Theories of Nuclear Moments

R. J. Blin-Stoyle
Clarendon Laboratory, Oxford England

CONTENTS

1. Introduction
2. Electromagnetic Multipole Moments
   2.1 Magnetic Dipole and Electric Quadrupole Moments
   2.2 Generalized Multipole Moments
   2.3 Selection Rules for Multipole Moments
3. Exchange Currents and Velocity Dependent Forces
   3.1 Exchange Currents
   3.2 Exchange Magnetic Moment Operators
   3.3 Velocity Dependent Forces
4. Nuclear Moments of H^2, H^3, and He^3
   4.1 Electromagnetic Moments of H^2
   4.2 Electromagnetic Moments of H^3 and He^3
5. Nuclear Models
   5.1 Introduction
   5.2 Particle Models
   5.3 Extreme Single-Particle Model
   5.4 Individual Particle Model
   5.4.1 Classification of Nuclear States
   5.4.2 Intermediate Coupling and Configurational Mixing
   5.5 The Collective Model
   5.5.1 Weak Coupling
   5.5.2 Strong Coupling
   5.5.3 Intermediate Coupling and Particle Forces
6. The Magnetic Moments of Odd-A Nuclei
   6.1 Magnetic Moments on the Extreme Single Particle Model.
   6.2 Magnetic Moments on the Individual Particle Model
   6.21 Intermediate Coupling and Interconfigurational Mixing
   6.3 Magnetic Moments on the Collective Model
   6.3.1 Comparison with Experiment
   6.3.2 The Value of g_R
   6.4 Exchange Currents and Velocity Dependent Forces
   6.4.1 Exchange Currents
   6.4.2 Velocity Dependent Forces
   6.5 Conclusions
7. Electric Quadrupole Moments of Odd-A Nuclei
   7.1 Electric Quadrupole Moments on the Extreme Single-Particle Model
   7.2 Electric Quadrupole Moments on the Individual Particle Model
   7.3 Electric Quadrupole Moments on the Collective Model
   7.4 Conclusions
8. Magnetic-Dipole and Electric-Quadrupole Moments of Odd-Odd Nuclei
   8.1 Moments of Odd-Odd Nuclei on Particle Models
   8.2 Moments of Odd-Odd Nuclei on the Collective Model
9. Moments of Excited States of Nuclei
10. Nuclear Magnetic Octupole Moments
11. Conclusions

Tables of Moments of Odd-A Nuclei

I. INTRODUCTION

THERE is now a considerable body of data available about the ground and low-excited states of nuclei. In particular, the spins, parities, and electromagnetic moments of the ground states of most nuclei are known and in many cases the latter have been obtained with considerable precision. This review is concerned with these electromagnetic moments and their interpretation in terms of current nuclear theories.

Experimentally it is found that the nuclear ground-state spins always satisfy the following rules:

(a) All nuclei with Z even and $(A-Z)$ even have zero spin. Here, Z and A are the nuclear charge and mass numbers, respectively.

(b) Nuclei with A odd have spin $(n+\frac{1}{2})\hbar$, where $n=0, 1, \cdots$ and $\hbar$ is Planck's quantum constant divided by 2π.

(c) Nuclei with Z odd and $(A-Z)$ odd have spin $n\hbar$ where $n=0, 1, \cdots$.

Now it is firmly established that an atomic nucleus with charge Ze and mass number A consists of an assembly of Z protons and $N=(A-Z)$ neutrons. For such an assembly the total angular momentum (i.e., the nuclear spin) is the vector sum of the orbital angular momentum of the nucleons together with their intrinsic spins. The orbital angular momentum is restricted to integral values while the intrinsic spins are $\frac{1}{2}\hbar$ and it therefore follows at once that for A odd the resulting spin is half-integral while for A even the spin is integral. However, the fact that nuclei with even numbers of protons and neutrons [case (a)] should always have zero spin in the ground state is not immediately apparent and only follows as a consequence of the form of nuclear forces.

It is further found experimentally that nuclei with nonzero spin have a magnetic dipole moment given by $\mu=gI\mu_0$ where g is known as the nuclear gyromagnetic ratio and μ_0 is the nuclear magneton (n.m.) defined as $e\hbar/2Mc$ (M is the proton mass); μ_0 has the value 5.0429×10^{-24} erg gauss^{-1}.

Nuclei with spin greater than or equal to 1 may also have an electric quadrupole moment which is a partial measure of the deviation of the nuclear charge distribution from spherical symmetry. In addition, nuclei with spin greater than or equal to 3/2 may have a magnetic octupole moment and a few cases in which this has been measured are known. Even higher electromagnetic multipole moments for nuclei with spin greater than 3/2 exist but so far none have been measured. In Secs. 2.1 and 2.2 precise quantitative definitions of the above multipole moments are given.

A detailed discussion of the measurement of nuclear spins and moments is given in the book by Ramsey.[1] Suffice it to say here that the methods can be divided into two broad groups, namely those which depend on the interaction of the nuclear moment with internal atomic or molecular fields and those which depend on the interaction of the nuclear moment with an applied external field. The former methods include measurements of atomic hyperfine structure and are generally not so accurate as the latter methods owing to uncertainties in the form and strength of the atomic fields. The latter methods involve the investigation of either deflections of particle beams in a magnetic field or resonance transitions, induced by an oscillating electromagnetic field, in an applied magnetic field; they enable accurate values for magnetic moments to be obtained. Atomic beam magnetic resonance methods have also been used to obtain magnetic octupole moments. It should also be mentioned here that in a few cases it has been possible to measure the moments of excited nuclear states by investigating the perturbation caused by external fields on the angular correlation between two successive nuclear radiations. The external fields are used to reorientate the spin of the intermediate nuclear state between the radiations and so modifies the angular correlation in a way which depends on the moments of this state and the form of the interaction (see review article by Frauenfelder[2]).

Use of these various methods has enabled many electromagnetic moments to be measured and it is clear that any satisfactory theory of nuclear structure must be able to account for these moments. At this point it is convenient to say a little about the present philosophy of nuclear theories. The nucleus is an exceedingly complicated structure consisting of many particles interacting together through the mechanism of a meson field. Even if the detailed form of the interaction was known present techniques would still not allow the solution of the resulting many-body problem. In point of fact there is little *a priori* information about the way nucleons interact within the nucleus. From nucleon-nucleon scattering experiments a certain amount of information is available as to the form of the interaction between two isolated nucleons. There is no reason to suppose, however, that this interaction is experienced between two nucleons when surrounded by many others and the possibility of many-body forces cannot be ignored. Nevertheless, it is interesting to see if nuclear properties can be interpreted in terms of two-body forces only.* As already pointed out, the many-body problem still cannot be solved even with this restriction. The procedure therefore has been to construct phenomenological models from semiclassical and simple quantum-mechanical considerations and to see to what extent these models can account for the experimental data. In particular any nuclear model must be able to predict the spin, parity, and electromagnetic moments of the ground states. This article is primarily concerned with a discussion of the success, or otherwise, of the various current models in this direction.

The layout of the review is as follows. In Sec. 2 expressions are obtained for the electromagnetic multipole operators and those general results for their expectation values are deduced which are independent of any particular model. Section 3 deals with the possible contribution to nuclear magnetic moments of exchange currents and velocity dependent forces. The nuclear moments of H^2, H^3, and He^3 are amenable to more exact calculations than those of heavier nuclei and are therefore discussed separately in Sec. 4. In Sec. 5 a survey of the nuclear models in current usage is given laying particular emphasis on those points which are relevant to the calculation of nuclear moments. Sections 6 and 7 are concerned with the magnetic dipole and electric quadrupole moments of odd-A nuclei and their interpretation in terms of nuclear models. The magnetic dipole and electric quadrupole moments of odd-odd nuclei are discussed in Sec. 8. In a few cases it has been possible to obtain information about the nuclear moments of excited nuclear states and the magnetic octupole moments of ground states; these aspects are mentioned in Secs. 9 and 10, respectively. The Appendix contains tables of nuclear moments and an indication of the nuclear structure capable of explaining them.

2. ELECTROMAGNETIC MULTIPOLE MOMENTS

2.1 Magnetic Dipole and Electric Quadrupole Moments

An atomic nucleus consists of an assembly of neutrons and protons confined to a region of space which is usually approximately spherical and with radius about 10^{-13} to 10^{-12} cm. The electromagnetic properties could be described completely by specifying the charge and current densities of this assembly. Only the protons can contribute to the former while, in addition to the proton

[1] N. F. Ramsey, *Nuclear Moments* (John Wiley and Sons, Inc., New York, 1953).

[2] H. Frauenfelder, *Beta and Gamma Ray Spectroscopy*, edited by K. Siegbahn (North Holland Publishing Company, Amsterdam, 1955), Chap. XIX.

* If many-body forces are of importance this approach might not be too unsatisfactory since the effect of the many body forces might be represented by an effective two-body force.

currents, the spin currents of both neutrons and protons can contribute to the latter.† Such a description is unwieldy, however, and it is more convenient to describe the electromagnetic properties in terms of the electromagnetic multipole moments.

As far as the magnetic dipole moment is concerned there are contributions from the orbital motion of the protons in the nucleus and from the spins of both neutrons and protons. The magnetic dipole operator $\boldsymbol{\mu}_{\text{op}}$ is then defined by

$$\boldsymbol{\mu}_{\text{op}}=(\boldsymbol{\mu}_{\text{orbital}})_{\text{op}}+(\boldsymbol{\mu}_{\text{spin}})_{\text{op}}, \tag{1}$$

where

$$(\boldsymbol{\mu}_{\text{orbital}})_{\text{op}}=\frac{e\hbar}{2Mc}\sum_{k=1}^{A} g_L{}^{(k)}\mathbf{L}^{(k)}, \tag{2}$$

$$(\boldsymbol{\mu}_{\text{spin}})_{\text{op}}=\frac{e\hbar}{2Mc}\tfrac{1}{2}\sum_{k=1}^{A} g_S{}^{(k)}\boldsymbol{\sigma}^{(k)}. \tag{3}$$

Here $\mathbf{L}^{(k)}$ and $\boldsymbol{\sigma}^{(k)}$ are the orbital angular momentum and Pauli spin operators for the kth nucleon and $g_L{}^{(k)}$ and $g_S{}^{(k)}$ are the orbital and spin gyromagnetic ratios. For a proton $g_L=1$ and $g_S=5.587$ while for a neutron $g_L=0$ and $g_S=-3.826$.

The magnetic moment is then obtained by calculating the expectation value of the z-component of $\boldsymbol{\mu}_{\text{op}}$ for the nuclear substate in which the spin is along the z-axis. Thus, if the wave function for a nucleus with spin I in the magnetic substate M is $\Psi_I{}^M$ then the magnetic moment μ is given by

$$\mu=gI=\int\Psi_I{}^{I*}(\boldsymbol{\mu}_{\text{op}})_z\Psi_I{}^I dV=\langle\mu_z\rangle_{M=I}. \tag{4}$$

The electric quadrupole moment operator Q_{op} is defined by

$$Q_{\text{op}}=\sum_{k=1}^{A} g_L{}^{(k)}(3z_k{}^2-r_k{}^2) \tag{5}$$

where $g_L{}^{(k)}$ is used formally to differentiate between neutrons and protons. The electric quadrupole moment is then conventionally defined as the expectation value of Q_{op} for the nuclear state $\Psi_I{}^I$:

$$Q=\int\Psi_I{}^{I*}Q_{\text{op}}\Psi_I{}^I dV. \tag{6}$$

Now $\Psi_I{}^{I*}\Psi_I{}^I$ represents the density distribution of the nucleus and it is quite easy to show that a prolate (cigar-shaped) charge distribution symmetrical about the z-axis will have a positive quadrupole moment while an oblate (disk-shaped) charge distribution gives rise to a negative quadrupole moment.

† No account is taken here of possible contributions from virtual meson currents etc. These currents can be viewed as modifying the intrinsic spin currents of the nucleons and also contributing a spatial current. They are discussed separately in Sec. 4.

2.2 Generalized Multipole Moments

The foregoing concepts of magnetic dipole and electric quadrupole moments can be generalized to higher multipole moments in the following way.[3]

We define the electric multipole operator of order λ as

$$Q_\lambda=e\sum_{k=1}^{A} g_L{}^{(k)}r_k{}^\lambda P_\lambda(\theta_k)$$

and the magnetic multipole operator of order λ as

$$M_\lambda=\frac{e\hbar}{2Mc}\sum_{k=1}^{A}[\nabla r_k{}^\lambda P_\lambda(\theta_k)]\cdot\left[g_L{}^{(k)}\frac{2}{\lambda+1}\mathbf{L}^{(k)}+\tfrac{1}{2}g_S{}^{(k)}\boldsymbol{\sigma}^{(k)}\right]$$

where $P_\lambda(\theta)$ is a Legendre function.

With these definitions we then see that

$\mu=\langle M_1\rangle_{M=I}$; magnetic dipole moment

$Q=\langle 2Q_2\rangle_{M=I}$; electric quadrupole moment.

It is to be noted that the operator Q_λ has parity $(-1)^\lambda$ while M_λ has parity $(-1)^{\lambda+1}$. Now only operators with even parity will have nonvanishing expectation values for a state of definite parity. But the ground state of a nucleus has a definite parity providing the center of mass is at rest. This therefore implies that only the even electric moments and the odd magnetic moments will be nonvanishing. The first two of these are μ and Q which have already been discussed. Next in order of importance is the magnetic octupole moment Ω and this has been defined by Schwartz[4] as

$$\Omega=-\int\Psi_I{}^I M_3\Psi_I{}^I dV. \tag{9}$$

No moments higher than Ω have been measured and no further reference to them will be made.

2.3 Selection Rules for Multipole Moments

Inspection of the expression for M_λ and Q_λ shows that both M_λ and Q_λ transform under the $D^{(\lambda)}$ representation of the group of real rotations of space. Further for a nucleus of spin I, $\Psi_I{}^M$ transforms under the $D^{(I)}$ representation. This means that the integrand as a whole in the expression for the expectation value of a multipole operator of order λ transforms under the product representation $D^{(I)}\times D^{(\lambda)}\times D^{(I)}$. But in order that the integral shall not vanish this product representation must contain $D^{(0)}$ and for a given λ, the condition for this is that $2I\geqslant\lambda$. Thus for a nonvanishing magnetic dipole moment $(\lambda=1)$, $I\geqslant\frac{1}{2}$; for a nonvanishing electric quadrupole moment $(\lambda=2)$, $I\geqslant 1$; for a nonvanishing magnetic octupole moment $(\lambda=3)$, $I\geqslant\frac{3}{2}$.

[3] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952).

[4] C. Schwartz, Phys. Rev. **97**, 380 (1955).

3. EXCHANGE CURRENTS AND VELOCITY DEPENDENT FORCES

3.1 Exchange Currents

According to current views nuclear forces result from the interaction of nucleons with mesons and since mesons are charged it follows that in a nucleus, in addition to the spin and space currents due to the nucleons, there will also be virtual meson currents present. Such currents will be referred to in the following as exchange currents.

The existence of exchange currents will clearly be of importance for the calculation of the electromagnetic properties of a nucleus and in particular they may be expected to modify the magnetic multipole moments. On the other hand, as suggested by Siegert[5] (Siegert theorem), exchange currents do not appreciably affect the electric multipole moments and these depend very largely on the proton distribution in the nucleus (see Foldy,[6] for example, where a discussion of this point is given). Since so little experimental data are available about magnetic multipole moments other than the dipole moment we shall only consider the effects of exchange currents with reference to the latter.

For any given meson theory of nuclear forces it should in principle be possible to calculate the properties of exchange currents. However, in view of the considerable difficulties with which the treatment and interpretation of meson theories are at present beset, it is more satisfactory to consider those aspects of exchange currents which are independent of any particular meson theory. That is, we consider a phenomenological theory of exchange currents in nuclei.

3.2 Exchange Magnetic Moment Operators

Osborne and Foldy[7] by applying certain invariance and symmetry restrictions on the possible forms of the contributions of exchange currents to the magnetic dipole operator have succeeded in considerably limiting these forms. In addition a further limitation can be imposed by taking account of the fact that the magnetic moment of the deuteron can be entirely accounted for without exchange effects (see Sec. 4 for a more detailed discussion of this point). The basic assumptions made about exchange effects in this approach can then be listed as follows.[8]

(a) The exchange effects can be described in terms of nucleon variables only.

(b) Only static two-body forces exist between nucleons. (The effect of velocity dependent forces is considered briefly in 3.3.)

(c) The exchange effects have a short range (i.e., the dependence on the interparticle distance is taken to be similar to that for nuclear forces).

(d) The exchange magnetic moment operator behaves as an axial vector under translation and rotation and changes sign under time reversal. This latter point is discussed by Kynch[9] and Caianiello.[10]

(e) The exchange magnetic moment operator changes sign under the interchange of neutrons and protons. This is to ensure that there is no exchange contribution to the deuteron magnetic moment.

With these restrictions there are only five possible forms for the exchange magnetic moment operator.[11–13]

$$\mathbf{M}_L=\frac{ie\hbar}{2Mc}\sum_{i,j}(\tau_3(i)-\tau_3(j))(\mathbf{r}_i\wedge\mathbf{r}_j)V(r_{ij},\boldsymbol{\sigma}_i,\boldsymbol{\sigma}_j)P_{ij},\tag{10}$$

$$\mathbf{M}_1=\frac{e\hbar}{2Mc}\sum_{i,j}(\tau_3(i)-\tau_3(j))(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\phi_1(r_{ij}),\tag{11}$$

$$\mathbf{M}_2=\frac{e\hbar}{2Mc}\sum_{i,j}(\tau_3(i)-\tau_3(j))(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\phi_2(r_{ij})P_{ij},\tag{12}$$

$$\mathbf{M}_3=\frac{3e\hbar}{2Mc}\beta^2\sum_{i,j}(\tau_3(i)-\tau_3(j))\times(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\cdot\mathbf{r}_{ij}\phi_3(r_{ij})\mathbf{r}_{ij},\tag{13}$$

$$\mathbf{M}_4=\frac{3e\hbar}{2Mc}\beta^2\sum_{i,j}(\tau_3(i)-\tau_3(j))\times(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\cdot\mathbf{r}_{ij}\phi_4(r_{ij})\mathbf{r}_{ij}P_{ij},\tag{14}$$

where P_{ij} is the space exchange operator for the particles i and j. $\tau_3(i)$ is the third component of the isotopic spin operator for the particle i and has the eigenvalues $\frac{1}{2}$ or $-\frac{1}{2}$ according as i is a neutron or a proton (see Sec. 5.41). In $\mathbf{M}_L$, $V(r_{ij},\boldsymbol{\sigma}_i,\boldsymbol{\sigma}_j)P_{ij}$ represents the actual charge exchange potential between the nucleons. However, in $\mathbf{M}_1\ldots\mathbf{M}_4$, the ϕ's are undetermined radial functions of range $1/\beta$.

