Electromagnetic Structure of Nucleons*

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The theoretical implications of various experiments relating to the electromagnetic structure of nucleons are examined in the light of current field theory. It is concluded either that the nucleon core is about three times as large as would be expected from intuitive considerations of meson theory, or that there is some inconsistency in the present field theory.

1. INTRODUCTION

INFORMATION about the internal structure of individual nucleons is contained in the results of a variety of experiments performed in recent years.[1] Those experiments in which the interaction with the nucleon is electromagnetic (or is thought to be so) are susceptible of a considerably more precise and unambiguous interpretation than those involving meson interactions. Of the former type (which alone will concern us here) the best known is that involving neutron scattering by atoms, which, when analyzed, gives information about the electron-neutron interaction (to be specific, its volume integral). The Lamb shift and the hyperfine splitting can also give such information, although much less precisely. The fact that the anomalous magnetic moments of the nucleons are equal and opposite has important implications for a meson model of the nucleons, of course. Recent experiments on the scattering of high-energy electrons by hydrogen and deuterium now give considerably more detailed and complete information about the proton and, to some extent, about the neutron. The aim of this paper is to examine the extent to which the results of the separate experiments can be combined into a consistent picture of nucleons as charge-current distributions. We make no claim for the originality of most of the theoretical ideas presented here; they have all appeared in various forms in the literature, and we have brought them together for the purpose of discussing these experiments. In Sec. 2 we present the phenomenology of the electromagnetic interaction of electrons and nucleons. This is interpreted in Sec. 3 in terms of a simple meson model of nucleon structure in which it is assumed that the "physical" nucleon is made up of "bare" nucleons and pions interacting in a charge-symmetrical manner. In Sec. 4 other, more speculative, ways of interpreting these results are suggested, and the implication of these ideas, and their effects on the interpretation of other experiments, are commented on. The Appendix, due to one of us (D. R. Y.), gives a general treatment of the charge-current density of particles of general spin.

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† The work was completed while this author was at the University of Paris.

[1] Some of the topics we shall discuss have been reviewed in Bethe and de Hoffmann's book *Mesons and Fields Vol. II* (Row, Peterson and Company, Evanston, 1955), pp. 289–299. In many cases we shall refer the reader to this excellent book for bibliographies.

2. PHENOMENOLOGY OF ELECTRON-NUCLEON INTERACTION

The scattering of an electron from a nucleon caused by their electromagnetic interaction[2] is represented by the Feynman diagram of Fig. 1. We write the matrix element for the process as

$$-4\pi i j_\mu^{p,n}(P',P)(1/q^2)j_\mu^e(k',k), \qquad (2.1)$$

which includes both the Coulomb interaction and the effect of the exchange of transverse photons.[3] The interpretation of this expression is as follows: the factor $(1/q^2)$ represents the propagation of a virtual photon of four-momentum q_μ between the electron and the nucleon where q_μ is the recoil momentum,

$$q_\mu = P_\mu' - P_\mu = -(k_\mu' - k_\mu). \qquad (2.2)$$

In the center-of-momentum frame ($q_0=0$), q^2 is given by

$$q^2 = (2k_c \sin\tfrac{1}{2}\theta_c)^2, \qquad (2.3)$$

where k_c and θ_c are the electron's momentum and scattering angle in this frame. The factor j_μ^e is the electron's charge-current density, which, assuming no internal structure, is given simply by

$$j_\mu^e(k',k) = -ie\bar{u}(k')\gamma_\mu u(k), \qquad (2.4)$$

where $u,\bar{u}$ are Dirac spinors for the electron. The charge-current density of the nucleon $j_\mu^{p,n}(P',P)$ (proton or neutron) includes all of the effects of the internal structure.[4] The purpose of the experiments we are discussing

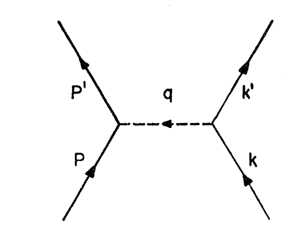

FIG. 1. Feynman diagram of the scattering of an electron by a nucleon caused by the exchange of a virtual photon.

[2] Scattering due to nonelectromagnetic interactions is discussed later in this Section.

[3] R. P. Feynman, Phys. Rev. 74, 939 (1948). Our notation differs from Feynman's somewhat, in that e, the electric charge, is given in unrationalized units, and the four-vector product $a\cdot b$ means $\mathbf{a}\cdot\mathbf{b}-a_0b_0$. Usually we put $\hbar=c=1$.

[4] Technically this is the vertex operator evaluated between free nucleon states.

is to determine information about this charge-current distribution beyond what is already well known from static experiments (the total charge and magnetic moment).

The problem has been simplified by Foldy,[5] and later, in more generality, by Salzman.[5] They show that the nucleon charge-current density must have the form

$$j_\mu^{p,n}(P',P)=ie\bar{v}(P')[\gamma_\mu F_1^{p,n}(q^2)+(\kappa^{p,n}/2M)\sigma_{\mu\nu}q_\nu F_2^{p,n}(q^2)]v(P). \quad (2.5)$$

The assumptions are (i) relativistic covariance, which means that $j_\mu^{p,n}$ transforms as a four-vector; (ii) a differential law of current conservation, which in momentum space is expressed as

$$(P'-P)_\mu j_\mu^{p,n}(P',P)=0; \quad (2.6)$$

and (iii), that the nucleon is a Dirac particle.[6] In Eq. (2.5), the quantities $v,\bar{v}$ are Dirac spinors for the nucleon, $\kappa^{p,n}$ is the anomalous magnetic moment of the nucleon in nuclear magnetons, and the rest of the symbols have their usual meanings. The functions $F_{1,2}^{p,n}(q^2)$ describe the internal structure, and in the static limit ($q^2\rightarrow 0$) take the value unity, except for $F_1^n(0)$, which is zero. As regards the uniqueness of the form of Eq. (2.5), it should be mentioned that there are other covariant expressions satisfying Foldy's assumption, e.g., the convection-current term $(P'+P)_\mu\bar{v}(P')v(P)$. However, such terms always can be expressed in the form (2.5) by the use of the Dirac equation,

$$(\gamma_\mu P_\mu - M)v(P)=0,$$
$$\bar{v}(P')(\gamma_\mu P_\mu' - M)=0. \quad (2.7)$$

The functions F_1 and F_2 are relativistic generalizations of the form factors characteristic of finite extension occurring in other experiments, for example, in the scattering of electrons from nuclei.[7] There, the form factor is simply the Fourier transform of a radial density function,

$$F(q^2)=\int f(r)\exp(i\mathbf{q}\cdot\mathbf{r})d^3r. \quad (2.8)$$

The function $f(r)$ comes from the product of the initial wave function of the scatterer at rest and the final wave function of the scatterer after it has absorbed the recoil momentum $\mathbf{q}$. For heavy nuclei, where the recoil velocity is negligible compared with c, nonrelativistic wave functions are sufficient, and $f(r)$ is just the static charge or magnetic-moment distribution. For the scattering from a nucleon, however, the situation is qualitatively different, in that for values of $\mathbf{q}$ large enough that the finite nucleon size may be detected, the recoil velocity is comparable with c. It should be emphasized, however, that it is always possible to analyze the experiments in terms of the invariant functions $F_1(q^2)$ and $F_2(q^2)$, and for intuitive convenience to define structure functions $f(r)$ as their Fourier transforms.[8] An accurate calculation of $f(r)$ from some theory would require a correct relativistic description of the internal state of the nucleon. Physically, $f(r)$ would contain the overlap of two wave functions, each Lorentz-contracted, but in different directions. (Actually the structure will be described by a relativistic many-body wave function, so the problem is in fact more complicated than this.) Thus, in relating f to the nucleon wave functions, effects of order v^2/c^2 (or q^2/M^2) are introduced. Consequently there will be a dependence of F on q^2 which is in a sense kinematic in origin, in addition to that coming from the finite extent of the internal wave functions. The essential point is that measurement of structure to within a distance d requires values of $|\mathbf{q}|$ of order $1/d$, and if absorption of this momentum causes relativistic recoil (i.e., if $|\mathbf{q}|>Mc/\hbar$) then intuitive concepts of static charge and current distributions are no longer vaild. Since we expect nuclear structure to extend a distance of order $\hbar/\mu c$, there should be a range of $|\mathbf{q}|$ values ($\mu c/\hbar<|q|<Mc/\hbar$) for which the interpretation in terms of static distributions has some validity. A correct relativistic theory for nucleon structure would, of course, avoid these difficulties by allowing a direct calculation of F as a vertex operator.

To the extent that it is possible to interpret F in terms of static charge-current distributions, it is instructive to make a nonrelativistic reduction of (2.5). This is done in the usual way by expressing the small components of the nucleon spinors in terms of the large components ϕ, which are independent of momentum. In the center-of-momentum frame, and for $|\mathbf{q}|\ll Mc/\hbar$, the components of $j_\mu^{p,n}$ become

$$j_0\simeq e(E/M)\phi_2^*\phi_1[F_1-(q^2/8M^2)(F_1+2\kappa F_2)], \quad (2.9a)$$

$$\mathbf{j}\simeq e[(\mathbf{P}'+\mathbf{P})/2M]\phi_2^*\phi_1F_1+(e/2M)\phi_2^*(i\boldsymbol{\sigma}\times\mathbf{q})\phi_1(F_1+\kappa F_2). \quad (2.9b)$$

In (2.9b), the first term represents the convection current, and the second the effect of the magnetic moment. In the expression for j_0, in (2.9a), there is, in addition to the expected term F_1, a kinematic term of order q^2 arising from the overlap of the two spinors. The importance of this term in connection with electron-neutron scattering was noted by Foldy.[9]

[5] L. L. Foldy, Phys. Rev. **87**, 688 (1952); G. Salzman, Phys. Rev. **99**, 973 (1955). An even more general derivation has been given by A. C. Zemach, reference 35. We thank Dr. Zemach for informing us of this work.

