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Theories of the Earth's Magnetism

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The earth's core may be assumed to consist of fluid metal surrounding a solid inner core which probably contains a source of heat to drive convection, but it is not possible at present to select between various possible types of convective motion in the fluid core. Types considered are characterized by some sort of radial flow streams and a tendency for the fluid to rotate on the average more rapidly near the axis to conserve angular momentum during the circulation. Though the actual flow may be quite complicated, proposed mechanisms for generating a terrestrial magnetic field are considered for some oversimplified flow patterns in an attempt to indicate what features of the flow may provide the most important possibilities for field generation. It is suggested that, without a field to absorb the energy, the flow would be accelerated indefinitely and would evolve through a succession of flow patterns, some of which would be expected to have the properties to generate a field capable of preventing further acceleration and prolonging the *status quo*, thus making it likely that the earth should have a field.

The generating mechanisms discussed include two induction theories, the dynamo theory of Elsasser and Bullard, which is discussed at length both in terms of velocity-current systems portrayed by elaborate models and in hydromagnetic terms, and the "twisted-kink" theory of Alvèn which is discussed only hydromagnetically. Each of these theories depends on amplifying an initial stray magnetic field up to a point where it dissipates all of the available energy, and is at least in this respect analogous to a conventional electrical generator but without a ferromagnetic core. Other mechanisms discussed depend either on the thermoelectric effect with junctions at the core-mantle interface or on a combination of thermoelectric and Hall effects in the core and mantle.

If the convective flow is rather irregular, the observed slow westward drift of the detailed pattern of the earth's field is attributed to the vanishing of the total torque on the core by the magnetic field threading through the core and mantle, as a result of an eastward drag on the outer part of the core rotating more slowly in space and a westward drag on the more rapidly rotating part of the core near the axis, with the presumption that the observed magnetic pattern is characteristic of the westward-drifting outer part. If the flow instead involves a jet stream, the flow in the jet may under some circumstances be expected to be eastward for reasons comparable to temperate-zone meteorology, so the magnetic field should exert a westward drag on it leading to the westward drift of the flow pattern.

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1. Introduction—The Earth and the Types of Theories

It must be made clear at the outset that the theory of the existence of terrestrial magnetism is in a very unsatisfactory state. To some tastes, this makes a review article on the subject undesirable, to others, particularly desirable because by pointing out some of the difficulties and explaining some of the competing possible physical effects even in oversimplified cases it may guide and stimulate further development, besides helping the casual reader appreciate the extent of the problem and the nature of some of the possible solutions that are being considered. The general question, "Why is the earth a magnet?" is surely sufficiently fascinating to merit such exploratory and perhaps wasteful attention in considerable detail. The possibility that the magnetism of other celestial bodies may have a related origin adds to the interest, but as it is hard enough to make dependable surmises about the interior of even this planet we inhabit, we shall not go further afield here.

The most pressing unsatisfactory feature at the present preliminary stage lies not in the realm of convincing ourselves that we must be concerned with the earth's core made of molten metal having physical properties in certain ranges, but rather in not knowing how such a fluid behaves dynamically under the combined influence of gravity, rotation, magnetic field, and a thermal or other source of inhomogeneity. The comparatively simple problem of a horizontal slab of fluid heated at the bottom in which one has simple Bernal cells of convection in the nonrotating case, becomes sufficiently complicated when rotation about a vertical axis is introduced, or for a conducting fluid when a vertical magnetic field is introduced, and especially when both are introduced; so that it is quite beyond the present state of the art to make dependable surmises about the nature of the dynamical solutions in the much more complicated geometry of the earth's core.

Perhaps the best we can do for the present is to try to understand in what ways some of the simplest imaginable flow patterns can produce a magnetic field, and then try to develop some feeling for how general such effects may be for other flow patterns which we can only vaguely anticipate. In particular, after appreciating that there are certain details of a fluid flow pattern that tend to build up a terrestrial magnetic field to a point where it will absorb all the energy available and thus limit the velocities in the flow pattern, we may propose that *the flow pattern as it evolves, accelerated by an energy source, will be very likely to encounter a stage in which it has field-producing features, at which point the evolution will be halted by the field's dissipation of the energy.* Without settling many of the details such a discussion can at least make it seem plausible that the earth should through most of its history be endowed with a magnetic field.

The transmission and reflection of longitudinal and transverse seismic waves at various depths have made possible the picture of the earth's interior as consisting of a core, mainly fluid but containing a solid inner core, surrounded by a thick, essentially solid mantle and a thin crust of familiar rocks. Comparison with meteorites and astronomical indications of the moment of inertia suggest that the core is largely iron and nickel, the mantle mainly the mineral known as olivene, $(Mg, Fe)_2SiO_4$. The seismological and other evidence has been reviewed in these pages by Elsasser (1950a).* Secondary evidence that there must be some fluid within the earth, having motions more rapid than would be expected in plastic flow, is found in the secular variation of the earth's magnetic field, and the fascinating, age-old problem of accounting for the existence of terrestrial magnetism is thus naturally associated with the existence of a fluid metallic core.

Because the magnetic needle has pointed nearly north as long as it has been known, we have the impression that the earth's field is steady. Because the line of zero declination was in Europe 400 years ago and is in America now, we have an indication of a fairly long-term drift of the detailed magnetic pattern. While there is order-of-magnitude agreement between this average long-term drift and the present rate of drift, the agreement is not close enough to indicate a steady rate of drift and indeed the rate seems to be varying. While the gross features of the earth's field are similar to those of a dipole near the center inclined slightly to the earth's axis of rotation, the deviations from such a dipole field may be attributed to several dipoles or current loops about on the surface of the core, and the whole pattern has a general westward drift (Vestine *et al.*, 1947; Runcorn, 1950). The shape of the pattern is itself not steady, but is characterized by localized regions of rapid change lasting several decades, the most violent one at present being near South Africa.

A longer look at the variations of the field is provided by a study of the magnetism of successive layers of sedimentary rocks and of lava flows, which suggests that the earth's main dipole field, while maintaining its axis close to the earth's axis, may have changed in direction at intervals of several tens of thousands of years. This reversal phenomenon is still disputed (Graham, 1953) because of the possibility of reversing the magnetism of some complex rocks by heating, but it seems possible to establish the reversal by observation of selected simpler rocks (Hospers, 1951; Runcorn, 1954) so it seems at present quite likely that reversals in polarity of the earth's field have taken place. The rock indicator saturates and it is thus not possible to follow the magnitude of the field. Within this limitation, the complete reversal seems to take place within three thousand years, so the field does not appear to vanish for any extended periods.

The problem is thus to account not for a steady field but for one which may gradually vary, remaining correlated with the earth's axis of rotation but perhaps even changing polarity. As for the westward drift, our knowledge of its westward direction extends over only part of a thousand years so it might, for all we know experimentally, also change direction. Since the observed drift is westward, it is gratifying that there is a satisfactory explanation of the westward drift, as is presented below. It would be embarassing if it were eastward.

In seeking a satisfactory theory of the existence of terrestrial magnetism, it is unfortunate, though perhaps a source of the fascination of the problem, that one cannot observe the properties of the materials with which one must deal, and is forced to rely on conjecture and remote extrapolation from laboratory conditions. There is as yet no single, outstandingly successful, theory of geomagnetism, and we shall discuss several theories, any one of which might be adequate and all of which may perhaps contribute significantly to the final phenomenon observed. The extent to which competing theories rely on conjecture concerning the

* References will be found at the end of the article listed in alphabetical order.

properties of materials must be taken into account in judging their relative plausibility.

The theories discussed depend, respectively, on the electric conductivity of the matter in the earth, on this and the thermoelectric power, on these and the Hall coefficient, and on the Nernst-von-Ettinghousen coefficient. Of these electric properties, surely conductivity is the simplest. The electromagnetic induction theories which depend only on the conductivity and the velocity pattern thus involve less controversial conjecture concerning electrical properties than do the other theories.

All the theories depend, of course, on the various properties of the earth's interior that contribute to the complicated flow pattern within the core, and this involves a large amount of additional conjecture. Even after making assumptions concerning such matters as viscosity and the distribution of heat sources, the equations of motion are too complicated to have been solved. One must rely on analogy and careful guesswork in selecting plausible flow patterns, and beyond this in estimating the probable magnetic effects of the flow.

After one has established the plausible existence of energy sources sufficient to maintain the earth's field, the outstanding problem in the induction theory is one of sign, the demonstration that the inductive reaction to an initial field may reasonably be expected to be regenerative to build up and perpetuate the original field, rather than degenerative to destroy it. The polarity of the original field is immaterial so there is nothing to preclude occasional reversals with changing flow patterns. So far as the theory has yet been developed, there is also nothing to preclude the development of degenerative flow patterns and consequent nonmagnetic eras which seem not to occur more than briefly. Either the induction is actually regenerative and primarily responsible for geomagnetism or else some other mechanism such as the thermoelectric effect makes the primary contribution, but even in the latter case, induction of one sign on the other must be present and profoundly modify the result, so it is in any case desirable to understand the factors contributing to the induction in seeking an ultimate solution of the problem. For these reasons, a rather extensive attempt is here made to present some of the details of the "dynamo" inductive process, for a typical flow pattern, in a comprehensible pictorial form.

2. Convection and Relative Rotation within the Core

Progressing out from the center of the earth, the presumably solid inner core extends to a radius of 1250 km, the fluid core from there to 3400 km, and the essentially solid mantle and crust to 6400 km. The inner core thus provides an inner boundary for the thick spherical shell comprising the fluid outer core, but the inner boundary is sufficiently small that in some parts of the discussion below it is ignored.

There are several sources of relative rotation of the fluid core within the mantle. The astronomical ones, the precession of the earth's axis (arising from differential gravitational effects on the earth's ellipticity) and the tidal deceleration of the earth's rotation, have been shown by Bullard (1949a, see also Elsasser, 1950b) to be negligible and we need not consider them here. We need consider only those motions which arise from convection in the core of an earth rotating with constant angular momentum.

The traditional source of convection, the one considered by Bullard (1949a), is thermal instability caused by an assumed radioactivity of the material within the core. The evidence from meteors and mineralogy indicates that almost all of the radioacitivity of the earth is in the crust on the outer surface of the mantle, but only about one percent of this in the central body within the core, as could reasonably be assumed, would provide enough power to maintain the earth's magnetic field. The amount of heat generated, even if stored in the inner part of the mantle without being conducted to the outside, would not produce unreasonably high temperatures in the lifetime of the earth. This would thus correspond to an earth that has been heating up on the inside since its formation.

Urey (1951) has assembled evidence for the view that the earth and other planets were formed by accretion of small cold particles with the heavy iron and nickel constituents, which are now mainly in the core, originally on or near the outside. He suggests that the process of settling of the denser materials toward the inside is not yet complete, and shows that this settling could produce an entirely adequate driving force for the convection in the core needed to explain geomagnetism, at the same time providing a source of heat. The inward flow of matter is thought to be of the order of 10^{10} g/sec and the energy developed 10^{19} erg/sec.

The inward transport of angular momentum is caused both by the inward *transport* of matter provided by this settling mechanism (which in itself tends to make an eastward drift of the matter in the core relative to the mantle) and by the *exchange* of matter between the inner and outer parts of the core in the course of any gravitationally driven convection (including thermal convection). The *exchange* of matter in convection is sufficient for our present discussion. We shall return to consideration of the smaller effect of the secular *transport* of matter when we discuss the problem of the observed westward drift.

In a rigid body, a gram of matter near the axis of roation has less angular momentum than a gram farther from the axis, their angular velocities being the same. In a rotating nonviscous fluid the exchange of matter between these regions by convection would be characterized by conservation of angular momentum of the bits of matter, which would make the average angular velocity of the matter near the axis greater than that further out. The same thing may be said in terms of Coriolis force: In the convection in the earth rotating eastward, a stream of fluid moving inward along any radius (except one along the axis of rotation), experiences a Coriolis force toward the east, and an outward stream a Coriolis force toward the west, making the parts of the fluid near the axis tend to rotate more rapidly toward the east than those farther out.

So long as there is gravitational instability, such as would arise from an internal source of heat in excess of that which can be carried away by conduction or from the settling of a dense constituent of matter as mentioned above, convection may be expected to take place perhaps involving approximately radial streams of flow and flow along the spherical surface of the fluid from the outward to the inward radial streams. These features may be constantly changing and may be accompanied by other complexities. A simple convection pattern involving two downward streams along the polar axis and two upward streams on the equator is shown in Fig. 1. A similar pattern of four approximately radial streams, but all of them on the equator, is shown in Fig. 2.

Whether the flow will take place in a large-scale convection pattern such as one of these or in a much more fine-scale turbulence depends on the relative magnitudes of various forces influencing the motion. In particular, we must consider in the equation of motion the following terms:

$$\rho\dot{\mathbf{v}} = \overset{(1)}{-\nabla p} + \overset{(2)}{\rho(\mathbf{g}+\boldsymbol{\omega}\times[\mathbf{r}\times\boldsymbol{\omega}])} + \overset{(3)}{2\rho\boldsymbol{\omega}\times\mathbf{v}} + \overset{(4)}{\mathbf{J}\times\mathbf{H}} - \overset{(5)}{\eta\nabla\times[\nabla\times\mathbf{v}]}. \quad (1)$$

The balance between the pressure gradient (1) and the gravitational term (2) provides the driving force for the convection, either through the interplay of temperature and thermal expansion to provide a temperature gradient in excess of the adiabatic gradient or through a supply of a denser (and perhaps more compressible) material at the top of the fluid. The Coriolis term (3) in a part of the fluid rotating about the central axis with angular velocity ω provides a deflecting force in the direction which would seem to favor turbulent circulation in the sense opposite to ω. The magnetic term (4),

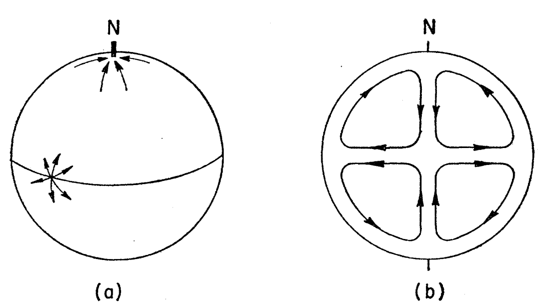


FIG. 1. A simple but implausible convection pattern in a polar section.

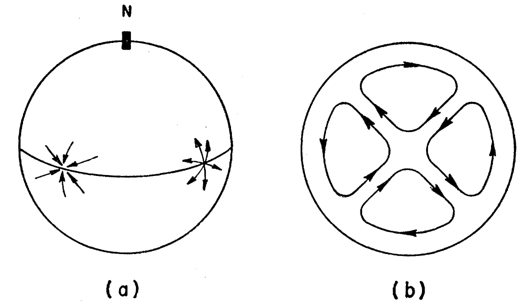


FIG. 2. A simple convection pattern in the equatorial plane.

in which permeability has been taken equal to unity because the mechanism we are discussing is not a ferromagnetic one, represents a magnetic drag on the motion of conducting matter through a magnetic field because of the interaction of the field H with the eddy currents J which it induces. The viscous term (5) ordinarily helps to maintain order in a fluid, since small-scale turbulence increases the importance of the derivative operator ∇ and thus increases the force necessary to drive the motion against this term. Without a magnetic field, turbulence ordinarily sets in when the ratio of the left member ($\rho\dot{v}$ in the preturbulent motion) to term (5), or rather twice this ratio, which may be written dimensionally

$$2\rho\dot{v}/(\eta v/l^2) \approx 2\rho l v/\eta = R \quad (2)$$

is very large, greater than 2000. R is well known as the Reynolds number. The known or plausible magnitudes of these quantities for the core of the earth are roughly

$\rho = 10$ g/cm^3, the density of the core,
$l = 2\ 10^8$ cm, the thickness of the fluid core, (3)
$\eta = 10^{-2}$ g cm^{-1} sec^{-1},
an estimated coefficient of viscosity,

and these would make $R \approx 2000$ for the very small velocity $v \approx 5\ 10^{-9}$ cm/sec. Since velocities about 10^7 times as large as this are needed merely to explain the mobility of the magnetic patterns on the earth, as is discussed further below, the motion would, without a magnetic field, have a very small-scale turbulence. A similar criterion for the expectation of turbulence in the presence of a magnetic field may only be stated even more roughly and qualitatively, and without the same empirical foundation. In the hydromagnetic case in which the conductivity σ is great enough to provide all the current demanded by the natural complexity of the field H, turbulence may be expected in general to twist the magnetic field into a pattern of a complexity comparable with that of the turbulence and represented by the same characteristic length l. In this case we may put the current density J equal to $\nabla\times\mathbf{H}/4\pi$ and the analog of the Reynolds number is the ratio of the first member to term (4) of Eq. (1):

$$\rho\dot{v}/|H\times[\nabla\times H]/4\pi| \approx 4\pi\rho v^2/H^2. \quad (4)$$

This is, incidentally, the ratio of kinetic energy to magnetic energy per unit volume. Unlike the Reynolds number, it does not contain the characteristic length l. If in rough analogy with viscosity we take 10^3 as the critical order of magnitude of this number, with $H=4$ gauss (the presumed value at the surface of the core) and $\rho=10$ g/cm^3, this allows v to become as great as 10 cm/sec before we might expect turbulence to set in. (If we take not 10^3 but 10 we get 1 cm/sec, which is just as good for our rough purposes.) With lower conductivity, the lines of H are not twisted so much and the amount of $\nabla\times\mathbf{H}$ that is developed is limited by the current J which, if we assume that the velocity pattern v is irregular enough that a return circuit is easily found, we may express roughly $\mathbf{J}\approx\sigma\mathbf{v}\times\mathbf{H}$. Equation (4) provides the relevant criterion unless

$$\sigma\mathbf{v}\times\mathbf{H}<\nabla\times\mathbf{H}/4\pi \quad \text{or} \quad 4\pi\sigma vl<1, \tag{5}$$

[where $\nabla\times\mathbf{H}$ and l are characteristic of the turbulence in question, as in (4)] in which case we have instead of (4) the larger quantity

$$\rho\dot{v}/|\sigma\mathbf{H}\times[\mathbf{v}\times\mathbf{H}]|\approx\rho v/\sigma lH^2. \tag{6}$$

We may presumably expect to have turbulence when either this or (4) exceeds some critical value such as 10^3, that is, either when the magnetic energy involved in twisting up the magnetic lines completely in the flow pattern is not great enough compared with the kinetic energy to impede turbulence, or when the conductivity is small enough to allow the flow pattern to slip through the magnetic field easily enough to permit turbulence. With a value of the conductivity typical of molten metals,

$$\sigma\approx10^5\ \text{ohm}^{-1}\ \text{cm}^{-1}=10^{-4}\ \text{sec cm}^{-2}, \tag{7}$$

the ratio (6) is greater than 10^3 when $l<(6\ \text{sec})v$, that is, for very small-scale turbulence at moderate velocities in the earth's core. Thus we find that unlike viscosity a magnetic field may, while inhibiting large-scale turbulence, permit a small-scale turbulence which is, however, in this case so microscopic as not to matter on a terrestrial scale. For velocities less than about 1 to 10 cm/sec we may expect essentially laminar flow in the large-scale convective patterns except for turbulence on a scale of a few cm or less which "slips through" the magnetic field, and which may facilitate the transport of heat into the edges of the convective pattern where it comes near the boundaries.

In the earth it thus seems that the complication of a magnetic field may in some ways simplify the flow by inhibiting turbulence. Let us now return to the elements of the problem by ignoring for a time both H and the turbulence. With thermally-driven convection alone, we expect upward and downward streams of fluid flow. With the Coriolis effects arising from rotation, we expect the matter on the inside, on the average, to have higher angular velocity than that on the outside, corresponding to approximately constant angular momentum per unit volume as the matter changes its distance from the axis of rotation. If that were all, we might then expect the upward and downward streams to be wound into ever-tighter spirals. At some degree of twisting we might expect such a pattern to break up, perhaps only to start over.

The problem here suggested is at least as complicated as the meteorology of the Earth's atmosphere, to which it has considerable similarity in spite of having different boundary conditions, and that problem has long resisted complete understanding even with rather complete access for observation. In imagining possible flow patterns within the Earth, we must at least make use of meteorological experience. Phenomena comparable to the jet stream and to the thermal wind with geostrophic flow may be important insofar as they are not suppressed by the magnetic field.

There are also important laboratory experiments on the effect of rotation on fluid flow, in some ways closely comparable with meteorological experience but with characteristic differences arising mainly from the different boundary conditions. Unfortunately it has not been possible in these experiments to reproduce or include a substitute for the spherically radial nature of the gravitational field which must be crucial for the flow pattern in the earth's core, but with due allowance for this and other shortcomings the experiments are still extremely suggestive.

The classical experiments of G. I. Taylor (1921, 1923) showed that with constant density and small viscosity [terms (4) and (5) of Eq. (1) absent, term (2) opposed by the static part of term (1), and term (3) dominant] there is a remarkable apparent rigidity of the fluid in the direction parallel to the axis of rotation (z-axis). For simplicity of explanation, let us consider the incompressible case in which in Eq. (1) the curl of term (2), as well as of term (1), vanishes, the force field in (2) being the gradient of a potential. With $\boldsymbol{\omega}$ a constant vector in the z-direction, taking the curl of the remaining terms of Eq. (1), the left member and the Coriolis term (3), gives

$$\tfrac{1}{2}\nabla\times\dot{\mathbf{v}}=\nabla\times[\boldsymbol{\omega}\times\mathbf{v}]=(\boldsymbol{\omega}\cdot\nabla)\mathbf{v}=-\omega(\partial/\partial z)\mathbf{v}. \tag{8}$$

In the first member, the total time derivative of the velocity of an element of matter relative to the rotating axes is

$$\dot{\mathbf{v}}=\partial\mathbf{v}/\partial t+(\mathbf{v}\cdot\nabla)\mathbf{v}, \tag{9}$$

of which the term $\partial\mathbf{v}/\partial t$ vanishes for steady flow. The remaining term is quadratic in $\mathbf{v}$ so for steady slow motions (that is, slow compared to the linear velocities induced by ω in a space-fixed frame) we have $(\partial/\partial z)\mathbf{v}\approx0$, indicating that the velocity pattern is not a function of z, or that line elements parallel to z move rigidly. With a horizontal boundary, as in a flat-bottomed vessel, we may then expect two-dimensional flow with $v_z=0$ throughout. This Taylor confirmed by moving a short cylinder about in a rotating dish of liquid and watching streams of ink flow around its projection in another plane. The dish had a flat bottom. Presumably a sloping bottom would impede the motion in the same way as do the vertical sides of the cylinder.

