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Electron Polarization, Theory and Experiment

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1. INTRODUCTION

WE may by the term polarization indicate the properties of electron beams resulting from a preferred orientation of the electron spin. These properties are analogous to some extent to the polarization of light. The electron spin was proposed by Goudsmit and Uhlenbeck on the basis of spectroscopic evidence within the framework of the old quantum theory. The nonrelativistic Pauli spin theory, the Dirac wave equation, and the more recent discovery of the anomalous magnetic moment of the electron and its explanation by quantum electrodynamics are the landmarks in the development of the theory of the spinning electron. The bulk of the evidence confirming the theory has been somewhat indirect, e.g., evidence concerning bound electrons. However, the existence of polarization properties for free electron beams is perhaps the consequence of the electron spin in which it shows up in the most direct way.

In this review paper, we shall discuss the formal description of electron polarization, the different ways of producing and detecting electron polarization, and experiments with polarized electrons. The topics which are of present interest will be dealt with most extensively. An earlier review of the subject was given by Rosenfeld (S 43)* (to which the author feels much indebted). Of other summarizing treatments given by Mott and Massey (S 49) and by Sommerfeld (S 39) the former deals extensively with polarization by Coulomb scattering at heavy nuclei, to which we refer therefore for a more extensive representation of the theory of this subject, than is given here in Sec. 3.

* See table of references at end of article.

2. FORMAL DESCRIPTION OF ELECTRON POLARIZATION; COMPARISON WITH PHOTON POLARIZATION; INFLUENCE OF SLOWLY VARYING ELECTROMAGNETIC FIELDS ON ELECTRON POLARIZATION

In quantum mechanics the description of the polarization of beams of electrons and photons is analogous to an appreciable extent (P 27, R 39, P 49, P 51a, P 51b, P 53h, P 54b, P 54c, R 55a). As we are concerned only with the polarization aspect of the waves, we shall compare plane waves of quanta with the same momentum **p** only. A definite state of polarization given by a wave function ψ may be written as

$$\psi=c_1\psi_1+c_2\psi_2, \tag{2.1}$$

in which ψ_1 and ψ_2 are two orthogonal wave functions, e.g., I for the case of *electrons* (a) two opposite spin orientations perpendicular to the momentum **p**, or (b) two opposite spin directions parallel and antiparallel to **p**; II for the case of *photons* (a) two plane polarized waves with perpendicular polarization planes (b) two right and left circularly polarized waves.

A wave function ψ of the form (2.1) characterizes a totally polarized beam. If we have a certain ideal polarization detector (e.g., counting photons with linear polarization in a certain plane only) the detector may be characterized by the wave function of the quanta to which it responds fully

$$\psi^{\text{det}}=c_1^{\text{det}}\psi_1+c_2^{\text{det}}\psi_2. \tag{2.2}$$

The probability of a response of the detector to a quantum given by (2.1) is

$$W=|\langle\psi^{\text{det}}|\psi\rangle|^2=|c_1^{\text{det}*}c_1+c_2^{\text{det}*}c_2|^2. \tag{2.3}$$

A partially polarized beam should not be represented by a single wave function, but by an "ensemble" of pure states, each characterized by one wave function. The quantum-mechanical description of such an "ensemble" is a *density matrix* (or *statistical operator*) ρ (see, e.g., R 38, R 39), which is a 2×2 matrix in our case of two fundamental states

$$\|\rho_{rs}\|=\begin{Vmatrix}\rho_{11} & \rho_{12}\\ \rho_{21} & \rho_{22}\end{Vmatrix}. \tag{2.4}$$

Here ρ is Hermitian, has eigenvalues positive or zero, and is normalized in such a way that

$$\rho_{11}+\rho_{22}=1. \tag{2.5}$$

The special case of a *totally polarized beam*, characterized by the wave function ψ (2.1) corresponds to a density matrix

$$\rho = \begin{Vmatrix} |c_1|^2 & c_1c_2^* \\ c_1^*c_2 & |c_2|^2 \end{Vmatrix}. \tag{2.6}$$

Similarly the detector given by ψ^{det} according to (2.2) can be characterized by a density matrix ρ^{det}. The probability W (2.3) of a response may now be expressed as the trace of the product of both density matrices

$$W = \mathrm{Tr}[\rho \quad \rho^{\text{det}}]. \tag{2.7}$$

The equality of (2.3) and (2.7) is easily verified. Generally the expectation value of an operator α for the state ρ is given by

$$\langle\alpha\rangle = \mathrm{Tr}[\alpha\rho]. \tag{2.8}$$

The density matrix (2.6) of a *pure state* can always be brought into the simple form

$$\rho = \begin{Vmatrix} 1 & 0 \\ 0 & 0 \end{Vmatrix}, \tag{2.9}$$

by a unitary transformation. However, the use of the density matrix is only essential, if we have a quantum-mechanical "mixture," representing an "ensemble" of states. It is always possible to diagonalize ρ, but generally one obtains (one can always choose $\rho' \geqslant \rho'' \geqslant 0$)

$$\rho = \begin{Vmatrix} \rho' & 0 \\ 0 & \rho'' \end{Vmatrix}, \tag{2.10}$$

instead of (2.9). While one may consider a *pure state* $\psi = c_1\psi_1 + c_2\psi_2$ as a "*coherent*" superposition of ψ_1 and ψ_2, a density matrix, specifying a quantum-mechanical *mixture*

$$\rho = \begin{Vmatrix} \rho' & 0 \\ 0 & \rho'' \end{Vmatrix} = \rho'' \begin{Vmatrix} 1 & 0 \\ 0 & 1 \end{Vmatrix} + (\rho' - \rho'') \begin{Vmatrix} 1 & 0 \\ 0 & 0 \end{Vmatrix}$$
$$= (1-P) \begin{Vmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{Vmatrix} + P \begin{Vmatrix} 1 & 0 \\ 0 & 0 \end{Vmatrix} \tag{2.11}$$

$(P = \rho' - \rho'')$ can be considered as the "*incoherent*" superposition of the unpolarized beam

$$\begin{Vmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{Vmatrix}$$

and the totally polarized beam

$$\begin{Vmatrix} 1 & 0 \\ 0 & 0 \end{Vmatrix}$$

with weights $(1-P)$ and P, respectively. We may call $P(0 \leqslant P \leqslant 1)$ the *degree of polarization*. A general density matrix is characterized by 3 independent parameters, e.g., P and the complex ratio c_2/c_1 specifying the polarization state of the totally polarized component, admixed to the unpolarized component. As a physical beam contains many quanta, a fourth parameter for the *total intensity* should be added for characterizing such a beam, as hitherto ψ as well as ρ represented the state of one quantum only. We may say that we have a *polarization measurement* of a physical beam with respect to a certain orthogonal basis (ψ_1, ψ_2) if we measure the intensities I_1 and I_2 for detector settings $\psi^{\text{det}} = \psi_1$ and $\psi^{\text{det}} = \psi_2$. We may express the result of a measurement by

$$I_1/I_2 = \rho_{11}/\rho_{22}, \tag{2.12}$$

so that

$$P(\psi_1,\psi_2) \equiv \rho_{11} - \rho_{22} = (I_1 - I_2)/(I_1 + I_2). \tag{2.13}$$

In order to specify the polarization completely, the 3 relevant parameters should be determined, e.g., by polarization measurements with respect to 3 linearly independent bases. For these bases we could take for photons:

(a) two states of linear polarization with perpendicular planes of polarization, (b) two other states of linear polarization making angles of $\pi/4$ with the planes of polarization in (a), (c) the two states of left and right circularly polarized light. (A)

A different way to characterize the state of polarization is by giving the expectation values of the Pauli σ matrices. We shall call them ζ_1, ζ_2, ζ_3. For a pure state we may write

$$\rho = \begin{Vmatrix} c_1c_1^* & c_1c_2^* \\ c_2c_1^* & c_2c_2^* \end{Vmatrix} = \tfrac{1}{2}(1 + \boldsymbol{\zeta}\cdot\boldsymbol{\sigma}), \tag{2.14}$$

with

$$\zeta_1 = c_1c_2^* + c_2c_1^*, \quad \zeta_2 = i(c_1c_2^* - c_2c_1^*),$$
$$\zeta_3 = |c_1|^2 - |c_2|^2. \tag{2.15}$$

To a 2-dimensional unitary transformation of the complex pair of numbers (c_1, c_2) corresponds a real 3-dimensional orthogonal transformation in the space of the $\boldsymbol{\zeta}$ vectors. For pure states $\boldsymbol{\zeta}$ is a unit vector, for mixtures $|\boldsymbol{\zeta}| < 1$, and it is easily shown that

$$P = |\boldsymbol{\zeta}|. \tag{2.16}$$

If the density matrix characterizing the polarization detector is also written in the form (2.14),

$$\rho^{\text{det}} = \tfrac{1}{2}(1 + \boldsymbol{\zeta}^{\text{det}}\cdot\boldsymbol{\sigma}), \tag{2.17}$$

the probability for detector response becomes

$$W = \mathrm{Tr}[\rho \quad \rho^{\text{det}}] = \tfrac{1}{2}(1 + \boldsymbol{\zeta}\cdot\boldsymbol{\zeta}^{\text{det}}), \tag{2.18}$$

which is valid for pure as well as for mixed states (note that $\mathrm{Tr}[\sigma_1] = \mathrm{Tr}[\sigma_2] = \mathrm{Tr}[\sigma_3] = 0$; $\mathrm{Tr}[1] = 2$). Orthogonal (pure) states are characterized by unit $\boldsymbol{\zeta}$ vectors of opposite direction. Up to this point, the formal developments are equally valid for electrons and photons. We shall now specify, and come to the point where distinctions should be made for both cases.

A. Specification to Electrons

For the case of electrons, it is of course well known that $\boldsymbol{\zeta}$ represents the direction of the electron spin (i.e., the spin angular momentum) in the nonrelativistic Pauli spin theory. Often $\boldsymbol{\zeta}$ is specified by the angles (χ,ω) in polar coordinates, so that

$$\zeta_1=\sin\chi\cos\omega,\quad \zeta_2=\sin\chi\sin\omega,\quad \zeta_3=\cos\chi, \tag{2.19}$$

so that

$$c_2/c_1=[\mathrm{tg}(\chi/2)]\exp(i\omega). \tag{2.20}$$

In the relativistic Dirac electron theory, we have 4-component wave functions instead of the 2-component wave functions of the Pauli spin theory. However, a definite polarization state can still be specified by (2.1), where ψ_1 and ψ_2 are two mutually orthogonal, positive energy solutions. Up to constant factors the (space components of the) spin angular momentum is (are) given by $\psi^*\boldsymbol{\sigma}\psi$, the magnetic moment by $\psi^*(-\beta\boldsymbol{\sigma})\psi$ (both measured in the laboratory system). The direction of $\boldsymbol{\zeta}$ according to (2.15) is given by $\psi^*[\frac{1}{2}(1-\beta)\boldsymbol{\sigma}]\psi$, which is not a part of a relativistic covariant. However, we shall continue to use the vector $\boldsymbol{\zeta}$ derived in this way in the Dirac theory. $\boldsymbol{\zeta}$ can be considered as the spin direction, if the electron is transformed to rest. The use of $\boldsymbol{\zeta}$ is convenient in the following respects: (a) $\boldsymbol{\zeta}$ can be calculated from the large components ψ_3 and ψ_4 of the Dirac wave function only; (b) the direction of $\boldsymbol{\zeta}$ remains unchanged if the electron is accelerated by an electric field (see later); (c) an unpolarized beam can be considered as an ensemble of electrons with spin $\boldsymbol{\zeta}$ pointing isotropically in all directions.

If we had taken the direction of the spin angular momentum or the magnetic moment in the laboratory system, (a), (b), and (c) would no longer be valid. We shall speak of *transverse polarization* of an electron beam if the direction of the spin is perpendicular to the momentum, of *longitudinal polarization* if the spin is parallel or antiparallel to the momentum. As 3 independent bases, for which we may determine $P(\psi_1,\psi_2)$ [compare (2.13)] in order to determine the state of polarization completely, we can choose [compare the similar situation for photons mentioned in the foregoing as (A)]

(a) two states of transverse polarization with opposite spin direction, (b) two other states of transverse polarization turned over an angle of $\pi/2$ relative to (a), (c) the two opposite states of longitudinal polarization (parallel and antiparallel). (B)

As we have just seen, the polarization of electron beams in Dirac theory may be expressed by 2×2 matrices, if one considers only positive energy states. However, one needs also a 4×4 representation in certain calculations of polarization effects by means of the Dirac equation and perturbation theory. This representation is defined in the following way. If

$$\psi=u\exp[(i/\hbar)\mathbf{p}\cdot\mathbf{x}], \tag{2.21}$$

represents a wave with polarization $\boldsymbol{\zeta}$ (normalized to one particle per unit volume), then we put ($\lambda,\mu=1,\cdots,4$ indicate the Dirac components; $(+)$ indicates that we deal with positive energy solutions)

$$P_{\lambda\mu}{}^{(+)}(\boldsymbol{\zeta})=u_\lambda u_\mu^*. \tag{2.22}$$

The explicit expression of $P^{(+)}(\boldsymbol{\zeta})$ may be written in the following way with the aid of the ρ and σ Dirac matrices (P 51d, P 53i, P 54c):

$$\begin{aligned}P^{(+)}(\boldsymbol{\zeta})=\tfrac{1}{4}[1&-(c\mathbf{p}/E)\cdot\boldsymbol{\zeta}\rho_1-(mc^2/E)\rho_3+\mathbf{K}\cdot\boldsymbol{\sigma}\\&-(c\mathbf{p}/E)\cdot\rho_1\boldsymbol{\sigma}+(c\mathbf{p}\times\boldsymbol{\zeta}/E)\cdot\rho_2\boldsymbol{\sigma}+\mathbf{J}\cdot\rho_3\boldsymbol{\sigma}],\end{aligned} \tag{2.23}$$

with

$$\begin{aligned}\mathbf{K}&=(mc^2/E)\boldsymbol{\zeta}+c^2\mathbf{p}(\mathbf{p}\cdot\boldsymbol{\zeta})/E(E+mc^2),\\ \mathbf{J}&=-\boldsymbol{\zeta}+c^2\mathbf{p}(\mathbf{p}\cdot\boldsymbol{\zeta})/E(E+mc^2).\end{aligned} \tag{2.24}$$

$E=(p^2c^2+m^2c^4)^{\frac{1}{2}}$ is the energy of the electron, m its rest mass. $P^{(+)}(\boldsymbol{\zeta})$ may also be written in a way which is clearly relativistically covariant.† For this we refer to the Appendix.

