Experimental Single-Particle Level Spacings in the Region of the $1f_{7/2}$ Shell

RUDI H. NUSSBAUM*

Physics Department, Indiana University, Bloomington, Indiana

A review is presented of experimental information about the relative position of single-particle states in odd-A nuclei in the region of the $1f_{7/2}$ shell. In Appendix I a discussion of the assignments is given for all the included level schemes. Some striking empirical regularities in nuclei with odd nucleon numbers >28 are mentioned in Appendix II.

1. INTRODUCTION

A study of regularities in the level structure of nuclei is of great importance for the interpretation of experimental data from nuclear reactions, radioactive decay schemes, neutron capture γ-ray spectra, and from other techniques, as well as for more detailed theoretical studies of nuclear models, to account for these regularities.

For the even-A nuclei this is best illustrated by the important results which are still being obtained by the detailed study of the systematics of excited states in even-even or odd-odd nuclei.

For the odd-A nuclei the investigation of the displacement of isomeric levels as a function of the even number of protons or neutrons yields an equivalent example.

Several studies have recently been made to derive more quantitative information about the internucleon interactions in medium weight nuclei, where the independent particle model offers a good description, from experimental level schemes.[1,2] These studies have so far been limited to the calcium isotopes because only for these nuclei sufficient data were available.

The scope of this review is to compile the present experimental data on odd-A nuclear level schemes in the region of the $f_{7/2}$ shell (where theoretical treatment is relatively easy because of its rather isolated position) to discern regularities as well as to point out the importance of extended experimental studies in this region to give a better foundation for continued theoretical research.†

2. INDEPENDENT-PARTICLE MODEL AND LOW LYING EXCITED STATES IN $f_{7/2}$ NUCLEI‡

2.0 The occurrence of the anomalous $5/2^-$ ground state spins in Mn^{55} and Ti^{47} was explained in terms of the interaction between the odd $f_{7/2}$ nucleons only, in the configuration $(f_{7/2})^{-3}$ for the protons and neutrons in Mn^{55} and Ti^{47}, respectively,[3] or by the interaction of *all* $f_{7/2}$ nucleons in the configuration $(f_{7/2})^5$ outside the 20–20 core in the discussion of Ti^{47} and V^{49} by Flowers.[4] In both treatments the effect is studied of increasing the range of nuclear forces on the position of the levels belonging to the configurations $(7/2)^n$, while Flowers included proton-neutron interaction in his investigation to explain a crossing-over of the $7/2^-$ and the $5/2^-$ levels. In his discussion Flowers[4] predicts an anomalous ground state spin $5/2^-$ for V^{49} as a possible test for his arguments. So far the spin of this nucleus has not yet been measured. The interesting implications for the spin assignments of the first two excited states of V^{49} and the ground state of Cr^{49} have been discussed previously.[5,6]

However, if the explanation for the occurrence of the anomalous *ground state* spins is valid, it will also bear consequences for the systematics of the low *excited states* of the $f_{7/2}$ nuclei. A systematic investigation was therefore started by the Amsterdam group.[5–7] (As far as this could be done with the technique of beta- and gamma-ray spectroscopy for cyclotron produced radioactive nuclei.)

2.1 In nuclei with 23 or 25 protons or neutrons showing *anomalous* $5/2^-$ ground state spin, a very low-lying $7/2^-$ first excited state is to be expected, and also the next higher $3/2^-$ level, belonging to the same configuration, should have a low excitation energy, compared to the first single-particle $p_{3/2}$ level.

2.2 Also, in the other $f_{7/2}$ nuclei with 23 or 25 protons or neutrons which have a *normal* $7/2^-$ ground state, the $5/2^-$ and $3/2^-$ levels should be found very close to the ground state, since only a relatively weak configuration interaction with higher levels should account for their occasional crossing over.

* Present address: CERN, Case postale 25, Aéroport, Genève 15, Switzerland.

[1] K. W. Ford and C. Levinson, Phys. Rev. 100, 1, 13 (1955).

[2] J. B. French and B. J. Raz, Phys. Rev. 98, 1523 (1955); Phys. Rev. (to be published).

† Part of this work has been previously published, references 5 and 8.

‡ In this paper "$f_{7/2}$ nuclei" refer to odd-A nuclei with an odd number of nucleons in the $f_{7/2}$ shell.

[3] D. Kurath, Phys. Rev. 80, 98 (1950); 91, 1430 (1953); I. Talmi, Helv. Phys. Acta 25, 185 (1952).

[4] B. H. Flowers, Phil. Mag. 45, 329 (1954).

[5] R. H. Nussbaum, *Nuclear Levels in the Neighbourhood of the $1f_{7/2}$ State*, thesis (Van Gorcum & Company N. V. Assen., The Netherlands, 1954).

[6] Nussbaum, van Lieshout, Nijgh, Ornstein, and Verster, Physica 20, 165 (1954).

[7] Nussbaum, van Lieshout, and Wapstra, Phys. Rev. 92, 207 (1953).

