Motion of Domain Walls in Ferrite Crystals

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I. INTRODUCTION

THE experience of Williams[1] and his collaborators has shown that it is possible to cut a sample from a single crystal of a magnetic material in such a way that the domain pattern includes only one movable wall. This wall moves as the sample is magnetized, and a linear relation exists between the magnetization of the sample and the distance between the wall and a fixed plane parallel to it.

We have used similar samples to study the motion of domain walls in ferrites, where eddy currents are a relatively unimportant source of damping. These samples are polygonal rings with each leg along an easy direction of magnetization. Since the (111) direction is the easy direction in ferrites, the rings are diamond shaped with their faces in the (110) plane as shown schematically by the solid lines in Fig. 1. The expected domain pattern consists of four stationary walls, one at each corner, and one movable wall all around the ring. This pattern is indicated by dotted lines in Fig. 1.

The results of our experiments can be understood very satisfactorily in terms of the theory of domain walls and their motion developed in the last few years. In fact, these results may be used to evaluate many of the parameters used in that theoretical development.

II. EXPERIMENTAL

A. Descriptive

The first experiment performed on these samples is designed to measure, by means of pulse techniques, domain wall velocity as a function of the field causing the wall to move. The sample is wound with a primary and a secondary winding. A square pulse of positive voltage is applied to the primary winding in series with a resistor which is large enough to keep the pulse rise time short. The rise time must be short compared to the time required for the field produced by the pulse to reverse the magnetization of the sample. On the other hand, since the pulse is applied for the purpose of reversing the magnetization of the sample, the length of the pulse must be longer than the time required for the reversal to occur. It must, therefore, be longer than the time required for the mobile wall to move from one side of the sample to the other under the field produced by the pulse. A second pulse of negative voltage is applied to the primary during each duty cycle in order to bring the wall back to its original position, so that the phenomenon may be observed repetitively. By synchronizing an oscilloscope sweep with the pulser, the signal induced in the secondary winding is observed while the applied pulse is on the primary. Since this signal is proportional to the velocity of the wall, it is constant during the application of a constant field. Irregularities in the crystal may cause this value to vary somewhat during the motion of the wall, as we shall see later, but the observer reads an average value.

The applied field due to the primary pulse is deduced from the current in the primary winding (measured by observing the voltage across the series resistor) using the solenoid formula $H_{\text{app}}=4\pi NI$. To obtain the relation between wall velocity and induced voltage per secondary turn we have

$$\text{Volts/turn}=d\Phi/dt\times10^{-8}=8\pi M_s(\Delta z/\Delta t)w_{\text{wall}}\times10^{-8}, \quad (1)$$

where $\Delta z/\Delta t$ is equal to domain wall velocity v, and w_{wall} is the width of the wall between the boundaries of the sample in the direction perpendicular to the direction of magnetization. The result of the experiment is thus a plot of ferromagnetic domain wall velocity *versus* the applied field. The slope of this plot measures a viscous damping coefficient for the motion of the domain wall.

The second experiment is the measurement of the complex initial permeability *versus* frequency. Standard bridge measurements are made of the impedance of a coil wound on the sample.

The interpretation of these experiments is, of course,

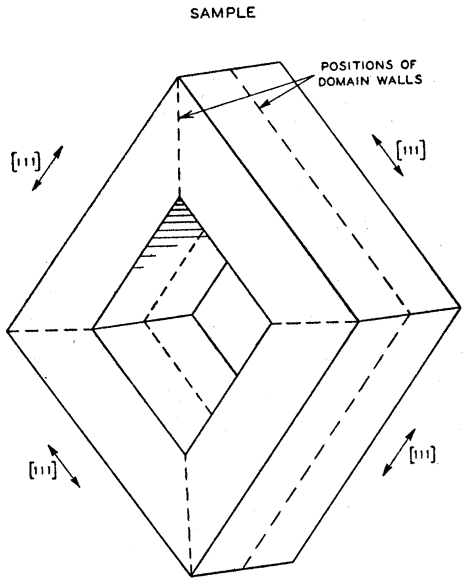

FIG. 1.

[1] H. J. Williams and W. Shockley, Phys. Rev. **75**, 178 (1949).