B. Specification to Photons

The developments of Eqs. (2.1)$\cdots$(2.18) are valid for photons as well as electrons. For photons the wave function ψ can be considered as a complex vector potential. In order to make a distinction from the case of electrons, we may replace (c_1,c_2) by (a_1,a_2), $\boldsymbol{\zeta}$ by $\boldsymbol{\xi}$, $\boldsymbol{\sigma}$ by $\boldsymbol{\omega}$. Hence, we may write (2.1) for photons in the form

$$\mathbf{A}\exp(i\mathbf{k}\cdot\mathbf{x})=a_1\mathbf{A}_1\exp(i\mathbf{k}\cdot\mathbf{x})+a_2\mathbf{A}_2\exp(i\mathbf{k}\cdot\mathbf{x}).$$

In order to agree with some optical conventions, we put

$$\omega_1=\begin{pmatrix}1&0\\0&-1\end{pmatrix},\quad \omega_2=\begin{pmatrix}0&1\\1&0\end{pmatrix},\quad \omega_3=\begin{pmatrix}0&-i\\i&0\end{pmatrix}, \tag{2.25}$$

so that, as for σ_1, σ_2, σ_3,

$$\omega_k{}^2=1,\quad \omega_1\omega_2=i\omega_3,\quad \text{etc.} \tag{2.26}$$

It follows that for a pure state

$$\rho=\left\|\begin{matrix}a_1a_1^* & a_1a_2^*\\ a_2a_1^* & a_2a_2^*\end{matrix}\right\|=\tfrac{1}{2}(1+\boldsymbol{\xi}\cdot\boldsymbol{\omega}), \tag{2.27}$$

with

$$\begin{aligned}\xi_1=|a_1|^2-|a_2|^2,\quad \xi_2=a_1a_2^*+a_2a_1^*,\\ \xi_3=i(a_1a_2^*-a_2a_1^*).\end{aligned} \tag{2.28}$$

We may represent the real vector $\boldsymbol{\xi}$ by means of two angles α and β

$$\left.\begin{aligned}\xi_1&=\cos2\beta\cos2\alpha,\\ \xi_2&=\cos2\beta\sin2\alpha,\\ \xi_3&=\sin2\beta.\end{aligned}\right\} \tag{2.29}$$

† We denote the scalar product of $\mathbf{a}$ and $\mathbf{b}$ by $\mathbf{a}\cdot\mathbf{b}$; the vector product by $\mathbf{a}\times\mathbf{b}$.

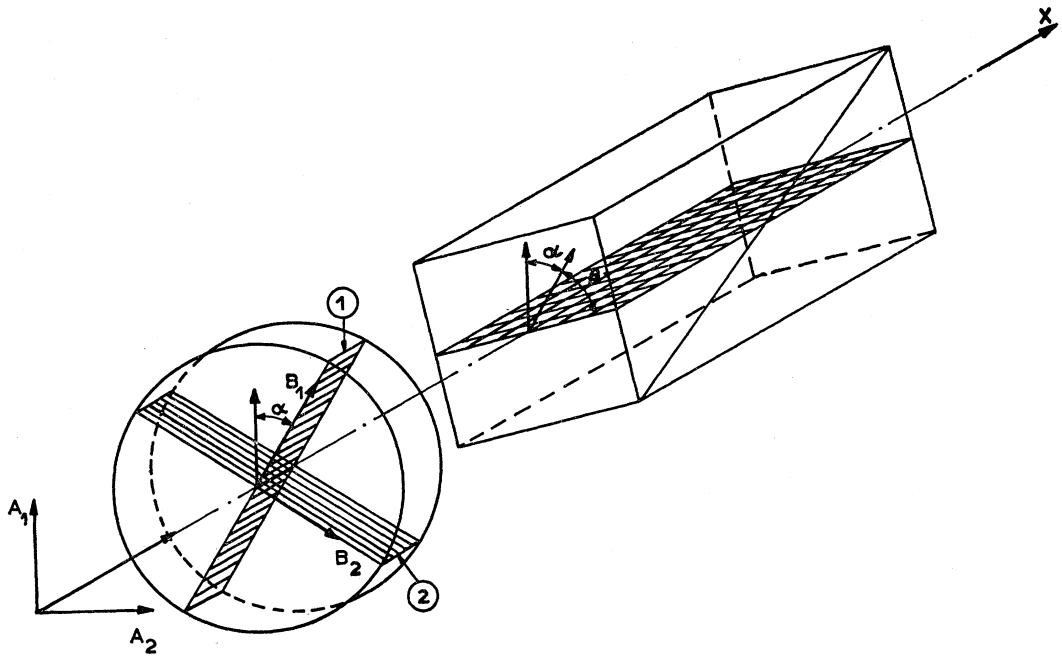

FIG. 1. Diagram of a complete analyzer for polarized light, consisting of a $\lambda/4$ plate and a Nicol prism. Light propagates "slow" through the $\lambda/4$ plate when the electric vector is in plane (1), "fast" when in plane (2); it is admitted by the Nicol when in cross-hatched plane. The state of polarization may be characterized by the angles α and β for the analyzer setting for optimum transmission and by the intensities transmitted for maximum and minimum transmission. (Figure from P 49.)

These angles are the angles which determine the setting of an ideal analyzer (consisting of a $\lambda/4$ plate and a Nicol prism) for light (see Fig. 1). This description of the polarization of photons goes back to Stokes (see P 1852 and P 49). The positive and negative unit vectors in the 1, 2, 3 directions may designate the polarization states (a), (b), (c) mentioned under (A). It should be stressed that the electron spin $\boldsymbol{\zeta}$ is a direction in physical space; the 3-dimensional vector $\boldsymbol{\xi}$ determines a "polarization space," which differs from ordinary physical space. The meaning of the directions of the coordinate axes in $\boldsymbol{\xi}$ space is not fixed by the choice of the coordinate system in physical space, but requires an additional fixation, for example, a choice of a certain plane (e.g., a plane of scattering in some process), such that $\mathbf{A}_1$ determines linear polarization in that plane. This has as a consequence that invariant formulas for physical processes may contain $\boldsymbol{\zeta}$ in invariant combinations with other vectors in physical space such as the momentum $\mathbf{p}$, giving, for example, $\mathbf{p}\cdot\boldsymbol{\zeta}$ or $\mathbf{p}\times\boldsymbol{\zeta}$, while the components of $\boldsymbol{\xi}$ occur separately (see, e.g., the formulas for Compton scattering in Sec. 4).

C. Expression of Transition Probabilities with the Aid of Density Matrices (P 51d, P 54b, P 54c)

We may make a few remarks concerning the expression of transition probabilities with the aid of density matrices (here again we do not specify to photons or electrons) for calculations including polarization effects. If a system is prepared at time $t=0$ in a state ρ_0 it will have developed in a time t to a state $\rho(t)$ (if the Hamiltonian is H)

$$\rho(t)=\exp[-iHt/\hbar]\rho_0\exp[iHt/\hbar]. \qquad (2.30)$$

The probability for having a response from a detector ρ^{det} at time t is now given by

$$\begin{aligned}P(0\rightarrow\text{det})&=\text{Tr}[\rho^{\text{det}}\rho(t)]\\&=\text{Tr}[\rho^{\text{det}}\exp(-iHt/\hbar)\rho_0\exp(iHt/\hbar)]. \qquad (2.31)\end{aligned}$$

In scattering experiments, we are interested in transition rates $R(0\rightarrow\text{det})=(1/t)P(0\rightarrow\text{det})$ (for small times t) determining cross sections. For transition rates, it is supposed that ρ^{det} is a kind of partial density matrix referring to the detection of scattered states, but not of unscattered states. The usual formula of perturbation theory may be expressed in density matrix notation as

$$R(0\rightarrow\text{det})=(2\pi/\hbar)d_f(E)\ \text{Tr}[\rho^{\text{det}}\Omega\rho_0\Omega^\dagger]. \qquad (2.32)$$

Here $d_f(E)$ is the density of final states; Ω is the perturbation part of the Hamiltonian (in first-order perturbation theory). Note the symmetry between ρ_0 and ρ^{det} in the formulas.

In calculating polarization effects in specific physical processes involving photons and electrons according to (2.32), ρ_0 and ρ^{det} are given in their polarization dependence by operators of the form (2.23) and (2.27), and for the result, traces have to be taken over Dirac ρ and σ matrices for electrons, and ω matrices for photons. A convenient feature of expressing the polarization dependence of a transition probability by $\boldsymbol{\zeta}$ and $\boldsymbol{\xi}$ vectors is that, if one has to average a result for some unobserved polarization, it is done simply by dropping the term in the $\boldsymbol{\zeta}$ or $\boldsymbol{\xi}$ of the relevant quantum; the result is linear in all $\boldsymbol{\zeta}$'s and $\boldsymbol{\xi}$'s (while it is quadratic when expressed in terms of (c_1,c_2)'s), and orthogonal states are determined by opposite $\boldsymbol{\zeta}$ or $\boldsymbol{\xi}$ directions. It is done less simply when using the (c_1,c_2) representation for polarization. However, in a result for photons the (a_1a_2) have the advantage over $\boldsymbol{\xi}$ in that they determine a direction in physical space, which may occur in simple invariant combinations with other physical vectors.

D. The Influence of Slowly Varying Electric and Magnetic Fields on Polarized Electron Beams

This influence on electron beams consists, in general, both of a change in the momentum vector $\mathbf{p}$ and in the spin direction $\boldsymbol{\zeta}$ (P 51c, P 28c, P 29b, P 29g, P 30b, P 35b, S 49, P 55b). We may give a quantitative quantum-mechanical derivation of these influences by looking, with an external electromagnetic field, for solutions of the Dirac equation, which become plane waves if the electric charge $e\rightarrow 0$. It is sufficient to consider only small deviations from plane waves (to first order in e), for in this way we obtain a differential law which determines entirely the "continuous refraction" and change of polarization in an electromagnetic field. We may give the results for the Dirac electron in the following way. We consider a plane wave progressing in the x direction. We consider separately transverse and longitudinal (i.e., perpendicular or parallel to $\mathbf{p}$) electric fields ($\mathfrak{E}$) and magnetic fields ($\mathfrak{B}$). We give the resulting change in kinetic momentum $\Delta\boldsymbol{\pi}$ (we have to distinguish between the kinetic momentum $\boldsymbol{\pi}$ and the total momentum $\mathbf{p}$—related by $\boldsymbol{\pi}=\mathbf{p}-(e/c)\mathbf{A}$—in case there is a vector potential), the angle $\Delta\gamma$ over which the beam is deflected, and the angle $\Delta\alpha$ over which $\boldsymbol{\zeta}$ is rotated, if we proceed over a distance x in the x direction.

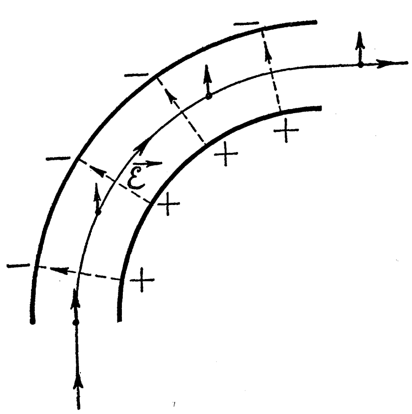

FIG. 2. Deflection of an electron beam in an electric field; nonrelativistic approximation. Dotted lines: electric field lines; short arrows: spin orientation. The polarization changes from longitudinal to transverse.

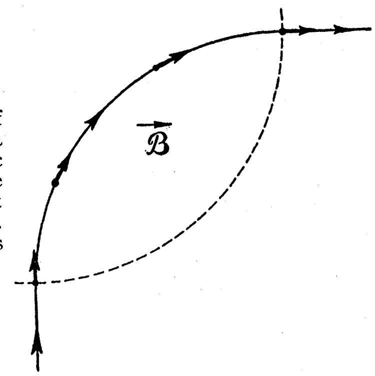

FIG. 3. Deflection of an electron beam in a magnetic field. Magnetic field perpendicular to the plane of the figure; short arrows: spin orientation. The polarization remains longitudinal.

(*a*) *Transverse Electric Field* $\mathfrak{E}$ *in the y Direction*

$$\Delta\pi_x=0, \qquad \Delta\pi_z=0,$$

$$\Delta\pi_y=(eE\mathfrak{E}/pc^2)x, \quad \Delta\gamma(\mathfrak{E})=(eE\mathfrak{E}/p^2c^2)x, \tag{2.33}$$

$$\Delta\alpha(\mathfrak{E}_\perp)=[e\mathfrak{E}/(E+mc^2)]x, \quad \text{rotation about the } z \text{ axis.} \tag{2.34}$$

It follows that

$$\Delta\alpha(\mathfrak{E}_\perp)/\Delta\gamma(\mathfrak{E})=E_{\mathrm{kin}}/E \quad \text{with} \quad E_{\mathrm{kin}}=E-mc^2. \tag{2.35}$$

Hence we see that in the nonrelativistic limit ($\Delta\alpha(\mathfrak{E}_\perp)/\Delta\gamma(\mathfrak{E})\rightarrow\frac{1}{2}(v/c)^2$ for $v\rightarrow 0$) the beam is deflected, and the spin direction remains the same. This is represented in Fig. 2. Hence we see that by deflecting a low-energy electron beam of longitudinal polarization over an angle of $\pi/2$, the polarization is transformed into transverse polarization. This could be compared with the transformation of circular polarization of light into linear polarization by means of a $\lambda/4$-plate. For relativistic energies of the electrons, a transformation of longitudinal into transverse polarization can also be achieved by a transverse electric field, but the deflection angle should then be $(\pi/2)/(1-E_{\mathrm{kin}}/E)$.

(*b*) *Longitudinal Electric Field* $\mathfrak{E}$

$$\Delta\pi_y=0, \quad \Delta\pi_z=0, \quad \Delta\pi_x=(eE\mathfrak{E}/pc^2)x, \tag{2.36}$$

$$\Delta\alpha(\mathfrak{E}_{\parallel})=0. \tag{2.37}$$

Hence, it follows that acceleration (or deceleration) by a longitudinal electric field leaves the electron polarization (direction of $\boldsymbol{\zeta}$) unchanged.

(*c*) *Transverse Magnetic Field* $\mathfrak{B}$ *in the z Direction*

$$\Delta\pi_x=0, \qquad \Delta\pi_z=0,$$

$$\Delta\pi_y=-(e\mathfrak{B}/c)x, \quad \Delta\gamma(\mathfrak{B})=-(e\mathfrak{B}/pc)x, \tag{2.38}$$

$$\Delta\alpha(\mathfrak{B}_\perp)=-(e\mathfrak{B}/pc)x, \text{ rotation about the } z \text{ axis,} \tag{2.39}$$

$$\Delta\alpha(\mathfrak{B}_\perp)/\Delta\gamma(\mathfrak{B})=1. \tag{2.40}$$

Hence, we see that a transverse magnetic field rotates the direction of the beam at the same rate as the electron spin (see Fig. 3), so that a magnetic field leaves, for example, a state of transverse polarization unchanged. This is valid for relativistic as well as unrelativistic energies.

(*d*) *Longitudinal Magnetic Field* $\mathfrak{B}$

$$\Delta\pi_x=0, \quad \Delta\pi_y=0, \quad \Delta\pi_z=0, \tag{2.41}$$

$$\Delta\alpha(\mathfrak{B}_{\text{II}})=-(e\mathfrak{B}/pc)x, \text{ rotation about the } x \text{ axis.} \tag{2.42}$$

The direction and magnitude of the momentum are not changed, but the electron spin precesses about the x axis. This is analogous to the rotation of the plane of polarization of light by a quartz plate (or solution of sugar).

The rotations of the electron spin mentioned in (*a*), (*b*), (*c*) were calculated directly from the Dirac equation. For the real electron, the g value is not quite equal to 2, and we have to multiply the precession rate of the spin (which is a consequence of its magnetic moment) by $g/2=1+\alpha/2\pi+\cdots$ (according to quantum electrodynamics). As a consequence, the spin precession in a magnetic field is slightly faster than the deflection of the orbital motion, so that a change from transverse polarization can occur, although only for about $\frac{1}{4}[g/2-1]^{-1}\approx 250$ "cyclotron" revolutions of the electron trajectory. The mathematical description of the motion of the electron with anomalous magnetic moment was considered in detail by Mendlowitz and Case (P 55b). For this purpose, an appropriate description may be achieved by using the Foldy-Wouthuysen transformation (see also P 54a).