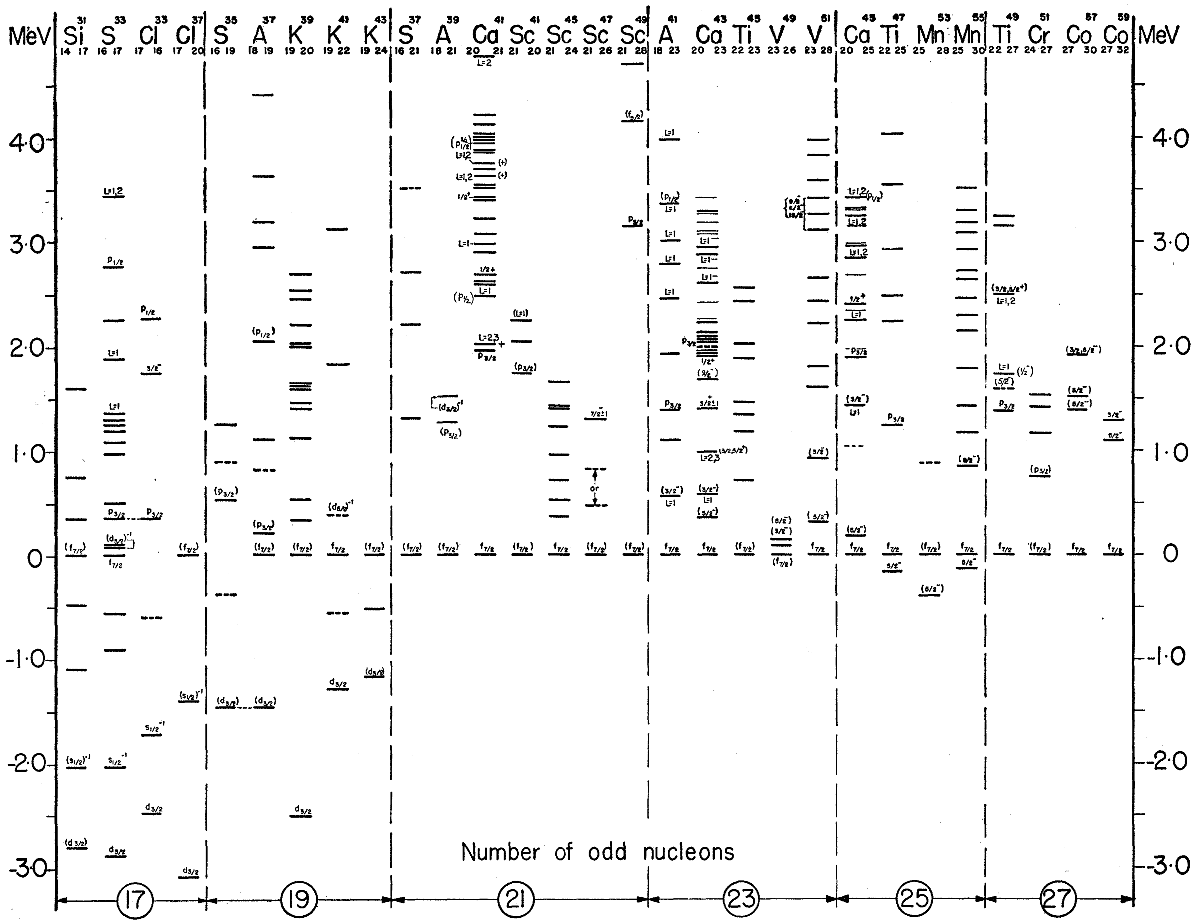

FIG. 1. Lower part of energy level schemes of odd-A nuclei with $17 \leqslant (N \text{ or } Z) \leqslant 27$ normalized to the position of the $1f_{7/2}$ level. (Exceptions are Cl^{33} and S^{35}, which are normalized to the position of the $2p_{3/2}$ level in S^{33} and the $1d_{3/2}$ level in A^{37}, respectively.) The spin and parity assignments are discussed in Appendix I for each nucleus. The symbol L refers to the l values obtained from angular distributions in stripping reactions.

2.3 The evidence for shell closures at 20 and 28 nucleons[8] suggests a rather isolated position of the $f_{7/2}$ state. Consequently, with only one particle (or one hole) in the $f_{7/2}$ shell, mainly single-particle excitation is to be expected for the lowest excited states, resulting in relatively high excitation energies for the lowest levels.§ Moreover, a comparison of Ca^{41}, Sc^{41}, Sc^{49}, and Ca^{47} with nuclei with 21 or 27 odd nucleons but an *additional* number of *even* nucleons in or outside the $f_{7/2}$ state should give some information about the importance of the interaction between the odd nucleon or hole of one kind and the even core of the other kind of nucleons. If this were negligible, as suggested in first approximation by Talmi and Kurath,[3] the level schemes of nuclei with 21 and 27 protons or neutrons should not depend much on the number of even nucleons outside the 20–20 core but should differ markedly from those of nuclei with 23 or 25 odd nucleons.

[8] See, e.g., M. Goeppert Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structures* (John Wiley & Sons. Inc., New York, 1955), Sec. V.7; H. Horie and A. Arima, Phys. Rev. **99**, 778 (1955).

§ For a discussion of the *jj*-coupling shell model for the Ca isotopes in particular, see references 1, 2 and 10. "Excitation energy" is used as a synonym for the energy of the first excited state.

3. LEVEL SCHEMES OF $f_{7/2}$ NUCLEI

3.0 In Fig. 1 the level schemes are given for all odd-A nuclei with odd nucleon numbers between 17 and 27, in which the position of the $f_{7/2}$ state, chosen as a normalizing level, is either identified or can be inferred from comparison with neighboring nuclei. The assignments of orbital and angular momentum values, labelling them as mainly "single-particle" levels, are based on relative cross-section measurements in (d,p) or (d,n) angular distribution experiments,[2] capture γ-ray intensity measurements,[9] Coulomb excitation, radioactive decay characteristics, or other techniques yielding information about spins and/or parities of levels. Cases where the assignments are based on less

[9] B. B. Kinsey and G. A. Bartholomew, Phys. Rev. **93**, 1260 (1954).

unambiguous information or on comparison of apparently similar level structures in neighboring nuclei are indicated by parentheses. A discussion of the individual assignments is given in Appendix I.

The most valuable information on level characteristics as far as the distinction between single- and many-particle levels is concerned is that obtained from angular distribution measurements of stripping reactions. The assignment of a definite l value is possible for many levels and from an analysis of the relative cross sections at the maxima of the angular distributions an unambiguous distinction can be made between states of mainly single-particle character (with seniority $s=1$) and mixed states ($s>1$). (For a detailed discussion see reference 2.) Those investigations like the recent extensive study of the Ca isotopes by the M.I.T. group[10,11] form the backbones of this study. They enable us to propose plausible assignments to levels of neighboring nuclei which are yet less extensively investigated, by comparison of their respective level schemes.

Before discussing the single-particle level spacing in particular, a few general remarks can be made from regularities shown in Fig. 1:

3.1 The excitation energy is appreciably higher in most of the nuclei with 20 or 28 even nucleons compared to their neighbors in Fig. 1 (see Cl^{37}, K^{39}, Ca^{41}, Sc^{41}, Sc^{49}, V^{51}, and Mn^{53}). This is also true for Ca^{43} and Ca^{45} if the $f_{7/2}-p_{3/2}$ distance is only considered and compared with that in A^{41} and Ti^{47}, respectively.

