REVIEWS OF MODERN PHYSICS VOLUME 27, NUMBER 3 JULY, 1955

From Professor Fermi's Notebooks

THE photographs on the following pages are sample pages from one of Professor Fermi's numerous notebooks which are being preserved in the Harper Memorial Library of the University of Chicago. The pages shown are the final ones of one of the books and include the beginning of the Index, which shows how carefully he organized his written material and also on what a wide range of subjects he was willing to make calculations.

We see that he had studied a design for a double-focusing deflecting magnet as well as the more abstruse question of the pseudoscalar nature of the charged pions.

The problem which he was investigating on May 16, 1952, concerned the relative probability of two modes of decay of the π^0 meson: one mode being the predominant one of the production of two gamma rays and the other the production of one gamma ray accompanied by an electron pair. Fermi had probably heard that this problem had already been worked out by Dalitz.[1] It was typical of him that he preferred to work out the solution for himself rather than read the paper in which this had been done. He needed only to know what the idea was. He could then usually work out his own solution in less time than he could follow that given by someone else.

He continues the problem into May 17, 1952, where he evaluates a complicated integral to calculate the total probability of pair production from an expression giving the angular distribution. It is furthermore typical that when confronted with this integral he plunged ahead and evaluated it numerically rather than seeking elegant mathematical transformations which would have reduced it to elementary forms.

Finally we see the answer—namely, that pairs should be produced in one out of 137 cases. The calculation has never been published which indicates again that probably he knew that it had already been done by some one else.

H. L. Anderson
Samuel K. Allison

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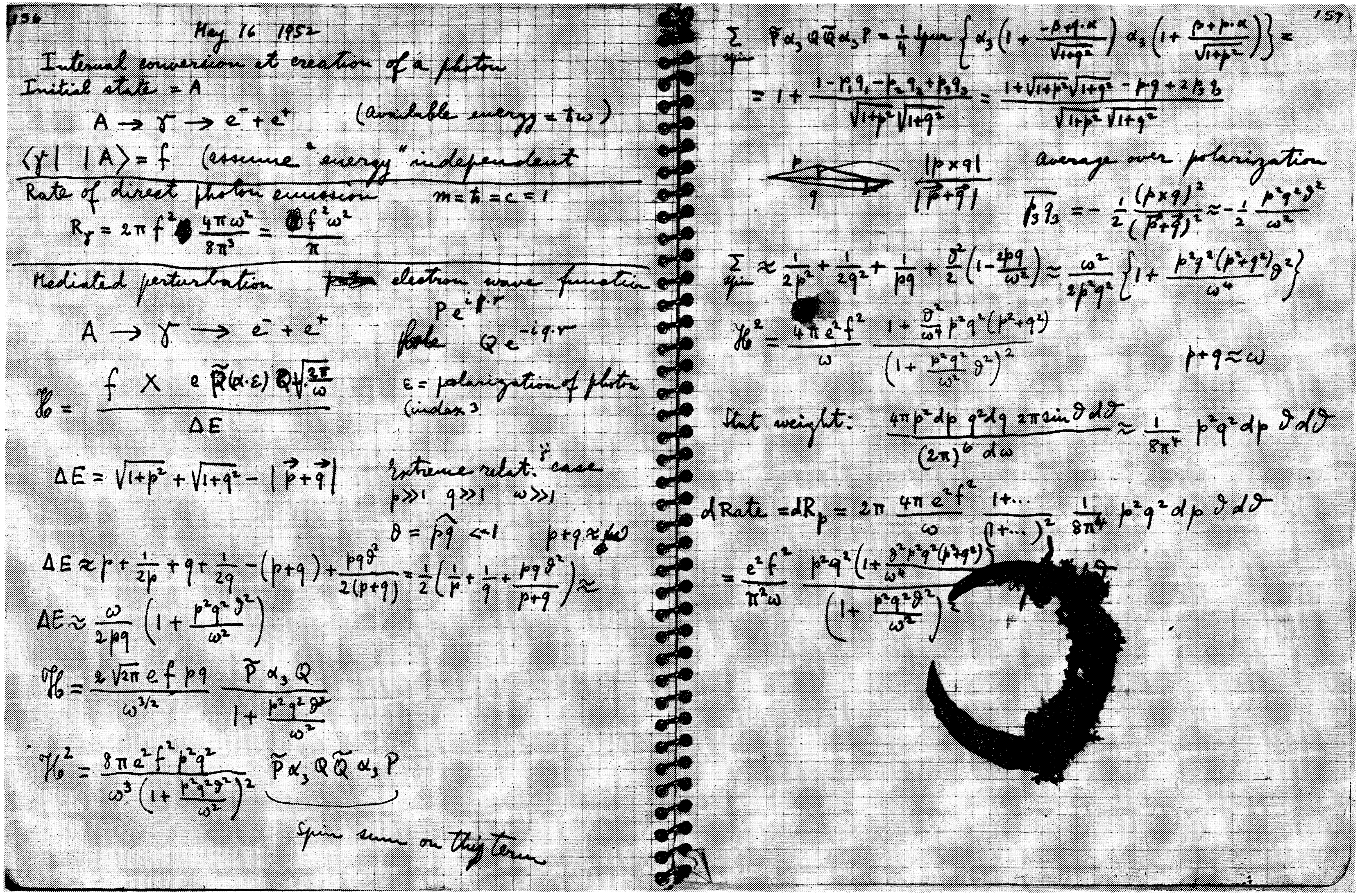
[1] R. H. Dalitz, Proc. Phys. Soc. (London) A64, 667 (1951).