$\mathbf{M}_L$ is referred to as the *space* exchange magnetic moment operator since it can be shown to result from a simple irrotational meson current flow between two nucleons, one nucleon acting as a source and the other as a sink, the source and sink having the same strengths. On the other hand, $\mathbf{M}_1\ldots\mathbf{M}_4$ are referred to as *spin* exchange magnetic moment operators.

The spin exchange operators can be given a very simple physical interpretation. They are all linear in the nucleon spins so that in the expression for the magnetic moment of a nucleus including spin exchange effects there will be two terms linear in $\boldsymbol{\sigma}$, one from (3) due to the intrinsic nucleon magnetic moment and one from $\mathbf{M}_1\ldots\mathbf{M}_4$. Thus the effect of the spin exchange

[5] A. J. E. Siegert, Phys. Rev. **52**, 787 (1937).
[6] L. L. Foldy, Phys. Rev. **92**, 178 (1953).
[7] R. K. Osborne and L. L. Foldy, Phys. Rev. **79**, 795 (1950).
[8] R. G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Company Inc., Cambridge, 1953).
[9] G. J. Kynch, Phys. Rev. **81**, 1060 (1951).
[10] E. R. Caianiello, Nuovo cimento **9**, 336 (1952).
[11] A. Russek and L. Spruch, Phys. Rev. **87**, 1111 (1952).
[12] M. Ross, Phys. Rev. **88**, 935 (1952).
[13] N. Austern and R. G. Sachs, Phys. Rev. **81**, 710 (1951).

moment is to modify the intrinsic nucleon moment by an amount dependent on the proximity of other nucleons. In particular $\mathbf{M}_3$ and $\mathbf{M}_4$ lead to a modification of the component of the intrinsic magnetic moment of a nucleon in a direction parallel to the line joining the nucleon to its disturbing neighbor. This has the effect of making the nucleon moments nonadditive.

3.3 Velocity Dependent Forces

If terms appear in the nuclear Hamiltonian which are dependent on the momenta of nucleons in the nucleus, then there can be additional contributions to the nuclear magnetic moment. In the presence of an electromagnetic field the standard prescription is that for all such terms in the Hamiltonian in which a proton momentum $\mathbf{p}$ appears, $\mathbf{p}$ should be replaced by $\mathbf{p}-e\mathbf{A}(\mathbf{r})/c$ where $\mathbf{A}(\mathbf{r})$ is the vector potential describing the electromagnetic field at the position $\mathbf{r}$ of the proton. Such a procedure then ensures that the condition of gauge invariance is fulfilled. Adopting this procedure with a two-body potential of the form $(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\cdot(\mathbf{r}_i-\mathbf{r}_j)\wedge(\mathbf{p}_i-\mathbf{p}_j)J(r_{ij})$, Austern and Sachs[13] have deduced the resulting contribution to the magnetic moment operator. This has a very complicated form, and so far no detailed estimates of its importance have been made.

However, it is possible that, considering one nucleon in the nucleus, the "smeared out" effect of such a potential resulting from all the other nucleons may be represented by a one-body spin-orbit potential of the form

$$-f(r)\boldsymbol{\sigma}\cdot\mathbf{L}=-f(r)\boldsymbol{\sigma}.(\mathbf{r}\wedge\mathbf{p}), \tag{15}$$

where $f(r)$ is a radial function and $\mathbf{L}$ is the single particle angular momentum operator (see Blin-Stoyle[14] where additional references are given).

Replacing $\mathbf{p}$ by $\mathbf{p}-e\mathbf{A}/c$ for a proton gives an interaction energy

$$e/cf(r)(\boldsymbol{\sigma}.\mathbf{r}\wedge\mathbf{A}). \tag{16}$$

For a constant magnetic field H, $\mathbf{A}$ can be written as $\mathbf{A}=\frac{1}{2}(\mathbf{H}\wedge\mathbf{r})$ and the interaction energy then has the form $-\boldsymbol{\mu}'\cdot H$ where

$$\boldsymbol{\mu}'=\frac{e}{2c}f(r)[-r^2\boldsymbol{\sigma}+(\mathbf{r}\cdot\boldsymbol{\sigma})\mathbf{r}]. \tag{17}$$

$\boldsymbol{\mu}'$ is therefore the operator for the additional magnetic moment to be associated with the motion of a proton in a spin-orbit potential.

4. NUCLEAR MOMENTS OF H^2, H^3, AND He^3

Although in general it is necessary to construct a model for the description of nuclear properties, it is possible to be a little more rigorous in the case of the particularly simple nuclei H^2, H^3, and He^3 which consist of only two or three nucleons. For this reason it is more convenient to treat them separately. The calculation and interpretation of their moments is discussed at length in the books on nuclear theory[3,8] and we shall therefore confine ourselves here to stressing the main points of the problem.

4.1 Electromagnetic Moments of H^2

The deuteron has spin 1 and therefore according to the rules set out in 2.3 it may have both a magnetic dipole moment and an electric quadrupole moment. Experimentally it is found that $\mu=0.857354\pm0.000009$ n.m. and $Q=(0.00274\pm0.00002)\times10^{-24}$ cm^2.

If only central forces were operative between nucleons, then the ground-state configuration of the deuteron would be $^3S_1$ so that its magnetic moment would be $\mu(H^2)=\mu(P)+\mu(N)$ where $\mu(P)$ and $\mu(N)$ are the intrinsic moments of the proton and neutron, respectively. This gives $\mu(H^2)=0.87975\pm0.00020$ and there is an obvious discrepancy well outside the experimental error. Further, for a $^3S_1$ state the charge distribution is spherically symmetrical and $Q=0$, again in conflict with experiment. These two discrepancies can be resolved simultaneously by assuming that, in addition to the central force, there is also a noncentral force (e.g., a tensor force) between nucleons. This would then cause an admixture of the $^3D_1$ state and the deuteron ground state would then be $^3S_1+{}^3D_1$. The admixture of the $^3D_1$ state results in a nonspherical charge distribution and therefore a finite Q and also modifies the magnetic moment. Indeed, an admixture of about 4% $^3D_1$ state is sufficient to account for the deuteron magnetic moment. The value of 4% is calculated, however, on the assumption that there are no relativistic corrections to the intrinsic magnetic moments of the proton and neutron and that there are no contributions from exchange currents. Estimates of the relativistic corrections[15,16] are in doubt both as to sign and magnitude but the indications are that they are of the order of a few percent. This means that the admixture of $^3D_1$ state to account for the magnetic moment cannot be set exactly at 4% but rather between say 2% and 6%. On the other hand there is little evidence of exchange effects but they cannot be completely ruled out.

Unfortunately a better estimate of the admixture of $^3D_1$ state cannot be obtained from the electric quadrupole moment even if exchange contributions are negligible.[17] The calculated quadrupole moment has the form[8]

$$Q=\frac{8\pi e}{5}\int[a_Sa_Df_S(r)f_D(r)-a_D{}^2f_D{}^2(r)]r^4dr$$

where a_S and a_D are the amplitudes of S and D states present and $f_S(r)$ and $f_D(r)$ are suitably normalized radial functions for these states. The quadrupole

[14] R. J. Blin-Stoyle, Phil. Mag. **46**, 973 (1955).

[15] R. G. Sachs, Phys. Rev. **72**, 91 (1947).
[16] G. Breit and I. Bloch, Phys. Rev. **72**, 135 (1947).
[17] J. Bernstein and A. Klein, Phys. Rev. **99**, 966 (1955).

moment clearly depends on the detailed behavior of the radial functions, and since these are related to the form of the inter-nucleon potential it is impossible to give definite values to a_S and a_D. It seems fairly certain, however, that it is possible to account for the electromagnetic properties of the deuteron in terms of a ground state consisting of an appropriate mixture of the $^3S_1$ and $^3D_1$ states.

4.2 Electromagnetic Moments of H^3 and He^3

H^3 and He^3 are mirror nuclei both having spin $\frac{1}{2}$. They will therefore have a magnetic dipole moment but no higher moments. Experimentally $\mu(H^3)=2.978643 \pm 0.000028$ n.m. and $\mu(He^3)=-2.127414\pm 0.000003$ n.m. Now $\mu(P)=2.79255\pm 0.000010$ n.m. and $\mu(N) = -1.91280\pm 0.00009$ n.m. and the closeness of $\mu(H^3)$ to $\mu(P)$ and of $\mu(He^3)$ to $\mu(N)$ indicates that the ground states of H^3 and He^3 are predominantly $^2S_{\frac{1}{2}}$. There is, nevertheless, a discrepancy and it is tempting to attribute it (as in the case of H^2) to the admixture of other states. However, it turns out[8,18,19] that no reasonable admixture of states can account for the anomalies. Calculations with two-body forces suggest that the ground state is primarily $^2S_{\frac{1}{2}}$ with a small admixture (about 4%) of $^4D_{\frac{1}{2}}$, whereas an admixture of about 40% $P_{\frac{1}{2}}$ state and little $D_{\frac{1}{2}}$ state is needed to give the correct magnetic moment.

It is possible, however, to give an explanation of the anomaly in terms of exchange effects. With an admixture of about 4% $^4D_{\frac{1}{2}}$ state the outstanding anomalies for H^3 and He^3 are about 0.27 n.m. and -0.27 n.m., respectively. It is satisfactory that the anomalies are equal and opposite since this is consistent with restriction (e) imposed in 3.2 as to the form of the exchange magnetic moment operator. Considering $\mathbf{M}_L$ and $\mathbf{M}_1 \ldots \mathbf{M}_4$ it turns out that the contribution to the moment from the space exchange operator $\mathbf{M}_L$ is negligible since its expectation value vanishes for the $^2S_{\frac{1}{2}}$ state.[19] On the other hand, each of $\mathbf{M}_1 \ldots \mathbf{M}_4$ has a finite expectation value for the $^2S_{\frac{1}{2}}$ state and can, either singly or in combination, account satisfactorily for the moment anomalies of both H^3 and He^3.[13] Villars[20] using a more sophisticated meson theoretical approach, has also interpreted these magnetic moment anomalies in terms of exchange effects of the form considered here.

5. NUCLEAR MODELS

5.1 Introduction

As pointed out in Sec. 1, for all but the very simplest nuclei, the task of obtaining exact wave functions representing the structure of a nucleus is prohibitive even with full knowledge of the internucleon potential energy. The approach adopted, therefore, is to construct nuclear models whose wave functions may bear some relation to those of the nuclei they represent and which can be used for the calculation of nuclear properties. (This point is discussed formally by Eden and Francis.[21]) In the recent section a brief survey is given of the models in current usage for the description of nuclear ground states. Such a survey to be complete requires a separate article and the present one must necessarily be cursory; attention is concentrated primarily on those aspects of the models relevant to the calculation of electromagnetic moments.

The models can be classified as particle models and the collective model. In the former attention is concentrated on the states of individual nucleons while in the latter collective effects are dominant.

5.2 Particle Models

The basic postulate of the particle models (detailed reviews of these models have been given for example by Flowers[22] and Pryce[23]) is that the interaction of any one nucleon within the nucleus with the remaining nucleons can be mainly represented by a static spherical potential well intermediate in shape between an oscillator and a rectangular well, the transition from the former to the latter proceeding as A increases. The states of individual nucleons can then be classified by a set of quantum numbers and determinantal wave functions can be constructed to represent the state of internal motion of the nucleus. The value of this model of the nucleus was not fully appreciated until 1948 when Mayer[24] and independently Haxel, Jensen, and Suess[25] proposed that in addition to the static well there is also a strong spin-orbit coupling force of the form $-f(r)\boldsymbol{\sigma}\cdot\mathbf{L}$ where $\mathbf{L}$ and $\boldsymbol{\sigma}$ are the orbital angular momentum and Pauli spin operators for a single nucleon. States of individual nucleons can then be classified according to the set of quantum numbers (n,l,j,m) where n is the total quantum number, l describes the orbital state, $j=l\pm\frac{1}{2}$ is the total angular momentum quantum number, and m takes the values j, $j-1$, $\ldots -j$, the energy of a given state being determined by n, l, and j.

In a given nucleus these (n,l,j) levels are then supposed to be filled up by protons and neutrons according to the Pauli exclusion principle so that the system has the lowest energy possible. As in the atomic case, this leads to the concept of shells and by suitably adjusting the shape of the well and the sign and strength of the spin-orbit coupling, it is a simple matter to account for the so-called "magic numbers" of nucleons 2, 8, 20, 28, 50, 82, and 126. Nuclei with these numbers of neutrons or protons are particularly stable and have distinctive properties which can be interpreted in terms of the complete filling of certain levels. Figure 1 shows an approxi-

[18] R. Avery and R. G. Sachs, Phys. Rev. **74**, 1320 (1948).
[19] R. Avery and E. N. Adams, Phys. Rev. **75**, 1106 (1949).
[20] F. Villars, Phys. Rev. **72**, 257 (1947); Helv. Phys. Acta. **20**, 476 (1947).

[21] R. J. Eden and N. C. Francis, Phys. Rev. **97**, 1366 (1955).
[22] B. H. Flowers, Progr. Nuclear Phys. **2**, 235 (1952).
[23] M. H. L. Pryce, Repts. Progr. Phys. **17**, 1 (1954).
[24] M. G. Mayer, Phys. Rev. **74**, 235 (1948).
[25] Haxel, Jensen, and Suess, Naturwiss. **35**, 375 (1948).

mate level pattern for nucleons and clearly indicates how the magic numbers arise. It is to be noted that the sign of the spin-orbit coupling has to be chosen so that the state $j=l+\frac{1}{2}$ lies below $j=l-\frac{1}{2}$.

For a completely filled shell or subshell the Pauli principle guarantees that the total angular momentum of the nucleons in this shell is zero. On the other hand, if the shell is only partially filled then a considerable degeneracy exists. However the representation of the internucleon interactions by a static potential well is clearly a crude approximation and it is to be expected that there will be some departure from this approximation which will manifest itself as a residual "effective" interaction between nucleons in the nucleus. This interaction need not be identical with that between free nucleons although it might be expected to have a similar form. It will have the effect of partially or completely resolving the degeneracy referred to previously. The actual manner of resolution in practice, however, is not always certain since, apart from the complexity of the necessary perturbation calculation, there is also the uncertainty as to the form of the effective internucleon potential. Nevertheless, it has been possible, using this particle approach, to obtain substantial agreement between theory and experiment about the properties of the ground and low-excited states of many nuclei.

5.3 Extreme Single-Particle Model

Mayer[24,26] in first postulating the ideas of the shell model made the simple but effective assumption that the internucleon interaction was such that an even number of neutrons or protons in a given level coupled to spin zero while an odd number coupled to the spin j of that level. Thus according to this scheme, all even-even nuclei should have spin zero and even parity (since an even number of particles in any one state must have even parity) as is observed experimentally. Nuclei with an even number of protons (neutrons) and an odd number of neutrons (protons) should have the spin and parity of the last odd neutron (proton) and this rule is also generally obeyed although there are a few exceptions. The spin of an odd-odd nucleus, however, cannot be predicted since the separate angular momenta of the odd neutron and odd proton can be combined to form several different resultant spins (see, however, the empirical coupling rules proposed by Nordheim[27]).

This model is particularly successful in accounting for the spins and parities of the ground states of odd-A nuclei and it is tempting to regard all the ground-state properties of such nuclei as vested, loosely speaking, in the state of the last odd nucleon; the remaining nucleons are then assumed to couple to zero spin and to form an inert core. The nuclear wave function can then be regarded as having the form $\Psi_j{}^m=\psi_{nlj}{}^m\Omega_0$ where Ω_0 is the core function and $\psi_{nlj}{}^m$ is a single-particle

[26] M. G. Mayer, Phys. Rev. **78**, 16 (1950); **78**, 22 (1950).
[27] L. W. Nordheim, Revs. Modern Phys. **23**, 322 (1951).

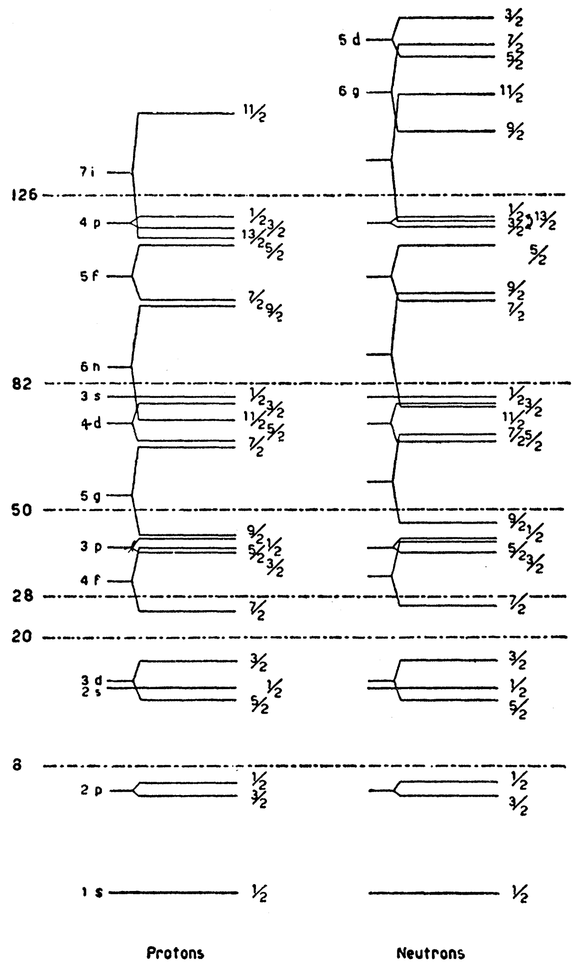

FIG. 1. Nuclear shell structure [from P. F. A. Klinkenberg, Revs. Modern Phys. **24**, 63 (1952)].

function.‡ Explicitly

$$\psi_{nlj}{}^m=\sum_\sigma C(l\tfrac{1}{2}j;\, m-\sigma\sigma)\chi^\sigma f_{nl}(r)Y_l{}^{m-\sigma}(\theta,\phi) \quad (18)$$

where $C(l\frac{1}{2}j;\, m-\sigma\sigma)$ is a Clebsch-Gordan coefficient, $\chi^\sigma(\sigma=\pm\frac{1}{2})$ is the nucleon spin function, $f_{nl}(r)$ is a radial function whose exact form is dependent on the shape of the potential well considered and $Y_l{}^{m-\sigma}(\theta,\phi)$ is a spherical harmonic. A function such as $\Psi_j{}^m$ can then be used, for instance, to calculate nuclear electromagnetic moments. Similarly a wave function describing an odd-odd nucleus can be constructed by combining two single particle functions to form a state of given total angular momentum (Sec. 8.1).