[6] This last restriction can be relaxed, as is shown in the Appendix so that Eq. (2.5) holds for any spin one-half particle, e.g. C^{13}. The Dirac spinors are used merely for convenience in representing j_μ in a covariant form.

[7] Hofstadter, Fechter, and McIntyre, Phys. Rev. **92**, 978 (1953); L. I. Schiff, Phys. Rev. **92**, 988 (1953).

[8] We mean here the three-dimensional Fourier transform, obtained by inverting Eq. (2.8).

[9] L. L. Foldy, Phys. Rev. **88**, 693 (1952). It might seem natural to define the spatial distribution of charge as the Fourier transform of the expression in square brackets in j_0. This is not unambiguous, however, as can be seen from Eqs. (A-22) and (A-23).

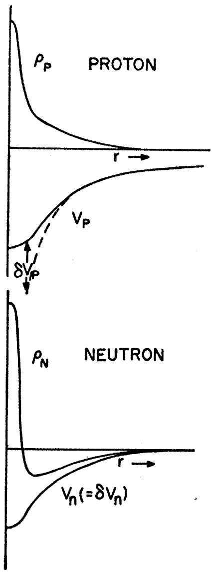

FIG. 2. Intuitive concepts of the neutron and proton charge densities and of the interactions potential between these particles and an electron. The quantities δV represent the departure of these potentials from their values for point nucleons.

For convenience, the functions F can be expanded in powers of q^2. The coefficient of q^2 is simply related to the mean-square radius of the distribution[10]:

$$F(q^2)\simeq 1-q^2\langle r^2\rangle/6+\cdots,\quad \langle r^2\rangle=\int r^2 f(r)d^3r. \quad (2.10)$$

Because of the relativistic complications discussed previously, there may be contributions to the coefficient of q^2 other than those coming from the finite extension. If this extension is of order $\hbar/\mu c$, they will not completely invalidate intuitive considerations based on static models. With these limitations in mind, we note that the mean-square radii of charge and moment distributions of the nucleons are, from Eqs. (2.9),

$$\begin{aligned}\langle r^2\rangle_{\mathrm{ch}}&=\langle r^2\rangle_1+(3/4M^2)[F_1{}^{p,n}(0)+2\kappa],\\ \langle r^2\rangle_{\mathrm{m}}&=[\langle r^2\rangle_1+\kappa\langle r^2\rangle_2]/(1+\kappa).\end{aligned} \quad (2.11)$$

In Fig. 2 are shown the usual concepts of the proton and neutron charge densities, and the interaction potentials between those particles and an electron. The quantities δV represent the departure of these potentials from their values for point nucleons, and they can be related to the mean-square radius by Poisson's equation,

$$\begin{aligned}\int\delta V d^3r&=(2\pi e/3)\int r^2\rho(r)d^3r\\ &=(2\pi e^2/3)\langle r^2\rangle_{\mathrm{ch}}.\end{aligned} \quad (2.12)$$

Electron-Neutron Interaction

Aside from the static limits, the first information about nucleon structure was obtained from experiments on the scattering of neutrons by atoms. Analysis of the results yields the volume integral of the electron-neutron interaction potential, Eq. (2.12). This quantity is conventionally represented by a constant potential, of strength V_0, extending out to a radius $r_0=e^2/mc^2$, the classical electron radius. (This convention is rather confusing since r_0 has nothing to do with this particular interaction.) Hughes *et al.*[11] find for V_0 the value -3860 ± 370 ev, and more recently Melkonian *et al.*[12] report the value -4165 ± 265 ev. Using the mean of these two results, and expressing it in terms of a radius by means of Eqs. (2.11) and (2.12), we find that $\langle r^2\rangle_{\mathrm{ch},n}=-(0.35\times10^{-13}\text{ cm})^2$. This is accounted for completely by the magnetic term of Eq. (2.11), so that

$$\langle r^2\rangle_{1,n}=(0.000\pm0.006)\times10^{-26}\text{ cm}^2. \quad (2.13)$$

Expressed in terms of potentials, as is customary, the magnetic contribution, first calculated by Foldy,[9] is -4070 ev, leaving for the V_0 associated with $\langle r^2\rangle_{1,n}$ the value 0 ± 200 ev. Had the neutron structure been comparable in extent to that found for the proton, a value of about 0.7×10^{-13} cm would be obtained for $\langle r^2\rangle_{1,n}{}^{\frac{1}{2}}$, or for V_0 about 16 000 ev! The conventional interpretation of this very surprising result is either that the radius associated with the Dirac term is very small, or that the structure, if extended, is almost neutral. In the light of our previous remarks on the meaning of $f(r)$, it is also possible (although perhaps unlikely) that the static charge-current distribution is extended, but that its contribution to $f(r)$ has been canceled fortuitously by relativistic corrections.

Another method for examining neutron structure has been suggested, and is being carried out, by Hofstadter.[13] In the inelastic scattering of electrons from the deuteron at high energies and large angles, binding effects are unimportant, and the neutron and proton scatter independently. Since the electron-proton scattering has been measured separately, the electron-neutron cross section can be deduced. This method can give information about $F_2{}^n$, and possibly also about $F_1{}^n$.

Electron-Proton Scattering

The extensive experiments on the scattering of high-energy electrons by hydrogen of Hofstadter *et al*,[14,15] have given much detailed information about the structure functions of the proton. The analysis uses the formula, first derived by Rosenbluth,[16] for the cross section for the scattering of a relativistic electron by a proton.

[10] Except for $f_{1,n}(r)$, all of the f's have unit volume integral because the corresponding $F(0)$ is unity. Because $F_{1,n}(0)=0$, the quantity $\langle r^2\rangle_{1,n}$ depends on the amount of charge displaced as well as on its radius. For reasonable distributions $\langle r^2\rangle_{1,n}$ will be negative, and the other quantities positive.

[11] Hughes, Harvey, Goldberg, and Stafne, Phys. Rev. **90**, 497 (1953).
[12] Melkonian, Rustad, and Havens, Bull. Am. Phys. Soc. Ser. II, **1**, 62 (1956).
[13] R. Hofstadter, Revs. Modern Phys. **28**, 214 (1956).
[14] R. N. McAllister and R. Hofstadter, Phys. Rev. **102**, 851 (1956).
[15] E. E. Chambers and R. Hofstadter, Phys. Rev. **103**, 1454 (1956).
[16] M. Rosenbluth, Phys. Rev. **79**, 615 (1950).

In the laboratory frame this can be expressed as

$$\sigma(\theta)=\sigma_{NS}(\theta)\{(F_1{}^p)^2+(q^2/4M^2)[2(F_1{}^p+\kappa^p F_2{}^p)^2 \times\tan^2\tfrac{1}{2}\theta+(\kappa^p F_2{}^p)^2]\}, \quad (2.14)$$

where σ_{NS} is the cross section for scattering by a point, spinless particle of mass M:

$$\sigma_{NS}(\theta)=e^4\cos^2\tfrac{1}{2}\theta/\{4k^2[1+2(k/M)\sin^2\tfrac{1}{2}\theta]\sin^4\tfrac{1}{2}\theta\}.$$

The terms in the square bracket in (2.14) arise from the magnetic moment of the proton. The interesting fact that one term involves the total moment while the other contains only the anomalous moment is related to the mixing of F_1 and F_2 in the expressions for charge and current density (2.9).

At the present experimental energies $F_1{}^2$ and $(F_1+\kappa F_2)^2$ are the dominant terms of Eq. (2.14). Since the dependence on angle and energy of the quantity in the curly brackets cannot be expressed in terms of $q(=2k_c \sin\frac{1}{2}\theta_c)$ alone, it is possible to separate the contributions from F_1 and F_2 by performing experiments at various energies and angles. If finite size effects are ignored, so that $F_1=F_2=1$, the first term has the very strong angular dependence and k^{-2} energy dependence characteristic of scattering in a Coulomb field. In contrast, the second term is approximately a constant, independent of energy and angle. Thus the charge scattering and hence the effect of finite charge extension is seen at small angles, while the magnetic moment size is apparent at large angles. The situation is illustrated in Figs. 3. and 4. The "total form factor" $\mathfrak{F}(\theta,k)$ defined by

$$\sigma(\theta)=\sigma_{\text{point}}(\theta)[\mathfrak{F}(\theta,k)]^2, \quad (2.15)$$

where σ_{point} is given by Eq. (2.14) with $F_1=F_2=1$, is plotted as a function of q^2. The three different proton models used for illustration assume either equal charge and moment radii, or else that one of these radii is zero.