156

May 16 1952

Internal conversion at creation of a photon

Initial state = A

$A \to \gamma \to e^- + e^+$ (available energy = $\hbar\omega$)

$\langle \gamma | \; | A \rangle = f$ (assume "energy" independent

Rate of direct photon emission $m = \hbar = c = 1$

$R_\gamma = 2\pi f^2 \frac{4\pi\omega^2}{8\pi^3} = \frac{f^2\omega^2}{\pi}$

Mediated perturbation electron wave function

$A \to \gamma \to e^- + e^+$

$P e^{ip\cdot r}$

~~hole~~ $Q e^{-iq\cdot r}$

$$\mathcal{H} = \frac{f \times e\,\tilde{P}(\alpha\cdot\varepsilon)\,Q\sqrt{\frac{2\pi}{\omega}}}{\Delta E}$$

ε = polarization of photon (index 3

$$\Delta E = \sqrt{1+p^2} + \sqrt{1+q^2} - |\vec{p}+\vec{q}|$$

Extreme relat.^{c} case

$p \gg 1 \quad q \gg 1 \quad \omega \gg 1$

$\vartheta = \widehat{pq} \ll 1 \qquad p+q \approx \omega$

$$\Delta E \approx p + \frac{1}{2p} + q + \frac{1}{2q} - (p+q) + \frac{pq\vartheta^2}{2(p+q)} = \frac{1}{2}\left(\frac{1}{p} + \frac{1}{q} + \frac{pq\vartheta^2}{p+q}\right) \approx$$

$$\Delta E \approx \frac{\omega}{2pq}\left(1 + \frac{p^2q^2\vartheta^2}{\omega^2}\right)$$

$$\mathcal{H} = \frac{2\sqrt{2\pi}\,e f pq}{\omega^{3/2}} \frac{\tilde{P}\alpha_3 Q}{1 + \frac{p^2q^2\vartheta^2}{\omega^2}}$$

$$\mathcal{H}^2 = \frac{8\pi e^2 f^2 p^2 q^2}{\omega^3\left(1+\frac{p^2q^2\vartheta^2}{\omega^2}\right)^2} \underbrace{\tilde{P}\alpha_3 Q\tilde{Q}\alpha_3 P}$$

Spin sum on this term

157

$$\sum_{\text{spin}} \tilde{P}\alpha_3 Q\tilde{Q}\alpha_3 P = \frac{1}{4}\,\text{Spur}\left\{\alpha_3\left(1 + \frac{-\beta + q\cdot\alpha}{\sqrt{1+q^2}}\right)\alpha_3\left(1 + \frac{\beta + p\cdot\alpha}{\sqrt{1+p^2}}\right)\right\} =$$

$$= 1 + \frac{1 - p_1q_1 - p_2q_2 + p_3q_3}{\sqrt{1+p^2}\sqrt{1+q^2}} = \frac{1 + \sqrt{1+p^2}\sqrt{1+q^2} - p\cdot q + 2p_3q_3}{\sqrt{1+p^2}\sqrt{1+q^2}}$$