The fact that the influence of (nearly homogeneous) electric and magnetic fields on polarized electron beams can simply be represented by a rotation of the spin vector $\boldsymbol{\zeta}$ about a certain axis has the following immediate consequence. If an unpolarized beam, which can be considered as an "ensemble" of electrons with $\boldsymbol{\zeta}$ pointing isotropically in all directions passes through such an electric or magnetic field, the distribution of $\boldsymbol{\zeta}$ directions remains isotropic; hence, the beam stays unpolarized.

One may think of obtaining polarized electrons by means of a kind of *Stern-Gerlach experiment* sending electrons through a strongly inhomogeneous magnetic field (although still varying on a macroscopic scale). However, in a well-known argument going back to Bohr and Mott (see T 29 and S 49 for a more precise quantitative discussion of this argument), it is shown that a splitting of an electron beam according to spin orientation cannot be attained in this way: the inhomogeneity of the magnetic field causes a spreading of the charged electron beam (the particles of the Stern-Gerlach experiment are electrically neutral), which is so large that the spreading arising from the different orientations of the magnetic moment in the inhomogeneous magnetic field is not detectable.

In analogy with the polarization of light by reflection at a mirror, one may think further of the possibility of obtaining polarized electron by reflection of a beam by a sudden change of potential (*Malus effect*). A discussion of this possibility in a number of theoretical papers led to a simple proof that it is also impossible to obtain electron polarization (P 29a, P 29e, P 30b, P 30d, P 31b, P 33b, P 35b, P 35c) in this way (this proof is reproduced in S 39).

Thus it appears, generally speaking, that all the foregoing proposals in which the electromagnetic fields are varying on a macroscopic scale are unable to produce electron polarization. However, in the next section it is extensively discussed how electron polarization can be achieved by an electric field varying on an atomic scale.

There is not necessarily a sharp distinction between a variation of the fields on a macroscopic and on a microscopic scale. A combination of very strong magnetic fields (of the order 10^4 gauss) and very weak electric fields (of the order 10^{-4} v) provides a kind of transition case where fields varying on a macroscopic scale, but small enough (10^{-4} v) to distinguish between discrete quantum states, may produce electron polarization according to proposals by Bloch and Dicke (see Sec. 7 for the further discussion).

3. POLARIZATION OF ELECTRONS BY COULOMB SCATTERING AT NUCLEI; THEORY AND EXPERIMENT

In the end of the last section, the impossibility of obtaining polarized electrons with various arrangements using electric or magnetic fields varying on a macroscopic scale, was discussed. It was first shown by Mott (T 29, T 32) that the Coulomb field of a nucleus may cause electron polarization if an electron is scattered in this field. Here we deal with a *strongly inhomogeneous* electric field varying *on a microscopic scale*. The physical cause underlying the polarization effect is that the scattering for a certain angle is affected by the spin orbit coupling, caused by the interaction of the magnetic moment of the electron with the magnetic field which a moving electron experiences in an electric field. The effect is calculated quantitatively by considering the solutions representing the scattering of an electron wave, according to the Dirac equation with a spherically symmetric electric potential. This potential is either taken to be the undisturbed Coulomb field of the nucleus, or this field corrected for the screening by the atomic electrons. The relevant solutions of this equation may be written, as to their asymptotic behavior, in the form (if the incident waves are propagating in the positive z direction)

$$\psi_\lambda=b_\lambda e^{ikz}+r^{-1}e^{ikr}u_\lambda(\vartheta,\varphi) \tag{3.1}$$

($\lambda=1, 2, 3, 4$ is the Dirac index. The asymptotic behavior for a pure Coulomb field differs somewhat from (3.1) in its radial dependence, see (S 49), but this is not essential for the following). If we put for convenience, $b_3=A$ and $b_4=B$, it may be generally shown that for a spherically symmetric potential u_3 and u_4 have the form

$$\left.\begin{aligned} u_3(\vartheta,\varphi)&=Af(\vartheta)-Bg(\vartheta)\exp(-i\varphi),\\ u_4(\vartheta,\varphi)&=Bf(\vartheta)+Ag(\vartheta)\exp(i\varphi).\end{aligned}\right\} \tag{3.2}$$

The functions $f(\vartheta)$ and $g(\vartheta)$ have to be determined from the detailed solution of the Dirac equation. It is important to note that (3.2) characterizes the scattering completely: the formula contains the directional dependence as well as the polarization dependence. We may, for instance, calculate the total intensity $I(\vartheta,\varphi)$ scattered in a direction (ϑ,φ) if the initial polarization is given by (A,B)

$$I(\vartheta,\varphi)=(|u_3|^2+|u_4|^2)/(|A|^2+|B|^2). \quad (3.3)$$

However, the result obtained in this way does not express the complete directional and polarization dependence of the scattering. We then have to include the initial polarization of the beam, as well as the polarization, which is observed after the scattering. The differential cross section including the dependence on both polarizations, may be expressed as $I(\mathbf{p}_1, \mathbf{p}_2, \boldsymbol{\zeta}_1, \boldsymbol{\zeta}_2)$, where $\mathbf{p}_1$ and $\mathbf{p}_2$ are the initial and final electron momenta, $\boldsymbol{\zeta}_1$ and $\boldsymbol{\zeta}_2$, the initial and final electron polarizations (we call further $\mathbf{n}_1$ and $\mathbf{n}_2$ the unit vectors in the direction of $\mathbf{p}_1$ and $\mathbf{p}_2$, so that $\mathbf{n}_1\cdot\mathbf{n}_2=\cos\vartheta$). We call the density matrices for the initial state and for the polarization sensitive detector of the scattered electron ρ_1 and ρ_2 respectively

$$\rho_1=\tfrac{1}{2}(1+\boldsymbol{\zeta}_1\cdot\boldsymbol{\sigma}), \quad (3.4)$$

$$\rho_2=\tfrac{1}{2}(1+\boldsymbol{\zeta}_2\cdot\boldsymbol{\sigma}). \quad (3.5)$$

The relation between (A,B) and (u_3,u_4) given by (3.2) may be expressed by means of an operator

$$\Omega=f(\vartheta)+ig(\vartheta)[\sigma_x\sin\varphi-\sigma_y\cos\varphi], \quad (3.6)$$

with the aid of which the polarization dependent cross section can be expressed in a form analogous to (2.32)

$$I(\mathbf{p}_1, \mathbf{p}_2, \boldsymbol{\zeta}_1, \boldsymbol{\zeta}_2)=\mathrm{Tr}[\rho_2\Omega\rho_1\Omega^\dagger]. \quad (3.7)$$

The result obtained by carrying out the traces over the σ matrices may be written in an invariant form, without any reference to a particular coordinate system

$$\begin{aligned}I(\mathbf{p}_1,&\mathbf{p}_2,\boldsymbol{\zeta}_1,\boldsymbol{\zeta}_2)\\ &=\tfrac{1}{2}\bar{I}(\vartheta)[1+\boldsymbol{\zeta}_1\cdot\boldsymbol{\zeta}_2]+\tfrac{1}{2}[D(\vartheta)/\sin\vartheta]\\ &\quad\times[\boldsymbol{\zeta}_1\cdot(\mathbf{n}_1\times\mathbf{n}_2)+\boldsymbol{\zeta}_2\cdot(\mathbf{n}_1\times\mathbf{n}_2)]\\ &\quad+\tfrac{1}{2}[F(\vartheta)/\sin\vartheta][-(\boldsymbol{\zeta}_1\cdot\mathbf{n}_2)(\boldsymbol{\zeta}_2\cdot\mathbf{n}_1)+(\boldsymbol{\zeta}_1\cdot\mathbf{n}_1)(\boldsymbol{\zeta}_2\cdot\mathbf{n}_2)]\\ &\quad+\tfrac{1}{2}[G(\vartheta)/\sin\vartheta]\{(\mathbf{n}_1\cdot\mathbf{n}_2)[(\boldsymbol{\zeta}_1\cdot\mathbf{n}_2)(\boldsymbol{\zeta}_2\cdot\mathbf{n}_1)\\ &\quad+(\boldsymbol{\zeta}_1\cdot\mathbf{n}_1)(\boldsymbol{\zeta}_2\cdot\mathbf{n}_2)]-[(\boldsymbol{\zeta}_1\cdot\mathbf{n}_2)(\boldsymbol{\zeta}_2\cdot\mathbf{n}_2)\\ &\qquad\qquad+(\boldsymbol{\zeta}_1\cdot\mathbf{n}_1)(\boldsymbol{\zeta}_2\cdot\mathbf{n}_1)]\}, \quad (3.8)\end{aligned}$$

with the abbreviations

$$\left.\begin{aligned}\bar{I}(\vartheta)&=|f|^2+|g|^2,\\ D(\vartheta)&=i(fg^*-f^*g),\\ F(\vartheta)&=fg^*+f^*g,\\ G(\vartheta)&=2|g|^2.\end{aligned}\right\} \quad (3.8a)$$

This formula has in its dependence on $\boldsymbol{\zeta}_1$ or $\boldsymbol{\zeta}_2$ a form as given by (2.18), which makes it easy to treat any case of partial polarization. Therefore, the form (3.8) represents the scattering and its polarization dependence in a very explicit form and the further discussion of polarization in single or double scattering can be made using (3.8) only, without coming back to the wave function (3.2). From (3.8) we may draw immediately the following conclusions:

1. If a polarized electron is scattered, which has a spin direction perpendicular to the plane of scattering before the scattering, this spin direction is not changed by the scattering.

2. If an unpolarized beam is scattered, it has a degree of polarization of

$$a(\vartheta)=D(\vartheta)/\bar{I}(\vartheta),$$

after the scattering and the direction of $\boldsymbol{\zeta}$ after the scattering is perpendicular to the plane of scattering.

3. The ratio of the intensities in both directions with a certain scattering angle ϑ for a given scattering plane for an incident beam with degree of polarization P (and $\boldsymbol{\zeta}$ perpendicular to the plane of scattering) is given by

$$[1+Pa(\vartheta)]/[1-Pa(\vartheta)].$$

4. The intensity after double scattering of an unpolarized beam under angles ϑ_1 and ϑ_2 is given by

$$I(\vartheta_1,\vartheta_2,\varphi)=1+\delta\cos\varphi;\quad \delta(\vartheta_1,\vartheta_2)=a(\vartheta_1)a(\vartheta_2) \quad (3.9)$$

(φ is the angle between the first and the second plane of scattering).

5. The determination of the unpolarized scattering and the normal double scattering experiment allow only the determination of two of the four real functions (namely; $\bar{I}(\vartheta)$ and $D(\vartheta)$), which may be formed from the two complex functions $f(\vartheta)$ and $g(\vartheta)$ according to (3.8a). However, it follows from (3.8) that the other two functions $F(\vartheta)$ and $G(\vartheta)$ are by no means unobservable in principle. A measurement might be carried out, e.g., by means of a triple scattering experiment such that one can say that in the second scattering the initial as well as the final polarization, ($\boldsymbol{\zeta}_1$ and $\boldsymbol{\zeta}_2$) are observed.

We have somewhat digressed on the consequences of (3.2) in order to facilitate the vizualization of the situation concerning the electron polarization by Coulomb scattering. (The presentation of the theory given here, makes use of discussions presented in P 51b and P 55b.) For the same purpose, it may be useful to consider the figure concerning the double scattering experiment [Fig. 4; for simplicity we take everywhere the scattering angle $\pi/2$; $a=a(\vartheta=\pi/2)$.] We take the points Q, R, S, T, U, and V in the same plane. The beams are scattered at R and T. The intensities are measured at U and V; the intensities of the different beams are given in Fig. 4; I, I', and I'' are constants. The relative intensities measured in U and V, when starting with an unpolarized beam may be obtained by incoherent superposition of both cases (a) and (b) in

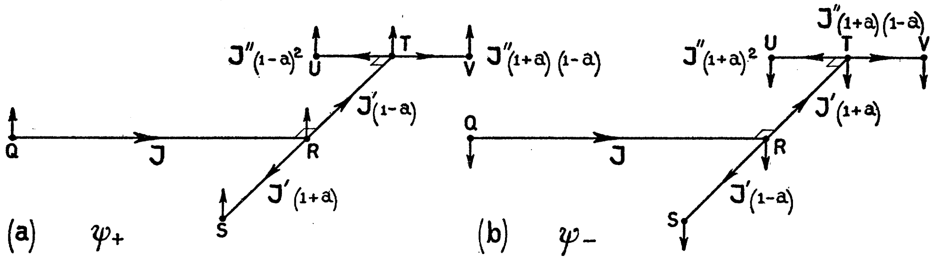

FIG. 4. The intensities are given for double scattering (at right angles) and spin orientations "up" and "down". The beams are scattered at R and T. Intensities for double scattering of an unpolarized beam are obtained by taking the sum so that different intensities result in U and V.

the figure. Thus we obtain

$$\left.\begin{aligned} I_U &= \tfrac{1}{2}I''[(1+a)^2+(1-a)^2]=I''(1+a^2),\\ I_V &= \tfrac{1}{2}I''[(1+a)(1-a)+(1+a)(1-a)]\\ &= I''(1-a^2), \end{aligned}\right\} \quad (3.10)$$

hence,

$$I_U/I_V=(1+a^2)/(1-a^2)=(1+\delta)/(1-\delta)$$

with

$$\delta=a^2, \quad (3.11)$$

being a special case of (3.9). We note that the intensity is larger in U than in V; the double scattering has according to (3.9) a minimum for $\varphi=0$, and a maximum for $\varphi=\pi$. This is in contrast with the double scattering experiment for photons (Barkla's experiment) where we have maxima for $\varphi=0$ and π, and minima for $\varphi=\frac{1}{2}\pi$ and $\frac{3}{2}\pi$.

We will limit ourselves to quote some results and to give references for the theoretical calculations providing numerical values for δ and a. Mott (T 29) calculated as the value for δ for double scattering with $\vartheta_1=\vartheta_2=\pi/2$ at nuclei with charge Ze

$$\delta\approx(\alpha Z)^2\beta^2(1-\beta^2)(2-\beta^2)^{-2} \quad (\beta=v/c). \quad (3.12)$$

This is only valid for $\alpha Z\ll 1$. It shows that $\delta\rightarrow 0$ for $\beta\rightarrow 0$ and $\beta\rightarrow 1$, which remains true for higher Z. Only for higher Z, the values of δ become sufficiently high to be easily observable. Numerical calculations are then required for precise values of δ. Values for δ as a function of v/c for $\vartheta_1=\vartheta_2=\pi/2$ and $Z=79$ (gold) were evaluated in 1932 by Mott (T 32), for $Z=80$ (mercury) in 1940 by Bartlett and Watson (T 39, T 40a), both for the undisturbed Coulomb field. Sauter (T 33) derived Mott's result (T 29) in a somewhat different way, and made an estimate of the influence of the screening by atomic electrons in a first approximation. The influence of screening was evaluated numerically by Bartlett and Welton (T 41a), by Massey and Mohr ($\vartheta_1=\vartheta_2=\pi/2$) (T 41b), by Mohr (T 43b) and by Mohr and Tassie (T 54), while Bartlett and Welton as well as Mohr considered the dependence of δ on the scattering angle.‡ We quote some theoretical results in Table I. It is seen that remarkably high values for the asymmetry for backward scattering angles and energies around 400 kev are predicted. Values for δ for positons were calculated by Massey (T 43a). The polarization effect for positons is much smaller than for negatons to the extent that it must be nearly impossible to establish the polarization effect for positons experimentally. Curves for δ and $a=\sqrt{\delta}$ are given in Figs. 5 and 6 as a function of the electron energy according to the calculations of Bartlett and Watson (T 39, T 40a) and Massey (T 43a). The accuracy of the older theoretical values given in the table and the curves should not be overestimated.