McAllister and Hofstadter[14] have measured the elec-

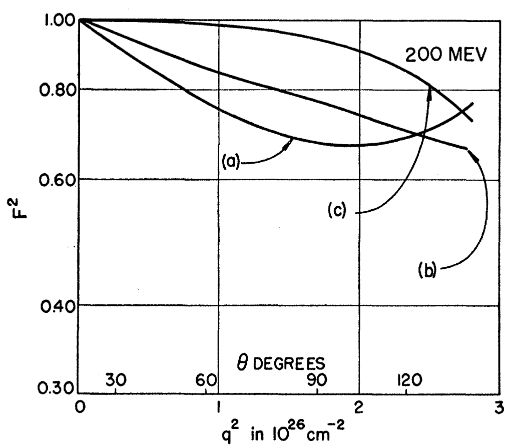

FIG. 3. A plot of the square of the total form factor $\mathfrak{F}$, defined by Eq. (2.15), *vs* q^2, at 200 Mev. Three different proton models are illustrated, corresponding to the following choices for the values of $\langle r^2\rangle_{1,p}{}^{\frac{1}{2}}$ and $\langle r^2\rangle_{2,p}{}^{\frac{1}{2}}$, in 10^{-13} cm: (a) 1.00 and 0.00; (b) 0.70 and 0.70; (c) 0.00 and 0.85.

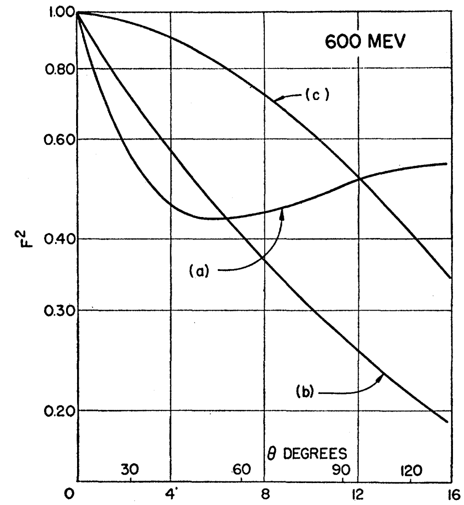

FIG. 4. A plot of $\mathfrak{F}^2$ *vs* q^2 at 600 Mev, for the same cases as in Fig. 3.

tron-proton scattering cross section at 100, 188, and 236 Mev, while more recently experiments at 200, 300, 400, 500, and 550 Mev have been carried out by Chambers and Hofstadter.[15] The detailed analysis of the experiments in terms of F_1 and F_2, and their associated distributions $f_1(r)$ and $f_2(r)$, has been carried out by Hofstadter and his colleagues, and here we shall quote from their results. A feature which makes the fitting with theory a little more flexible than Figs. 3 and 4 might indicate is that the absolute values of the experimental cross sections are not known, although the relative normalization of results for the various energies is known for the latter series of runs. In their analysis, the above authors use the simplifying assumption that the analytic forms of $f_1(r)$ and $f_2(r)$ are identical. The best agreement is found to occur for shapes which are not singular at the center, and which drop off fairly rapidly at large radius. The best fit is then given for equal radii; their actual value depends a little on the choice of shape, but an average value is

$$(\langle r^2\rangle_{1,p})^{\frac{1}{2}}=(\langle r^2\rangle_{2,p})^{\frac{1}{2}}=(0.77\pm0.10)\times10^{-13}\text{ cm}. \quad (2.16)$$

It is possible to have slightly different radii, but neither can be less than about 0.6×10^{-13} cm, or greater than about 1.5×10^{-13} cm. For the case of equal radii the results are illustrated in Fig. 5, where $4\pi r^2 f(r)$ is plotted against r. Chambers[17] has also considered the possibility of different analytic forms for $f_1(r)$ and $f_2(r)$. He finds that it is still not possible to relax the above limits on the radii. The general conclusions are that the proton radii are as given in Eq. (2.16), and that there is no concentration of charge or magnetic moment at the center.

There are possible corrections that might be made to the above analysis, but they are quite unimportant. By

[17] E. E. Chambers, Ph. D. dissertation, Stanford University (1956) (unpublished).

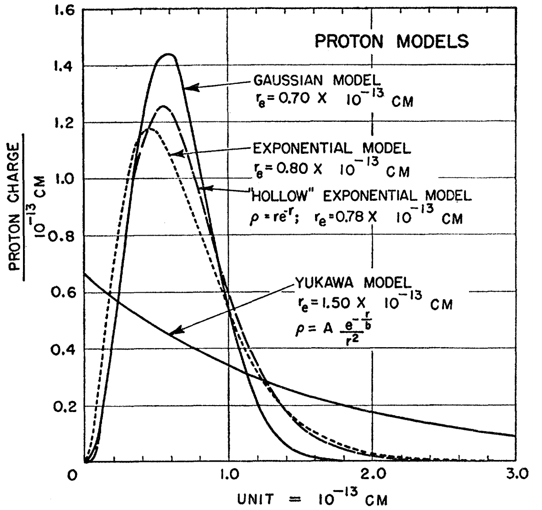

FIG. 5. Graphical representation of proton distribution functions, from the analysis of Chambers and Hofstadter.[15] For the assumption that $f_1(r)=f_2(r)$, these authors find that the Gaussian, exponential and hollow exponential models all give a good fit to the experiments. The Yukawa model, which is suggested by the theory of scalar mesons weakly coupled to nucleons, is given for comparison only; it is in fact inadmissible as a fit to the data. What is plotted *vs* radius r is $4\pi r^2 f(r)$, so that these curves have equal area.

comparing with the results of a partial wave analysis of the scattering process,[18] it is found that the Born approximation used by Hofstadter *et al.* gives cross sections accurate to about 0.25%. The effect on predicted radii is completely negligible. A complication which might influence the interpretation of the form factors F arises from the possibility of virtual excitation of an isobaric state of the proton, corresponding to the resonance in the cross section for photoproduction of mesons. Drell and Ruderman[19] have estimated that the effect of this on the predicted radii is less than about half a percent. The radiative correction of Schwinger[20] has of course been taken into account by Hofstadter *et al.* in their handling of the experimental data.[21] Throughout the analysis it has been assumed that the functions $F(q^2)$ are analytic and "smooth" over the regions of q examined. The electron-proton and electron-neutron experiments are made over completely different ranges of q. It is conceivable, therefore, that the functions F are so peculiar that extrapolation from the one range of q to the other in the simple way that has been assumed is unjustified. A theory that would predict such functions would itself be very peculiar, and we do not consider the possibility further.

Electron-Deuteron Scattering

There exists at present no relativistic theory for the binding of nucleons of finite size to form a nucleus. In the nonrelativistic limit, the solution of the Schrödinger equation describing the interaction of a group of point nucleons is presumably to be interpreted as giving the distribution in space of the centers of mass of the extended nucleons. A difference between this calculated distribution and the measured charge distribution could then be ascribed to finite nucleon size. Use of the nonrelativistic theory unfortunately forces us to ignore the relativistic effects discussed at the beginning of this section.

Experiments on the elastic and inelastic electron-deuteron scattering have been carried out by McIntyre and Hofstadter.[22,23] Following the analysis of Schiff,[24] Jankus[25] has made extensive calculations of both processes, investigating the effect of various assumptions about the neutron-proton potential, and taking into account the effects of magnetic dipole and electric quadrupole moments. Calculations of elastic scattering for particular potentials have been carried out by McIntyre,[23] Bernstein,[26] and Ravenhall.[27] For the detailed comparison of the experiments with these calculations, we quote from the work of McIntyre. He finds that even with the most favorable choice of potential, the experimentally observed charge distribution is significantly more extended than that given by the above calculations. His results are illustrated in Fig. 6.

In the nonrelativistic approximation these calculations give the distribution in space $|\psi_D(r)|^2$ of the centers of mass of the neutron and proton. Neglecting distortions of the nucleon structure due to the binding, the observed deuteron charge density is then

$$\rho(r)=\int[f_{1,p}(|r-r'|)+f_{1,n}(|r-r'|)]\times|\psi_D(r')|^2d^3r'. \quad (2.17)$$

The form factor for electron-deuteron scattering is thus

$$F(q^2)=[F_1{}^p(q^2)+F_1{}^n(q^2)]F_D(q^2), \quad (2.18)$$

where $F_D(q^2)$ is the form factor calculated directly from $|\psi_D(r)|^2$. The factor contributed by the nucleon structure is for small q just

$$[\cdots]\simeq[1-(q^2/6)(\langle r^2\rangle_{1,p}+\langle r^2\rangle_{1,n})+\cdots]. \quad (2.19)$$

On the basis of the naive meson theory discussed in the next section, it was expected that the proton and

[18] Yennie, Ravenhall, and Wilson, Phys. Rev. **95**, 500 (1954), and unpublished calculations.

[19] S. D. Drell and M. A. Ruderman, Phys. Rev. (to be published).

[20] J. Schwinger, Phys. Rev. **76**, 790 (1949).

[21] As in other electron-scattering experiments, the radiative correction is quite large (~20%) but its variation over the angular range used is small (~3%), so that it is not an important correction. The physical situation is different from that considered by Schwinger, in that the proton can emit and absorb photons; but the extra contributions due to this, even that arising from interference with the electron contributions, are negligible.

[22] J. A. McIntyre and R. Hofstadter, Phys. Rev. **98**, 158 (1955).

[23] J. A. McIntyre, Phys. Rev. **103**, 1464 (1956).

[24] L. I. Schiff, Phys. Rev. **92**, 988 (1953).

[25] V. Z. Jankus, Phys. Rev. **102**, 1586 (1956).

[26] J. Bernstein, unpublished calculations. We thank Dr. Bernstein for communication of these results.

[27] Unpublished calculations.

