Controlled Fusion Research—An Application of the Physics of High Temperature Plasmas

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I. INTRODUCTION

ACHIEVEMENT of the controlled release of fusion energy on a large scale will represent a truly permanent solution to mankind's expanding need for energy sources. Among the several light elements which might be used as fuel in a fusion reactor, the deuterium in the world's oceans alone would sustain an energy production rate 1000 times the world's present capacity for more than a billion years. However, the technical problems to be solved seem great indeed. When made aware of these, some physicists would not hesitate to pronounce the problem impossible of solution.

Scattered references to the problem of fusion power have appeared in the literature for many years. It is known that several countries are working on the problem, and at the 1955 Geneva Conference H. J. Bhabha of India ventured the prediction that the problem will be solved in twenty years. In the fall of 1955, Chairman Strauss of the U. S. Atomic Energy Commission announced officially that the Commission was supporting, under the code name of Project Sherwood, a long-range research program aimed toward achieving controlled fusion for peacetime uses. It was disclosed that the major portion of the experimental work was being carried out at three sites (Princeton University, and two laboratories operated for the U. S. Atomic Energy Commission by the University of California: the Los Alamos Scientific Laboratory and the University of California Radiation Laboratory at Livermore), and that supporting research was being carried out at Oak Ridge and New York University. Although 1951 was given as the beginning date for Project Sherwood, the problem of fusion power had actually been under study at Commission Laboratories since before the end of World War II. Much of the basic physical theory needed was formulated at Los Alamos by Edward Teller, Enrico Fermi, James Tuck, and others. Some of the results of this theory were later to be applied in the Los Alamos Sherwood experimental program, initiated and directed by J. L. Tuck. Also, early in 1951, Princeton University astrophysicist Lyman Spitzer, Jr., unaware of the classified Los Alamos work, conceived a different approach to a fusion reactor from others then under consideration in this country and submitted his idea to the U. S. Atomic Energy Commission. The Commission in turn initiated support under Spitzer's direction of what is now Project Matterhorn at Princeton. Shortly thereafter Herbert York, of the University of California Radiation Laboratory in Berkeley, having learned of the work of the Los Alamos and Princeton groups, suggested several new approaches. Thus, along with his job of organizing the new Livermore laboratory, York formed a small experimental group, with the author of this article as head, for the purpose of exploring and expanding on these new approaches to the problem.

In June of 1952, T. H. Johnson, Director of the Research Division of the U. S. Atomic Energy Commission, convened a classified conference on Controlled Thermonuclear Reactions in Denver, Colorado. Under the chairmanship of Edward Teller, the conference produced a summary of the problem and the then most promising attacks on it. The great hopes and equally great technical difficulties of the controlled fusion problem were brought into focus, and under the Research Division, coordination and expansion of the Commission's controlled fusion research was begun.

Since the Denver meeting, many additional conferences have been held covering the experimental and theoretical aspects of the problem. At present the Sherwood scientific program is guided by a national steering committee, which has as its members: Edward Teller of the University of California, James L. Tuck, of the Los Alamos Scientific Laboratory, Lyman Spitzer, Jr., of Princeton University, and William Brobeck of the University of California Radiation Laboratory. The coordination and administration of the over-all program was placed under the direction of Amasa S. Bishop, head of the Sherwood branch of the Division of Research.

In many of its parts this article will represent merely a distillation of the work of the many physicists who have been contributing diligently to the Sherwood effort. The thorough understanding of the basic physical principles of the controlled fusion reaction which exists today is in large part a measure of their individual and collective genius. Undergirding the entire field is, of course, the earlier work of the astrophysicists, who may now look forward to the possibility of practical peacetime applications of ultra-high temperature processes, which have in the past only been known to exist in the stars, or in the core of the atomic bomb.

II. MOTIVATIONS

It has been said that the achievement of a controlled fusion reaction represents one of the great scientific challenges of the century. This fact alone would provide sufficient stimulus to the physicists of today to tackle the problem with vigor. But a broader look will show that the motivations are much deeper. Sobering studies of population trends and the great humanitarian needs for improving the health and standard of living of the majority of the world's people show the need for an astounding sustained growth in power production capabilities in the world.[1]

If these extrapolations prove to be valid, fossil fuels such as coal and oil would fall far short of satisfying the demands in less than a century. Even the recoverable solar energy incident on all the accessible land area of the earth would scarcely equal some predictions of the power requirements a century from now. If the predicted demand is to be satisfied and maintained, there remains only nuclear fission or nuclear fusion as sources of large quantities of energy. It is estimated that the energy reserve of fissionable fuels which are economically processable is roughly 25 times that of fossil fuels. Large though this reserve is, it would represent only a relatively few decades of energy needs in the possible world of a century from now. By comparison, the energy requirements of that future era could be supplied for a billion years by fusion of only the deuterium in the oceans. Fusion fuel is essentially inexhaustable.

Another significant comparison is in the cost and cost-trends of nuclear and fossil fuels. Even by the extraction methods in use today the per-unit-energy cost of deuterium as a fuel is only a few percent of the cost of coal. While the unit cost of all other fuels including fissionable materials will become increasingly higher as lower grade reserves are tapped, the cost of deuterium recovered from sea water can be expected to drop or at least remain roughly constant indefinitely.

Again in the framework of the possible world of a century from now, the prospect of supplying some hundreds of billions of kilowatts of power from fission reactors alone is not a pleasant one from the standpoint of radiation accidents and disposal of radioactive wastes. At such levels of power production some 10^{13} curies of long-lived radioacitve solid and gaseous fission products would be produced per year and would have to be safely disposed of. The full impact of this problem will of course not be felt in this generation, so that these extrapolations clearly do not influence the present desirability of constructing power generating plants based on the fission reaction.

The same questions of explosion hazard and radioactive waste disposal can be asked about a fusion reactor. There the most probable answer is that no discernible explosion hazard will exist, and by the nature of the reactions involved no appreciable amount of radioactive waste is likely to be produced.

To anticipate later discussions, the problem of achieving a controlled thermonuclear reaction is to heat a suitable nuclear fuel to kinetic temperatures of 100 000 000°C or more, and then to controllably confine it somehow at these temperatures for a sufficiently long time to permit the fuel nuclei to undergo fusion—with the consequent release of energy in excess of the losses from the reaction region. The excess energy flux would then be available to be harnessed as useful power. The generation of fusion power differs greatly from that of fission power. No initial heating or heat insulation of fissionable fuel is needed to cause a nuclear fission reaction to become self-sustaining, whereas heating and confinement of the fuel are the prime problems of the fusion reaction.

Although reasonably good estimates can be made of the operating expenses of a fission power plant, no estimate of similar accuracy can be made for a fusion reactor. It can probably be said, however, that there is no reason to believe that the cost of fusion power cannot be made competitive with other sources, given sufficient development, especially in the light of rising costs for all other types of fuel.

One important possibility implicit in the nature of the controlled fusion reaction is that methods can be envisaged by which much of the fusion energy could be extracted directly in electrical form. This possibility could introduce a revolutionary reduction in the cost and complexity of electrical power generating installations, through elimination of the conventional thermal cycle.

In summary, it appears that the continued growth of civilization as we know it will demand the achievement of practical fusion power within a period substantially less than one hundred years. It further appears that a primary fuel for fusion power, deuterium, is essentially inexhaustible and that even its utilization on a grand scale should present no problems of radioactive waste disposal. Beclouding this rosy prospect are the truly formidable scientific and technical problems yet to be solved in achieving a practical controlled fusion reactor.

III. WHAT IS A CONTROLLED FUSION REACTOR?

The answer to the question "what is a controlled fusion reactor?" is best given today by merely saying that it is a device within which appropriate isotopes of light elements could be caused to undergo nuclear fusion, the end result being the controlled production and extraction of useful quantities of energy, in excess of that required to operate the device. Ingenious suggestions have been advanced for defining some possible forms such a device might take. Study of these devices and the associated physical phenomena and apparatus important to their performance is the reason for the existence of Project Sherwood. Some of the physical problems common to any research efforts in the field will be sketched in this article. In this section, some general aspects of the problem will be discussed which are predominately related to the nuclear reactions themselves.

[1] P. C. Putnam, *Energy in the Future* (D. Van Nostrand Company, Inc., New York, 1953).

TABLE I. Fusion reactions.

(1)	$D+D\rightarrow He^3$	$+n+$	3.25	Mev
(2)	$D+D\rightarrow T$	$+p+$	4	Mev
(3)	$T+D\rightarrow He^4$	$+n+$	17.6	Mev
(4)	$He^3+D\rightarrow He^4$	$+p+$	18.3	Mev
(5)	$Li^6+D\rightarrow 2\,He^4+$		22.4	Mev
(6)	$Li^7+p\rightarrow 2\,He^4+$		17.3	Mev

Nuclear Fusion Reactions

Among the nuclear reactions which appear promising for a controlled fusion reactor are those involving the various isotopes of hydrogen, helium, and lithium. Some of these reactions are listed in Table I.

Reactions (1) and (2) occur with roughly equal probability. Reactions (3) and (4) are interesting because of their high energy yield and also because they involve the reaction products of (1) and (2).

The cross sections for reactions (1), (2), (3), and (4) have been measured down to low energies. Figure 1 summarizes some recent published data on these reactions.[2]

The strong influence of the Coulomb barrier on the cross section at low energies is to be noted. This effect would, of course, appear even more pronounced for reactions (5) and (6), making the practical utilization of these reactions more difficult.

Energy Balance Considerations

Since reaction cross sections not far below the maximum values are achieved (particularly for reaction (3)) in the range of energies between 10 and 100 kev, whereas the energy yield per reaction is several Mev, it is apparent that a substantial margin exists for the production of a net energy gain. That is to say, even if only a few percent of a group of energetic deuterons and tritons were to undergo fusion, more reaction energy would be produced that the total kinetic energy of the original group.

As is well known, the energy balance still falls far short of permitting a net energy gain by simply bombarding a target of, say, deuterium with a beam of deuterons. In such a case the bulk of the kinetic energy of the impinging beam is dissipated uselessly by ionization, radiation, and energy transfer to the atomic electrons of the target, resulting in a yield of only 10^{-5}, or about 10^{-4} of that necessary to produce a favorable energy balance.

A possible answer to this problem, taking a leaf from the book of the astrophysicist, is that the entire fuel charge must be heated to a kinetic temperature sufficient to produce a substantial reaction rate by virtue of the mutual collisions of these fuel nuclei. These kinetic temperatures correspond to mean particle energies of tens or perhaps even hundreds of kilo electron volts.* At such temperatures all matter is completely ionized, and ionization losses, dominant in the example of the deuteron beam incident on a target, are negligible. Achievement of a net power balance under such conditions then becomes primarily a problem of competition between the nuclear energy produced within the volume of the reacting fuel charge and the energy lost through its outer surface. Here again the astrophysicists will answer that, since the volume to surface ratio increases with the radius, sufficiently large reacting regions can always maintain a net energy balance, *viz.*, the sun and the stars. For earthlings this method of solving the problem is not a very attractive one and other means must be found. These means must consist in picking out the most favorable fusion reactions and in creating especially effective circumstances for the reduction of surface energy losses, not only from the standpoint of achieving a favorable energy balance, but also of preventing the material walls surrounding the reaction region from being vaporized.

As has been said, the heating of matter to ultra-high kinetic temperatures results in a state of complete ionization, i.e., a gas composed of free electrons and ions in equal numbers, charge-wise. In this report, a fusion fuel in such a state is called a *plasma* after the original definition by Langmuir. The study of the dynamics of totally ionized gases or plasmas is a new and important field of physics which is receiving much attention in

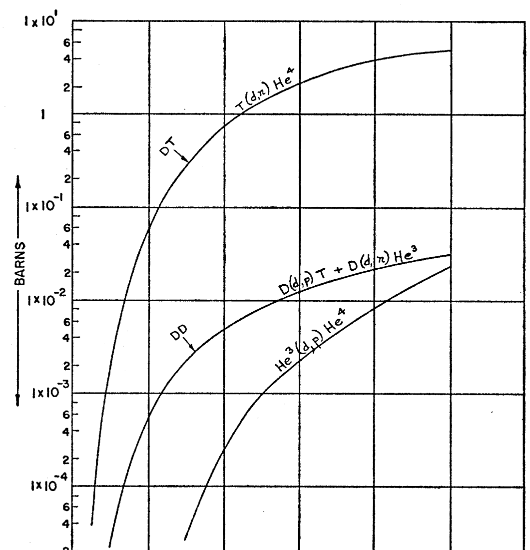

FIG. 1. Nuclear fusion reaction cross sections as a function of relative particle energy.

[2] Arnold, Phillips, Sawyer, Stovall, and Tuck, Phys. Rev. 93, 483 (1954).

* For a Maxwellian particle distribution the mean particle energy $\bar{W}$ is equal to $(\frac{3}{2})kT$. In discussions of fusion reactions it is convenient to speak of kinetic temperatures in terms of kilo electron volts. In these units, 1 kev kinetic temperature $=1.16\times10^{7\circ}$ Kelvin, and is $\frac{2}{3}$ of the mean particle energy in a Maxwellian distribution appropriate to that temperature.

connection with astrophysical problems. This is perhaps not surprising when it is realized that all but a tiny fraction of the matter in the universe exists in the plasma state. Controlled fusion research represents an attempt to apply astrophysical plasma dynamics on an earthly scale.

Theory of Binary Reactions

The fusion reaction is a binary one, i.e., two-body collisions are involved. In the case of a fusion reaction in a heated plasma, continuous mutual collision processes are involved between all the particles. It will be useful to review the theory of such processes.

Suppose that a group of fuel ions with a kinetic temperature T are mutually colliding. The probability that two fuel ions will react in passing close to each other is, of course, describable in terms of a mutual reaction cross section σ, which is a function of the relative velocity between the ions, v_{12}. The probability per unit time that a given ion of type 1 will react with another of type 2 will be given by the product of the reaction cross section σ, the relative velocity v_{12}, and the particle density of atoms of type 2, n_2 ions/cm^3. Since there will exist a distribution of relative velocities, rather than a fixed relative velocity, the product σv_{12} must be averaged over the distribution appropriate to the kinetic temperature of the plasma. Thus the rate per particle of type 1 is

$$R_1 = n_2 \langle \sigma v_{12} \rangle_{\text{Av}}. \tag{1}$$

The total reaction rate per unit volume is then found by multiplying R_1 by the particle density of ions of type 1

$$R_{12} = n_1 R_1 = n_1 n_2 \langle \sigma v_{12} \rangle_{\text{Av}} \text{ reactions/cm}^3\text{/sec.} \tag{2}$$

If ion types (1) and (2) are identical (as in the DD reaction), then the expression becomes

$$R_{11} = \tfrac{1}{2} n^2 \langle \sigma v \rangle_{\text{Av}} \text{ reactions/cm}^3\text{/sec.} \tag{3}$$

The reaction power density is simply the reaction rate times the energy release per reaction, W_{12} or W.

$$p = n_1 n_2 \langle \sigma v_{12} \rangle_{\text{Av}} W_{12}, \tag{4}$$

or

$$p = \tfrac{1}{2} n^2 \langle \sigma v \rangle_{\text{Av}} W. \tag{5}$$

Where the fuel plasma velocity distribution is known, the computation of $\langle \sigma v \rangle_{\text{Av}}$ is straightforward, though lengthy. Where a Maxwellian velocity distribution can be assumed, useful analytic expressions can be derived which predict $\langle \sigma v \rangle_{\text{Av}}$ at low and medium energies.