p q $\frac{|p \times q|}{|\vec{p}+\vec{q}|}$

Average over polarization

$$\overline{p_3q_3} = -\frac{1}{2}\frac{(p\times q)^2}{(\vec{p}+\vec{q})^2} \approx -\frac{1}{2}\frac{p^2q^2\vartheta^2}{\omega^2}$$

$$\sum_{\text{spin}} \approx \frac{1}{2p^2} + \frac{1}{2q^2} + \frac{1}{pq} + \frac{\vartheta^2}{2}\left(1 - \frac{2pq}{\omega^2}\right) \approx \frac{\omega^2}{2p^2q^2}\left\{1 + \frac{p^2q^2(p^2+q^2)}{\omega^4}\vartheta^2\right\}$$

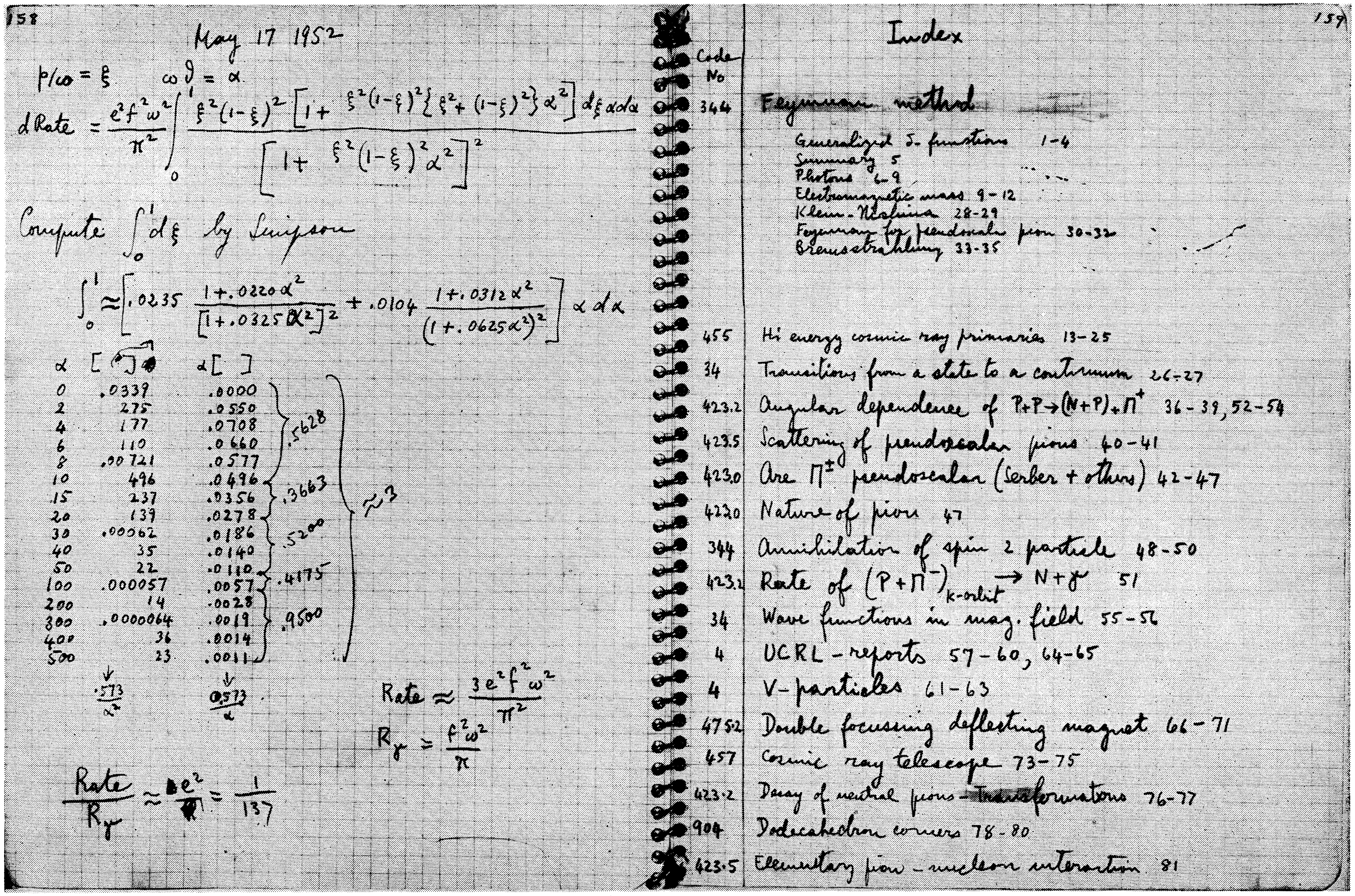
$$\mathcal{H}^2 = \frac{4\pi e^2 f^2}{\omega} \frac{1 + \frac{\vartheta^2}{\omega^4}p^2q^2(p^2+q^2)}{\left(1 + \frac{p^2q^2}{\omega^2}\vartheta^2\right)^2}$$