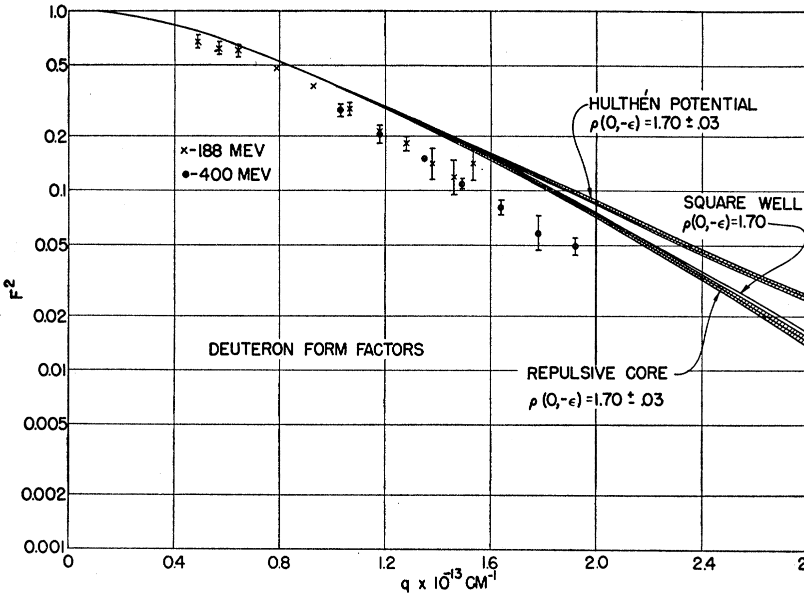

FIG. 6. Experimental and theoretical results of McIntyre on elastic electron-deuteron scattering at 188 and 400 Mev. The theoretical curves include only deuteron S-state contributions; magnetic dipole and electric quadrupole effects are small and would in any case increase F^2. This figure illustrates the fact that the experimental results predict a more extended charge than that obtained assuming point nucleons.

neutron contributions to the q^2 term would cancel, leaving no correction to (2.18) due to the nucleon structure. On the other hand, use of the results of the experiments on the free nucleons means that Eq. (2.19) reduces to the proton form factor. In fact, the analysis of McIntyre strongly favors the latter choice, and can thus be regarded as confirming the difference in size of the neutron and proton. Alternatively, the assumption of nucleon sizes as obtained from the other experiments would give valuable information about the deuteron wave functions.

A possible source of error in the above treatment, associated with the use of the first Born approximation, has been studied by Schiff.[28] In the second Born approximation there is, as well as the usual contribution in which the nucleus stays in its ground state, also a dispersion contribution associated with the possibility of virtual excitation. He shows that the sum of these two effects is of order 1/137 of the elastic scattering, and so is quite unimportant. For the special case of the deuteron, a more exact calculation has been carried out by Volk and Malenka,[29] which confirms his result for the total second Born contribution.

At present, little is known about the accuracy of the nonrelativistic approximation, and of the effect of binding on the nucleon structure. A start to the calculation of relativistic corrections has been made by Blankenbecler,[30] who has treated the deuteron phenomenologically as an elementary vector particle, in analogy with the electron-proton calculation of Rosenbluth.[16] Estimates of mesonic corrections by Bernstein[31] will be discussed in the next section.

[28] L. I. Schiff, Phys. Rev. **98**, 756 (1955).
[29] H. S. Volk and B. J. Malenka, Phys. Rev. (to be published).
[30] R. Blankenbecler, unpublished calculation.
[31] J. Bernstein, Phys. Rev. (to be published).

Hydrogen Spectra

There is known to be a discrepancy of 0.6 Mc between the experimental and theoretical values of the Lamb shift for the $2s_{\frac{1}{2}}$ and $2p_{\frac{1}{2}}$ levels of hydrogen and deuterium.[32,33] Part of this can be ascribed to the finite proton size: according to Eqs. (2.11), (2.16), and (2.12), this produces a shift in the $2s_{\frac{1}{2}}$ level of 0.1 Mc,[34] reducing the discrepancy to 0.5 Mc. In deuterium most of the effect of finite size has already been taken into account with a term corresponding to $F_D(q^2)$ of Eq. (2.18).[33] Because of the small neutron size, the finite proton size gives an additional shift of 0.1 Mc in deuterium, the same as in hydrogen.

According to Zemach,[35] the hydrogen hyperfine structure can be used in combination with other experiments to determine a mean electromagnetic radius $\langle r\rangle_{\rm em}$ of the proton: present experimental values lead to the result that $\langle r\rangle_{\rm em}<0.5\times10^{-13}$ cm. This mean radius depends on both the charge and the moment distributions, the relationship among the distributions being

$$\langle r^2\rangle_{\rm em}=\langle r^2\rangle_{\rm ch}+\langle r^2\rangle_{\rm m}.$$

For the shapes predicted by the electron-scattering experiments, which are not peaked at the center, $\langle r\rangle_{\rm em}$ should not be much smaller than $(\langle r^2\rangle_{\rm em})^{\frac{1}{2}}$. The electron-scattering values for the radii predict, however, that $(\langle r^2\rangle_{\rm em})^{\frac{1}{2}}$ is approximately 1.0×10^{-13} cm. There is an apparent disagreement between these two

[32] Triebwasser, Dayhoff, and Lamb, Phys. Rev. **89**, 98 (1953); this paper contains references to the earlier work of Lamb and co-workers in this field.
[33] E. E. Salpeter, Phys. Rev. **89**, 92 (1953).
[34] The reliability of the perturbation theory has been confirmed by considering exact solutions of the Dirac equation containing a potential of the kind shown in Fig. 2.
[35] A. C. Zemach, Phys. Rev. (to be published). We thank Dr. Zemach for informing us of this work.

determinations of the proton size. We regard electron scattering, however, as an inherently more accurate method for examining nucleon structure.

Nonelectromagnetic Interactions

The preceding discussion, and in fact all previous analyses of electron scattering, have assumed a purely electromagnetic interaction between the electron and the scatterer. Although the agreement between experiment and this theory is remarkably close,[13] it is still desirable to consider the possibility of nonelectromagnetic interactions. As is well known, there are five possible nonderivative interactions,

$$\begin{array}{ll} \bar{u}uV^s & \text{(scalar)}, \\ \bar{u}\gamma_\mu u V^v & \text{(vector)}, \\ \bar{u}\gamma_\mu\gamma_\nu u V^t & \text{(tensor)}, \\ \bar{u}\gamma_5\gamma_\mu u V^{pv} & \text{(pseudovector)}, \\ \bar{u}\gamma_5 u V^{ps} & \text{(pseudoscalar)}, \end{array} \tag{2.20}$$

where the potentials V are of short range. The presence of the vector interaction would not modify the preceding phenomenological analysis, since the finite size effects appear in just this form. For example, the potentials δV of Fig. 2 are $\bar{u}\gamma_4 u V^v$ in this notation. It would alter, however, the interpretation of F_1 and F_2 in terms of nucleon structure; we shall return to this point later. An example of the tensor interaction is given by the electron's anomalous magnetic moment; the Schwinger radiative correction to scattering includes it automatically. For simplicity we will consider first the scalar interaction.

The scalar interaction differs from the interaction with an electrostatic potential by the factor of the Dirac matrix β. For low energy electrons β is effectively unity, and the two interactions will have the same effect. For example, a change of 0.5 Mc in the Lamb shift, mentioned earlier, would require a volume-integral for V^s of 8×10^{-39} Mev cm^3. At high energies ($E\gg Mc^2$) it turns out that there is no interference between the two interactions, and the scattering cross section can be written

$$\sigma(\theta)=\sigma_{\text{em}}(\theta)+\sigma_s(\theta),$$

$$\sigma_s(\theta)=\left[\int V^s d^3r/4\pi\hbar c\right]^2 q^2[F(q^2)] \times[1+2(\hbar k/Mc)\sin^2\tfrac{1}{2}\theta]^{-2}, \tag{2.21}$$

where $F(q^2)$ is the form factor for V^s. By comparing (2.21) with (2.14), we see that σ_s has the same form as the term in σ_{em} involving the total moment, except for an extra factor of q^2. Thus the effect of σ_s is to reduce the observed magnetic moment size. For example, a scalar interaction with the above volume integral would reduce $\langle r^2\rangle_{\text{m}}$ by $(1.7\times10^{-13}\text{ cm})^2$! If, more plausibly, we were to assume that $\langle r^2\rangle_{\text{m}}$ is really $(1.0\times10^{-13}\text{ cm})^2$, then the fact that the phenomenological analysis has given its value as $(0.8\times10^{-13}\text{ cm})^2$ could be ascribed to the presence of a scalar interaction with volume-integral 3×10^{-39} Mev cm^3.

It is probably true that all of the interactions (2.20) behave similarly, i.e., they would all have effects on the cross section indistinguishable from those coming from the finite electromagnetic sizes. It seems clear that there is no interference between the vector interaction and any of the others. Thus, except for a possible anomalous vector interaction, the effect of the interactions (2.20) is to increase the cross section, and, therefore, to reduce the apparent electromagnetic size. From a theoretical point of view, such interactions would be very unpleasant, and we do not regard the possibility very seriously.

3. MESON-THEORETICAL IDEAS ABOUT NUCLEON STRUCTURE

The discussion of this section will be based on the assumption that "physical" nucleons are made of "bare" nucleons and pions interacting in a charge-symmetrical manner. The results of the phenomeonlogical analysis of the previous section will be examined from this point of view in as general a way as possible. Various calculations of nucleon structure, based on particular meson theories, can then be compared with these conclusions. Some effects associated with the presence of K mesons will also be discussed.