The flow in the atmosphere and perhaps that in the earth's fluid core are driven by thermal convection, and recent very interesting experiments by Hide (1953) at Cambridge study the flow of a rotating liquid driven by thermal convection. These experiments and a variety of related ones have also been carried out by Fultz and his associates at the University of Chicago. Hide used a deep flat-bottomed vessel rotating about a vertical axis with the liquid confined between inner and outer cylindrical boundaries as shown in Fig. 3(a), the inner boundary being hot and the outer cold, or vice versa. With slow rotation, the convective pattern would be a toroidal loop up near the inner wall, for example, down along the outer, and the outward flow along the top surface was observed to be not radial but spiral as a result of the small Coriolis term. With slightly more rapid rotation, however, the flow is observed to become horizontal, displaying a characteristic two-dimensional flow pattern to transport heat between the inner and outer cylinders. Aside from viscous end effects near the bottom, these patterns are of the same general shape in all horizontal cross sections, with velocity within the pattern varying with height. The orientation of the pattern may also vary with height in a spiral fashion forming "fronts" leaning forward. Here the conditions differ from Taylor's, the inhomogeniety being essential to the thermal driving force of the convection, and $\partial \mathbf{v}/\partial z$ is definitely not zero.

Most of Hide's photographic observations show only the top surface pattern, being made by photographing a floating anisotropic power. A typical pattern is shown in Fig. 3(b), and it is immediately seen that it differs very essentially from the simple imagined convection pattern of Fig. 2 in having a rather narrow "jet stream" which, while weaving radially inward and outward as does the pattern of Fig. 2, does not loop back on itself but continues to circulate in the same sense around the central axis and joins up with itself only after it has gone the whole way around [see Fig. 3(c)]. The fairly small fraction of the fluid constituting the jet stream moves rapidly, and the sectors of fluid isolated in the loops of the jet stream are impelled to rotate in both senses in the manner of idler wheels, but relatively slowly (the outer ones frequently hardly rotate at all) so that together they account for about as much radial flow of matter as does the jet stream. The narrowness of the jet stream may perhaps be accounted for in these experiments by the limited conduction of heat across the streams where they contact the hot and cold boundaries (though the corresponding meteorological phenomenon appears to be more dynamic in origin).

The explanation of this flow and the sense of flow, as given by Hide, has much in common with the meteorological theory of the "thermal wind." It is dependent

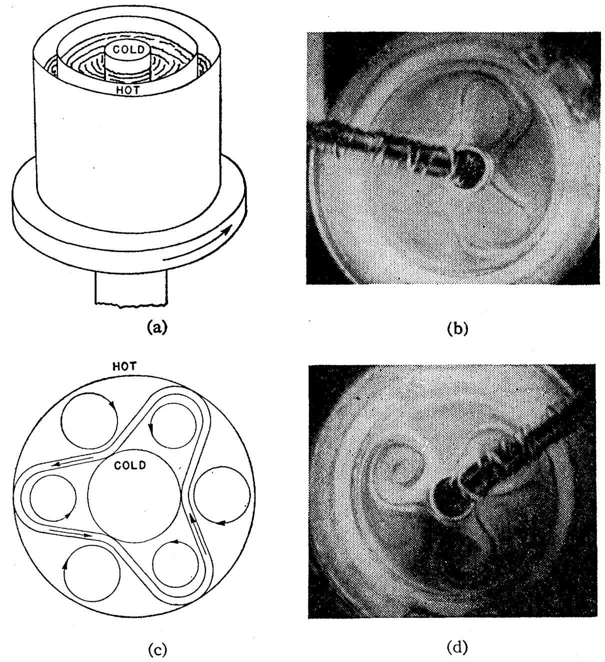


FIG. 3. Laboratory experiment exhibiting convection with a jet stream.

on having gravity (or a strong component thereof) parallel to the axis of rotation. In the rigid rotation (about a vertical axis) of a dish of homogeneous fluid, there are equipotential surfaces of the potential giving rise to the gravitational-plus-centrifugal-force term (2), the parabolic free surface being one of them. We may discuss the flow in one of them, or for simplicity in moderately slow rotation we may temporarily neglect the centrifugal force and consider the flow in horizontal planes.

Along the bottom of the vessel where the flow vanishes the pressure is then approximately independent of radius. With the inhomogeneity introduced by heating the outer cylinder, the lowered density due to thermal expansion up along the outer cylinder makes the pressure decrease less rapidly there than up along the cold inner cylinder, so that any level above the bottom (or more accurately on any paraboloid that was an equipotential surface before the thermal expansion) the pressure is higher at the outer boundary than at the inner. This requires a general circulation in the same direction as the rotation so that the Coriolis force may support the pressure difference (adding to to the effective centrifugal force), and indeed the jet flow is observed to be "forward" with the outer boundary hot, cold, or "backward" with the temperatures reversed. The patterns drift in the direction of the jet flow, as is discussed further below. This is a model of the temperate-zone circulation, with the arctic cold boundary at the center and horse-latitude warm boundary outside, in which the prevailing winds and weather patterns indeed move eastward, or "forward."

Since that "thermal wind" explanation accounts in a general way only for the direction of circulation, we may further consider the effect of the centrifugal force in impelling the jet flow. With the hot boundary outside, we have in a horizontal plane the centrifugal-force analog of a thermally unstable atmosphere, hot on the outside toward which centrifugal force is pushing it, in a way to drive any circulation which interchanges the hot and cold portions of the fluid. We may say that the warmer and thicker inward stream "floats" toward the inside more insistently than does the colder and thinner outward stream. We really have no reason to expect the flow to be so accurately horizontal as this, and indeed we run into trouble if we continue to expect this in the case with the temperatures reversed, for we see that the fluid is then stable with no flow. It is, however, unstable relative to a sort of spiral motion that departs from the horizontal plane, a compromise between the circulation in planes through the axis it would have without rotation and the tendency toward horizontal motion introduced by rapid rotation. The vertical sheet of fluid that constitutes the jet is then presumably becoming thicker and thinner at various parts as a result of fairly small vertical components of motion within it, but still larger than permitted at the upper free surface and especially at the lower boundary. The "idler wheel" regions are long rollers in which a considerable degree of vertical rigidity is provided by the Taylor effect, though this is no longer completely effective in an inhomogeneous fluid.

Let us now consider why the general flow pattern has a drift in the same circulatory direction as that in which the jet flows. To understand the steady state of the "thermal wind" we need only note that while most of the temperature drop is across the jet stream, part of it is across the "idler wheel" regions, so the same argument applies to their drift as to the jet flow. One may still attempt to trace some of the torques involved in achieving this steady state. We may make the simplifying assumption that the main motivating force of the convection acts on the jet flows, so the adjacent regions are truly "idler wheel." (The neglected thermal drive of the "idler wheels" merely increases the force of the argument.) If the pattern had no drift in the rotating coordinate system, the direction in which it would start to drift would be determined by the total frictional torque acting on the fluid as it flows past the inner and outer boundaries. With the direction of the jet flow taken as "forward," the frictional force exerted on the jet where it rubs past the boundary is backward, whereas that exerted by the surface on each of the "idler wheels" is forward, since they are rolling over backward where they make contact with the surface. If the flow pattern is essentially stationary, the torque exerted on an "idler wheel" by the boundary is equal and opposite to that exerted on it by the jet stream wrapped around its other side. The shape of the pattern appears to be such that the jet stream makes effective contact with the "idler wheels" (which are actually deformable columns of fluid and not really circular) over a much greater fraction of its length than it does with the boundary, and we may therefore expect the frictional drag between jet stream and "idler wheels" to be greater than that between jet stream and boundary. Thus the torque exerted by the boundary forward on the "idler wheels" is greater than that exerted backward on the jet stream, and the resultant forward torque would start the fluid pattern drifting forward. Alternatively, we may think of the pattern as having achieved a steady forward drift which reduces the frictional drag on the "idler wheels" and increases that on the jet stream, making the two equal.

In the meteorology of the temperate and polar zone of the earth's atmosphere, the isobars are approximately lines of flow because of the phenomenon of "geostrophic flow," or the effect of the important Coriolis term in orienting the pressure gradient normal to the flow lines. The daily charts of isobars of the Northern Hemisphere (at the "500-millibar level") present a varying pattern complicated by the geography of the continents, but they frequently include an irregular multilobed jet-stream pattern vaguely similar to Fig. 3(b), and in his thesis Hide is able to present an unusually symmetrical weather chart carefully selected to be strikingly similar to a pattern observed in the laboratory. If we wish to assume that the two patterns have the same cause, the polar region and the equatorial belt taking the place of the cold and hot boundaries, we must conclude that the varying vertical component of ω through the temperate zone apparently does not drastically effect the pattern. There is, however, a theory of the progressing waves advanced by Rosby, which depends essentially on this variation of ω_r, and it may be that both causes contribute to the observed result.

Figure 3(b) illustrates a simple, steady, three-lobed flow pattern, but Hide also finds more complex behavior. As ω increases the number of lobes increases. Under some conditions he also observes a characteristic periodic change in the nature of the flow pattern which he calls "vascillation." Starting with a flow similar to Fig. 3(c), the jet stream seems to break off at each of the points where it approaches the inner boundary and, instead of proceeding forward along that boundary, winds back around the inner "idler wheel," as shown in Hide's photograph Fig. 3(d). After this it returns to the original pattern with a continuous jet stream, and the cycle is repeated. With a steady flow similar to Fig. 3(c), the "idler wheels" are so thick compared to the jet stream that it seems there would not be time for heat absorbed from the outer wall to penetrate well into the "idler wheel" and out again toward the inner wall half a revolution later. Thus the outer "idler wheels" tend to remain hot and the inner ones cold, creating local instabilities. Vascillation apparently constitutes an equalization mechanism whereby the matter constituting the jet stream is periodically exchanged with that constituting the "idler wheels." It obviously provides a wealth of opportunity to discuss variations in the pattern of the earth's magnetic field.

Enough has been said to suggest that rather complicated flow patterns may be expected in the earth's interior. It is possible that they may have some slight similarity to some of those described here, but the complications introduced by the magnetic field, as well as the spherically radial gravitational field, will surely make serious alterations. Both the magnetic field and the spherical boundaries will greatly alter the influence of the Taylor effect. For the sake of trying to explore and explain the types of electromagnetic phenomena that may arise, we shall at first ignore most of the hydrodynamic complexity and undertake a lengthy discussion of the simple flow pattern of Fig. 2, realizing that it may have in common with the actual flow no more than the existence of inward columns of fluid flow at some azimuths, and outward columns at others.

II. INDUCTION THEORIES

3. Development of the Dynamo Theory

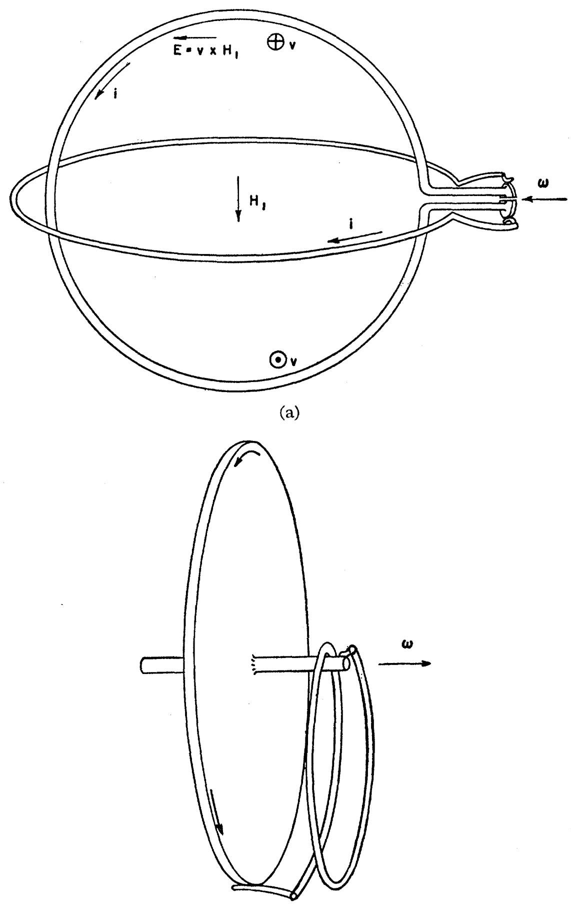
An early suggestion of Larmor (1919) that the earth's magnetism might arise from electromagnetic induction in the fluid motions within the earth's core was inadequate (Cowling, 1934) in the detail of its original form. From it there have developed two essentially separate induction theories, the induction or "dynamo" theory of Elsasser and Bullard which we consider first, and the "twisted kink" theory of Alvèn which is discussed below (Section 11). Elsasser (1946–1947) initiated the development of the dynamo theory by pointing out the importance of the toroidal field, and later Bullard (1949) suggested a possible way of completing the regenerative cycle, into an at least somewhat plausible theory of which the main outlines are fairly clear. The quantitative details remain as yet incompletely considered and are clouded by our ignorance of the physical properties of the earth's interior. The main appeal of the theory in the form in which Bullard has tentatively proposed it, is that almost the simplest imaginable convective motions, those of Fig. 2, within the fluid core of the earth seem as though they would rather probably give rise to inductive effects, if they could be calculated, of the correct sign to regenerate an accidental initial field and bring it up to a magnitude determined by the power provided by the driving forces of the convection. In this latter respect the proposed inductive effects are quite similar to those in a series-wound dynamo when the polarity of the connection between the field coils and the armature brushes is properly suited to the direction of rotation. The dynamo postulated within the earth is more complicated than a simple series-wound generator, as we shall see in detail. It is more nearly similar to a generator equipped with an auxiliary or field generator, in which the current from the armature of the main generator feeds the field coils of the auxiliary generator and the armature of the auxiliary in a regenerative fashion feeds the main field coils. One of the main problems is to show that the convective currents in the earth are "hooked up" in a regenerative, not a degenerative fashion. With a pair of terminals reversed, such a system becomes degenerative and the currents and magnetic fields vanish. A start on the problem of the existence of terrestrial magnetism can be made by trying to understand what types of convective patterns and what types of subsequent inductive effects seem to contribute to regeneration. The problem is much more complicated than the system of generators because the earth's core is presumably built without any insulators and the possible current loops analagous to the field coils are shorted out by alternative paths in parallel, making it necessary to estimate which paths are most important in contributing to the final regenerative or degenerative result.

Attempts at mathematical analysis of the inductive effects are extremely difficult and appear as yet not to have yielded a definite result, in spite of valiant efforts by Bullard and his co-workers to find a solution of Maxwell's equations for an assumed flow pattern. The trouble seems to be that, even after one has, within the simplified general pattern of Fig. 2, selected a sample detailed radial dependence of the velocity distribution, it is necessary in a power series expansion to make a sample selection of the terms in the expansion, and one cannot be sure that the sampling of the terms is adequate. It is to be hoped that further progress along those lines may be made.

In order to see what the nature of the inductive process may be, and what local features of it may be most important, we here resort instead to a graphic approach in which by means of a model we attempt to trace the electric and subsequent magnetic effects of various volume elements of the flow pattern.

4. The Simple Generators

By way of preparation for a discussion of the magnetic induction effects within the earth's fluid core, let us review the workings of the simplest model of a conventional series-wound generator, as depicted in Fig. 4(a). It is a device for regenerating and amplifying an initial accidental field (which is usually supplied in practical generators by the remnant magnetization of a ferromagnetic environment not present in this simple model). It consists of two loops of wire, one rotating within the other, connected to one another by means of brushes on a commutator arranged to maintain the correct polarity of the connection. With the commutator connection as shown, it will regenerate only if rotating in such a way that the top of the rotating coil is moving into the paper. Then an accidental field $\mathbf{H}_1$ downward, for instance, induces an electromotive force $\mathbf{E}=\mathbf{v}\times\mathbf{H}_1$ toward the left in the top and toward the right in the



(a)

(b)

FIG. 4. Two simple generators.

bottom of the rotating coil, and thus a current J which continues in such a direction through the stationary horizontal coil to produce a downward magnetic field in the same direction as the original accidental field. Whether it is large enough to regenerate the original field depends on the conductivity σ of the wire, the angular velocity ω and the dimensions. An order- of-magnitude estimate suffices for illustration, in which the inhomogeneity of H and numerical factors near unity are neglected. The electromotive force is about $Ea=avH=\omega a^2H$, the resistance is about $a/\sigma d^2$, so that the induced field may be written

$$H_2 \approx J/a \approx (\omega a^2 H_1)(\sigma d^2/a)/a = \omega\sigma d^2 H_1.$$

[The same thing may be taken directly from one of Maxwell's equations

$$\nabla\times\mathbf{H}=4\pi\mathbf{J}\approx\sigma\mathbf{v}\times\mathbf{H},$$

since $\nabla\times H$ in the stationary wire is of order of magnitude H_t/d, and H_t, the tangential field at the edge of the wire, is about equal to $(a/d)H_2$.] The condition for $H_2=H_1$ leading to successful regeneration is thus that $\omega=1/(\sigma d^2)$. With $\sigma=10^{-4}\ \mathrm{cm^{-2}\ sec^{-1}}$, $d=1$ mm, as in wire used in ordinary generators, this requires $\omega\approx10^6\ \mathrm{sec^{-1}}$, which seems rather fast. However, in an ordinary generator, this figure is cut down by perhaps a factor of 1000 by increasing the number of turns, and a factor 1000 by use of a ferromagnetic circuit, and then increased by a factor 100 by introducing an external resistor, giving a reasonable angular speed of 100 $\mathrm{sec^{-1}}$. Such a series winding is, of course, not the usual one. Without the ameliorating effects of ferromagnetic saturation, it has the characteristic that it does not regenerate until it reaches the critical speed, then uses all the power available without permitting that speed to be exceeded, since beyond that speed the field would successively amplify itself to infinity.

The critical angular speed scales as $1/d^2$. In a conducting circuit within the fluid core of the earth, if we had a commutator there, the conductor would be very fat with $d\approx10^8$ cm $=10^9$ mm. Thus the estimated critical angular speed would be reduced by a factor 10^{18} to about $10^{-12}\ \mathrm{sec^{-1}}$, implying linear speeds of the order of 10^{-4} cm/sec. This gives hope that a considerably less efficient generator might require speeds 100 or 1000 times as great, such as seem to be required by the rate of westward drift of the nondipole geomagnetic field.

Another simple generator is shown in Fig. 4(b). In it the commutator is replaced by the even simpler sliding contacts on the axle and on the edge of the rotating disk. It functions by the same principle of electromagnetic induction, which in this case amounts to the fact that the magnetic field in the stationary coordinate system transforms into an electric plus magnetic field in the coordinate system of the conducting material which makes up the rotating disk. It, too, regenerates a stray magnetic field only if it rotates sufficiently rapidly in the sense indicated.

5. Drag of Magnetic Lines in Moving Conductors

The induction effects implied by the Maxwell equations

$$\nabla\times\mathbf{H}=4\pi\mathbf{J}=4\pi\sigma(\mathbf{E}+\mathbf{v}\times\mathbf{H}).\quad \nabla\cdot\mathbf{H}=0. \qquad (10)$$

$$\nabla\times\mathbf{E}=-\dot{\mathbf{H}} \qquad \nabla\cdot\mathbf{E}=4\pi\rho, \qquad (11)$$

bring it about that magnetic lines of force tend to stay with the material in a good conductor, lagging behind the material at a rate determined by the imperfect conductivity. A simple and instructive example of this phenomenon is provided by the consideration of three conducting infinite slabs moving transversely to an exterior applied field H as indicated in Fig. 5. The motions of the slabs are in the vertical direction z, and the applied field is in the horizontal direction y. Everything is uniform in the xz-plane, and the only nonvanishing derivative is d/dy. A steady state is assumed without accumulated charges ρ, and from Eq. (11) we take $E=0$. From Eq. (10) we then have

$$dH_y/dy=0, \quad H_y=\mathrm{const}, \qquad (12)$$

and

$$dH_z/dy=-4\pi\sigma H_y v_z=4\pi J_x, \qquad (13)$$

which has opposite sign in the adjacent slabs. The lines of force are thus parabolas, joined at the boundaries as indicated to satisfy the boundary condition of a horizontal field outside the conductors, which also requires that the velocities v_2 and v_1 be related to the thickness of each outer slab a, the half-thickness of the inner slab b, and their respective conductivities σ_a, σ_b, thus

$$v_2/v_1=\sigma_1 a/\sigma_2 b, \tag{14}$$

and that

$$(H_z)_{\max}=4\pi H_y\sigma_1 v_1 a=4\pi H_y\sigma_2 v_2 b \tag{15}$$

$$Z=2\pi\sigma_2 v_2 b(a+b), \tag{16}$$

where Z is the maximum over-all lateral deflection of a line of force. As the velocities and the conductivity increase, the lines of force are dragged increasingly far downward and crowded together more nearly parallel to the interfaces between slabs as H_z increases. The force per unit area of the magnetic drag acting between the slabs, is the number of lines of force crossing the unit area times the z-component represented by one of them, or

$$F=H_y(H_z)_{\max}=4\pi\sigma H_y^2 v_1 a=4\pi\sigma H_y^2 v_2 b, \tag{17}$$

while no vertical force is exerted across the outer boundary of the outer slab with the particular boundary condition we have imposed.

It is sometimes convenient to think of the magnetic lines being dragged by the moving conductor as a result of the first and second members of Eq. (13), without reference to the current, and sometimes convenient to think of the current induced by the motion according to the second and third members of the same equation, then to think of the secondary field, in this case H_z, induced in turn by this current according to the first and third members of Eq. (13). The two procedures are, of course, equivalent and we shall use them interchangeably. The former procedure, ignoring the currents, is convenient for problems of hydromagnetics where the conductivity is effectively infinite because its greater simplicity facilitates intuitive reasoning and makes possible the treatment of some problems that otherwise would not have been undertaken. In the present application the conductivity is finite and this procedure involves curvatures in three dimensions which may to some readers seem less tangible than the lengthier discussion in terms of currents. For this reason and for the historical reason that it has not been presented elsewhere the hydromagnetic approach is postponed to Sec. 10, and we first undertake to understand the regenerative process in the procedure that involves a multiplicity of currents and frequent recourse to a familiar right-hand rule for the sign of the induction.