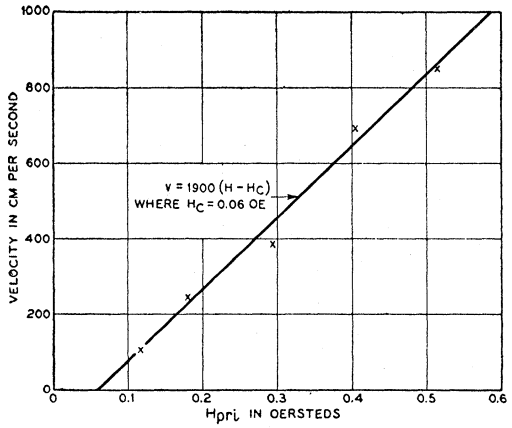

FIG. 2. Domain wall velocity *vs* applied field.

based on the domain pattern indicated by the dotted lines in Fig. 1. Observations on the actual domain pattern of each sample are, therefore, desirable. These observations have thus far proved the most difficult part of the experiments, but the essential features of the ideal domain pattern have been observed.

As a further check to indicate how freely the movable wall actually can move in the sample, a hysteresis loop is also obtained on the recording fluxmeter.[2] If the wall does move freely, the loop should have sides which are almost vertical, of course.

B. Results

The first experiments were done on a sample cut from a crystal of Fe_3O_4.[3] These data are shown in the accompanying figures which indicate the nature of the results.

Figure 2 shows the result of the velocity *versus* applied field experiment. We see that, since the line through the data does not pass through the origin, the field effective in moving the wall is less than the applied field. This difference varies somewhat as the wall traverses the sample, since the signal induced in the secondary winding is observed to vary, but the average is by definition the coercive force, H_c. The value of H_c is given by the intercept of the line drawn through the data.

Figure 3 shows the data on complex permeability *versus* frequency. The magnitude of these values of permeability is so large that we can be quite sure that it is due almost entirely to domain wall motion.

Figure 4 shows the hysteresis loop obtained on the Fe_3O_4 sample. The essential features of the domain pattern were observed using the same technique as that of Williams, Bozorth, and Shockley.[4] These observations are discussed in more detail in a previous publication.[5]

The nickel ferrite results are broadly similar, but they are not as good, and must be regarded as preliminary. $NiFe_2O_4$ is, of course, a more complicated substance chemically, and the crystals available thus far are not as perfect as the Fe_3O_4 crystal. Under chemical attack they show some signs of a variation in chemical composition with position in the crystal, and the composition of the crystal from which the best sample was cut corresponded to the formula $(NiO)_{0.91}$-$(FeO)_{0.09}Fe_2O_3$. As a result of these inhomogeneities, the hysteresis loop was not as square as is desirable, and as a result of the differential chemical attack, the domain pattern on this sample could not be observed.

The data on domain wall velocity *versus* applied field seem reasonably satisfactory, however. The signal in the secondary winding had a sharp peak at the start of the applied pulse which we attribute to the motion of walls around inhomogeneities, but a fairly constant signal of the sort expected followed the peak, and the observations were therefore made in this flat region. The data followed a straight line of the form $v=20\,000$ $(H-1.5)$ where v is in centimeters/sec, and H is in oersteds. Data on other samples indicate that the 20 000 is accurate to something like 25–50 percent. Since this value is considerably higher than the value for magnetite given in Fig. 2, we see that the damping of wall motion in nickel ferrite is considerably less than that in magnetite.

The data on permeability *versus* frequency show indications that only part of the movable pattern has been observed. There is a substantial contribution which shows no frequency dependence up to 20 mc/sec, which was the highest frequency at which data could be taken. There is a contribution, however which fits a relaxation curve much like Fig. 2. This contribution gives a permeability at zero frequency of 32, and its relaxation frequency is approximately 7 mc/sec. This is consistent with the smaller damping in nickel ferrite indicated by the domain wall velocity *versus* applied

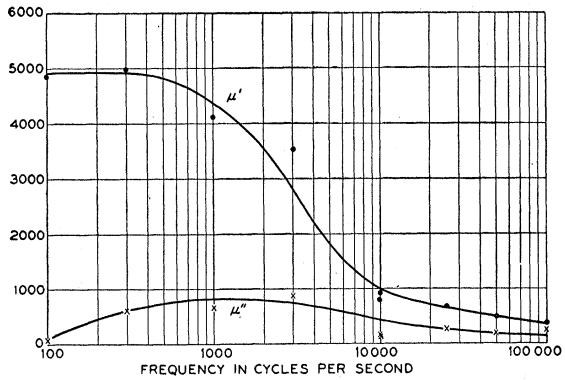

FIG. 3. μ', μ'' *vs* frequency.

[2] P. P. Cioffi, Phys. Rev. **67**, 200 (1945).

[3] J. Smiltens, J. Chem. Phys. **20**, 990 (1952); Technical Report 49. Laboratory for Insulation Research, M.I.T., December, 1951. The authors wish to express their gratitude to Professor A. von Hippel of the Laboratory for Insulation Research for providing a crystal of Fe_3O_4.