3.2 A comparison of the excitation energies of Ca^{41}, Sc^{41}, Sc^{49}, and Ca^{49} suggest a more rigid structure of the 20–28 core compared to the 20–20 core. This is also indicated by the difference in the excitation energy of Ca^{40} (3.35 Mev) and Ca^{48} (3.83 Mev).[10] The same feature might explain the difference in the splitting of the $(f_{7/2})_{7/2,\,5/2,\,3/2}{}^3$ level triplet in Ca^{43} and V^{51}. Further evidence for such an effect might be the occurrence of low-lying *positive* parity levels in Ca^{41} and Ca^{43}. In effect in Ca^{43} the energy required to break up the 20 nucleon configuration is about half of that necessary to lift an $f_{7/2}$ neutron into the first level of the next shell ($p_{3/2}$). It is to be kept in mind, however, that the gain in pairing energy may play an important part here. On the other hand, in Ca^{41} the first positive parity level is just *above* the $p_{3/2}$ level, thus indicating that the presence of the two extra neutrons in Ca^{43} favors a decoupling of the 20–20 nucleon core. One might think of this mechanism in terms of stabilization effects as suggested by DeShalit and Goldhaber.[12]

3.3 We find the predictions made in Sec. 2.1 and 2.2 confirmed in all investigated cases. The features discussed in Sec. 2.3 are also clearly shown in the level schemes: The first excited states in Ca^{41} and Sc^{49} are the single-particle $p_{3/2}$ states with an excitation energy of several Mev. No direct experimental evidence for the single-particle nature of the first excited state in Sc^{41} is so far available. Since the excitation energy is practically equal for the two mirror nuclei, it seems justified, however, to assume a similar character for the two states. Unfortunately, no data are yet available for Ca^{47}.

In the cases where the even number of nucleons do not fill a shell completely, the picture is as follows:

3.4 An increasing number of even nucleons outside the 20 shell seems to depress the lower levels and probably to compress the whole level scheme.‖ See e.g. Sc^{41} *vs* Sc^{45}. (Unfortunately, no levels are known in Sc^{43} and only β-decay data are known for Sc^{47}.) A measurement of the nature of the Sc^{45} levels by means of $Ca^{44}(d,n)Sc^{45}$ angular distributions would be of great interest to determine the strength of the even-odd nucleon interaction. A comparison between Ca^{43} and Ti^{45} seems not to be in agreement with this trend. It has to be kept in mind, though, that the resolution in the $Sc^{45}(p,n)Ti^{45}$ measurements is much lower than in the $Ca^{43}(p,p')$ experiments. So it is well possible that the actual level density in Ti^{45} is greater than that shown by the present data. In the same way V^{49}–V^{51} and Ti^{49}–Cr^{51} may be compared. Less difference seems to be indicated between the level schemes of Ca^{45} and Ti^{47}. It would also be interesting to compare Ti^{49} with Sc^{43}.

3.5 Angular distribution measurements have shown that there are three or more levels below about 4 Mev which have a single-particle p-level character on account of their high relative cross section in Ca^{41} and A^{41}. The same is found in Cr^{53} with a $p_{3/2}$ ground state (see Sec. 4). This is theoretically difficult to understand. It also introduces an uncertainty for the p-doublet splitting (see also Ca^{41}, Appendix I).

3.6 In A^{41}, Ca^{43}, and Ca^{45}, a low-lying $l=1$ level is found which on account of its low stripping cross section can be described as the $(f_{7/2})_{3/2}{}^3$ state with a small admixture of the single-particle $(f_{7/2})_0{}^n p_{3/2}$ $(n=2,4)$ configuration.[2]

However, no stripping is observed for the $(f_{7/2})_{5/2}{}^{\pm 3}$ states which is in agreement with a reasonable estimate for the cross section for these levels, assuming a similar admixture of the single-particle $(f_{7/2})_0{}^n f_{5/2}$ $(n=2,4)$ state.[2]

3.7 Although many of the levels investigated by (d,p) or (d,n) angular distribution measurements show clear Butler distributions, superimposed on an apparently isotropic background (probably mainly from compound-nucleus formation), several proton groups in the $Ca(d,p)$ measurements show distributions which do not allow an unambiguous orbital momentum assignment, except if one allows different values for the

[10] C. M. Braams, *Energy Levels of Calcium Isotopes*, thesis (Excelsior, Oranjeplein 96, The Hague, The Netherlands, 1956).

[11] Braams, Bockelman, Browne, Buechner, Cobb, and Guthe (unpublished); see also C. K. Bockelman, Bull. Am. Phys. Soc. Ser. II, **1**, 223 (1956) and reference 2.

[12] A. DeShalit and M. Goldhaber, Phys. Rev. **92**, 1211 (1953).

‖ This possible explanation has also been suggested by B. J. Raz, (private communication).

nuclear radius to be used for different levels of the same nucleus.[11] The 2.01 and 0.99 Mev levels in Ca^{41} and Ca^{43}, respectively, have distributions between those expected for $l=2$ and $l=3$. In Ca^{43} the 0.99 Mev level has definitely positive parity (see Appendix I) and so has most probably the 2.01 Mev state in Ca^{41}. This feature of an $l=2$ distribution, shifted to larger angles, or requiring a smaller nuclear radius to fit the theoretical curve, might be characteristic for these positive parity levels, all arising from a decoupling of the 20-nucleon configuration. Similar ambiguous distributions halfway between the theoretical $l=1$ and $l=2$ curves occur for some levels in Ca^{41}, Ca^{45}, and Ti^{49}. Not enough information is available to identify those levels with one or the other l value. Future refinements in the stripping and compound-nucleus theories may cast some light on these irregularities.

3.8 The theoretical studies of the level structure of $f_{7/2}$ nuclei in j–j coupling also predict relatively low-lying levels with high spin values $9/2^-$, $11/2^-$, and $15/2^-$ from the $(f_{7/2})^{\pm 3}$ configuration[1–4]. Braams[10] found evidence for a possible high spin level at 1.678 Mev in Ca^{43}. There are three more levels below the 2.048 $p_{3/2}$ level in Ca^{43} which do not show observable stripping characteristics and among which the high spin assignments could be distributed. Angular momentum selection rules, however, will make a definite identification very difficult. A favorable and rather unique case is the observation of $V^{50}(n,\gamma)V^{51}$ neutron capture γ rays. Since the capturing state in V^{50} has either spin[13] $11/2^+$ or $13/2^+$, strong $E1$ transitions should be found to any existing $9/2^-$, $11/2^-$, or $15/2^-$ state.[9,13] Bartholomew and Kinsey[14] found three γ rays, two of which fit into the level scheme of V^{51} as obtained from $V^{51}(p,p')$ reactions.[13] The higher energies of these states (3.11, 3.26, and 3.41 Mev) compared to their probable energy in Ca^{43} (below 2 Mev) might be due to the 28 neutron configuration in V^{51} as pointed out in Sec. 3.2. Evidence for a possible $9/2^-$ state has also been found in Mn^{55} (see Appendix I).