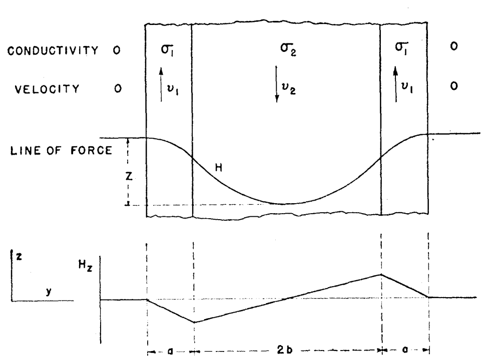


FIG. 5. Bending of lines of force by moving conducting slabs.

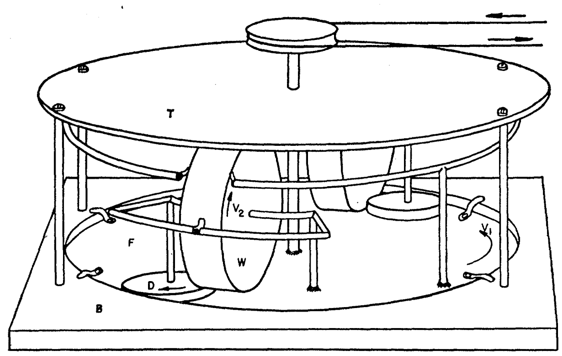


FIG. 6. A fanciful merry-go-round as a model for the fluid velocities in the earth's core which are considered important to the inductive process in the dynamo theory.

6. Visual Aids for Describing the Velocities, Currents and Fields Involved in the Dynamo Theory

The earth may conveniently be described as a magnetic merry-go-round, a rather complicated carrousel. Such a caricature is, in fact, a useful pedagogical tool in attempting to picture the relative three-dimensional motions postulated in a combination of convection and shearing rotation within the earth's fluid core, and in visualizing the four successive steps of magnetic induction to which they give rise in the regenerative process of the dynamo theory. It seems desirable to be more specific concerning the question, "What moves relative to what?", than previous published remarks have been (Bullard, 1949), and subsequently, "What current induces what field where?"

The carrousel shown in Fig. 6 helps to picture and point out the places and relative motions. It represents the northern hemisphere only, and would have to be mirrored in the equatorial plane (containing B and F) to represent the interior of the whole earth. The top T and the base B and the posts that connect them represent a conducting spherical shell within which rotates a fluid sphere. The fluid sphere with convection currents similar to those of Fig. 1 is represented in Fig. 6 by the rotating machinery inside. (The slightly more complicated machinery representing the convection pattern of Fig. 2 is discussed later, at the end of this section.) The floor F of the carrousel, driven by the pulley above, rotates in a direction indicated by the arrow v_1. (We shall see that the result is not affected by this arbitrary

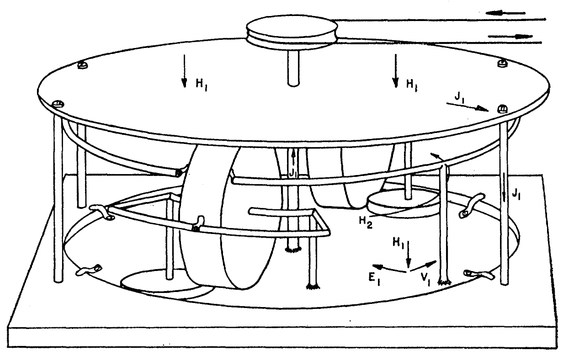


FIG. 7. First step in the inductive process.

choice of the conventional positive angular velocity vector, upward along the z-axis.) The thick cylindrical wheel W is mounted on F by a framework and is driven by the drive-wheel D which rubs on its lower edge and on the inside of the base B. The wheel W thus rotates about a horizontal axis in the sense shown by the arrow v_2, while it rides around on the carrousel. The velocity v_1 thus represents the rotation and v_2 the convection.

The fluid core of the Earth is, of course, a continuous conducting medium, providing conducting paths in all directions everywhere. On the carrousel, by means of rods and sliding contacts, we have pictured those typical conducting paths to which we wish to call attention for purposes of explanation. The possibility of pointing them out in this way facilitates subsequent discussions of the relative importance of various paths in the regenerative process.

Figure 7 shows the first of the four steps in the regenerative process. The accidental initial field H_1, which we wish to regenerate, is assumed downward to correspond to the situation in the earth. This is quite arbitrary and a matter of definition: The earth's north pole is a magnetic south pole because it attracts a "north-seeking pole," or north pole of a compass, which defines the direction of a magnetic field outside the earth. Inside the core of the earth we have the return circuit from geographic north to south. The velocity v_1 of the floor F traversing the field H_1 induces the electric field $E_1 = v_1 \times H_1$ which gives rise to the current J_1 through the complete circuit indicated, across the floor F, up the central shaft, across the top T, and down the outer post and through a sliding contact between B and F. This is, of course, typical of the current at all

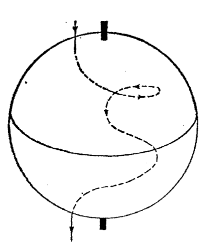


FIG. 8. Dragging of lines of force into a toroidal form, result of the first step.

four center posts in the carrousel, or, indeed, at all azimuthal angles in the earth, and gives rise to the "toroidal" field H_2 which forms closed circles about the central axis, eastward in the northern hemisphere (Fig. 7) and westward in the southern hemisphere. This is the step that was pointed out and emphasized by Elsasser (1947), and is fundamental to any process of inductive regeneration. The vector sum of H_1 southward and H_2 eastward is a field in a general southeasterly direction, so the net result of this first step is to drag the lines of force around in a spiral toward the south east in the northern hemisphere and to let them spiral back toward the south west in the southern hemisphere, as suggested in Fig. 8, which is taken from Fig. 16 of Elsasser's review (1950a). This represents a solid sphere rotating within a conducting spherical shell, very similar to its two-dimensional analog discussed above in connection with Fig. 5.

The second step of induction is shown in Fig. 9. The toroidal field H_2 passes through the conducting wheel W and in the outer part of this wheel the upward

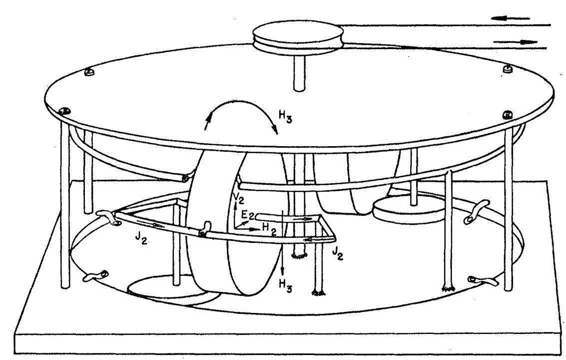


FIG. 9. Second step in the inductive process.

velocity v_2 traversing H_2 induces the inward electric field E_2 (in the coordinate system of the conductor W) which drives the current J_2 in such a way as to produce a magnetic field H_3 downward on the right side of W (and also upward on the left side of W, though this is not indicated in such detail). The circuit of J_2 rides around on the carrousel, and H_3 is a magnetic field in this moving coordinate system. It loops up through the top T on the left side of W and down through it on the right side as indicated.

Figure 10 shows the third step. The coordinate system of H_3 moves eastward with a velocity represented at some radius by v_1. An equivalent way of saying this is that the top T moves with the reversed velocity similarly represented by $-v_1$ relative to the coordinate system of H_3, and thus traverses H_3 toward the left in such a way as to induce an outward electric field E_3 at a point in the top T to the right of the wheel W, where the vertical component of H_3 is downward as indicated. This produces eddy currents J_3 in loops both ways around this region, but the important loop

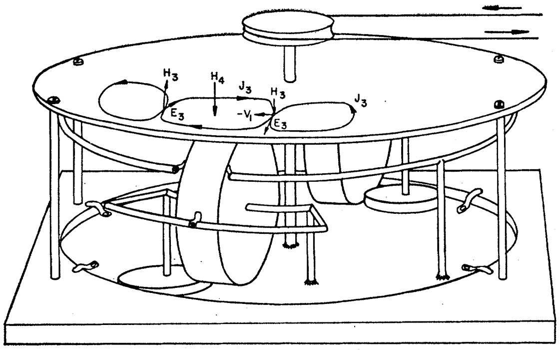


FIG. 10. Third step in the inductive process.

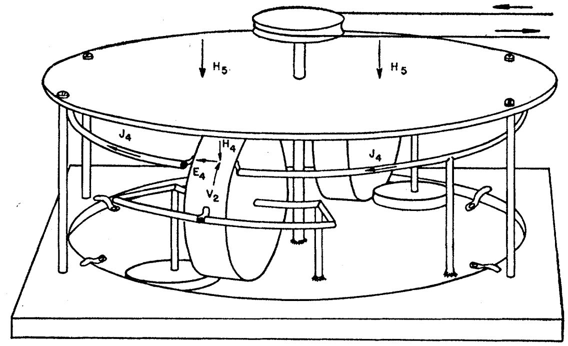


FIG. 11. Fourth step in the inductive process, which reproduces the original field.

is the one which includes the inward E_3 over on the left side of W. This loop produces a downward H_4 in the central plane of the wheel W. We thus see that, from the point of view of an attendant riding on F, the upper part of the field H_3 has been dragged backward (westward) in this step.

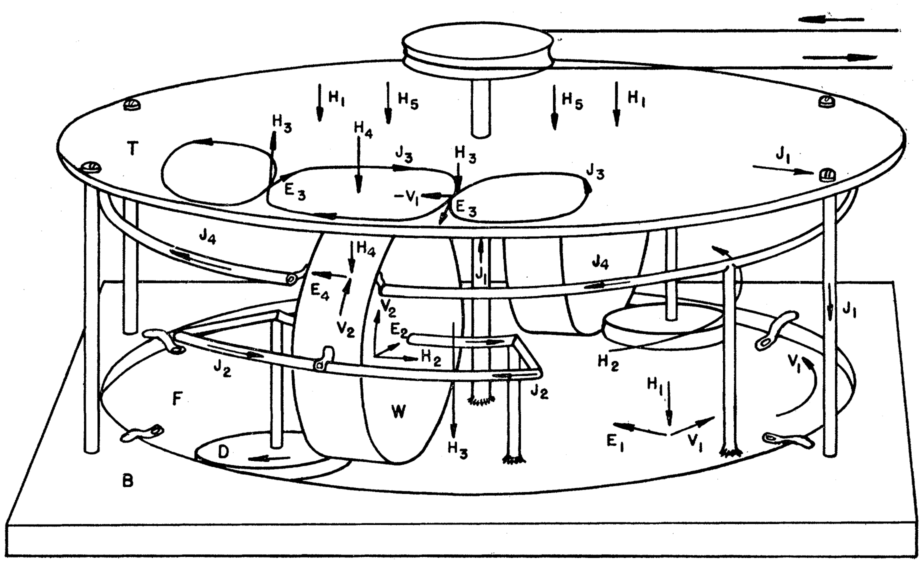
The fourth and last step of the regenerative process is shown in Fig. 11 and all four steps are summarized in Fig. 12. The field H_4 is traversed by the outer half of the top of W with a velocity v_2 inward, inducing a westward electric field $\mathbf{E}_4$. This (in series with a similar field induced in the similar wheel shown mounted opposite and possibly other such wheels not shown) drives the current $\mathbf{J}_4$ around a large loop, which produces H_5 downward, in the same sense as the assumed original field $\mathbf{H}_2$, which is thereby regenerated if the magnitude is great enough. We thus see, by comparison with our previous discussion of a simpler generator, that this curious carrousel would work as a dynamo if large enough and driven fast enough. To show that the earth's interior would do likewise requires much further discussion, a little of which follows.

We note that here (and also in the process we have described in Section 7) two of the four steps involve v_1 and the other two v_2, so the regeneration is quadratic in v_1 and in v_2 and *independent of the direction* of either. Thus the wheel W turning in the opposite way would produce the same final current $\mathbf{J}_4$.

The convection with four currents in the equatorial plane, two up and two down, sketched in Fig. 2 is roughly represented in the northern hemisphere by a carrousel with four wheels W, two of them rotating in the opposite sense. This motion sketched in Fig. 2 is the case considered specifically by Bullard (1949a) in his pioneer discussion of the sign of the regeneration. Our discussion of the carrousel which follows his of the sphere closely for the first two steps in Figs. 12 (and 14), is motivated mainly by the impression that his qualitative discussion, especially of the subsequent steps, is too brief for easy comprehension and much less specific than befits the inherent interest of the dynamo process.

Our explicit discussion has been confined to the northern hemisphere. The patterns we have considered are symmetrical relative to reflection in the equatorial

FIG. 12. The four steps of the regenerative process, assembled.



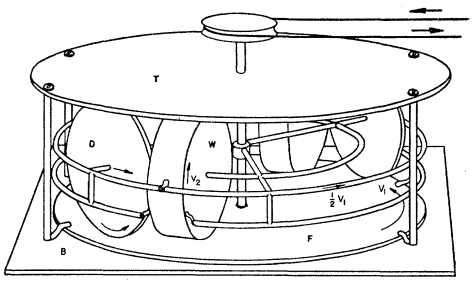


FIG. 13. A dynamo model invoking horizontal shear.

plane. The fact the regeneration is independent of the direction of H_1 is then sufficient to establish the applicability of the discussion to the southern hemisphere. More explicitly, the vertical component of v_2 has the opposite direction in the southern hemisphere and all other motions are the same, while H_2 has the opposite sign (Fig. 8). These each enter the sequence of four steps only once, so there are two changes of sign in the chain from H_1 to H_5, which leaves the sign of the regeneration unchanged in going to the southern hemisphere.

7. The Compound Carrousel—A Model Incorporating Internal Rotational Shear†

Let us now discuss another pedagogical contraption which illustrates another type of relative motion. The simple carrousel discussed above may be used as an analog of a sphere which is solid so far as its rotation is concerned, but with convection within it, rotating within a conducting shell, with electrical contact between them. In turn, this may be used as a rather crude model of the interior of the earth. In a rotating fluid fluid sphere experiencing some sort of frictional or magnetic drag with its surroundings, it would be expected that, at a given distance ρ from the axis of rotation, the fluid near the equatorial plane might rotate fastest, that is, there would be rotational shear across planes parallel to the equator.

To represent this shearing motion within the core we make our carrousel one step more complicated, as shown in Fig. 13. In it the top T, the base B, and the floor F are much the same as before, but the drive-wheel D is

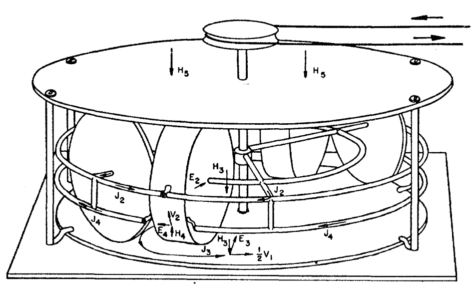


FIG. 14. The second, third, and fourth inductive steps involving horizontal shear.

† The casual reader may omit Secs. 7 and 8 without serious loss of preparation for following sections.

mounted on a horizontal radial axis which is part of an intermediate framework which supports the thick wheel W. The drive-wheel D rolls on T and on F in such a way that the intermediate framework, which includes the axle of d, rotates about the central vertical axis with just half the angular velocity with which F rotates, as indicated in Fig. 13 by the vector $\frac{1}{2}v_1$. The drive-wheel D also rubs against W in such a way as to impart to it the same rotation as before, as represented by the typical velocity $\mathbf{v}_2$.

Steps 1 and 2 are the same as before, up to the formation of $\mathbf{J}_2$ and $\mathbf{H}_3$. Steps 3 and 4 are indicated on Fig. 14. In step 3, the vertical component of $\mathbf{H}_3$ penetrates the floor F downward on the right of the wheel W and upward on the left. The floor F moves to the right with a rotation indicated by $\frac{1}{2}\mathbf{v}_1$ *relative to the loop in which* $\mathbf{J}_2$ *flows to make* $\mathbf{H}_3$, and this gives rise to the electric field $\mathbf{E}_3$ inward along a radius of F on the right of W and outward on the left. The current $\mathbf{J}_3$ thus circles around the outer part of the bottom of W in such a way as to make the magnetic field $\mathbf{H}_4$ upward. (Again a matter of the motion dragging $\mathbf{H}_3$, this time forward, to become $\mathbf{H}_4$.) In this field $\mathbf{H}_4$, the outward motion $\mathbf{v}_2$ of the bottom of W as a fourth step induces the electric field $\mathbf{E}_4$ to the left, which again makes $\mathbf{J}_4$ in the large core-encircling loop to produce $\mathbf{H}_5$ in a sense to regenerate $\mathbf{H}_1$.

Both of the processes of regeneration suggested by Figs. 12 and 14 could contribute simultaneously to the magnetization of the earth, along with others and in competition with several similar processes of degeneration, all with this one combination of motions. Bullard (1949a) pointed out that there are also several possible combinations of motions which may regenerate the original solenoidal field.

8. Some Other Current Loops

As yet the sides of our carrousel have been left open so that we may see in and also for the historical reason that the relative motions thus represented correspond most closely to Bullard's original explanation of the regenerative process and its probable sign. The core of the earth is surrounded by a sphereical shell which extends all around it, not just on the top and bottom, so we may improve our carrousel by surrounding it by a cylindrical shell to cover the sides, as shown in Fig. 15 with part of the cylinder cut away. The motions in the interior are the simple ones of Fig. 6 [not compound as in (Fig. 13)].

A very important current loop is the one shown carrying $\mathbf{J}_2$ approximately in a vertical plane, giving rise to $\mathbf{H}_3$ inward along the cylindrical radius. This $\mathbf{H}_3$ is dragged toward the west (in the rotating coordinate system) making $\mathbf{H}_4$ as the result of eddy currents in the conducting cylindrical shell as indicated, and in Step 4 this $\mathbf{H}_4$ is traversed by the wheel W with $\mathbf{v}_2$ upward, inducing $\mathbf{E}_4$ toward the west as before and thus regenerating $\mathbf{H}_1$. This loop for $\mathbf{J}_2$ together with the one

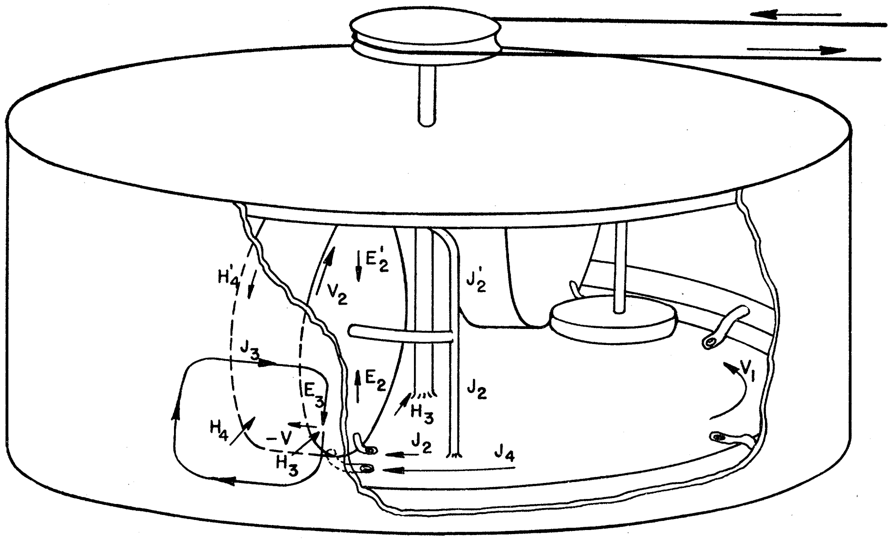


FIG. 15. Some current loops involving the equatorial regions of the mantle.

shown in Fig. 9 may be represented by an oblique one emphasizing the outward-and-upward motion of the convection suggested in Fig. 2, but the outward motion is more important than the upward motion because only part of the outward-moving material flows upward, while part flows horizontally as it approaches the surface. Thus the $\mathbf{J}_2$ of Fig. 15 is expected to be more important than the $\mathbf{J}_2$ of Fig. 9 in the regenerative process. (Its prototype in the earth is also geometrically more favorably situated, as will be clear after discussion of the westward drift.)

In the carrousel of Fig. 15, the current $\mathbf{J}_2'$ arising from the motion of the upper half of W circulates in the opposite sense, and this is the only reversal in the associated four steps, so this constitutes a competing degenerative process. The same is true of the inner half of W in Fig. 9, though the corresponding current loop is not there drawn. These loops in the carrousel do not cast serious doubt on the adequacy of the regenerative process in the earth because the parts of the motion involved do not correspond so directly to motions we wish to assume in the earth, in a simplified convective pattern similar to Fig. 2, rather than Fig. 1. We use the carrousel to help visualize motions within the earth, but it also contains some motions which we do not want to assume even in a simplified pattern in the earth because they are so strongly suppressed by the Coriolis force.

9. The Inapplicability of Cowling's Theorem

One of the serious objections that may be raised to the dynamo theory of geomagnetism as here presented is that it is too complicated to be called elegant. One is requested to twist his wrist several ways and almost stand on his head in the course of trying to understand it. An induction mechanism by means of a simpler and more elegant configuration of currents or lines of force would be preferable if one could find it. A sort of lower limit to the complexity is provided by Cowling's theorem (Cowling, 1934), which was developed to show that the very simple induction mechanism originally proposed by Larmor (in connection with sun spots) was too simple to provide a self-sustained field in the earth, though a comparable simple rotation may be of interest in concentrating a field in sunspots which is maintained by a more complicated mechanism elsewhere in the sun.

In reporting the induction mechanism outlined above as a possible source of the earth's magnetism, it is necessary that we show that it is not subject to the contradiction of Cowling's theorem. It is hoped that this discussion of the theorem and its inapplicability will also serve to bring the reader into a mood of willingness to consider a fairly complicated mechanism by showing that for an induction mechanism to work, some such degree of complexity is necessary to evade the theorem.