$p + q \approx \omega$

Stat weight: $\frac{4\pi p^2 dp\, q^2 dq\, 2\pi \sin\vartheta\, d\vartheta}{(2\pi)^6\, d\omega} \approx \frac{1}{8\pi^4} p^2q^2\, dp\, \vartheta\, d\vartheta$

$$d\text{Rate} = dR_p = 2\pi \frac{4\pi e^2 f^2}{\omega} \frac{1+\dots}{(1+\dots)^2} \frac{1}{8\pi^4} p^2q^2\, dp\, \vartheta\, d\vartheta$$

$$= \frac{e^2 f^2}{\pi^2\omega} \frac{p^2q^2\left(1 + \frac{\vartheta^2p^2q^2(p^2+q^2)}{\omega^4}\right)}{\left(1 + \frac{p^2q^2\vartheta^2}{\omega^2}\right)^2} \vartheta\, d\vartheta$$



158

May 17 1952

$p/\omega = \xi \qquad \omega\vartheta = \alpha$

$$d\,\text{Rate} = \frac{e^2 f^2 \omega^2}{\pi^2} \int_0^1 \frac{\xi^2(1-\xi)^2\left[1+\xi^2(1-\xi)^2\{\xi^2+(1-\xi)^2\}\alpha^2\right]}{\left[1+\xi^2(1-\xi)^2\alpha^2\right]^2}\, d\xi\, \alpha\, d\alpha$$

Compute $\int_0^1 d\xi$ by Simpson

$$\int_0^1 \approx \left[.0235\,\frac{1+.0220\alpha^2}{[1+.0325\alpha^2]^2} + .0104\,\frac{1+.0312\alpha^2}{(1+.0625\alpha^2)^2}\right]\alpha\, d\alpha$$

α	$[\;]$	$\alpha[\;]$	
0	.0339	.0000	.5628
2	275	.0550	
4	177	.0708	
6	110	.0660	
8	.00721	.0577	
10	496	.0496	.3663
15	237	.0356	
20	139	.0278	.5200
30	.00062	.0186	
40	35	.0140	
50	22	.0110	.4175
100	.000057	.0057	.9500
200	14	.0028	
300	.0000064	.0019	
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	$\downarrow$ $\frac{.573}{\alpha^2}$	$\downarrow$ $\frac{0.573}{\alpha}$	≈ 3