In reference (2) it is shown that reaction cross sections at low energies can be fitted accurately by a Gamow barrier penetration expression. For the total DD reaction the expression is (energies in kev, cross sections in barns, 1 barn$=10^{-24}$ cm^2)

$$\sigma_{\text{DD}} = \frac{288}{W} \exp[-45.8 W^{-\frac{1}{2}}]. \tag{6}$$

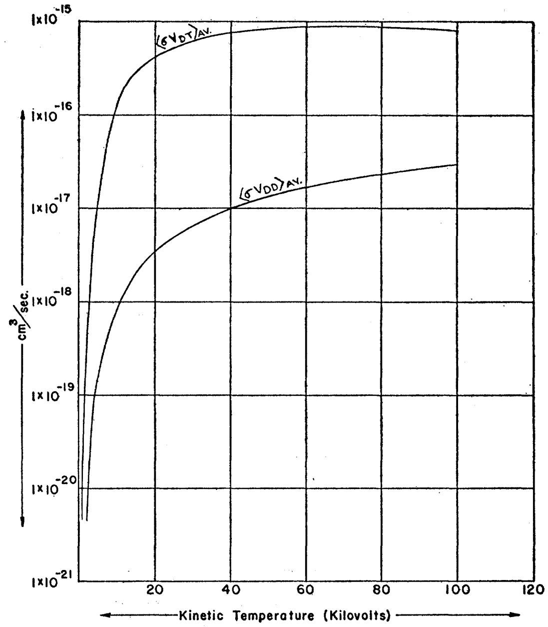

FIG. 2. $\langle \sigma v_{\text{DD}} \rangle_{\text{Av}}$, $\langle \sigma v_{\text{DT}} \rangle_{\text{Av}}$—reaction rate parameters for a Maxwellian particle distribution.

The value of $\langle \sigma v \rangle_{\text{Av}}$ can be obtained by integrating the product of the above expression and the particle velocity distribution over all possible relative velocities. For a Maxwellian distribution with a temperature T in kilovolts, the result obtained is†

$$\langle \sigma v_{\text{DD}} \rangle_{\text{Av}} = 260 \times 10^{-16} T^{-\frac{2}{3}} \exp[-18.76 T^{-\frac{1}{3}}] \qquad T < 50 \text{ kev.} \tag{7}$$

A similar but somewhat more complicated expression can be obtained for the DT reaction. The values of $\langle \sigma v_{\text{DD}} \rangle_{\text{Av}}$ and $\langle \sigma v_{\text{DT}} \rangle_{\text{Av}}$ for Maxwellian particle distributions are shown in Fig. 2 for the energy range 1 to 100 kev.

It is interesting to note that at very low temperatures (say below 5 kev) the important reacting particles are those few with energies several times the mean particle energy. This is a consequence of the exceedingly steep falloff of the reaction cross section at low energies.

A rough estimate of this effect for the DD reaction may be obtained merely by locating the maximum of the function

$$\exp[-45.8 W^{-\frac{1}{2}} + W T^{-1}], \tag{8}$$

which is the product of the exponential terms in the reaction cross section and Maxwellian velocity distribution expressions.

By differentiation, the value of W_m, the energy of ions giving the maximum contribution to the reaction rate, can be found. Expressed as a ratio of W_m to the

† See G. Gamow and E. Teller, Phys. Rev. **53**, 608 (1938) for a derivation of the functional form. The numerical values above were obtained by C. Leith (private communication).

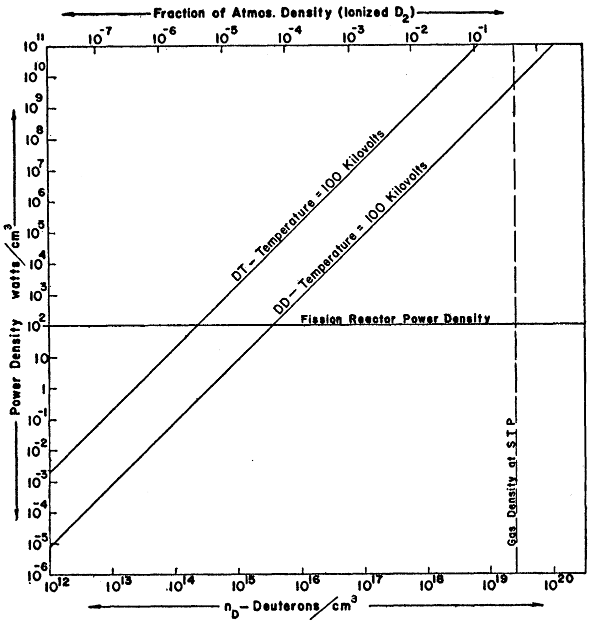

FIG. 3. DD and DT total reaction power density as a function of deuteron particle density (temperature = 100 kilovolts).

temperature T in kilovolts, this is

$$\frac{W_m}{T} = \frac{8.1}{T^{\frac{1}{3}}}. \tag{9}$$

Thus at $T=1$ kev this expression indicates that most of the reactions are contributed by particles with energies about eight times the kinetic temperature energy, i.e., at low temperatures the tail of the Maxwellian wags the dog.

Reaction Power Density

It is of interest to calculate representative reaction powers, mean reaction times, and reaction mean free paths for some possible situations. Consider the DD and DT reactions, supported at a temperature of 100 kev. (This temperature is chosen merely because the cross sections for both DD and DT are well-developed and slowly varying at 100 kev, so that the figures quoted are not particularly temperature sensitive. It will later be shown that operation at temperatures an order of magnitude lower is at best marginal.) From the curves the value of $\langle \sigma v_{\mathrm{DD}} \rangle_{\mathrm{Av}}$ is seen to be about 3×10^{-17} cm³/sec. The mean reaction energy for the total reaction is, from Table I $(3.25+4) \div 2 = 3.6$ Mev or about 6×10^{-13} joule/reaction. Thus the mean total reaction power density is

$$p_t = \tfrac{1}{2} n_{\mathrm{D}}^2 \times (3 \times 10^{-17}) \times (6 \times 10^{-13})$$
$$= 9 \times 10^{-30} n_{\mathrm{D}}^2 \text{ watts/cm}^3 \ (T = 100 \text{ kev}). \tag{10}$$

The important feature of the expression is that it depends on the *square* of the deuteron density.

In Fig. 3, p_t is plotted against the particle density of the fuel. The horizontal line at 100 watts/cm³ (2800 kilowatts per cubic ft) represents a typical power density which might be achieved in a fission power reactor. The dashed vertical line at $n_{\mathrm{D}} = 2.7 \times 10^{19}$ marks the particle density of a gas at STP.

Also plotted is the reaction power density for a 50% mixture of D and T at a temperature of 100 kilovolts, calculated from Eq. (2) and Fig. 2.

Examination of the curves for DD and DT shows that power densities of the same order of those in fission reactors are achieved at fuel particle densities of about 10^{-4} and 10^{-5} times atmospheric densities, respectively. These particle densities are perhaps surprisingly low, since they are almost as low as are encountered in many laboratory vacuum systems (10^{-3} mm Hg $\approx 10^{-6}$ atmosphere). Now it is clear that an exact parallel cannot be drawn between the problems of pressure and heat transfer in a fission reactor which limit the working power density, and the related problems in a fusion reactor. Nevertheless, similar limitations on power transfer will appear in any continuously operating fusion reactor. It follows that continuous operation at a temperature of 100 kev and at densities even approaching atmospheric densities seems impossible. A measure of this is the fact that a power of 500 000 kilowatts represents about the power output of a large steam-powered electrical generating plant. With deuteron particle densities equal to atmospheric density, and at 100 kev temperature, Fig. 3 indicates that this power is equalled by the DD reaction in a reacting volume of only 0.03 cubic centimeter! At the same time the gas kinetic pressure exerted by the fuel would be about 10^7 atmospheres or 1.5×10^8 psi.

At the other extreme are the consequences of operating at too low a fuel density. Many proposed schemes for producing thermonuclear power fail, not qualitatively but quantitatively, merely because insufficient density can be achieved. When the density has fallen to 10^{12} particles/cm³ for example, the curves show that DD power has fallen to only about 10^{-5} watt/cm³, which is far too low to be economically interesting. A density of 10^{12} particles/cm³ is typical of plasma densities generated in ordinary gas discharges.

Reaction Mean Free Path—Energy Division

Operation of a fusion reactor at lower or higher temperatures than 100 kev is surely conceivable. However, it will be shown in a later section that operation below a certain minimum temperature is not possible. Having specified $\langle \sigma v \rangle_{\mathrm{Av}}$ one may find the mean lifetime of a fuel ion. From Eq. (1) the collision rate per particle is $R_1 = n \langle \sigma v \rangle_{\mathrm{Av}}$. Thus the mean lifetime before reaction is $\tau = 1/R_1 = 1/n\langle \sigma v \rangle_{\mathrm{Av}}$. This is plotted in Fig. 4 as a function of the deuteron particle density, n_{D}, for DD and DT at 10 kev and at 100 kev. The indicated points are located at densities appropriate to a power density of 100 watts/cm³ (fission reactor values). From these it can be seen that in what might constitute typical cases the mean lifetime of a particle before reaction could be many seconds. It follows that the mean life of a fuel ion before it is lost from the reaction region must not be too short compared to this time in order to maintain the power balance.

The significance of a necessarily long reaction time can be more fully appreciated from the curves for the reaction mean free path, $\lambda = 1/n\bar{\sigma}$. These are plotted in Fig. 5, also for DD and DT at 10 and 100 kev. It is to be noted that if the power density is assumed equal to 100 watts/cm^3 the mean free path for DD reactions at 100 kev, for example, is about 5×10^9 cm, or a distance about equal to the circumference of the earth!

Thus far nothing has been said about the immediate fate of the energy produced when a fusion reaction takes place. The kinetic energy of the reaction is of course divided between the product particles, with the lion's share going to the lighter particle. If the lighter particle happens to be a neutron it will almost surely leave the reacting region (which has low density) and deposit its kinetic energy elsewhere. Charged reaction products may or may not leave the reacting region depending on the circumstances. In any event, the total particle energy of the charged reaction product is all that is available internally to supply energy losses and to sustain the reaction. The partition of energy between charged particles and neutrons is listed below for DD and DT reactions (neglecting initial kinetic energies).

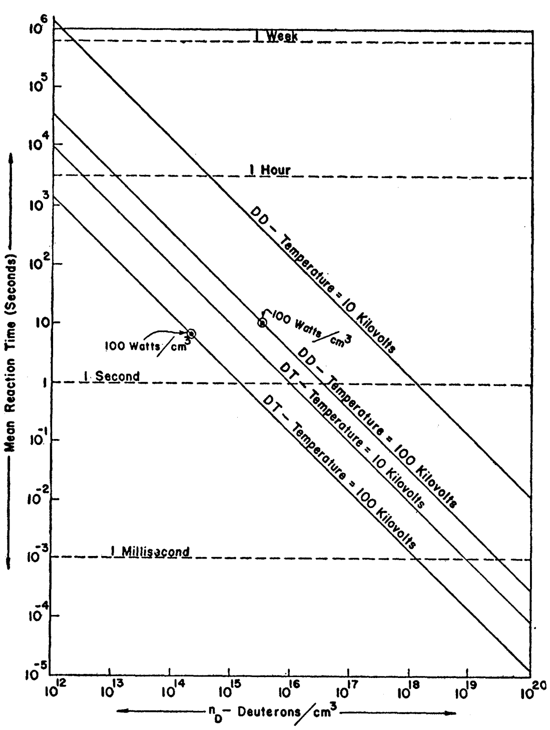

FIG. 4. Mean reaction time at various temperatures as a function of deuteron particle density.

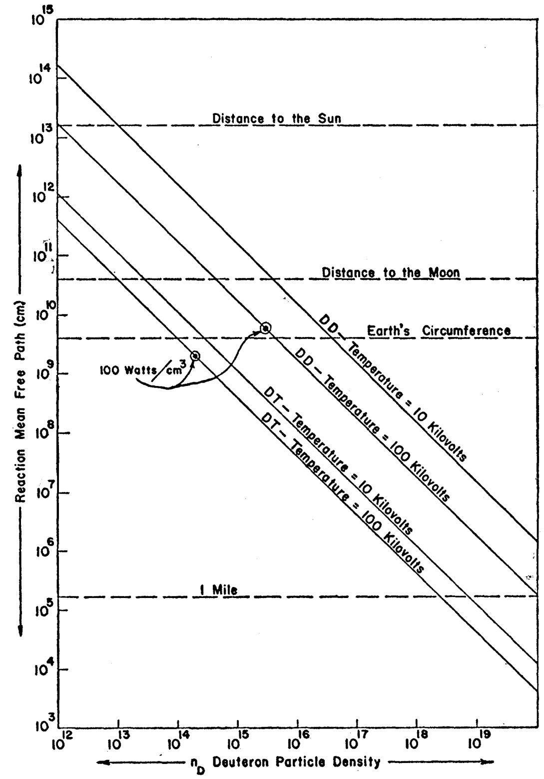

FIG. 5. Reaction mean free paths as a function of deuteron particle density.

$$\left.\begin{array}{l}\mathrm{DD}\rightarrow(\mathrm{T}+1.0\ \mathrm{Mev})+(p+3.0\ \mathrm{Mev})\\ \mathrm{DD}\rightarrow(\mathrm{He}^3+0.8\ \mathrm{Mev})+(n+2.45\ \mathrm{Mev})\end{array}\right\}\begin{array}{l}\text{Roughly equal prob-}\\ \text{ability of occurrence.}\end{array}$$

$$\mathrm{DT}\rightarrow(\mathrm{He}^4+3.6\ \mathrm{Mev})+(n+14.1\ \mathrm{Mev})$$

In the case of the total DD reaction, on the average 66% of the reaction energy is transferred to charged reaction products and 34% to neutrons. Thus the *internal* power density generated by the DD reaction is 0.66 times the value given by Eq. (10) and the DD curve of Fig. 3. In the case of the DT reaction only 20% is transferred to charged products and 80% is imparted to neutrons.