[4] Williams, Bozorth, and Shockley, Phys. Rev. **75**, 155 (1949).

[5] J. K. Galt, Phys. Rev. **85**, 664 (1952).

field experiment and with the high coercive force of this sample (1.5 oersteds) which leads us to expect a high α. These data are probably somewhat less accurate than those on wall velocity *versus* applied field.

III. THEORY

A theoretical treatment of the data divides itself rather naturally into two parts. First, we may correlate the data in terms of an equation of motion for unit area of domain wall. In doing this we determine the constants of motion (mass, viscous resistance, and stiffness) of unit area of wall. Secondly, we can show the relation between the mass and viscous resistance of unit area of domain wall and the constants which characterize the ferromagnetic material in general. This last step is essentially an application of the recent work of Becker,[6] Döring,[7] and Kittel.[8]

Consider unit area of a 180° domain wall between two regions of saturated material. Such a system has an equation of motion for small amplitudes of the applied applied magnetic field H which may be written

$$m\ddot{z}''+\beta\dot{z}'+\alpha z=2M_sH, \tag{2}$$

where z is the displacement of the domain wall along its normal, m is its mass per unit area, β is a parameter measuring viscous resistance, and α is a stiffness parameter, which has meaning only for small fields such as those used in initial permeability measurements. When fields larger than the coercive force are applied, as in the experiment on wall velocity *versus* applied field, the term containing α disappears and the effective field inside the material is less than the applied field by an amount equal to the coercive force; this is shown by the data given in Sec. II. These results are quite reasonable when one remembers the spikes which pull back on the wall, in the experiments of Williams and Shockley for small wall motions, and snap off entirely if the wall moves a large distance.

As these remarks indicate, under the conditions of the experiment in wall velocity, Eq. (2) takes the form

$$\beta\dot{z}'=2M_s(H_{\text{app}}-H_c). \tag{3}$$

This relation obviously fits the data given in Sec. II.

Let us now consider the initial permeability data. We have for the relation between z and μ,

$$\mu=\Delta B/\Delta H=\Delta\Phi/A_{\text{coil}}\Delta H=8\pi M_szw_{\text{wall}}/A_{\text{coil}}H, \tag{4}$$

where w_{wall} is the width of the domain wall between the boundaries of the sample in a direction perpendicular to the magnetization and A_{coil} is the cross-sectional area of the coil around the sample. The general solutions of Eq. (2) for a sinusoidal applied field will not be reproduced here, because we note that Eq. (2) can be further simplified since we observe a relaxation mechanism

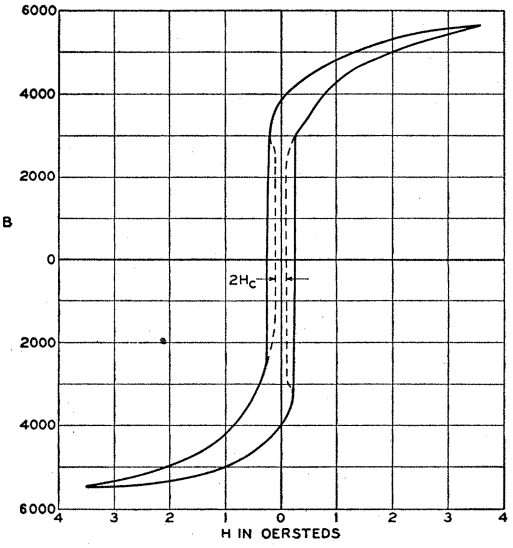

FIG. 4. Hysteresis loop.

(see Sec. II). The values of α, β, and m will be calculated later; suffice it to say here that the order of magnitude of m is such that the first term on the left in Eq. (2) is negligible, and we may write

$$\beta\dot{z}'+\alpha z=2M_sH, \tag{5}$$

which we may solve to give, if $H=H_0e^{j\omega t}$,

$$z=\frac{2M_sH_0}{\alpha}\left[\frac{1}{1+\omega^2\beta^2/\alpha^2}-\frac{j\omega\beta/\alpha}{1+\omega^2\beta^2/\alpha^2}\right]e^{j\omega t}. \tag{6}$$

We shall derive values of α and β later by fitting Eqs. (3) and (6) to the data.

We now come to the second part of the theoretical problem. Starting with the now familiar equation of motion for the magnetization M in a small volume first used by Landau and Lifshitz[9] and taking advantage of the recent work of Kittel,[8] Becker,[6] and Döring,[7] we derive the relations between β and m and the fundamental constants which characterize the ferromagnetic material.