The assumption of charge symmetry implies a relationship between proton and neutron structure. Some of the component states of physical nucleons are represented pictorially in Fig. 7. Charge symmetry requires that, whenever a proton state contains a charged meson, there is a corresponding neutron state with a meson of the opposite sign of the charge. (If the interaction is also *charge-independent*, the amplitudes of the states

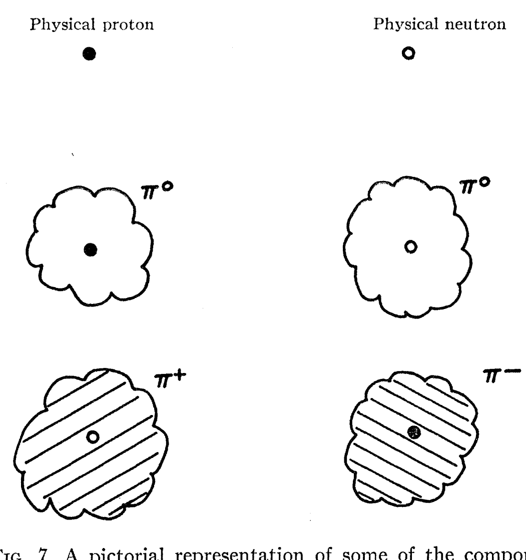

FIG. 7. A pictorial representation of some of the component states of physical nucleons, following the ideas of weak-coupling meson theory. The bare proton is indicated by the black dot, the bare neutron by the open circle.

containing neutral mesons are related to those for the charged mesons.) This is clearly true not only for those component states containing one meson, but for all states. Following Sachs,[36] we can use this result to eliminate the meson part of the nucleon charge density; if we add the proton and neutron charge densities, the meson part cancels completely. The resulting charge distribution, which is a combination of the bare nucleon parts of the proton and neutron charge densities, is called the core distribution;

$$\begin{aligned}\rho_c(r)&=\rho_{c,p}(r)+\rho_{c,n}(r)\\&=\rho_p(r)+\rho_n(r).\end{aligned}\tag{3.1}$$

It includes nucleon pairs as well as single nucleons. From (2.11), the mean square radius of ρ_c is given by

$$\langle r^2\rangle_c=\langle r^2\rangle_{1,p}+\langle r^2\rangle_{1,n}+(3/4M^2)[1+2(\kappa^p+\kappa^n)].\tag{3.2}$$

There may be some doubt about the treatment of the Foldy terms, but fortunately they tend to cancel each other. The experimental values discussed in Sec. 2 then give

$$\langle r^2\rangle_C{}^{\frac{1}{2}}=0.77\times10^{-13}\text{ cm}.\tag{3.3}$$

For comparison, the nucleon compton wavelength is 0.21×10^{-13} cm, and the charge radius of the proton calculated in the above way is 0.84×10^{-10} cm. Thus the core radius is three and one-half times the nucleon Compton wavelength! The results of Chambers and Hofstadter[15] on the shape of the proton charge distribution, which indicate no concentration of charge at small distances, are consistent with this result, and would be inconsistent with a small core radius.

It is very difficult to understand this result. The picture we are considering for nucleon structure would attribute core size to the recoil of the nucleon upon emission of mesons, since the bare particles are assumed to have no intrinsic extension. If the momenta of the emitted mesons are small compared with Mc, by a simple velocity argument the core size will be about one-seventh ($1/7\simeq\mu/M$) of the size of the meson cloud. Under these circumstances the meson cloud extends a distance $\hbar/\mu c$, so that the core should be no bigger than $\hbar/Mc$. If, on the other hand, the virtual mesons are emitted with momenta comparable with Mc, then, because of the relativistic increase in the meson's mass, the core and the meson cloud will be about equal in size, this size being, however, around $\hbar/Mc$. For the states with more than one meson present, the situation is not clear to us. From a semiclassical standpoint, there seem to be a number of possibilities, of which we give two ex-extremes. As a starting point we assume a recoilless theory in which all of the mesons have the same wave function, centered about the fixed source. A possible approximation to the problem with recoil is to neglect its effect on the state function, and to calculate the core radius by assuming the relation

$$MR+\mu(r_1+r_2+\cdots+r_n)=0,\tag{3.4}$$

where R is the coordinate of the bare nucleon, and r is the coordinate of the ith meson, all measured from the position of the center of mass. It is then easy to show that

$$\langle R^2\rangle=\bar{n}(\mu/M)^2\langle r^2\rangle_{\text{meson}},\tag{3.5}$$

where $\bar{n}$ is the average number of mesons present. Clearly this result cannot be correct if $\bar{n}$ is large. It seems to us that a more reasonable assumption is that the meson wave functions are centered on the instantaneous position of the bare nucleon. The result is to replace $\bar{n}$ of Eq. (3.5) by the average value of

$$n/(1+n\mu/M)^2.\tag{3.6}$$

Since this quantity has an upper limit of about $M/4\mu$, $\langle R^2\rangle$ is close to $(\hbar/Mc)^2$. Although these semiclassical arguments are difficult to justify, the second seems to us the more valid. A correct quantum-mechanical version of Eq. (3.4) will contain field operators, which may link states containing different numbers of mesons, so that the final result may differ from what we have given here. A similar situation with regard to the meson cloud has been pointed out by Sachs.[36] On the other hand, many-meson effects on the meson cloud in the Chew-Low theory do not change the one-meson results qualitatively,[37] and the same result may hold with respect to the core.[38]

[36] R. G. Sachs, Phys. Rev. **87**, 1100 (1952).

[37] S. B. Treiman and R. G. Sachs, Phys. Rev. (to be published); S. Fubini, Nuovo cimento **3**, 1425 (1953); H. Suura (private communication). We are informed by Dr. F. Zachariasen that, due to a computational error, the numerical values given in his calculation [Phys. Rev. **102**, 295 (1956)], which are in disagreement with those of the aforementioned authors, are incorrect.

[38] A different opinion on the size of the nucleon core has been put forward by Tamm [I. Tamm, International Congress on Theoretical Physics, University of Washington (1956), reported by N. N. Bogoliubov] and is as follows. In strong-coupling theory the spatial distribution of nucleon-antinucleon pairs will be closely the same as the meson distribution. If a pair annihilates again, it will not contribute to the nucleon core. Those processes in which the antinucleon of the pair annihilates with the original nucleon do contribute, however, and the large cross section for antiproton annihilation in nuclei observed at Berkeley suggests that they can happen even for pairs created relatively far out in the meson distribution. Thus the presence of many bare-nucleon pairs in the physical nucleon can explain a core distribution which is as large as the meson distribution. These qualitative arguments are, we think, open to question on several counts. That the pair distribution follows the meson distribution will only be true for very strong coupling, where there are so many virtual mesons present that an additional amount of many times $2Mc^2$ added to the energy denominator is inappreciable. Yet the work done on the Chew-Low theory would suggest that the coupling is weak. The application of the experimental results relating to physical nucleon pairs to the annihilation of bare nucleons is not obviously justifiable. Also, it is difficult to believe that the pair contribution can be so closely the same as the meson distribution that the electron-neutron interaction cancels from an expected 10 000 volts down to 200 volts. To settle these questions, it is very necessary to make reliable calculations of the core size. As a final note of comfort, it can be pointed out that if Tamm's suggestion is correct, the true theory is a strong-coupling theory involving the emission of many nucleon pairs, and all meson calculations which have neglected pairs are of little value.

From a theoretical point of view, the correct procedure would be to calculate the appropriate vertex operator with a relativistic field theory, taking account of renormalization. The resulting form factor would include all of the relativistic effects discussed in the last section, and which the above physical arguments ignore. Unfortunately, present techniques allow calculation only in the lowest orders of perturbation theory. A summary of the results of such calculations is contained in Sec. 46 of reference 1. Fried[39] has examined the electron-neutron interaction using pseudoscalar coupling in second order perturbation theory. Using $g^2/\hbar c=13.5$, a "reasonable" value, one finds his formula gives 1300 ev for the specific interaction. Combining this work with Rosenbluth's calculations of electron-proton scattering using the same theory and approximation,[16] one obtains for this value of $g^2/\hbar c$ a core radius of 0.38×10^{-13} cm. Thus the theory to this approximation predicts a not unreasonable value for the electron-neutron interaction, but the core radius is too small by a factor two. The reliability of such calculations can be gauged by looking at their predictions for the anomalous nucleon magnetic moments. There the second-order calculation gives for the ratio $-\mu_n/(\mu_p-1)$ about seven, instead of its observed value close to unity. Although the fourth-order calculations improve the situation somewhat, the results are so different from those in second order as to indicate that no valid conclusions can be drawn from such low-order calculations. It is conceivable that if a correct calculation could be done the results would agree with the experiments. We feel, however, that this is unlikely, and that either the physical model used is basically wrong, or the phenomena can be explained in some other way.