Cowling's theorem in its simplest form refers to the situation shown in Fig. 16(a), in which attention is directed to a closed ring, indicated by a double line, about the axis of cylindrical symmetry of the system. Through this ring thread magnetic lines of force H, surrounding the ring in closed loops. (The ring is then at a maximum or minimum of the vector potential.) The theorem states that the currents induced around the ring by possible motions of the conducting medium in the magnetic field cannot maintain the field, in particular in the immediate vicinity of the loop, so the inner closed line of force will soon disappear, and then the successively larger ones will collapse about it.

In proving the theorem one applies to a very small loop of H about the ring the familiar line and surface integrals

$$\int \mathbf{J}\cdot d\mathbf{S} = \oint \mathbf{H}\cdot d\mathbf{s} \equiv H_0 s. \tag{18}$$

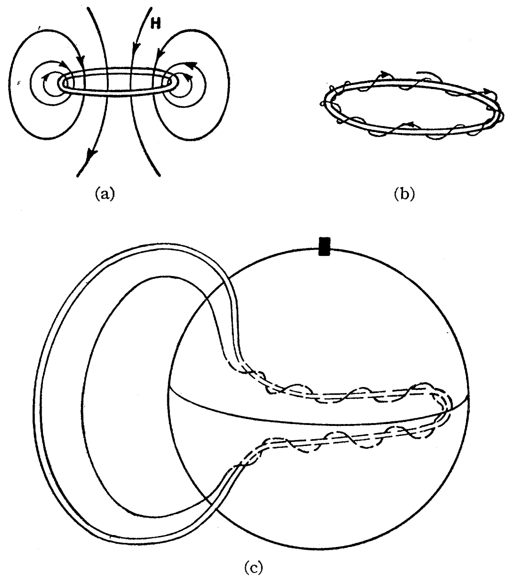


FIG. 16. Twisting of bundles of lines of force is encountered only in complex geometrical situations.

Here s is the distance around the loop (*not* the ring) and H_0 is thus an average field around it. If we neglect the contribution of potential differences to the current (as we may in this case because of the cylindrical symmetry assumed also for the flow pattern), we have

$$\mathbf{J}=\sigma\mathbf{v}\times\mathbf{H}<\sigma v_{\max}H_0. \tag{19}$$

Here $v_{\max}$ is the greatest velocity at any point in the loop. The first integral in Eq. (18) is thus less than $\sigma v_{\max}H_0S$, where S is the area of the loop, so we have the inequality

$$H_0 s<\sigma v_{\max}H_0 S. \tag{20}$$

Since S is quadratic in s, the inequality is not satisfied in the limit for a small loop with finite $\sigma v_{\max}$, which shows that the current is insufficient to maintain the field. It also happens that H_0 decreases as the loop over which it is an average gets smaller (as follows from Eq. (18)), but this cancels out in Eq. (20).

The more general form of Cowling's theorem applies to the situation of Fig. 16(b). Here again a closed ring is discussed so that the integral of grad V around it shall vanish, but this time the nearby lines of force are twisted about it, always threading through it in the same direction. Because of the happenstance just mentioned, that the component of H normal to the ring and circulating around it vanishes in the limit as one approaches the ring, the ring itself is a line of force. Even without cylindrical symmetry, Eq. (19) still applies to an average around the ring of the component of J along the ring, and the theorem applies that the field threading through the ring cannot maintain itself by induction arising from velocity of the medium. If the twisting lines did not always thread the ring in the same direction, then $\int H\cdot ds$ in Eq. (18) would be at some places positive, some negative, and (20) could be satisfied in the average by cancellation.

In the terrestrial inductive process as we have been describing it, the relevant magnetic loop analogous to the ring of Fig. 16(b) is sketched schematically in Fig. 16(c). It loops around outside the conducting core and dips into it in such a way as to bend (at least part way) around the axis like a curved hairpin. The bundle of adjacent lines of force, one of which is shown in Fig. 16(c), twists around the two prongs of the curved hairpin. Corresponding to the fact that the final toroidal current, J_4 of Figs. 11, 14, and 15, is toward the west in both hemispheres, these lines of force twist around in one sense in the northern hemisphere and in the opposite sense in the southern hemisphere, from the point of view of one going around the loop. Topologically, they wind up on one leg of the hairpin and unwind on the other, in a manner in which originally separate continuous bands could be intertwined. For this reason Cowling's theorem does not apply.

10. The Dynamo Theory from the Hydromagnetic Point of View

Hydromagnetics, or magnetohydrodynamics, as it has usually been called, is in many cases greatly simplified by forgetting about induced currents and considering only the tendency of lines of force to be dragged along by moving conducting material. Even though it may seem plausible that the current loops we have portrayed in Figs. 7–14 and particularly Fig. 15 may be the most important ones, it would make the theory seem simpler and additionally convincing to see from this hydromagnetic point of view that the lines of force are kept from collapsing by being dragged along by the rotational and convective streams of matter. In the discussion of Fig. 8, we have already seen this for Step 1. The next steps are a little more involved and it is not so easy to make them as graphic, but with all their complexity they provide a true portrayal of the main regenerative process proposed in the dynamo theory. From the discussion of Cowling's theorem, it is clear that we must discuss not just one line of force, but a bundle of lines of force, and consider how they become twisted about one another. Consider the part in the northern hemisphere, the upper leg of the "hairpin" in Fig. 8 and Fig. 16(c). The bundle of lines of force H in Fig. 17(a) is pushed upward (i.e., outward along the radial direction ρ, toward the right in the diagram) and northward by the convective current $\mathbf{v}$, which has components v_ρ and v_z, as shown in Figs. 17(b) and 17(d). By way of a simple model (in which a gradual rotational shear is replaced by a sudden shear) we consider the fluid not rotating, but contained in a conducting spherical shell which rotates so as to have a westward velocity $-v_1$ at the surface. This drags toward the west the upward (ρ) component of the bulge, as shown in an exaggerated fashion in Fig. 17(c). Even the lines that do not bulge out into the surrounding shell are thus affected by the crowding together of the lines on the east side of the bulge. For a given ρ a little inside the surface, the component v_ρ has a maximum in the equatorial plane, while v_z is there zero and has a maximum value some distance above this plane. In the equatorial plane, v_ρ approaches zero near the surface. The effect we are about to describe depends on both v_ρ and v_z, so is most important where **v** is directed near the 45° direction and fairly near the surface. Let us consider the effective **v** in this direction having maximum value along some line, and consider a bundle of lines of force H which before deflection is centered on the line of this maximum **v**, as suggested by Fig. 17(f). It is deflected along ρ and z, but less along ρ than z in the central azimuthal plane of the convection current, because of the geometrical effect shown in Fig. 17(c), (the side of the bulge being not so far out as the extreme of the bulge, while the z deflection is not thus modified down in this corner near the equator). The maximum **v** now pushes near the outer edge of this bundle of lines of force, in such a way as to give it a net counterclockwise twist, as in Fig. 17(g). The corresponding twisting moment is clockwise in the southern hemisphere. (For a downward convection current there is an inward bulge and **v** is reversed; the twist is still counterclockwise in the northern hemisphere, clockwise in the southern.) If we consider the lines of force to be effectively "anchored to the wall" where they penetrate through the conducting mantle (by the local eddy currents therein), then we see that these twisting moments are just what is needed to twist up the bundle of lines of force in the manner shown in Fig. 16(c).

The cumulatively important part of the twisting occurs in the long stretches of the bundle, between the azimuthal planes of the convection currents where the twisting torque is applied, just as in the consideration of currents in Fig. 15 (for example) the important regenerating current $\mathbf{J}_4$ flows in a large loop almost around the sphere as a result of electric fields confined

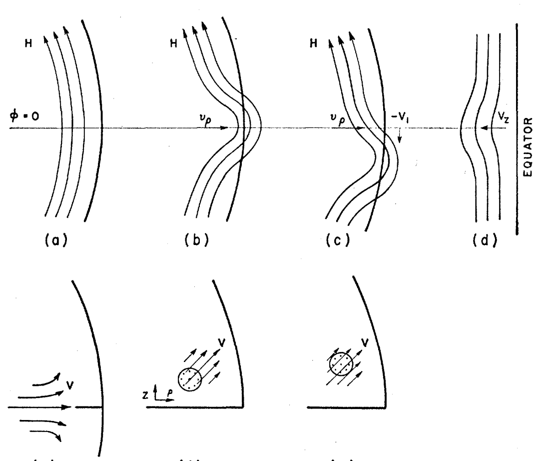


FIG. 17. Hydromagnetic equivalent of the dynamo theory.

to the vicinity of the radial convection streams. Within the bulge shown in Figs. 17(b) and 17(c) there exist local twistings and turnings in both senses which appear to be unimportant because they affect so short a length of the bundle, corresponding to the local currents set up by $\mathbf{E}_4$ in Fig. 14, for example, in the small space between $\mathbf{E}_4$ and the outside of the conducting region in a return loop not shown in the figure. In addition to the central bundle selected in Fig. 17(f), there are other bundles on either side of the maximum **v** on which twisting torques act even before the displacement, but these tend to cancel out between the two sides and the change in torque induced by the bulging displacement is in the same direction as in the representative central bundle.

To complete the schematic hydromagnetic picture we must now imagine the irregularly-shaped loop in Fig. 16(c) repeated several times around the sphere in such a way that the "hairpin" part of each penetrates into the region between the two branches of the next "hairpin." Each of these twisted bundles making up a "hairpin" is defined by the way it passes through a particular upward or downward convective stream, so there is not a continuum but a discrete set of them that need not intertwine where they are locally adjacent. The reason that the external field represented by the outside part of the loop cannot collapse to nothing is then that it surrounds the "hairpin" of the next loop, in which the twisting mechanism explained above is operating to maintain a curl H. Thus as a result of motions not possessing cylindrical symmetry, we have an over-all situation in which a circulating magnetic field is prevented from collapsing in the equatorial zone, and this part of the field can then support the outer part of the field which threads through this ring-shaped region somewhat in the manner of Fig. 16(a).

11. The "Twisted Kink" Theory

When a rather stiff cord or a bundle of rubber strands is twisted it relieves the local twisting strain by looping itself into a kink. Alvèn (1950) has suggested that the tendency of twisted bundles of magnetic lines of force to do likewise may form a basis for inductive regeneration of the earth's magnetic field. In particular, he points out the possibility that the adjacent lines of force can join together as the kink closes, allowing the column and loop each to close up and separate from one another. The mechanism of the twisted kink in a magnetic field has been treated analytically by Lundquist (1951) and more recently by Dungey and Loughhead (1953), who show that there is another type of instability that may set in besides kinking, but do not show that it would preclude kinking, since they investigate only the initial small deviation from cylindrical symmetry.

A possible way in which the kinking may contribute to the regeneration of the earth's field is suggested in Fig. 18. We assume that the jet stream or other con-

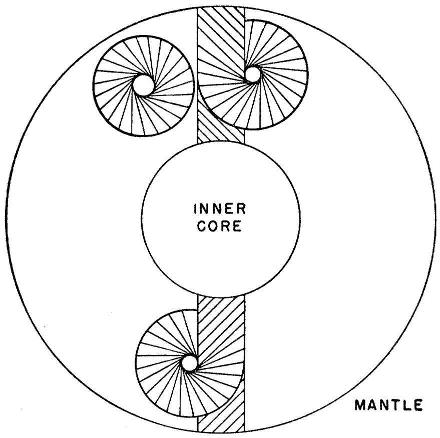


FIG. 18. Formation of magnetic kinks.

vective pattern will, through frictional drag provided mainly by local magnetic fields, cause the inner core to rotate faster than the mantle. As in any inductive theory, we start with (the z-component of) an initial stray field, however small. The bundle passing through the core tends to concentrate as a thick twisted bundle about the axis. Here we may apply Alvèn's process of kink formation and subsequent breaking away of a loop, as in Fig. 18. Once out on its own in the fluid core, the magnetic loop begins to decay with a characteristic relaxation time. From Maxwell's equations (10) and (11) we have the wave equation $\nabla\times[\nabla\times H]=-4\pi\sigma\dot{H}$ which order-of-magnitude-wise is simply

$$\dot{H}/H=-1/4\pi\sigma l^2$$

and defines the relaxation time, the time characteristic of the exponential decay of a field in a conducting block of linear dimensions l, as

$$T\approx 4\pi\sigma l^2\approx 10^{-3}(l/\mathrm{cm})^2 \text{ sec} \tag{21}$$

if we use the conductivity (7). For a body the size of the core, this gives about 10^{14} sec $=3\cdot 10^6$ years, or for a body ten times smaller, as one of the loops arising from kinking might be, about 10^{12} sec. For a tightly twisted loop, it appears that this would not be very much less in a fluid than in a solid, though we do not have an adequate treatment of this question and the answer may depend on the magnetic environment. The conductivity might be less than (7) and the time nearer 10^{11} sec. This is still much greater than the time of rotation of the inner core at a peripheral speed of 1 cm/sec, comparable to that of a jet stream, for example.

We may thus assume that a separated loop as in Fig. 18 may last during the subsequent generation of several more such loops by further twisting and kinking. Once ten or more of them have been generated in this fashion, they might be expected by squeezing each other from the side to cause an expansion of the size of the magnetic loops, until eventually the loops from the northern and southern hemispheres might join, making more lines of force thread through the inner core to be further built up by the same process. This forms what appears to be a fairly promising basis for the regeneration of the earth's field, though much more careful consideration is required to validate the picture. It is not at all clear what effect the convective streams in the equatorial region would have on the expanding loops. Alvèn in his discussion considered only the events in this region and suggested that the twisting exerted here by the convective streams would form kinks in this region which might in turn be stretched by the convective streams in such a way as to regenerate the original field.

III. THE WESTWARD DRIFT AND OTHER DETAILED CONSIDERATIONS

12. Convection and Magnetic Drag Driving the Westward Drift

We turn now to the problem of the westward drift of the geomagnetic patterns on the earth's surface (Vestine *et al.* (1947); Bullard (1949a); Elsasser (1950a)). Discussion of further details of the regenerative process of the dynamo theory will depend on details of the mechanism of westward drift. On the other hand, the mechanism of westward drift is largely independent of the details of the regenerative process; it depends merely on the same coexistence of rotation and convection within the fluid core, and on the observed end result of the regenerative process, the main (dipole) component of the earth's field threading through the fluid core. It is even independent of the nature of the origin of the field, whether it is primarily inductive or thermoelectric, for example, and thus, unfortunately, does not help us to choose between these theories.

Physical theories are considered well founded when many observed facts are deduced from few hypotheses. The geophysical phenomena of the deep interior are remarkable for their scarcity and remoteness, which makes it difficult to come to definite conclusions concerning the behavior of the core of the earth. Seismology has made important strides to delineate the boundaries of spherical shells of various density and rigidity characteristics, and astronomical data make suggestions concerning their contribution to the total moment of inertia. The magnetic phenomena consist not only of the primary effect, the existence and declination of the main dipole, but also of a bewildering wealth of detail concerning deviations from a simple dipole field and their secular variation. It is perhaps too much to hope, at least of the simple outlines of a theory, that it account for much of the smaller detail. It is gratifying that the mere existence of radial convection currents, which provides the basis of the induction theories here outlined but might also coexist with some other origin of the main field, provides an entirely natural and very simple explanation of the most striking feature of the secular variation, the westward drift of the general pattern of the detailed nondipole field. This drift takes place at the rate of about 0.18° per year.

The westward direction of the drift may seem at first sight enigmatic because the earth rotates toward the east, and the Coriolis forces in a rotating convective fluid are in such a direction as to make the inner part rotate faster than the outside, that is, eastward relative to the outside in the case of the earth. However, the detailed structure of the field observed at the surface may be presumed to arise from motions in the outer part of the core, and this is expected not only to move westward relative to the inner part of the core, but also to move westward relative to the mantle, as we shall see immediately.

We have seen that the rotational motion within the core is expected to be determined largely by a balance between Coriolis and magnetic forces. The Coriolis forces are virtual forces in a rotating (terrestrial) coordinate system which serve to conserve angular momentum in a stationary (astronomical) coordinate system. If there were no magnetic drag or other forces acting within the fluid, the radial convection would bring about a state of motion wherein the angular momentum (about the axis in the stationary system) per unit mass would be the same at all distances from the axis of rotation of the core. (In this nonrigid rotation, the axis is defined as a line of particles which are stationary in space.) Constant angular momentum per unit mass at various radii would imply tangential velocity inversely proportional to the distance from the axis, or angular velocity inversely proportional to the square of this distance, much greater near the axis than further out. With magnetic or viscous drag or both, this tendency for the inner part (i.e., the part nearer the axis) to rotate faster than the outer part persists to a lesser extent, the extent depending upon a balance between Coriolis forces and the drag forces.

With magnetic drag forces, the axis of rotation of the core would be expected (Inglis, 1941) to coincide rather closely (within a few degrees) with the axis of rotation of the mantle, even if the boundary of the core were spherical, in spite of the gradual change in direction of the axis of rotation of the mantle described as the 27 000 year precession arising from quadrupole-moment torques exerted by the sun and moon on the oblate mantle. For an oblate boundary between the fluid and solid, there is a theorem by Poincaré which, although proved (Lamb, 1919) only for an infinitesimal angle between the axes, appears to show that the two axes would coincide in spite of such a precession caused only by torques on the mantle. Furthermore, quadrupole torques acting on the core, with an ellipticity similar to that of the mantle, tend to give it independently approximately the same rate of precession as the mantle. We may thus take the axes of rotation of the core and mantle to be the same.

In an earth rotating with constant angular velocity Ω, and in the limit for small magnetic and other drag, one would then expect the angular velocity of the core to be a function of the distance ρ from the axis of rotation,

$$\omega(\rho)=\Omega B^2/\rho^2. \tag{22}$$

If the length B in the constant of proportionality ΩB^2 is less than the radius A of the core, then there is a cylinder of radius B which is the locus of points at rest relative to the mantle as shown in Fig. 19(b). With small drag, the only possible steady state is one in which B has the value, $B<A$, for it is only thus that the total torque exerted by the mantle on the core, or by the core on the mantle, can vanish. The inner part of the core, with $\rho<B$, rotates faster than the mantle, or eastward relative to the mantle, and the outer part, with $\rho>B$, rotates more slowly than the mantle, or westward relative to the mantle, as suggested in Fig. 19(a). Thus the inner part exerts a torque of one sign (north) and the outer part a torque of the opposite sign (south) on the mantle, which can make a total of no angular acceleration in the steady state. Expressed otherwise, the inner part of the core experiences a southward torque from the mantle and a northward torque from the outer part of the core which must cancel one another, while the outer part of the core is subject to equal and opposite torques from the inner part and from the mantle. If the steady state has not been reached, on sees easily that the angular acceleration is in the direction to approach the steady state. With strong drag forces, $\omega(\rho)$ will vary with ρ less rapidly than in Eq. (22), but the same sort of division of the core into two oppositely rotating parts must take place in the steady state.

An attempt to picture the mechanism of the magnetic drag responsible for the westward drift of the outer part of the core is made in Figs. 19(b) and (c), where the deflection of a typical line of force in the inner part of the core is shown on the left and in the outer part of the core on the right. The torques exerted by the drags in opposite directions on the core as a whole balance one another. In order to appreciate the balance of torques on the two parts of the core separately, one must imagine a torque acting between the two parts of the core across the cylindrical boundary. This torque is associated with a net inward flow of angular momentum with the convection, angular momentum having been imparted to the inward flowing material by the drag in the outer part, and vice versa.

For the sake of an estimate, let us assume that the field in the core is homogeneous and parallel to the axis, which makes it the field of a dipole in the mantle. In analogy with the discussion of Fig. 5, we may take the magnetic drag force per unit area of projection on the equatorial plane, Fig. 19(c), to be roughly proportional to the relative velocity of the core and mantle at a given ρ. The vanishing of the total torque exerted by the mantle on the core may then be written

$$\int_0^A (\Omega-\omega(\rho))\rho^2 d\rho = \Omega\int_0^A (1-B^2/\rho^2)\rho^2 d\rho = 0 \quad (23)$$

by use of the approximation in Eq. (22), which then gives

$$B = 3^{-\frac{1}{2}}A. \quad (23.1)$$

The fraction of the area of the spherical boundary between the core and mantle which bounds the outer part of the core is thus $(2/3)^{\frac{1}{2}} = 0.82$, and only the remaining 18 percent of the outer surface of the core is found in the "polar caps" of Fig. 19(b) at the ends of the approximately cylindrical inner part of the core. Thus a very broad equatorial band of the outer surface of the core drifts westward in this approximation.

If we assume that the observed drifting pattern of the earth's field is caused by local irregularities such as vortices moving with the material just beneath the surface of the core, the mantle is thick enough to obscure to some extent the external observation of these irregularities in the small "polar cap" so the observed secular variation is dominated by a westward drift characteristic of the broad equatorial band. This does not correspond in detail with the conclusion of Vestine that the motion of the nondipole field is consistent with an angular velocity independent of latitude. It must also be remembered that the Taylor-effect tendency for convective flow parallel to the equatorial plane tends to bring the convective streams, and thus the turbulent irregularities, to the surface in the equatorial belt. The polar caps might thus be quite without distinguishable character and all contributions to the irregularity of the pattern might come from the westward-drifting equatorial belt. However, the independence of latitude cannot be claimed with great accuracy, the patterns studied being sufficiently vague and ephemeral as to leave some freedom of interpretation, and it is not yet clear that the observations contradict the expectations from this simplified model. On the other hand, the slight refinements of the model discussed below, in Sec. 16 reduce the theoretically expected dependence on latitude.

A still more crude model for the discussion of the westward drift could consist of the two parts of the core pictured in Fig. 19(b) as solids sliding on one another and within the solid mantle, one rotating forward and the other backward relative to the mantle. Then one needs to think of the torques mentioned above acting on these rigid rotating bodies.