For a long time (until 1942) *experimental results* on double electron scattering failed to show any polarization effect. Careful experiments by Dymond (E 32, E 34a), G. P. Thomson (E 34b), and Richter (E 37) gave negative results. The same is true for earlier experiments (P 28a, P 29c, P 30a, P 32a), or the asymmetries which were obtained in these experiments are now considered to have been of instrumental origin. The difficulty in the experiments is to be sure that one observes real single scattering. The experiments were mainly done with thin gold foils. A polarization which is smaller than for single scattering, may originate in the following ways: (a) *inelastic scattering* with ionization or excitation of the scattering atom; (b) *exchange scattering;* if the scattered electron changes its position with an atomic electron, the polarization will be lost; (c) *multiple scattering;* if the final scattering angle is obtained by a succession of small angle scatterings, the polarization will be negligible as in small angle scattering nearly no polarization effect exists; (d) *plural scattering;* we shall speak of plural scattering if the final

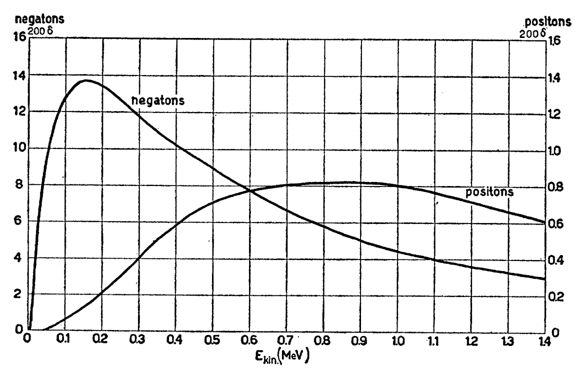

FIG. 5. The asymmetry percentage 200δ in a double scattering experiment as a function of the energy for negatons and positons ($Z=80$) [after the data calculated by Massey (T 43a), Bartlett and Watson (T 40a)]. Scattering angles: 90°.

‡ Extensive calculations of the polarization asymmetries by means of the Univac computer were made by N. Sherman (T 56) for $Z=13$, 48, and 80 and for scattering of electrons at point nuclei.

TABLE I. Theoretical results for the asymmetry (200δ) in a double scattering experiment for a number of scattering angles $\vartheta_1=\vartheta_2=\vartheta$, at different electron energies E_{kin} for scattering by gold ($Z=79$) and mercury ($Z=80$).

Author		E_{kin} \ ϑ	45°	60°	75°	90°	105°	120°	135°	150°	165°
Bartlett and Welton ($Z=80$)	(T 41a)	100 kev			1.8	9.6	19.8	22.6	24.0	17.2	5.2
Mohr and Tassie ($Z=79$)	(T 54)	121 kev	0	1	6	16	25	33	32	25	11
Mohr ($Z=79$)	(T 43b)	392 kev	10	1	0	16	70	90	140	190	10
Sherman ($Z=80$)	(T 56)	128 kev ($\beta=0.6$)	0.0	0.8	5.1	14.7	26.9	36.0	34.9	22.7	7.1
		205 kev ($\beta=0.7$)	0.0	1.0	5.2	14.0	26.5	38.0	41.0	30.0	10.2
		341 kev ($\beta=0.8$)	0.1	1.0	4.5	11.7	23.1	36.8	45.9	39.8	15.8
		661 kev ($\beta=0.9$)	0.1	0.7	2.7	7.2	15.3	27.8	43.1	51.0	28.9

scattering is the result of two scatterings over rather large angles. Examples of the geometry involved in plural scattering are shown in Fig. 7. The contribution by plural scattering to the total scattering is appreciable in particular if the first scattering occurs in the direction of the foil and if the two scattering angles are both smaller than the total scattering angle (as the differential cross section increases rapidly for smaller angles). The influence of the causes (a), (b), and (c) for depolarization were discussed by Rose and Bethe (E 39b). A criterion for the foil thickness that multiple Coulomb scattering is negligible compared with single scattering was given earlier by Wentzel (R 22). It follows that scattering foils of thicknesses of the order 10^{-5} cm, as were used in the above-mentioned experiments, are thin enough for the first three causes of depolarization to be negligible. The negative results of the earlier experiments are now considered to be a consequence of the depolarizing influence of plural scattering for foils used in the reflection position under an angle of $\pi/4$. [Kikuchi (E 39a) obtained positive results with a polarizer foil as thick as 10^{-3} cm; it is now believed that his asymmetry was of instrumental origin.] Before recognizing the influence of plural scattering, it was investigated whether nuclear potentials would be possible, which would explain the negative results for the double scattering experiment (T 35, T 40b). It was found

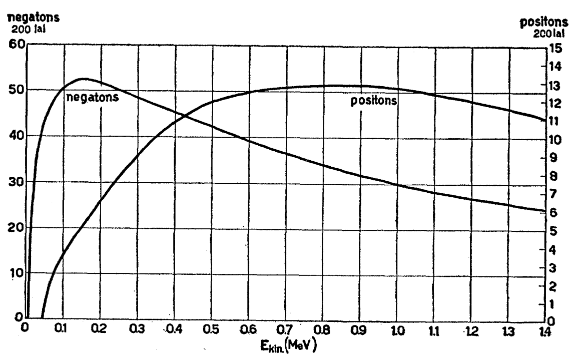

FIG. 6. The asymmetry percentage $200\,|a|$ in a single scattering experiment of a totally polarized beam as a function of the energy for negatons and positons ($Z=80$) (after the same calculations as Fig. 5). Scattering angle: 90°.

that the required forms for the nuclear potential were rather implausible. Landau stressed the importance of multiple scattering for depolarization (E 40b), but the greater importance of plural scattering seems now to be certain.

The importance of plural scattering, and the appreciable deviation from unity of the reflection-transmission ratio, were first studied and recognized by Chase, Cox, and Goertzel (E 40a, E 43a) and by Pethukhov and Vyshinsky (E 41). A further study of it was made by Ryu [(E 50, E 53b); the discussion in E 48 contains an error]. We may call the ratio of the real scattering intensities (including plural scattering) to the intensity, which would exist for single scattering only, $1+r$ and $1+t$ for the reflection and transmission position, respectively. If scattering over $\pi/4$ shows a negligible polarization effect compared to scattering over an angle of $\pi/2$, the quantity δ characterizing the asymmetry in double scattering will be reduced to an apparent magnitude of $\delta/(1+r)^2$ and $\delta/(1+t)^2$. The ratio $R=(1+r)/(1+t)$ was determined experimentally under several conditions. Shull, Chase, and Myers (E 43b) obtained $R=1.55$ at 400 kev for a foil of 4.1×10^{-5} cm thickness and Ryu, Hashimoto, and Nonaka (E 53b) obtained $R=1.4$ at 100 kev for a foil of 5×10^{-6} cm thickness (see the references for further data). Theoretical estimates can give a rough explanation of these values. Having the importance of plural scattering in mind, the first successful double scattering experiment was made in 1942 by Shull, Chase, and Myers (E 43b). In order to avoid the plural scattering depolarization the more recent experiments use the foils in positions b, d, or e (Fig. 7). It is seen from the figure that a plurally scattered electron, in these cases, must have suffered at least one scattering over an angle of $\pi/2$ or more, so that for these electrons also a substantial polarization effect should be expected. It is not easy to eliminate all instrumental asymmetries in the experiments. In this respect it is advantageous to use a symmetric position for the polarization detector foil with two counters (see Fig. 7 positions d and e).

We have summarized the most important of the experimental data on the asymmetry in double scattering experiments in Table II. Comparing the theoretical

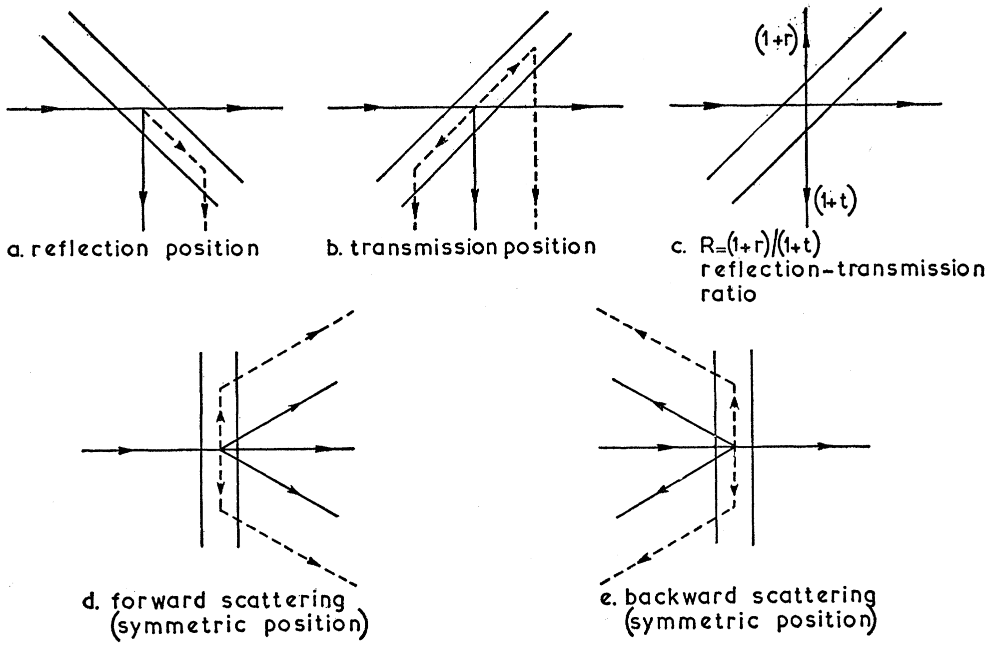

FIG. 7. The geometrical situation concerning plural scattering for different positions. Drawn lines: incident and singly scattered electrons; dotted lines: plural scattering with same total scattering angle; most important if the first scattering is in the plane of the scattering foil.

and experimental values of the Tables I and II and Fig. 5, we see the following. The value of δ obtained by Shull c.s. at 400 kev for $\vartheta_1=\vartheta_2=90°$ is in good agreement with the theory. The experiments of Ryu c.s. show qualitative agreement with the theory in the following respects: δ increases with the energy and with the scattering angle in the region for ϑ and E_{kin} for which he measured. No theoretical values taking screening into account are available for all values of ϑ and E_{kin}, for which he determined δ, but it is clear that quantitatively the increase of δ with ϑ, found by Ryu, is far less marked than found in the theoretical calculations; further, his experimental values are generally lower than the theoretical ones, sometimes even by a factor two. In the experiments, the influence of plural scattering was decreased by a favorable geometry as explained before. But it may be that part of the discrepancy can be explained by a correction, still to be made because of plural scattering. However, it is improbable that this would explain the discrepancy completely (E 53b). The results with $\vartheta_2=78°$ quoted in Table II seem rather high compared with the theoretical values of Table I. The approximate proportionality of a with Z, according to the theory, agrees within experimental error with the results of Louisell, Pidd, and Crane.

In view of these results, a continued experimental and theoretical investigation of this effect would certainly be desirable. In particular, it would be important to have: (a) an experimental investigation of δ for $\vartheta=90°, \cdots, 150°$ for $E=400$ kev, in order to check the high values of δ calculated for these parameters; (b) more precise theoretical values for δ as a function of both E and ϑ, and a careful theoretical study of the dependence of δ on the screening field.

Scattering by a thin gold foil might be used to *detect the polarization of electrons*, which is produced by some other method than scattering (see Sec. 5). In such other cases, the available intensity of the polarized electron beam will often be rather limited. This may easily cause an intensity problem for measuring the scattered electrons. With respect to the intensity, the arrangement *d*, Fig. 7, such as used by Louisell, Pidd, and Crane, may be the most favorable. The single scattering occurs in a somewhat forward direction (higher cross section), and even the use of somewhat thicker foils may be possible, because the plurally scattered electrons should also show quite some polarization effect here. On the other hand, backward scattering (position *e*, Fig. 7), may have the advantage of a larger polarization effect, but it has a smaller intensity (see Table I). Hence, the choice of position *d* or *e* should be made after more careful theoretical and experimental investigations as indicated above. Further, the counting rate may be increased by using rather large solid angles for the counters. In this way, it might be possible to count fractions up to 10^{-3} or 10^{-4} of the electrons incident on the detector foil of a good polarization detector.

We may mention here the negative results of quite a number of experiments done in the years 1929–1935, which tried to detect electron polarization effects by double reflection at mirrors or double scattering by

TABLE II. Experimental results for the asymmetry in double scattering experiments with scattering angles ϑ_1 and ϑ_2 and at electron energy E.

Observer	Scattering foils: Element and position of foil	Thickness	Electron energy E_{kin} (kev)	Scattering angles ϑ_1	ϑ_2	Observed asymmetry 200δ
Shull, Chase, and Myers (E 43b)	Au–Au transm. pos. under 45°	4.1×10^{-5} cm	400	90°	90°	12 ± 2
Shinohara and Ryu (E 49)	Au–Au (a) (b)	5×10^{-6} cm	90	90°	78°	9.0 ± 0.6
Ryu[c] (E 50, E 52a, E 52b, E 53b, E 53c)	Au–Au (a) (b)	5×10^{-6} cm	60	105°	105°	7.9 ± 2.2
			80			9.9 ± 1.6
			100			13.2 ± 2.2
			120			14.7 ± 1.7
			60	120°	120°	9.1 ± 2.2
			80			11.8 ± 2.0
			100			14.0 ± 1.3
			120			17.8 ± 0.8
			60	135°	135°	7.3 ± 1.0
			80			13.5 ± 1.0
			100			14.3 ± 1.1
			120			16.2 ± 2.1
Louisell, Pidd, and Crane (E 53a, E 54)	Au–Au (a) (b)	0.135 mg/cm^2	420	90°	78°	8.9 ± 1
	Au–Ag (a) (b)	0.135 mg/cm^2 0.23 mg/cm^2	420	90°	78°	5.5 ± 1

[a] Transmission position under about $90°-\frac{1}{2}\vartheta_1$ with the beam.
[b] Perpendicular to the beam.
[c] Ryu measured far more numerous values of 2δ; however, the values given here cover about the range of variables which he used. $Z=79$ for Au; $Z=49$ for Ag.

Debye-Scherrer diffraction (P 29d, P 29f, P 30c, P 30e, P 32b, P 33d, P 34a, P 34b, P 35a).§ These negative results were understood theoretically by the study of electron scattering by a periodic electric field (crystal lattice) in a number of papers (P 28b, P 31a, P 31b, P 33a, P 33c, P 35c). Polarization effects vanish in first approximation. A polarization effect exists for the second approximation, but this depends on the dimensions of the crystal and is vanishingly small, except for crystals which consist of such a small number of atoms, that we come back to Coulomb scattering by atoms.

4. POLARIZATION RELATIONS IN COMPTON SCATTERING, CORRELATION BETWEEN ELECTRON POLARIZATION AND CIRCULAR POLARIZATION OF PHOTONS (P 38, P 49, P 54b, P 54c, P 54d)

In phenomena involving photons and electrons, a close correlation generally can be expected between electron polarization and circular polarization of photons, as both these polarizations are associated with a spin angular momentum. This can be very well illustrated by the polarization effects in Compton scattering. We may express the differential cross section for Compton scattering, including all polarization effects, in the following form:

$$d\sigma/d\Omega=r_0^2(k^2/k_0^2)\Phi(\mathbf{k}_0,\mathbf{k},\boldsymbol{\xi}^0,\boldsymbol{\xi},\boldsymbol{\zeta}^0,\boldsymbol{\zeta}), \tag{4.1}$$

where $r_0=e^2/mc^2$ is the classical electron radius, $\mathbf{k}_0$ is the initial photon momentum, $\mathbf{n}_0=\mathbf{k}_0/|\mathbf{k}_0|$, $\mathbf{k}$ is the final photon momentum, $\mathbf{n}=\mathbf{k}/|\mathbf{k}|$, $\boldsymbol{\xi}^0$ is the polarization vector for the initial photon, $\boldsymbol{\xi}$ is the polarization vector for the final photon, $\boldsymbol{\zeta}^0$ is the polarization vector for the initial electron, $\boldsymbol{\zeta}$ is the polarization vector for the final electron.