To simplify the discussion, in this and all other sections of this paper where the DD reaction is considered, the possible contribution of secondary fusion reactions involving the charged reaction products themselves (i.e., T and He^3) is neglected.

IV. COMPETING PROCESSES

The processes in which energy is lost are in competition with the nuclear yields. The role of these processes is crucial. Some of these loss processes are common to any system. Among these is radiation from the electron cloud which, together with the fuel ions, forms the fuel plasma. If the more usual situation of radiation equilibrium were to be blindly assumed, the radiation flux from a reacting region of the fuel would be given by the usual blackbody relationship $I=\sigma T^4$ ergs/cm²/sec. At 10 kev (about 10^8 degrees Kelvin), this radiation flux would amount to some 10^{21} watts/cm²! We can safely conclude that no controlled fusion device can operate in a state of radiation equilibrium between the particles and the radiation field. However, radiation equilibrium is only achieved when the mean free path for internal absorption of the radiation in question is smaller than the dimensions of the system. Fortunately this condition is far from satisfied in the plasma of a fusion reactor, whence the use of the phrase "kinetic temperature" rather than merely "temperature" to describe the energy state of a plasma.

The situation is similar to that in the tenuous outer parts of the sun's corona. Although it has been found that the kinetic temperature of the electron gas in the corona is of the order of 10^6 degrees Kelvin, the actual radiation from the sun is effectively dominated by deeper lying layers and corresponds to that from a blackbody at a temperature value of only about 5000°K. In the outer parts of the corona, the mean free path for the important components of the self-radiation is much larger than the thickness of the layer so that radiative equilibrium is not attained.

Effective Radiation Rate—Minimum Temperature for a Self-Sustaining Fusion Reaction

For systems where dimensions are small compared to the mean free path for absorption of photons at an energy corresponding to the kinetic temperature, a rough rule is that the actual radiation from the medium will be of the order of the blackbody value diminished by the ratio of dimensions of the system to the mean free path for absorption of the photons. At 10^8 degrees Kelvin the absorption mean free path for plasma densities of, say, 10^{15} particles per cm³, is about 10^{20} cm so that the reduction is of the same order. More precisely stated, the dominant radiation process from the plasma will be ordinary bremsstrahlung or x-ray emission occurring when the fast-moving electrons of the plasma are deflected by the Coulomb field of the fuel ions. The theory of this radiation is discussed by Heitler[3] and others. Upon averaging over a Maxwellian electron distribution an expression can be found for the radiation power density emitted as x-rays from a completely ionized gas (T_e = electron temperature in kilovolts).

$$p_r=0.54\times10^{-30}Z^2n_e^2T_e^{\frac{1}{2}} \text{ watts/cm}^3. \tag{11}$$

For a hydrogenic plasma $Z=1$ and $n_e=n_i$ (n_e= electron particle density, n_i= ion particle density). So that for the DD reaction for example,

$$p_r=0.54\times10^{-30}n_\mathrm{D}^2T_e^{\frac{1}{2}} \text{ watts/cm}^3. \tag{12}$$

Comparison with Eq. (5) for the rate of power generation shows that both vary as n_D^2. If electron and ion temperatures are assumed equal, it is seen that even in the absence of all other losses (except neutrons) a controlled fusion reaction sustaining itself from its internal energy generation cannot be accomplished below a certain minimum temperature, which is independent of the density. Since $\langle\sigma v\rangle_\mathrm{Av}$ for the reaction increases exponentially with temperature while the radiation rate increases only as $T^{\frac{1}{2}}$, power production may exceed radiation above a critical temperature T_c at which $p_r=p_i$ (we count only the *charged* reaction products of the DD reaction since neutrons will escape). We find from Eqs. (5), (7), and (12) the ratio of fusion to radiated power

$$p_i/p_r=1.92\times10^4(T)^{-7/6}\exp[-18.76T^{-\frac{1}{3}}]. \tag{13}$$

Setting $p_i/p_r=1$, one finds T_c to be 35 kilovolts. The value of T_c for DT is considerably lower, about 4 kev.

Because of the Z^2 dependence of p_r, it is apparent that the presence of even a small amount of ionized high-Z elements could greatly increase the radiated power density. It follows that high purity is a prime requirement for the plasma of a fusion reactor. For this and other reasons utmost attention must be paid to the role of the material walls of the reactor in introducing impurities through bombardment and heating effects which might otherwise be considered unimportant.

Need for Confinement—Possible Modes of Operation

Radiation from the plasma electrons thus establishes a minimum temperature of operation of a fusion reactor, but does not introduce any specific limitations on size of the reaction region or on number density of the plasma (so long as radiative equilibrium is not approached), since both radiation power density and fusion power density vary in the same way with particle density. We have seen that, in a continuous operating reactor, power density limitations would probably appear, which in turn limit the particle density. It will be shown in a later section that the mean free path for ordinary collisions between particles of the plasma is very long when the plasma is at thermonuclear temperatures. But unless other means are used, collisions between the particles represent the only mechanism inhibiting their escape from the reaction region. Thus at densities such as 10^{15} particles per cm³ there is no *a priori* reason why a hot fuel ion may not escape from the reactor immediately.

It is clear that a dilemma exists in probing further into the concept of a controlled fusion reaction. Two general ways out of the difficulty can be mentioned. One possibility is that the idea of a continuously reacting plasma must be abandoned and a transient mode of operation be substituted. Here high densities would be used so that, for a short time at least, collisional diffusion velocities would be sufficiently slow to prevent the

[3] W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, New York, 1954), third edition.

plasma from escaping rapidly enough from the region in which fusion is taking place to quench the reaction. The logical extrapolated limit of this process is a hydrogen bomb, which is not exactly "controlled" in the usual sense of the word. The second general attack is to interpose between the material walls of the reactor and the plasma some force field capable of exerting a pressure, so that the reaction heat is not immediately quenched by contact with a low temperature region. Even at reduced densities it is clear that the high temperatures of the fuel implies that substantial pressures must be sustained. For example, at 100 kev kinetic temperature and a total particle density of 6×10^{15} particles per cm^3, the kinetic pressure exerted by the fuel would be about 1000 atmospheres or 15 000 lb/in.^{2}. A gravity field is clearly too weak, except when applied on a stellar scale. The outstanding remaining possibility is the electromagnetic field, which is capable of transferring momentum and thus can exert a force. Since the fusion fuel will be essentially totally ionized, the charged particles constituting the fuel are capable of direct interaction with such a field. The gamut of possibilities here is large and their discussion is clearly outside the scope of this article, but brief mention of an example will be made in the next section.

In summary, a controlled fusion reactor must operate far from radiative equilibrium, under which condition radiation losses are limited to ordinary bremsstrahlung. At a sufficiently high operating temperature reaction power will exceed radiation, permitting the achieving of a self-sustaining fusion reaction if no other significant losses are present. At the same time, to overcome excessive losses of particles (and thus energy) by direct transport out of the reaction region, either a high density (and therefore highly transient) mode of operation must be contemplated, or containment for useful lengths of time by some type of electromagnetic field must be sought.

V. AN EXAMPLE OF A POSSIBLE MEANS FOR MAGNETIC CONFINEMENT—THE PINCH EFFECT

A classic example of an electromagnetic field which can act to confine a group of charged particles is the so-called "pinch effect"—the self-constriction of a group of charged particles moving in such a way as to produce a unidirectional current. The effect is another example of the familiar fact that parallel circuits carrying current in the same direction attract each other. A theory of the pinch effect was first set forth by Bennett[4] and amplified by Tonks[5] and Tonks and Allis.[6] Some experimental work on the subject has been reported in the literature[7,8] and recently a more direct connection

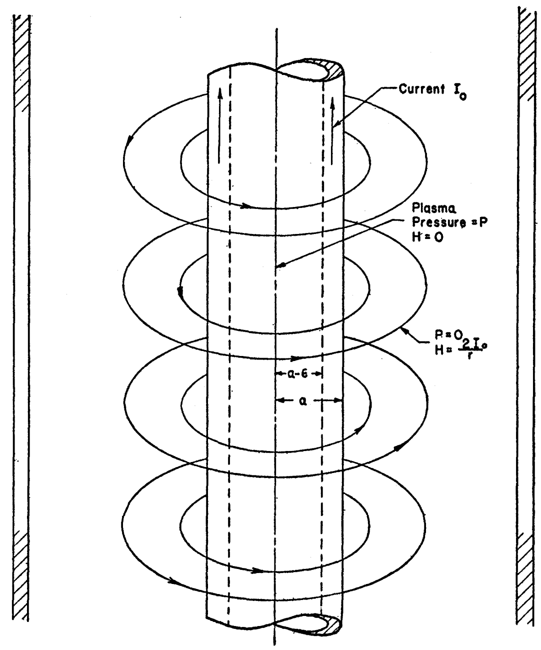

FIG. 6. Schematic representation of pinch effect in a plasma.

has been cited between the pinch phenomenon and controlled fusion research.[9,10]

The theoretical work of Bennett and others showed that in the ordinary pinch effect at high currents the conducting column would become closely concentrated at a central axis, producing a plasma column and surrounding magnetic field as shown in Fig. 6. The fact that the current becomes pinched is direct evidence that the carriers are being confined in the radial direction by the effect of the magnetic field.

To illustrate the principles behind magnetic confinement of this type let us adopt an over-simplified picture of the pinch and derive from it the conditions for kinetic pressure balance between plasma and the effects of the pinch magnetic field. We assume, as shown in Fig. 6 that the pinched current is confined to a thin cylindrical shell or boundary of outer radius a and thickness ϵ. The plasma filled region inside the shell will be characterized by a uniform density n_i of singly charged ions per cm^3 and n_e electrons per cm^3, and a kinetic temperature T. n_i will be taken equal to n_e (necessarily so as will be seen later). Since it is a gas, the plasma within exerts a kinetic pressure $P=(n_i+n_e)kT$. Outside the shell the density and thus the kinetic pressure is equal to zero. The pressure difference must be supplied by a gradient of the magnetic field.

Macroscopically, the balancing force is just the body

[4] W. H. Bennett, Phys. Rev. **45**, 890 (1934).
[5] L. Tonks, Trans. Electrochem. Soc. **72**, 167 (1937).
[6] L. Tonks and W. Allis, Phys. Rev. **56**, 360 (1939).
[7] A. A. Ware, Trans. Roy. Soc. **A243**, 863 (1951).
[8] S. W. Cousins and A. A. Ware, Proc. Phys. Soc. (London) **A64**, 159 (1951).

[9] *Nucleonics* (December, 1955), p. 23, discusses Los Alamos Controlled Fusion Research Program under J. L. Tuck and discloses investigation of properties of the pinch effect.
[10] *Nucleonics* (February, 1956), p. 42, includes review of unclassified work on pinch effect at Tufts and University of Southern California.

force $\mathbf{j}\times\mathbf{H}$ dynes acting on each unit volume of the current carrying region, where $\mathbf{j}$ is the current density. In this case the force is of course exerted only within the thin shell carrying the current. Within the conducting shell itself the field will fall from its value of $2I_0/a$ at the outside, to zero at the inner surface, while at the same time the particle density is assumed to rise from zero at the outside, to its central value, at the inner surface. If we assume that current is distributed uniformly through the layer, with thickness ϵ, then at any point in the shell itself the field will have the value

$$H=2I/r, \tag{14}$$

where I is the total current within the radius r. Thus, if the thickness of the shell is ϵ,

$$I(r)=I_0\left(\frac{r+\epsilon-a}{\epsilon}\right), \quad (a-\epsilon)<r<a.$$

Let $r=a-x$, $0<x<\epsilon$, then

$$I(x)=I_0\left(1-\frac{x}{\epsilon}\right), \tag{15}$$

$$H(x)=\frac{2I_0}{a-x}\left(1-\frac{x}{\epsilon}\right)\approx\frac{2I_0}{a}\left(1-\frac{x}{\epsilon}\right), \ \epsilon\ll a. \tag{16}$$

The current density j is the total current divided by the area of the shell

$$j=\frac{I_0}{2\pi a\epsilon}, \ \epsilon\ll a. \tag{17}$$

The total (inward) force per unit area exerted by the shell is the integral of $\mathbf{j}\times\mathbf{H}$ through the shell. Here $\mathbf{j}$ is perpendicular to $\mathbf{H}$ so that

$$\int_0^\epsilon (jH)dx=\left(\frac{2I_0}{a}\right)\left(\frac{I_0}{2\pi a\epsilon}\right)\int_0^\epsilon\left(1-\frac{x}{\epsilon}\right)dx$$
$$=I_0^2/(2\pi a^2). \tag{18}$$

This force must just balance the kinetic pressure of the plasma $P=(n_i+n_e)kT$

$$I_0^2/(2\pi a^2)=P=(n_i+n_e)kT, \tag{19}$$

i.e.,

$$I_0^2=2NkT, \tag{20}$$

where N is the total number of particles per centimeter of length of the confined column of radius a. This relationship is useful in establishing the current necessary for pinch confinement. Note that the expression for I_0 is independent of the radius of the pinch.

Magnetic Pressure

Remembering that $H_0=2I_0/a$, we find that (19) may be rewritten as

$$[H_0^2/8\pi]_{\text{outside}}=P_{\text{inside}}. \tag{21}$$

The role of the magnetic field appears here in its true light. The external magnetic field region balances the kinetic pressure on the region inside by a magnetic pressure of magnitude $H_0^2/8\pi$. ($H_0^2/8\pi$ is also the energy density in the magnetic field.) The same relations above can be simply rearranged to yield the result that within the layer

$$\nabla\left(\frac{H^2}{8\pi}+P\right)=0,$$

i.e.,

$$(H^2/8\pi)+P=\text{constant}=(H_0^2/8\pi). \tag{22}$$

This relationship is valid where curvature of the magnetic field lines can be neglected, as in the present case. It has appeared frequently in the literature of plasma physics.[11,12] It is important to note that the magnetic pressure exerted is a direct result of the interaction of plasma currents and the magnetic field, and that this pressure results only where the plasma currents act to diminish the field, i.e., where the medium has properties which could loosely be called diamagnetic. The example chosen, where the currents producing the magnetic field are excluded from a conducting region, can only occur in a perfect conductor, but the illustration could also represent a situation which might persist for a short time in a plasma of finite electrical conductivity.

To illustrate the strength of magnetic field required for magnetic confinement let us return to the former example of a DD reaction carried out at 100 kev temperature and a deuteron particle density of 3×10^{15} per cm^3.

Assume that the electron temperature and ion temperature are approximately equal. As in the former example the plasma pressure is $P=(n_i+n_e)kT=6\times10^{15}\times(1.6\times10^{-7})=10^9$ dynes/cm^2 or about 1000 atmospheres (15 000 psi). From this

$$H_0^2/8\pi=10^9, \quad H_0=1.6\times10^5 \text{ gauss}.$$

If the radius of the plasma cylinder is 10 cm, for example, Eq. (19) shows that the required current is about 8×10^5 emu, or 8×10^6 practical amperes.