3.9 In Fig. 1 also are included the level schemes of nuclei with 17 and 19 protons or neutrons. The reason for this is mainly to compare the single-particle $f_{7/2}$, $f_{5/2}$, $p_{3/2}$, and $p_{1/2}$ distances found in these cases with those found in the $f_{7/2}$ nuclei. Unfortunately, only little complete information from stripping reactions is known so far. In Cl^{33} and S^{35} the $f_{7/2}$ levels are not identified. The level schemes have, therefore, arbitrarily been normalized to the positions of the $p_{3/2}$ and the ground state $d_{3/2}$ in S^{33} and A^{37}, respectively (indicated by dotted lines between the respective states). No detailed discussion will be presented here about these schemes although, where possible, the data are included in the presentation of single-particle level distances in Sec. 5.

4. LEVEL SCHEMES OF NUCLEI WITH 29 PROTONS OR NEUTRONS. THE $f_{5/2}$ LEVEL

4.0 A review of the level schemes for nuclei with 29 protons or neutrons is shown in Fig. 2. They are included here mainly to investigate the distance between the $p_{3/2}$ ground state and the single-particle $f_{5/2}$ and $p_{1/2}$ states. In all but one (Ca^{49}) case there are probably some many-particle states among the lower excited states due to the presence of an even number of nucleons outside the closed 28 shell. Although very little unambiguous data are available here, a few general remarks can be made:

4.1 The excitation energy decreases markedly when pairs of protons are added to the 20-proton and (28+1)-neutron configuration. (Compare with Ca^{41}, Ca^{43}, and Ca^{45}, Fig. 1.)

4.2 An important question is the position of the $f_{5/2}$ state relative to the $p_{3/2}$ state. The assumption has frequently been made in papers on shell model interpretations[8] that these two levels should be very close together. The argument is mainly based on the evidence that Rb^{85} has a $5/2^-$ ground state and a $3/2^-$ first excited state at 150 kev while in Rb^{87} the spins are reversed in order and the excitation energy is 400 kev.

It is, however, by now well known that nuclei in this region of N or Z have a great number of very low-lying states which by no means can be interpreted as single-particle states; in fact, no experiments have been done yet which could distinguish between single-particle and many-particle levels. E.g., in Zn^{67} with 37 neutrons and 30 protons there are three $5/2^-$ levels and three $3/2^-$ levels below 900 kev excitation energy. Rietjens and van den Bold[15] propose $f_{5/2}$ both for the Zn^{67}

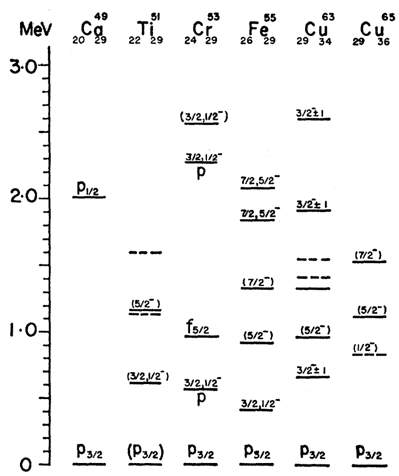

FIG. 2. Energy levels below 3 Mev in odd-A nuclei with 29 protons or neutrons. A discussion of the spin and parity assignments is given in Appendix I.

[13] Way, King, McGinnis, and van Lieshout, "Nuclear level schemes," US-AEC Rept. TID-5300 (1955).

[14] G. A. Bartholomew and B. B. Kinsey, Phys. Rev. **89**, 386 (1953).

[15] L. H. Th. Rietjens and H. J. van den Bold, Physica **21**, 701 (1955).

ground state and for the 595-kev level, while another $5/2^-$ level is as close as 182 kev to the ground state. It is, therefore, suggested that an interpretation in terms of single-particle states is not well justified in this case nor in the case of the Rb isotopes which have 37 protons. A study of the levels of the nuclei with only *one* nucleon outside the closed 28-shell should give a more reliable measure for the $p_{3/2}-f_{5/2}$ distance.

The most favorable nucleus in this class for studying single-particle levels is Ca^{49}. Although angular distributions have so far only been analyzed for the ground and the first excited state, it is certain that the $f_{5/2}$ level is *at least* 3.6 Mev above the $p_{3/2}$ ground state.[10] However, recent angular distribution measurements[16] in $Cr^{52}(d,p)Cr^{53}$ have identified the single-particle $f_{5/2}$ level at 0.97 Mev. This depression relative to Ca^{49} might be accounted for by the four protons outside the 20 core. (See Sec. 3.4.) In four of the six given level schemes of Fig. 2 a $5/2^-$ assignment is excluded for the first excited state by the experimental data (see Appendix I). The lowest possible $5/2^-$ level in these cases occurs as second excited state at about 1 Mev, while the first excited state might be the $p_{1/2}$ level (for the $p_{3/2}-p_{1/2}$ doublet splitting see Sec. 5.3).

It has been stated previously[2,5,10,17] that no identification of the single-particle $f_{5/2}$ state has been possible so far in any of the investigated $f_{7/2}$ nuclei. However, the β decay of Ca^{49} leading to Sc^{49} sets a *lower limit* of 4 Mev to the position of the $f_{5/2}$ level,[18] resulting also in a lower limit of 1 Mev for the $p_{3/2}-f_{5/2}$ separation (see Appendix I) in this (20+1) proton and 28-neutron nucleus. In the decay of Fe^{59} into Co^{59}, however, a $5/2^-$ state lies at 1.10 Mev, 190 kev below a $3/2^-$ level. Since here we have 2 neutrons outside the 28 shell it is

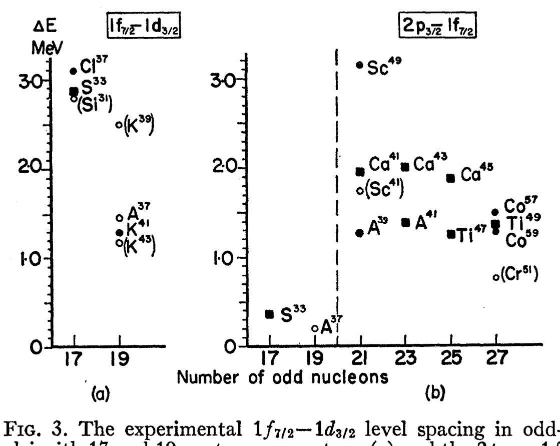

FIG. 3. The experimental $1f_{7/2}-1d_{3/2}$ level spacing in odd-A nuclei with 17 and 19 neutrons or protons (a) and the $2p_{3/2}-1f_{7/2}$ separation in odd-A nuclei with $17 \leqslant (N \text{ or } Z) \leqslant 27$ (b). (See Sec. 5.1 and 5.2.) The meaning of the symbols used is given in Sec. 5.0.