Clearly the extreme single-particle model is a considerable oversimplification of the state of a nucleus. It is nevertheless a convenient reference model and accounts for the bulk of the spin and parity assign-

‡ Strictly $\Psi_j{}^m$ should be completely antisymmetrical, but since we are interested in the expectation values of single particle operators no exchange terms can arise.

ments of nuclear ground states. The fact that high spins (e.g., 9/2, 11/2, 13/2) predicted by this model are not generally observed as ground states can be accounted for by the introduction of the additional concept of "pairing energy." The idea is that when two nucleons in a given level are paired off to spin zero they gain (negative) pairing energy whose magnitude increases with the angular momentum of the particular level. This means that, energetically, when the various levels are being filled, it may be more favorable for nucleons to be paired off in high angular momentum states at the expense of raising a nucleon from an energetically lower filled level with smaller spin. Such a nucleus will then have the spin of the lower level. Apart from this type of spin anomaly, there are a few others which cannot be interpreted in terms of the single particle model. For example Na^{23} has 12 neutrons and 11 protons and there are nominally 3 protons in the $1d_{5/2}$ state so that on the single particle model Na^{23} should have spin 5/2. The measured spin is 3/2 which can only be satisfactorily interpreted as the coupling of several particles in the $d_{5/2}$ state to spin 3/2. This and similar anomalies can probably be accounted for by relaxing the extreme restrictions of the single particle model as described in the following section.

5.4 Individual Particle Model

Calculations with any reasonable type of effective internucleon potential indicate that the conditions imposed in the extreme single particle model are far too restrictive. The less restrictive assumption is made in the individual particle model that the nuclear ground state properties are now interpreted in terms of *all* the nucleons which lie outside closed shells. For example Cl^{35} has one $1d_{5/2}$ proton and two $1d_{5/2}$ neutrons outside closed shells. The neutrons can couple to angular momentum 0 or 2 on the individual particle model§ whereas on the extreme single-particle model only the former is allowed. Thus as far as the ground state is concerned this means that perturbation procedures must be used to obtain the lowest energy state of a number of particles (those outside closed shells) interacting through an effective internucleon potential. This is manifestly a problem of some magnitude. Fortunately the labor in its solution can be considerably reduced in many cases, particularly for light and medium-heavy nuclei, by introducing a system of classification of nuclear states according to various symmetry properties.

5.4.1 Classification of Nuclear States

The classification of nuclear states to be discussed is important as will be seen, because specifically nuclear forces have a short range and are approximately charge independent. Because of electrostatic forces and the neutron proton mass difference the nuclear Hamiltonian is not completely charge independent. However, for light and medium-heavy nuclei, Radicati[28,29] has shown that charge independence is a good approximation and it is therefore convenient in the nucleus to treat neutrons and protons on an equal footing. This is conveniently done by assigning an "isotopic spin" $t=\frac{1}{2}$ to a nucleon and denoting a neutron by the substate $m_t=+\frac{1}{2}$ and a proton by the substate $m_t=-\frac{1}{2}$. Formally the isotopic spin is then treated exactly as the ordinary intrinsic spin and a nucleon now has associated with it a two-component isotopic spin function as well as an ordinary spin function. It can then be shown|| that a correctly antisymmetrized function representing an assembly of nucleons is one which is completely antisymmetric under interchange of space, ordinary spin and isotopic spin variables. Such a state in jj-coupling will then be characterized by J, M, and T, M_T where J is the total angular momentum, M its z-component, T the total isotopic spin, and M_T its z-component. The importance of the concept of charge independence is that formally this corresponds to the statement that the nuclear Hamiltonian is invariant with respect to rotations in isotopic spin space so that $(T^2)_{\mathrm{op}}$ and $(T_3)_{\mathrm{op}}$ commute with it. This means that T and M_T are "good" quantum numbers and that nuclear states can be classified in terms of them. Even if nuclear forces are not charge independent M_T is still a good quantum number since it is related to the charge Z by $Z=\frac{1}{2}(A-2M_T)$ which is necessarily a constant of the motion.

Now for two nucleons the properties of a state are completely described by J, M, T, and M_T; for more than two nucleons however, this is generally not so and it is necessary to introduce other concepts in order to classify states more completely. This further classification is particularly simple when the nucleons under consideration are "equivalent" particles.[30] By "equivalent" particles is meant particles which have the same values of n, l, and j, each having a wave function $\psi_{nlj}{}^m\omega^{m_t}$. To classify the states in jj-coupling two new quantum numbers s and t are introduced referred to respectively as the *seniority number* and the *reduced isotopic spin* which characterize the properties of the wave function for n particles (say) under symplectic transformation.¶ For complete understanding of these ideas a sound knowledge of group theory is required and reference should be made to the papers by Flowers[30–32] for further details. Suffice it to restate here that the wave function for n equivalent particles in the states (nlj) can be written as $\Phi(j^nTM_T; st; \alpha JM)$

§ The spin states 1 and 3 are not allowed by the exclusion principle.

[28] L. A. Radicati, Proc. Phys. Soc. (London) **A66**, 139 (1953).

[29] L. A. Radicati, Proc. Phys. Soc. (London) **A67**, 39 (1954).

|| A detailed discussion of the isotopic spin formalism is given by Sachs.[8]

[30] B. H. Flowers, Proc. Roy. Soc. (London) **A212**, 248 (1952).

¶ A symplectic transformation is one which leaves invariant the antisymmetric bilinear form $\Sigma_m(-)^{m-\frac{1}{2}}\psi_i(m)\psi_k(-m)$ where $\psi_i(m)$ is a single-particle wave function for the particle i.

[31] A. R. Edmonds and B. H. Flowers, Proc. Roy. Soc. (London) **A214**, 515 (1952).

[32] A. R. Edmonds and B. H. Flowers, Proc. Roy. Soc. (London) **A215**, 120 (1952).

where α represents any further labels necessary to completely classify a state.

The virtue of this classification of states is as follows.[31,32] In the limit of short range forces the energies of the different possible states of the n particles are determined primarily by n, T, s, and t; that is, s and t are nearly good quantum numbers. In particular for a given T the lowest level will have $s=0$ or 1 according as n is even or odd and this imples that $J=0$ or j according as n is even or odd. Thus the main feature of the single particle model is reproduced in this more general approach. In addition for finite range forces there is every indication that spin anomalies of the type occurring in Na^{23} can also be satisfactorily explained. Using such n particle ground state configurations it is then possible to calculate the expectation values of electromagnetic moment operators.

This system of classification, of course, is only of use when the neutrons and protons are filling the same shells, that is, for light and medium-heavy nuclei. Now for light nuclei, the extreme jj-coupling approximation is not completely satisfactory (see Sec. 5.42) and for very light nuclei it appears that the wave functions are nearer the LS-extreme. In this approximation the total orbital and total spin angular momenta L and S are good quantum numbers and the spin-orbit coupling is assumed to be small. Fortunately, as has been shown, for example by Wigner,[33] Racah,[34] Jahn,[35–37] and Flowers,[38] it is possible to classify states according to their symmetry properties under permutation of the component particles. The classification is again of a complicated nature and reference should be made to the foregoing papers for details. One can, however, state the following general result for the ground states of nuclei in LS-coupling. If internucleon forces are mainly short-range Wigner and Majorana (i.e., ordinary plus space exchange) then even-even nuclei should be in ^1S-states and odd-A nuclei should be in the state resulting from the addition of a single particle to a spherically symmetrical even-even core. Thus for odd-A nuclei an effectively extreme singe-particle model is again predicted.

Now in the case of heavy nuclei the situation is much more complicated. For such nuclei electrostatic effects are important as is shown by the fact that as A increases so does the ratio of neutrons to protons. Thus, in heavy nuclei, the isotopic spin is no longer a good quantum number and in addition, because of the preferential filling of neutron levels, the neutrons and protons outside closed shells are no longer equivalent particles. A simple classification of states can therefore no longer be given. Only in the region of Pb^{208} is it possible to make more than superficial individual particle type calculations of the distribution and properties of nuclear energy levels.[39,40] Lead-208 is a doubly (magic) closed shell nucleus so that the immediately neighboring nuclei can be described primarily in terms of one or two particles (or holes) and the possible states are completely classified according to their angular momenta. In addition the jj-extreme seems to be a good approximation in this region.

5.4.2 Intermediate Coupling and Configurational Mixing

As pointed out in the previous paragraph, for very light nuclei the LS-coupling scheme seems, if anything, closer to reality than the jj-scheme. In the former the single-particle spin-orbit coupling force is neglected while in the latter the residual effective two-body internucleon forces are neglected. There is every indication that neither coupling scheme can give a completely satisfactory description of nuclear properties and that the situation lies somewhere between the two moving from the LS-side towards the jj-side as A increases. The situation is then referred to as intermediate coupling.

To investigate the structure of nuclear levels under these circumstances the procedure is as follows. A basic set of states is taken and the energy matrix of the nuclear Hamiltonian (which now contains two-body interactions and a spin-orbit potential) is formed. The diagonalization of this matrix then enables the energy eigenvalues and the structure of the various states (in particular the ground state) to be obtained. For light nuclei it is usual to take the basic set of states to be those for LS-coupling (see, for example, the calculations by Lane[41–44]) and to express the results in terms of an intermediate coupling parameter which measures the relative importance of the two types of coupling. It is found for light nuclei that many ground- and excited-state properties can be well represented by an intermediate coupling situation. Few calculations have been performed for heavier nuclei but here it is probably more convenient to start from the jj-extreme.[45,46]

We have seen that the residual effective internucleon forces lead to mixing of the two components of a spin-orbit doublet and the situation of intermediate coupling. They will also lead to mixing of other configurations which have the same parity and angular momentum as the original one and this effect, referred to as configurational mixing, will be of particular importance when the configurations are energetically close to one another.

[33] E. P. Wigner, Phys. Rev. **51**, 106 (1937).
[34] G. Racah, Phys. Rev. **76**, 1352 (1949).
[35] H. A. Jahn and H. van Wieringen, Proc. Roy. Soc. (London) **A209**, 1952 (1951).
[36] H. A. Jahn, Proc. Roy. Soc. (London) **A201**, 516 (1950).
[37] H. A. Jahn, Proc. Roy. Soc. (London) **A205**, 192 (1951).
[38] B. H. Flowers, Proc. Roy. Soc. (London) **A210**, 497 (1952).
[39] M. H. L. Pryce, Proc. Phys. Soc. (London) **A65**, 773 (1952).
[40] D. M. Brink, Proc. Phys. Soc. (London) **A67**, 757 (1954).
[41] A. M. Lane, Proc. Phys. Soc. (London) **A66**, 977 (1953).
[42] A. M. Lane and L. A. Radicati, Proc. Phys. Soc. (London) **A67**, 167 (1954).
[43] A. M. Lane, Proc. Phys. Soc. (London) **A68**, 189 (1955).
[44] A. M. Lane, Proc. Phys. Soc. (London) **A68**, 197 (1955).
[45] R. J. Blin-Stoyle, Proc. Phys. Soc. (London) **A66**, 1158 (1953).
[46] R. J. Blin-Stoyle, and M. A. Perks, Proc. Phys. Soc. (London) **A67**, 885 (1954).

For instance Elliott and Flowers[47] have shown that to account for the properties of mass 18 and 19 nuclei configurational mixing between the near lying $2s$ and $1d$ shells has to be taken into account. As the number of possible configurations grows this effect is expected to become increasnigly important. In particular, for nuclei far removed from a closed-shell structure the situation becomes completely out of hand and present techniques are quite incapable of dealing with the problem from the individual particle point of view.

5.5 The Collective Model

For nuclei in the region of closed shells it is to be expected that the equilibrium shape of the nucleus is approximately spherical and that the nucleons move in an essentially spherical potential well. On the other hand, in regions far removed from closed-shell configurations this is no longer the case and the effect of the many "loose" nucleons is that the nucleus finds it energetically more favorable to take up a nonspherical shape. There can then be *collective* oscillations about this equilibrium shape which become increasingly important the larger the distortion. These oscillations will modify the effective nuclear field and so be strongly coupled to the motion of the nucleons. Thus, near closed shells, nuclei should be fairly satisfactorily described by an individual particle approach, whereas far away from closed shells collective motion is of importance and, in fact, it transpires that a relatively simple description of nuclei in such regions can be given in terms of collective coordinates (see the review by Bohr and Mottelson[48]). Such regions are, moreover, just those where the purely individual particle description is completely inadequate.

The most important oscillations are those of order 2 associated with an ellipsoidal type deformation. For such a deformation the nuclear surface can be defined by $R(\theta)=R_0(1+\beta Y_2^0(\theta))$ where θ is measured relative to axes fixed in the nucleus, β is a measure of the deformation and R_0 is the equilibrium radius. β can be expressed as $4/3(\pi/5)^{\frac{1}{2}}\Delta R/R_0$, where ΔR is the difference between the major and minor axes of the ellipsoid.

The oscillations of order 2 can be quantized according to standard field theoretic treatments[49] and this process leads to the concept of quantized surface vibrations (surfons) of energy $\hbar\omega$ and each having an angular momentum of two units. The energy $\hbar\omega$ of a surfon can be estimated on hydrodynamical grounds from the known properties of the nuclear fluid evidenced in the semiempirical mass formulas and is found to decrease with increasing A ($\hbar\omega$ varies roughly from 10 Mev to 1 Mev between $A\sim 10$ and $A\sim 250$). After quantization β is found to be linearly related to creation and annihilation operators for surfons. For a small number ($N\leqslant 3$) of surfons, the state of the nuclear surface is then completely defined by N and R where R is the total angular momentum of the surface.

Now in addition to the vibrations around the equilibrium shape, for large deformations the system may rotate as a whole preserving both its shape and internal structure. The rotation, however, is not a rigid rotation and the moment of inertia $\mathbf{I}$ associated with it is generally no greater than and usually much less than one-half the moment of inertia for rigid rotation.[50] Associated with this rotation is a sequence of rotational energy levels with energies given by

$$E_{\rm rot}=\frac{\hbar^2}{2\mathbf{I}}[J(J+1)-J_0(J_0+1)],$$

where J and J_0 are the spins of a given level and the ground state, respectively. Such levels have been well identified in the regions $150<A<190$ and $A>225$ which are both well away from closed shells. Details of rotational levels, transition probabilities between such levels etc. are given in the article by Bohr and Mottelson[48] where further references can be found.

From the point of view of nuclear moments we are mainly concerned with the collective aspects of nuclear ground states and in particular with the angular momentum coupling schemes. For their description it is convenient to consider the two extremes of *weak* and *strong* coupling.

5.5.1 Weak Coupling

Weak coupling applies in the regions not far removed from closed shells where the deformation is relatively small. Under these conditions the bulk of the nucleons contribute only to the collective motion of the nucleus; on the other hand the particles in the last filled levels must be treated on a separate footing. For a deformation β the particles can be assumed to move in a distorted potential of the form

$$V[r(1+\beta Y_2^0(\theta))]\simeq V(r)+\beta Y_2^0(\theta)(dV(r)/dr)$$

for small β. dV/dr is only expected to be appreciable near the nuclear surface and the term in β can be regarded as representing a particle-surface coupling. It has the effect of coupling particle states which have the same parity and which differ in angular momentum by not more than two units. In particular it couples the two components of a spin-orbit couplet. Since β is linear in the surfon creation operator it follows that to first order a single-particle state can have admixed a state representing one surfon and the same or a different particle state. Estimates of the amplitudes of such admixtures can be made using the hydrodynamical

[47] J. P. Elliott and B. H. Flowers, Proc. Roy. Soc. (London) **A229**, 536 (1955).
[48] A. Bohr and B. R. Mottelson, *Beta and Gamma Ray Spectroscopy*, edited by K. Siegbahn (North Holland Publishing Company, Amsterdam), Chap. XVII.
[49] A. Bohr and B. R. Mottelson, Dan. Mat. Fys. Medd. **26**, No. 14 (1952).
[50] A. Bohr, Kgl. Danske Videnskab. Selskab Mat.-fys. Medd. (to be published, 1955).

model.[51] Physically this corresponds to a sharing of the total angular momentum of the nucleus between the particle and the surface.

5.5.2 *Strong Coupling*

In the strong coupling approximation, which is associated with large β and to be expected in the regions $150<A<190$; $A>225$, the situation is analogous to that of linear molecules. The surface will generally be axially symmetric and the individual particles are coupled separately to the symmetry axis in states characterized by their component of angular momentum Ω_i along the symmetry axis (Fig. 2 where $\Omega=\sum_i\Omega_i$). It should be noted that because of the axial symmetry the particle states $+\Omega_i$ and $-\Omega_i$ are degenerate and that energetically particles will fill pairwise in these states. The surface also rotates as a whole and this rotation is characterized by quantum numbers I, K, and M where I is the total angular momentum of surface plus particles, K its projection on the symmetry axis, and M its projection on a fixed axis in space. In the ground state it can be shown that $K=\Omega$ (Fig. 2) and that R, the surface angular momentum, is perpendicular to the symmetry axis.[51]

For large β the coupling of different shell model states to the surface may be considerable and the nucleon states can no longer be classified according to their j-values. The only "good" quantum numbers for the nucleons are now Ω_i and parity. A perturbation procedure is usually no longer valid and to obtain the forms of the nucleon wave functions, solutions have to be obtained for particle motion in an ellipsoidal potential. Such solutions have recently been given by Moszkowski[52] and Nilsson.[53]

5.5.3 *Intermediate Coupling and Particle Forces*

The situation in many nuclei will clearly be intermediate between the weak and strong coupling extremes. In addition there is the further complication in all cases of the effect of the residual effective interparticle forces. Many nuclei, then, require a highly complex description and no simple coupling scheme is available. Calculations for intermediate coupling for particular cases have been given[51] but so far the situation in the very complicated cases can only be described in a qualitative fashion.

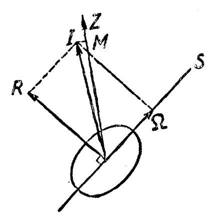

FIG. 2. Coupling scheme for strongly deformed nuclei.

[51] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab. Mat.-fys. Medd. **27**, No. 16 (1953).
[52] S. A. Moszkowski, Phys. Rev. **99**, 803 (1955).
[53] S. G. Nilsson, Kgl. Danske Videnskab. Selskab. Mat.-fys. Medd. **29**, No. 16 (1955).

TABLE I. The Schmidt values for nuclear magnetic moments.