Thus far it has not been necessary to discuss the specific microscopic mechanism by which the magnetic field effects containment of the plasma. It is clear that the qualitative effect in the example above is that ions and electrons attempting to escape radially from the boundaries will be turned around upon entering the region of strong magnetic field. The infinitesimal currents produced by the reflection of the charged particles from the boundary must integrate to a self-consistent set of macroscopic currents which produce the given magnetic field configuration. These concepts can serve as an introduction to the problem of *stability* of the confined plasma, which is of great importance.

[11] L. Spitzer, *Physics of Fully Ionized Gases* (Interscience Publishers, Inc., New York, 1956).

[12] H. Alvén, *Cosmical Electrodynamics* (Oxford University Press, New York, 1950).

Pinch Instability—Longitudinal Confinement

Although it has been shown above that a state of pressure equilibrium can be established in pinch confinement, it has not been shown that a stable equilibrium is possible for the simple pinch. In matter of fact it is not. In a now classic paper Kruskal and Schwarzchild[13] have shown that a self-constricting current flowing in a plasma is unstable against "kinking" perturbations. That is to say, if the pinched column undergoes an infinitesimal localized lateral displacement, displacements of length greater than the column diameter will grow exponentially with time, until the column is disrupted. The situation is something like the instability of a long thin rotating shaft supported only at its ends. It arises physically from the fact that the magnetic field (and therefore the magnetic pressure) is greater near the concave side of a curved current carrying conductor than it is on the convex side. This can be seen from the illustration in Fig. 7, where it is seen that the magnetic field lines become crowded together on the concave side of the column and spread apart on the convex side. The instability predicted by Kruskal and Schwarzchild is a special case of a more general type of hydromagnetic instability. Instabilities of this type have the property that the time of growth by a factor e of an unstable perturbation is about equal to the transit time of an ion going a distance equal to the "wavelength" of the perturbation. Thus short "wavelengths" grow most rapidly and at thermonuclear temperatures the predicted growth times are disturbingly short. (A 100-kev deuteron has a speed of about 300 centimeters per microsecond.) In order to convert the simple pinch effect into a means for producing controlled fusion reactions, ways would have to be found to circumvent this fundamental instability. One conceivable way is to establish the pinch within a time shorter than the instability growth time. If in such a case a high enough instantaneous reaction power were attained, a net energy yield might still be achieved.

To complete the picture of the simple (quasi-stationary) pinch effect as a possible confinement means, it should be mentioned that the field configuration shown in the example of Fig. 6 provides no effective means of longitudinal confinement of the plasma. In the experiments of Cousins and Ware,[9] some of the work of Tuck at Los Alamos and the work at Tufts and the University of Southern California[10] toroidal induced discharges were used which provide in principle a way to reduce these losses, by closing the end of the discharge on itself.

VI. PHENOMENA IN A COMPLETELY IONIZED GAS

Problems in the physics of completely ionized gases have received considerable study in recent years. Although of obvious importance to controlled fusion reactions, any detailed treatment of this subject is outside the scope of this article and the reader is referred to the literature for further information.[11,12] However, some of the simple and fundamental features of this "fourth state of matter" will be reviewed in this section.

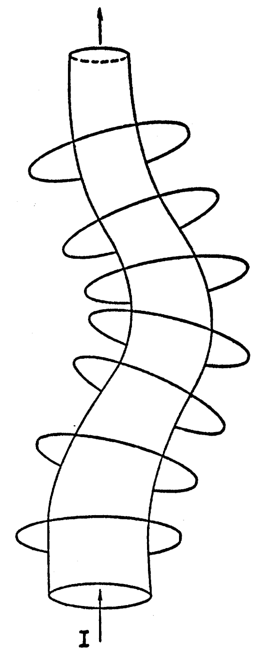

FIG. 7. "Kink" instability of the pinch effect.

A plasma is governed in its gross behavior by three dominant processes: (1) long-range collective electrostatic or space-charge-like effects produced by the plasma itself; (2) short-range collisional interactions between individual particles of the plasma; and (3) interaction between the individual particles and externally applied electromagnetic fields. Strictly speaking, (1) and (3) are coupled effects, since the plasma may profoundly alter the character of an external applied field through collective effects. Nevertheless the separation suggested may often represent a reasonable approximation to the actual behavior.

Charge Equality

Process (1) acts most strikingly in enforcing a state of charge equilibrium in a plasma. That is to say any tendency for the charge density to depart from neutrality will result in large electrostatic fields which oppose the change. To illustrate this effect it is only necessary to calculate the electrical field which would exist around a sphere of plasma with both electron and ion density equal to 10^{15} particles/cm^3 if the electrons of the plasma were somehow to be suddenly removed. If the radius of the sphere were 1 cm for example, Gauss' theorem gives for the field

$$E=\frac{Q}{r^2}=(4\pi/3)\times10^{15}\times4.8\times10^{-10}$$
$$=2\times10^6 \text{ stat volts/cm or}$$
$$600 \text{ million volts/cm!}$$

[13] M. Kruskal and M. Schwarzchild, Proc. Roy. Soc. (London) A, **223**, 348 (1954).

Even a departure of only 1 part in 10^5 from charge equality would give rise to a field of 6000 volts/cm near a sphere of plasma of 1 cm radius or 600 000 volts/cm near a sphere of 1 meter radius. Effective equality of electron and ion charge density must be assumed as a condition in any operating fusion reactor.

Electrical Conductivity

Another obvious property possessed by a cloud of freely moving negative and positive charges is that of electrical conductivity. As in any conducting medium the numerical value of the conductivity depends inversely on the collision rates between the carriers and other neighboring particles. In a hot plasma (as opposed to ordinary conductors) collision rates become smaller as temperature increases and thus the electrical resistivity *decreases* with increasing temperature. The theory of the resistivity of a fully ionized plasma[11] shows that it varies inversely with the temperature to the three-halves power. For a hydrogenic plasma the numerical value is given approximately by the expression (T in kilovolts)

$$\rho_0 = \frac{3\times 10^{-6}}{T^{\frac{3}{2}}} \text{ ohm-cm.} \tag{23}$$

Thus at 100 kev the theoretical resistivity is about 3×10^{-9} ohm-cm—less than 1% of that of copper at room temperature. This expression strictly applies only when the effective dimensions of the plasma are large compared to a mean free collision path of electrons in the plasmas. At low densities and high temperatures this condition may be far from satisfied. Nevertheless for other reasons the qualitative result of low resistivity remains valid in many cases of practical interest. It follows that in many situations a hot plasma behaves somewhat like a superconductor, i.e., it can exclude or diminish externally generated magnetic fields (as in the example of the pinch) and it can act to "short out" charge accumulations or electric fields. These qualitative results are not particularly sensitive to the plasma density.

In the presence of a magnetic field, the theoretical steady state resistivity is slightly altered for conduction in a direction across the field lines and is essentially unaltered for conduction along the field lines. Transient effects may however be radically different, depending on the time scale.

Plasma Oscillations

Another property of a fully ionized gas which owes its origin to space-charge phenomena and high effective conductivity is the phenomenon of electrostatic plasma oscillations. Simple forms of these oscillations which do not depend on a magnetic field being present for their existence were first predicted and observed by Tonks and Langmuir. These (longitudinal) oscillations of the plasma are associated with the fact that a small momentary average displacement of charges of one sign with respect to the other in the plasma will establish an electric field in such a direction as to oppose the displacement. Inertial effects of the displaced charges then provide a mechanism for periodic oscillations about the point of zero average charge displacement. The frequency of these oscillation is determined by the mass of the displaced charges and the "spring constant" of the electric field produced by the displacement. Plasma oscillations of this type may either be electronic or ionic in nature, i.e., the frequency is determined by electronic or ionic masses. Ionic oscillations are much lower in frequency than electron oscillations and are not as easy to identify. Considerable theoretical work has been done in recent years on the subject of plasma oscillations, much of it by Soviet investigators.[14] The reader is referred to the literature for more complete details.[11,12,15,16]

It is important to note that the frequency of electronic plasma oscillations really represents a measure of the time of response of the electron gas of the plasma to a time varying electric field, whether it be the self-field resulting from a separation of charges or an externally applied field. Thus the plasma frequency appears ubiquitously in the descriptions of many phenomena in a plasma. Disregarding a correction for the effect of thermal velocities[14,15] one finds the electron plasma frequency is

$$\omega_p = 4\pi n_e e^2/m \text{ radians/sec,} \tag{24}$$

or

$$f_p = 9\times 10^3 n_e^{\frac{1}{2}} \text{ cycles/sec.}$$

Thus at $n_e = 10^{15}$,

$$f_p = 2.7\times 10^{11} \text{ cycles/sec.}$$

Transient electrical fields which are applied in a time much longer than $1/\omega_p$ are dominated by the electrical conductivity effects discussed earlier. If they occur in a shorter time, the inertial effects of the plasma dominate and it may have little influence. Thus it is that ordinary light passes freely through a plasma, whereas radio waves may be strongly reflected or attenuated. In matter of fact the effective dielectric constant of a plasma (in cases where collisional and magnetic field effects may be neglected) may be written as

$$K = 1 - \left(\frac{\omega_p}{\omega}\right)^2 = 1 - \frac{4\pi n_e e^2}{m\omega^2}, \tag{25}$$

an expression no doubt familiar to students of classical dispersion theory. Here ω is the angular frequency of the electromagnetic wave propagated through the plasma.

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[14] L. Landau, J. Phys. U.S.S.R. **10**, 25 (1946).
[15] D. Bohm and E. P. Gross, Phys. Rev. **75**, 1851 and 1864 (1949).
[16] D. Gabor, Proc. Roy. Soc. (London) **213**, 73 (1952).

The index of refraction is

$$\nu=\frac{1}{\sqrt{K}}=\frac{1}{\left[1-\left(\frac{\omega_p}{\omega}\right)^2\right]^{\frac{1}{2}}}, \tag{26}$$

corresponding to phase velocities *greater* than the velocity of light. Unattenuated propagation of an electromagnetic disturbance can occur only if $\omega>\omega_p$, a situation resembling that in a wave guide.

Hydromagnetic Waves

The introduction of a magnetic field has little effect on electromagnetic wave propagation in a plasma if both ω and ω_p are much greater than ω_c, the electron cyclotron frequency. Also if the electric vector of the electromagnetic wave is parallel to H, propagation of the wave as predicted by Eq. (26) is relatively unaffected, whatever the frequency. When ω or ω_p are comparable to or smaller than ω_c the situation is no longer simple, and complicated dispersion effects result.

In the presence of a magnetic field new types of waves are also possible which are of fundamental importance. These are the so-called magnetohydrodynamic or hydromagnetic waves described by Alfvén.[12] In their simpler forms these waves are transverse, propagate along magnetic lines of force, and may be thought of as similar to the transverse oscillations of loaded elastic bands (the field lines). The loading is of course supplied by the ions (and electrons) of the plasma and is related to a very important qualitative concept of the motion of plasma particles in the presence of a magnetic field. This concept is that for motions which occur sufficiently rapidly compared to interparticle collision frequencies (but not too rapidly compared to periods of rotation of the particles), charged particles act as though bound to magnetic lines of force (more precisely to certain special magnetic surfaces).‡ Since, as noted, magnetic field lines act very much like mutually repelling elastic bands, transverse vibrational or torsional waves may be propagated along the direction of the field lines at speeds determined by the mass loading per unit length (i.e., the ion density) and a force constant determined by the strength of the magnetic fields. The speed of propagation of these waves is given by the Alfvén speed

$$v_A=\left(\frac{H^2}{4\pi\rho}\right)^{\frac{1}{2}}. \tag{27}$$

ρ is equal to $(n_iM_i+n_em_e)\approx n_iM_i$. Squaring both sides of (27) and dividing by the square of the velocity of light one obtains

$$\left(\frac{v_A}{c}\right)^2=\frac{2(H^2/8\pi)}{\rho c^2}. \tag{28}$$

This expression is thus equal to the ratio of twice the magnetic energy density to the particle rest energy per cm^3. In plasmas of practical interest for controlled fusion reactions this ratio is much less than 1, i.e., the waves are much slower than light waves.§

One reason for describing the hydromagnetic oscillations is that they illustrate an important attribute of a hot plasma in a magnetic field, i.e., that of "sticking" to magnetic lines of force (or at least to magnetic surfaces). Also, unstable modes of these same oscillations can be important in the general problem of the stability of a plasma confined by a magnetic field. For example, (28) can be seen to represent the square of an index of refraction of the plasma for Alvén waves, so that the reciprocal of the quantity $(v_A/c)^2$ itself resembles an effective dielectric constant of the medium for some types of hydromagnetic disturbances. This dielectric constant may be very large in cases of practical interest.

Collision Processes

It has already been shown that the plasma of a controlled fusion reactor could not exist in equilibrium with its own radiation field. Furthermore it obviously must not exist in thermal equilibrium with surrounding material walls. Thus such a confined plasma may not even approach kinetic thermodynamic equilibrium in the usual sense. For example, the existence of a magnetically confined plasma must be considered as a nonequilibrium state, since confining electrodynamic forces may only exist if electrical currents flow within the plasma. Since a plasma is not a perfect conductor these confining currents will decay with time in an isolated plasma system, or may be maintained in a steady state only by a continuous input of energy, either kinetic or electromagnetic. Just as in the case of ordinary conductors, collisions provide the dissipative means and determine the time rate of approach to equilibrium. However, in contrast to ordinary conductors, collision rates in a plasma become slower as the temperature is increased, thus facilitating magnetic containment.

In a totally ionized gas, scattering, or collisional, processes are almost entirely ascribable to the Coulomb fields of the bare nuclei and free electrons of the plasma. Because of the well-known infinite effective range of a Coulomb field, it is usual to divide the particle Coulomb interactions into two general types, a division which has already been indicated. The combined effect of all particles beyond a certain cutoff or screening distance is lumped into a collective space-charge-like interaction where only gross charge effects are important. On the other hand, within this cutoff distance it is physically acceptable to discuss "collisions" as discrete independent events, even though in general many particles will lie within the unscreened region. The screening length ("Debye length") is determined in effect by the

‡ W. A. Newcomb has constructed a comprehensive theoretical treatment of the *motion of magnetic lines*, which is reproduced in Princeton University Observatory Technical Report No. 1 (1955), and which rigorously treats this concept.

§ F. de Hoffmann and E. Teller, Phys. Rev. **80**, 692 (1950) have shown that in the limit of very high magnetic fields or low density Alfvén waves become ordinary light waves.