In order to fix the meaning of $\boldsymbol{\xi}^0$ and $\boldsymbol{\xi}$, we have to specify the meaning of the 1- and 2-states for the initial and final photon. We shall choose

$$\left.\begin{aligned}&\mathbf{A}_1{}^0=\mathbf{A}_1=(\mathbf{k}_0\times\mathbf{k})/|\mathbf{k}_0\times\mathbf{k}|,\\&\mathbf{A}_2{}^0=(\mathbf{k}_0\times\mathbf{A}_1{}^0)/|\mathbf{k}_0|;\quad \mathbf{A}_2=(\mathbf{k}\times\mathbf{A}_1)/|\mathbf{k}|.\end{aligned}\right\} \tag{4.2}$$

In this way, the 1- and 2-directions specify photons, linearly polarized in and perpendicular to the plane of scattering.

The complete expression for the cross section is complicated, because so many independent vectors are involved. $\Phi(\mathbf{k}_0,\mathbf{k},\boldsymbol{\xi}^0,\boldsymbol{\xi},\boldsymbol{\zeta}^0,\boldsymbol{\zeta})$ is a linear function of the polarization vectors, and can be separated into 16 terms, according to the 16 different ways of choosing sets of polarization vectors. We shall write

$$\Phi=\Phi_0+\Phi_1+\Phi_2+\Phi_3+\Phi_4, \tag{4.3}$$

§ It may be useful to say that we did not mention a number of papers of this period by E. Rupp, stating positive results, as it appeared later that they were unreliable; see E. Rupp, Z. Physik 95, 801 (1935) and C. Ramsauer, Z. Physik 96, 278 (1935).

where

$$\left.\begin{aligned}&\Phi_0 \text{ is independent of polarizations,}\\ &\Phi_1=\Phi_1(\xi^0)+\Phi_1(\xi)+\Phi_1(\zeta^0)+\Phi_1(\zeta),\\ &\Phi_2=\Phi_2(\xi^0,\xi)+\Phi_2(\zeta^0,\zeta)+\Phi_2(\xi^0,\zeta^0)+\Phi_2(\xi,\zeta)\\ &\qquad\qquad+\Phi_2(\xi^0,\zeta)+\Phi_2(\xi,\zeta^0).\end{aligned}\right\}\quad(4.4)$$

Φ_3 and Φ_4 depend on 3 and 4 polarization vectors, respectively.

In a feasible experiment, not all of the polarizations will be observed; at present, only experiments in which at most two polarizations are involved have been performed. To obtain the cross section for some definite experiment, one should average over unobserved initial polarizations and sum over unobserved final polarizations. Since $-\boldsymbol{\xi}$ and $-\boldsymbol{\zeta}$ represent states orthogonal to $\boldsymbol{\xi}$ and $\boldsymbol{\zeta}$, the averages and sums merely cancel the unobserved terms. For example, if we observe $\boldsymbol{\xi}^0$ and $\boldsymbol{\zeta}$, the experimental cross section is

$$d\sigma_{\text{exp}}/d\Omega=\tfrac{1}{2}r_0^2(k^2/k_0^2)[\Phi(\xi^0,\ \xi,\ \zeta^0,\ \zeta)+\Phi(\xi^0,\ -\xi,\ \zeta^0,\ \zeta)+\Phi(\xi^0,\ \xi,\ -\zeta^0,\ \zeta)+\Phi(\xi^0,\ -\xi,\ -\zeta^0,\ \zeta)],\quad(4.5)$$

or

$$d\sigma_{\text{exp}}/d\Omega=2r_0^2(k^2/k_0^2)\times[\Phi_0+\Phi_1(\xi^0)+\Phi_1(\zeta)+\Phi_2(\xi^0,\zeta)].\quad(4.6)$$

The results found for $\Phi_0,\cdots,\Phi_2$ are the following (units are used in which $mc^2=1$, $\hbar=1$, $c=1$; it is assumed that the electron is initially at rest):

$$\Phi_0=\tfrac{1}{8}[(1+\cos^2\vartheta)+(k_0-k)(1-\cos\vartheta)],\quad(4.7)$$

$$\Phi_1(\xi_0)=\tfrac{1}{8}\xi_1{}^0\sin^2\vartheta,\quad(4.8)$$

$$\Phi_1(\xi)=\tfrac{1}{8}\xi_1\sin^2\vartheta,\quad(4.9)$$

$$\Phi_1(\zeta^0)=\Phi_1(\zeta)=0,\quad(4.10)$$

$$\Phi_2(\xi^0,\xi)=\tfrac{1}{8}[(1+\cos^2\vartheta)\xi_1{}^0\xi_1+2\cos\vartheta(\xi_2{}^0\xi_2+\xi_3{}^0\xi_3)+(k_0-k)\cos\vartheta(1-\cos\vartheta)\xi_3{}^0\xi_3],\quad(4.11)$$

$$\Phi_2(\xi^0,\zeta^0)=-\tfrac{1}{8}\xi_3{}^0(1-\cos\vartheta)\boldsymbol{\zeta}^0\cdot(\mathbf{k}_0\cos\vartheta+\mathbf{k}),\quad(4.12)$$

$$\Phi_2(\xi,\zeta^0)=-\tfrac{1}{8}\xi_3(1-\cos\vartheta)\boldsymbol{\zeta}^0\cdot(\mathbf{k}_0+\mathbf{k}\cos\vartheta),\quad(4.13)$$

$$\Phi_2(\xi^0,\zeta)=-\tfrac{1}{8}\xi_3{}^0(1-\cos\vartheta)[\boldsymbol{\zeta}\cdot(\mathbf{k}_0\cos\vartheta+\mathbf{k})-(1+\cos\vartheta)(k_0+k)\times(k_0-k+2)^{-1}\boldsymbol{\zeta}\cdot(\mathbf{k}_0-\mathbf{k})],\quad(4.14)$$

$$\Phi_2(\xi,\zeta)=-\tfrac{1}{8}\xi_3(1-\cos\vartheta)[\boldsymbol{\zeta}\cdot(\mathbf{k}_0+\mathbf{k}\cos\vartheta)-(1+\cos\vartheta)(k_0+k)\times(k_0-k+2)^{-1}\boldsymbol{\zeta}\cdot(\mathbf{k}_0-\mathbf{k})],\quad(4.15)$$

$$\begin{aligned}\Phi_2(\zeta^0,\zeta)=\tfrac{1}{8}\{&(\boldsymbol{\zeta}^0\cdot\boldsymbol{\zeta})[(1+\cos^2\vartheta)+\tfrac{1}{2}(k_0-k)\sin^2\vartheta]\\&-\tfrac{1}{2}(k_0-k)[\boldsymbol{\zeta}^0\cdot(\mathbf{n}_0+\mathbf{n})\boldsymbol{\zeta}\cdot(\mathbf{n}_0+\mathbf{n})\\&+\boldsymbol{\zeta}^0\cdot(\mathbf{n}_0\times\mathbf{n})\boldsymbol{\zeta}\cdot(\mathbf{n}_0\times\mathbf{n})]\\&+\tfrac{1}{2}(k_0+k)(1+\cos\vartheta)(\boldsymbol{\zeta}^0\times\boldsymbol{\zeta})\cdot(\mathbf{n}_0\times\mathbf{n})\\&+(1+\cos\vartheta)(k_0-k)(k_0-k+2)^{-1}\boldsymbol{\zeta}\\&\cdot(\mathbf{k}_0-\mathbf{k})\boldsymbol{\zeta}^0\cdot(\mathbf{n}_0+\mathbf{n})\\&-(1+2\cos\vartheta-\cos^2\vartheta)(k_0-k+2)^{-1}\boldsymbol{\zeta}\\&\cdot(\mathbf{k}_0-\mathbf{k})\boldsymbol{\zeta}^0\cdot(\mathbf{k}_0-\mathbf{k})\}.\end{aligned}\quad(4.16)$$

We discuss the three polarization effects in Compton scattering involving polarized electrons, which seem to be nearest to an experimental test.

A. Production and Detection of Circular Polarization of γ Quanta by Compton Scattering with Polarized Electrons

In this respect the cross sections $d\sigma(\xi^0,\zeta^0)$ and $d\sigma(\xi,\zeta^0)$ are of importance. As an example, we give

$$d\sigma(\xi^0,\zeta^0)/d\Omega=\tfrac{1}{2}r_0^2(k^2/k_0^2)[(1+\cos^2\vartheta)+(k_0-k)(1-\cos\vartheta)+\xi_1{}^0\sin^2\vartheta-\xi_3{}^0(1-\cos\vartheta)\boldsymbol{\zeta}^0\cdot(\mathbf{k}_0\cos\vartheta+\mathbf{k})].\quad(4.17)$$

In case we investigate the cross section of circularly polarized photons (without linear polarization) and the electron polarization vector is parallel (or antiparallel) to k_0 the cross section simplifies to

$$d\sigma(\xi^0,\zeta^0)/d\Omega=\tfrac{1}{2}r_0^2(k^2/k_0^2)\times[(1+\cos^2\vartheta)+(k_0-k)(1-\cos\vartheta)-P(1-\cos\vartheta)\cos\vartheta(k_0+k)].\quad(4.18)$$

P is the product of the degrees of polarization of the photon and the electron (P is positive for a left circularly polarized photon and an electron spin parallel to $\mathbf{k}_0$). We may write this cross section as

$$d\sigma/d\Omega=d\sigma_0/d\Omega+Pd\sigma_1/d\Omega,\quad(4.19)$$

where $d\sigma_0$ is the Klein-Nishina expression without polarization and $d\sigma_1$ gives the part sensitive to spin and circular polarization. Integration over $d\Omega$ gives for the

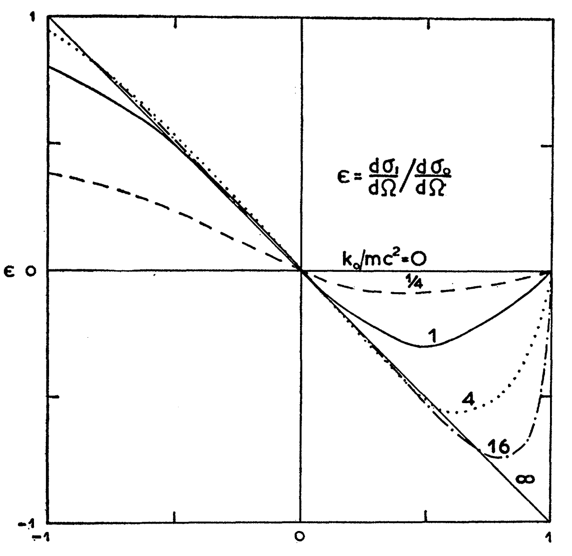

FIG. 8. The fractional change in the differential Compton cross section resulting from entirely polarized initial electrons with spin in the direction of the incident circularly polarized photon. The change is plotted as a function of the scattering angle θ of the photon. (Figure from P 53d.)

polarization dependent part of the total cross section

$$\sigma_1/2\pi r_0^2 = \frac{1+4k_0^2+5k_0^2}{k_0(1+2k_0)^2} - \frac{1+k_0}{2k_0^2}\ln(1+2k_0). \quad (4.20)$$

In Fig. 8, the ratio $(d\sigma_1/d\Omega)/(d\sigma_0/d\Omega)$ is represented as a function of angle showing the change of sign of the polarization effect in the differential cross section for forward and backward directions. Figure 9 shows the polarization dependent part σ_1 in the total Compton scattering cross section. The change in sign occurring in σ_1 for 1.25 mc^2=0.65 Mev can be understood from the difference in sign for $d\sigma_1/d\Omega$ for forward and backward directions, and from the fact that the forward Compton scattering becomes more and more preponderant for higher energies.

The experimental possibilities of detecting effects in Compton scattering depending on $\boldsymbol{\zeta}^0$ require the availability of polarized electrons at rest for the initial state.

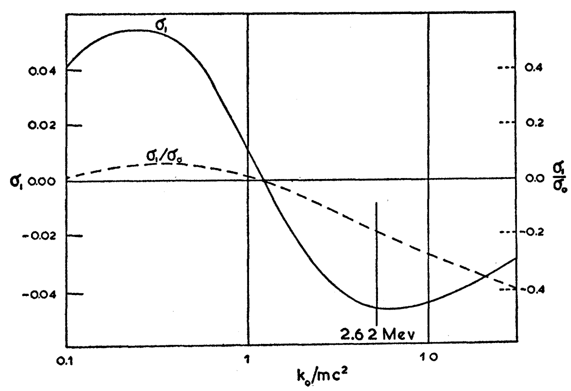

FIG. 9. The polarization-sensitive part σ_1 of the total Compton cross section for the same polarizations as in Fig. 8 (in units $2\pi r_0^2$). The part independent of polarization is σ_0. (Figure from P 53d.)

These are available in magnetized ferromagnetic materials. The binding of these electrons does not play an essential role at the energies which are mostly used in the study of the Compton effect. Unfortunately the degree of polarization P averaged over all electrons of the atom is never very high. For iron one has $P\approx 2/26 \approx 8\%$ at saturation of the magnetization. The circular polarization-polarized electron effect in Compton scattering was first detected experimentally by Gunst and Page (P 53d) in 1953.‖ They measured the differences in transmission T of an iron bar 30 cm long and 3.8 cm in diameter for 2.62-Mev γ rays (of ThC″) for magnetized and unmagnetized iron. For a bar of length L there arises a difference in transmission for magnetized and unmagnetized iron with initially unpolarized photons which can be understood as follows: taking the unmagnetized transmission as unity, the left and right

‖ A positive result for the circular polarization effect in Compton scattering reported earlier by Clay and Hereford (P 52b) is now considered to have been of instrumental origin (private communication).