Bullard, Freedman, Gellman, and Nixon (1950) have discussed the westward drift in terms of a somewhat similar, though in reality quite different, model. They likewise consider rigid inner and outer part of the core sliding on one another, but separated by a sphere rather than a cylinder of radius B. They point out the tendency of the inner part to rotate more rapidly than the outer part, and can conveniently consider the induction mechanism to proceed within these two parts, since the outer part is analogous to the top T of the carrousel in Figs. 6–12. They point out that current $\mathbf{J}_1$ induced in the first step (Fig. 7) would also leak out into the more weakly conducting mantle and would couple with the field $\mathbf{H}_1$ produced there by the currents in the inner part of the core in such a way as to make a westward drag on the core resulting in a westward drift. This model may also be discussed in close analogy to the linear problem of Fig. 5: v_2 downward corresponds to the eastward motion of the inner sphere of the core, v_1 upward to the westward drift of the surrounding spherical shell of the core, and the magnetic drag across the boundary between conductors compensates the inward flow of angular momentum. The horizontal emergence of the line of force in Fig. 5 corresponds to the lack of torque between the core as a whole and the mantle, when averaged over all radii ρ. In the spherical problem, the core moves too fast to permit normal emergence at small ρ, and at large ρ the lack of penetration through the core makes a deviation from normal emergence in the opposite direction out into the conducting mantle. The weakness of this spherical model is that one can see no mechanism in nature to provide the analog of the rigidity of the spherical shell $B<r<A$ while it rotates westward relative to the mantle, and without this rigidity the model does not even approximate the motion it is intended to represent.

13. Speed of the Westward Drift

The observed speed of the westward drift is very much less than the maximum that would be possible according to the mechanism proposed. If the Coriolis term (3) of Eq. (1) were very much larger than the others, we would have Eq. (22) as a good approximation, whence, with Eq. (23.1), we find for the angular speed of the almost-free relative motion between mantle and adjacent fluid of the core at the equator,

$$|\Omega(A)-\Omega| = (2/3)\Omega = (2/3)2\pi\ \text{day}^{-1} = 5\times10^{-5}\ \text{sec}^{-1}. \quad (24)$$

The angular speed of the observed westward drift is, as already mentioned, about $0.18°/\text{yr} = 10^{-10}\ \text{sec}^{-1}$. If the westward drift is to be attributed to the motion of the fluid core at all (and there appears to be no other possibility), the rapid relative rotation in Eq. (24) must be very effectively suppressed (to the extent of a factor $f \approx 2\times10^{-6}$) by strong coupling forces between the mantle and the various parts of the core (or directly between the parts of the fluid core). This still leaves valid the explanation of the westward direction of the drift.

If the magnetic assumption applying to Eq. (4) were strictly valid, that is, if the magnetic lines of force were frozen in the conducting matter of the core and mantle, then as long as any relative rotation continued, the

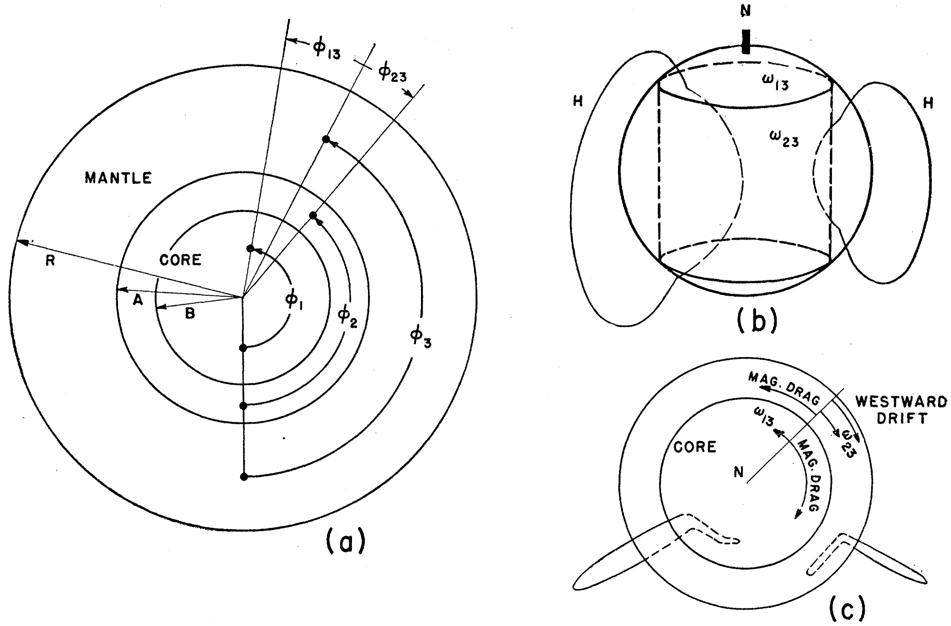


FIG. 19. Differential rotation and magnetic drag in the core as a source of the "westward drift."

lines of force would continue to be wrapped further around the axis in the manner suggested by Fig. 8 or by Eq. (15) with $\sigma=\infty$, making the restraining force approach infinity. Thus with infinite conductivity the relative rotation would eventually stop and there would be no westward drift.

The westward drift is analogous to v_2 in Eq. (17) in the linear slab problem, and its existence is allowed by the finite product of conductivity and dimensions, analogous to σb of Eq. (17). The rotating sphere problem differs from the linear slab problem in having electric charges built up on the surface, and in the fact that the plane of the paper containing a loop of the line of force shown in Fig. 5 is wrapped about an axis (Fig. 8). In Fig. 5 the lines of force slip through the matter only as a result of the curvature shown in that plane, but the third dimension of the spherical problem makes it possible for them to slip toward the axis of rotation and thus oppose the rotation of the inner part more than that of the outer part of the core. This introduces another tendency for a westward drift, and increases the fraction of the core that experiences a westward motion relative to the mantle. We neglect this effect in pushing the analogy so far as to use Eq. (17) for an order-of-magnitude estimate of the forces involved in the spherical problem (Eq. (28), below).

Bullard, Freeman, Gellman, and Nixon (1950) [see also Bullard (1949a), Sec. 6] have made an estimate of the rate of change of angular momentum and of the forces required in their spherical-shell model of the westward drift, and it seems appropriate to make a similar estimate, which need be no more than an order-of-magnitude estimate, for the flow across the cylindrical boundary shown in Fig. 19. With $B=3^{-\frac{1}{2}}A=0.58A$ from Eq. (23.1), the inner cylindrical part contains 46 percent of the volume. Thus the volume of the sphere is divided approximately into two halves by the cylindrical boundary. If the matter in the two halves is interchanged while rotating at approximately the angular velocity of the earth Ω, the gain of angular momentum by the material put into the outer half is

$$\Delta L=\Omega\Delta I=0.56I\Omega=0.56(8\pi/15)SA^5\Omega, \tag{25}$$

about half the angular momentum of the core.‡ The rate of change of angular momentum of this material is then

$$\dot{L}=\Delta L/T, \tag{26}$$

where T is the mean time of interchange of the material in the two halves.

We presume that this angular momentum is imparted by the torque of magnetic forces, the magnitude of which depends on the relative velocity of the core and the mantle. It also depends on how much the magnetic field can "slip through" the material of the mantle (or of the core if it should have the lower conductivity). This relative tangential velocity $v_t(\rho)$ we may for simplicity assume to be less than that of the free convective rotation (24) by a constant factor f:

$$v_t=f\Omega((B^2/\rho^2)-1)\rho, \tag{27}$$

(though in reality the strongly retarded distribution may be more nearly linear than quadratic in $1/\rho$.) At a given $\rho<B$ at one end of the inner cylindrical part we may think of the line of force represented by the left-hand loop in Fig. 19(b) as curving and slipping through the core as through the inner slab of Fig. 5, through the conducting layer of the mantle just as in the outer slab of Fig. 5, and passing normally out into outer space

‡ Here we use S for density, keeping ρ for cylindrical radius.

as a part of the dipole field we observe. We approximate the force per unit area, $F(\rho)$, from the linear problem, Eq. (17),

$$F(\rho)=4\pi\sigma H^2 v_2(A^2-\rho^2)^{\frac{1}{2}}\approx 4\pi\sigma H^2 v_2 A$$
$$=4\pi\sigma H^2 A\,(C\sigma_c/(C\sigma_c+A\sigma))v_t\approx 4\pi\sigma_c C H^2 v_t. \quad (28)$$

Here C is the thickness (corresponding to a of Fig. 5) and σ_c the conductivity of the conducting layer moving with the mantle; A and σ refer to the core; v_t corresponds to v_1+v_2 in Fig. 5, and from Eq. (14) we have

$$v_2=(C\sigma_c/(C\sigma_c+A\sigma))v_t\approx(C\sigma_c/A\sigma)v_t, \quad (29)$$

for the case wherein the $C\sigma_c/A\sigma\ll 1$. We have used this approximation in Eq. (28).

In the coordinate system of Fig. 5 the field in the outside space is purely magnetic and perpendicular to the surface. In the terrestrial analog this system is moving at $-v_1$ relative to the observer sitting on the slab (mantle), in whose system there is according to the Lorentz transformation also an electric field $E\sim(v_1/c)H$, but v_1 is so small compared to the velocity of light c that this is presumably unobservable (of the order of 10^{-13} volt/cm).

The moment of force on the two rounded ends of the cylindrical region is

$$\dot{L}=4\pi\int_0^B F(\rho)\rho^2 d\rho\approx 4\pi^2 B^4 C\sigma_c H^2 f\Omega, \quad (30)$$

and from Eqs. (25), (26), (30) and $B=3^{-\frac{1}{2}}A$, that is, equating the magnetic torque to the rate of change of angular momentum in the interchange, we have

$$1/T\approx\tfrac{1}{2}f(C\sigma_c/A\sigma)\sigma H^2/S\equiv\tfrac{1}{2}fg\sigma_0 H^2/S. \quad (31)$$

Here we have introduced a "geoconductive ratio" g defined as the effective conductivity of the mantle relative to that of a normal metal as thick as the core, $g=(C\sigma_c/A\sigma)(\sigma/\sigma_0)=(C\sigma_c/A\sigma_0)$, where σ_0 is the conductivity of a typical normal metal, 10^{-4} sec cm^{-2}. Since there are so many uncertainties that we cannot hope to derive the rate of the westerly drift with significant accuracy, we have used it to determine $f\approx 2\cdot 10^{-6}$ as discussed in connection with Eq. (24). With $\sigma_0=10^{-4}$ sec cm^{-2}, $H^2=10$ gauss$^2=10$ g cm^{-1} sec^{-2}, and a density $S=10$ g cm^{-3}, we have as a unit of reciprocal time in Eq. (31)

$$\sigma_0 H^2/S=10^{-4}\text{ sec}^{-1}, \quad (32)$$

with which, and with $f=2\cdot 10^{-6}$,

$$T=g^{-1}10^{10}\text{ sec}=g^{-1}\,300\text{ years}. \quad (33)$$

The time required for the interchange, that is for half the volume of the core to flow across half the cylindrical boundary, $2\pi B(A^2-B^2)^{\frac{1}{2}}$ with $B^2=0.37A^2$, at the mean radial speed $\bar{v}_r$ is

$$T\approx 0.7A/\bar{v}_r. \quad (34)$$

This with Eq. (33) gives us

$$\bar{v}_r\approx 7\quad 10^{-11}gA\text{ sec}^{-1}=0.024\text{ g cm/sec}. \quad (35)$$

This is the mean radial speed necessary to make the interchange of matter between the two parts require the magnetic torque on the inner cylinder estimated by use of the observed westerly drift, not just the radial speed obtained simply by equating the speed of convection to the speed of the westerly drift. With $g\approx 1$, this mean radial velocity is, however, of the same order of magnitude as the tangential speed $v_t=10^{-10}A$ sec^{-1} ≈ 0.03 cm/sec corresponding to the observed westward drift (cf. remarks below Eq. 24):

$$\bar{v}_r=0.7gv_t\approx 0.7v_t. \quad (36)$$

Thus a conductivity comparable to ordinary metals in a thick layer of the mantle, would suffice, with this simple model, to let the mean flow velocities within the flow pattern in the core be about as great as the drift velocity of the pattern relative to the mantle. Such a high conductivity makes the relaxation time of the earth's field rather long, a million years or more as we have seen in Sec. 11 [Eq. (21)], too long to be compatible with some of the local variations of the earth's field which seem to take place in a time scale of more nearly a thousand years. If the conductivity of the mantle (or of the core) is an order of magnitude smaller, so as to act as a less effective brake on the relative rotation of the inner and outer parts of the core, the velocities within the flow pattern must be correspondingly slower in order not to drive the relative rotation of the inner and outer parts of the mantle much faster than the observed drift. More rapid flow is permitted if instead of averaging over the material of the core we consider the flow concentrated in a jet stream, as discussed in Sec. 18.

The rate of flow of matter across the interface implied by Eqs. (34) and (35) is of the order of $10^{17}g$ g/sec, which even with a small value of g is still so large compared to the flow in Urey's settling process, 10^{10} g/sec mentioned at the end of Sec. II, that the transport of angular momentum by the settling matter may be neglected. That is, the convective matter is much more abundant than the settling matter which may be driving the convection.

The extent of the spiralling of the lines of force in the manner of Fig. 8 may be estimated by use of Eq. (16), which we express as follows [using Eqs (27) and (29)] in adapting it to the notation of the spherical problem:

$$Z=2\pi\sigma_2 v_2 b^2\rightarrow 2\pi\sigma(C\sigma_0/(C\sigma_0+A\sigma))$$
$$\times f\Omega\rho(1-B^2/\rho^2)(A^2-\rho^2). \quad (37)$$

For a typical radius $\rho=B/2$, and still with $g=1$, this is evaluated approximately

$$Z/A=10^4. \quad (37a)$$

The lateral excursion Z of the lines is thus determined by the hold the conductivity of the mantle has on them.

With $g=10^{-1}$ the lines of force are wrapped around the axis something like 100 times, though the number would be considerably reduced by the radial migration of the lines toward the axis which is neglected in this approximation.

14. The Dynamo Theory as Modified by the Westward Drift

An unsavory aspect of the dynamo theory of terrestrial magnetism is that it involves nonanalytic relative motions in a continuum, and it is not quite easy to assure oneself that one has considered all of them and properly assessed their relative contributions. With the conventional positive rotation our exploratory discussions of the regeneration have treated explicitly a core rotating eastward within the mantle. It has appeared plausible that the relative motions most important to the dynamo action are those near but not in the equatorial plane, including the rotational shear relative to the equatorial belt surrounding the core represented by the cylindrical shell of Fig. 15. This primary contribution to the regeneration was discussed in connection with Figs. 15 and 17. Other significant contributions appear to be those involving shear between stacked disks, as discussed in connection with Figs. 12 and 13. Only the parts of these models far from the axis were considered to correspond roughly to convection in the earth consisting mainly of rising and falling currents near the equatorial plane (Fig. 2). Thus in either case it is mainly the outer part of the motion that matters, so the fact that the inner and outer parts rotate in opposite senses in keeping with our discussion of the westward drift does not greatly alter the regenerative process. That the important outer part rotates toward the west rather than the east within the mantle is also unimportant, as we have seen from the remark that the process is quadratic in v_1, or in the hydromagnetic considerations of Fig. 17, from the fact that the twist is the same for either sense of relative rotation.

No matter whether the rotational velocity varies as in Eq. (27) or less drastically with distance from the axis, there must be some rotational shear across cylindrical elements throughout the volume, of sign opposite to that encountered at the outer boundary. While this is deserving of more complete treatment, it is not expected that this should contribute overwhelmingly to the regeneration because we have found reason to emphasize the upward and northward part of the convective flow (outward and upward in the diagram), and this presumably exists only near the outer boundary where the upward convection is dispersed horizontally. (If there is stratification of the convection, a sort of large-scale turbulence in layers divided by cylindrical surfaces, the horizontal spreading of upward and downward streams at such a surface would tend to cancel.)

The problem of making a quantitative estimate of the field produced by a given driving force of the convection is considerably modified by this conception of the westward drift. Even the choice of a model to simplify the calculations may depend on one's judgement of the conductivity of the mantle, on the parameter g. While it would be most interesting to have it shown analytically that velocity distribution of a simple convective pattern does act as a dynamo, it must be remembered that the patterns that have yet been discussed are too simple to be compatible with the westward drift.

15. Estimate of the Fields and Energies Involved

A basic question about the general problem of accounting for the earth's field is the adequacy of the available energy source to maintain the field. A rough order-of-magnitude estimate indicates that it is possible to postulate a heat source within the earth supplying plenty of power, or alternatively that adequate power is involved in Urey's hypothesis of the settling of denser matter. As is discussed further in Sec. 17, the thermal source at the center of the earth may well be of the order of magnitude $S\approx 10^{20}$ ergs/sec. The energy resident in the earth's field is at least of the order of magnitude $(H^2/8\pi)(4\pi A^3/3)\approx 10^{26}$ ergs, calculated here simply as the part inside of the core of radius A if the surface field $H\approx 4$ oersteds persists within. Without an energy source the field would, with a relaxation time $T=10^{14}$ sec given by (21), lose energy at a rate of about 10^{12} ergs/sec and a source at least this great is required to maintain it. We thus see roughly that we have a generous factor of about 10^8 to spare to take account of the various inefficiency factors, thermodynamic (Carnot), electromagnetic or thermoelectric, and mechanical, and of the excess of the internal toroidal field H_2 over the surface field H.

Let us now look in more detail at the energy dissipation specifically in the induction theory. One indication of the magnitude of the "toroidal" field H_2 is given by an analysis of typical details of the secular variation. Bullard (1948) has shown that the secular variation during the past century in the region of rapid change near South Africa may be accounted for by the development during this time of a dipole moment $M\approx 2\ 10^{24}$ gauss cm^3 just beneath the surface of the core. One mechanism for the development of such a dipole moment is the turning over of a mass of conducting fluid which was originally threaded horizontally by H_2. That is, a turbulent surface eddy twists the lines of force. Assuming 400 km as a reasonable magnitude for the diameter of such a surface eddy in the earth, and that the velocities in it are comparable to those suggested by the westward drift, Bullard has concluded that H_2 is of the order of magnitude of $10H_1$. Since this involves somewhat arbitrary choice of eddy size and indicates only conditions near the surface, and since another mechanism such as the transport of a magnetic monopole, which is equivalent to the development of a magnetic dipole, might be involved, we prefer to leave

H_2 as an undetermined parameter in discussing the magnitudes which might be encountered in the regenerative process.

As we have seen, the process is sufficiently complicated to involve several unknown factors, and we here attempt a discussion which is intended to be merely illustrative of the sorts of considerations that must be involved, and cannot be considered in any way quantitative. In cases wherein the magnetic lines are essentially frozen to the matter the methods of hydromagnetics which ignore the currents are particularly appropriate for quantitative estimates, but here where the slipping of the lines through the matter is essential to the process, we find it preferable to include the currents in the discussion.

We consider the process typified by Figs. 7 and 13, with the crucial part of the regenerative process taking place near the equator. The process consists of four steps, $i=1\ldots 4$. In a typical step, H_i and v_i induce an electric field $E_i \approx v_i H_i$. This field acts throughout a roughly cylindrical region of length a_i and area A_i, for example, and drives a current which outside the cylinder may, in a typical step, loop back from one end to the other of the cylinder in all azimuths so that only a fraction, say h, of the current through the cylinder is effective as J_i in the current loop, giving rise to H_{i+1} in which we are interested. Since the current returns through an area much greater than A_i, most of the resistance of the circuit is in the cylinder and the current through it is

$$J_i/h \approx E_i \sigma_i A_i = v_i H_i \sigma_i A_i. \tag{38}$$

The magnetic field produced by a current J_i in a loop of area a_i^2 at a distance along its axis a_i is about

$$H_{i+1} = 2^{-\frac{1}{2}} \pi J_i / a_i \approx 2 h v_i H_i \sigma_i A_i / a_i \equiv k_i H_i. \tag{39}$$

There are so many quantities of which the estimates are uncertain that we introduce an ignorance parameter h whenever it is needed to represent a number which we expect should probably lie between $\frac{1}{10}$ and $\frac{1}{2}$. In Eq. (39) we put $a_i = hA$, $A_i = h^2 A^2$, A being still the radius of the core. If further we put $v_i \approx v_2 \approx v$ in keeping with an assumption of a simple pattern of convection as discussed in connection with Eq. (36) and take σ everywhere the same, we have

$$H_{i+1} = 2h^2 v \sigma A H_i. \tag{40}$$

As the result of the four-step regenerative process we have

$$H_5 = H_1 = (2h^2 v \sigma A)^4 H_1. \tag{41}$$

Thus v and σ are related to each other and through h to the geometry of the convection by the necessary condition that the dynamo shall function in a regenerative manner:

$$2h^2 v \sigma A = 1. \tag{42}$$

If we consider v to be of the order of magnitude of the velocity of westward drift, $v \approx 0.03$ cm/sec, and express conductivity in terms of the typical value for metals $\sigma_0 = 10^{-4}$ sec cm^{-2}, we find from Eq. (42), $\sigma \approx 10^{-3} \sigma_0 / 2h^2$, a not unreasonable figure.

The power dissipated electrically as heat in each step is

$$W_i \approx E_i a_i (J_i / h) = h^3 \sigma v^2 A^3 H_i^2, \tag{43}$$

and in the sum over the four steps it is probable that one step may predominate because of the variation of H_i from step to step, which is expected as a consequence of having different effective values of h^2 in each step. From Eq. (43) with Eq. (42) we thus estimate the total power

$$W = \tfrac{1}{2} h v A^2 H_m^2 = \tfrac{1}{2} h (H_m / H_1)^2 W_0, \tag{44}$$

where H_m is the largest of the four values of H_i. H_1 is about 4 gauss, and $v \approx v_t = 10^{-10} A$ sec^{-1}, and we introduce as a convenient unit of power

$$W_0 = v A^2 H_1^2 = 5\ 10^{16} \text{ ergs sec}^{-1}. \tag{45}$$

The power available is probably somewhat greater than this unit. Urey gives as the power available from his setting process $10\ 10^{21} (\Delta\rho/\rho)$ ergs sec^{-1}, where $\Delta\rho$ is the effective difference between the densities of the settling material and its environment as it settles and compresses across a large pressure difference. Since densities may differ and change considerably, it would not be surprising to find $(\Delta\rho/\rho)$ as large as 10^{-2} even at these high pressures, which would then lead us to expect $H_m \approx 10 H_1$ or somewhat greater. The power that might be available in thermal convection has been discussed by Frankel (1945) and by Bullard (1949a) and in Elsasser's review article (1950a). The figure given in the latter, 1.1×10^{-6} cal/sec per cm^2 of the surface of the central body (radius 1250 km), corresponds to 10^{19} ergs/sec, the same order of magnitude as is reasonable from the settling process, and again Eq. (44) suggests $H_m \approx 10 H_i$. If H_m were H_2, this would be in keeping with Elsasser's original emphasis on the first as an amplifying step and with Bullard's conclusion from the secular variation, and in no way contradicts the explanation of the regenerative process.