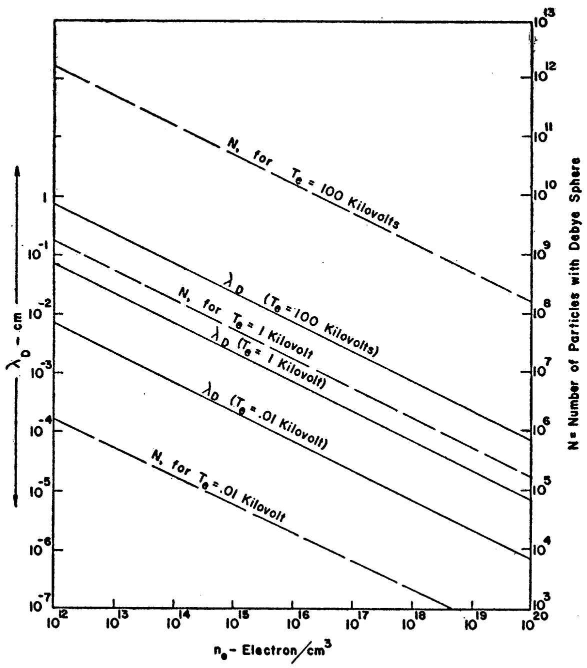

FIG. 8. Debye length and number of particles within a Debye sphere as a function of particle density.

distance beyond which the surrounding electrons of the plasma can, by their collective motion, screen off the Coulomb field of a particular charge from that of another one moving by it. Thus the Debye length is closely related to the minimum response time of the plasma to a localized electrical transient, i.e., the ordinary plasma frequency. This time is of course about equal to $1/\omega_p = (m/4\pi n_e e^2)^{\frac{1}{2}}$. The distance, λ, that the average electrons of the plasma can move in this time in attempting to screen the field is about equal to the mean electron velocity times the above time, i.e.,

$$\lambda = \left(\frac{\frac{3}{2}kT_e}{m}\right)^{\frac{1}{2}} \cdot \left(\frac{m}{4\pi n_e e^2}\right)^{\frac{1}{2}} = \left(\frac{kT_e}{(8/3)\pi n_e e^2}\right)^{\frac{1}{2}}. \quad (29)$$

λ is thus a crude estimate of the screening distance. It is more usual to use Debye's theoretical value (derived in connection with the theory of solids) which is slightly smaller:

$$\lambda_D = \left(\frac{kT_e}{4\pi n_e e^2}\right)^{\frac{1}{2}}. \quad (30)$$

This quantity only appears in the argument of a logarithm in all quantities of physical interest, so that its exact numerical value is not important. In Fig. 8 λ_D is plotted as a function of n_e for various electron temperatures. Also plotted are the average number of charges lying within the Debye sphere (a sphere with radius equal to the Debye length). It can be seen that any particular particle may actually interact with many others in passing through a distance equal to the Debye length. For this reason it will turn out that the effect of "distant" collisions (far compared to nuclear dimensions, but still within a Debye length) is more important than close encounters in producing scattering of any particular charged particle as it passes through the plasma.

First consider close collisions. Now the collision cross section for a deflection of one charged particle by another is given by the classical Rutherford relation.[17] For a "large" deflection owing to a single close collision, an easy way to find the order of magnitude of the collision cross section is to find the distance of closest approach between two charged particles (at which the mutual Coulomb potential energy is about equal to the initial relative kinetic energy of the colliding particles). Particles which pass about this close together will evidently be scattered through a large angle, and the cross section for such a process will be about equal to the area of a disk with a radius equal to the above distance. Thus we set initial particle relative kinetic energy W equal to the mutual electrostatic potential energy. For two singly charged particles then,

$$W = e^2/r_c,$$

with r_c being the distance of closest approach of classical particles. Hence

$$\sigma_c \approx \pi r_c^2 = \pi e^4/W^2. \quad (31)$$

Expressing W in kilovolts, we find

$$\sigma_c \approx \frac{6\times 10^{-20}}{W^2}\ \mathrm{cm}^2. \quad (32)$$

The cross section σ_c is seen to vary inversely with the square of the relative energy. At 100 kev relative energy it is about 6 barns, i.e., about 160 times the DD reaction cross section at the same relative energy. At much higher energies the above relationship underestimates the cross section because of direct nuclear interaction effects.

To calculate the effect of distant collisions we may assume that the deflecting effect of each such collision is small and is random in its direction relative to the motion of the scattered particle. Now the incremental impulse given to a particle in such encounters is the mean force times the mean time of an encounter

$$F(\Delta t) = \Delta(Mv) = \Delta p, \quad (33)$$

(Δp = impulse; p = momentum). The force is just that arising from the moving charge interacting with the electric field of the "background" particle with which it is "colliding." Thus $F = Ee$, and Δt is about equal to the time that the particle spends in going through a distance equal to its distance of closest approach as shown in Fig. 9. Thus $\Delta t \approx r/v$ and $E \approx e/r^2$. (v = particle

[17] N. F. Mott and H. S. W. Massey, *Theory of Atomic Collisions* (Oxford University Press, New York, 1949).

velocity.)

$$\Delta p \approx Ee\cdot r/v = e^2/rv. \quad (34)$$

Since the distant encounters produce momentum transfers which are essentially random in direction, the mean square momentum change will be proportional to the number of such encounters times the square of the momentum change per encounter. As the scattered particle moves a distance L it "collides" with a number of equal mass particles lying in a shell at distance r given by the particle density times the volume of such a shell: $dN_L = n(2\pi rLdr)$, thus for random encounters,

$$\langle(\Delta p)^2\rangle_{\text{Av}} = (nL2\pi rdr)\cdot(e^2/rv)^2, \quad (35)$$

we can find the total momentum change in path length L by integrating over all collisions, i.e., integrating over r between the distance of closest and most distant approach.

$$\langle(\Delta p)^2\rangle_{\text{Av}} = \frac{2\pi e^4}{v^2} nL\log(q) \text{ where } q = (r\max/r\min). \quad (36)$$

Let us divide both sides by the square of the initial momentum of the scattered particle $p^2 = (Mv)^2$.

$$\frac{\langle(\Delta p)^2\rangle_{\text{Av}}}{p^2} = \frac{\pi e^4}{2(\frac{1}{2}Mv^2)^2} nL\log(q). \quad (37)$$

When $\langle(\Delta p)^2\rangle_{\text{Av}}$ has increased to the point where it is comparable to p^2, we can assume that the particle has been scattered through a large angle, such as 90° (or will have had its energy changed by a substantial amount). In other words, if L is thought of as the mean free path for large angle scattering of a charged particle by distant collisions, then L is found by setting $\langle(\Delta p)^2\rangle_{\text{Av}}/p^2 = 1$

$$1 \approx \frac{\pi}{2}\frac{e^4}{(\frac{1}{2}Mv^2)^2} nL\log(q) = \frac{\pi}{2}\frac{e^4}{W^2} nL\log(q).$$

But $L = 1/n\sigma$ from the meaning of a cross section. Thus we find for the effective cross section for large angle scattering or substantial energy transfer due to distant collisions the expression

$$\sigma_d \approx \frac{\pi e^4}{W^2}\frac{\log(q)}{2}. \quad (38)$$

It is seen that σ_d is just σ_c multiplied by the factor $\log(q)/2$. The factor q is very large but bounded. It is given roughly by the ratio of the greatest distance of

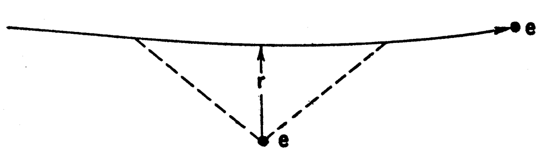

FIG. 9. Collision geometry for distant collisions.

interaction between two colliding particles and their distance of closest approach. The former distance is evidently set by the Debye length. The latter is about equal to r_c calculated above, if quantum effects are neglected. Thus $q \approx \lambda_D/r_c$ which is a number of order 10^9 for typical plasma densities and temperatures. Thus $\log(q) \approx 20$, and is relatively insensitive to changes in plasma density and temperature. If we disregard corrections for relative energy of the background particles, we find $\sigma_d \approx 10\sigma_c$. Thus the accumulative effects of distant collisions are about an order of magnitude more effective in producing scattering of a charged particle than are close encounters. With energy W in kilovolts,

$$\sigma_d \approx \frac{6\times 10^{-19}}{W^2}\text{ cm}^2. \quad (39)$$

This relationship may be expected to hold approximately for encounters between equal mass particles, or where the scattered particle is lighter than the scattering particles. Thus ion-ion, electron-electron, and electron-ion scattering effects are roughly estimable from σ_d above. Detailed and more accurate computations of this type, for the case of stellar encounters, have been made by the astrophysicist Chandrasekhar.[18] Spitzer[11] has extended these calculations to the case of a fully ionized plasma.

One of the important consequences of the work of Chandrasekhar and of Spitzer is in the introduction of the notion of collisional *relaxation times* of a plasma. It is clear that the mean time for a large scattering in angle or change in momentum of a particle interacting with a background of other particles is a measure of the rate of attempted approach of a nonequilibrium plasma distribution to equilibrium.

Two separate cases are of particular interest. The first is concerned with the time for a particular energetic ion or electron (the "test particle") to be scattered in direction (or changed in energy) by encounters with other like particles.

The cross section σ_d calculated above may be used to estimate these relaxation times.‖ These times are, respectively, t_{ii}, the mean collision time for an ion scattered by distant encounters with other ions, t_{ee}, a similar time for an electron scattered by electrons, and t_{ei}, the time for scattering of an electron by distant encounters with ions. Evidently $t_{ei} \approx t_{ee}$ of recoil effects are neglected. Thus (subscripts i and e refer to ions and electrons respectively)

$$t_{ii} = \frac{1}{n_i\sigma_d v_i}, \quad (40)$$

$$t_{ee} = \frac{1}{n_e\sigma_d v_e} \approx t_{ei}. \quad (41)$$

[18] S. Chandrasekhar, *Principles of Stellar Dynamics* (University of Chicago Press, Chicago, 1942).

‖ For more precise values the reader is referred to the original papers by Chandrasekhar and Spitzer.

Because an electron has about 60 times the velocity of a deuteron of equal energy, $t_{ii}\approx 60t_{ee}$ for comparable particle energies. By inserting the value of σ_d above one may express the energy exchange times as a function of background particle density and scattered particle energy in kilovolts. Using the mass of the deuteron for M, so that $v_i=3\times 10^7\sqrt{W}$ cm/sec, we have

$$t_{ii}=6\times 10^{10}\frac{W^{\frac{3}{2}}}{n_i}\text{ seconds,} \tag{42}$$

$$t_{ee}=10^{9}\frac{W_e^{\frac{3}{2}}}{n_e}. \tag{43}$$

Note that the relaxation times become *longer* at high energies. (These equations should be viewed only as estimates of the relaxation times, since differences of a factor of 2 or more with respect to accurately derived values are possible.)

From the equations it can be seen that, for example, at $n_i=3\times 10^{15}$ and $W=150$ kev, $t_{ii}=0.04$ second. (The corresponding mean free path is about 1×10^7.) From Fig. 4 of Sec. III, the mean reaction time for DD at 100 kilovolts kinetic temperature is seen to be 10 seconds or about 250 times the relaxation time of a deuteron at the mean particle energy appropriate to that temperature. This implies that at plasma temperatures of about 100 kilovolts a deuteron must on the average undergo a considerable but not extremely large number of effective scattering events before reacting. At lower energies this number is much larger.

The second case of interest pertains to the effect of collisions of an ion with background electrons. In this case two limiting situations are possible: (a) the ion energy is much less than the mean electron energy and (b) the ion energy lies above the mean electron energy. In case (a) the ion will gain energy by encounters with the more rapidly moving electrons at a rate essentially determined by the electron-ion collision rate and the mean energy transfer per collision. In (b), the ion will lose energy to the electrons, even though their mean velocity is greater than that of the ion.

The rate for (a) may be estimated by methods similar to those used to determine σ_d. Since the ions move very slowly compared to the average electron velocity, the problem is very much like the classical Brownian motion problem. As fast electrons pass in the vicinity of the ion they give it an incremental momentum kick of the kind described in the calculation of σ_d, except that here the mean electron velocity must be used.

$$\Delta p\approx e^2/r\bar{v}_e, \tag{44}$$

where $\bar{v}_e$ is the mean electron velocity.

The number of such collisions in time Δt from electrons passing nearby, say within a ring of area $dA=2\pi r dr$, is effectively proportional to the mean number of electrons per second passing through such a ring. From elementary kinetic theory this number can be set approximately equal to $3n_e\bar{v}_e dA$. Assuming that the encounters are random and integrating over r one obtains

$$\langle(\Delta p)^2\rangle_{\text{Av}}=\frac{6\pi n_e e^4}{\bar{v}_e}\log(q)\Delta t. \tag{45}$$

Since $(1/2)m\bar{v}_e^2\approx(3/2)kT_e$, and $\langle(\Delta p)^2\rangle_{\text{Av}}=2M\langle\Delta W\rangle_{\text{Av}}$, substitution of these quantities in (45) yields for the rate of energy gain by an ion bombarded by energetic electrons ($W\ll(3/2)kT_e$, all units cgs)

$$\frac{dW}{dt}\approx 3\pi\log(q)\frac{n_e e^4}{(3m_e kT_e)^{\frac{1}{2}}}\cdot\left(\frac{m}{M}\right). \tag{46}$$

This expression shows that the rate of gain of energy of an ion is independent of its energy (as long as it is small compared to the mean electron energy) and that it varies inversely with the square root of the electron temperature. This qualitative fact is of some interest in connection with the possibility of heating "cold" ions by collision with "hot" electrons. Since it is often easier to impart energy directly to the electrons of a plasma than to the ions, a possible heating cycle would be to heat electrons and then allow them to impart energy to the ions by collisions. This is likely feasible at energies of a few volts, but at fusion temperatures the energy transfer rate becomes slower and makes the heating less effective.