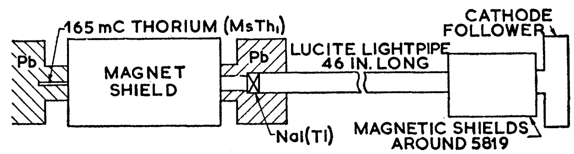

FIG. 10. Arrangement of equipment in the polarization experiment by Gunst and Page. The variation of the transmission of γ rays from a strong source through an iron bar with the magnetization is measured (change of total Compton scattering cross section). (Figure from P 53d.)

circularly polarized components (l.c.p. and r.c.p.) of the unpolarized beam give exponential factors in the transmission for the magnetized bar which sum to

$$\tfrac{1}{2}\exp[-NL\nu\sigma_1]+\tfrac{1}{2}\exp[+NL\nu\sigma_1] = \cosh[NL\nu\sigma_1]\approx 1+\tfrac{1}{2}(NL\nu\sigma_1)^2, \quad (4.21)$$

where N=number of atoms/cm^3; ν=number of polarized electrons per atom. (See Fig. 10.) It follows from (4.21) that we can write approximately for the relative change of the transmission T by magnetization

$$[T(\nu)-T(0)]/T(0)=\tfrac{1}{2}(NL\nu\sigma_1)^2. \quad (4.22)$$

It follows further that the transmission ratio of l.c.p. relative to r.c.p. photons is given by

$$R=\exp[2NL\nu\sigma_1]. \quad (4.23)$$

The measured change in counting rate amounted to $0.59\pm 0.09\%$, which gives with $\nu=2.06$ an experimental

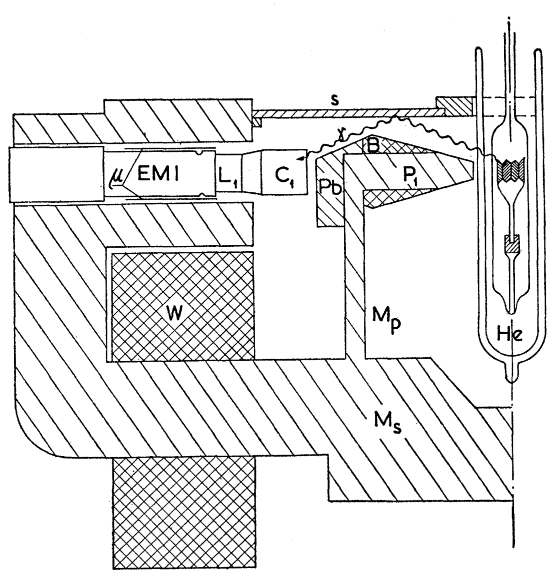

FIG. 11. Diagram of the apparatus used in the Kamerlingh-Onnes Laboratory in Leyden for producing and measuring circularly polarized γ rays. The change of the differential Compton cross section is measured by changing the relative orientation of the circular polarization of the γ rays emitted from the polarized nuclei in the cryostate and the direction of magnetization of the scattering iron S. The magnet M_p (with coil B) determines the direction of polarization of the nuclei; the magnet M_s (with coil W) determines the direction of magnetization of S. The γ rays are detected by the NaI(Tl) crystal C_1 with photomultiplier EM_1. (Figure from P 55e.)

value of $\sigma_1/\pi r_0^2 = 0.089 \pm 0.007$ in good agreement with the theoretical value 0.093. It follows from (4.23) that the γ rays transmitted through the bar must have had a degree of circular polarization $P \approx 10\%$ (at an intensity of about 10 counts/sec).

The same polarization effect was used in 1955 at the Kamerlingh-Onnes Laboratory in Leyden (P 55f) to detect the circular polarization of the 1.17- and 1.33-Mev γ rays from polarized Co^{60} nuclei. (See Fig. 11.) A source of 110 μC Co^{60} was contained in a crystal cooled down to 0.006°K by adiabatic demagnetization. In this way a degree of circular polarization as high as 75% was attained. It was detected by the polarization effect in the forward Compton scattering differential cross section. The measured changes in counting rate when changing the relative orientation of circular polarization and polarized electrons amounted up to 3%. The polarization effect could be measured with an accuracy of 10%. The agreement of the theoretical value (including the mechanism of polarization for the nuclei in the crystal) with experiment was satisfactory. In 1955 Trumpy also reports a detection of the circular polarization of gamma rays from polarized neutron capture by means of a transmission measurement analogous to that of Gunst and Page (P 55e).

B. Production of Polarized Electrons by Compton Scattering of Circularly Polarized Photons

The relevant cross section is

$$\begin{aligned} d\sigma(\boldsymbol{\xi}^0,\boldsymbol{\zeta})/d\Omega = {}& \tfrac{1}{4}r_0^2(k^2/k_0^2)\{(1+\cos^2\vartheta) \\ & +(k_0-k)(1-\cos\vartheta)+\xi_1^0\sin^2\vartheta \\ & -\xi_3^0(1-\cos\vartheta)[\boldsymbol{\zeta}\cdot(\mathbf{k}_0\cos\vartheta+\mathbf{k}) \\ & -(1+\cos\vartheta)(k_0+k)(k_0-k+2)^{-1}\boldsymbol{\zeta}\cdot(\mathbf{k}_0-\mathbf{k})]\}. \end{aligned} \tag{4.24}$$

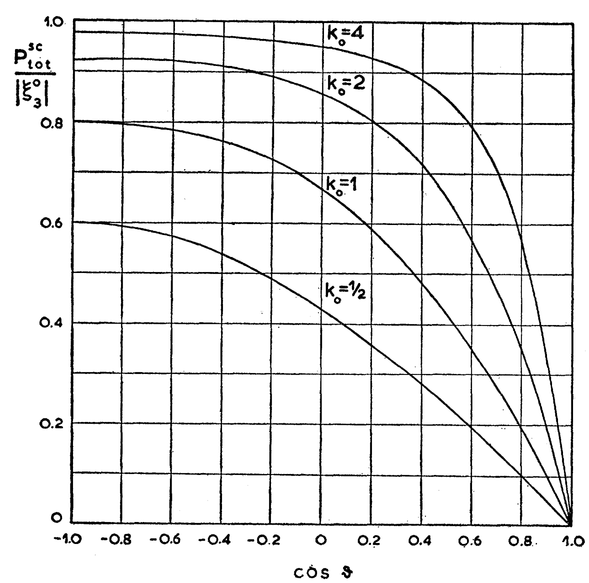

FIG. 12. Production of polarized electrons by Compton scattering of (entirely) circularly polarized photons at unpolarized electrons. Total degree of polarization of the final electrons as function of the scattering angle of the photons.

If we write this cross section as

$$d\sigma(\boldsymbol{\xi}^0,\boldsymbol{\zeta})/d\Omega = S[1+\boldsymbol{\zeta}\cdot\boldsymbol{\zeta}^{sc}], \tag{4.25}$$

we can say that $\boldsymbol{\zeta}^{sc}$ represents the state of polarization of the Compton scattered electron. We write

$$\boldsymbol{\zeta}^{sc} = P_p^{sc}\,\mathbf{e}_p + P_p^{sc}\,\mathbf{e}_q, \tag{4.26}$$

where $\mathbf{e}_p$ and $\mathbf{e}_q$ are the unit vectors in the directions of the perpendicular vectors

$$\mathbf{p} = \mathbf{k}_0 - \mathbf{k} \quad \text{and} \quad \mathbf{q} = \mathbf{k} - \mathbf{p}(\mathbf{k}\cdot\mathbf{p})/p^2. \tag{4.27}$$

For diagrams of P_p^{sc} and P_q^{sc} (which might be called degree of transverse and longitudinal polarization) we refer to P 54d. The total degree of polarization $P^{sc} = [(P_p^{sc})^2 + (P_q^{sc})^2]^{\frac{1}{2}}$ is plotted in Fig. 12. All the data were calculated for $\xi_1^0 = 0$ and $\xi_3^0 = 1$ (complete circular polarization) of the initial photon.

C. The Polarization Correlation between the Initial and Final Electron

If we observe the polarizations of the initial and final electron, we may write the relevant cross section as

$$\begin{aligned} d\sigma(\boldsymbol{\zeta}^0,\boldsymbol{\zeta})/d\Omega &= 2r_0^2(k^2/k_0^2)[\Phi_0+\Phi_2(\boldsymbol{\zeta}^0,\boldsymbol{\zeta})] \\ &= F(1+\boldsymbol{\zeta}\cdot\boldsymbol{\zeta}^{sc}). \end{aligned} \tag{4.28}$$

This defines the polarization vector $\boldsymbol{\zeta}^{sc}$ of the scattered electron and its degree of polarization $P^{sc} = |\boldsymbol{\zeta}^{sc}|$. We note the following consequences of the general formulas: (a) in the classical limit $k_0 \approx 0$, $P^{sc} \approx 1$, and $\boldsymbol{\zeta}^{sc} \approx \boldsymbol{\zeta}^0$, (b) for forward scattering of the photon, $\cos\vartheta = 1$, $P^{sc} = 1$, and $\boldsymbol{\zeta}^{sc} = \boldsymbol{\zeta}^0$, (c) if $\boldsymbol{\zeta}^0$ is perpendicular to the plane of scattering, where $\zeta_\perp^0$ is the absolute value of $\boldsymbol{\zeta}^0$ then

$$\left.\begin{aligned} P^{sc} &= [(1+\cos^2\vartheta)+(k_0-k)(1-\cos\vartheta)]^{-1} \\ &\qquad\qquad \times(1+\cos^2\vartheta)\zeta_\perp^0, \\ \boldsymbol{\zeta}^{sc} &= P^{sc}\boldsymbol{\zeta}^0. \end{aligned}\right\} \tag{4.29}$$

Curves for P^{sc} according to (4.29) are shown in Fig. 13. Analogous to Compton scattering one may also expect correlations between electron polarization and circular polarization of the gamma radiation in other quantum processes involving photons and electrons, such as bremsstrahlung, pair production, and positon annihilation.

5. METHODS OF PRODUCING POLARIZED FREE ELECTRONS

We want to enumerate and discuss concisely the possibilities which, in principle, can produce polarized free electrons.

(a) The only method up to now, in which it was shown experimentally that polarized electrons were produced, is the *Coulomb scattering at heavy nuclei* discussed in Sec. 3.

(b) *Compton electrons ejected by circularly polarized photons* will, in general, have an appreciable polarization (see Sec. 4). These photons could be obtained by transmission of γ quanta of a very strong source through a bar of magnetized iron or from polarized radioactive nuclei emitting γ quanta (see Sec. 4). Numerical values for this effect according to the theory are available. The main problem is whether it would be feasible to obtain experimentally intensities, which allow the measurement of the electron polarization.

(c) *Compton electrons ejected by photons from magnetized iron* (or another ferromagnetic material; see Sec. 4 for the theory). The difficulty for observing this effect will again be the experimental problem of intensities. Moreover, the degree of polarization will be at most about 8% (the average degree of polarization of the electrons in iron magnetized to saturation), so will not be very easily measurable even at a sufficient intensity.

(d) *Ejection of bound polarized electrons from magnetized iron* could be performed by other methods than Compton scattering. We may think of 1. *the photoelectric effect* (see P 30b), 2. *cold emission*, 3. *secondary emission or scattering of electrons at magnetized iron.* It seems that these methods have not been considered in much detail theoretically. An attractive feature of the photoelectric effect is that it seems possible to obtain a higher degree of polarization for the ejected electrons than the average one for all electrons. By choosing a suitable photon energy, photoelectric emission of the unpolarized electrons of the deeper lying shells would not be possible, so that the average of the polarization for the photoelectrons may be appreciably higher.

It was suggested implicitly that the three methods mentioned above would not change too much the polarization of the bound electrons during the process of ejection. We may make an order of magnitude estimate of the chance of a spin flip in the photoelectric effect according to Mott (S 49). Let us suppose that we have an electron bound in the ground state of the Coulomb field of a nucleus with change Ze. It has a velocity of the order

$$v \approx \alpha Z c. \tag{5.1}$$

The couple acting in an electric field $\mathfrak{E}$ on the magnetic moment $\mu = e\hbar/2mc$ of an electron moving with velocity $v \approx \alpha Z c$ is of the order

$$M = (v/c)\mathfrak{E}\mu = \alpha Z \mathfrak{E}(e\hbar/2mc) = (e^3/2mc^2)\mathfrak{E}Z. \tag{5.2}$$

For a spin flip the spin angular momentum has to change by an amount $\hbar$, which may occur in a time T of the order of $\hbar/M$

$$T \approx \hbar/M = 2\hbar mc^2/Z\mathfrak{E}e^3. \tag{5.3}$$

We want to compare this time with the time t in which the field $\mathfrak{E}$ has to act in order to liberate the electron from the bound state in the Coulomb field with binding energy $Z^2(e^4m/\hbar^2)$ corresponding to a momentum $p \approx me^2Z/\hbar$. The time t required for a force $e\mathfrak{E}$ to cause

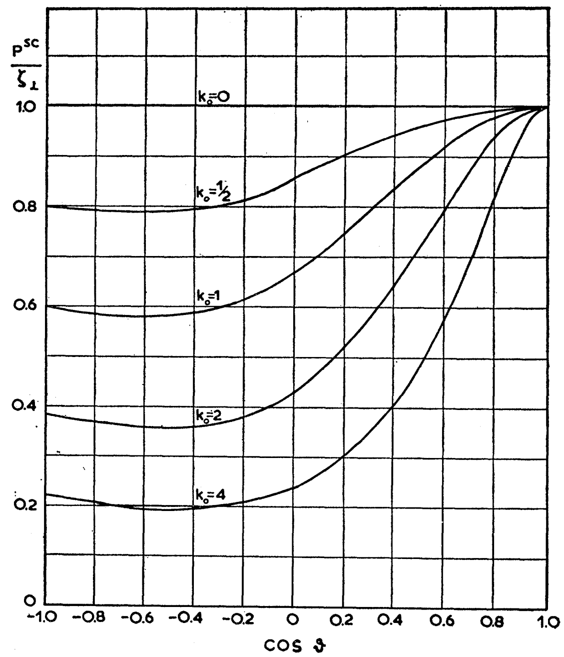

FIG. 13. Production of polarized electrons by Compton scattering of unpolarized photons at electrons which are initially completely polarized perpendicular to the plane of scattering. Degree of polarization of the final electrons as function of the scattering angle of the photons.

a momentum change p is of the order

$$t \approx p/e\mathfrak{E} = Zem/\mathfrak{E}\hbar. \tag{5.4}$$

Comparing T and t we see that

$$T/t \approx (\alpha Z)^{-2}. \tag{5.5}$$

Hence, for low Z we may conclude that $T \gg t$, so that the spin has "no time to flip" during the photoemission. We can say that the photoelectric effect is a result of the electric vector and that the influence of the photon angular momentum (polarization) on the electron polarization is negligible. This holds, e.g., for visible light causing photoemission of outer electrons of the atom. The estimate (5.5) shows that T may become of the same order as t for high Z and electrons of the lower bound states. Hence, it might be that, for example, some electron polarization would result for photoelectrons liberated from high Z elements by circularly polarized γ quanta.

(e) In addition to magnetized iron, another source of *bound polarized electrons* can be provided by an optical method, which was proposed by Kastler (P 50) and realized by Kastler c.s. (P 52a) and by Dicke and Hawkins (P 53e, P 55a). If we have, e.g., sodium vapor in a magnetic field, the Zeeman splitting causes different energies for the different magnetic quantum numbers m_F. By exciting an appreciable fraction of the atoms by irradiating the vapor with radiation of the resonance frequency in the direction of the magnetic field, an appreciable "polarization" of the atoms is obtained.

This entails a polarization of bound electrons and of the nuclei. The polarized electrons of the excited state may be ejected by absorption of a second photon. If the energy of the second photon is chosen sufficiently low, so that only excited electrons can be ejected, one may even attain totally polarized (low-energy) electrons. As two photons have to be absorbed by the same atom, it is again difficult to obtain a sufficient intensity of electrons. This method was indicated and realized by R. H. Dicke (1950, private communication), who obtained 10^5 electrons/sec in this way, with a computed polarization of 20–30%. However, he did not succeed in detecting the polarization. As Coulomb scattering at heavy nuclei is only a suitable detector of polarization for electrons of at least, say, 50 kev, one should either first accelerate the low-energy polarized electrons to such an energy, or look for a detector of electron polarization effective for low-energy electrons.