$$\text{Rate} \approx \frac{3e^2 f^2 \omega^2}{\pi^2}$$

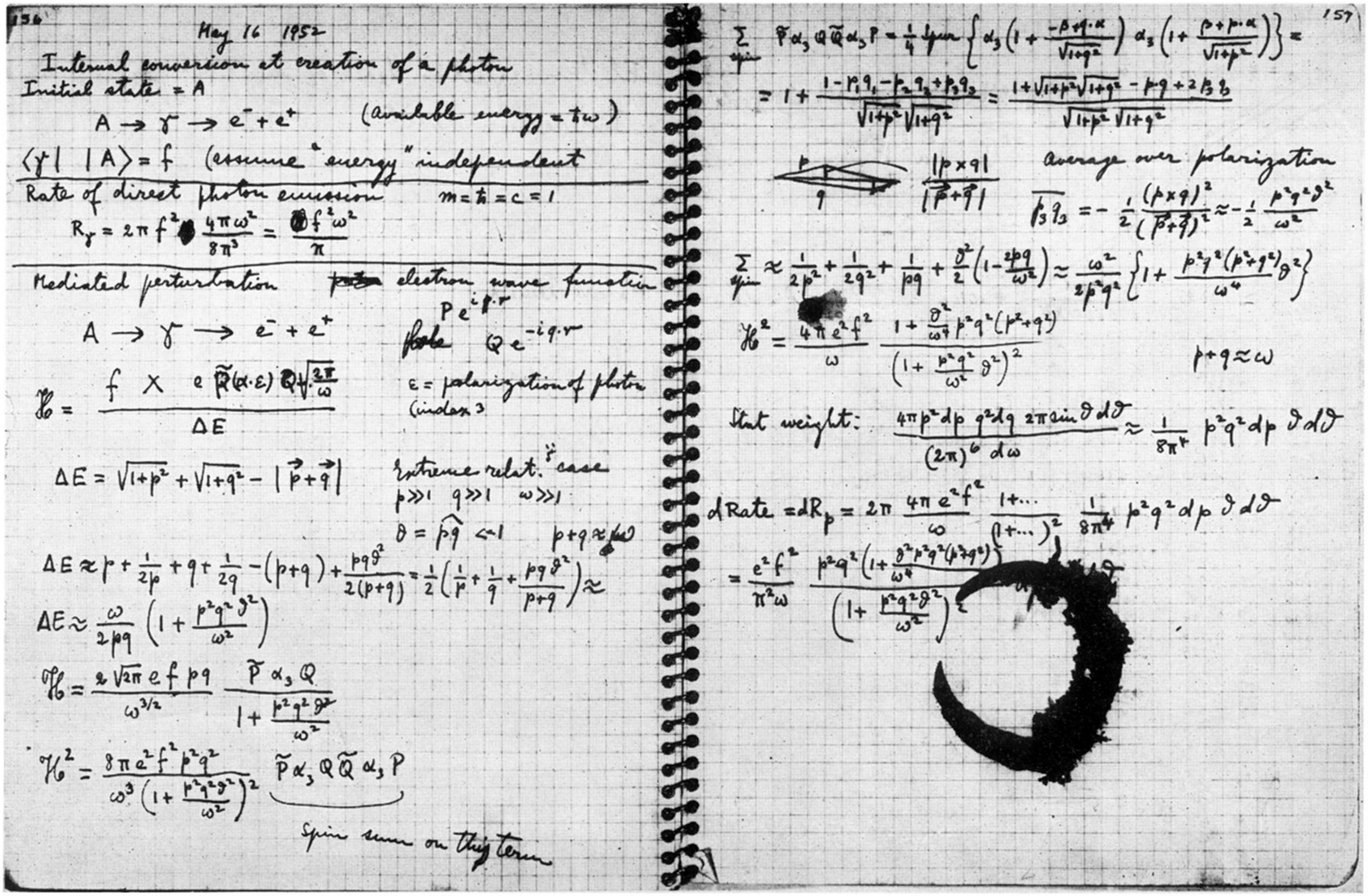
$$R_\gamma = \frac{f^2\omega^2}{\pi}$$

$$\frac{\text{Rate}}{R_\gamma} \approx e^2 = \frac{1}{137}$$

159

Index

Code No	
344	Feynman method
	Generalized δ-functions 1-4 Summary 5 Photons 6-9 Electromagnetic mass 9-12 Klein-Nishina 28-29 Feynman for pseudoscalar pion 30-32 Bremsstrahlung 33-35
455	Hi energy cosmic ray primaries 13-25
34	Transitions from a state to a continuum 26-27
423.2	Angular dependence of $P+P\to(N+P)+\Pi^+$ 36-39, 52-54
423.5	Scattering of pseudoscalar pions 40-41
423.0	Are $\Pi^\pm$ pseudoscalar (Serber + others) 42-47
422.0	Nature of pion 47
344	Annihilation of spin 2 particle 48-50
423.2	Rate of $(P+\Pi^-)_{k\text{-orbit}}\to N+\gamma$ 51
34	Wave functions in mag. field 55-56
4	UCRL-reports 57-60, 64-65
4	V-particles 61-63
475.2	Double focusing deflecting magnet 66-71
457	Cosmic ray telescope 73-75
423.2	Decay of neutral pions-~~Transformations~~ 76-77
904	Dodecahedron corners 78-80
423.5	Elementary pion-nucleon interaction 81