In the discussion of electron-ion collision effects, the other limiting case of interest is that where the ion energy is large compared to the mean electron energy. Here the result, not as easily obtainable by approximate methods, is that even though the mean electron velocity may be greater than the ion velocity, the energy transfer is in fact dominated by those few electrons of the distribution which have a lower velocity than the ion, and the effect of these collisions is always to reduce the ion energy until it reaches the mean electron energy. Spitzer and others have calculated exact expressions for energy transfer between ions and a Maxwellian electron distribution. A form is given below that encompasses both of the cases here considered.¶ This expression, is for a hydrogenic plasma: (cgs units)

$$\frac{dW}{dt}=4\pi\sqrt{2}\log(q)\frac{n_e e^4}{(\pi mkT_e)^{\frac{1}{2}}}\left(\frac{m}{M}\right)\left(1-\frac{W}{\frac{3}{2}kT_e}\right). \tag{47}$$

When the ion energy W is much less than $\frac{3}{2}kT_e$, this expression reduces to the same form as the approximate equation (46) above, apart from a numerical factor of $(3\pi/32)^{\frac{1}{2}}$ or 0.54. For this limiting case, inserting numerical values into (47), expressing W and kT_e in kilovolts, and setting M equal to the mass of a deuteron,

¶ M. H. Johnson (private communication).

one obtains

$$\frac{dW}{dt}=1.4\times10^{-12}\frac{n_e}{T_e^{\frac{1}{2}}}\text{ kilovolts/sec.} \tag{48}$$

At $n_e=3\times10^{15}$ and $T_e=0.1$ kilovolt (such as might be obtained in a high current gas discharge), a low energy ion will gain energy at a rate of 1.3×10^4 kilovolts/second so that a 1 volt ion may double its energy in about 0.1 μsec. However, at the same density and 100 kev the rate is 430 kilovolts/sec; a 10-kev ion will double its energy only after 0.023 sec.

When $W\gg kT_e$, (47) becomes (energy and temperature in kilovolts)

$$\frac{1}{W}\frac{dW}{dt}=-0.9\times10^{-12}\frac{n_e}{T_e^{\frac{3}{2}}}. \tag{49}$$

This may be integrated immediately to yield

$$W=W_0e^{-t/\tau_e}, \tag{50}$$

where $\tau_e=1.1\times10^{12}(T_e^{\frac{3}{2}}/n_e)$. Collisions with a "cold" electron distribution are seen to exponentially "damp" the energy of a fast ion with a time constant proportional to the three-halves power of the electron temperature, so that the ion approaches asymptotically to the energy $\frac{3}{2}kT_e$, in agreement with classical equipartition considerations. At $n_e=3\times10^{15}$ and an electron temperature of 10 volts (typical of ordinary discharges), τ_e is only 0.3 microsecond or about 3×10^{-7} times the mean reaction time for DD at 100 kilovolts temperature. This is of importance to certain problems discussed in Sec. VIII.

One of the interesting consequences of the general equation (47) is that at high plasma temperatures the "equilibrium" electron temperature may lag somewhat behind the ion temperature. If no appreciable energy is being fed into the plasma from reactions or external sources, the electron temperature will be determined by a balance between the rate of bremsstrahlung radiation and the rate of collisional energy transfer from the ions, found by combining Eqs. (47) and (12). For a deuterium plasma the resulting temperature difference is given approximately by the equation (temperatures in kilovolts)

$$T_i-T_e=4.4\times10^{-3}T_e^2. \tag{51}$$

Thus if $T_i=100$ kev, T_e would asymptotically approach 75 kev in the absence of other energy exchange processes.

Particle Motions

At fusion temperature the mean free path for collisions of ions and electrons in a reacting plasma will likely be very long, as already noted. In this case the motion of an ion or electron in the plasma through distances of the order of the apparatus dimensions will be largely dominated by electrodynamic forces. Each charged particle of the plasma will move nearly in accordance with the usual equation of motion of a particle in an electromagnetic field, where here the field is the resultant of the self-consistant superposition of externally generated fields and the space-charge and motional fields produced by the plasma itself. A sufficiently weak plasma may have a relatively minor effect on an externally generated field and a picture of its behavior may be deduced from the motion of individual particles in the applied field.

The equation of motion of a charged particle in an electromagnetic field is, in Gaussian units,

$$m\frac{d\mathbf{v}}{dt}=e\left(\mathbf{E}+\frac{\mathbf{v}}{c}\times\mathbf{H}\right). \tag{52}$$

Electric forces with a component parallel to the motion may produce a change in kinetic energy. Magnetic forces always act in a direction perpendicular to the motion and thus produce curvature but no change in energy.

If $\mathbf{E}=0$ and $\mathbf{H}$ is constant in space and time, the motion consists of the superposition of an arbitrary constant velocity component along the magnetic lines and a rotation at the cyclotron angular frequency, $\omega_c=eH/mc$. In other words the orbits described are helices parallel to the direction of the field. Because of their opposite sign of charge, ions and electrons rotate in opposite directions, with frequencies in the ratio of (m/M), respectively. For the same rotational energy, ions execute orbits which are $(M/m)^{\frac{1}{2}}$ times the diameter of the electron orbits (about 60 times in the case of deuterons). The product of magnetic field and ion orbit radius of curvature is for deuterons (rotational energy in kilovolts).

$$H\rho_c=6.4\times10^3\sqrt{W_\perp}\text{ gauss-cm.} \tag{53}$$

Also for a deuteron,

$$\omega_c=4.8\times10^3H\text{ radians/sec.} \tag{54}$$

When an electric field is present with a component perpendicular to the magnetic field the particle path consists of a helical motion and a superposed drift at constant velocity in a direction perpendicular both to the magnetic field and the transverse electric field component. When this drift velocity is small compared to the particle orbit velocity the actual motion may be thought of as arising from a transverse drift of the instantaneous center of rotation (guiding center) of the particle. This important drift velocity is given by the equation

$$\mathbf{v}_0=c\frac{\mathbf{E}\times\mathbf{H}}{H^2}=c\frac{E}{H}, \tag{55}$$

if E and H are perpendicular. Note that its direction and magnitude are independent of the magnitude or sign of charge, or the mass of the particle.

If it is remembered that even in a static magnetic field a moving observer will see an electric field, the drift velocity $\mathbf{v}_0$ may be readily deduced from the equation of motion. Assume that the actual particle motion consists of a uniform velocity $v_0' \ll c$ plus a variable part v_1, so that $\mathbf{v}=\mathbf{v}_0'+\mathbf{v}_1$, i.e., viewed from a coordinate system moving at velocity $\mathbf{v}_0'$, the particles' velocity is $\mathbf{v}_1$. In such a moving coordinate system there will appear in addition to the applied electric field a motional electric field $\mathbf{E}_m=(\mathbf{v}_0'\times\mathbf{H})/c$. In the moving frame therefore the equation of motion is

$$m\frac{d\mathbf{v}_1}{dt}=e\left(\mathbf{E}+\mathbf{E}_m+\frac{\mathbf{v}_1\times\mathbf{H}}{c}\right). \tag{56}$$

Pick for the velocity v_0' that velocity where $\mathbf{E}_m=-\mathbf{E}$, so that $\mathbf{E}+\mathbf{E}_m=0$. In this frame there is no electric field and the orbits are again helices. Picking the velocity

$$\mathbf{v}_0'=\mathbf{v}_0=c\frac{(\mathbf{E}\times\mathbf{H})}{H^2},$$

then one finds

$$\mathbf{E}_m=\frac{(\mathbf{E}\times\mathbf{H})\times\mathbf{H}}{H^2}=-\mathbf{E},$$

since $\mathbf{H}$ and $\mathbf{E}$ are perpendicular.

The drift velocity $c(E/H)$ has another important meaning in connection with a magnetic field which varies with time. Suppose that a uniform magnetic field produced by a long solenoid is increasing with time. In this case an electric field appears which consists of circular lines of electric force centered on the axis of the solenoid. The magnitude of this field in the laboratory frame is found from the usual transformer equation

$$\oint E\cdot dl=\frac{-1}{c}\int\left(\frac{dH}{dt}\right)\cdot dA, \tag{57}$$

i.e., at a radius r

$$2\pi rE=\frac{-\pi r^2}{c}\frac{dH}{dt}. \tag{58}$$

At any radius r the drift velocity $v_0=c(E/H)$ may be evaluated.

$$v_0=dr/dt=c(E/H)=-(r/2)(dH/dt)(1/H), \tag{59}$$

i.e.,

$$(1/r)(dr/dt)=-\tfrac{1}{2}(1/H)(dH/dt). \tag{60}$$

This can be integrated directly to give

$$\frac{r}{r_0}=\left(\frac{H_0}{H}\right)^{\frac{1}{2}}, \tag{61}$$

or

$$\pi r^2H=\pi r_0^2H_0, \tag{62}$$

showing that an inward motion of any part of the region at the local drift velocity $c(E/H)$ preserves flux between the moving point and the axis—i.e., the guiding centers of particles moving at the local drift velocity stay on the surface of some collapsing flux tube of the magnetic field.

In the presence of magnetic field gradients more complicated motion of the particles will occur. When the gradients are small the motion can be predicted by simple relationships. The case of a magnetic field with a gradient perpendicular to the local direction of field lines is of particular interest. The qualitative effect of such a gradient is shown in Fig. 10. Where the field is strong, radii of curvature are smaller than average; where weak they are larger. Thus a cycloidal "walk" results, with the guiding centers of positively and negatively charged particles moving oppositely and perpendicular to the direction of the gradient. In a plasma this results in a tendency for charge separation to occur, which in turn can produce local electric fields. The eventual result may be that the plasma drifts as a whole in the "downhill" direction of the transverse gradient. The details of the actual resultant plasma drift motion are best treated by the macroscopic equations of motion of the plasma, for example in the form discussed by Spitzer.[11] The velocity of the individual particle drifts (if uninhibited by collective effects) is[12]

$$v_d=\frac{\rho_c v_\perp}{2}\frac{\nabla_\perp(H)}{H}, \tag{63}$$

where $\nabla_\perp(H)$ is the gradient of the scalar magnitude of H in a plane perpendicular to H, ρ_c is the radius of curvature of the particle, and $v_\perp$ is its rotational velocity component. The expression is valid only if $v_d\ll v_\perp$, i.e., $(\rho_c/2)(\nabla_\perp(H)/H)\ll 1$. The latter condition merely implies that the fractional change in magnetic field strength across an orbit shall be small.

Drifts which might be called "gyroscopic" in origin can arise from gravitational or centrifugal forces. These forces lead to drift motions perpendicular to the applied force and in opposite directions for ions and electrons. In a gravity field with a component $g_\perp$ perpendicular to the magnetic field the drift is[12]

$$v_g=\frac{g_\perp}{\omega_c}, \tag{64}$$

ω_c is the cyclotron frequency of the particle in the

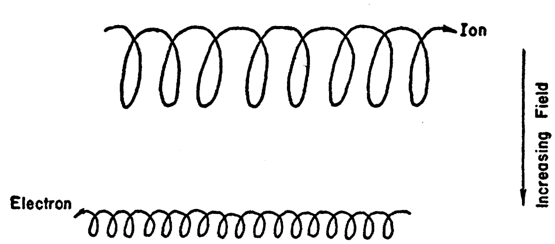

FIG. 10. Particle drifts in a magnetic field having a gradient transverse to the direction of the field.

magnetic field. For high magnetic fields this drift velocity is small.

If particles are moving along lines of force which are curved a centrifugal acceleration will arise which will also result in a drift, calculable from (64) above by replacing $g_\perp$ by the centrifugal acceleration.[11] Suppose that the particle is executing a helical path along the curved field lines with a velocity component along the field lines of magnitude $v_\parallel$. Then if the bundle of magnetic lines are curved with a radius of curvature R, the centrifugal acceleration is $v_\parallel^2/R$, so that

$$v_c = v_\parallel^2/R\omega_c. \tag{65}$$

In contrast to gravitational drifts, this drift velocity may be substantial. Note that the guiding center drifts v_d, v_g, and v_c are oppositely directed for ions and electrons and may give rise to charge separation and consequent electric field effects. In many such cases the end result may be that the plasma takes up a motion in the direction of weakest magnetic field—i.e., it tends to be repelled from regions of strong magnetic field.

This apparent tendency for charged particles moving in a magnetic field to be repelled from regions of strong magnetic field—i.e., to behave diamagnetically—is also exhibited by particles moving in a magnetic field with a gradient parallel to the direction of the field lines. The repelling of charged cosmic-ray particles by the dipole magnetic field of the earth is a well-known effect. Fermi has applied similar concepts also to explain the origin of cosmic rays.[19] The repelling effect of a positive field gradient parallel to the magnetic field lines is most easily explained in terms of one of the so-called "adiabatic invariants" of the motion of a charged particle. Since particles moving in a static magnetic field spiral about lines of force and are acted on only by forces perpendicular to their motion, the angular momentum associated with their rotational motion about a line of force should be an approximate constant of the motion. Also, the rotational motion of a charge must generate a magnetic moment which will also be a constant of the motion. This can be shown from simple considerations.

The usual cyclotron equation for balance between centrifugal and magnetic forces on a circularly moving particle is

$$\frac{mv_\perp^2}{r} = \frac{Hev_\perp}{c}, \tag{66}$$

which can be rewritten as

$$\frac{\frac{1}{2}mv_\perp^2}{H} = \frac{e}{2mc}(mv_\perp r), \tag{67}$$

$mv_\perp r$ is the angular momentum of the rotational motion, call it α; $\frac{1}{2}mv_\perp^2$ is the rotational energy of the particle; $e/2mc$ is a constant. Thus

$$\frac{\frac{1}{2}mv_\perp^2}{H} = \frac{W_\perp}{H} = \frac{e}{2mc}(\alpha) = \mu, \tag{68}$$

where μ is the magnetic moment in ergs/gauss. The constancy of α implies the constancy of μ and thus $W_\perp/H$ represents an adiabatic invariant of the motion. An "adiabatic invariant" is here to be understood as a quantity which is constant for slow changes in the magnetic field at the particle during a single rotation. $W_\perp/H$ has also been shown to be an invariant for magnetic fields varying slowly with time.[12,20]

For charged particles moving freely in a static magnetic field another invariant of the motion is the total kinetic energy, i.e., in moving along magnetic field lines the sum of rotational and translational energy should be a constant. If the magnetic field strengths at two different regions (1) and (2) encountered by the particle are H_1 and H_2 respectively, then conservation of energy requires that

$$W_\parallel(1) + W_\perp(1) = W_\parallel(2) + W_\perp(2). \tag{69}$$

The invariance of μ also implies that

$$\frac{W_\perp(1)}{H_1} = \frac{W_\perp(2)}{H_2}, \tag{70}$$

whence

$$W_\parallel(2) = W_\parallel(1) - W_\perp(1)\left[\frac{H_2}{H_1} - 1\right], \tag{71}$$

if $H_2/H_1 > 1$, then $W_\parallel(2) < W_\parallel(1)$. Evidently the particle slows down its velocity component along the field lines in moving into a stronger field. If the initial $W_\parallel$ is not too great a particle may be turned around by a positive field gradient, which acts therefore somewhat like a potential. Since force is the negative gradient of potential, differentiation of (71) yields an approximate equation for axial motion of a particle spiraling along a magnetic line.

$$F_z = -\mu\frac{\partial H_z}{\partial z}, \tag{72}$$

where z is parallel to the local direction of the magnetic field lines.

It is apparent from the equations above that particles which move with too small a component of rotational energy will not be turned around by the field.