(f) *The β radiation emitted from polarized β-radioactive nuclei will, in general, be polarized.* (P 51d). Certain nuclei can be polarized in crystals cooled down by adiabatic demagnetization to a few hundredths of a degree (see, e.g., R 55b). If the β-radioactive nucleus changes its nuclear spin (which has a certain direction before the emission) during the β decay, this angular momentum is transferred to the electron and the neutrino emitted in this process. This gives rise to a preferential direction for the spin of the emitted electron. The calculation of the degree of polarization can be made according to the general principles of the theory of β radioactivity, making use of perturbation theory and Dirac wave functions, and following the general principles indicated in Sec. 2. In particular, the 4×4 matrix (2.23) is used. As an example, we give the transition probability for β emission of an electron with energy E (including the rest mass) and momentum $\mathbf{p}$, using a detector set for spin direction $\boldsymbol{\zeta}^{\text{det}}$ (this formula is derived for pure Gamow-Teller interaction)

$$P(E,p,\boldsymbol{\zeta}^{\text{det}})=(G^2/16\pi^4)pEq^2\left|\int\boldsymbol{\sigma}\right|^2[1+(A/E)\boldsymbol{\eta}\cdot\boldsymbol{\zeta}^{\text{det}} +[A/E(E+1)](\boldsymbol{\eta}\cdot\mathbf{p})(\boldsymbol{\zeta}^{\text{det}}\cdot\mathbf{p})] \quad (5.6)$$

(units are used such that $\hbar=1$, $c=1$, $m=1$). Here G is the Fermi coupling constant; $\int\boldsymbol{\sigma}$ the nuclear matrix element, q the neutrino momentum; the unit vector $\boldsymbol{\eta}$ is the axis of polarization of the nuclei (axis of rotational symmetry for the nuclear orientation). A is proportional to the nuclear polarization. If I_i and I_f are the initial and final nuclear spins, we have

$$A=\begin{cases} f_1, & \text{if}\quad I_i=I_f+1 \quad (I_i\geqslant 1) \\ f_1/(I_i+1), & \text{if}\quad I_i=I_f \quad (I_i\geqslant\frac{1}{2}) \\ -f_1I_i/(I_i+1), & \text{if}\quad I_i=I_f-1 \quad (I_i\geqslant 0)\end{cases} \quad (5.7)$$

where

$$f_1=\sum_{M=-I_i}^{I_i}(M/I_i)a_M \quad (-1\leqslant f_1\leqslant 1), \quad (5.8)$$

while a_M is the probability that the initial nucleus has the magnetic quantum number M with respect to the axis $\boldsymbol{\eta}$. (We normalize such that

$$\sum_{M=-I_i}^{I_i}a_M=1).$$

We may write the result (5.6) as in Sec. 2 as a multiple of $\frac{1}{2}[1+\boldsymbol{\zeta}\cdot\boldsymbol{\zeta}^{\text{det}}]$. The value of $\boldsymbol{\zeta}$ for the spin of the electrons emitted with momentum $\mathbf{p}$ determined in this way is

$$\boldsymbol{\zeta}=A[\boldsymbol{\eta}/E+(\boldsymbol{\eta}\cdot\mathbf{p})\mathbf{p}/E(E+1)]. \quad (5.9)$$

Hence we have, in particular:

(1) $\mathbf{p}\perp\boldsymbol{\eta}$ (electrons emitted perpendicular to the nuclear polarization axis; $E\approx1$ for low-energy electrons in our units)

$$\boldsymbol{\zeta}=(A/E)\boldsymbol{\eta}\begin{pmatrix}\text{transverse polarization,}\\ \text{degree of polarization: } P=A/E\end{pmatrix}. \quad (5.10)$$

(2) $\mathbf{p}\|\boldsymbol{\eta}$ (electrons emitted in the direction of the nuclear polarization axis)

$$\boldsymbol{\zeta}=A\boldsymbol{\eta}\begin{pmatrix}\text{longitudinal polarization,}\\ \text{degree of polarization: } P=A\end{pmatrix}. \quad (5.11)$$

The experimental situation is such that values for f_1 and A of order unity can be attained, e.g., for Co^{60} nuclei. Hence, one can say that the electrons emitted in the nuclear polarization experiments with these nuclei are highly polarized. The detection of the electron polarization has not yet been attempted and is difficult because (a) only β rays from the surface of the crystal come out of the source; (b) since the source is in a cryostat in these experiments, the detection of the polarization should occur within the cryostat or the β rays should come out of the cryostat, both of which are difficult experimentally; (c) it seems difficult to obtain an intensity of the electrons sufficient to detect the polarization.

(g) By a *combined use of very strong magnetic fields* (10^3 to 10^4 gauss) *and very weak electric potentials* (10^{-5} to 10^{-4} volt), it may be possible to trap electrons of a definite polarization. These polarized electrons may then be removed from the trapping potential, retaining their polarization. Although space charge severely limits the number of electrons which can be trapped at a time, a rapid repetition of the trapping and "blowout" procedure may provide a current of polarized electrons which is sufficient for many experiments: say 30 electrons could be trapped at a time and the procedure could be repeated 3000 times a second; then an electron current of about 10^5 electrons per second would result. They are essentially low-energy electrons, although they may be accelerated, of course, to higher energies, retaining their polarization. Detailed procedures for trapping electrons in this way were made by Bloch (P 53a) and Dicke; see Sec. 7 for the details.

6. METHODS OF DETECTING ELECTRON POLARIZATION

Since in any experiment on electron polarization a polarizing device as well as an analyzer will be required, we shall now also enumerate the possibilities for analyzers.

(a) The only method which has been successful up to now in experiments for detecting electron polarization is the *Coulomb scattering at heavy nuclei* discussed in Sec. 3.

(b) Another phenomenon which will depend to some extent on the polarization of the incident electrons is the *scattering by magnetized iron*. One may try either to detect an effect in the total cross section (transmission through a foil of magnetized iron) or in the differential cross section for a scattering angle favorable for sensitivity to the polarization effect. It should be expected that the quantum-mechanical exchange scattering, which provides the dependence on the relative orientation of the spin polarization of the incident and the bound electron, is most effective at lower energies than method (a). An attempt to use this effect was made by R. H. Dicke in 1950 (private communication) who produced polarized electrons according to method 5(e), and tried to detect the polarization of the electrons (after acceleration) by measuring the transmission through a thin ($\approx 10^{-6}$ cm) magnetized iron foil. A theoretical estimate of the influence of the polarization led one to expect a small but measurable effect, which could not be detected however. An early attempt following this line of thought was made by Myers and Cox in 1929. They sent β rays of Ra through two foils of magnetized iron, in which the relative direction of magnetization was changed (P 29h). They failed to detect any observable effect in the transmission.

(c) It may be possible to *measure the spin angular momentum* carried by electrons with longitudinal polarization *in a mechanical way* (L. Marton, private communication). If such electrons fall on a suspended disk, a torque may be measured as a consequence of the polarization. The condition of a torque arising from the linear momentum of the incident electrons and an asymmetric geometry should be avoided. This will be most feasible for low-energy electrons.

(d) When exciting atoms by means of *exchange scattering with low-energy polarized electrons*, the excited state may be polarized (have an oriented total angular momentum). The atom may be placed in a magnetic field, and pass to a lower level with emission of *circularly polarized light* (E. S. Dayhoff, private communication). If circular polarization is detected for the light, it follows that the incident electrons were polarized.

It is remarkable that it is more difficult to think of methods of detecting polarization, which seems apt for realization, than of methods of producing polarization. Except for the four proposals just mentioned, scarcely any other method has been devised. In the proposals for resonance experiments by Bloch and Dicke (discussed in Sec. 7), the detection (as well as the production) makes use of the combination of strong magnetic and weak electric fields, but this method of detection depends on the specific energies with which the polarized electrons are produced there, and cannot be considered as a *general* method of detecting electron polarization.

7. EXPERIMENTS WITH POLARIZED ELECTRONS; DETERMINATION OF THE g FACTOR OF THE FREE ELECTRON

In the preceding sections, different possibilities for production and detection of polarized free electrons were discussed. In addition to production and detection, one may make further experiments with the polarized electrons. We may summarize the various possible purposes of different experiments as follows: (1) Testing the theory concerning the polarization of electrons (see, e.g., Sec. 3). (2) Determination of the g factor of the free electron (see later). (3) Indirectly investigating the degree of polarization of bound electrons, a quantity useful for knowledge of the solid state. [The circular polarization experiment in Compton scattering described in Sec. 4 allows, in principle, the determining of the average degree of polarization of the bound electrons. Similar information might be obtained if one could measure, e.g., the degree of polarization of electrons ejected by photoeffect from magnetized iron (see Sec. 5).] (4) Detection of the polarization of β rays from polarized β-radioactive nuclei, which may give useful information for nuclear physics and concerning the mechanism of the polarization of the nuclei (see Sec. 5).

Up to now, the experiments have had the first two aims. In the rest of this section, we shall discuss in more detail experimental procedures which were carried out or proposed in order to determine the g value of the free electron. This is of special interest in relation to testing the value $g=2(1+\alpha/2\pi+\cdots)$ given by quantum electrodynamics, which deviates by about 1 part in a thousand from the $g=2$ of simple Dirac theory. This value was found to be in agreement with the g value determined for electrons bound in an atom, but it would be of interest to have an independent determination for the free electron in order to have an independent check, perhaps even of higher accuracy.

Two main directions of approach can be distinguished in the proposals for measuring the g value of the free electron.

(a) Between the production and the detection of the electron polarization *a constant magnetic field* is used; the precession angle of the magnetic moment in the magnetic field is measured.

(b) The *frequency* at which a *spin flip* occurs is determined by a *resonance experiment* in a constant magnetic field. For this purpose, a radio-frequency field is applied after the electrons are produced in a definite

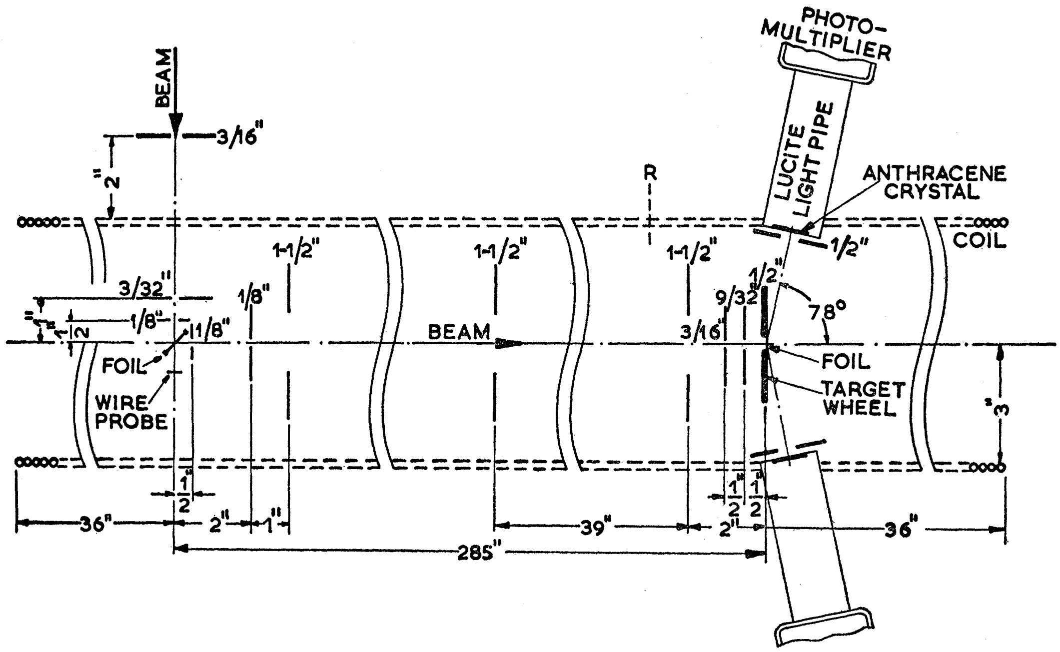

FIG. 14. Diagram of the double scattering experiment of Louisell, Pidd, and Crane, with a magnetic field between the two scatterings. The electron spin precesses in a plane perpendicular to the direction of the beam. (Figure from P 54e.)

state of motion and polarization and before they hit the detector, which may detect the spin flip.

(a.1) An experiment of this kind was performed by Louisell, Pidd, and Crane (P 54e, P 53f, P 53g) in the following way. A double scattering experiment was performed with 420-kev electrons with gold foils of 0.135 mg/cm^2 as polarizer and analyzer foils (compare Fig. 14). The polarization was attained by scattering through an angle of 90°, using the polarizer under 45° in the transmission position. The scattered beam hit the analyzer foil perpendicularly; the two electron counters were placed so as to have a scattering angle of 78°. These counters were scintillation counters consisting of an anthracene crystal, Lucite light pipe, and photomultiplier. Polarizer and analyzer foil were separated by a distance of 725 cm; the electrons went through a brass tube of 15 cm diameter; a single layer of copper tubing on the brass tube provided a solenoid through which a current of about 60 amp, provided a homogeneous longitudinal magnetic field of about 120 gauss. This magnetic field caused a precession of the electron spin of about 5 complete turns ($\approx 1800°$). The precession could be measured by turning the analyzer counters about the axis of the brass tube. In this way, the phase of the polarization asymmetry was determined for several solenoid currents. Instrumental asymmetries were eliminated by comparing the counting rates occurring for a polarizer foil of aluminum (low Z) which gives only a negligible electron polarization. In addition to the spin precession, the magnetic field also causes a periodic focusing of the slightly divergent beam (aperture 2.25°). Focusing occurs at distances in which the cyclotron (orbital) motion perpendicular to the axis of the tube has made an integral number of revolutions. Hence, if the g value were 2, the focusing would occur at the same distances at which the spin precession had completed an integral number of turns. It follows that we can determine g by comparing the spin precession and the electron focusing, which can be measured independently. By making such a relative measurement, it is not necessary, for example, to make an absolute calibration of the magnetic field. The experiment resulted in a determination of the g value with an accuracy of half a percent: $g = 2.00 \pm 0.01$.

(a.2) A proposal for an experimental setup, which may allow an accuracy of about 1 in 10^5, was given by Crane (P 53c, and private communication) and its construction has begun. Instead of using a continuous electron current, a pulsed beam of electrons is used, which is trapped in a constant magnetic field (a kind of betatron field) so as to make 1000 to 10 000 revolutions before being scattered the second time. The number of revolutions for a pulsed beam may be determined from the time of flight τ. As we explained in Sec. 2, the anomaly of the electron magnetic moment may cause the electron polarization to change from transverse polarization to longitudinal polarization (and reverse) in about 250 revolutions. If the magnetic field is arranged in this way, one should expect a result for the measured asymmetry as a function of the number of revolutions as represented in Fig. 15. $1/N_0$ will give the deviation of $g/2$ from unity. So if τ can be measured to

1%, the g factor may be determined to about 1 part in 10^5.

A thorough theoretical investigation of different aspects of the double scattering experiment with a magnetic field was carried out by Case and Mendlowitz (P 53b, P 55b, P 55c). In particular, they considered the depolarization, which may occur for polarized electrons carrying out many cyclotron revolutions. They conclude that a careful experiment may allow a determination of the g factor to at least one part in 10^5.

(b.1) A *resonance experiment* might be possible if the electrons in a double scattering experiment could be trapped for a sufficient number of cyclotron revolutions in a magnetic field between the two scatterings (see S 43 and P 51c). In order to obtain an accuracy for the g value of 1 part in 10^5, one would like to have at least 10^5 cyclotron revolutions in this experiment. It is not necessary here to use a pulsed beam as no time-of-flight measurement would be required. However, it seems most attractive in connection with the trapping problem to use some periodic device. As is clear from the above the resonance frequencies for the cyclotron motion ω_c and for the spin flip ω_s will be very close. By a relative measurement of both frequencies, one obtains directly the value for $g/2$. In order to obtain a good measurement of ω_s, it will be necessary that its determination should not be disturbed by the close-lying strong resonance at ω_c. This may be difficult experimentally and require a cyclotron motion which is disturbed very little.