Various calculations of neutron and proton form factors have been made using current cut-off meson theories. In these theories the bare nucleon is treated as a fixed source, and the meson-nucleon interaction is modified to include a cut-off function which suppresses the high momenta. Because the mesons, being pseudoscalar, are emitted into p states, their wave function falls off rapidly outside the source, and its size depends sensitively on the source or cut-off distribution. The cut-off function found by Chew[40] to give agreement with other meson experiments (e.g., meson-nucleon scattering meson photoproduction, etc.) leads to a meson cloud of reasonable size ($\sim 0.7\times 10^{-13}$ cm). In order to obtain agreement with the observed nucleon sizes, Salzman[41] has assumed arbitrarily that the core extends as far as the meson cloud. As our previous arguments suggest, it seems unlikely that present meson theories can lead to such a large core. A very necessary improvement, which has not yet been made to our knowledge, is the calculation of the core size with this theory. As a first step it may be sufficient to treat the nucleon nonrelativistically.

It has been assumed previously that these static theories do not include the Foldy terms. A proof of this, given by Salzman,[41] involves a two-component reduction of Dirac spinors, taken in the limit as M tends to infinity, but with the anomalous nucleon moment held at its observed value. A different treatment of the charge-current density, given in the appendix, considers charge and magnetic moment as independent quantities. From this point of view the static calculation of the charge density should give all of the interaction, including what from the other viewpoint are called the Foldy terms. It seems to us that all nonrelativistic theories are ambiguous in their treatment of these terms.

An interesting suggestion about the effect of heavy mesons on nucleon structure has been made by Sandri.[42] Because of the "strangeness" selection rule, the K-meson cloud surrounding the nucleon will contain only positively-charged and neutral particles. Thus in adding proton and neutron charge densities to make the core distribution, the K-meson contributions do not cancel. As Sandri points out, s-state mesons extend further than p-state mesons of the same mass. However, the relatively large K-meson mass together with the small coupling constant lead to only a small addition to the core size [about $(\hbar/Mc)^2$ to $\langle r^2\rangle_c$].

The conclusions about the nucleon core are confirmed by the electron-deuteron scattering. In fact, from Eqs. (2.18) and (3.1), we see that the elastic scattering is a process which, so far as its dependence on nucleon size is concerned, automatically measures just ρ_c. Of the various corrections to this simple nonrelativistic picture, Bernstein[31] has considered the fact that the binding is due to the exchange of mesons, rather than to an instantaneous potential. He identifies the state of two physical nucleons, with no mesons being exchanged, with the phenomenological wave function. The additional contribution to the scattering comes from the state in which one meson is being exchanged. The scattering from the exchanged mesons vanishes by charge symmetry; the effect comes from scattering by the nucleons in this state. The nucleon distribution in the one-meson state is much more peaked at the center than for the no-meson state, and as a result the cross section is enhanced for large q. Bernstein calculates the effect to be more than 10% at the largest q values shown in Fig. 6. It is not clear to us what the phenomenological Schrödinger wave functions are an approximation to, and even if the identification with the no-meson state is valid, the calculation has neglected the effect of states containing two mesons on the one-meson state, which is presumably as important as the effect Bernstein has examined. There may well be an effect of this kind, however.

[39] B. D. Fried, Phys. Rev. **88**, 1142 (1952).
[40] G. Chew, Phys. Rev. **95**, 1669 (1954).
[41] G. Salzman, Phys. Rev. **99**, 973 (1955); Phys. Rev. (to be published).
[42] G. Sandri, Phys. Rev. **101**, 1616 (1956).

4. ALTERNATIVE INTERPRETATIONS OF THE EXPERIMENTS

Since it is not clear that orthodox meson theory will be able to explain the experimental results, we consider in this section possible alternative interpretations.

Charge Symmetry

Present experimental evidence for belief in charge symmetry and charge independence is discussed in Sec. 30 of reference 1. Charge symmetry is a very plausible hypothesis that, according to experimental results, is clearly true to a good approximation, although it is impossible to make a very accurate quantitative check. A possible solution to the dilemma about nucleon structure is that charge symmetry holds for interactions over large distances, but breaks down completely for short distances. This would imply that the fundamental interaction is not charge symmetric; that the large distance effects are charge symmetric is then difficult to understand. The stronger assumption of charge independence contains charge symmetry. It is used in all current theories of phenomena involving nucleon and pions, and plays a fundamental role in theories of the strange particles. It would be a pity to throw out such a beautiful and simplifying hypothesis if any other way can be found out of the difficulty.

Current Conservation

In deriving Foldy's result, Eq. (2.5), it was necessary to assume a differential law of current conservation. Now we might imagine that, since the nucleon has a complicated structure, charge may not be conserved in small regions, but only as a whole. Although such a modification is objectionable because it violates gauge invariance, for the sake of completeness we consider it briefly.

Current conservation is expressed by Eq. (2.6), which in momentum space says that j_μ should be orthogonal to the momentum transfer q_μ. The simplest and most general charge-current density which violates this condition is

$$j_\mu' = -ieq_\mu \bar{v}(P')v(P). \tag{4.1}$$

In the static limit this implies a radial current flow, but gives no contribution to the charge density. If (4.1) is inserted into the matrix element (2.1), the result is zero because of the conservation law obeyed by the electron. Thus, in order to obtain an electron-proton interaction, we must assume that the electron also violates current conservation. Following this through, we arrive at an additional cross section of the same form as $\sigma_s(\theta)$, shown in Eq. (2.21). Thus, as with the anomalous interactions discussed in Sec. 2, violation of charge conservation will not help in interpreting the experiments.

Electromagnetic Interaction

We can easily explain all of the experiments in terms of a modification of quantum electrodynamics at small distances. The high-momentum cutoffs introduced into the theory by Feynman[43] to suppress the ultraviolet divergences have just this effect. Following Feynman, we may introduce this cutoff as a modification in the photon propagator,

$$1/q^2 \rightarrow C(q^2)/q^2. \tag{4.2}$$

From Eqs. (2.1) and (2.5), the effect on the phenomenological analysis is seen to be the replacement of any form factor F by CF; hence these experiments cannot be used to separate finite size effects from those of a possible modification in the Coulomb law. The physical reason for this is that the scattering involves not the charge distribution, whose extension is characterized by F, but its electromagnetic potential V, which depends also on the force law; with the modification (4.2), even a point charge could give a potential of the form shown in Fig. 2.

Because the observed form factors CF_1 and CF_2 in the electron-proton scattering are essentially the same, it is possible to ascribe all of the effects to C, although we expect some intrinsic nucleon size. (If the form factors had been unequal, the two effects would be to some extent distinguishable, in that an upper limit could be put on the radius associated with C, and a lower limit on the actual finite extension.) The modification does not affect the results for the neutron charge radius, since the product $C(q^2)F_{1,n}(q)^2$ is still approximately $(1/6)q^2\langle r^2\rangle_{1,n}$ for small q. For the other experiments—electron-deuteron scattering, Lamb shift, and hyperfine splitting—the effects of finite nucleon size can be reinterpreted as being caused partly or entirely by a modification in the Coulomb law. Such an interpretation makes the reconciliation of the various nucleon properties much easier. With the assumption that, for example, the meson cloud extends only to about $\hbar$/Mc, it is quite plausible that the core and meson distributions are similar enough that the electron-neutron interaction can be as low as 200 ev, especially since the effect of K mesons can be important at this small distance. At the same time the meson clouds in the neutron and proton are charge symmetric, so the near equality of κ^p and $-\kappa^n$ is maintained.

It is hard to devise experiments which would clearly distinguish between a modification in quantum electrodynamics and the effect of finite nucleon size. The most obvious possibility is high-energy electron-electron scattering, but the laboratory energy required to obtain a center-of-momentum energy of 100 Mev is 40 Bev! Processes which involve only real photons, such as Compton scattering, are not affected by a change in the photon propagator [since $C(k^2=0)=1$], although

[43] R. P. Feynman, Phys. Rev. 74, 939 (1948).

particle structure will alter the process. Other electrodynamic processes, such as bremsstrahlung and pair production, usually involve such a small momentum transfer that nuclear size itself is unimportant. In estimating the electromagnetic effects in proton-proton scattering, and in the properties of light nuclei, the two alternatives have slightly different effects, but they would be masked by the greater uncertainty in our knowledge of nuclear forces.

A modification in the Coulomb law would alter slightly the results of other experiments. For example, the radii of nuclear charge distributions deduced from mu-mesic atom level structure and high-energy electron-nucleus scattering would be reduced slightly. For mean square radii the effect is given by

$$\langle r^2\rangle_{\text{obs}}=\langle r^2\rangle_{\text{charge}}+\langle r^2\rangle_c.$$

There would also be some alteration in the nuclear surface thickness.[44]

The theoretical implications of such a modification in quantum electrodynamics have been discussed by Feynman,[45] in an article reviewing the present situation in fundamental theoretical physics, and we will recapitulate some of the points discussed. The renormalization view of the theory regards the cutoffs as mathematical devices to eliminate divergences, with no physical consequences. In contrast to this, Feynman's viewpoint is that they are the manifestation of effects not included in the present theory, which we do not know how to describe in a more fundamental way. According to the former view the theory cannot be used to calculate quantities which depend sensitively on the cutoff—the neutron-proton mass difference, for example. From the latter view, such quantities can be used to give information about the cutoff. Feynman's results for the $N-P$ mass-difference correspond to a cutoff of the same order of magnitude as that required for the electron scattering.