156

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157

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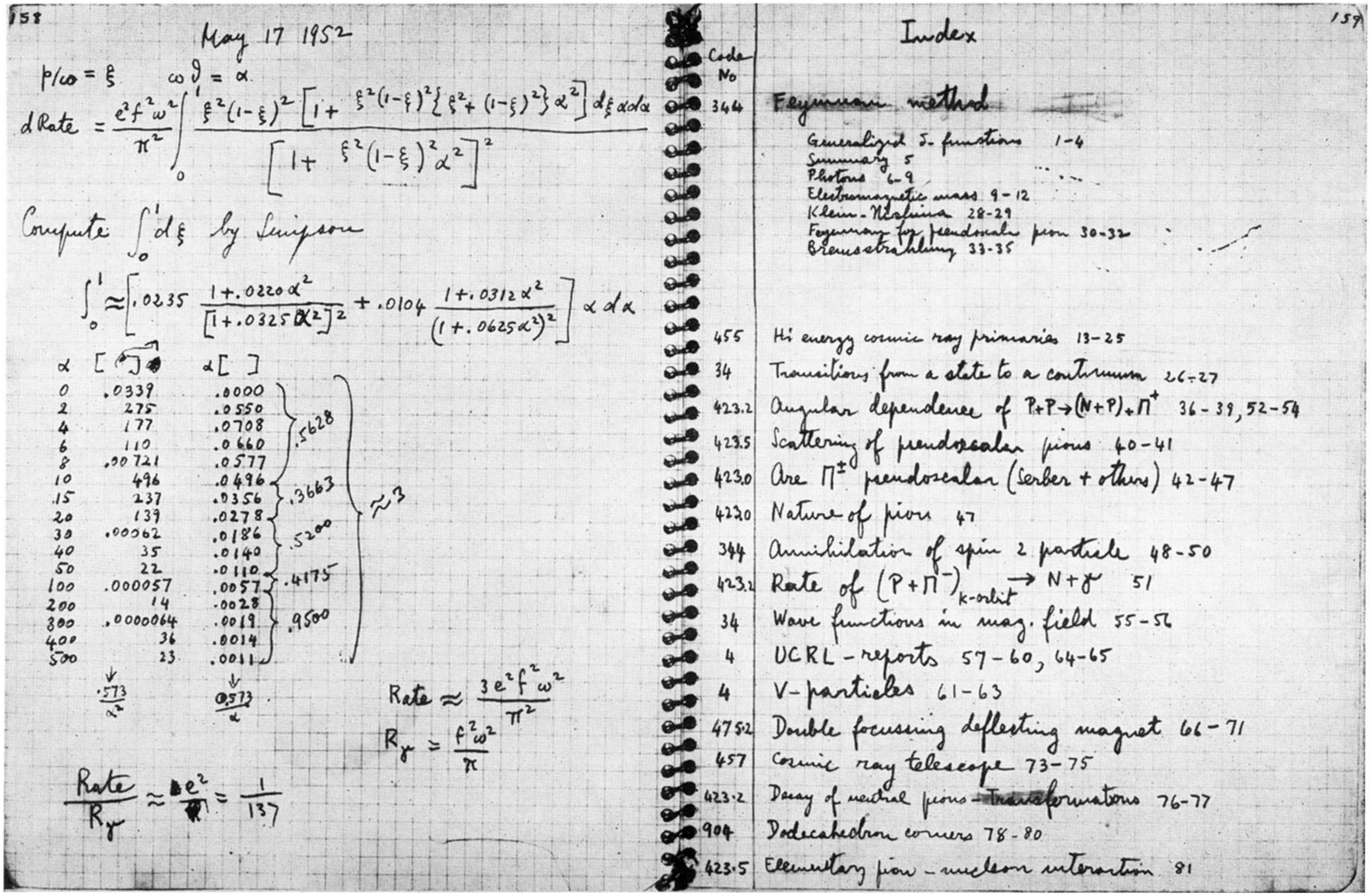

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Index