Davis, Phys. Rev. **97**, 1205 (1955).