A determination of which H_i should be the largest, H_m, or, just beyond this, an estimate of the earth's external field from the power available, would involve more careful estimates of h. It should be labeled h_{ij} to distinguish the various ways in which it enters: in Eq. (39) we call it h_{i1}, representing the inefficiency of E_i in producing J_i, and put $a_i = h_{i2} A$, $A_i = h_{i3}^2 A$. Then Eq. (41) would be, more generally, $H_5 = H_1 \pi_i k_i$ with $k_i = \sigma_i v_i A h_{i1} h_{13}^2 / h_{12}$ and instead of Eq. (42) we would have

$$\prod_{i=1}^{4} k_i = 1 \tag{46}$$

Thus instead of having the product, Eq. (42), equal to unity for all steps, we will have k_i larger than unity for at least one (amplifying) step and smaller for others. The preliminary discussion of Fig. 7 would make it seem that the first step should be the most strongly amplifying one because the magnetic lines thread through the whole moving sphere in such a way that h_{12} and h_{13} might be expected to be about unity, and h_{11} in the preliminary discussion appears to be less than unity only because E_1 exists only part way around the circuit, whereas in the other steps the motion of only a small part of the sphere may be involved and there exist ineffective circuits competing for J_2. Our further discussion has shown that this simple conclusion is dependent on the presence of fairly good conductivity in the mantle but not more than is reasonable to assume. The ratio H_2/H_1 is roughly equal to Z/A of Eq. (37a), which, by neglecting unknown pressure effects on conductivity, we estimated to be about 10^3 or 10^4. This leaves room for quite a lot of deamplification by some of the other steps as seems consistent with the smaller amounts of moving matter and the competitive geometries involved.

16. Inclination of the Magnetic Axis

Each of the induction theories, the dynamo theory and the twisted-kink theory provides a natural connection between the mechanical and magnetic axes of the earth; in their simplest form they seem to make the two axes parallel because of the manner in which H_2 is formed from H_1 by the dragging around of the lines of force by the differential rotation associated with the mechanical axis. The 19° inclination of the magnetic axis is then to be explained as some sort of a perturbation. We shall see that in the dynamo theory it is almost too easy to obtain a sufficiently large perturbation.

So long as the convection currents have a symmetry that leaves them unchanged by reflection in the equatorial plane, no inclination is to be expected, and the models we have as yet discussed, with the convection currents centered in the equatiorial plane (and possibly the axis itself as in Fig. 1), have had this symmetry. It seems likely that the threading of the lines of force through the convection loops will tend to bring about a symmetrical configuration, unless the conductivity of the mantle (through which the lines of force must slip, as mentioned in Sec. 12) is appreciably nonuniform. We shall here see that in the dynamo theory there is an amplification of any chance, small deviation from symmetry such that a small deviation of the flow pattern can cause a large magnetic inclination.

We consider a small deviation from this symmetry, with two of the opposite radial convection streams, for example, outward, deviating from the equatorial plane by an angle α, as shown in Fig. 20. The thickness of the stream moving out with velocity v_2 is hA. At a distance ρ from the axis it is lifted a distance $z=\alpha\rho$ in the linearly varying field

$$H_2(z)\approx H_2 z/A, \tag{47}$$

where H_2 is now the maximum value of the toroidal field which occurs at a distance almost A from the equatorial plane. The electric field induced by this change from the symmetrical position is

$$E_2'(\rho)=v_2H_2(z)=(\alpha\rho/A)v_2H_2. \tag{48}$$

This gives rise to an electric current density approximately $\sigma E_2'(\rho)$ within the stream which is to be multiplied by the width of the stream hA and by $d\rho$ to give the current flowing across the stream between ρ and $\rho+d\rho$. Of this a fraction h flows around the sphere as part of J_2', the rest shunting back around the convection stream above and below the plane of Fig. 20. We assume as an approximation that it flows circularly, so that the current circulating between ρ and $\rho+d\rho$ is

$$J_2'(\rho)d\rho\approx h^2\alpha\sigma v_2H_2\rho d\rho. \tag{49}$$

If treated as a current in a circular wire in the plane of Fig. 20, this contributes to the field at the surface of the sphere (at a point A along the x-axis) $2\pi\rho(\rho^2+A^2)^{-1}\times J_2'(\rho)d\rho$ which when integrated gives for the field at the surface from the equivalent transverse dipole

$$H_x'=2\pi(1-\pi/4)h^2\alpha A\sigma v_2H_2 = 1.4h^2\alpha A\sigma v_2H_2\approx\alpha H_2. \tag{50}$$

The last expression can be considered only an order-of-magnitude estimate as it involves cancelling out h^2 in substituting Eq. (42). The inclination of the main magnetic dipole moment μ of the earth from true north is in this approximation

$$\phi=H_x'/H_1=\alpha H_2/H_1. \tag{51}$$

Since the first step in the regenerative process is probably an amplification, with $H_2\gg H_1$, we probably have $\phi\gg\alpha$ so that only a very small deviation of the convection streams from symmetry about the equatorial plane is required to account for the observed magnetic inclination.

One might hope for a more casual theory of the inclination, relating it to other phenomena, but lacking it we may at least note that the inclination could thus

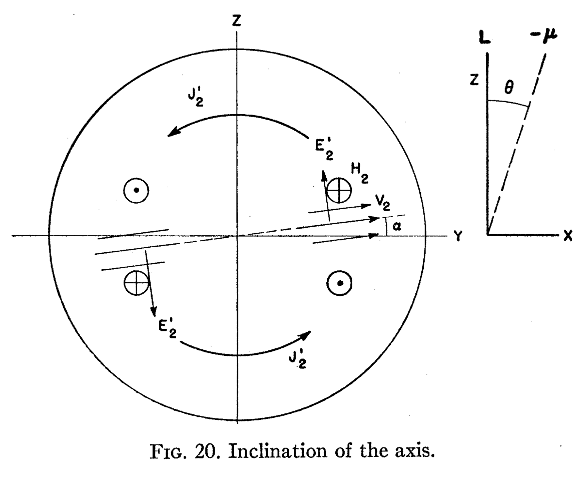


FIG. 20. Inclination of the axis.

perhaps be the chance result of rather small inhomogenities.

Inhomogeneities in the mantle, perhaps associated with differential cooling under the continents and oceans, might seem more likely than spontaneous unsymmetrical behavior on the core. The westward drift has been attributed to competition between various torques exerted between the mantle and core, with lines of force slipping through the material of the mantle in various directions, and a greater ease of slipping through one part of the mantle than another could provide another reason for the inclination of the axis.

17. The Possibility of a Jet-Stream Flow Pattern in the Earth's Core

To avoid excessive complexity in presenting some of the inductive effects that might be important, we have in Secs. 6–15 confined the discussion primarily to an admittedly oversimplified flow pattern within the core, essentially that of Fig. 2. From simple analogy with the observations cited in Sec. 2 it seems likely that there is actually a more complicated flow pattern perhaps involving a jet stream.

We have seen that the stability of a jet stream without a magnetic field is associated with its effectiveness in transporting heat without requiring transport by conduction over more than a very short path laterally into the narrow stream. A magnetic field, if sufficiently strong, may provide an additional reason for the stability of a jet stream in a conducting fluid. If a rubber band is stretched around the end of a right cylinder, but not across a diameter, it has a tendency to slip off. Correspondingly, if we have a cylindrical stream of conducting fluid moving parallel to the axis within a surrounding conductor, a magnetic line of force traversing the cylinder and not passing through the center has a tendency to be squeezed out, as suggested in Fig. 21. At high velocities or with large dimensions this effect, analogous to a "skin effect," greatly reduces the magnetic drag opposing the motion in the cylinder. A simple dimensional estimate such as we have made above indicates that the characteristic velocity for this effect is $v \approx (10^3\ \mathrm{cm}/L)$ cm/sec, where L is the thickness of the stream, so that the magnetic field may be expected to be squeezed out of any large stream within the core. When the magnetic field is squeezed out of the moving matter, the question then arises into what space is it repelled, and this, of course, depends on the flow pattern and may help determine which flow patterns are possible.

In Sec. 2 we have discussed the "thermal wind" explanation of the eastward flow and drift in the meteorological case with cold at the center of a cylindrical vessel rotating eastward. This explanation is dependent on the assumption of sticking at the bound surface at the bottom, which through the rather

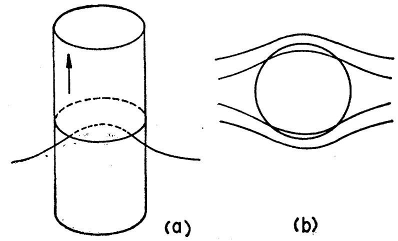


FIG. 21. Magnetic lines avoid a flow stream.

small viscosity keeps the motion slow near the bottom and thus permits practically no Coriolis contribution to a radial pressure gradient at low altitude. This is the basis for assuming equal pressures at the inner and outer edges of the bottom surface, and in this process friction is not negligible at least in the bottom layers. It has been mentioned that with cold in the center we have a natural thermal convection in a horizontal plane driven by the centrifugal force, but that with a warm boundary in the center, the flow pattern must deviate from the horizontal plane. This distinction is of interest in connection with the principle of constancy of average angular momentum per unit volume with varying radius in a nonviscous rotating convective fluid, of which use was made in the more general considerations of Sec. 2. In the explanation of the thermal wind with cold at the center, no use is made of viscous drag between the upper layers and the constancy of average angular momentum with radius must apply to the motion confined in each horizontal slab (if the motion is indeed horizontal). That this principle is consistent with the simple geometrical situation, typified by Fig. 3(c), is apparent from the fact that the rapidly moving matter in the jet stream constitutes a larger fraction of the matter at small radii than at large, making the average tangential speed greater at smaller radii. With a thin jet stream and with the outer radius much larger than the inner, this requires that the flow in the jet stream be much faster than the velocity $\Omega\rho$ due to the rotation. With the hot boundary at the center, the thermal wind explanation requires circulation in the opposite direction, as is also observed, and the motion is no longer thermally driven if it is purely horizontal, so there must be transport of momentum across horizontal boundaries and the conservation of angular momentum within any horizontally bounded region no longer applies. The simple qualitative geometrical consideration of flow in a plane (Fig. 3(c) with v reversed) still leaves conservation possible, but only if the jet speed, which is now oppositely directed to $\Omega\rho$, is much greater in magnitude so that the average tangential speed is in the direction of the jet flow, backward. So great a jet speed m need not be, and apparently is not, achieved, and similarly in the case first discussed with cold at the center, there is no assurance of horizontal flow and no need to satisfy the simple conservation requirement at all exactly. Insofar as there are deviations, they are transmitted up from the closed boundary at the bottom by viscosity and by vertical components of flow.

In the earth's core there is no question about the source of the thermal drive, if thermal it be with a source of heat at the center, since the inwardly directed radial component of gravity provides ample motivation for the hot part of the stream to float toward larger ρ (so long as the thermal gradient exceeds the adiabatic gradient). But even with the jet stream thus driven, and at the same time retarded by a magnetic field, some of the thermal-wind considerations might apply to help determine what type and direction of motion may be stable.

Hide (private communication) has suggested that the direction of flow permitted in the earth's core may be determined by comparison with his cylindrical experiment, with a careful use of inversions and neglecting the ρ component of gravity even though it is important in driving the convection. Let us replace the spherical core by a cylindrical one, similar to Hide's experimental arrangement, Fig. 3(a), but with a lid on it in contact with the top surface of the liquid, and with gravity pointing downwards in the top half, upwards in the bottom half, as the z-component of gravity does in the earth [left side of Fig. 22(a)]. The two halves differ in two factors, the direction of gravity and the apparent rotation looking down on the appropriate pole, and with these two reversals the resultant phenomena are the same in the two halves. The boundary between them in the equatorial plane thus acts as a free surface for both bodies of fluid. The upper half differs from the laboratory experiment, Fig. 3(a), only in the interchange of free and bound surfaces between top and bottom, for gravity is down in each. The lower half differs from Fig. 3(a) only in the direction of gravity, for the bound surface is on the bottom for each case.

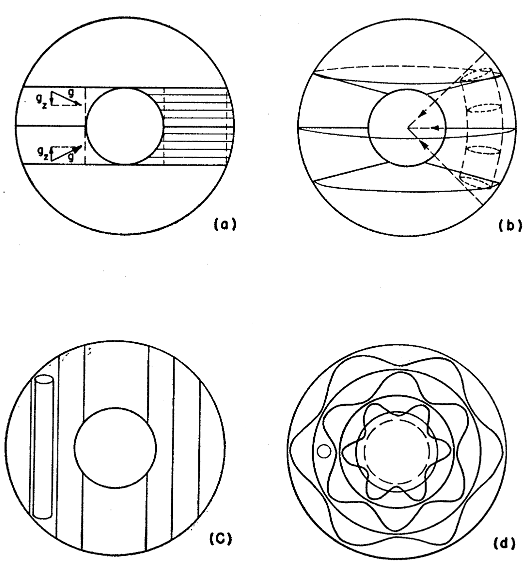


FIG. 22. Various flow patterns.

Thus each half has one mechanical reversal from Fig. 3(a). But there is also a thermal reversal, for we must now make the center of our model hot if it is to correspond to thermally driven convection in the earth. These two reversals leave the direction of flow the same as in Fig. 3(a), or the same as in the temperate-zone atmosphere, eastward.

Since the magnetic drag, though strong, provides no preference for eastward or westward flow, it seems reasonable to presume that this is the dominant effect in determining the direction of stable flow.

The complexities introduced by the spherical boundaries of the core and spherically radial gravitational field may affect the motion in several ways. With no Coriolis force, we would expect convective streams along the spherical radius r, in patterns of which Fig. 1 is a simple special case. With a dominant Coriolis force alone we might expect the thermal convection to take place mostly in planes normal to the axis, at least in the region between the dotted lines on the right side of Fig. 22(a). The Taylor effect would lead one to expect the fluid above and below this region to follow the convective pattern in part, but there are difficulties in trying to fit the curved upper and lower surfaces and also the surface of the core. With limited influence of the Coriolis force a compromise something like Fig. 22(b) might perhaps be reached, involving not quite cylindrically radial flow, with convective columns (like the "idler wheels") either bent, as shown, to try to fit the boundary, or bent in another direction to try to accommodate the "hairpins" of the magnetic field. Another possibility is a cylindrically radial stratification, as suggested in Fig. 22(c). Such stratification is observed by D. Fultz in a laboratory experiment with a fluid container of this spherical symmetry rotating rapidly about a vertical axis. The tendency of the Taylor effect to induce vertical rigidity of the flow pattern is less flagrantly violated if the flow pattern is confined to a fairly thin cylindrical shell of which the ends, while not flat, do not deviate from flatness as much as in a thicker shell. Among the possibilities for the transport of heat out through this sequence of shells would be a jet-stream flow pattern within each shell, as suggested in Fig. 22(d). Such a pattern might break up into a collection of cylindrical vortices with a tendency to avoid radial drifts because of the change of length required, though a change in length of such a vortex in a nonideal fluid is possible, with a corresponding change in thickness and angular velocity about its own axis. The modifications of any of these patterns by a magnetic field might, of course, be quite profound.

18. Influence of a Jet-Stream Flow Pattern on the Mechanism of the Westward Drift

Let us consider a jet-stream flow pattern similar to Fig. 3(c), or even Fig. 22(d), near the equatorial plane of the fluid core, perhaps somehow fading off gradually toward the poles. Some of the places where a magnetic field parallel to the axis could thread through this pattern without being squeezed out of moving matter are the inner core and the "idler wheels." If these bodies continue to rotate in spite of the magnetic field, there will be a tendency to concentrate the field in twisted bundles of magnetic lines near the axes of rotation, in a form of bending within these smaller bodies somewhat resembling the "hairpins" discussed on a larger scale in connection with Figs. 8 and 18. (In the smaller bodies the migration toward the axis would be relatively more important and the lines would probably not twist around the axis so many times as estimated for the simple flow in the whole core, even without the instability that leads to kinking.) The torques exerted on the core as a whole by the twisting of these bundles contributes to the drift of the pattern but only in a small way, both because they are of opposing signs and because of the comparatively small radii of the bundles acting as "lever arms" of the torques.

We have mentioned that it is possible to postulate an energy source about 10^8 times as strong as needed for a generator of high efficiency, but we have no assurance that the energy source is this great. The property of a dynamo like the one here discussed to build up the field to such a strength as to use up all the driving energy, with whatever inefficiency it may have, makes it seem likely that the field would slow down but not stop the rotation of the central core and the "idler wheels," and become somewhat bunched near the rotation axes.

Besides these twisted bundles through the rotating regions, magnetic lines are to be expected, perhaps trapped only temporarily as they are pushed through, in the space between the idler wheels and the jet stream, penetrating into the edges of each. The main contribution to the drift of the flow pattern would be expected to come from the drag on these magnetic lines, since this drag acts with the lever arm ρ to produce a large torque about the z-axis. We have seen in Sec. 16 that there is reason to expect the jet flow to be eastward. This flow is analogous to the paddle wheels of a river boat always moving aft and pushing the boat forward, in this case pushing the fluid core with its flow pattern westward. There are various sources of resistance to the general rotation of the core with its flow pattern, analogous to the hull friction of the boat, in particular the frictional or local magnetic drag between the outer "idler wheels" and the mantle where they come in contact.

We keep the primary result of the "thermal wind" mechanism, that the jet flow should be eastward, because the magnetic field does not introduce a preference. The secondary result of the "thermal wind" mechanism is that the drift also should be eastward, but this is opposed and presumably overcome by the dominating magnetic effect arising from the drag on the eastward jet flow.

The mechanism of the westward drift is then rather similar to that discussed in Sec. 12 and Fig. 19, where the fluid core was considered to be divided into two parts, the volume inside and the volume outside of a cylinder of radius B. Here instead we again divide the volume into two parts, the jet stream and the rest of the fluid. The jet stream flows eastward for reasons associated with keeping the angular momentum per unit volume of the inner half equal to that of the outer half. The magnetic drag on the jet-stream part is westward, so the general drift must be westward in order that the drag on the rest of the fluid may be eastward, permitting the two magnetic torques to balance in the steady state. In this case the entire jet stream pattern drifts westward and there is no contradiction with Vestine's conclusion that the angular velocity of westward drift appears to be independent of latitude, as discussed in Sec. 12.

The type of flow pattern suggested in Fig. 22(c) and (d), which divides the fluid core into a number of increments of cylindrical radius $d\rho$ and may contain a jet stream in each $d\rho$, may be expected in a general way to tend to establish a flow with the average angular momentum per unit volume independent of ρ, as previously discussed in Sec. 12 Eq. (22), ff, since the drag between adjacent sections provides for a radial transport of angular momentum. The general remarks there made about the westward drift might then apply, with the irregular pattern of the observed field arising from the outer section $d\rho$ which drifts westward.

19. Influence of a Jet-Stream Flow Pattern on the Induction Process

The salient feature of the induction theory that became clear from the discussion of Secs. 6, 8, and 10, especially in connection with Figs. 15 and 17, is that it depends not only on the existence of upward and downward streams in the equatorial plane, such as are present also in jet-stream patterns, but also in an essential way on the spreading out (or drawing together) of these streams at the surface in such a way as to provide a north-south component of velocity, such as is absent from a strictly two-dimensional jet-flow pattern. Since the presence of a magnetic field, the inhomogeneity, and the spherical geometry all induce deviations from the two-dimensional flow of the Taylor effect, it is in a general way not to be excluded that there may be some flattening of the jet stream as it comes against the surface to provide the essential north-south component flow. From the above discussion it appears not to matter if the spreading is more nearly in the shape of a cobra's hood than of a mushroom. It is, however, crucial that it should be the outward stream that spreads and the inward one which contracts, corresponding to the way in which, for example, $\mathbf{v}$ in Fig. 17(f) points upward to the right. The change in sign of just one component of this velocity would make the difference between a regenerative and a degenerative induction process.

It would require a quite drastic deviation from the almost-two-dimensional flow patterns we have been discussing to provide sufficient spreading of the outward stream near the surface. The deviation suggested Fig. 22(b), with flow in shallow cones is, for example, not enough. Here the outward stream in such a cone still makes an angle with the radius vector at any point such as to have a component toward the equator, whereas a component toward the pole is required. Also the pattern suggested in Fig. 22(c) and (d) does not contain the north-south component required for the dynamo process, but would provide a rotation of the inner core as a basis for the twisted-kink process.

It was mentioned at the beginning of Sec. 16 that a magnetic field should encourage the formation of a jet stream. The tendency to form a large Elsasser toroidal field H_2 as suggested by Figs. 8 and 19 may be quite strong, perhaps as a result of a transient period of haphazard radial transport before establishment of a jet-stream pattern (as is observed in the laboratory demonstrations of jet streams). Such a field would favor a z-direction stratification of ρ-direction flow, and thus might give rise to a jet stream or other patterns of radial flow in the equatorial plane and perhaps others in planes parallel to it (e.g., just two others about 1000 km north and south to conduct the heat away from the polar regions of the inner core) separated by intervals dz in which H_2 is strong and the flow inhibited. The magnetic lines could then weave between the regions of radial flow to form hairpins approximately as in Fig. 8. The radial streams might then have an approximately round cross section, like the body of a cobra, spreading out like the cobra's hood where the flow approaches the surface and turns in the direction of H_2 so as not to be impeded by it. It is in this extreme, in which the magnetic field completely suppresses the Taylor effect and associated "thermal wind" tendencies for flow resembling two-dimensional flow, that the Bullard dynamo induction process may perhaps be realized.