The constancy of μ has another interesting corollary. Since $W_\perp/H = \text{constant}$ and $H\rho_c \sim W_\perp^{\frac{1}{2}}$ from Eq. (53), it follows that $\rho_c^2 H = \text{constant}$, i.e., the flux through the orbit circle is an approximate constant of the motion.

Diffusion Across A Magnetic Field

Although of heuristic value, the picture of a plasma as composed of an assembly of independently moving

[19] E. Fermi, Astrophys. J. **119**, 1 (1954).

[20] G. Hellwig, Z. Naturforsch. **10a**, 508 (1955).

charged particles subject only to simple adiabatic laws is an over-simplification. For many problems the macroscopic equations of the plasma should be used.[11] An example of a problem which is most simply analyzed by the macroscopic or hydrodynamical equations is the diffusion of charged particles across a strong magnetic field (i.e., where the particle pressure is negligible compared to the magnetic pressure).

Microscopically, the diffusion of charged particles across a magnetic field is to be understood as a two-dimensional random walk of particles at a rate determined by their mutual collision frequency and with a step length roughly equal to the cyclotron radius. Thus, qualitatively, the diffusion rate will be lowered either, by (a) raising magnetic field (smaller orbit diameter), or (b) increasing temperature or reducing density (lower collision frequency).

In a quiescent plasma composed of electrons and identical ions the diffusion may originate either from electron-ion, ion-ion, or electron-electron collisions. Simon [21] has recently calculated the relative importance of these processes and has apparently resolved a paradox. The paradox is that a first-order treatment predicts that collisions between like particles lead to no net diffusion (since the center of mass remains fixed in such collisions).

The usually dominant term is the effect of electron-ion collisions for which the diffusion velocity is given by.[11]

$$v_p = \frac{-c}{\sigma}\frac{\nabla P}{H^2} \text{ cm/sec,} \tag{73}$$

where σ is the electrical conductivity of the plasma in cm^{-1} (emu) and P is the plasma pressure. Expressed as a function of temperature in kilovolts for a deuterium plasma this is approximately

$$v_p \approx \frac{6.3\times 10^3}{T^{\frac{3}{2}}}\frac{\nabla P}{H^2}. \tag{74}$$

If P is assumed to be some constant small fraction β of the magnetic pressure term $H^2/8\pi$, and the characteristic distance associated with ∇P is denoted by L, then

$$v_p \approx \frac{1.5\times 10^5}{T^{\frac{3}{2}}}\left(\frac{\beta}{L}\right) \text{ cm/sec.} \tag{75}$$

For $T=100$ kilovolts for example $v_p \approx 150(\beta/L)$ cm/sec corresponding to a very slow diffusion rate, and a correspondingly great reduction in heat transport across the field.

Simon[21] has shown theoretically that under certain circumstances diffusion owing to collisions between like particles can be of importance. However, in this case the diffusion rate varies as H^{-4} and is not simply proportional to ∇P.

[21] A. Simon, Phys. Rev. **100**, 1551 (1955).

The expressions above are derived under the assumption that transport of particles across a magnetic field arises only from mutual collisions. However, some study several years ago at the University of California Radiation Laboratory[22] of the diffusion of ions across the plasma of arc discharges in a magnetic field seemed to indicate that a more rapid diffusion than predicted above could occur. It was postulated by Bohm and co-workers at the Laboratory that random electrostatic fields arising from turbulent types of plasma oscillations were responsible for these excessive drifts. Bohm proposed an expression for the diffusion velocity; (temperature in kilovolts)

$$v_B = 6\times 10^9 T_e \frac{\nabla n}{n}\cdot\frac{1}{H} \text{ cm/sec.} \tag{76}$$

This depends directly on electron temperature and inversely only as the first power of magnetic field. It predicts a much higher diffusion loss rate than (74) above and correspondingly greater heat transport.

Some recent experimental work by Simon and Neidigh (unpublished) at ORNL seems to indicate that the Bohm diffusion mechanism, if it exists, may be associated with highly unsymmetric plasma systems. Their work, carried out in arc discharges with axial symmetry, seems to bear out the $1/H^2$ dependence of (74). However, the assumption of a nonturbulent plasma may be far from the truth in many situations of interest to controlled fusion research. An example is the instability of the pinch effect, mentioned in Sec. V.

Compression of a Plasma

To conclude the discussion of phenomena in totally ionized media, mention should be made of the effects associated with compression of a plasma. Depending on the nature of the compression and its time-scale a plasma can act either as a one-, two-, or three-dimensional gas. This unusual circumstance arises because processes which are slow compared to the individual particle motions may still be rapid compared to the collisional relaxation times of the plasma particles. When this is the case the gas kinetic degrees of freedom of the plasma are effectively decoupled from each other. If such a plasma is confined and adiabatically compressed in one dimension only, then the energy in this degree of freedom will increase in accordance with the adiabatic law for a one-dimensional gas. For compression across a magnetic field two degrees of freedom are involved and the behavior is as a two-dimensional gas. In general, for a gas with number of degrees of freedom f, the variation of kinetic temperature T is given by

$$T_f \sim n^{\gamma-1}, \tag{77}$$

where T_f is to be understood as the generalized kinetic

[22] A. Guthrie and R. K. Wakerling, *The Characteristics of Electric Discharges* (McGraw-Hill Book Company, Inc., New York, 1949).

temperature associated with the compression. Also,

$$\gamma=(2+f)/f, \tag{78}$$

and n is the particle density.

For $f=1$, $\gamma=3$, and $T_1 \sim n^2$. This fact is noted by Fermi in his theory of the origin of cosmic rays,[19] where the acceleration mechanism he proposed arises from the repeated reflection of energetic particles from two approaching magnetic clouds at a variable distance L from each other, so that $T_1 \sim 1/L^2$.

When the compression is two-dimensional, $f=2$ and $\gamma=2$, so that $T_2 \sim W_\perp \sim n$. When the compression is adiabatic and is carried out in a time long compared to the relaxation times, $f=3$, $\gamma=5/3$, and $T \sim n^{\frac{2}{3}}$ as in an ordinary gas.

The existence of adiabatic compressional processes in a plasma confined by a magnetic field suggests strongly that such processes may be reversible and therefore could be useful for the extraction of energy from a reacting plasma by its expansion against the magnetic field. It is evident that one result of this expansion could be the generation of electrical energy in external coupled electrical circuits, possibly at very high thermodynamic efficiencies.

Nonadiabatic, i.e., irreversible, shock-hydrodynamic effects can also occur in a plasma. Here the physical situation is considerably complicated by the presence of a magnetic field and new types of phenomena can occur. De Hoffmann and Teller[23] have treated some of these, and new papers are beginning to appear on the subject. This facet of plasma physics is a new and interesting one, and is of relevance to some aspects of controlled fusion research, for example the transient pinch effect mentioned in Sec. V.

Summarizing, a plasma at thermonuclear temperatures may be qualitatively pictured as a mixture of two charged particle gases—the ions and the electrons. These two gases are only weakly coupled through collisions, in the small, but strongly coupled in the large through Coulomb forces. Depending on the time scale and nature of application of external electromagnetic forces, either gas may behave as a one-, two-, or three-dimensional ideal gas for compressional processes. Through exhibiting a high effective electrical conductivity, the plasma will tend to move in such a way as to minimize electric fields existing in frames of reference locally at rest with respect to the plasma. This latter property can be identified with a tendency for the plasma to preserve constant flux in its motion, and thus locally to "stick" to magnetic lines of force. As a consequence of this, the guiding centers of the individual particles of a sufficiently weak plasma will tend to move on the surface of flux tubes of a magnetic field, as long as the field varies slowly (but not too slowly) with time. Diffusion arising from collisional effects tends to destroy this situation, so that the diamagnetism of a plasma is of necessity a nonequilibrium phenomenon. In addition, the presence of induced particle drifts may give rise to charge separation and electric fields. These drift effects in turn may thus be self-perpetuating so that in particular cases the plasma may exhibit hydromagnetic instabilities.

[23] F. de Hoffmann and E. Teller, Phys. Rev. 75, 1851 and 1864 (1949).

VII. SCALING LAWS—POWER BALANCE

A moment's reflection will show that the phenomena of nuclear fusion and chemical fusion are similar in many ways. Both release energy through a combination or rearrangement of the reactive particles induced by close collisions. Both can be self-sustaining only under favorable physical circumstances; namely, attaining a minimum ignition temperature, exceeding a minimum quantity of fuel, and establishing a sufficiently low rate of energy loss to the surroundings to prevent quenching the reaction. If too small a fire is laid in the fireplace it will not catch; it is difficult to ignite a single piece of coal lying in a grate. These everyday observations about chemical fusion have their counterparts in the nuclear fusion reaction. In this section some general "scaling laws" which may apply to fusion reactors are discussed. Seen in their light, the first achievement of a detectable rate of thermonuclear reactions in a hot plasma might represent much less of a feat than the subsequent attainment of an energetically self-sustaining reaction. The search for practical nuclear fusion power will not only depend on establishing theoretical operability, but also in showing that a self-sustaining reactor utilizing the proposed principles would not be too large to be constructed or require unattainable physical conditions for operation.

The achievement of a net power balance from a fusion reactor will depend on a favorable competition between the nuclear energy produced and two types of energy loss processes, "direct" and "indirect." Direct energy losses are those ascribable to escape of radiation or energetic particles from the reacting region. Indirect energy losses are those resulting from external inefficiencies connected with heating and confining the plasma. Failure to control direct losses results in quenching the reaction. Excessive indirect losses would prevent achievement of a closed cycle capable of producing a net power output, even though the internal reaction might be satisfactorily efficient. This latter situation resembles what would happen if the electrical generator on an automobile broke down while it was running. The internal combustion cycle of the automobile engine would continue only until the ignition battery ran down and the spark plugs ceased to fire.

Scaling Laws For Power Output

To illustrate how power balance scaling relationships might be deduced for realistic schemes, let us make a simplified analysis of an unrealistic one—the stationary pinch example of Sec. V. We shall suppose that the Kruskal-Schwarzchild instability does not exist and that the pinch is made so long that the losses to the end electrodes are unimportant. In steady state simple scaling relationships may be found for the nuclear power output. If the radius is R cm, the fusion power per unit length for DD will be

$$p=\tfrac{1}{2}n_D^2\langle\sigma v\rangle_{\text{Av}}W\cdot\pi R^2. \tag{79}$$

Now the pressure is proportional to the square of the magnetic field, in accordance with Eq. (21), from which it follows that $n_D\sim H_0^2/T$. Neglecting the numerical constants, one finds the variation of p with radius, temperature and magnetic field from (79) above to be

$$p\sim\left(\frac{\langle\sigma v\rangle_{\text{Av}}}{T^2}\right)R^2H_0^4. \tag{80}$$

Some conclusions can be drawn from the expression. Firstly, if losses were small, the most efficient operating temperature would be determined by the maximum point of the function $(\langle\sigma v\rangle_{\text{Av}}/T^2)$. This function has a broad maximum in the vicinity of 10 to 15 kev for both the DD and DT reactions. Since these temperatures are above the bremsstrahlung critical temperature, T_c, (see Sec. IV) of the DT but below that for the DD reaction they represent a possible operating temperature region for the one, but not for the other. Secondly, the strong dependence of power output on magnetic field is noteworthy, pointing up the advantages of the use of high confining fields, and the probable impracticability of operating any controlled fusion device at small fields. Since Eq. (80) was derived from the simple concept of magnetic pressure, similar relationships for the fusion power scaling laws can be expected to occur in the analysis of more complicated examples of magnetic confinement than the simple pinch discussed in this article.

Without a detailed analysis it is not possible to set down more complete expressions for the power loss scaling laws. A few results can, however, be found which are not especially dependent on the example.

Direct Losses

It has already been shown that the ratio of bremsstrahlung loss to fusion power is independent of density and dimensions. Thus this part of the power balance is unaffected by scaling in physical dimensions or magnetic field (at constant temperature). Other direct losses may arise from the escape of particles out of the system by diffusion. Simple diffusion processes are characterized by a transport velocity which is proportional to the gradient of the density. For example, expressions (74) and (76) of Sec. VI for the rate of diffusion of particles across a magnetic field both depend on the gradient of the particle density, though having a different functional dependence on magnetic field.

As a simple example, consider systems which are long compared to their transverse dimensions, and assume that upon scaling in radial dimensions the relative spatial distribution of the plasma is preserved. In such a case the power loss per unit length through radial diffusion will be proportional (a) to the plasma boundary area per unit length, (b) the plasma diffusion velocity, (c) the particle density and, (d) the mean particle energy (proportional to the kinetic temperature).

$$p_L\sim Rv_DnT. \tag{81}$$

If the radial diffusion arises from classical collisional effects, so that Eq. (74) of Sec. VI, applies this becomes

$$p_L\sim\frac{n^2}{T^{\frac{1}{2}}H_0^2}, \tag{82}$$

independent of the radius (since gradient $n\sim n/R$ under a uniform dimensional scaling). p_L is also seen to depend on n^2 as does p, since it also results from a binary process-collisional diffusion.

Eliminating the particle density through its dependence on confining magnetic field (see Eq. (21)) one obtains

$$p_L\sim\frac{H_0^2}{T^{5/2}}. \tag{83}$$

A kind of a figure of merit for the competition of nuclear power with direct losses other than radiation can be found by dividing p by p_L

$$\frac{p}{p_L}\sim(\langle\sigma v\rangle_{\text{Av}}T^{\frac{1}{2}})R^2H_0^2. \tag{84}$$

Where direct particle losses by collisional radial diffusion are important (84) shows that high temperature operation is favored, and that radial dimension and strength of the magnetic field are of equal and considerable importance in determining the attainment of a favorable power balance.

Indirect Losses

The calculation of indirect losses is obviously more intricate. It would involve the assessment of the power loads in every auxiliary associated with the fusion reactor, as well as more immediate sources of energy loss such as joule heating of conductors (whether plasma or metallic). In a closed cycle requiring electrical energy to produce and sustain the fusion reaction, the efficiency of conversion of thermal energy derived from the fusion reaction to electrical energy could be an important factor.