(b.2) A different proposal for a *resonance experiment* with low-energy electrons was made by Bloch (P 53a).¶ As in the preceding proposal, the g value would be determined by measuring the ratio ω_c/ω_s. This method makes use of the stationary states of an electron in a homogeneous magnetic field; if we have a field of magnitude $\mathfrak{B}$ in the z direction, the energies of the eigenfunctions of the Hamiltonian are characterized by 2 quantum numbers l and m_s (we give the result in the

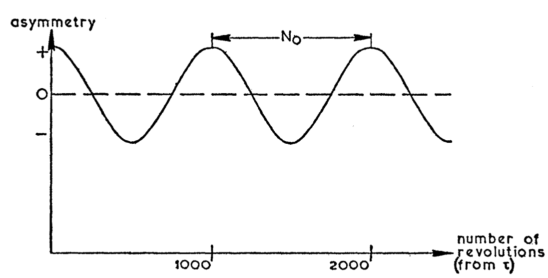

FIG. 15. Asymmetry to be expected in a different double scattering experiment proposed by Crane, for measuring the anomalous magnetic moment of the electron. The electron performs many revolutions in a magnetic field, which changes the polarization from transverse to longitudinal in about 250 revolutions. Hence the observed asymmetry should vary with a period of about 1000 revolutions, which period N_0 is a direct measure for the deviation of g from its Dirac value.

¶ In the following discussion, the author has also profited from a private communication of Dr. O. Frisch who is also considering this experiment.

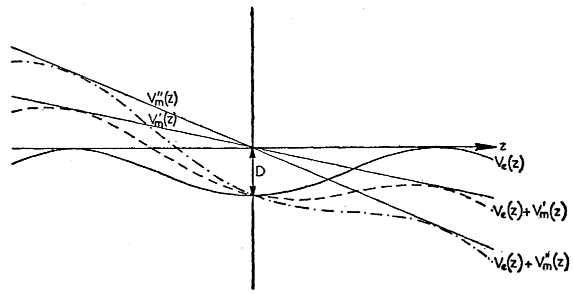

FIG. 16. Electric trapping potential V_e and magnetic "blow-out" potentials V_m' and V_m'' in the experiment proposed by Bloch for measuring the magnetic moment of the electron. A magnetic "blow-out" potential V_m'' occurring as a result from a gradient in the magnetic field (and dependent on the state of motion of the electron) will blow out all electrons from the electric trap V_e; a magnetic potential V_m' would leave part of the electrons in the trap.

nonrelativistic limit)

$$E_{l,\,m_s}=\frac{p_z^2}{2m}+(2l+1+gm_s)\mathfrak{B}\mu_0, \tag{7.1}$$

$l=0,\ 1,\ 2\cdots$ is a quantum number related to the orbital motion of the electron in the x, y directions.

$m_s=\pm\frac{1}{2}$ is a quantum number for the spin angular momentum in the z direction.

μ_0 is the Bohr magneton.

The wave functions are limited in the x and y directions, but not in the z direction; the motion in the z direction can be confined to a limited region by an electric trapping potential (see later) such that p_z is negligible. For high l, solutions are obtained corresponding to electrons moving in circles in the magnetic field. For low l, e.g., $l=0$, 1, one obtains smeared out wave packets. For $l=0$, we have a wave packet, which is largely confined to a region with radius $r\approx 0.8\times 10^{-5}$ cm for $\mathfrak{B}=1000$ gauss. Neglecting the term $p_z^2/2m$ in (7.1), we have a system of energy levels which is degenerated for $g=2$:

$$E_{l,\,m_s}=\left\{\begin{array}{llll} 0 & \text{for} & l=0, & m_s=-\frac{1}{2} \\ 2\mu_0\mathfrak{B} & \text{for} & l=0, & m_s=+\frac{1}{2} \\ & & \text{and}\ \ l=1, & m_s=-\frac{1}{2} \\ 4\mu_0\mathfrak{B} & \text{for} & l=1, & m_s=+\frac{1}{2} \\ & & \text{and}\ \ l=2, & m_s=-\frac{1}{2} \end{array}\right\}. \tag{7.2}$$

For $\mathfrak{B}=1000$ gauss: $\omega_c\approx\omega_s\approx 1.8\times 10^{10}$ sec^{-1}; $\hbar\omega=2\mu_0\mathfrak{B}=10^{-5}$ ev. The degeneracy is removed by the anomaly of the magnetic moment. If the magnetic field is slowly varying with z, the energy acts as a magnetic potential energy pushing the electron to the value of z with the lowest energy

$$V_m{}^{l,m_s}=[2l+1+gm_s]\mathfrak{B}(z)\mu_0. \tag{7.3}$$

Suppose we apply, in addition, an electric trapping potential

$$V_e=-e\varphi(z), \tag{7.4}$$

where $\varphi(z)$ may approximate near the origin

$$\varphi(z) \approx \alpha z^2 - \tfrac{1}{2}\alpha(x^2+y^2), \tag{7.5}$$

or for a more extended region near the z axis

$$\varphi(z) \approx \alpha z^2 - \beta z^4 - \tfrac{1}{2}\alpha(x^2+y^2) + 3\beta z^2(x^2+y^2) + \tfrac{3}{8}\beta(x^2+y^2)^2, \tag{7.6}$$

which represents a potential turning down again some distance from the origin so that V_e has a well-defined depth D. (See Fig. 16.) If we have the two potentials V_m and V_e simultaneously, and if an appreciable gradient $\partial\mathfrak{B}/\partial z$ is established, part of the electrons may be "blown out" of the electric trap by the magnetic field gradient (it may even be that all states except $l=0$, $m_s = -\frac{1}{2}$ are removed) (see Fig. 16).

The experiment will consist of a repetition of the following cycle (a homogeneous constant field remains throughout the experiment).

(i) *Trapping phase;* the electric trapping potential (which is maintained in stages (ii), (iii), (iv) and a gradient $\partial\mathfrak{B}/\partial z$ are established in such a way that electrons in states up to a certain l', $m_s = +\frac{1}{2}$, and $l'' = l'+1$, $m_s = -\frac{1}{2}$ are trapped.

(ii) *The gradient $\partial\mathfrak{B}/\partial z$ is removed;* the same electrons remain trapped.

(iii) *A radio-frequency field is applied,* which will cause transitions $l' \rightarrow l'+1$, $l'' \rightarrow l''+1$ at a resonance frequency ω_c and transitions $m_s = -\frac{1}{2} \rightarrow m_s = +\frac{1}{2}$ at a resonance frequency ω_s.

(iv) *The gradient $\partial\mathfrak{B}/\partial z$ is again established,* in case of a resonance ω_c, the states $l'+1$, $m_s = +\frac{1}{2}$, and $l''+1$, $m_s = -\frac{1}{2}$ will be "blown out"; for a ω_s resonance, the state l'', $m_s = +\frac{1}{2}$ will be "blown out." By observing the blown out electrons, after acceleration, by an electron multiplier, one may observe the resonance frequencies ω_c and ω_s.

The experiment has many difficulties: the depth D of the trapping potential should be very small (of the order 10^{-5} v). Such potentials at the axis may be attained by a cylindrical enclosure (say of radius 5 cm) divided into rings with potential differences partly as small as 0.02 v. Only a small number (say 5 to 10), electrons can be trapped at a time (space charge is one of the limiting factors). However if the cycle is repeated at a rate of, e.g., 180 or more per second, a sufficient number of "blow-out" electrons may be ejected so that the resonances can be observed. For this purpose, it is also necessary that the states of motion of the trapped electrons have a sufficient lifetime: the vacuum should be good enough (10^{-7} mm Hg) so that collisions are rare, and the energy exchange through radiation with the surroundings has to be considered. But in spite of the difficulties the experiment may be quite feasible.

(b.3) A *resonance experiment* rather analogous to (b.2) was proposed and tried experimentally by R. H. Dicke in 1947–1949 (private communication). This proposal also has a trapping phase, a phase in which the radio-frequency is applied, and an ejection phase. However, the details of the trapping procedure (in which carefully adjusted electric potentials and strong magnetic fields are also used) are different. The experiment gave sharp cyclotron resonances but failed to give spin resonances; the latter fact could only be understood by an involved consideration of details of the experiment.

8. POLARIZATION OF POSITONS

Positons as well as negative electrons can be polarized. Coulomb scattering by heavy nuclei provides a means for producing and detecting polarization for positons as well as for negatons. This effect was calculated for positons by Massey (T 43a, see also Sec. 3), but found to be much smaller than for negatons. This can be qualitatively understood by remembering that the region most effective for the polarization effect is the neighborhood of the nucleus. The positons penetrate less in this region than the negatons because of the opposite charge. The other methods of producing or detecting polarization for electrons given in Secs. 5 and 6, cannot be applied to positons, with the exception of β rays from polarized radioactive nuclei (e.g., one knows quite well how to polarize the β^+ emitters Mn^{52}, Co^{56}, Co^{58}).

One may ask whether special methods of detecting positon polarization can arise from the annihilation process of polarized positons. If positronium is formed with polarized positons, the ^3S state will have a preferential spatial orientation (the ^1S state is spherically symmetric). Annihilation of polarized positons stopping in matter containing polarized bound electrons may show effects depending on the relative spin orientation. However, it seems difficult to use one of these possibilities for the detection of positon polarization.

9. CONCLUDING REMARKS

The subject of polarization of the free electron deals with the very fundamental aspect of matter which is formed by the fact that the electron has a spin. In spite of the fact that the theory of a number of important consequences was treated thoroughly about 1930, it was 1942 before it became clear that theory and experiment are at least qualitatively in agreement in this field. It is gratifying that the uneasiness, which lasted for some time because of the negative results of the experiments on electron polarization was satisfactorily resolved. Generally speaking, the experiments on electron polarization are far from easy, and numerous theoretical predictions in the field were not yet checked by experiment. In spite of its intrinsic interest, this field of research has always been somewhat out of the flow of the main efforts in experimental research probably because physicists had already gained a firm belief in the quantum mechanics of the spinning electron on the basis of other experimental data. We all hope that the experiments to determine the g factor of the free electron with a high accuracy will be successful in the next few years, so relating this field with the recent developments of quantum electrodynamics.

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APPENDIX. RELATIVISTIC NOTATION FOR THE ELECTRON SPIN PROJECTION OPERATOR (P 53i, P 54c, P 55d)

The spin projection operator $P^{(+)}(\boldsymbol{\zeta})$ may be written in a clearly relativistically covariant form with the aid of Dirac γ matrices. Take instead of u_λ the positive energy solution w_λ with the same spin direction but normalized to one particle per unit volume *in the rest system*. If we write

$$\bar{P}_{\lambda\mu}{}^{(+)}(\boldsymbol{\zeta})=-w_\lambda\bar{w}_\mu(\bar{w}_\mu=w_\mu{}^*\rho_3), \tag{A1}$$

then we have the relation,

$$\bar{P}^{(+)}(\boldsymbol{\zeta})=-P^{(+)}(\boldsymbol{\zeta})\rho_3(E/mc^2), \tag{A2}$$

$\bar{P}^{(+)}(\boldsymbol{\zeta})$ may be written in the form

$$\bar{P}^{(+)}(\boldsymbol{\zeta})=\tfrac{1}{4}[1-(i/mc)p_\mu\gamma^\mu-s_\mu\gamma^{(\mu)}-m_{\mu\nu}\gamma^{(\mu\nu)}], \tag{A3}$$

with

$$p_\mu=[\mathbf{p},(i/c)E], \tag{A4}$$

$$\left.\begin{aligned} s_\mu&=(\mathbf{s}',i(\mathbf{p}\cdot\boldsymbol{\zeta})/mc),\\ \mathbf{s}'&=\boldsymbol{\zeta}+(\mathbf{p}\cdot\boldsymbol{\zeta})\mathbf{p}/m(E+mc^2),\end{aligned}\right\} \tag{A5}$$

$$\gamma^{(\mu)}=(i\gamma^2\gamma^3\gamma^4,\ i\gamma^3\gamma^1\gamma^4,\ i\gamma^1\gamma^2\gamma^4,\ -i\gamma^1\gamma^2\gamma^3). \tag{A6}$$

p_μ is a 4-vector; s_μ is a (pseudo) 4-vector. The expressions $\bar{\psi}\gamma^{(\mu)}\psi$ and $\bar{\psi}\gamma^{(\mu\nu)}\psi$ are a (pseudo) 4-vector and a 4-tensor. A product of two antisymmetric 4-tensors $T_{\mu\nu}$ and $S_{\mu\nu}$ may be written with the aid of two pairs of 3-vectors $(\mathbf{F},\mathbf{G})$ and $(\mathbf{F}',\mathbf{G}')$

$$T_{\mu\nu}S_{\mu\nu}=\mathbf{F}\cdot\mathbf{F}'-\mathbf{G}\cdot\mathbf{G}'. \tag{A7}$$

For $m_{\mu\nu}$, these 3-vectors are

$$\left.\begin{aligned} \mathbf{F}&=\mathbf{m}'=(E/mc^2)\boldsymbol{\zeta}-(\mathbf{p}\cdot\boldsymbol{\zeta})\mathbf{p}/m(E+mc^2),\\ \mathbf{G}&=-(\mathbf{p}\times\boldsymbol{\zeta})/mc.\end{aligned}\right\} \tag{A8}$$

The corresponding parts of $\gamma^{(\mu\nu)}$ are given by

$$\left.\begin{aligned} \mathbf{F}'&=(i\gamma^2\gamma^3,\ i\gamma^3\gamma^1,\ i\gamma^1\gamma^2),\\ \mathbf{G}'&=(-\gamma^1\gamma^4,\ -\gamma^2\gamma^4,\ -\gamma^3\gamma^4).\end{aligned}\right\} \tag{A9}$$

The vectors $\mathbf{s}'$ and $\mathbf{m}'$ are the directions for the spin angular momentum and the magnetic moment, respectively, of an electron with momentum $\mathbf{p}$ (for $\mathbf{p}=0$, one has $\boldsymbol{\zeta}=\mathbf{s}=\mathbf{m}$) having $\boldsymbol{\zeta}$ as spin direction in the coordinate system in which the electron is at rest. If a wave function ψ transforms according to

$$\psi'=S\psi, \tag{A10}$$

for a Lorentz transformation the corresponding transformation for $\bar{P}^{(+)}(\boldsymbol{\zeta})$ is given by

$$\bar{P}^{(+)\prime}(\boldsymbol{\zeta})=S\bar{P}^{(+)}(\boldsymbol{\zeta})S^{-1}. \tag{A11}$$

$\bar{P}^{(+)}(\boldsymbol{\zeta})$ can be considered as the projection operator for positive energy and spin $\boldsymbol{\zeta}$; we may write it in a factorized form

$$\bar{P}^{(+)}(\boldsymbol{\zeta})=\bar{P}^{(+)}\bar{P}(\boldsymbol{\zeta})=\bar{P}(\zeta)\bar{P}^{(+)}, \tag{A12}$$

where $\bar{P}^{(+)}$ is the projection operator for positive energy, and $\bar{P}(\boldsymbol{\zeta})$ the projection operator for spin direction $\boldsymbol{\zeta}$. These operators may be written in the following form:

$$\left.\begin{aligned} \bar{P}^{(+)}&=\tfrac{1}{2}[1-(i/mc)p_\mu\gamma^\mu],\\ \bar{P}(\boldsymbol{\zeta})&=\tfrac{1}{2}[1-s_\mu\gamma^{(\mu)}].\end{aligned}\right\} \tag{A13}$$

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