[16] A. J. Elwyn and F. P. Shull, Bull. Am. Phys. Soc. Ser. II, 1, 281 (1956).
[17] R. H. Nussbaum, Bull. Am. Phys. Soc. Ser. II, 1, 16 (1956).
[18] *Nuclear Data Cards* and *Nuclear Science Abstracts* (U. S. National Research Council).

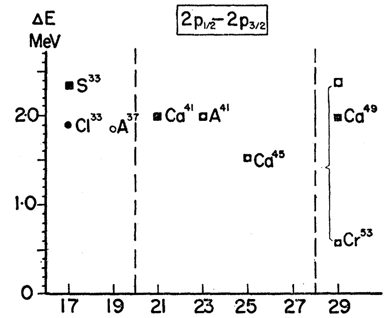

FIG. 4. The experimental $2p_{1/2}-2p_{3/2}$ doublet splitting in odd-A nuclei with $17 \leqslant (N \text{ or } Z) \leqslant 27$. (See Sec. 5.0 and 5.3.)

not clear, though, whether or not one or both of these levels have single-particle character.

Possible explanations for the failure to observe the $f_{5/2}$ level in the Ca isotopes have been discussed in references 2 and 10.

5. SINGLE-PARTICLE LEVEL DISTANCES

5.0 From the data given in Figs. 1 and 2 we have plotted in Figs. 3, 4, and 5 the single-particle level distances. Points obtained from angular distribution and relative cross-section measurements in stripping reactions are indicated by squares.¶ Open circles stand for measurements of the yield at 90° from stripping reactions. Due to the unknown contributions from compound nucleus formation which is more pronounced at such large angles, l-value assignments on the basis of this yield must be considered quite uncertain unless additional information is available (see discussion in Appendix I). Full circles stand for levels for which the spin and parity can be assigned from decay characteristics, neutron capture γ-ray intensities, or other spin determining experiments. Also given in parentheses are those levels for which no direct spin or parity measurement has been done but which could be tentatively assigned on the basis of comparison with neighboring level schemes.

5.1 $f_{7/2}-d_{3/2}$ Separation

In Fig. 3a, the $f_{7/2}-d_{3/2}$ separation is given for 17 and 19 nucleons. It is seen that the addition of 2 neutrons or protons depresses the $f_{7/2}$ level considerably except in K^{39}, which might be explained by its closed-shell neutron configuration.

5.2 $p_{3/2}-f_{7/2}$ Separation

In Fig. 3b, the $p_{3/2}-f_{7/2}$ separation is plotted. It is found that this splitting is quite constant in the $f_{7/2}$ nuclei. There is a rather constant difference between the Ca isotopes, all having a closed 20-proton shell, and the other nuclei with an incompletely filled shell of the

¶ In some cases some ambiguity exists about the assignment. This is indicated by open squares.

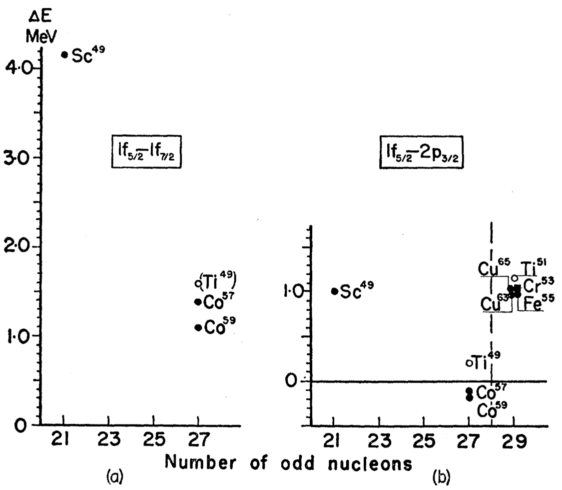

FIG. 5. The experimental $1f_{5/2}-1f_{7/2}$ doublet splitting (a) and the $1f_{5/2}-2p_{3/2}$ spacing in odd-A nuclei with $21 \leqslant (N \text{ or } Z) \leqslant 29$ (b). (See Sec. 5.0, 5.4 and 5.5.)

even nucleons. Sc^{49} exceeds the Ca separation by more than 1 Mev, probably due to its tighter bound 28-neutron configuration (see Sec. 3.2).

5.3 $p_{1/2}-p_{3/2}$ Doublet

It was mentioned in Sec. 3.5 that some ambiguity exists for the assignment of the $p_{1/2}$ levels in Ca^{41}, A^{41}, and Cr^{53} due to the existence of more than two p levels with single-particle relative cross sections. However, some arguments presented in Appendix I favor the assignment given in Fig. 4 for Ca^{41}. The choice in the case of A^{41} has been influenced by a comparison with Ca^{41}. The point for Ca^{45} actually denotes a lower limit (see Appendix I) for the position of the $p_{1/2}$ level. Apart from these uncertainties, however, it is remarkable that a more or less constant doublet-splitting is found in S^{33}, Cl^{33}, Ca^{41}, and Ca^{49}.

5.4 $f_{5/2}-f_{7/2}$ Doublet

In Fig. 5a, the very meager information is given about the f-doublet splitting. The points only represent a *lower limit* for this splitting as discussed in Sec. 4.2.

5.5 $f_{5/2}-p_{3/2}$ Separation

It has been discussed in Sec. 4.2 that only in Cr^{53} the $f_{5/2}-p_{3/2}$ separation is known with the best obtainable certainty. The clustering of points for 20 protons or neutrons around the same energy value in Fig. 5b supports their assignment. Also in Sc^{49} the separation seems to be the same. However, the points at 27 odd nucleons are much less reliable since no evidence for a single-particle character of the levels is as yet available for these nuclei.