Odd proton nuclei				Odd neutron nuclei			
$j=l+\frac{1}{2}$		$j=l-\frac{1}{2}$		$j=l+\frac{1}{2}$		$j=l-\frac{1}{2}$	
State	μ_s	State	μ_s	State	μ_s	State	μ_s
$s_{1/2}$	+2.793			$s_{1/2}$	−1.913		
$p_{3/2}$	+3.793	$p_{1/2}$	−0.264	$p_{3/2}$	−1.913	$p_{1/2}$	+0.638
$d_{5/2}$	+4.793	$d_{3/2}$	+0.124	$d_{5/2}$	−1.913	$d_{3/2}$	+1.148
$f_{7/2}$	+5.793	$f_{5/2}$	+0.862	$f_{7/2}$	−1.913	$f_{5/2}$	+1.366
$g_{9/2}$	+6.793	$g_{7/2}$	+1.717	$g_{9/2}$	−1.913	$g_{7/2}$	+1.488
$h_{11/2}$	+7.793	$h_{9/2}$	+2.624	$h_{11/2}$	−1.913	$h_{9/2}$	+1.565
$i_{13/2}$	+8.793	$i_{11/2}$	+3.560	$i_{13/2}$	−1.913	$i_{11/2}$	+1.619

6. THE MAGNETIC MOMENTS OF ODD-A NUCLEI

We now consider the values of the nuclear magnetic moments of odd-A nuclei and investigate their possible interpretation in terms of the foregoing nuclear models. The procedure to be adopted is to show to what extent the different models are capable of accounting for the observed magnetic moments. No attempt is made to discuss every measured moment. In the appendix at the end of the article, however, nuclear moments are tabulated and an indication is given of how each moment can be interpreted in terms of one or another nuclear model.

6.1 Magnetic Moments on the Extreme Single-Particle Model

On the extreme single-particle model described in Sec. 5.3, the nuclear ground-state properties are vested in a single particle whose state can be written in the form $\psi_{nlj}{}^m$. Using this wave function to calculate the expectation value of the magnetic dipole operator (1) as in (4) we see that only one term in the sum given there contributes, namely the one corresponding to the single particle. A straightforward calculation gives

$$\mu_s=gj=\tfrac{1}{2}j\left[(g_L+g_S)+(g_L-g_S)\frac{l(l+1)-\frac{3}{4}}{j(j+1)}\right], \quad (19)$$

where $j=l\pm\frac{1}{2}$ and g_L and g_S are the orbital and spin g-factors for the odd nucleon and take the values given in Sec. 2.1. In terms of the neutron and proton magnetic moments $[\mu(N)=-1.913,\ \mu(P)=2.793]$ Eq. (19) can be written

$$\text{odd neutron}\quad j=l+\tfrac{1}{2},\ \mu_s=\mu(N),$$

$$j=l-\tfrac{1}{2},\ \mu_s=\frac{-j}{j+1}\mu(N),$$

$$\text{odd proton}\quad j=l+\tfrac{1}{2},\ \mu_s=(j-\tfrac{1}{2})+\mu(P),$$

$$j=l-\tfrac{1}{2},\ \mu_s=\frac{j}{j+1}[(j+\tfrac{3}{2})-\mu(P)].$$

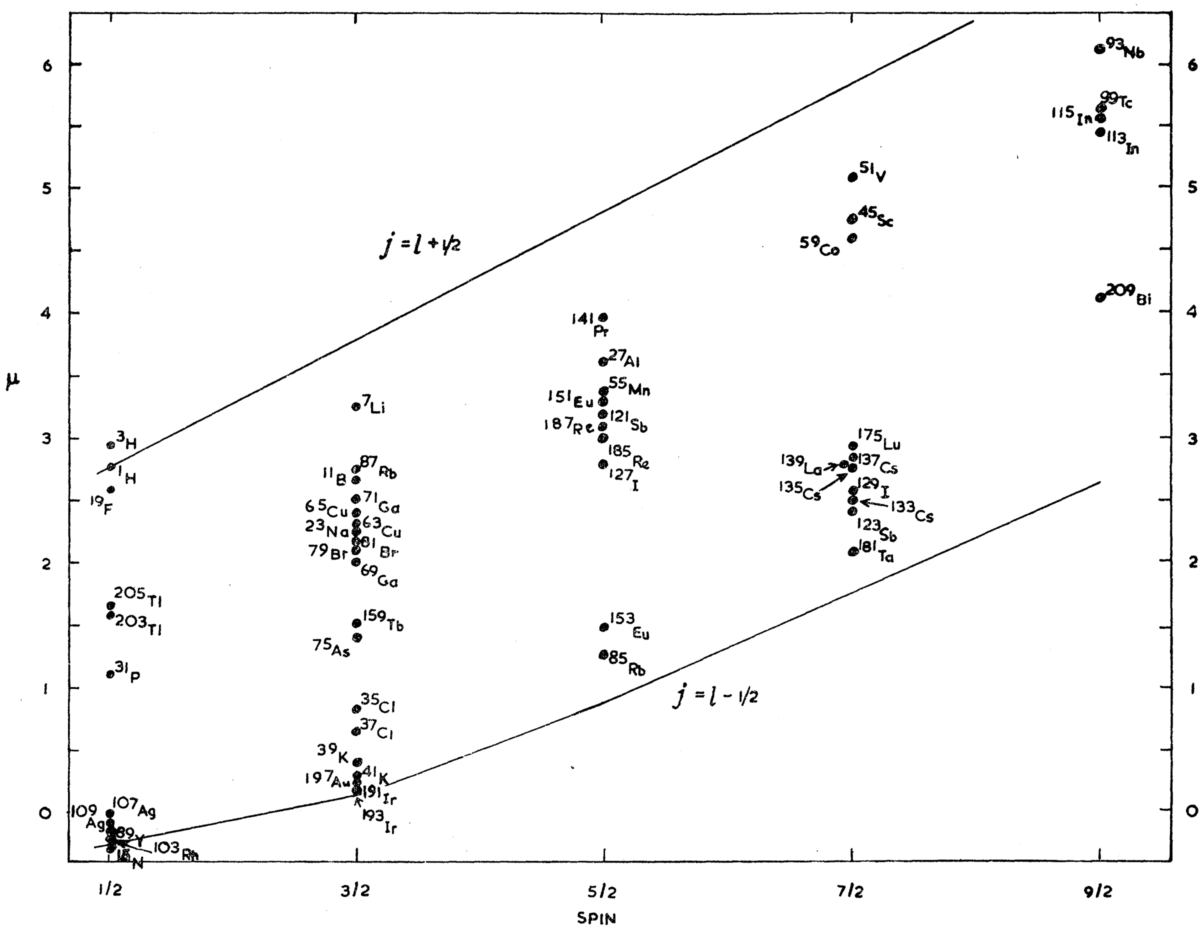

FIG. 3. Schmidt diagram for odd-proton nuclei.

The values of μ_s for the different cases are given in Table I.

It is usual to plot curves of μ_s against j for neutrons, and protons, such curves being referred to as Schmidt diagrams and the resulting lines as the Schmidt[54] lines. The Schmidt diagrams are given in Figs. 3 and 4 on which are also plotted the measured magnetic moments of odd-A nuclei. Of course the Schmidt lines only have any meaning at points corresponding to allowed values of j, that is at half-integer values of j.

Inspection of the Schmidt diagrams shows that there are several general points to be noticed.

(1) The moments of most nuclei deviate from the Schmidt lines by amounts varying between about $\frac{1}{2}-1\frac{1}{2}$ n.m. On the other hand, lines drawn to represent the average deviations are roughly parallel to the Schmidt lines.

(2) Apart from H^3, He^3, N^{15}, C^{13} the deviations of the magnetic moments are all inwards from the Schmidt lines.

(3) The average deviation of odd proton nuclei is a little larger (about 20%) than the average deviation of odd neutron nuclei.

(4) The only nuclei which do not deviate by more than about 0.2 n.m. are (apart from H^1, N^1, H^3, He^3) O^{17},K^{39}, K^{41} and all $p_{\frac{1}{2}}$ nuclei.

Although the Schmidt lines do not then agree in general with the observed magnetic moments, there is clearly a qualitative correspondence and it is possible to associate most moments with one or the other line. This enables parity allocations to be assigned to nuclear ground states according to which line the nucleus corresponds. Such parity assignments are almost invariably correct. Apart from $p_{\frac{1}{2}}$ nuclei the others which have moments laying close to the Schmidt lines have associated with them a closed shell structure in which case it is to be expected that the single particle model is a fair approximation. In this connection, however, it is significant that Bi^{209} which is a "double closed shell plus one" nucleus deviates considerably (by 1.4 n.m.) from the single-particle value.

The fact that in general there are large deviations may be attributed to two possible causes. Firstly, the single-particle model wave function is certainly not the

[54] T. Schmidt, Z. Physik **106**, 358 (1937).

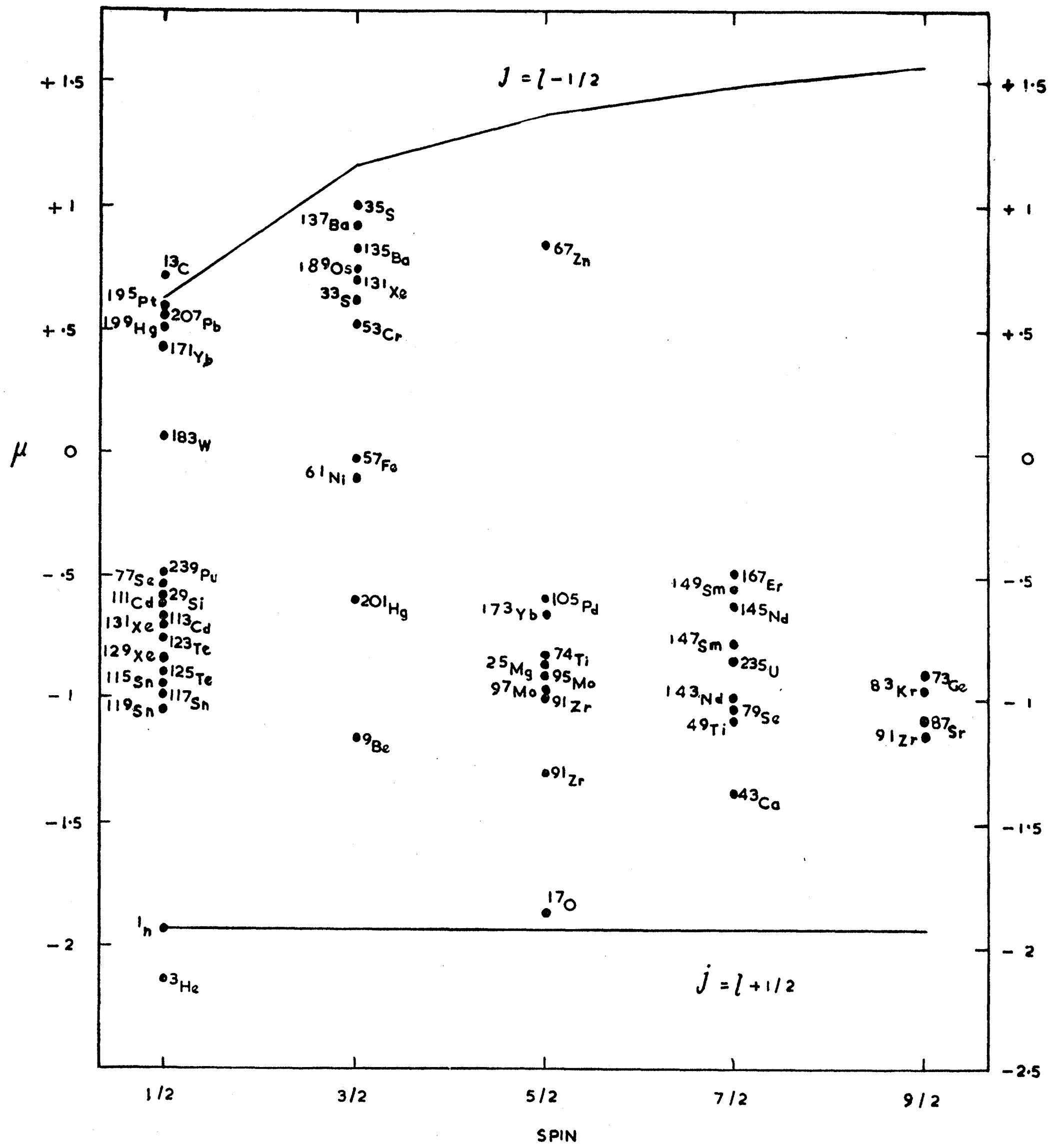

FIG. 4. Schmidt diagram for odd-neutron nuclei.

correct nuclear wave function and therefore no surprise should be occasioned by the lack of agreement. In subsequent paragraphs we shall see to what extent the deviations can be accounted for by using the more refined wave functions implied by the other models already discussed. Secondly, there is the possibility that a large part of the deviations might be attributed to exchange current effects. As pointed out in Sec. 3.2 the spin exchange currents effectively modify the nucleon spin g-factors and if this modification is a decrease the nucleon intrinsic moment may be quenched in the nucleus. If this is the correct explanation, however, it is then difficult to explain the fact that the moment of O^{17}, say, lies so close to the free nucleon Schmidt line. This point is discussed in Sec. 6.4.

6.2 Magnetic Moments on the Individual Particle Model

In the individual particle model the angular momentum of a nucleus is shared between all the nucleons outside closed shells in a manner dependent on the form of the internucleon interaction. Under these circumstances, although the spin is generally that predicted by the single particle model, it is to be expected that the magnetic moment will be different. For light and medium-heavy nuclei when states can be classified according to their symmetry properties, it is possible to obtain relatively simple expressions for the magnetic moments.[55–58]

[55] M. Mizushima and M. Umezawa, Phys. Rev. **83**, 463 (1951).
[56] M. Mizushima and M. Umezawa, Phys. Rev. **85**, 37 (1952).

TABLE II. The magnetic moments of light nuclei on the individual particle model.

Atom	Z	N	A	Configuration	Isotopic spin	μ_s	μ_{Ind}	μ
Li	3	4	7	$(p_{3/2})^3$	$\frac{1}{2}$	3.79	3.04	3.26
Be	4	5	9	$(p_{3/2})^{-3}$	$\frac{1}{2}$	−1.91	−1.16	−1.18
B	5	6	11	$(p_{3/2})^{-1}$	$\frac{1}{2}$	3.79	3.79	2.69
				$(p_{3/2})^{-3}(p_{1/2})_n{}^2$	$\frac{1}{2}$	3.79	3.04	
C	6	7	13	$(p_{1/2})^1$	$\frac{1}{2}$	0.64	0.64	0.70
N	7	8	15	$(p_{1/2})^{-1}$	$\frac{1}{2}$	−0.26	−0.26	−0.28
O	8	9	17	$(d_{5/2})^1$	$\frac{1}{2}$	−1.91	−1.91	−1.89
F	9	10	19	$(s_{1/2})^3$	$\frac{1}{2}$	2.79	2.79	2.63
Mg	12	13	25	$(d_{5/2})^{-3}$	$\frac{1}{2}$	−1.91	−0.64	−0.96
Al	13	14	27	$(d_{5/2})^{-1}$	$\frac{1}{2}$	4.79	4.79	3.64
				$(d_{5/2})^{-3}(s_{1/2})_n{}^2$	$\frac{1}{2}$	4.79	3.52	
Cl	17	18	35	$(d_{3/2})^3$	$\frac{1}{2}$	0.13	0.26	0.82
Cl	17	20	37	$(d_{3/2})^{-3}$	$\frac{3}{2}$	0.13	0.13	0.68
Sc	21	24	45	$(f_{7/2})^5$	$\frac{3}{2}$	5.79	5.10	4.76

Thus, if a configuration has an even number N_e of neutrons (protons) and an odd number N_0 of protons (neutrons) in equivalent states outside closed shells, then for the ground state with isotopic spin $T=(N_e-N_0)/2$ and lowest seniority ($s=1$), the nuclear moment can be written $\mu_{\mathrm{Ind}}=\alpha\mu_n+\beta\mu_p$ where $\alpha+\beta=1$ and

$$\beta=N_e/(2j+2)(2T+2), \qquad \text{for} \quad N_0>N_e,$$

$$\beta=(2j+1-N_e)/(2j+2)(2T+2), \quad \text{for} \quad N_0<N_e.$$

In the foregoing expressions μ_n and μ_p are the magnetic moments which a single neutron and a single proton would have in the state under consideration, respectively. A selection of calculated moments is given in Table II and it can be seen that the agreement between theory and experiment is improved and that in some cases quite large deviations from the Schmidt lines are predicted. For heavier nuclei the situation is much more complex and so far no calculations of this type have been reported.

In spite of the improvement brought about by the above approach there are still many cases where these considerations have no effect. Thus, for spin $\frac{1}{2}$ nuclei, there can be no coupling with an even number of particles in an unfilled shell in the above manner since, by the exclusion principle, the even nucleons must couple to angular momenta 0, 2, 4 etc. and only the 0 state can couple to spin $\frac{1}{2}$ to give a resultant spin $\frac{1}{2}$; this is just the assumption made in the single-particle model. On this scheme then, all spin $\frac{1}{2}$ nuclei should still have the single-particle values. This is approximately true for $p_{\frac{1}{2}}$ nuclei, but $s_{\frac{1}{2}}$ nuclei, on the other hand, deviate by large amounts (e.g., the P^{31} deviation is 1.48 n.m.). Further, on this scheme, since Bi^{209} has only one particle outside "magic" closed shells, it should have the Schmidt value and this is manifestly contrary to observation.

Thus it seems that although the agreement can be improved in some cases by taking account of all nucleons outside closed shells, there are still many anomalies.

6.2.1 Intermediate Coupling and Interconfigurational Mixing

It has already been pointed out that, certainly for light nuclei, the angular momentum coupling scheme appears to lie in between the jj- and LS-extremes. For example, Lane[41,43,44] has been able to explain many nuclear properties of the light nuclei with configurations $(1p)^n$ where $n=1$, 2, 3, 4, and 9 (lithium isotopes, C^{13} and N^{13}) in terms of an intermediate coupling situation. In particular, the magnetic moments of these nuclei can be exactly reproduced.

For heavier nuclei the jj-extreme seems to be the better approximation. Nevertheless, it transpires that even a small deviation from the jj-extreme can considerably affect the magnetic moment.[45,46,59] This deviation from jj-coupling can be regarded as the partial mixing of the two components of a spin-orbit doublet. If one assumes that the mixing is small, which appears to be the case for heavier nuclei, the nuclear wave function for a nucleus of spin I can be written symbolically as

$$\Psi_I=\chi_I+\sum_p \alpha_p\phi_{p,I}$$

where χ_I represents a simple shell-model configuration (e.g. the extreme single-particle configuration) and the $\phi_{p,I}$ represent admixed configurations, such configurations being characterized by the variable p. The magnetic moment of the nucleus is then obtained by calculating the expectation value of the magnetic moment operator (1) by use of this function. For small α_p the most important contributions to the magnetic moment will be those linear in the α_p, and the condition that contributions of this kind should occur is that χ_I and $\phi_{p,I}$ differ at most by one single-particle state and that then the orbital state is the same. Thus the only possible type of admixed configuration which satisfies this condition is one in which a single nucleon is transferred from a state $l_{j=l+\frac{1}{2}}$ to $l_{j=l-\frac{1}{2}}$. Three possibilities have to be considered.