IV. THERMOELECTRIC AND OTHER THEORIES

20. Properties of Matter at High Pressures

In units used in the study of atomic structure, the hydrostatic pressure throughout the core of the earth is of the order of magnitude one electron-volt per cubic angstrom. It is capable of supplying enough energy per atom to excite the atom from one electron state or band to the next, but this is not exactly what it does. Instead it is expected to compress the atom enough to broaden and raise the bands.

At high pressure a metal is apt to go into a body-centered cubic lattice in which the volume attributed to each atom may be approximated by a sphere of radius r_s. The behavior of the energy bands as a function

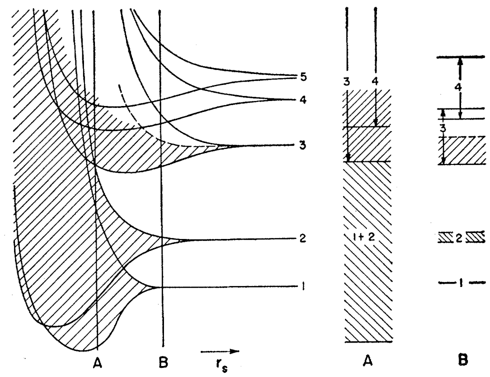


FIG. 23. Electron energy bands as affected by compression.

of r_s is represented schematically in Fig. 23. At large r_s the levels are the atomic energy levels. As the atoms are brought together, first the upper unoccupied levels start to spread out into bands, then the level occupied by the outermost electrons, and at still smaller r_s the lower electron states fully occupied by some of the inner electrons start to spread out into bands. The spreading may be thought, on a simple Heitler-London picture, for example, to arise from the fact that there are many states of the polyatomic system corresponding to one state of each atom, and that these degenerate states of the system start to "repel" each other because of the electron interactions when the wave functions of the corresponding electrons in the atoms begin to overlap. Thus the smaller the radius of the outer part of an electron wave function in an atom, the closer the atoms must come before the electron level starts to spread out into a band.

As the atoms are brought closer together, another effect is the gradual rise of the centroid of the band to higher energies. This is analogous to the way the quantized states in a box are elevated in energy as the box is made smaller. More kinetic energy is prescribed as the atoms are squeezed into a smaller space. The cohesion and equilibrium radius for ordinary metals are determined by the fact that the highest occupied band is only partly filled, so the electrons may settle predominately in the depression in the lower half of the band.

As pressure is applied, at the radius at which it has imparted to the atom one or a few electron volts, most of this energy will be accounted for by the rise in energy of the electrons in the highest occupied band, though the contribution of the rise of the next lower band may also be significant. At the compressed radius, some of the lower bands will have begun to spread out, and the empty band next above the first occupied band may have begun to overlap it, as $r_s = A$ in Fig. 23. At normal pressures, $r_s = B$, there is a gap between these two bands and the material is a conductor or an insulator depending on whether or not the first occupied band is partly or completely filled, that is, on whether or not the electrons at the top of the "Fermi sea" may become conducting to other states in the same band, by being excited only a little by an electric field. If the gap between the two levels is not much wider than kT, or if it is divided into small jumps of this order by intermediate levels provided by impurtiy atoms, the substance is a semiconductor with conductivity increasing with increasing temperature T. Thus we see that the application of pressure of the order of magnitude available can change an insulator or poor semiconductor into a good semiconductor or a conductor by reducing or eliminating the gap.

The magnitude and sign of the thermoelectric effect depends in a complicated way on the difference between the behavior of the electrons in the conduction bands of the two materials, and on the matching of the bands at the junctions. It is affected by the distribution of the density of levels in energy throughout that portion of the bands to and from which electrons may be thermally excited. The distribution of level density is, of course, different in the two materials, and the amount of thermal excitation is different at different parts of each because of the temperature gradient. While these effects are complicated enough that we cannot expect to find them completely calculated, it appears that the general nature of the thermoelectric process is not greatly altered by the application of high pressure. The tabulated thermoelectric powers of the metals (relative to Pb) near room temperature range in absolute magnitude from about 0.003 to 300 microvolts per °C difference between the two junctions, with only about one-fourth as many above 10 μv/°C as below. The few exceptionally large values may be expected to arise from cases in which the bands in the two metals happened to fit together in some particularly favorable way, perhaps with the top of a band in one metal just overlapping the bottom of one in the other, for instance. High pressure spreads out the bands, causing them to overlap and lose some of their distinctive features, and high temperature smears out the surface of the "Fermi sea," and averages over a wider region, so we may expect, in general, that the main effect of high pressure and high temperature will be to seek the average of thermoelectric properties and avoid extreme values. Thus it is a reasonable guess that the thermoelectric power encountered between two different substances deep in the earth will be comparable with the representative, not the exceptionally large, laboratory values, say 1 to 10 μv/°C.

The Hall effect depends in sign presumably on whether the conduction is primarily attributable to electrons or "holes" (that is, the empty states not occupied by electrons at the top of an almost-filled band). Since the pressure tends to make the relevant bands overlap, it voids the concept of "holes" in the simplest sense, and therefore might be considered to favor the probability of a negative Hall effect, due to electrons. However, the distinction is not quite so simple, and depends on the derivative of level density at the energy of the conduction electrons. The magnitude of the Hall coefficient depends on the effective mean free path of the electrons, that is, on how far they go before they suffer a collision and forget about their recent magnetic deflection, and this may vary with energy. The high temperature means that one must average over a quantity of varying sign and magnitude over some energy range, which again should tend to avoid extreme values and give relatively small values of the Hall coefficient, still, however, of the order of magnitude of most laboratory values. The effective mean free path is shorter at higher temperature because of the greater disorder in the lattice, and this again tends to make both the Hall coefficient and the conductivity deep in the earth somewhat small compared to laboratory values, though this effect may be compensated by the ordering effect of high pressure.

21. Thermoelectric Effect as a Possible Origin of the Earth's Field

Because of the easy distinction of the presumably metallic fluid core and the solid mantle which may be assumed to consist of minerals not very different from those we know near the surface of the earth, particularly olivene, $(Mg, Fe)_2SiO_4$, the first modern attempts to treat the origin of the earth's magnetism did so on the assumption that the primary conductivity takes place in the core, while the mantle was assumed to be almost an insulator and to conduct only enough to attenuate the fluctuations of the fields produced in the core. With this assumption, Elsasser (1939) proposed that geomagnetism might have its origin in thermoelectric currents within the core. The convection within the core was assumed to be driven by a radioactive heat source near the center and to consist of upward and downward streams, in a pattern perhaps similar to Fig. 2 though a smaller-scale convection was favored in the discussion. The thermocouple was obtained by assuming that the warmer upward stream had also a somewhat different material composition from the downward cool stream, perhaps as a result of random inhomogeneities in composition. The effect of the Coriolis force tends to keep the circulatory motion parallel to the equatorial plane, and the otherwise random inhomogeneities in the temperature-composition pattern are then expected to produce a magnetic moment parallel to the axis of rotation. Difficulties were encountered [Inglis and Teller (1940)] in permitting large enough velocities for a sufficient Coriolis force and large enough temperature differences for a sufficient thermoelectric force without requiring more heat transport than seemed compatible with the observed flow through the crust.

The more recent realization that the conductivity in the mantle can be great enough to contribute to the primary process makes it possible to improve on this early mechanism, but the same difficulties still make

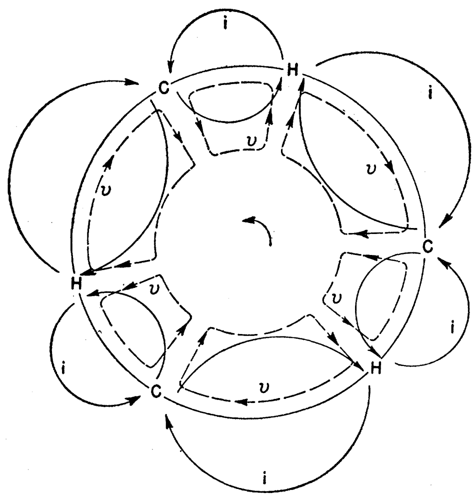


FIG. 24. Possible thermoelectric currents.

it implausible that this is the main source of the earth's magnetism, as we shall see. In Fig. 24, a pattern is drawn of six convective loops in the equatorial plane, and it is assumed that the loops rotating counter to the earth's rotation are larger than the others because they are favored by the Coriolis force. Alternatively, we could just as well consider the jet-stream pattern of Fig. 3(c), in which the angle between the upward and downward streams of a given loop where they make contact with the mantle is less than the angle occupied by the adjacent "idler wheel" region. Let us now consider the thermocouples formed by the mantle as one conductor and the core as the other. The hot and cold junctions at the surface of the core are marked H and C, respectively, in Fig. 24. If the thermoelectric power of the mantle relative to the core is positive, current flows in the six loops as shown, the three larger loops clockwise and the three smaller ones counterclockwise.§ There is thus a north-south asymmetry that may be expected to give rise to a net magnetic dipole moment parallel to the axis.

The upward flow of heat through the crust is observed to be about $1.5 \cdot 10^{-6}$ cal cm^{-2} sec^{-1} (Jeffreys 1952, page 282) which, if assumed to apply also to suboceanic flow, amounts to $8 \cdot 10^{12}$ cal/sec for the whole earth. This may be accounted for entirely by radioactivity in the crust. Thus we have no indication that there is any upward heat flow through the outer mantle. We have no reliable indication of its temperature. Urey's mechanism for convection driven by the settling of heavier constituents provides a very diffuse source of heat, spread out wherever there is viscous friction, or electric dissipation from magnetic drag opposing the large-scale convection. A reasonably efficient thermoelectric generator requires a more concentrated source of heat. Meteoric samplings, presumably of a shattered planetary mass perhaps similar to the earth's interior, make it seem unlikely that it contains a concentrated radioactive source of heat, but this is not direct proof. The solid inner core of radius 1250 km within the fluid core would be a favorable place to have a source of heat. Elsasser (1950, page 28) has postulated that UO_3 and ThO_2 might provide such a source, but Urey (1951, page 270) objects that it is difficult to imagine any geochemical process which would result in the segregation of these oxides.

For the sake of examining the thermoelectric theory, let us simply assume as large a heat source in the inner core as may be needed, and still not contradict directly observed terrestrial data such as the flow through the crust. We may assume that the earth started out fairly cool and has been heating up from the center ever since, so long as the heat wave has not penetrated out to the surface. The heat conductivity over the large distances is so small and the heat capacity of the large mass so large that this process will permit a somewhat greater heat transport across the fluid core than is observed up through the crust. It is thus not necessary to assume a steady state as was done by Inglis and Teller (1940).

We assume a steady source of heat in the inner core, some of which is used to heat up the core to a temperature which throughout the fluid core is almost independent of radius because of the convection. Let the rest of it, the rate of flow of heat up through the inner surface of the mantel, be S. If an inner layer of the mantle of thickness $r-R_C$, density ρ and specific heat s, is heated to an average temperature T in the age of the earth t, we have roughly

$$St = 4\pi R_C^2(r-R_C)\rho sT. \tag{52}$$

If this heat flow passes into the mantle by conduction rather than convection, there will be a temperature gradient at the inner surface somewhat larger than, but of order of magnitude, $T/(r-R_C)$, and with heat conductivity λ, we have

$$S = 4\pi R_C^2 \lambda T/(r-R_C). \tag{53}$$

In terms of electric conductivity σ which is expected to be not so strongly dependent on temperature we have, from the Wiedemann-Frantz relation (Chapman and Cowling 1952, page 316)

$$\lambda = 2.44\ 10^8 T\sigma \text{ g cm}^3 \text{ sec}^{-4} \text{ deg}^{-2}. \tag{54}$$

Combining these three equations we obtain a rough estimate of the electrical conductivity σ required for an inner layer of the mantle of thickness $(r-R_C)$ to be heated to a temperature T by thermal conduction in the age of the earth:

$$\sigma = (r-R_C)^2(\rho s/2.4\ 10^8 Tt) \text{ g}^{-1} \text{ cm}^{-3} \text{ sec}^4 \text{ deg}^2. \tag{55}$$

§ Runcorn (1954) has discussed a thermal and current distribution in terms of a single Legendre polynomial $P_1{}^m(\theta)e^{im\phi}$ (depending upon further interaction with the flow to change an "electric-mode" field into a "magnetic-mode" field), but it should be noted that the distribution of Fig. 24 cannot be approximated in this way without introducing additional symmetry, which would nullify the proposed effect.

In evaluating this roughly we may take $\rho s = 1$ cal deg^{-1} cm^{-3} and $t = 10^{17}$ sec. Because, as was mentioned in the discussion of Sec. 20, the pressure may be assumed to have spread the upper occupied electron energy bands in olivene enough to wash out the distinction between conductor and semiconductor, and it is plausible that the conductivity might have a value even as high as one characteristic of very good metallic conductors, $\sigma = 10^6$ ohm^{-1} cm^{-1} $= 10^{-3}$ cm^{-2} sec, ten times as high as in Eq. (7). The temperature at the surface of the core cannot be higher than the melting point of silicate rocks there. At laboratory pressures they expand on melting so high pressure inhibits melting. At zero pressure the melting point is about 1300°C and at the pressure of the surface of the core Uffen (1952) has estimated an upper limit of 6000°. For purposes of orientation, we find from Eq. (55) that the conductivity required so that the inner half of the mantle, with $(r-R_C) = 1500$ km, may be heated by conduction to half of this temperature, 3000°, is about

$$\sigma = 10^{-5} \text{ cm}^{-2} \text{ sec.} \tag{56}$$

It is thus very likely that the conductivity of the lower half of the mantle is great enough to have heated it to a rather high temperature, if a sufficient source of heat is available in the core. At the lower pressures of most of the outer half of the mantle, the conductivity would be expected to be considerably lower than that of metallic conductors, and perhaps less than Eq. (56), and without transport by conduction alone this could prevent the heat from penetrating too abundantly out to the surface even if the inner half were heated to, say, 3000°.

With the enormous dimensions available in the earth, plastic flow is considered to be a likely mechanism of heat transport throughout most of the mantle, in spite of its being sufficiently solid to transmit rapidly transient transverse seismic waves. If there is no regularly stratified chemical inhomogeneity to inhibit convection, this mechanism could limit the temperature of the inner part of the mantle to rather smaller values, perhaps near 2000° (Verhoogen, 1954).

For the sake of all reasonable generosity to the thermoelectric theory of terrestrial magnetism, let us assume that the inner half of the mantle has been heated to a temperature of 4000°. The rate of heat flow required to do this in the age of the earth is

$$S = 2\times 10^{26} \text{ cm}^3 \rho s T/t = 1.5\times 10^{13} \text{ cal/sec,} \tag{57}$$

which is only about twice the flow out through the outer crust. Since most of the latter must be attributed to radioactivity in the crust, we thus gain perhaps a factor ten over a permissible steady-state assumption.

In order to obtain as large a thermoelectric effect as possible with this limited heat flow, we assume that a circulation in the core similar to that in Fig. 24 takes place in quite narrow filaments, of diameter D, cross-sectional area D^2. Each rising filament heats a considerable area, of order of magnitude R^2, of the core-mantle interface. Presumably the conduction in the mantle is poorer than in the core (perhaps because of less local convection within the layer) and the temperature at the interface at any point will be nearly that of the stream in the core. We may thus take the same temperature difference ΔT between the centers of the junctions H and C in Fig. 24, and between the rising and descending filaments in the core. The temperature of the interface will vary continuously between the limits at H and C, so in effect we have two large thermal junctions, each of area about R^2, with an effective difference of temperature something like $\Delta T/2$.

We then have a thermoelectric current approximately

$$I \approx R\sigma Q \Delta T/2 \tag{58}$$

going around a circuit of area roughly R^2, to make a magnetic moment

$$\mu \approx IR^2 = R^3 \sigma Q \Delta T/2 \tag{59}$$

for each of the six loops. Because of the cancellation between the loops, the total dipole moment may be comparable to that for one loop. Here Q is thermoelectric power, σ is electric conductivity, and R the radius of the core, 3400 km. Taking the representative values $Q = 10^{-5}$ volt deg^{-1} $= 10^3$ g$^{\frac{1}{2}}$ cm$^{\frac{3}{2}}$ sec^{-2} deg^{-1} and $\sigma = 10^{-3}$ cm^{-2} sec, we have

$$\mu = 2\times 10^{25} \Delta T \text{ g}^{\frac{1}{2}} \text{ cm}^{\frac{5}{2}} \text{ sec}^{-1} \text{ deg}^{-1} = 10^{26} \text{ dyne}^{\frac{1}{2}} \text{ cm}^2,$$

the latter being the observed value of the earth's moment. We thus estimate the order of magnitude of the temperature difference required to be

$$\Delta T = 5°\text{C}, \tag{60}$$

or perhaps more because of possible further inefficiency of the process.

If for the product of the density and specific heat of the core fluid we again take $\rho s = 1$ cal deg^{-1} cm^{-3}, the net upward heat transport by N pairs of upward and downward streams, each of speed v and area D^2, is

$$ND^2 v\rho s \Delta T = 6\times 10^{17} N (D/R)^2 v \text{ cal/cm.} \tag{61}$$

If we equate this to the heat flow of Eq. (57), taking $N = 3$, we find the speed

$$v = 10^{-5} (R/D)^2 \text{ cm/sec.} \tag{62}$$

This is an upper limit: if ΔT is larger than Eq. (60), v must be correspondingly smaller in order not to carry too much heat.

Unless the stream diameter D is very small, this is a small velocity and the question arises whether the Coriolis force is large enough to have any appreciable effect on the motion. The most favorable case is the limit in which Eqs. (60) and (62) apply. The Coriolis force per unit volume arising from the earth's angular speed $\Omega \approx 10^{-4}$ sec^{-1} is

$$\begin{aligned} F_C = 2\rho v \Omega &= 2\times 10^{-9} (R/D)^2 \rho \text{ cm sec}^{-2} \\ &= 2\times 10^{-8} (R/D)^2 \text{ dyne cm}^{-3}, \end{aligned} \tag{63}$$

with Eq. (62) and $\rho=10$ g cm^{-3}. A fair indication of the other forces acting on the fluid is obtained by estimating the buoyant driving force, which is the difference between term (2) of Eq. (1) for the upward and downward streams,

$$F_D=\alpha\rho g\Delta T=3\times10^{-2}\rho \text{ cm sec}^{-2},$$

with the coefficient of expansion $\alpha=10^{-5}$ deg^{-1} and with $g=600$ cm sec^{-2}. In the complicated local balance between the terms of Eq. (1), it is not clear how large the Coriolis term would have to be to introduce an appreciable asymmetry in the convection pattern. The Coriolis force acts in a direction normal to the driving force, and is compensated directly rather by gradients of the pressure that are built up, while the driving force is compensated more directly by magnetic drag and perhaps viscous forces. It seems safe to assume that the term C, if not quite as large as the others, will have to be within an order of magnitude of them to have any appreciable influence. Putting $F_C=F_D/10$ we find that the stream diameter D must be very small,

$$D=10^{-3}R. \tag{64}$$

(This large v is required only by the thermoelectric theory; an induction theory allows ΔT, and thus the driving force per unit volume to be much smaller and a smaller v in a thicker stream may still leave the Coriolis force significant.)

Though the very narrow stream required by the thermoelectric theory seems extreme, the crowding of the stream by the magnetic field discussed in connection with Fig. 21 might perhaps provide a sufficiently effective mechanism to confine the flow to such fine filaments. It could also provide the drag to prevent further acceleration, and it is needed for this since even with such narrow streams ordinary viscosity is not sufficient. If the flow just outside of the filaments should remain locally laminar, the retarding force per unit volume of the filament would be about

$$F_V=V\eta/D^2=10^{-11}\eta \text{ cm}^{-1}\text{ sec}^{-1}. \tag{65}$$

For this to equal the driving force, $F_V=F_D$ with $\rho=10$ g cm^{-3} would require an enormous coefficient of viscosity, 3 10^{10} g cm^{-1} sec^{-1}, about that for cold pitch and much larger than the usual values for liquid metals, 10^{-2} to 10 g cm^{-1} sec^{-1}.

22. Combined Thermoelectric and Hall Effects

Thus we see that rather extreme assumptions are required to make a thermoelectric theory of the earth's field adequate, either an extreme geometry or extreme and implausible values of some of the physical properties of matter such as the thermoelectric coefficient Q. These unpleasant requirements arose from a desire to have the Coriolis force large enough to influence the flow, to attain an asymmetry in the spacing of the hot and cold junctions so as to avoid cancellation of the contributions to the earth's magnetic moment. An equivalent asymmetry may be supplied by invoking the Hall effect on the currents in the mantle, in a comparatively simple and attractive scheme suggested by Tatel, Tuve, and Vestine (Vestine, 1954). It requires temperature differences of the order of magnitude given by Eq. (60), but the velocities may be as small as or smaller than Eq. (62) with $D\approx R$, since the process is not critically dependent on Coriolis force. Just as with the thermoelectric effect alone, the streams must carry enough energy to supply the Peltier heat, but the general thermodynamic argument mentioned above indicates that they can.

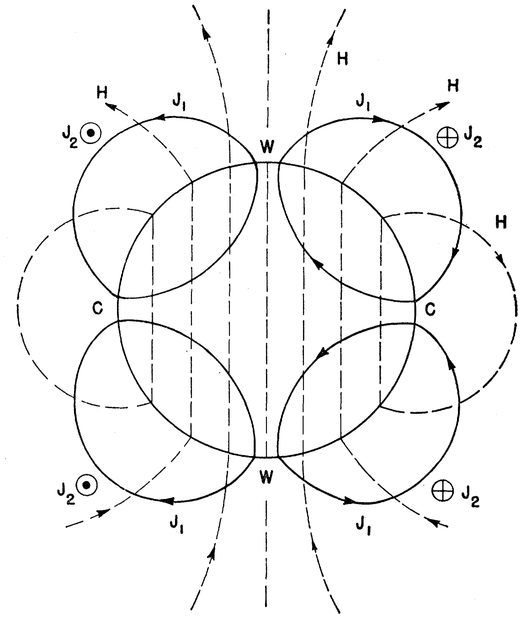


FIG. 25. Hall effect influencing thermoelectric currents.