6. SUMMARY AND CONCLUSIONS

6.1 The excitation energy generally decreases with the addition of pairs of the even kind of nucleons. There is evidence for a strong interaction between the even and the odd nucleons (Sec. 3.4 and 4.1).

6.2 The 20–28 nucleon core seems to be more strongly bound than the 20–20 core (Sec. 3.2).

6.3 Nuclei with 23 or 25 protons or neutrons have on the average an excitation energy of about 1/5 of the energy of the first excited single-particle state. In some cases, the three lowest levels can be identified as belonging to the $(f_{7/2})^{\pm 3}$ configuration (Sec. 3.3).

6.4 In some nuclei more than two p levels are found below 4 Mev which apparently have single-particle character (Sec. 3.5).

6.5 The $(f_{7/2})_{3/2}{}^{\pm 3}$ state is sometimes found to have a small admixture of the $(f_{7/2})_0{}^n p_{3/2}$ $(n=2,4)$ single-particle state (Sec. 3.6).

6.6 Positive parity levels from decoupling of the 20-nucleon configuration seem to show Butler distributions between that for $l=2$ and $l=3$ (Sec. 3.7).

6.7 There is some evidence for the existence of the predicted high spin states in nuclei with an $(f_{7/2})^{\pm 3}$ ground state configuration (Sec. 3.8).

6.8 The experimental level distances are summarized in Table I. The average is given for a group of

TABLE I. Experimental single-particle level distances (see Sec. 6.8).

Levels	ΔE(Mev)	Number of nucleons Protons	Neutrons
$1f_{7/2}-1d_{3/2}$	3.1	17	20
	~2.8	14,16	17
	2.5	19	20
	~1.3	{19 18}	{22,24 19}
$2p_{3/2}-1f_{7/2}$	~0.3	{16 18}	{17 19}
	~1.9	{21 20}	{20 21,23,25}
	~1.3	{18 22 27}	{21,23 25,27 30,32}
	3.1	21	28
$2p_{1/2}-2p_{3/2}$	~2	{16 17 18}	{17 16 19}
	~1.8	{18 20}	{23 21,25}
	2	20	29
	0.6 or 2.3[a]	24	29
$1f_{5/2}-1f_{7/2}$	$\geqslant 4$	{18 21 20}	{23 28 21}
	>3.4	20	23,25
	$\geqslant \sim 1.3$	{22 27}	{27 30,32}
$1f_{5/2}-2p_{3/2}$	>2.6	18	23
	>2	20	21
	$\geqslant 1$	{20 21}	{23 28}
	>1.5	20	25
	(~0)?	{22 27}	{27 30,32}
	$\geqslant 1$	{22,24,26 29}	{29 34,36}
	$\geqslant 3.6$	20	29

[a] See Sec. 5.3.

nuclei with about equal values from graphs 3–5. The nuclei included for each value are indicated by their proton and neutron numbers.

6.9 It would be of great interest to investigate the changes brought about in the very promising results for the predicted level scheme of Ca^{43} (Ford and Levinson[1]) by revision of the assumed single-particle level structure of Ca^{41} as a basis of their calculations. They assumed, e.g., the position of the $f_{5/2}$ state to be at 2.01 Mev. The recent analysis of the data of the $Ca^{40}(d,p)Ca^{41}$ reaction[2,11] however, fixes this level as a positive parity state (see Appendix I) while the $f_{7/2}-f_{5/2}$ splitting might well be of the order of 4 Mev or more (Sec 4.2). There is also some evidence that the 3.95 Mev level in Ca^{41} is the $p_{1/2}$ state rather than the 2.47 Mev level as assumed in the calculations (Appendix I). Moreover, there is some doubt now about the existence of a 0.84-Mev level[13,10] in Ca^{43} as reported by Lindqvist and Mitchell,[19] which in the calculations has been identified with the expected $9/2^-$ level. Similar calculations for other nuclei in this region would be very valuable to check the consistency of the theory as well as to obtain further information about the values of entering parameters.

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APPENDIX I. DISCUSSION OF LEVEL ASSIGNMENTS

In order to avoid too many references we refer mainly to the well-known extensive compilations in references 13, 18, and 20. Only additional discussion of direct interest to the proposed assignment is given here. We refer also to the discussion given in reference 5.

Si^{31}

The assignment to the 0.76-Mev level is in agreement with Holt and Marsham's tentative assignment.[21] The position of the $f_{7/2}$ state is inferred from comparison with S^{33}.

S^{33}

Assignments from Holt and Marsham's work[21] and neutron capture γ spectra.[13,18] A close triplet has been found by Paris *et al.*[18] between 2.87 and 2.97 Mev, which was not resolved in Holt and Marsham's work. Therefore, one of the three close-lying levels has been assumed to be the $d_{5/2}^{-1}$ in order to explain the existence of a 2.9 Mev γ ray[20] in the decay of Cl^{33}.

Cl^{33}

Assignments from $^{32}S(d,n\theta)Cl^{33}$. Since the $f_{7/2}$ level is not known, the level scheme is arbitrarily normalized to the position of the $p_{1/2}$ level in S^{33} in Fig. 1.

Cl^{37}

Work by Schiffer *et al.*[18] and Morinaga and Bleuler.[18]

S^{35}

Levels from Paris *et al.*[18] The 1.99 Mev level is excited by neutron capture γ rays (Groshev[18] and Kinsey[13]) and is, therefore, probably $p_{3/2}$. A 1.10 Mev level is suggested by the neutron capture γ spectra. A corresponding proton group might well be obscured by the broad C^{12} peak in Paris' spectrum. This first excited state would have a comparable position with that in K^{41} and K^{43}. The ground state is arbitrarily matched to that of A^{37} in Fig. 1.

A^{37}

Assignments from comparison with S^{33} and consideration of relative yields of the $A^{36}(d,n)A^{37}$ reaction.[20]

K^{39}

Levels from work by Sperduto and Buechner,[18] Schwartz *et al.*[18] and Schiffer *et al.*[18] Assignment of the first excited state from comparison with neighboring nuclei only.