(1) For $I=l+\frac{1}{2}$ nuclei, if there is more than one nucleon in this state, the admixed configurations can be formed by transferring one nucleon to the state $l_{j=l-\frac{1}{2}}$.

(2) For $I=l-\frac{1}{2}$ nuclei, if there is more than one hole in this state, the admixed configurations can be formed by transferring one nucleon from the state $l_{j=l+\frac{1}{2}}$ to the state $l_{j=l-\frac{1}{2}}$.

(3) For all nuclei there can be a transfer of one nucleon from a different state $l'_{j'=l'+\frac{1}{2}}$ to $l'_{j''=l'-\frac{1}{2}}$.

Using a delta-function interaction between nucleons it is found by application of first-order perturbation

[57] B. H. Flowers, Phil. Mag. 43, 1330 (1952).

[58] G. Racah, Report of the Birmingham Conference on Nuclear Physics, p. 8 (1953).

[59] A. Arima and H. Horie, Progr. Theor. J. Phys. 11, 509 (1954).

theory[45,46] that in cases (1) and (2) the deviation $\Delta\mu$ of the magnetic moment from the Schmidt value has the form $\Delta\mu = A_I{}^l \xi_l (g_S - g_L)$ where $A_I{}^l$ has a known dependence on l and I and g_S and g_L are the spin and orbital g-factors of the odd nucleons. ξ_l depends on the form and strength of the internucleon interaction and on the magnitude of the spin-orbit splitting. For Bi^{209} ($l=5$, $I=9/2$) a delta-function interaction is probably a fair approximation and ξ_5 can be roughly estimated to lie between -0.25 and -0.5 which gives a correction to the magnetic moment of the required order of magnitude (to agree exactly with experiment $\xi_5 = -0.43$ and this implies about 5% admixture of the $h_{11/2}$ state into the $h_{9/2}$). For other nuclei it is more satisfactory to treat ξ_l as a parameter, which, since it depends only on l, should have approximately the same value for $I = l + \frac{1}{2}$ and $I = l - \frac{1}{2}$ nuclei and should also be roughly independent of whether the nucleus is an odd proton or an odd neutron type.

Considering all those nuclei whose magnetic moments can be treated by the foregoing methods values of ξ_l can be chosen to fit the experimentally observed mean deviations. The values of ξ_l so obtained are $\xi_1 = -0.38$, $\xi_2 = -0.26$, $\xi_3 = -0.20$, $\xi_4 = -0.24$, $\xi_5 = -0.43$. In Figs. 5 and 6 it can be seen that the general trend of the deviations is reproduced by using these parameters. In addition, the values of ξ_l required are all reasonable.

In case (3) $\Delta\mu = C_{Ij'j''}{}^{ll'} \eta_{ll'}$, where $C_{Ij'j''}{}^{ll'}$ is again known and depends on the interaction and spin-orbit splitting. For the different possible admixtures $C_{Ij'j''}{}^{ll'}$ lies between 0 and 1.2 for $I = l - \frac{1}{2}$ nuclei and between 1.6 and 2.4 for $I = l + \frac{1}{2}$ nuclei in agreement with the observation that the latter nuclei have larger deviations from the Schmidt lines than the former. To explain the observed deviations when admixtures of this kind are responsible, values of $\eta_{ll'}$ between -0.2 and -0.5 are required and these again are quite reasonable. An important point emerges that for $p_{\frac{1}{2}}$ nuclei $C_{Ij'j''}{}^{ll'}$ is identically zero so that $\Delta\mu = 0$. This is in agreement with point (4) of Sec. 6.1. On the other hand $s_{\frac{1}{2}}$ nuclei are predicted to have large deviations, mainly through the admixture of configurations in which $d_{\frac{5}{2}}$ nucleon is transferred to the $d_{\frac{3}{2}}$ state. This is also in agreement with observation.

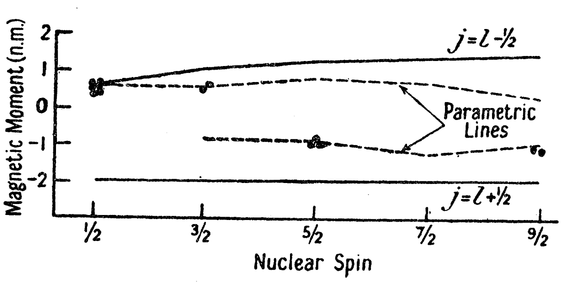

FIG. 5. Deviations of the magnetic moments of odd-proton nuclei from the Schmidt lines resulting from interconfigurational mixing [from R. J. Blin-Stoyle and M. A. Perks, Proc. Phys. Soc. (London) **A67**, 885 (1954)].

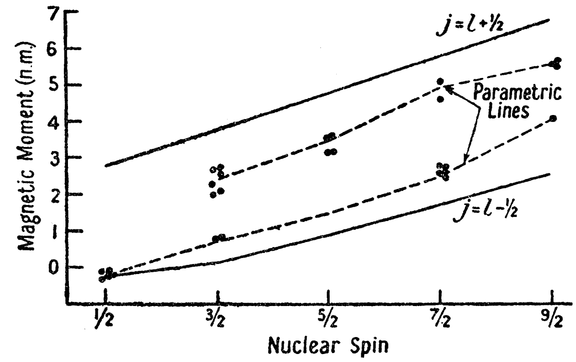

FIG. 6. Deviation of the magnetic moment of odd-neutron nuclei from the Schmidt lines resulting from interconfigurational mixing [from R. J. Blin-Stoyle and M. A. Perks, Proc. Phys. Soc. (London) **A67**, 885 (1954)].

Although the calculations related to the foregoing ideas are crude, the important point emerges that nuclear magnetic moments are extremely sensitive to admixtures to the zero-order shell-model wave function which contribute in first order to the magnetic moment (e.g., a 5% admixture changes the Bi^{209} moment by 1.4 n.m.). In a few cases, however, the distribution of nucleons among the various shell-model states is such that no type of admixture can lead to a contribution to the magnetic moment linear in its amplitude of mixing. Under these circumstances it is to be expected that the magnetic moment will lie close to the Schmidt value. The condition for this is that the nucleus should consist of doubly closed shells in both the LS- and the jj-sense plus or minus an odd nucleon. The only nuclei which satisfy this condition (other than $p_{\frac{1}{2}}$ nuclei which should have a small deviation in any case) are O^{17}, F^{17}, Ca^{41}, Ca^{39}, K^{39}. Of these nuclei, only the magnetic moments of O^{17} and K^{39} are known and the deviations of both are particularly small being 0.02 and 0.27 n.m., respectively. On the other hand a nucleus which has doubly closed shells in the jj-sense only plus or minus an odd particle can have a magnetic moment differing considerably from the single particle value. Thus, by an approach of this kind it is possible to account semi-quantitatively for the magnetic moments of most nuclei.

It should be noted that deviations from admixtures of types (1) and (2) are larger than those from type (3) and that in addition the deviation is directly proportional to $g_L - g_S$ for the odd nucleons. Now, apart from the g-factors, it might be expected that the deviations are the same whether the odd group of nucleons are neutrons or protons providing their number and total angular momenta are the same. Taking into account the different g-factors and denoting deviations from the Schmidt lines by $\Delta\mu_n$ and $\Delta\mu_p$ for odd neutron and odd proton nuclei, respectively, we then have for admixtures of types (1) and (2),

$$|\Delta\mu_p| / |\Delta\mu_n| = |g_L{}^p - g_S{}^p| / |g_L{}^n - g_S{}^n| \approx 1.20.$$

This is of the order of magnitude of the observed ratio of the deviations noted in point (3) in Sec. 6.1 and seems to indicate that admixtures within the odd nucleon configurations are indeed the most important. De Shalit[60] has pointed out that if the deviations are interpreted solely in terms of nucleons from the odd group then the preceding relation holds generally independent of the coupling scheme. In this connection it is a striking fact as stressed by Schawlow and Townes[61] and Volkov[62] that the deviations of nuclear moments for pairs of odd-A nuclei with the Z of one member of the pair equal to the N of the other member are in general very close. This can again be understood if the deviations result from configurational mixing within the group of odd nucleons. Volkov,[62] Brennan and Volkov,[63] and Davidson[64] have discussed this point at some length.

6.3 Magnetic Moments on the Collective Model

Although many nuclear magnetic moments can be interpreted satisfactorily in terms of simple configurational mixing of the type described in Sec. 6.2.1, there are nevertheless some moments which can only be accounted for by a collective approach and indeed it will be seen that the collective model can give an alternative explanation of the magnetic moments in many cases. Particularly striking examples of nuclear moments which can only be explained by the collective model are those of the two isotopes Eu^{151} and Eu^{153}. From the point of view of the shell model they differ only by the the addition of two neutrons to the $h_{9/2}$ state and this is not expected to affect the magnetic moments appreciably; nevertheless, their respective moments are 3.6 n.m. and 1.6 n.m. and a satisfactory explanation can only be given in terms of the collective model (q.v.).

Consider now a system consisting of a single particle and a distorted core. A straightforward generalization of (1), (2), and (3) yields for the magnetic moment operator for this case

$$\boldsymbol{\mu}_{\text{op}}=\frac{e\hbar}{2Mc}(\tfrac{1}{2}g_S\boldsymbol{\sigma}+g_L\mathbf{L}+g_R\mathbf{R}), \tag{20}$$

where $\mathbf{R}$ is the angular momentum operator for the core and g_R is the appropriate g-factor. If the nucleus is uniformly charged, then[65]

$$g_R=Z/A. \tag{21}$$

In weak coupling when j, the particle total angular momentum, is expected to be a fairly good quantum number $\boldsymbol{\mu}_{\text{op}}=(g_J\mathbf{J}+g_R\mathbf{R})$ and

$$\begin{aligned}\mu_C&=\langle g_JJ_z+g_RR_z\rangle_{M=I}\\&=g_JI-(g_J-g_R)\langle R_z\rangle_{M=I}.\end{aligned} \tag{22}$$

[60] A. De Shalit, Phys. Rev. **90**, 83 (1953).
[61] A. L. Schawlow and C. H. Townes, Phys. Rev. **82**, 268 (1951).
[62] A. B. Volkov, Phys. Rev. **94**, 1664 (1953).
[63] J. G. Brennan and A. B. Volkov, Phys. Rev. **97**, 1380 (1955).
[64] J. P. Davidson, Phys. Rev. **85**, 432 (1952).
[65] K. Way, Phys. Rev. **55**, 963 (1939).

Bohr and Mottelson[51] (see also Kerman,[66] Foldy and Milford,[67,68] Davidson and Feenberg,[69] and a semi-empirical approach by Osborn and Klema[70]), have estimated the way in which $\langle R_z\rangle_{M=I}$ varies as the coupling of the nucleon to the surface increases and for the nuclear ground state $j=I$ they find that $\langle R_z\rangle_{M=I}$ increases monotonically from zero for zero coupling to $I/(I+1)$ in the strong coupling limit. Thus, in weak coupling the magnetic moment is little different from the single particle value μ_s. On the other hand, in the strong coupling limit

$$\mu_C=\mu_s-(g_J-g_R)I/(I+1) \quad \text{for} \quad I=j>\tfrac{3}{2} \tag{23}$$

$$=g_J\frac{I^2}{I+I}+g_R\frac{I}{I+I}. \tag{24}$$

For $I=j=\frac{3}{2}$ care has to be taken since a degeneracy exists between the states $\Omega=\frac{3}{2}$, $\Omega=\frac{1}{2}$ where Ω is the component of the nucleon angular momentum along the symmetry axis [see Sec. 5.5.2 and also Bohr and Mottelson[51] (Appendix III)]. No simple expression for the moment exists in this case and only an approximation can be made.

For $I=j=\frac{1}{2}$ there is no direct coupling to the surface and $\mu_c=\mu_s$. The values of μ_c in the strong coupling limit are plotted as dotted lines in Figs. 7 and 8 where it can be seen that for $j=l+\frac{1}{2}$ nuclei the agreement between theory and experiment is considerably improved.

For $j=l-\frac{1}{2}$ there is little change. However in this strong coupling limit there is expected to be a strong admixture due to the asphericity of the potential well of the spin-orbit coupling partner which will considerably modify the result, particularly as there will be first-order contributions to the moment just as in Sec. 6.2.1.

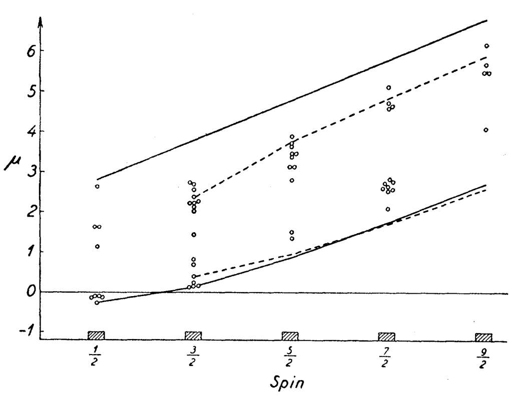

FIG. 7. Deviation of the magnetic moments of odd-proton nuclei from the Schmidt lines on the strong coupling mode [from Aage Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab. Mat.-fys. Medd. **27**, No. 16, 42 (1953)].

[66] A. K. Kerman, Phys. Rev. **92**, 1176 (1953).
[67] L. L. Foldy and F. J. Milford, Phys. Rev. **80**, 751 (1950).
[68] F. J. Milford, Phys. Rev. **93**, 1297 (1954).
[69] J. P. Davidson and E. Feenberg, Phys. Rev. **89**, 856 (1953).
[70] R. K. Osborn and E. D. Klema, Phys. Rev. (to be published).

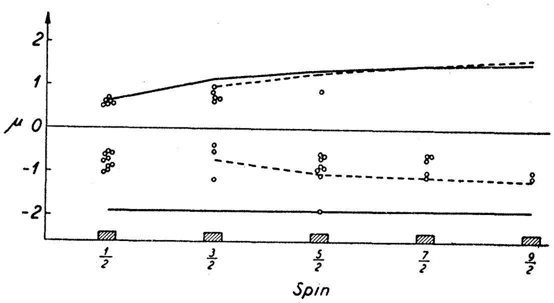

FIG. 8. Deviation of the magnetic moments of odd-neutron nuclei from the Schmidt lines on the strong coupling model [from Aage Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab. Mat.-fys. Medd. **27**, No. 16, 42 (1953)].

The contribution to the magnetic moment from admixtures of this and other types can be estimated by perturbation procedures[51] but it is more satisfactory to obtain solutions directly for the wave functions of particles moving in an ellipsoidal potential with spin-orbit coupling.[52,53,71] One can then obtain an expression for g_Ω the g-factor associated with a particle state in the deformed nucleus and Eq. (24) is replaced by

$$\mu_C=\frac{I^2}{I+I}g_\Omega+\frac{I}{I+I}g_R \quad \text{for} \quad I=\Omega\neq\tfrac{1}{2}. \tag{25}$$

For the state $\Omega=K=\frac{1}{2}$ the situation is more complicated since the surface coupling is not a direct effect but results indirectly through the coupling of different orbital states (see Sec. 6.3.1).

6.3.1 Comparison with Experiment

From the foregoing discussion it is clear that the magnetic moment can be affected in two ways. The angular momentum of the nucleus may be shared between the core and the external particles and, in addition there may be admixtures of near lying external particle states. If there are no near lying states of spin equal to or greater than that of the odd particle, then there will be no appreciable admixtures and only the former effect is important. Assuming strong coupling there is then an unambiguous value for the nuclear moment and for nuclei of this type ($j=l+\frac{1}{2}$) agreement between theory and experiment is generally very good.[51]

For $s_{\frac{1}{2}}$ nuclei there is a strong interaction between the $s_{\frac{1}{2}}$ and the $d_{\frac{3}{2}}$ and $d_{\frac{5}{2}}$ states which may lead to a large deformation and considerable modification of the moments as is observed. On the other hand for $p_{\frac{1}{2}}$ nuclei the collective effects are small thus accounting for the observed small deviations of $p_{\frac{1}{2}}$ nuclei.

All the deviations so far discussed can equally well be accounted for by configurational mixing in the individual particle model (Sec. 6.2.1) and it is probable that to a certain extent we have two alternative ways of looking at the same problem. The equivalence is not complete since, for example, it is quite impossible to explain the anomalous moment of Bi^{209} on the collective model. It is to be expected, however, that in the regions $150<A<190$ and $A>225$ the collective description is more satisfactory. In this region the nuclei are highly distorted as is evidenced, for instance, by their large electric quadrupole moments (see Sec. 7.3) and therefore can be well described by the strong coupling approximation. The main problem then is to obtain the value of g_Ω. The most satisfactory method of obtaining g_Ω is to consider an ellipsoidal well having a distortion appropriate to the particular nucleus under consideration; this can be deduced from the electric quadrupole moment, quadrupole transition probability between excited states etc. The particle levels are then filled up to obtain the lowest energy configuration subject to the restrictions of the exclusion principle (i.e., pairwise filling in the different Ω states). This means that g_Ω will then be that for the last odd particle. For a given odd number of nucleons the state of the last particle depends critically on the nuclear distortion. Thus for Eu^{151} and Eu^{153}, which should be adequately accounted for by this description, although both isotopes have the same number of odd particles, their distortions are considerably different since their quadrupole moments are respectively 1.2×10^{-24} cm^2 and 2.5×10^{-24} cm^2. It is thus possible to account for their magnetic moments in terms of two very different g_Ω's resulting from the fact that the wave functions for the odd particle depend sensitively on the deformation.[71] Similar calculations have also been applied to Th^{169}.[72]

6.3.2 The Value of g_R

In the strong coupling approximation the magnetic moment of an odd-A nucleus is dependent on the two parameters g_Ω and g_R. Now g_Ω can be obtained from a model as described above but the resulting value is bound to be a little uncertain since, for instance, no account is taken in that model of the residual inter-nucleon forces. It is possible however, in one or two cases, to determine g_Ω and g_R unambiguously from experimental data. A typical example is the nucleus Ta^{181}. The ground-state spin is 7/2 and the magnetic moment is 2.1 n.m. so that Eq. (25) gives one relation between g_Ω and g_R. Further, the radiation emitted by the excited rotational levels obtained by Coulomb excitation has been studied in detail.[73,74] This radiation is interpreted in terms of $M1+E2$ admixtures and, knowing the $E2$ intensities from the standard theory of collective oscillations, the $M1$ intensities can be determined. The latter intensity determines $(g_\Omega-g_R)^2$, and according to the sign of $g_\Omega-g_R$ two possible solutions for g_Ω and g_R are obtained. This analysis can be applied to both the

[71] K. Gottfried, thesis, Massachusetts Institute of Technology, June, 1955.

[72] B. R. Mottelson and S. G. Nilsson. Z. Physik **141**, 217 (1955).

[73] T. Huus and J. Bjerregord, Phys. Rev. **92**, 1579 (1953).