Vacuum polarization affects the photon propagator in the manner indicated by Eq. (4.2), but in the opposite direction to that of a finite size. Feynman brings about the finite size modification described by Eq. (4.2) by introducing "heavy photons." Because their potential must at short distances cancel the Coulomb potential, it is unfortunately necessary that their coupling constant be imaginary. As has been discussed by Feynman, this leads to very fundamental difficulties with regard to conservation of probability. In fact it seems to be impossible to obtain the finite-size effects required from a consistent, point-interaction theory. General arguments lead to an expression for the renormalized photon propagator of the form[46]

$$D_F(q^2)=q^{-2}\left\{1-q^2\int_0^\infty \sigma(\kappa^2)d\kappa^2/\kappa^4(q^2+\kappa^2)\right\}^{-1}, \quad (4.3)$$

where $\sigma(\kappa^2)\geqslant 0$. The form factor must then be of the general form

$$C(q^2)=\left\{1-\int_0^\infty \sigma(\kappa^2)d\kappa^2/\kappa^4 +\int_0^\infty \sigma(\kappa^2)d\kappa^2/\kappa^2(q^2+\kappa^2)\right\}^{-1}. \quad (4.4)$$

Even though it may not be possible to expand the integral of Eq. (4.4) in ascending powers of q^2, it is clear that the whole expression is an increasing function of q^2, and so cannot represent a finite-size effect.

Thus, unless it is possible for pion-nucleon theory to explain the large core size, it seems necessary to make a fundamental revision of present electrodynamic theory. Theoretical arguments concerned with the consistancy of the theory have been advanced by many authors[47] for such a revision, and the nucleon-size experiments may be the first experimental manifestation of this need. It may be necessary to describe nucleons by nonlocal fields, or even to alter our usual concepts of space at small distances.‡

Finite Electron Size

As can be seen from Eq. (2.1), the experiments could also be explained in terms of a finite electron size. Most of the remarks made in connection with the modification of the Coulomb law apply here also. In particular, electron-electron scattering at ultra-high energies would distinguish this possibility from the others. The main objection to this explanation is that there is no reason why the Dirac electron theory should break down at this particular wavelength, since it is clearly valid to wavelengths considerably shorter than $\hbar/mc$, its natural length. We do not regard it as a very likely explanation of the experiments.

[44] For the nuclei examined in Hahn, Ravenhall, and Hofstadter [Phys. Rev. **101**, 1131 (1956)], the nuclear surface thickness is decreased by about 13%, while the value of c (the point where the charge distribution has dropped to a half of its central value) is increased by about one percent.

[45] R. P. Feynman, Anais acad. brasil. Ciênc. **26**, 1 (1954).

[46] Section 25c of reference 1; G. Källén, Helv. Phys. Acta **25**, 417 (1952); H. Lehmann, Nuovo cimento **11**, 342 (1954); J. S. Schwinger, Lectures at Stanford, 1956. We would like to acknowledge the clarification of these points produced by Professor Schwinger's stimulating lectures.

[47] G. Källén, *Proceedings of the CERN Symposium* (Geneva, Switzerland, 1956), Vol. 2, p. 187; L. Landau and I. Pomeranchuk, Doklady Akad. Nauk. U. S. S. R. **102**, 489 (1955) [a review of the work of Landau and colleagues on this subject is given in Pomeranchuk, Sudakov, and Ter-Martirosyan, Phys. Rev. **103**, 784 (1956)]; J. S. Schwinger, reference 46.

‡ *Note added in proof.*—A recent determination of the electron's magnetic moment by P. A. Franken and S. Liebes, Jr. [Phys. Rev. **104**, 1197 (1956)] gives the result that $(\mu_e/\mu_0)_{\text{exp}}=1+(\alpha/2\pi)+(0.7\pm 2.0)(\alpha^2/\pi^2)$. The value of the third term predicted by quantum electrodynamics [R. Karplus and N. Kroll, Phys. Rev. **77**, 536 (1950)] is $-2.973\ (\alpha^2/\pi^2)$. The discrepancy may perhaps be another indication of the breakdown of quantum electrodynamics.

5. SUMMARY

An examination has been made of the present experimental situation regarding the electromagnetic structure of nucleons. It is difficult to understand the remarkable difference in charge radius between the neutron and the proton. Relativistic effects are not expected to be too important, and current meson theories which are charge symmetric seem to us unable to explain the difference. It may be that our physical considerations have leaned too heavily on weak-coupling concepts and results, but a calculation which does not make this approximation, and which at the same time does not neglect recoil, has not yet been made. Apart from this, there seem to be two relatively simple explanations: (i) that charge symmetry does not hold for very small distances; or (ii), that quantum electrodynamics fails at high energies—in other words, that the interaction between two charges is not $(1/r)$ at very small distances. The first alternative would destroy the simplicity of present charge-independent field theories. The second would require a fundamental alteration of present field theory.

ACKNOWLEDGMENTS

In collecting the ideas which form the body of this paper we have benefited greatly from discussions with very many people, both at Stanford and elsewhere. We should like to acknowledge particularly the interest of Professor Robert Hofstadter and Dr. J. A. McIntyre, whose experiments stimulated this investigation. We wish to thank also Professor L. I. Schiff and Professor W. E. Lamb, Jr., for several helpful conversations and Professor Sidney D. Drell for a critical reading of the manuscript.

APPENDIX. GROUP-THEORETICAL TREATMENT OF CHARGE-CURRENT DISTRIBUTIONS OF RELATIVISTIC PARTICLES

There are well-known group-theoretical arguments that nonrelativistic particles of definite spin and parity can possess only certain electric and magnetic multipole moments. In this appendix those arguments will be extended to relativistic particles, and it will be shown in particular that for a spin one-half particle the most general expression of the current density is given by Eq. (2.5).

We shall be interested primarily in matrix elements of the current-density operator between states of definite momentum and spin projection

$$J_\mu(\mathbf{P}',m';\mathbf{P}m)\delta(P'-P-q)=\langle\mathbf{P}',m|j_\mu(q)|\mathbf{P},m\rangle. \quad \text{(A-1)}$$

The physical states of the particle, $|\mathbf{P}m\rangle$, $|\mathbf{P}'m'\rangle$, include all of the effects of the interaction which produces its internal structure; here we need use only the information that these states form a basis for a representation of the inhomogeneous Lorentz group corresponding to a definite spin. The operator $j_\mu(q)$ is the Fourier transform of the current-density operator

$$j_\mu(q)=(2\pi)^{-4}\int j_\mu(x)e^{-iq_\lambda x_\lambda}d^4x. \quad \text{(A-2)}$$

The δ function representing energy and momentum conservation arises from the fact that $j_\mu(x)$ may be expressed

$$j_\mu(x)=e^{iP_\lambda x_\lambda}j_\mu(0)e^{-iP_\lambda x_\lambda}, \quad \text{(A-3)}$$

where P_λ is the energy-momentum operator for the system. In momentum space, charge conservation is expressed by

$$q_\mu j_\mu(q)=0. \quad \text{(A-4)}$$

This imposes a condition on the matrix elements of j_μ

$$(P'-P)_\mu J_\mu(\mathbf{P}'m';\mathbf{P}m)=0. \quad \text{(A-5)}$$

Under a homogeneous Lorentz transformation the state vectors $|\mathbf{P}m\rangle$ and the operators $j_\mu(q)$ transform in a definite way (which fortunately we need consider explicitly only in special simple cases). Thus a knowledge of $J_\mu(\mathbf{P}'m';\mathbf{P}m)$ for one pair of momenta $\mathbf{P}$, $\mathbf{P}'$ and all m, m' will determine J_μ for all other $\mathbf{P}m$, $\mathbf{P}'m'$ that can be reached from the first set by a Lorentz transformation. The totality of such pairs is given by

$$(\bar{P}'-\bar{P})^2=(P'-P)^2=q^2. \quad \text{(A-6)}$$

Thus, for a given q^2, the current density is characterized by at most $4(2s+1)^2$ independent constants: in fact, as we shall see, the actual number of such constants is $(2s+1)$. For a given value of q^2 it now seems appropriate to study the properties of the current density for some particularly simple values of $\mathbf{P}$, $\mathbf{P}'$. The simplest choice seems to be

$$\mathbf{P}'=-\mathbf{P}=\tfrac{1}{2}\mathbf{q}. \quad \text{(A-7)}$$

For simplicity we orient the z axis along $\mathbf{q}$; then since q_0 necessarily vanishes for this combination of momenta, Eq. (5) reduces to

$$J_z(\tfrac{1}{2}\mathbf{q},m';-\tfrac{1}{2}\mathbf{q},m)=0. \quad \text{(A-8)}$$

It will also be convenient to take the direction of spin quantization along the z axis. Then under a rotation about the z axis through an angle ϕ, the states and the operators j_μ will transform according to

$$R_z(\phi)|\pm\tfrac{1}{2}\mathbf{q},m\rangle=e^{im\phi}|\pm\tfrac{1}{2}\mathbf{q},m\rangle \quad \text{(A-9)}$$

$$R_z(\phi)j_0(q)R_z^{-1}(\phi)=j_0(q) \quad \text{(A-10a)}$$

$$R_z(\phi)[j_x(q)\pm ij_y(q)]R_z^{-1}(\phi)=e^{\pm i\phi}[j_x(q)\pm ij_y(q)]. \quad \text{(A-10b)}$$

From this we can easily deduce that (displaying only the m dependence)

$$J_0(m',m)=A(m)\delta_{mm'} \quad \text{(A-11a)}$$

$$J_\pm(m',m)=J_x(m',m)\pm iJ_y(m',m)=B_\pm(m)\delta_{m'm\pm1}. \quad \text{(A-11b)}$$