K^{41}

The 1.290 Mev level has been assigned $f_{7/2}$ from the β-decay data of A^{41} (Schwarzschild *et al.*[18]). The 1.67 Mev state is probably $d_{5/2}^{-1}$ from the relative intensity of the $K^{40}(n,\gamma)K^{41}$ γ ray A″ in Kinsey and Bartholomew's[20] work.

K^{43}

Levels from Schwartz *et al.*,[18] tentative assignments from comparison only.

S^{37}

Level scheme from Bellamy and Flack's[18] $A^{40}(n,\alpha)S^{37}$ reaction.

A^{39}

Levels and assignment from Scott and Segel's[18] $K^{39}(n,p)A^{39}$ and Penning's *et al.*[18] $Cl^{39}(\beta^-)A^{39}$ investigation.

Ca^{41}

See references 2, 10, and 11. Since the work by Elwyn and Shull[16] has shown that the $f_{5/2}$ level gives the expected angular distribution and relative cross section in $Cr^{52}(d,p\theta)Cr^{53}$ and considering the facts that the relative cross section for the 2.01 Mev level in Ca^{41} is a factor 7 too low for an $f_{5/2}$ level, that it does not have the right angular distribution, and that there is a similar level in Ca^{43} at 0.99 Mev which has definitely *positive* parity, positive parity is also assigned to the 2.01 Mev level. This assignment

[19] T. Lindqvist and A. C. G. Mitchell, Phys. Rev. **95**, 1535 (1954).
[20] P. M. Endt and J. C. Kluyver, Revs. Modern Phys. **26**, 95 (1954).
[21] J. R. Holt and T. N. Marsham, Proc. Phys. Soc. (London) **A66**, 167 (1953).

would also agree qualitatively with the observed intensities of the neutron capture γ rays by Demidov[18,10,11] and Kinsey.[13] There are two levels in Ca^{41} which could be interpreted as $p_{1/2}$ levels, one at 2.47 and one at 3.95 Mev, from their stripping characteristics and the intensity of the neutron capture γ rays feeding them. Bockelman (private communication) pointed out that the reduced width of the 1.95 $p_{3/2}$ level is equal to that for the 3.95 Mev level, but 3 times as large as that for the 2.41 Mev state. Also, the intensity ratio between protons in the stripping reaction and γ rays in the neutron capture γ spectrum leading to the 3.95 Mev level is equal to the same ratio for the 1.95 Mev state while it is a factor *two* greater for the 2.47 Mev level.[22] Moreover, a p-doublet splitting of about 2 Mev is also found in Ca^{49}, Cl^{33}, and S^{33} supporting our proposed assignment.

Sc^{41}

Levels known from Plendl and Steigert's $Ca^{40}(d,n)Sc^{41}$ results.[18] Assignments from comparison with Ca^{41} and relative yields of the neutron groups.

Sc^{45}

Levels known from (p,p') and (n,n') reactions.[18]

Sc^{47}

The 1.3 Mev level has been assigned $7/2^-\pm 1$ on account of the $\log ft$ value 6.0 for the β^- transition of Ca^{47} to this state. A positive parity, however, is as well possible, considering the positive parity level at about the same position in Ca^{43}. The order of the 0.48–0.83 Mev γ rays in the decay of Ca^{47} is not known. A discussion of the anomalous high $\log ft$ value for the expectedly allowed $Ca^{47}(\beta^-)Sc^{47}$ ground state transition has been given in reference 13 and by King.[23]

Sc^{49}

Level assignments are given in accordance with recent data on the $Ca^{49}(\beta^-)Sc^{49}$ decay.[18]

A^{41}

Assignments from Burrows *et al.*[24] from $A^{40}(d,p)A^{41}$. The assignment $p_{1/2}$ to the 3.36 Mev level is not unambiguous. While the 1.39 Mev state has a relative neutron capture probability $\Lambda=100$,[21] the higher $l=1$ states (2.46, 2.79, and 3.01 Mev) have $\Lambda\simeq 18$, and the 3.36 and 3.98 Mev levels have $\Lambda=28$. Since $\Lambda/2J_f+1$ should be nearly a constant for the single-particle states,[21] only the two highest of the aforementioned levels are likely to be $p_{1/2}$ states, although Λ would be expected to be $\simeq 50$. The choice of the 3.36 Mev level as $p_{1/2}$ in Figs. 1 and 4 is based only on the established p-doublet splitting of $\simeq 2$ Mev in S^{33}, Cl^{33}, Ca^{41}, and Ca^{49}.

Ca^{43}

Assignments from $Ca^{42}(d,p)Ca^{43}$.[2,10,11] (For a comparison with neutron capture γ rays see reference 5.) A previously reported γ ray of 1.05 Mev[13] in the decay of Sc^{43} could not be reconfirmed in recent experiments.[25] On the other hand, since the 0.991-Mev level is fed by the most intense β^- branch in the decay[18] of K^{43}, there is no doubt left about its assignment of positive parity. The position of the $p_{1/2}$ level in Ca^{43} is not yet known. However, it is quite certain that its position must be above the energy region studied so far.[10,11] This would make the p-doublet splitting >1.5 Mev in agreement with the values found in S^{33}, Ca^{41}, A^{41}, and Ca^{49}.

Ti^{45}

Levels from (p,n) reactions.[18]

V^{49}

See references 5 and 6.

V^{51}

Levels from reference 13. Assignments based on decay data of Cr^{51} and V^{51}. (See also Sec. 3.8.)

Ca^{45}

Levels and assignments from references 2, 10, and 11.

Ti^{47}

No angular distributions have been measured in $Ti^{46}(d,p)Ti^{47}$ except for the 1.40 Mev level.[13]

Mn^{53}

Assignments from β^+-decay data only.[13]

Mn^{55}

Levels and assignments from references 13 and 18. It is found in $(n,n'\gamma)$ experiments[13] that the γ ray de-exciting the 0.98 Mev level via a cascade to the 0.13 Mev $f_{7/2}$ level is about 20 times as strong as the direct transition from the 0.97 Mev level. Since the ground state is $5/2^-$, this branching ratio might indicate a $9/2^-$ spin for the 0.97 Mev level. Recent measurements by Beghian *et al.*[18] have confirmed this feature. (See Sec. 3.8.)