[74] T. Huus and C. Zapancic, Kgl. Danske Videnskab. Selskab. Mat.-fys. Medd. **28**, No. 1 (1953).

TABLE III. Experimental values of g_Ω and g_R.

Nucleus	μ	Transition (kev)	$(g_\Omega - g_R)^2$	g_Ω	g_R
Ta^{181}	2.1	137	0.202	0.70	0.25
				0.50	0.95
		166	0.284	0.72	0.19
				0.48	1.02
Au^{197}	0.19	277	0.149	−0.061	0.32
				0.25	−0.14

transitions ($11/2 \rightarrow 9/2 \rightarrow 7/2$) that occur in Ta^{181} and has also been applied to Au^{197}.[75] The possible values of g_Ω and g_R are given in Table III. Now, in Ta^{181}, the sign of $g_\Omega - g_R$ can be determined from the form of the angular correlation of the two successive radiations since the $M1-E2$ interference term which occurs in the expression for the angular correlation depends on $g_\Omega - g_R$ rather than on its square. The observed angular correlation implies that the lower values of g_R are the correct ones. Thus g_R would appear to vary between 0.19 and 0.32. These values should not be taken too seriously, but it does seem that g_R is probably smaller than 0.40 which is obtained for Ta^{181} by use of Eq. (21). This discrepancy may be understood in terms of the possible difference between the radii of the proton and neutron distribution in the nucleus. Let us make the crude assumption, which may be approximately valid for nuclei having large moments of inertia, that the moment of inertia of Ta^{181} results from the rigid rotation of a surface layer of nuclear fluid. If R_i is the inner radius of this layer (assumed spherical) and R_n and R_p are the radii of the neutron and proton distributions, then

$$g_R = \frac{Z}{A}\left[\frac{Ax^2-(1-y)(Zx^2+N)}{y(Zx^2+N)}\right], \tag{26}$$

where $x = R_p/R_n$ and $y = I/I_{\text{rigid}}$, I being the measured moment of inertia and I_{rigid} the moment of inertia for rigid rotation of the whole nucleus. For Ta^{181} $I/I_{\text{rigid}} \sim 0.4$ and taking $R_p = 1.2A^{\frac{1}{3}} \times 10^{-13}$ cm and $R_n = 1.4A^{\frac{1}{3}} \times 10^{-13}$ cm gives $g_R = 0.23$. Thus, this rather crude calculation indicates that a difference in neutron and proton radii is capable of explaining the deduced value of g_R.

6.4 Exchange Currents and Velocity Dependent Forces

Hitherto no account has been taken of possible contributions to nuclear moments from exchange currents and velocity dependent forces. For H^3 and He^3 it was found that a reasonable explanation of the magnetic moments was forthcoming only if such effects were taken into account and it is therefore important to see if large contributions of this type are to be expected in the case of heavier nuclei.

6.4.1 Exchange Currents

Bloch[76] has pointed out that the general trend of nuclear magnetic moments can be interpreted as a partial quenching of the intrinsic nucleon moments (see also De Shalit,[77] Klinkenberg,[78] and Candler[79]). In almost every case the observed magnetic moments of odd-A nuclei can be explained by a single-particle calculation with the intrinsic nucleon moment lying between the free nucleon moment and the completely quenched moment (i.e., $\mu(N)=0$, $\mu(P)=1$). However, such an explanation, if it is to be accepted, must be put on a sounder theoretical footing especially in view of the fact that it seems possible to account for nucleon moments solely by use of more refined wave functions for the nucleus.

Attempts to do this have been made by various authors. Miyazawa[80] uses a phenomenological form for $\mathbf{M}_L$ the space exchange magnetic operator and deduces the spin-exchange contribution meson theoretically. He then calculates the exchange contribution to the magnetic moment of an odd particle interacting with a spherically symmetrical core, the core being represented by a Fermi gas. Russek and Spruch[11] on the other hand represent all the exchange operators phenomenologically and use a shell model to describe the core. In both cases it is claimed that by suitable choice of the form and strength of the exchange operators, the deviation of nuclear moments from the Schmidt values can be explained. Ross[12] also uses a phenomenological approach but in addition makes the highly reasonable but restrictive assumption that the combination of exchange operators finally chosen must also account for the H^3 and He^3 magnetic moment anomalies. With this restriction it then transpires that the resulting anomaly in all cases is always about $\frac{1}{4}$ n.m. and *outside* the Schmidt lines.

As Ross points out, many-body forces may become important for heavier nuclei in which case the restriction to two-body operators in the expressions for the exchange moments [Eqs. (10) to (14)] is unrealistic. However, a many-body effect is expected to vary smoothly from one nucleus to another, whereas the observed magnetic moments show large fluctuations. In addition, if exchange moments are important it is difficult to understand the fact that nuclei such as O^{17} do not deviate from the Schmidt lines. The general conclusion therefore seems to be that although a small deviation ($\frac{1}{4}$ n.m.) in nuclear magnetic moments is to be expected because of exchange current effects, the cause of the main deviations must lie elsewhere.

[75] P. H. Stelson and F. K. McGowan, Phys. Rev. **99**, 112 (1955).

[76] F. Bloch, Phys. Rev. **83**, 839 (1951).
[77] A. De Shalit, Helv. Phys. Acta. **24**, 296 (1951).
[78] P. F. A. Klinkenberg, Physica **17**, 715 (1951).
[79] C. Candler, Proc. Phys. Soc. (London) **A64**, 999 (1951).
[80] H. Miyazawa, Progr. Theoret. Phys. **6**, 801 (1951).

6.4.2 *Velocity Dependent Forces*

There is the additional possibility that velocity dependent forces will cause a modification of nuclear magnetic moments. The only calculations so far reported in this connection are related to the spin-orbit coupling associated with the shell model.[81,82]

By Eq. (17) the additional contribution to the magnetic moment for a neutron is zero and for a single proton the operator for the additional moment is

$$\boldsymbol{\mu}_{\mathrm{op}}'=\frac{e}{2c}f(r)[-r^2\boldsymbol{\sigma}+(\mathbf{r}\cdot\boldsymbol{\sigma})\mathbf{r}], \qquad (17)$$

where $f(r)$ is the radial dependence of the spin-orbit potential. Calculating the expectation value for this operator for a simple shell-model state [Eq. (18)] gives for the additional contribution to the magnetic moment for a state with total angular momentum j,

$$\mu'=\mp\langle r^2f(r)\rangle\frac{2j+1}{j+1}\text{n.m} \quad \text{for} \quad j=l\pm\tfrac{1}{2} \qquad (27)$$

where $\langle r^2f(r)\rangle$ is the average of $r^2f(r)$ over the nucleon orbit. From the known magnitude of the spin-orbit splitting, Jensen and Mayer[81] roughly estimate the deviation to be of the order $\mp 0.13(2j+1)(j+1)$ n.m. The correction is in the right direction (i.e., inwards from the Schmidt lines) but is much smaller than the observed deviations and is, of course, not applicable to odd neutron nuclei. A similar estimate is also given by Marty.[82]

6.5 Conclusions

The first general conclusion that can be drawn from the foregoing discussion is that the main deviations of the nuclear magnetic moments of odd-A nuclei from the Schmidt lines must be attributed to configurational mixing within the single-particle model of the nucleus. In particular, configurations which contribute in first order to the magnetic moment are of considerable importance and by appealing to such configurations it is possible to account for most deviations. However, in the regions where the nucleus is considerably distorted the configurational mixing cannot be described adequately in terms of a particle picture and one has to turn to the collective model. This model then enables one to account for the outstanding deviations. Contributions from exchange currents and velocity dependent forces appear to be an order of magnitude smaller than the observed deviations and are therefore only important in regions where the nuclear wave function is known accurately (e.g., for H^3 and He^3). Taking them into account however, and the accompanying uncertainty as to their exact values means that it should not be expected to estimate the nuclear magnetic moments of odd-A nuclei to an accuracy greater than about $\frac{1}{4}-\frac{1}{2}$ n.m.

In the appendix tables are given of all known nuclear magnetic moments of odd-A nuclei, their deviations from the single particle value and an indication as to the probable cause of the deviations.

7. ELECTRIC QUADRUPOLE MOMENTS OF ODD-A NUCLEI

We now investigate the ways in which the various nuclear models can account for the observed electric quadrupole moments. For electric quadrupole moments the situation is simplified in one respect that by the Siegert theorem (3.1) exchange currents do not contribute in any important way. This means that the experimental results must be interpreted very largely in terms of nuclear wave functions.

7.1 Electric Quadrupole Moments on the Extreme Single-Particle Model

On this model the nuclear properties are represented by a single-particle wave function of the form given in (18) and the quadrupole moment is obtained by calculating the expectation value of Q_{op} [Eq. (5)] by use of this wave function. The calculation is straightforward and yields

$$\begin{aligned} Q=Q_j&=-\frac{2j-1}{2(j+1)}\langle r^2\rangle \quad \text{for an odd proton nucleus,} \\ Q&=ZQ_j/A^2 \qquad\qquad \text{for an odd neutron nucleus,} \end{aligned} \qquad (28)$$

where j is the total angular momentum of the single particle. The neutron quadrupole moment is much smaller than Q_j and is a recoil effect. By $\langle r^2\rangle$ is meant the average value of r^2 for the nucleon orbit and this is usually replaced by $3R_0^2/5$ where R_0 is the nuclear radius.

Now R_0 varies between $\sim 2\times 10^{-13}$ cm for the lightest nuclei to $\sim 10^{-12}$ cm for the heaviest so that on the single-particle model we expect odd proton nuclei to have quadrupole moments varying between $\sim 10^{-26}$ cm^2 to $\sim 6\times 10^{-25}$ cm^2. On the other hand the quadrupole moments of odd neutron nuclei should vary between $\sim 3\times 10^{-28}$ cm^2 and $\sim 1\times 10^{-25}$ cm^2. Inspection of the table of quadrupole moments given in the appendix shows that the single-particle model is singularly deficient in its ability to account for the moments. Rosenfeld[83] has stressed the following significant facts. First, the quadrupole moment of a nucleus having one hole (i.e., a shell filled apart from one nucleon) is approximately equal, but of opposite sign, to that of the corresponding nucleus having one particle in the shell. Second, the quadrupole moments of odd neutron nuclei can be just as large as those of odd proton nuclei which

[81] J. H. D. Jensen and M. G. Mayer, Phys. Rev. **85**, 1040 (1952).
[82] C. Marty, J. phys. radium **15**, 783 (1954).
[83] L. Rosenfeld, Physica **17**, 461 (1951).

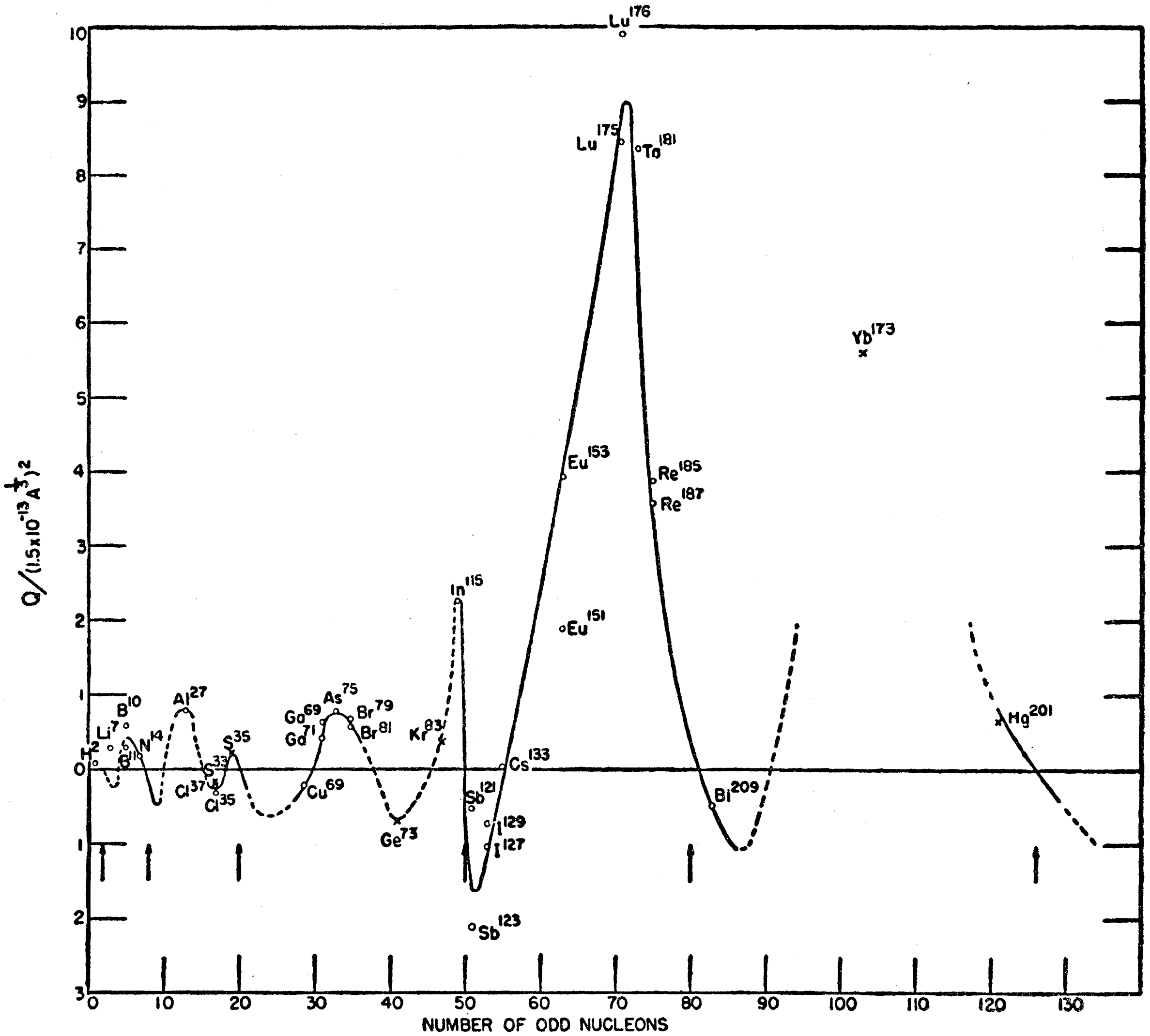

FIG. 9. Plot of Q/R_0^2 against the number of odd nucleons [from Townes, Foley, and Low, Phys. Rev. **76**, 1415 (1949)].

clearly indicates that the nuclear angular momentum must be shared with protons. Third, some quadrupole moments are 10 to 20 times larger than the expected single particle values. These latter moments are those of nuclei roughly in the regions $150<A<190$, $A>225$ and necessarily imply that the quadrupole moment is associated with a deformation in which a large number of particles are sharing and must therefore be interpreted in terms of the collective model (see Sec. 7.3). None of the foregoing points can be accounted for on the extreme single-particle model.

However, as pointed out by Gordy,[84] Hill,[85] Townes *et al.*,[86] and Rosenfeld,[83] there is a definite correlation between quadrupole moments and nuclear shell structure. This is strikingly demonstrated in Fig. 9 in which Q/R_0^2 is plotted against the number of odd nucleons. At the magic numbers (indicated by arrows) it can be seen that there is always a change in sign of the quadrupole moment and that the magnitudes of the quadrupole moments in these regions are of the order to be expected for a single particle.

7.2 Electric Quadrupole Moments on the Individual Particle Model

The fact that the angular momentum of a nucleus is shared among all the particles outside closed shells in the individual particles model at once points to an explanation of the fact that odd neutron nuclei have electric quadrupole moments. The model also predicts that the quadrupole moment of a "hole" will be equal and opposite to that of a particle in the same state. This latter point is a specific example of a more general result. Thus if the quadrupole moment of, say, n

[84] W. Gordy, Phys. Rev. **76**, 139 (1949).
[85] R. D. Hill, Phys. Rev. **76**, 998 (1949).
[86] Townes, Foley, and Low, Phys. Rev. **76**, 1415 (1949).

equivalent protons (each in the single-particle state with total angular momentum j) is calculated using a correctly antisymmetrized wave function corresponding to a total angular momentum j with $n-1$ nucleons coupled to zero angular momentum, the following result is obtained for Q (reference 8, Appendix IV):

$$Q=\left(1-\frac{2n-2}{2j-1}\right)Q_j \tag{29}$$

for $1 \leqslant n \leqslant 2j$. In particular for a hole ($n=2j$) we see that $Q=-Q_j$. For all other values of n other than 1 or $2j$, Q is less than Q_j and is negative for a shell less than half-filled and positive for a shell more than half-filled. This is only in fair agreement with observation since there seems to be a predominance of positive quadrupole moments.[87]

In the case of an odd neutron nucleus in which there are also "loose" protons outside closed shells, it is to be expected that the nucleus will have an appreciable electric quadrupole moment. Thus for light and medium-heavy nuclei whose states can be approximately classified according to their symmetry properties the expectation value of Q_{op} can be calculated at once using the appropriate wave function. Flowers[56] gives as a particular example that of three nucleons in the states j for which $I=j$, $T=\frac{1}{2}$ and $s=1$. He finds

$$Q(1p+2n)=Q_j(4j+5)/(6j+6)$$

and

$$Q(\mathrm{l}n+2p)=Q_j(2j+7)/(6j+6).$$

In both cases $Q\approx 0.7\langle r^2\rangle$ so that the quadrupole moment "induced" on an odd neutron by two protons is about the same as that of one proton in the presence of two neutrons. This agrees with observations in many cases. For heavier nuclei effects of this sort will also occur but cannot be expressed in a simple fashion.

In addition, for all nuclei, there can be interconfigurational mixing leading to contributions to the quadrupole moment linear in the amplitude of the admixture. Such effects have been discussed by Perks[88] and Horie and Arima.[89] These authors point out that in addition to the admixtures which contribute linearly to the magnetic moment and the quadrupole moment, there can be admixtures of other states (for which $l'=l\pm 2$ where l is the orbital angular momentum of the shell-model state) which also give linear contributions. In fact, as far as quadrupole moments are concerned, the former admixtures are unimportant whereas the latter can give contributions having several times the order of magnitude of the single particle quadrupole moment. Thus the admixture accounting for the large magnetic moment anomaly of Bi^{209} does not cause a large deviation in its quadrupole moment from the single particle value. This is in fact observed.

From the foregoing discussion it seems that the individual particle model with interconfigurational mixing can account for many nuclear quadrupole moments. On the other hand it does not appear to give a satisfactory account of the very large quadrupole moments observed and these must be interpreted by the collective model.

7.3 Electric Quadrupole Moments on the Collective Model

On the collective model of the nucleus a quadrupole moment is to be associated both with the deformed core (Q_s) and also with the particles outside the core (Q_p). Thus

$$Q=Q_p+Q_s \tag{30}$$

and it is with Q_s that we are mainly concerned since this term, involving so many more protons, is expected to be much larger than Q_p.