One scaling law which should apply to any magnetic confinement scheme has to do with the power required to maintain a magnetic field. If a current density j flows in a conductor (either the plasma of the pinch example, or the external conductors of other possible magnetic confinement methods), the power dissipated per unit volume of the conductor is equal to $\rho_0 j^2$ where ρ_0 is the resistivity. The magnetic field produced by any conductor system is proportional to j times a linear dimension of the system. For simplicity consider a cylindrical system with a length great compared to its diameter. We may then write the scaling relationship for a magnetic field produced by the conductor system

$$H_0 \sim jR_c, \tag{85}$$

where R_c is the radius of the conductor system. Thus

$$H_0^2 \sim j^2 R_c^2. \tag{86}$$

Now the total joule heat dissipated per unit length in the system is proportional to the unit volume dissipation rate $\rho_0 j^2$ times the volume per unit length of the system, i.e.,

$$p_j \sim \rho_0 j^2 R_c^2 \sim \rho_0 H_0^2,$$

independent of the diameter of the system. Thus for the ratio of p/p_j there is obtained (since $R_c \sim R$):

$$\frac{p}{p_j} \sim \left(\frac{\langle\sigma v\rangle_{\text{Av}}}{T^2}\right)\frac{1}{\rho_0} R^2 H_0^2. \tag{87}$$

Just as in the case of competition with the direct losses from particle diffusion, competition between fusion power output and joule heat losses in a long system is favored by increasing the radius or the magnetic field. It is interesting to note that in cases where volume joule heat dissipation, $\rho_0 j^2$, is the limiting factor in the design of a magnetic field coil, Eq. (85) shows that high magnetic fields are most easily obtained in large diameter systems.

Other Scaling Factors

To conclude discussion of scaling factors mention should be made of material and radiation problems. It has already been noted that heat transfer and other problems must eventually introduce limitations in the practical achievable power density of a fusion reactor. Attractive though it seems to dream of constructing a compact nuclear fusion engine about the size of an unabridged dictionary, limitations on power density introduced by the properties of materials would alone represent a serious barrier. In addition, all fusion fuels of present practical interest can undergo reactions which produce neutrons. Thus a fusion reactor producing any substantial amount of power would probably have to be surrounded by several feet of radiation shielding. This would make the dreamed-for miniature reactor somewhat less economically attractive even if it were technically feasible.

Another factor obviously important to the achievement of a favorable power balance is the choice of reacting fuel. Here the significant choices are as between the DD reaction and the DT reaction. The DT reaction exhibits a much larger reaction cross section than DD, as seen from Fig. 1. The total energy release per reaction is also substantially higher. However, among the various factors which must be taken into account in weighing the relative advantages of DT or DD as primary fuels, consideration should be given to the fact that most of the kinetic energy in the DT reaction appears in the neutrons. In the case of the DD reaction a large fraction of the reaction energy may eventually appear in the form of charged particles and might therefore be recoverable by direct conversion to electrical energy through interaction with confining fields.

VIII. FRUITLESS APPROACHES

Controlled fusion research presents an unusual two-edged challenge to the physicist. While demanding great ingenuity in the formation of qualitative ideas to solve problems of heating and confinement, lack of diligent scrutiny of the *quantitative* aspects of the problem can render the most ingenious of plans completely worthless. By way of illustration of this, some simple examples of what appear to be fruitless approaches to controlled fusion will be discussed. It is often risky to attempt to prove general theorems about the impossibility of performing certain experiments. Nevertheless, confronted with the first and second laws of thermodynamics few physicists today engage in a search for perpetual motion machines. (It is hoped that the construction of a practical fusion reactor does not fall into a similar category of endeavor!)

Earnshaw's Theorem—"Electric Containment"

The fuel of a fusion reactor is made up of charged particles and charged particles are influenced by electrostatic fields. Thus someone might suggest that a fusion reactor be constructed from a set of charged electrodes, forming an electrostatic "cage" for the plasma, which could thus be held free from contact with the material electrodes by the action of electrostatic fields.

Such a proposal fails on at least three counts, two qualitative and one quantitative. Firstly, Earnshaw's theorem from classical electrostatics shows that no position of stable equilibrium for even a single charged particle can be formed by the field of arbitrarily disposed charged conductors. Secondly, any electrostatic field which would represent a potential well for one sign of charge would represent a "hill" for the other sign. The third objection has to do with magnitude of the "pressure" which can be exerted by an electric field. Just as in the case of the magnetic field this pressure is limited by the energy density of the field, which for electric fields is $(E^2/8\pi)$ (stat volts/cm)^2 or ergs/cm^3. Choosing the former example of a plasma density of 6×10^{15} particles per cm^3 and a temperature of 100 kev, one discovers that E turns out to be 1.6×10^5 stat volts/cm or 4.8×10^7 volts/cm! This is an enormous field, and since the "electric pressure" varies as the square of the field there is not much hope in finding an operating point of the slightest practical interest, even if the previous objections did not apply.

Target Bombardment

It has already been mentioned that, although nuclear fusion reactions can be produced copiously in the laboratory by merely bombarding say, a deuterated target with a beam of high-energy deuterons, this does not offer a promising avenue to fusion power because of the low yield of reactions per incident deuteron. Most of the energy of the incoming particle is expended uselessly in ionization of the target atoms, which promptly radiate the lost energy by recapture of the electrons and by other similar processes. A frequently suggested improvement on this idea is to use the plasma of an ordinary gas discharge as the bombarded target, thereby eliminating the problem of ionization losses. Here the idea is qualitatively good, but fails quantitatively. Firstly, most ordinary gas discharge plasmas are not completely ionized and ionization and other losses still could predominate over reactions. Secondly, the electron temperature of most such discharges is of order 0.01 kev, usually established by processes occurring near the boundaries of the discharge. The expression (50) of Sec. VI then shows that the mean time for energy loss from an incoming "hot" ion by collisions with the "cold" electrons would be about $10^9/n$ seconds. Thus if $n=10^{15}$, for example the time for the ion energy to be substantially reduced is only 10^{-6} second, which is very short compared to the mean reaction time at these densities. The idea might have more merit if the electron temperature were substantially increased—however, this then becomes essentially the same problem as that of containing a hot self-reacting plasma, which if solved will also do the job.

Colliding Beams

A variation on the theme of bombarding a target with an ion beam is the suggestion that reactions be produced by directing two ion beams at each other, obtaining reactions from the mutual collisions. Such a suggestion has the merit that the problem of getting high enough ion energies is apparently solved and, by confining the ion in a beam, collisions with walls are avoided, at least for a time. This scheme perishes from the numbers. A very intense ion beam might have a current density approaching 0.1 ampere per cm^2. The density of ions in such a beam is calculated by dividing the charges per second per cm^2 by the velocity. For 100-kev deuterons this yields a particle density of 2×10^9 ions/cm^3. Application of Eq. (10) of Sec. III gives a reaction power density of 4×10^{-11} watt/cm^3!

Many other examples similar to those above could be cited. All illustrate the unusual importance of combining quantitative feasibility with qualitative operability as criteria in judging proposals for achieving controlled fusion reactions.

IX. PLASMA DIAGNOSTICS

One of the tantalizing things for the experimental physicist attempting to undertake an orderly study of phenomena going on in the midst of a hot plasma is the difficulty of performing measurements. If he were studying, say, the solid state properties of quartz, he would first order some quartz from a chemical supplier. He would then devise certain experimental tests and bring to bear in the course of the experiments a number of well-understood techniques, such as optical and electrical measurements or even acoustical methods. Moving the piece of quartz from one experimental setup to another, he could assemble data from which he could obtain a precise picture of the properties of the material quartz.

Contrast this situation with that of the physicist engaged in controlled fusion research. Firstly, he will find it rather difficult to order a liter or so of hot plasma from his supplier. The material he is to study must be manufactured in the course of the experiment. Secondly, he will find that the nearer he comes to success the fewer will be the number of feasible experimental techniques which may be applied to the problem. The best of these—the detection of neutrons from the fusion process—will properly only occur well along on the trail, and may even be misleading, since neutron producing fusion reactions can arise from entirely spurious processes. These considerations greatly intensify the need for sharpening of the observational and critical senses of the experimental physicist. Space does not permit an extensive discussion of the physics of plasma diagnostic measurements. Some example methods can be described however, which will illustrate the attacks involved. Types of measurements discussed are: (1) optical or spectroscopic, (2) electromagnetic interactions, and (3) nuclear reaction measurements.

Spectroscopic Measurements

The controlled fusion problem is perhaps most nearly a direct descendant from the science of astrophysics. It is therefore natural that astrophysical measurement techniques be applied to study controlled fusion reactions. Until the advent of radio-astronomy, optical measurements were the only means of obtaining astrophysical data. However, in a successful fusion reactor the plasma must be essentially totally ionized, so that light from atomic processes is totally negligible. It would appear that spectroscopic measurements will be of minor use in the later stages of controlled fusion research. However, in the earlier stages of such research, where temperatures are still not up to the fusion point, and total ionization has not yet been achieved, spectroscopic measurements can give much useful information. An idea of the purity and composition of the plasma can be obtained by studying emission spectra. By studying the broadening of individual spectral lines, information on the plasma ion density and its temperature can be obtained. In a plasma the passage of ions near to each other results in localized electric fields which in turn produce a fluctuating Stark broadening[24] and the amount of this broadening is related to the plasma density. Also, motion of an emitting ion relative to the observer will give rise to a doppler shift of the emitted spectral line. If light is being received from a number of such ions and their direction of motion is random, an additional broadening of the line will result. This may be distinguished from that owing to Stark broadening and may be used to determine the ion temperature. Ordered or gross turbulent motions of the plasma may also produce doppler shifts. Thus care must be used in interpreting this kind of spectroscopic data, especially since emitting atoms may comprise a nonrepresentative sample of the plasma.

Electromagnetic Measurements—Microwaves

Measurement of electromagnetic effects produced by the plasma is probably the most important class of plasma diagnostic techniques. Some of these are derived from the older field of gas discharge physics, to which fusion research owes a considerable debt. Especially in recent years electromagnetic measurements of gas discharge phenomena have reached a point of considerable refinement. The brilliant experimental and theoretical work of Allis and Brown and their collaborators at Massachusetts Institute of Technology in using microwave techniques to analyze discharges has an application in fusion research. In the MIT work plasma properties are usually deduced from the detuning of cavity resonators containing the discharge plasma. Such a detuning effect is to be understood most simply in terms of the simple dielectric constant of a plasma defined in Sec. VI, Eq. (25)

$$\begin{aligned} K &= 1-(4\pi ne^2/m\omega^2)=1-(f_p{}^2/f^2) \\ &= 1-8.1\times10^7(n/f^2). \end{aligned} \tag{88}$$

Since the MIT group found it convenient to use cavities operating at a wavelength of 10 cm ($f=3000$ mc) in their work, Eq. (88) shows that most of their measurements had to be restricted to electron densities below about 10^{11} particles per cm^3. Larger size experimental chambers would have required even lower operating densities.

Though it can be seen that because of size and density limitations microwave cavity techniques cannot often be used in controlled fusion research, a related microwave technique can be employed. By using millimeter wave techniques the density at which K becomes negative (and thus $\sqrt{K}$ becomes imaginary) can be increased considerably. Then by constructing a microwave interferometer in which an extended path through the plasma forms one leg of the interferometer, the electron density can be measured and plotted as a function of time or other variables. The state of the millimeter microwave art still does not permit covering the whole spectrum of interesting plasma densities by this method but much useful information can nevertheless be obtained.

[24] J. Holtsmark, Physik. Z. 25, 73 (1924).

Electromagnetic Measurements—Induction Effects and Probes

It has already been noted that a plasma possesses properties which might be described as "diamagnetic" and that its high electrical conductivity may often result in a "freezing-in" of lines of magnetic force within the plasma. For this reason the presence of a confined plasma and its internal electrical currents can profoundly alter the nature of magnetic fields existing near the plasma. Motion of the plasma can then induce voltages in external circuits which can in turn be used to infer something about the motion of the plasma body and possibly its density and temperature. For example, in the pinch effect of Sec. V, if the pinch becomes smaller or larger as a function of time, this will be evidenced by an effect on the external circuit.

Consider a linear pinch discharge as in Fig. 6 of Sec. V. The voltage which will be measured from one end to the other will be the sum of resistive and inductive effects.

$$V=RI+\frac{d}{dt}(LI). \tag{89}$$

Let us suppose that resistive effects are small. Then

$$V=\frac{d}{dt}(LI)=L\frac{dI}{dt}+I\frac{dL}{dt}. \tag{90}$$

V, I, and dI/dt can be measured externally. There remains an equation which can be solved for L as a function of time. But in this case the geometry is known, so that L is a known function of the radius of the pinched discharge (for current flowing on the surface, as in the example). Thus from externally measured voltages and currents the dimensions of a transient pinched discharge can be determined as a function of time. More detailed information can be obtained about the pinch from pickup loops appropriately placed in the field of the pinch currents.

Information about electrostatic fields surrounding the plasma can sometimes be gleaned by the use of electric probes, as in classical gas discharge physics. Here however the situation is not as clean, and the need for minimizing physical contact between the confined plasma and the probe make this technique of limited applicability.

Nuclear Measurements

Measurement of the neutrons resulting from fusion reactions in a plasma would certainly be a satisfying way to diagnose the condition of the plasma. Under the proper circumstances such measurements could give information on the plasma temperature, its absolute density, and its spatial distribution. If the plasma is composed of deuterium and is at a temperature below about 10 kilovolts, the temperature dependence of the reaction rate would make neutron yield a useful thermometer. A clear-cut variation of the neutron yield with the square of the plasma density could strongly suggest that the neutrons were of thermonuclear origin, whereas a linear dependence on density would probably indicate that neutrons were merely being produced by target bombardment. This and other pitfalls await careless interpretation of neutron production in controlled fusion experiments. For example, in some cases, motional or other electric fields could produce acceleration of ions in such a way as to give rise to a neutron yield which is not of thermonuclear origin.

The general problem of plasma diagnostics is one of the most important and challenging aspects of controlled fusion research. Unfortunately it is most difficult in the early stages of the research and will become easier only when the end is in sight.

X. CONCLUDING REMARKS

This article is an attempt to present some of the physical background and practical problems of a research field which is yet in its infancy. The intent has not been to impress readers with the difficulty and complexity of controlled fusion research but rather to assemble in one place some of the important pieces of a jig-saw puzzle yet to be put together. The challenge of the field to the scientific mind should be tremendous, and the goal to be won is of the highest worth. It is the firm belief of many of the physicists actively engaged in controlled fusion research in this country that all of the scientific and technological problems of controlled fusion will be mastered—perhaps in the next few years. Several different and novel approaches to the problem, involving both transient and steady-state methods, are being studied in this country. No doubt similarly promising research is being carried out in other laboratories throughout the world.

Out of the pursuit of this problem a new and fertile field of experimental and theoretical physics is arising. From a thorough understanding of the physics of ultra-high temperature plasmas and their interaction with electromagnetic fields one can hope not only for the achievement of controlled fusion power, but also, as a result of this increased knowledge of nature, there will no doubt arise new and important applications to other fields of science and technology.

That an early success in achieving a self-sustaining controlled fusion reaction would lead to economically competitive fusion power in the near future is highly unlikely. Still, in the fusion reaction are implicit new dimensions—those of power obtained, possibly by direct electrical conversion, from an inexpensive, safe, and virtually inexhaustible fuel. These possibilities will surely someday play a dominant role in shaping the world of the future.