Ti^{49}

A level at 1.59 Mev is suggested by the existence of a 6.53 Mev plus a 1.59 Mev neutron capture γ ray[13] adding up to the correct neutron binding energy in Ti^{49} of 8.12 Mev. The relatively low intensity of the 6.53 Mev line is in qualitative agreement with an assignment $3/2^+$, $5/2^+$, or $5/2^-$. Because of the lack of evidence for an $f_{5/2}$ level in the $f_{7/2}$ nuclei, a positive parity state might be considered in view of corresponding levels in Ca^{43} and Ca^{41}. On the other hand, a $5/2^-$ level is found close to a $3/2^-$ level in Co^{59}. The other assignments are made from (d,p) angular distribution measurements in combination with consideration of the intensity of neutron capture γ rays.[13]

Cr^{51}

The assignment $p_{3/2}$ to the 0.75 Mev level is based on the existence of a relatively strong 8.50–0.74 neutron capture γ-ray cascade[13] although the energy fit is not too good. This assignment is weakly supported by the absence of a γ ray in the decay of Mn^{51} from this level, although the upper limit on its intensity is so far only 14%.[13]

Co^{57}

Spin assignments from the decay[13] of Ni^{57}.

Co^{59}

Assignments from the decay[13] of Fe^{59}.

Ca^{49}

Assignments from reference 10. The weak γ-ray G in Kinsey's neutron capture γ-ray spectrum[13] might be a ground state transition in Ca^{49}.

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22 A. M. Lane and D. H. Wilkinson, Phys. Rev. **97**, 1199 (1955).
23 R. W. King, Bull. Am. Phys. Soc. Ser. II, **1**, 16 (1956).
24 Burrows, Green, Hinds, and Middleton, private communication by J. R. Holt.
25 R. W. Hayward, private communication to J. B. French.

Ti^{51}

No angular distributions have been measured in the $Ti^{50}(d,p)Ti^{51}$ experiments. A 6.34 Mev γ ray in Kinsey's neutron capture γ spectrum[13] fits into the level scheme leading to the 0.61 Mev first excited state. This supports our assignment. No γ ray is found to either member of the possibly double level at 1.5 Mev[13] in qualitative agreement with the assignment $5/2^-$ to this level. Further support for this is given from the level order in Cr^{53}.

Cr^{53}

The $Cr^{52}(d,p)$ reaction has been measured by Elwyn *et al.*[21] and is supported by neutron capture γ-ray measurements.[13] Both the 0.57 and the 2.29 Mev states show stripping characteristics in agreement with a p-state assignment.[2] The assignment for the 2.68 Mev state is supported by (n,γ) data. Recent measurements on the decay of 1.7-min V^{53} seem also to fit well this level scheme.[26]

Fe^{55}

Two capture γ rays seem to fit well into the scheme leading to the 0.42 and the 0.94 Mev levels, respectively, with about equal intensity. However, from the β^+-decay data of Co^{55} a p level seems to be excluded for the 0.96 Mev state. Although $p_{1/2}$ for the 0.42 Mev state would be in agreement with the Co^{55} decay, it does not agree with the low intensity of the capture γ ray to this state. Moreover, in (p,n) reactions by Stelson and Preston[13] the cross section for the formation of the 0.42 Mev state is much lower than that for the $p_{3/2}$ ground state. This level might, therefore, have a many-particle structure in disagreement with the nature of the corresponding level in Cr^{53}. Angular distribution measurements in $Fe^{54}(d,p)Fe^{55}$ could clear this point up. The assignments of the higher levels are given in agreement with the Co^{55} decay.

Cu^{63}

Level assignment in agreement with β decay and Coulomb excitation data.[13,27]

Cu^{65}

The assignments are consistent with the decay data of Zn^{65} and Ni^{65}. A recently reported level at 0.815 Mev, found in Coulomb excitation,[28] so far could not be confirmed[29] from the β^--decay of Ni^{65}. A $1/2^-$ spin ($p_{1/2}$?) would explain that it is neither excited from Ni^{65} nor from Zn^{65}.

[26] A. W. Schardt and B. J. Dropesky, Bull. Am. Phys. Soc. **1**, Ser. 2, 162 (1956) and private communication.

[27] For the Zn^{63} β^+ decay: R. van Lieshout and R. W. Hayward (private communication).

APPENDIX II. SOME EMPIRICAL REGULARITIES IN NUCLEI WITH 31 UP TO 39 ODD NUCLEONS

A surprising regularity in the level spacing of some nuclei with 31, 33, and 37 neutrons has been pointed out by Wapstra.[30] There are now five cases of odd-A nuclei in this group in which the level spacing follows the sequence $E=n^2\times E_0$, n being an integer and E_0 some basic energy value.

In Table II this regularity is shown for the five nuclei Fe^{57},[13] Ni^{59},[17] Ni^{61},[30] Zn^{67},[17] and Ga^{69}.[31]

TABLE II. Level sequence $E=n^2\times E_0$.

	${}_{26}Fe_{31}{}^{57}$		${}_{28}Ni_{31}{}^{59}$		${}_{28}Ni_{33}{}^{61}$		${}_{30}Zn_{37}{}^{67}$		${}_{31}Ga_{38}{}^{69}$	
n	E kev	E_0	E kev	E_0	E kev	E_0	E kev	E_0	E kev	E_0
1	14	14	...	...	...	...	...	...	...	...
2	...	...	360	90	70	18	92	23	...	...
3	133	15	870	97	...	...	182	20	324	36
4	...	...	1310	82	280	18	388	24	576	36
5	350	14	2150	86	(460)	18	595	24	878	35
6	...	...	...	...	650	18	870	24	1340, 1120	37, 31
7	...	...	...	...	940	19	...	...	1720, 1530	35, 31
8	...	...	...	...	1220	19	...	...	1920, 2040	30, 31

It might be pointed out that these levels cannot be interpreted as rotational bands on account of their spin values. The form of regularity, however, suggests some kind of collective effect which is not apparent in nuclei with an incompletely filled 28–28 core. The data of Ge^{69} indicate the possible existence of "bands" with different values of E_0. So far no good explanation for this feature is known and more data will have to be accumulated.

[28] G. M. Temmer (private communication).

[29] Suri and Nussbaum (unpublished).

[30] Nussbaum, Wapstra, Bruil, Sterk, Nijgh, and Grobben, Phys. Rev. **101**, 905 (1956).

[31] Nussbaum and Suri (to be published); Ge^{69} is independently being investigated by H. J. van den Bold (private communication).