Suppose that the shape of the core is defined by $R(\theta)=R_0[1+\beta Y_2{}^0(\theta)]$, where θ is measured with respect to axes fixed in the core. If the nuclear charge distribution is uniform, the quadrupole moment of the core relative to the above axes is given by $Q_0=3/(5\pi)^{\frac{1}{2}}(ZR_0{}^2\beta)$. The quadrupole moment Q_s measured with respect to axes fixed in space will not, however, be equal to Q_0 because of the particle-surface coupling which results in a precession of the core angular momentum about the total nuclear spin axis. It is usual to express Q_s in terms of Q_0 in the following way:

$$Q_s=P_Q(x)Q_0, \tag{31}$$

where $P_Q(x)$ is referred to as the projection factor. x is related to the strength of the particle-surface coupling and the nuclear deformability. It is given explicitly by

$$x=\left(\frac{5}{16\pi}\right)^{\frac{1}{2}}\frac{1}{I^{\frac{1}{2}}}\frac{k}{(\hbar\omega C)^{\frac{1}{2}}}.$$

Here k is a measure of the strength of the particle-surface coupling and is generally taken to be of the order 40 Mev; C measures the nuclear deformability and for a deformation of order 2 is given by

$$C=4R_0{}^2S-\frac{3}{10\pi}\frac{Z^2e^2}{R_0}, \tag{32}$$

where S is the nuclear surface tension estimated as $15.4A^{\frac{2}{3}}/4\pi R_0{}^2$ Mev. For most nuclei C is taken to be about 50 Mev. $\hbar\omega$ is the phonon energy (see Sec. 5.5).

For weak coupling, $x\ll 1$ and $P_Q(x)\approx 1$ and then decreases monotonically to the strong coupling limit $P_Q=I(2I-1)/(2I+1)(2I+3)$ this expression being valid for $I>\frac{3}{2}$.** For $I=0$ or $\frac{1}{2}$, P_Q vanishes so that

[87] S. A. Moszowski and C. H. Townes, Phys. Rev. **93**, 306 (1953).
[88] M. A. Perks, Proc. Phys. Soc. (London) **A68**, 1083 (1955).
[89] H. Horie and A. Arima, Phys. Rev. **99**, 778 (1955).

** $I=3/2$ has to be treated as a special case.

TABLE IV. The moments of odd-odd nuclei. μ_{calc} and μ_{emp} are obtained from (35) using the free and empirical nucleon g-factors, respectively.

Atom	Z	N	A	Spin	μ	Proton state	Neutron state	μ_{calc}	$g_{emp}(p)$	$g_{emp}(n)$	μ_{emp}	Q
H	1	1	2	1	0.9	$s_{1/2}$	$s_{1/2}$	0.9				2.7×10^{-3}
Li	3	3	6	1	0.8	$p_{3/2}$	$p_{3/2}$	0.6[a]				
B	5	5	10	3	1.8	$p_{3/2}$	$p_{3/2}$	1.9	2.0	−0.8	1.8	0.13
N	7	7	14	1	0.4	$p_{1/2}$	$p_{1/2}$	0.4	−0.6	1.4	0.4	0.02
Na	11	11	22	3	1.7	$d_{5/2}$	$d_{5/2}$	1.7	1.5	−0.3	1.8	
Na	11	13	24	4	1.7	$d_{5/2}$	$d_{5/2}$	2.3	1.5	−0.3	1.5[b]	
Cl	17	19	36	2	1.3	$d_{3/2}$	$d_{3/2}$	0.8	0.6	0.7	1.3	−0.018
K	19	21	40	4	−1.3	$d_{3/2}$	$f_{7/2}$	−1.7	0.3		−1.5	
K	19	23	42	2	−1.1	$d_{3/2}$	$f_{7/2}$	−1.7	0.3	−0.4	−1.0	
V	23	27	50	6	3.4	$f_{7/2}$	$f_{7/2}$	3.3	1.5	−0.3	3.6	
Mn	25	29	54	2	5.1	$f_{7/2}$	$p_{3/2}$	6.2	1.5	−0.3	4.8	
Co	27	31	58	2	3.5±0.3	$f_{7/2}$	$p_{3/2}$	6.2	1.3	0	3.9	
Co	27	33	60	5	3.8	$f_{7/2}$	$p_{3/2}$	3.9	1.3	−0.3[c]	4.1	
Cu	29	35	64	1	±0.4	$p_{3/2}$	$f_{5/2}$	0.9	1.5	0.4	−0.5	
Rb	37	49	86	2	−1.7	$f_{5/2}$	$g_{9/2}$	−2.1	0.5	−0.2	−1.7	
Cs	55	79	134	4	3.0	$g_{7/2}$	$d_{3/2}$	2.2	0.7	0.6	2.8	
La	57	81	138	5	3.7	$g_{7/2}$	$d_{3/2}$	2.2	0.8	0.6	3.7	3.15±0.45
Lu	71	105	176	$\geqslant 7$	4.2±0.8							8

[a] In LS-coupling $\mu=0.9$.
[b] μ_{emp} obtained with configuration $\{[d_{5/2}(p)]_{3/2}{}^{3}[d_{5/2}(n)]_{5/2}{}^{-1}\}_4$.
[c] $g_{emp}(n)$ difficult to estimate.

although the core may be distorted the observed quadrupole moment is zero as is expected from general symmetry considerations (see Sec. 2.3).

Q_0 can be estimated by treating the core hydrodynamically and finding the value of β for which the configuration of core plus particle has minimum energy.[51,90]

Neglecting terms higher than first order in β the value we obtain is

$$Q_0=\left[\frac{-3}{4\pi}\right]\left[\frac{2I-1}{2(I+1)}\right]\left[\frac{k}{C}\right][ZR_0^2]. \tag{33}$$

For more than one particle outside the core the situation becomes confused since the way in which levels are filled by the particles depends in a complicated fashion on both the distortion and the internucleon forces. Calculations by Moszkowski and Townes[87] neglecting internucleon forces indicate that states with positive quadrupole moments are lower in energy than those with negative quadrupole moments in the regions of large distortions. This then accounts for the predominance of positive quadrupole moments. However, the situation is very complicated and only rough estimates of nuclear quadrupole moments can be made. Such estimates have been given by Van Wageningen and de Boer,[91] Bohr and Mottelson,[51] and Moszkowski and Townes[87] who find that there is qualitative agreement and in particular that large quadrupole moments are to be expected away from closed shells. However, the indications are that the hydrodynamical description of the core is a poor approximation to reality. Thus, in this approximation no account is taken of the expected rigidity of the core in the region of closed shells and Bi^{209} and O^{17}, for instance, are predicted to have quadrupole moments about 15 and 30 times larger respectively than their measured values.

[90] J. Rainwater, Phys. Rev. **79**, 432 (1950).
[91] R. Van Wageningen and J. de Boer, Physica **18**, 369 (1952).

7.4 Conclusions

As with magnetic moments the values of quadrupole moments can be accounted for if the condition is relaxed that the nucleus is to be represented by the extreme single particle model. Thus, configurational mixing within the individual particle model or weak surface coupling in the collective model probably account for all quadrupole moments except those very large ones that can only be interpreted as a strong collective distortion of the core. Unfortunately it does not seem at present possible to give more than a qualitative estimate of these latter moments. With magnetic moments in strong coupling the measured quadrupole moment in principle enables g_Ω to be estimated and the magnetic moment to be calculated. On the other hand, the mechanism of the distortion has to be investigated to calculate the quadrupole moment and this is not an easy problem since it involves detailed knowledge of the core structure.

Tables in the appendix give measured quadrupole moments, their ratio to the single particle value and probable interpretation in terms of nuclear models.

8. MAGNETIC DIPOLE AND ELECTRIC QUADRUPOLE MOMENTS OF ODD-ODD NUCLEI

8.1 Moments of Odd-Odd Nuclei on Particle Models

The simplest particle description of an odd-odd nucleus is to attribute the nuclear properties to an odd proton and an odd neutron each in its appropriate shell model state. If these states are denoted as in Eq. (18)

TABLE V. Magnetic moments of odd-odd nuclei on the collective model.

Atom	Z	N	A	Spin	μ	Proton state	Neutron state	Ω_p	Ω_n	μ_c
B	5	5	10	3	1.8	$(p_{3/2})^{-1}$	$(p_{3/2})^{-1}$	3/2	3/2	1.8
N	7	7	14	1	0.4	$p_{1/2}$	$p_{1/2}$	1/2	1/2	0.4 to 0.5[a]
Na	11	11	22	3	1.8	$(d_{5/2})^3$	$(d_{5/2})^3$	3/2	3/2	1.7
Na	11	13	24	4	1.7	$(d_{5/2})^3$	$(d_{5/2})^{-1}$	3/2	5/2	1.1
K	19	21	40	4	−1.3	$(d_{3/2})^{-1}$	$f_{7/2}$	1/2	7/2	−1.1
K	19	23	42	2	−1.1	$(d_{3/2})^{-1}$	$(f_{7/2})^3$	−1/2	5/2	−0.7
Co	27	31	58	2	3.5±0.3	$(f_{7/2})^{-1}$	$(p_{3/2}, f_{5/2})^3$	7/2	−3/2	3.6[b]
Co	27	33	60	5	3.8	$(f_{7/2})^{-1}$	$(p_{3/2}, f_{5/2})^3$	7/2	3/2	3.6[c]
Rb	37	49	86	2	−1.7	$(f_{5/2})^{-1}$	$(g_{9/2})^{-1}$	5/2	9/2	−1.6

[a] Possible $p_{3/2}$ admixture is taken into account.
[b] Value for predominantly $f_{5/2}$ neutron state.
[c] Value for predominantly $p_{3/2}$ neutron state.

by $\psi_{n_pl_pj_p}{}^{m_p}$ and $\psi_{n_nl_nj_n}{}^{m_n}$, respectively, and the total spin is I, then the nuclear wave function can be written

$$\Psi_I{}^M=\sum_{m_pm_n} C(Ij_pj_n;\, m_pm_n)\psi_{n_pl_pj_p}{}^{m_p}\psi_{n_nl_nj_n}{}^{m_n}. \quad (34)$$

Using this function to calculate the expectation value of the magnetic moment operator [Eqs. (1) and (4)] one obtains

$$\mu=\tfrac{1}{2}\left[(g_p+g_n)I+(g_p-g_n)\frac{j_p(j_p+1)-j_n(j_n+1)}{I+1}\right], \quad (35)$$

where g_p and g_n are the g-factors of the odd proton and neutron, respectively, and are given by Eq. (19).

Fair agreement between theory and experiment is obtained if this formula is used to calculate magnetic moments. Theoretical and experimental results are compared in Table IV where it can be seen that the agreement is best for light nuclei. As pointed out by Talmi,[92] in the case of nuclei such as Li^6, B^{10}, N^{14}, Na^{22} etc. in which the odd proton and odd neutron are presumably in the same shell-model states a large discrepancy is not to be expected. For these cases $j_p=j_n$ and $\mu=\frac{1}{2}(g_p+g_n)I$ so that even though there may be large deviations in g_p and g_n modifying the corresponding odd-A moments, these deviations are found empirically and theoretically (see Sec. 5) to be in opposite directions and therefore tend to cancel in the odd-odd nucleus. Inspection of Table IV shows that the larger deviations are in nuclei in which the odd neutron and odd proton are in different states.

Of course for a very light nucleus like Li^6 the jj-extreme is not expected to be good. In the LS-extreme Li^6 should be predominantly in the $^3S_1$ state and the magnetic moment will then be approximately the same as that of the deuteron i.e., 0.88 n.m. There is obviously better agreement with this value and Lane[43] has shown that in intermediate coupling exact agreement is obtained.

For nuclei in which the neutron and proton states are different, Schwartz[93] has put forward the tentative suggestion that formula (35) should still be used to calculate the magnetic moment but that g_p and g_n should now represent the *empirical* g-factors. That is, for the nucleus (Z,N) g_p and g_n are to be taken as the measured g-factors for the neighboring odd-A nuclei $(Z, N-1)$ and $(Z-1, N)$, respectively, where Z and N are the proton and neutron numbers in the odd-odd nucleus. The success of this approach can be seen in Table IV but it can only be justified if (a) the deviations of odd-A magnetic moments from the Schmidt values are due largely to interactions of the odd nucleon with the core and (b) if jj-coupling is an adequate approximation. Now we have already seen that it is possible to account for the magnetic moments of odd-A nuclei by appealing to point (a) and for heavy nuclei (b) is probably a good approximation and it would seem that his approach would give a fairly reliable estimate of the magnetic moments of odd-odd nuclei. Unfortunately the g-factors for immediately adjacent odd A nuclei are not always known; however, Schwartz[93] finds that using the g-factors of nearby nuclei in the same state still gives reasonable agreement. It seems then that, as for odd-A nuclei, the magnetic moments of odd-odd nuclei can also be interpreted in terms of an individual particle approach with interconfigurational mixing, the latter being implied by using the empirical g-factors.

Using Eq. (34), the expectation value of the quadrupole moment operator can also be calculated. The following value for the quadrupole moment of the nucleus on the particle model is then obtained:

$$Q=\frac{(2I+1)!}{2j_p!}\left[\frac{(2j_p-2)!(2j_p+3)!}{(2I-2)!(2I+3)!}\right]^{\frac{1}{2}} \times W(j_pIj_pI;j_n2)(-)^{(j_n-j_p-I)}Q_{j_p}, \quad (36)$$

where Q_{j_p} is the quadrupole moment of a proton in the state j_p and $W(j_pIj_pI;j_n2)$ is a Racah coefficient. From the form of (36) it is clear that Q is always less than or equal to Q_{j_p}.

In Table IV are given the quadrupole moments of odd-odd nuclei and in all cases quoted there other than Lu^{176} the observed quadrupole moments are of the order to be expected for a particle description allowing for the possibility of a certain amount of interconfigura-

[92] I. Talmi, Phys. Rev. **83**, 1248 (1951).
[93] H. M. Schwartz, Phys. Rev. **89**, 1293 (1953).

TABLE VI. The magnetic moments of excited states of nuclei.

Nucleus	State	Spin	Parity	g	μ	Reference
Cd^{111}	247 kev	5/2	+	$-(0.34\pm0.09)$	$-(0.85\pm0.22)$	a
Cd^{111}	247 kev	5/2	+	$-(0.31\pm0.01)$	$-(0.78\pm0.03)$	b
Cs^{134}	128 kev	8	−	0.14 ± 0.001	1.10 ± 0.01	c
Ta^{181}	480 kev	5/2	+	1.2 ± 0.12	3.0 ± 0.30	d
Pb^{204}	1.274 Mev	4	+	$+0.054\pm0.005$	0.22 ± 0.02	e
Np^{237}	69 kev	5/2	+	0.8 ± 0.2	2.0 ± 0.5	f

[a] Aeppli, Albers-Schonberg, Bishop, Frauenfelder, and Heer, Phys. Rev. **84**, 370 (1951).
[b] W. Zobel and R. M. Steffen, Phys. Rev. **98**, 1186 (1955).
[c] V. W. Cohen and D. A. Gilbert, Phys. Rev. **95**, 569 (1954); L. S. Goodman and S. Wexler, Phys. Rev. **95**, 570 (1954).
[d] S. Raboy and V. E. Krohn, Phys. Rev. **95**, 1689 (1954).
[e] V. E. Krohn and S. Raboy, Phys. Rev. **97**, 1017 (1955).
[f] V. E. Krohn, T. B. Novey, and S. Raboy, Phys. Rev. **98**, 1187 (1955).

tional mixing. For example the quadrupole moment of Li^6 can be exactly accounted for in intermediate coupling.[43] However, the large quadrupole moment of Lu^{176} must be attributed to a collective distortion of the core.

8.2 Moments of Odd-Odd Nuclei on the Collective Model

Bohr and Mottelson[51] have discussed the magnetic moments of odd-odd nuclei on the collective model and obtain for the strong coupling moments the values given in Table V where the configuration, defined by Ω_p and Ω_n is also given. The moments in this table are calculated in a straightforward fashion from Eq. (25) in which for g_Ω is used

$$g_\Omega=\frac{1}{\Omega}[\Omega_n g_\Omega(n)+\Omega_p g_\Omega(p)], \tag{37}$$

where $g_\Omega(n)$ and $g_\Omega(p)$ are the g-factors for the odd neutron and odd proton moving in an ellipsoidal potential (see Sec. 6.3). As stressed in Sec. 6.3 these are generally not known and the values used in Table V are those obtained for a pure j-state. In the strong coupling limit these will be considerably altered due to the mixing of other j-states so that the agreement in the table should not be expected to be too good. In general, it seems that for odd-odd nuclei out of the rotational region the situation is one of intermediate coupling. Lu^{176} is expected to have the strong coupling moment but so far its configuration has not been identified or its moment calculated. Its large electric quadrupole moment is consistent with the fact that it lies in the rotational region $150<A<190$.

9. MOMENTS OF EXCITED STATES OF NUCLEI

As mentioned in Sec. 1 it has been possible in a few cases to estimate the magnetic dipole and electric quadrupole moments of excited nuclear states. Those moments that have been measured are given in Table VI.

So far only very rough estimates have been given for nuclear electric quadrupole moments since there is always considerable uncertainty as to the form and strength of the electric fields interacting with the nucleus.

For Cd^{111} the magnetic moment is similar to that for other $d_{5/2}$ odd neutron nuclei and can be explained as interconfigurational mixing. The moment of Cs^{134} is satisfactorily interpreted as an admixture of the $(d_{5/2}, h_{11/2})_8$ and $(g_{7/2}, h_{11/2})_8$ configurations, the terms in brackets representing the odd proton and odd neutron configurations, the total angular momentum in each case being 8. Tantalum-141 probably has spin 5/2 and the moment is in agreement with those for other $d_{5/2}$ nuclei. A mixture of states is probably sufficient to account for the moment of Pb^{204} although the moment is rather small.

10. NUCLEAR MAGNETIC OCTUPOLE MOMENTS

The nuclear magnetic octupole moments which have so far been deduced are given in Table VII. The values given there have been obtained from various sets of experimental results by Schwartz.[4]

Now the values to be expected on the single-particle model are obtained by using the single-particle wave function (18) to calculate the expectation value of $-M_3$ as in Eq. (9). This has been done by Schwartz[4] who obtains

$$\Omega=\mu_0\frac{3(2j-1)(j+2)}{2(2j+4)(2j+2)}[(j-3/2)g_L+g_S]\langle r^2\rangle, \qquad \text{for} \quad j=l+\tfrac{1}{2}$$

$$\Omega=\mu_0\frac{3(2j-1)(j-1)}{2(2j+4)(2j+2)}[(j+5/2)g_L-g_S]\langle r^2\rangle. \qquad \text{for} \quad j=l-\tfrac{1}{2}$$

Lines similar to the Schmidt lines can be drawn for odd proton and odd neutron nuclei by plotting $\Omega/\mu_0\langle r^2\rangle$ against j. In Fig. 10 are given the two lines for odd proton nuclei; also included are the values of $\Omega/\mu_0\langle r^2\rangle$ for the nuclei given above with $\langle r^2\rangle$ replaced by $3R_0^2/5$. It is striking that the observed octupole moments bear a similar relation to the single-particle lines as that of magnetic dipole moments to the Schmidt lines. Presumably configurational mixing of some kind can also account for these deviations. So far, however, no calculations on this problem have been made and are probably not worth while until more experimental data are available.