Further information can be obtained by making the following combination of transformations: (i) space inversion, which leaves m, m' unchanged, but reverses the sign of $\mathbf{q}$ and the spatial components of j_μ; (ii) rotation of 180° about the y axis, which reverses the sign of m, m', but restores the original sign of $\mathbf{q}$ and the x component of j_μ. Since this combination of operations must leave the matrix element unchanged (the specific effect of the parity operation on the states of the particle cancels out because the particle has definite parity), we find the relations

$$A(m)=A(-m)=A(|m|) \tag{A-12a}$$

$$B_\pm(m)=-B_\mp(-m). \tag{A-12b}$$

The minus sign arises from the fact that $R_y(\pi)|\frac{1}{2}\mathbf{q},m\rangle =(-1)^{s+m}|-\frac{1}{2}\mathbf{q},-m\rangle$, according to the usual conventions. The reality properties of the operators

$$j_0{}^*(\mathbf{q})=j_0(-\mathbf{q});\ \mathbf{j}^*(\mathbf{q})=\mathbf{j}(-\mathbf{q}), \tag{A-13}$$

in combination with a space inversion, lead to the further relations

$$A^*(m)=A(m) \tag{A-14}$$

$$B_\pm{}^*(m)=-B_\mp(m+1). \tag{A-15}$$

Thus the A's are real. The reality properties of the B's are not fixed by the present considerations alone, but it can be shown by time-reversal arguments[48] that $B_+(m)=B_+[(m+1)$ for the usual choice of phase of the angular momentum states. When this is combined with Eqs. (A-12b) and (A-15), it is seen that the B's are also real. No additional information can be obtained by considering rotations about the x and y axes since, in contrast to the situation in the nonrelativistic case, a preferred axis ($\mathbf{q}$) enters the definition of the spin states.

We may now illustrate these results for the few lowest spins.

Spin Zero

The m label may be omitted, and for the special choice of momenta we have simply

$$J_0=A(q^2),\quad \mathbf{J}=0. \tag{A-16}$$

These may be expressed in covariant form,

$$J_\mu(P';P)=(1/2M)(P+P')_\mu F(q^2), \tag{A-17}$$

where

$$F(q^2)=[M/(M^2+\tfrac{1}{4}q^2)^{\frac{1}{2}}]A(q^2). \tag{A-18}$$

This is just the form for the current density of a Klein-Gordon particle modified by a form factor $F(q^2)$, which may represent possible internal structure. We emphasize again that it is not necessary to assume the particle to be a Klein-Gordon particle, but only that it is a spin-zero particle.

Spin One-Half

The nonvanishing matrix elements may be written out explicitly

$$J_0(\tfrac{1}{2};\tfrac{1}{2})=J_0(-\tfrac{1}{2};-\tfrac{1}{2})=A(q^2), \tag{A-19}$$

$$J_+(-\tfrac{1}{2};\tfrac{1}{2})=-J_-(\tfrac{1}{2};-\tfrac{1}{2})=B(q^2). \tag{A-20}$$

In order to write these in covariant form, it is convenient to introduce Dirac spinors which transform in the same way as the states $|\mathbf{P}m\rangle$; these are the usual quantities $v_{\mathbf{P}m}$, which for convenience we take to have the relativistic normalization

$$\bar{v}_{\ m}v_{\mathbf{P}m}=1. \tag{A-21}$$

The form of current density given in Eq. (2.5) has the right transformation properties and the arbitrarines necessary to fit the two functions A and B; it is therefore one possible way of writing Eqs. (A-19) and (A-20) covariantly. Any other covariant forms can always be reduced to Eq. (2.5); this analysis shows also that the most general covariant expression for a current which is not conserved is given by Eq. (4.1). The relation between A, B and F_1, F_2 is

$$\begin{aligned} A(q^2)&=e[F_1(q^2)-\kappa(q^2/4M^2)F_2(q^2)],\\ B(q^2)&=2e(|q|/2M)[F_1(q^2)+\kappa F_2(q^2)]. \end{aligned} \tag{A-22}$$

We may also define a "charge" form factor by

$$F_{\text{ch}}(q^2)=[M/(M^2+\tfrac{1}{4}q^2)^{\frac{1}{2}}]\times[F_1(q^2)-\kappa(q^2/4M^2)F_2(q^2)]. \tag{A-23}$$

The factor $M/(M^2+\frac{1}{4}q^2)^{\frac{1}{2}}$ is the reciprocal of the usual E/M factor in the relativistic charge density [see Eqs. (A-17) and (A-18)].

We can now see why there is an ambiguity in calculating the nucleon's charge distribution according to a fixed source meson theory. In the limit $M\to\infty$, there is no distinction between $A(q^2)/e$, $F_1(q^2)$ and $F_{\text{ch}}(q^2)$; however, for finite M we do not know with which quantity the static charge distribution is to be associated. The difference between the first and second possibilities is just the Foldy term, which is not negligible. The difference between the first and third possibilities is associated with the Lorentz contraction of the charge and it is comparatively unimportant for the analysis of the present paper.

Spin One

A new feature arises in the case of spin one in that it is now necessary to specify two constants in order to

[48] This possibility was suggested by Professor S. D. Drell. For a discussion of time reversal, see S. Watanabe, Revs. Modern Phys. **27**, 40 (1955). See also F. Coester, Phys. Rev. **89**, 619 (1953).

determine the charge density completely. The new constant is clearly associated with the possibility of an electric quadrupole moment, so we write the charge density in the form

$$J_0(m';m)=A_1(q^2)\delta_{m'm}+A_2(q^2)(m^2-\tfrac{2}{3})\delta_{m'm}. \quad \text{(A-24)}$$

The second term has been so chosen that it vanishes upon averaging over m. Only one constant is needed to specify the current density in the special Lorentz frame.

This result has applications to electron scattering from deuterium. R. Blankenbecler[30] has performed such a calculation by expressing the current density in a covariant form using the β-matrix formalism.

Spin Three-Halves and Higher

One new constant is needed to describe the current density of a spin three-halves particle. It is associated with the possibility of a magnetic octupole moment and occurs explicitly because $J_+(\frac{3}{2},\frac{1}{2})$ and $J_+(\frac{1}{2},-\frac{1}{2})$ are not related to each other by group-theoretical considerations. In this way, every increase of the spin by one-half will result in the possible addition of an electric or magnetic multipole moment.

Cross-Section Formula

In practical applications we often have to evaluate quantities of the form

$$\langle J_\mu{}^*J_\nu\rangle_{\rm Av}=\frac{1}{2s+1}\sum_{mm'} \times J_\mu{}^*(\mathbf{P}m;\mathbf{P}'m')J_\nu(\mathbf{P}'m';\mathbf{P}m). \quad \text{(A-25)}$$

In the special Lorentz frame it is easy to see that

$$\begin{aligned}\langle J_0{}^*J_0\rangle_{\rm Av}&=\bar{A}^2,\\ \langle J_0{}^*J_i\rangle_{\rm Av}&=0,\\ \langle J_i{}^*J_j\rangle_{\rm Av}&=\begin{cases}\bar{B}^2\delta_{ij}, & i=x,y,\\ 0, & i=z,\end{cases}\end{aligned} \quad \text{(A-26)}$$

where

$$\begin{aligned}A^2&=\frac{1}{2s+1}\sum_m |A(m)|^2,\\ B^2&=\frac{1}{2}\frac{1}{2s+1}\sum_m |B_+(m)|^2.\end{aligned} \quad \text{(A-27)}$$

Defining $\bar{P}_\mu=P_\mu+P'_\mu$, Eq. (A-26) may be expressed in the covariant form

$$\langle J_\mu{}^*J_\nu\rangle_{\rm Av}=\bar{A}^2[\bar{P}_\mu\bar{P}_\nu/(-\bar{P}^2)] +\bar{B}^2[\delta_{\mu\nu}-(q_\mu q_\nu/q^2)-(\bar{P}_\mu\bar{P}_\nu/\bar{P}^2)]. \quad \text{(A-28)}$$

In order to calculate the cross section, a similar sum must be carried out over the electron spins; the result is

$$\langle j_\mu{}^*j_\nu\rangle_{\rm Av}\langle J_\mu{}^*J_\nu\rangle_{\rm Av}=(e^2/2m^2)\{(\bar{A}^2+\bar{B}^2) \times[-q^2-(\bar{P}\cdot\bar{k})^2/\bar{P}^2]+2\bar{B}^2q^2\}, \quad \text{(A-29)}$$

where $\bar{k}_\mu=k_\mu+k_\mu'$. From this the cross section may be obtained directly in any coordinate system by inserting the proper expressions for incident flux, density of final states, etc.

Possible Extensions of the Method

The same method may be applied to the matrix elements for pair production with only changes of detail. In this case $q^2<0$ and it is convenient to choose the frame in which $\mathbf{q}=0$. Then the matrix elements of j_0 vanish, while those of j_z do not. Because the range of q^2 is different from that for scattering, group theory seems to impose no relationship between pair production and scattering. However, if these matrix elements are used directly in higher order perturbation calculations, the result will not usually be gauge invariant (in spite of charge conservation!), and it will be necessary to introduce extra terms to maintain gauge invariance. At present the method seems fruitful only in lowest order perturbation theory.

We may also employ these techniques to calculate the form of the matrix element for systems making a transition (for example, a nuclear transition or photoproduction of pions). For any specific case the procedure would be quite clear, so we shall not present any general rules. Suffice it to say that the result is the relativistic generalization of the usual multipole description of radiative transitions.