Let us assume an even simpler convective situation that we have been considering, with simply a cold belt C, around the equator and a warm region W, at each of the poles of the core-mantle interface. This could come about because the velocities are so low that Coriolis effects are negligible compared to the magnetic field which inhibits the sort of flow we have been discussing in the equatorial plane but permits a modified Bernal cell flow between the inner core and the polar regions of the mantle, parallel to the magnetic field. The opposite situation, with cold at the pole and a warm belt at the equator, could be established if the Coriolis effects are more influential than the field. Either situation is as good as the other for the first step of the process. In this step it is only the symmetry that matters unless we go farther and worry about the signs of the thermoelectric and Hall coefficients which are difficult to guess from experience confined to the laboratory. In Fig. 25, the circle represents the core-mantle interface, and the system of currents J_1 is set up by the thermocouple. With this system of currents, the Hall effect in the mantle (where it is expected to be much more important than in the molten metal of the core) if it happens to have the appropriate sign can produce a regenerative process. A stray field H produces Hall currents J_2 in circles about the axis in the directions indicated (cross, into the paper: dot, out), which tends to regenerate the part of the field threading through it.

Examined in a little more detail, the scheme is not quite as simple as it seems at first. Since a circle of current J_2 contributes only to the field threading through it, attention should be drawn to the question "What current supports the field immediately surrounding the equatorial circle?" In Fig. 25 are indicated successive loops of H about the point on the equator marked C. For each such loop we have

$$\int \mathbf{H}\cdot d\mathbf{s} = \int \mathbf{J}_2\cdot d\mathbf{S}, \qquad (66)$$

where the first is a line integral around the loop and the second is a surface integral over the area enclosed by it. Considering these quantities as functions of the distance r from C (after separating off a common angle factor in the neighborhood of C where the geometry is simple), we may take $H(r)$ proportional to the Hall current $J_2(r)$ that it produces, $H(r)=k^{-1}J_2(r)$, and consider $|\mathbf{dS}|=dr\int|\mathbf{ds}|$. We then find

$$J_2(r)=k\int_0^r J_2(r')dr' \quad \text{or} \quad J_2=Ae^{kr}. \qquad (67)$$

Here k is essentially the Hall constant divided by the conductivity and has the sign of the Hall constant. Depending on whether k is positive or negative, the circulating current either builds up or decays with increasing r from the value of $J_2=A$ at the appropriate point near the equator at which H vanishes. This $r=0$ value of J_2 is not supported by the Hall effect. With a Hall constant of the appropriate sign (positive for the J_1 shown in Fig. 25), the effect here considered can merely act as an amplifier of an initial J_2 at the equatorial point $r=0$, and then only if this initial J_2 has the appropriate sign (the one indicated in Fig. 25 for the stray H there shown, or the opposite one if H is reversed).

The rectifying action of this last condition provides the necessary north-south asymmetry, to take the place of the Coriolis force used in Sec. 22. Consider the case in which the equatorial plane, while in general perhaps warmer than the poles, has convection and temperature differences within it. If in Fig. 24 the Coriolis influence is relaxed so much that there are hot and cold junctions equally spaced around the equator, then the thermoelectric current i at or very near the equator would have directions alternating with longitude, and only in alternate sectors the amplification is described by Eq. (67) would be effective (unless the stray H is more complex, alternating between the sections, in which case it would be expected to be unstable and grow in some sections at the expense of the others). At this stage we would have a net magnetic moment of the earth, but with a field more dependent on longitude than observed at the surface. As a further step, the amplified J_2 arising from the Hall emf in the favored sectors might overwhelm the primary current i near the equator in the unfavored sectors, leading to amplification in those sectors also and smoothing out the observed field.

Though various other motions and effects would be superposed on this thermal mechanism, it does not appear that they would much modify it unless they are so strongly degenerative as to nullify it. In particular, if there is any convection parallel to the equatorial plane, the mechanism of the westward drift (Sec. 12) would be operative and tend to make toroidal magnetic fields in the core, the "hairpins" of Fig. 16. The schematic straight lines of H drawn in Fig. 25 would be bent also by the ρ components of the convection. This flow might include a jet stream warming the equatorial belt more than the polar regions, when averaged over longitude.

23. Regeneration by the Electric Field Induced by Compression

Quite independent of the complexities of fluid flow within the core and even independent of the rotation of the earth, there are possibilities for a relatively simple regenerative mechanism for maintaining the earth's field arising from the gravitational field within the enormous mass of the earth, and the consequent electric fields. In a series of early papers on the subject, Gunn (1929) has suggested some of these possibilities. He discusses in particular the motion of conduction electrons in an electric or gravitational field crossed by a magnetic field and their contribution to regenerating the magnetic field. In more recent unpublished remarks he has suggested that the source of the electric field is probably the temperature gradient (Nernst effect), but it seems that an even more potent source of electric field is to be found in the compression caused by gravity, and it will suffice to present this as perhaps the most important representative of the theories of this general type.

In laboratory physics one makes a habit of thinking that electric fields cannot exist within an ordinary conductor in which no current is flowing, but with thermal gradients or in a conductor so large that gravity is important this is no longer true. Gravity acts on the nuclei but practically not at all on the electrons, and thus tends to cause a separation of positive and negative charges. This tendency is strongly opposed by the attraction between nuclei and electrons, accounting for the high stability of atoms, but in a large conductor such as the interior of the earth, a small partial separation may be effected and electric fields may be developed in the process.

Consider in Fig. 26 the small elements of volume V_1 and V_2 containing identical collections of atoms (or single atoms), but with V_2 deeper and more highly compressed than V_1. The conduction electrons are piled up to a higher kinetic energy or "Fermi energy" in V_2 than in V_1, that is $E_{F2} > E_{F1}$. This is a rough way of speaking of the phenomenon illustrated in Fig. 23, in connection with which it was mentioned that the pressure well within the earth amounts to an energy per atomic volume of the order of magnitude of one electron volt.

By way of a simple estimate of this quantity, a column of matter 1 cm square and 3400 km high with an average density of 7 g/cm^3 in an average gravitational intensity $(2/3)g \approx 700$ cm/sec^2 exerts a pressure at the bottom of 1.7×10^{12} erg/cm$^3 = 10^{24}$ ev/cm^3. Ordinary iron at zero pressure, for example, with a density 7.85 g/cm^3 contains 8.7×10^{22} atoms/cm^3. If the atoms were compressed to twice this density at the center of the earth, or about 1.7×10^{23} atoms/cm^3, this pressure would correspond to 6 ev per atomic volume.

Simple quantization in a box of one electron per atomic volume shows that the kinetic energy per electron is

$$KE_{\mathrm{Av}} = (3/40)h^2/mV^{\frac{2}{3}} = 2.25\ 10^{-15}\ \mathrm{ev}\cdot(\mathrm{cm}^2/V^{\frac{2}{3}}), \quad (68)$$

where V is the atomic volume and h is Planck's constant. The atomic volume for ordinary iron, $V=(1/8.7)10^{-22}$ cm^3 makes $KE_{\mathrm{Av}}=4.4$ ev. Reducing V to half this increases KE_{Av} to 7 ev, an increase of 2.6 ev per atomic volume, which is about half of the pressure energy 6 ev per atomic volume estimated for the center of the earth if iron were compressed by a factor two. Thus about two "conduction electrons" per atom would be required in order that their increase in kinetic energy would account for this compressibility. Actually, iron has more valence electrons than this but they are not all equivalent to conduction electrons. The compression reduces their potential energy by squeezing them closer to the nucleus at the same time that it increases their kinetic energy, in a manner in which even Fig. 23 is a simplified representation, so the net effect may be about the same as the increase of kinetic energy of about two simple "conduction electrons" of which the potential energy is ignored. It is, on this rough theoretical basis, plausible that so drastic a compression should take place. Empirical evidence seems to require almost this much.

For the sake of a rough estimate, let us assume that most of the mantle is conductor—our treatment will apply about as well to the core alone, or to the two together considered as one large conductor. In a deep element of volume V_2, the Fermi energy E_{F2} exceeds E_{F1} higher up by one or two electron volts, say 2 ev. In such a situation, there is no dam to support the higher electron level at V_2 until an electric field develops by the migration of a very small fraction of the conduction

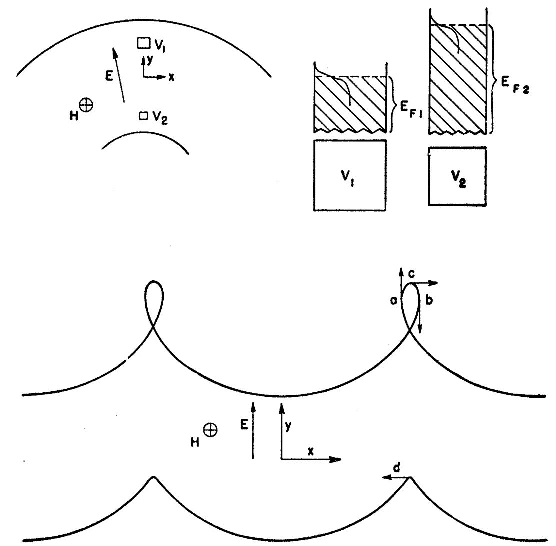


FIG. 26. Effect of radial electric field arising from pressure gradient.

electrons. The migration stops when the electric field, or difference in electric potential, has lifted the bottom of the "Fermi sea" of electrons at V_1 until the top levels coincide at the two positions. This requires a potential difference $(E_{F2}-E_{F1})/e=2$ volts between V_1 and V_2, the region around V_1 having acquired a net negative charge and around V_2 a net positive charge, say Q in the whole inner part of the mantle. We thus have concentric spherical shells, the inner one with Q, the outer one with $-Q$, and an electric field confined to the region between them of magnitude $E=Q/r^2$ ≈ 2 volts/10^8 cm $=7\ 10^{-10}$ esu/cm, with r varying near $2\ 10^8$ cm, requiring $Q\approx 10^{-4}$ abcoulomb or $6\ 10^{15}$ electron charges, about 10^{-10} e per atomic volume.

The top of the electron sea is fuzzy because of the high temperature, as indicated by the short curves near the upper right corner of Fig. 26 representing the shape of the Fermi distribution in energy. The "tailing" of the distribution extends over about kT which for $T=3000°$ is about $\frac{1}{4}$ ev. This is enough smaller than $E_{F2}-E_{F1}$ to show that the temperature is only a secondary cause of electron migration.

Perhaps the simplest of the many inadequate "theories" of the earth's magnetism is the old suggestion that this charge distribution Q rotating with the earth has a magnetic moment. The outer, negative, shell, of course, predominates and incidentally gives a moment of the observed sign, with the magnetic north (-seeking) pole at the geographic south. The magnetic field at the center of a loop of radius r with this charge flowing around it at angular velocity Ω is $Q\Omega/r\approx 10^{-17}$ oersteds for the earth with $\Omega=10^{-5}$ sec^{-1}, $r\approx 10^8$ cm, about 10^{-18} too small!

Let us assume that there is an initial stray magnetic field H in the negative z-direction (into the paper in Fig. 26) and discuss the possibility of regeneration by the electron motion in this and the electric field E in the positive y-direction (radially outward). The path of a (hypothetical) free electron in such crossed fields is a cycloid, the superposition of a uniform rotation on a uniform translation, since there is a moving coordinate system with its $v_x=-cE/H$, in which the magnetic force due to the translation just compensates the electric force and relative to which the motion is therefore circular, with $Hr'=\mathrm{m}v'c/e$. There are two possible types of cycloidal motion as shown in Fig. 26, one with loops in which the translational $v_x<v'$, the speed of the rotational motion in the moving system and one without loops in which $v_x>v'$.

The statistical-mechanical concept of electrons traveling in free paths between collisions with atoms provides a useful approximation to the more refined discussions in terms of the states in energy bands and the interaction of these with lattice irregularities. Comparison of the two approaches shows that the mean free path, λ, is to be taken to have the order of magnitude $5\ 10^{-7}$ cm for a very good conductor under ordinary conditions (Seitz, 1940, page 184). This is much less than the period (in x) of the cycloids encountered in the fields with which we are concerned. The curvature of each electron free path will, however, correspond to some fairly short segment of a cycloid. We make the usual assumption that the free paths start in random directions, and we provisionally neglect any dependence of λ on energy. The problem is to average over the x-displacements of all these free paths of length λ, and it would not be surprising if the general tendency of the cycloid to sweep toward the negative x-direction should also appear in this average, as it obviously must for long λ.

Instead of averaging properly over angles, let us examine the four simplest examples, with electrons leaving their collisions in the positive and negative x- and y-directions. With the initial velocity v_0 up or down, at point a or b on the upper cycloid in Fig. 26, the energy is changed in a path λ by $\pm\lambda eE$ so the average energy during the flight is

$$\tfrac{1}{2}m\bar{v}^2=\tfrac{1}{2}mv_0^2\pm\lambda eE/2, \qquad (69)$$

and roughly we may take the average radius of curvature to be

$$\bar{\rho}=mc\bar{v}/eH=\rho_0(1\pm\lambda eE/2mv_0^2\cdots). \qquad (70)$$

In either case the average x-component of the excursion is $x=\bar{\rho}(1-\cos(\lambda/\rho)=\lambda^2/2\bar{\rho}$ and the average over paths starting at a and b is

$$(\bar{x}_a+\bar{x}_b)/2=\lambda^3eE/4mv_0^2\rho_0=\lambda^3e^2EH/4m^2cv_0^3 \qquad (71)$$

primarily toward the right in Fig. 26 and against the general cycloidal drift because of the greater curvature of the path starting at a than at b. Consider now the paths starting at c and d in Fig. 26, with v_0 horizontal. Here we have

$$1/\bar{\rho}=\partial^2y/ds^2=(\partial^2y/\partial t^2)/v_0^2=(-E\pm v_0H/c)e/mv_0^2 \qquad (72)$$

in which cases $\bar{x}=\pm\bar{\rho}\sin(\lambda/\bar{\rho})=\pm\lambda(1-\lambda^2/6\bar{\rho}^2)$ and we have

$$(\bar{x}_c+\bar{x}_d)/2=-\lambda^3e^2EH/3m^2cv_0^3 \qquad (73)$$

toward the left in Fig. 26 and with a coefficient $\frac{1}{3}$ slightly larger in magnitude than the coefficient $\frac{1}{4}$ in Eq. (71). Thus the average over the four starting points is

$$\bar{x}=-\lambda^3e^2EH/24m^2cv_0^3, \qquad (74)$$

indeed in the direction of the general cycloidal drift. We use this as a rough estimate of the average over-all directions of v_0. In electron experiencing v_0/λ collisions per second has a corresponding drift velocity $(v_0/\lambda)\bar{x}$ and contributes $(e/c)(v_0/\lambda)\bar{x}/R^2$ to the magnetic field at the center of the earth. Taking two electrons per atomic volume $V=10^{-23}$ cm^3 throughout a volume of, say, R^3 with $R=1.5\times10^8$ cm we have for the order of magnitude of the regenerated field

$$H=KH,\quad K=(\lambda^2/12v)(e^2/mc^2)(eER/mv_0^2) \approx 2\times10^{-3}. \qquad (75)$$

In evaluating K we have also used $e^2/mc^2=2.8\ 10^{-13}$ cm, $eEr=2$ ev, $\mathrm{m}v_0^2=\frac{1}{2}$ ev, the value of $2kT$ for about 3000°. In this rough estimate, the mechanism proposed thus lacks a factor 500 of being regenerative. The sign, at least, appears to agree with observation (and leaves no room for the controversial reversals).

This estimate contains many roughly guessed factors, but still makes it seem extremely unlikely that a mechanism of this general type could work. Perhaps the greatest uncertainty is in the effect of very high pressures on effective mean free path, particularly in minerals, but in making the estimate we have already been fairly generous in extrapolating from known data for metals, taking a value of λ for silver, about 7 times as great as for iron at zero pressure. Extrapolation from data at laboratory pressures (Bridgeman, 1949) would indicate an increase of λ for iron by about a factor 3 at the pressures near the earth's center at room temperature, but this expected increase is more than offset by extrapolation of the decrease with temperature.

There is a possibility that the effect is considerably enhanced by the chemical discontinuity between the core and mantle. Though contact difference of potential at zero pressure is commonly of the order of magnitude one volt, comparable to the two volts we have just used arising from pressure difference in a homogeneous medium, it seems likely that the contact potential will be considerably enhanced by very high pressure such as encountered at the surface of the core. If the effective number of conduction electrons per atom in one substance (iron-nickel alloy) is half that in another (olivene), the Fermi energy may be raised by the pressure twice as much in the first as the second. It does not seem likely that this type of contribution will yield more than a factor ten and it may have the wrong sign because the lighter atoms of olivene have fewer electrons in the closed shells to oppose compression than does iron.

It has been noted that this mechanism is not dependent on the rotation of the earth, but if it were the primary source of the regeneration the direction of the field could still be coupled to the axis of rotation by the differential rotation of core and mantle, and the general mechanism of the westward drift already discussed would still apply.

24. Inadequate Generation by Differential Rotation

Among the other theories depending on the mean free path in transport phenomena under peculiar circumstances, there is one depending on the radial gradient of horizontal velocity that is of some slight present interest because it has fairly recently been given an experimental test which shows it to be inadequate. Consider the component circulating about the axis, v_ϕ, of the velocity of the core fluid as a function of distance from the axis, $v_\phi(\rho)$. In familiar statistical-mechanics fashion, the electrons passing a point at ρ carry in addition to a random motion the velocity characteristic of the place predominantly near $\rho\pm\lambda$ at which they had their last collision, and this velocity may be expanded

$$v_\phi(\rho\pm\lambda)=v_\phi(\rho)\pm\lambda\partial v_\phi/\partial\rho+(\lambda^2/2)\partial^2 v_\phi/\partial\rho^2. \quad (76)$$

There will thus be, on the average, a drift velocity proportional to the last of these terms and a current $i=eN\lambda^2\partial^2 v_\phi/\partial\rho^2$ giving a magnetic field $H\approx i\rho$ at the center of a thick, doughnut-shaped loop which scales as v_ϕ/ρ if we take λ and the electron density N to be the same in the cases compared. The experiment (Lochte-Holtgreven, 1953, and personal communication) involves a vigorously churned pot of mercury with $\rho\approx 1$ cm and $v_\phi\approx 100$ cm/sec. A field of about 10^{-3} oersted was observed. Scaling this to the earth with the $1/\rho$ factor only, even taking this too-high value of v_ϕ for the earth, gives a result too small by a factor 10^{-12}.

25. Concluding Remarks

The more natural title "Conclusion" for this section must unfortunately be avoided because the present state of knowledge permits no definite conclusion. In the light of our very limited knowledge of the earth's interior, we have examined the strong points and the weaknesses of a number of proposals, and cannot with finality select any one as alone plausible. We are struck not by the paucity but by the wealth of possible origins of the earth's magnetism within the framework of Nature's laws as we ordinarily postulate them, and thus find it quite unnecessary to give heed to any possibility of a new law to account for just this phenomenon (such as Blackett's hypothesis of a universal relationship between rotating matter and magnetic moment, for which empirical backing has vanished with newer astronomical measurement).

The induction theories are concerned primarily with details of the fluid motions within the earth's core and it seems extremely likely from what has been said that there must be a considerable variety of hypothetical flow patterns that will cause regeneration, and many others that will not. It is likely that there is an entirely adequate source of energy to maintain the field with the rather low efficiency that would be expected of such a mechanism. The evolution of flow patterns that would be induced by the accelerations possible with such a source would be expected to encounter some flow pattern that would generate a magnetic field and be stabilized by the decelerating influence of that field. Thus even without any comprehensive understanding of the details it seems likely that a mechanism similar to one of the two induction theories, or to a combination of them, should be providing us with the earth's field unless the field is already provided by some even more favorable mechanism (in which case the evolution might be stopped by the decelerating influence of the field before reaching a regenerative flow pattern).

The two induction theories, while quite similar in a general way, differ markedly in the fact that the dynamo theory contemplates that the magnetic field will modify the flow pattern mainly by opposing the flow along paths it might follow without the field, whereas the twisted kink theory, while opposing acceleration of the general flow pattern, may also cause local disturbances in the flow by carrying matter along with the kink-forming process. For the dynamo theory with a given flow pattern, the main question is one of sign, whether it is regenerative or degenerative, and only incidentally whether it is strongly enough regenerative. For the twisted kink theory there is essentially no question of sign; it is regenerative if the twisting is rapid enough to make kinks at all, but perhaps not strongly enough regenerative to overcome both the tendency of the kinked loops once formed to attenuate and the possible degenerative tendency of any dynamo action (and thermoelectric action for that matter) that may be present.

Among the other theories, the combination of thermoelectric and Hall effects seems the most attractive. Here again it is a question of sign whether it is contributing to, perhaps being primarily responsible for, the earth's field or opposing it.

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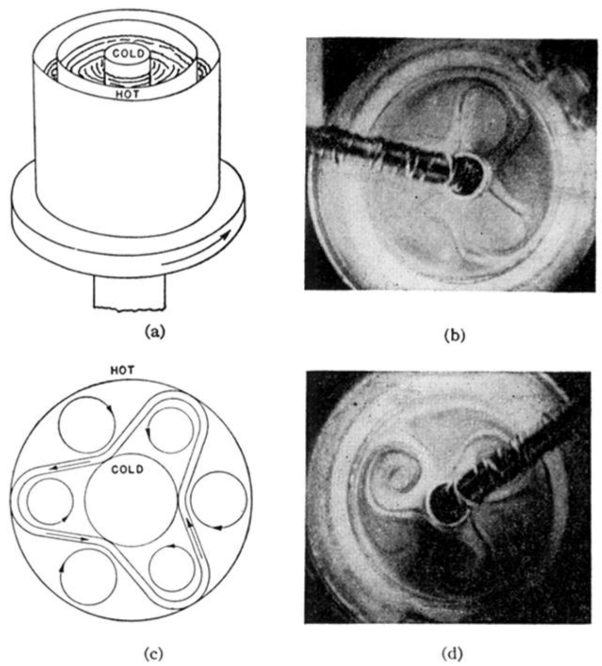

FIG. 3. Laboratory experiment exhibiting convection with a jet stream.