TABLE VII. Nuclear magnetic octupole moments.

Nucleus	Spin	Shell model state	$\Omega(\times10^{24}\,cm^2)$	Reference
Ga^{69}	3/2	$p_{3/2}$	0.107 ± 0.004	a
As^{71}	3/2	$p_{3/2}$	0.146 ± 0.004	b
In^{115}	9/2	$g_{9/2}$	0.31 ± 0.01	c
I^{127}	5/2	$d_{5/2}$	0.17 ± 0.03	c

[a] R. T. Daly Jr., and J. H. Holloway, Phys. Rev. **96**, 539 (1954).
[b] P. Kusch and T. G. Eck, Phys. Rev. **94**, 1799 (1954).
[c] Jaccarino, King, Satten, and Stroke, Phys. Rev. **94**, 1798 (1954).

11. CONCLUSIONS

In this review an attempt has been made to discuss the interpretation of the available data on nuclear moments in terms of current theories of the nucleus. As stressed in the introduction such theories for all but the very simplest nuclei are theories of semiempirical nuclear models whose wave functions may be used to calculate nuclear moments. It is apparent from the foregoing discussion that one nuclear model alone is not capable of accounting for all the nuclear moments. The following general points about the models used can be made.

(1) The extreme single-particle model of the nucleus in which the ground state properties are vested in a single particle can account for the spins and parities of the vast majority of nuclei. Although this model does not give the magnetic dipole and magnetic octupole moments correctly its predictions are in agreement with their general trend. On the other hand, most electric quadrupole moments do not agree with this model and in many cases the theoretical quadrupole moment is an order of magnitude smaller than the measured one.

(2) The individual particle model with configurational mixing, in which the nucleus is now represented by several particles in states which are approximate eigenfunctions of a Hamiltonian including residual effective internucleon interactions, is capable of explaining all moments apart from those of nuclei lying in the approximate ranges $150<A<190$ and $A>225$.

(3) It is essential to appeal to collective motion to explain the large nuclear quadrupole moments in the range $150<A<190$ and $A>225$ and also to account for magnetic moments in these regions. Further, a collective approach in many cases can furnish an alternative explanation to that given by the individual particle model.

Thus a combination of the collective and individual particle aspects of nuclear motion seem sufficient to account satisfactorily for all nuclear moments. For odd-A nuclei in all but the simplest cases it seems impossible to estimate nuclear magnetic dipole moments to an accuracy greater than $\frac{1}{4}$ to $\frac{1}{2}$ n.m. owing to the uncertainty in the nuclear wave function and the contributions from exchange currents and velocity dependent forces. On the other hand, using empirical g-factors, a fairly good estimate of the magnetic moments of odd-odd nuclei can be made. The situation with nuclear quadrupole moments is more uncertain and apart from a few light nuclei it is impossible to make more than a qualitative estimate.

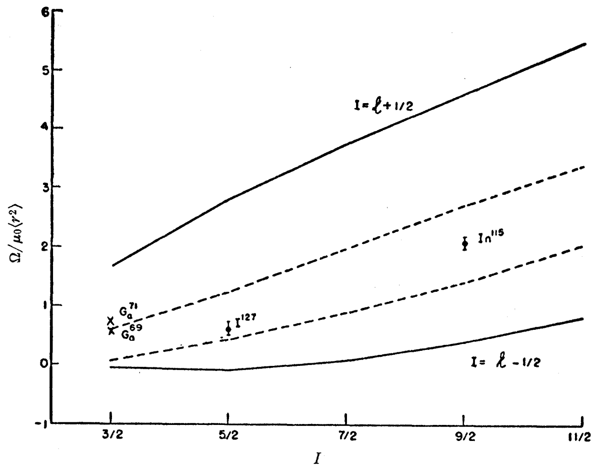

FIG. 10. Magnetic octupole moments of odd-proton nuclei (—— $g_s=5.58$; ---- $g_s=2.00$) [from Charles Schwartz, Phys. Rev. **97**, 380 (1955)].

A final point which should be stressed is that nuclear moments (particularly the magnetic moments) are very sensitive to certain types of admixture in the nuclear wave function. This means that nuclear moments cannot in general be taken as a good guide to the purity or otherwise of nuclear states. If the deviation in the moment is small there may nevertheless be large admixtures of states which do not appreciably affect the moment. On the other hand, a large deviation may only mean a small admixture of states to which the moment is particularly sensitive.

ACKNOWLEDGMENTS

The author is indebted to Professor H. Primakoff and Dr. G. R. Satchler for reading the manuscript and making helpful suggestions for its improvement.

(Appendix to this article is on pages 100–101.)

APPENDIX

The electromagnetic moments of odd proton nuclei.

Nucleus	Z	N	A	Spin	Shell-model state	μ	μ_s	$\Delta\mu$	Q	Q_p	Q/Q_p	References
H	1	0	1	1/2	$s_{1/2}$	2.8	2.8	0	0	0	1	
H	1	2	3	1/2	$s_{1/2}$	3.0	2.8	−0.2	0	0	1	a
Li	3	4	7	3/2	$p_{3/2}$	3.3	3.8	0.5	−0.02±0.02	−0.01	2±2	b
B	5	6	11	3/2	$p_{3/2}$	2.7	3.8	1.1	0.05	0.02	2	b
N	7	8	15	1/2	$p_{1/2}$	−0.3	−0.3	0.0	0	0	1	c
F	9	10	19	1/2	$s_{1/2}$	2.6	2.8	0.2	0	0	1	d
Na	11	12	23	3/2		2.2			0.1			e
Al	13	14	27	5/2	$d_{5/2}$	3.6	4.8	1.2	0.15	0.04	4	b, f, g
P	15	16	31	1/2	$s_{1/2}$	1.3	2.8	1.5				b, f, g
Cl	17	18	35	3/2	$d_{3/2}$	0.8	0.1	0.7	−0.08	−0.04	2	b, f, g
Cl	17	20	37	3/2	$d_{3/2}$	0.7	0.1	0.6	−0.06	−0.04	1.5	b, f, g
K	19	20	39	3/2	$d_{3/2}$	0.4	0.1	0.3				c
K	19	22	41	3/2	$d_{3/2}$	0.2	0.1	0.1				b, f, g
Sc	21	24	45	7/2	$f_{7/2}$	4.8	5.8	1.0				b, f, g
V	23	28	51	7/2	$f_{7/2}$	5.1	5.8	0.7	0.3	0.08	4	b, f, g
Mn	25	30	55	5/2		3.5			0.5			b, f, g
Co	27	30	57	7/2	$f_{7/2}$	4.6	5.8	1.2				b, f, g
Co	27	32	59	7/2	$f_{7/2}$	4.6	5.8	1.2	0.5	0.08	6	b, f, g
Cu	29	34	63	3/2	$p_{3/2}$	2.2	3.8	1.6	−0.16	−0.06	3	b, f, g
Cu	29	36	65	3/2	$p_{3/2}$	2.4	3.8	1.4	−0.15	−0.06	3	b, f, g
Ga	31	38	69	3/2	$p_{3/2}$	2.0	3.8	1.8	0.23	0.06	4	b, f, g
Ga	31	40	71	3/2	$p_{3/2}$	2.6	3.8	1.2	0.15	0.06	3	b, f, g
As	33	42	75	3/2	$p_{3/2}$	1.4	3.8	2.4	0.3	0.06	5	b, f, g
Br	35	44	79	3/2	$p_{3/2}$	2.1	3.8	1.7	0.33	0.06	5	b, f, g
Br	35	46	81	3/2	$p_{3/2}$	2.3	3.8	1.5	0.28	0.06	5	b, f, g
Rb	37	44	81	3/2	$p_{3/2}$	2.0	3.8	1.8				b, f, g
Rb	37	48	85	5/2	$f_{5/2}$	1.4	0.9	0.5	0.31	0.10	3	b, f, g
Rb	37	50	87	3/2	$p_{3/2}$	2.8	3.8	1.0	0.14	0.07	2	b, f, g
Y	39	50	89	1/2	$p_{1/2}$	−0.1	−0.3	0.1				
Nb	41	52	93	9/2	$g_{9/2}$	6.2	6.8	0.6				b, f, g
Tc	43	56	99	9/2	$g_{9/2}$	5.7	6.8	1.1	0.3	0.14	2	b, f, g
Rh	45	58	103	1/2	$p_{1/2}$	−0.0	−0.3	0.2				b, f, g
Ag	47	60	107	1/2	$p_{1/2}$	−0.1	−0.3	0.2				b, f, g
Ag	47	62	109	1/2	$p_{1/2}$	−0.1	−0.3	0.1				b, f, g
Ag	47	64	111	1/2	$p_{1/2}$	−0.1	−0.3	0.1				b, f, g
In	49	64	113	9/2	$g_{9/2}$	5.5	6.8	1.3	1.14	0.15	7	b, f, g
In	49	66	115	9/2	$g_{9/2}$	5.5	6.8	1.3	1.16	0.15	8	b, f, g
Sb	51	70	121	5/2	$d_{5/2}$	3.4	4.8	1.4	−0.5	−0.13	4	b, f, g
Sb	51	72	123	7/2	$g_{7/2}$	2.5	1.7	0.8	−0.7	−0.15	5	b, f, g
I	53	74	127	5/2	$d_{5/2}$	2.8	4.8	2.0	−0.8	−0.13	6	b, f, g
I	53	76	129	7/2	$g_{7/2}$	2.6	1.7	0.9	−0.4±0.2	−0.14	3±1	b, f, g
I	53	78	131	7/2	$g_{7/2}$				−0.4	−0.14	3	b, f, g
Cs	55	76	131	5/2	$d_{5/2}$	3.5	4.8	0.7				b, f, g
Cs	55	78	133	7/2	$g_{7/2}$	2.6	1.7	0.9	−0.003	−0.14	0	b, f, g
Cs	55	80	135	7/2	$g_{7/2}$	2.7	1.7	1.0				b, f, g
Cs	55	82	137	7/2	$g_{7/2}$	2.8	1.7	1.1				b, f, g
La	57	82	139	7/2	$g_{7/2}$	2.8	1.7	1.1	0.9	0.15	6	b, f, g
Pr	59	82	141	5/2	$d_{5/2}$	3.9	4.8	0.9	−0.05	−0.13	0	b, f, g
Eu	63	88	151	5/2	$d_{5/2}$	3.6	4.8	1.2	1.2	0.14	9	b, f, h
Eu	63	90	153	5/2	$d_{5/2}$	1.6	4.8	3.2	2.5	0.14	18	b, f, h
Tb	65	94	159	3/2	$d_{3/2}$	1.5±0.4	0.1	1.4±0.4				b, f, i
Tm	69	100	169	1/2	$s_{1/2}$	−0.2	2.8	3.0				b, f, i
Lu	71	104	175	7/2	$g_{7/2}$	2.9±0.5	1.7	1.2±0.5	5.7±0.3	0.18	32±1	b, f, i
Ta	73	108	181	7/2	$g_{7/2}$	2.1	1.7	0.4	4.3±0.4	0.18	24±2	b, f, i
Re	75	110	185	5/2	$d_{5/2}$	3.2	4.8	1.6	2.8	0.16	18	b, f, i
Re	75	112	187	5/2	$d_{5/2}$	3.2	4.8	1.6	2.6	0.16	16	b, f, i
Ir	77	114	191	3/2	$d_{3/2}$	0.2	0.1	0.1	1.5	0.11	14	b, f, j
Ir	77	116	193	3/2	$d_{3/2}$	0.2	0.1	0.1	1.5	0.11	14	b, f, j
Au	79	118	197	3/2	$d_{3/2}$	0.1	0.1	0	0.6	0.12	5	b, f, j
Tl	81	122	203	1/2	$s_{1/2}$	1.6	2.8	1.2				b, f, g
Tl	81	124	205	1/2	$s_{1/2}$	1.6	2.8	1.2				b, f, g
Bi	83	126	209	9/2	$h_{9/2}$	4.0	2.6	1.4	−0.4	−0.2	2	b, f, g

[a] Exchange effects.
[b] Intermediate coupling.
[c] "Doubled closed (LS) shell minus one" nucleus.
[d] Probably mainly $(s_{1/2})_{1/2}{}^3$ configuration.
[e] Probably $(d_{5/2})_{3/2}{}^3$ configuration.
[f] Configurational mixing.
[g] Weak surface coupling.
[h] Transition from weak-to-strong surface coupling.
[i] Strong surface coupling.
[j] Transition from strong-to-weak surface coupling.

The electromagnetic moments of odd neutron nuclei.

Atom	Z	N	A	Spin	Shell-model state	μ	μ_s	$\Delta\mu$	Q	Reference
n	0	1	1	1/2	$s_{1/2}$	−1.9	−1.9	0		
He	2	1	3	1/2	$s_{1/2}$	−2.1	−1.9	−0.2		a
Be	4	5	9	3/2	$p_{3/2}$	−1.2	−1.9	0.7	0.02	b, c
C	6	7	13	1/2	$p_{1/2}$	0.7	0.6	−0.1		b
O	8	9	17	5/2	$d_{5/2}$	−1.9	−1.9	0.0	−0.004	d, e
Mg	12	13	25	5/2	$d_{5/2}$	−0.9	−1.9	1.0		b, f, g
Si	14	15	29	1/2	$s_{1/2}$	−0.6	−1.9	1.4		b, f, g
S	16	17	33	3/2	$d_{3/2}$	0.6	1.1	0.5	−0.06	b, f, g
S	16	19	35	3/2	$d_{3/2}$	1.0	1.1	0.1	0.06	f, g, h
Ca	20	23	43	7/2	$f_{7/2}$	−1.3	−1.9	0.6		b, f, g
Ti	22	25	47	5/2		−0.8				b, f, g
Ti	22	27	49	7/2	$f_{7/2}$	−1.1	−1.9	0.8		b, f, g
Cr	24	29	53	3/2	$p_{3/2}$	−0.5	−1.9	1.4		b, f, g
Zn	30	37	67	5/2	$f_{5/2}$	0.9	1.4	0.5		b, f, g
Ge	32	41	73	9/2	$g_{9/2}$	−0.9	−1.9	1.0	−0.2±0.1	b, f, g
Se	34	41	75	5/2					0.9	b, f, g
Se	34	43	77	1/2	$p_{1/2}$	0.5	0.6	0.1		
Se	34	45	79	7/2		−1.0			0.7	b, f, g
Kr	36	47	83	9/2	$g_{9/2}$	−1.0	−1.9	0.9	0.15	b, f, g
Sr	38	49	87	9/2	$g_{9/2}$	−1.1	−1.9	0.8		b, f, g
Zr	40	51	91	5/2	$d_{5/2}$	−1.3	−1.9	0.6		b, f, g
Mo	42	53	95	5/2	$d_{5/2}$	−0.9	−1.9	1.0		b, f, g
Mo	42	55	97	5/2	$d_{5/2}$	−0.9	−1.9	1.0		b, f, g
Pd	46	59	105	5/2	$d_{5/2}$	−0.6	−1.9	1.3		b, f, g
Cd	48	63	111	1/2	$s_{1/2}$	−0.6	−1.9	1.3		b, f, g
Cd	48	65	113	1/2	$s_{1/2}$	−0.6	−1.9	1.3		b, f, g
Sn	50	65	115	1/2	$s_{1/2}$	−0.9	−1.9	1.0		b, f, g
Sn	50	67	117	1/2	$s_{1/2}$	−1.0	−1.9	0.9		b, f, g
Sn	50	69	119	1/2	$s_{1/2}$	−1.0	−1.9	0.9		b, f, g
Te	52	71	123	1/2	$s_{1/2}$	−0.7	−1.9	1.2		b, f, g
Te	52	73	125	1/2	$s_{1/2}$	−0.9	−1.9	1.0		b, f, g
Xe	54	75	129	1/2	$s_{1/2}$	−0.8	−1.9	1.1		b, f, g
Xe	54	77	131	3/2	$d_{3/2}$	0.7	1.1	0.5	−0.12	b, f, g
Ba	56	79	135	3/2	$d_{3/2}$	0.8	1.1	0.3		b, f, g
Ba	56	81	137	3/2	$d_{3/2}$	0.9	1.1	0.2		b, f, g
Nd	60	83	143	7/2	$f_{7/2}$	−1.1	−1.9	0.8	∼1	b, f, g
Nd	60	85	145	7/2	$f_{7/2}$	−0.6	−1.9	1.3	∼1	b, f, i
Sm	62	85	147	7/2	$f_{7/2}$	−0.8	−1.9	1.1		b, f, i
Sm	62	87	149	7/2	$f_{7/2}$	−0.6	−1.9	1.3		b, f, i
Gd	64	91	155	3/2		−0.31				b, f, j
Gd	64	93	157	3/2		−0.38				b, f, j
Er	68	99	167	7/2	$f_{7/2}$	−0.5	−1.9	1.4	10.2	b, f, j
Yb	70	101	171	1/2	$p_{1/2}$	0.45	0.6	0.2		b, f, j
Yb	70	103	173	5/2	$f_{5/2}$	−0.7	1.4	2.1	3.9	b, f, j
W	74	109	183	1/2	$p_{1/2}$	0.1	0.6	0.5		b, f, j
Os	76	113	189	3/2		0.7				b, f, j
Pt	78	117	195	1/2	$p_{1/2}$	0.6	0.6	0.0		
Hg	80	117	197	1/2	$p_{1/2}$	0.5	0.6	0.1		
Hg	80	119	199	1/2	$p_{1/2}$	0.5	0.6	0.1		
Hg	80	121	201	3/2	$p_{3/2}$	−0.6	−1.9	1.4	0.6	b, f, g
Pb	82	125	207	1/2	$p_{1/2}$	0.6	0.6	0.0		
U	92	143	235	7/2		−0.8				b, f, j
Pu	94	145	239	1/2		±0.4				b, f, j
Pu	94	147	241	5/2		±1.4				b, f, j

[a] Exchange effects.
[b] Intermediate coupling.
[c] Recoil quadrupole moment =0.008.
[d] "Double closed (LS) shell plus one" nucleus.
[e] Recoil quadrupole moment =0.001.
[f] Configurational mixing.
[g] Weak surface coupling.
[h] $(d_{3/2})_{3/2}{}^3$ configuration considerably reduces the effect of b.
[i] Transition from weak-to-strong surface coupling.
[j] Strong surface coupling.