The equation of motion is

$$d\mathbf{M}/dt=\gamma[\mathbf{M}\times\mathbf{H}]-(\lambda/M^2)[\mathbf{M}\times(\mathbf{M}\times\mathbf{H})], \tag{7}$$

where γ is the gyromagnetic ratio, and λ is a parameter characteristic of a given ferromagnetic material which is determined by the magnitude of the damping effects in the motion of M and thus determines the magnitude of the last term on the right in (7). The power dissipated per unit volume is $\mathbf{H}\cdot(d\mathbf{M}/dt)$, which is

$$\mathbf{H}\cdot d\mathbf{M}/dt\cong\lambda H_e^2, \tag{8}$$

where $\mathbf{H}=\mathbf{H}_0+\mathbf{H}_e$, $\mathbf{H}_0$ is the applied field and $\mathbf{H}_e$ is a field associated with the motion of the wall as Becker[6]

[6] R. Becker, J. phys. et radium **12**, 332 (1951).
[7] W. Döring, Z. Naturforsch. **3a**, 374 (1948).
[8] C. Kittel, Phys. Rev. **80**, 918 (1950); J. phys. et radium **12**, 291 (1951).
[9] L. Landau and E. Lifshitz, Physik. Z. Sowjetunion **8**, 153 (1935).

TABLE I. Values of β.

Substance	α	β (from $\mu_{initial}$)	β (from wall vel vs H)	β (corrected for eddy current effects)
Fe_3O_4	9.1×10^3	0.5	0.484	0.406
$NiFe_2O_4$	7.2×10^5	0.016	0.026	0.026

has shown. It exists only in the wall and is perpendicular to the wall. The value of this field is determined from the precessional angular velocity of the spins in the moving wall by means of the Larmor relation. It is

$$\mathbf{H}_e=-(\mathrm{v}/\gamma)(\partial\theta/\partial z), \tag{9}$$

where γ is the gyromagnetic ratio and θ is the rotational angle of the spins in a 180° wall. In the theory of the domain wall[10] it is shown that

$$\partial\theta/\partial z=[g(\theta)/A]^{\frac{1}{2}}, \tag{10}$$

where A is a measure of the exchange energy per unit volume due to gradients in the direction of the magnetization as given by Eq. (11):

$$\text{Exchange energy/unit vol.}=A[(\nabla\alpha_1)^2+(\nabla\alpha_2)^2+(\nabla\alpha_3)^2]. \tag{11}$$

Here α_1, α_2, α_3 are the direction cosines of the magnetization. $g(\theta)$ is the anistropy energy density:

$$g(\theta)=K_1(\alpha_1^2\alpha_2^2+\alpha_2^2\alpha_3^2+\alpha_3^2\alpha_1^2), \tag{12}$$

expressed in terms of θ. K_1 is the first-order anistropy constant.

If we use Eqs. (9) and (10) in Eq. (8) and integrate over z to get the power dissipated for unit area of wall, we have

$$\int_{-\infty}^{\infty} H\cdot\frac{dM}{dt}dz=2H_0M_sv=(\lambda v^2/\gamma^2A^{\frac{1}{2}})\int_0^{\pi}[g(\theta)]^{\frac{1}{2}}d\theta, \tag{13}$$

where we have used Eq. (10) to transform from integration over z to integration over θ as well as to evaluate Eq. (9). We may now write

$$v=\frac{2M_s\gamma^2A^{\frac{1}{2}}}{\lambda\int_0^{\pi}[g(\theta)]^{\frac{1}{2}}d\theta}H_0. \tag{14}$$

This is the desired relation between v and H_0 which is to be compared with Eq. (3). In this way we find

$$\beta=(\lambda/\gamma^2A^{\frac{1}{2}})\int_0^{\pi}[g(\theta)]^{\frac{1}{2}}d\theta. \tag{15}$$

We derive m from the energy of motion of the wall, which we equate to $\frac{1}{2}mv^2$. This was first done by Becker.[6] He pointed out that this energy was equal to $(1/8\pi)\int H_e^2dV$, and by means of Eq. (9) he was able to write

$$m=(1/4\pi\gamma^2)\int_{-\infty}^{\infty}\left(\frac{\partial\theta}{\partial z}\right)^2dz. \tag{16}$$

If we use Eqs (10), (11), and (12), this becomes

$$m=(1/4\pi\gamma^2A^{\frac{1}{2}})\int_0^{\pi}[g(\theta)]^{\frac{1}{2}}d\theta. \tag{17}$$

It will be noted that the above analysis neglects the effect of eddy currents. If this effect is not negligible (it is not negligible in Fe_3O_4), Williams, Shockley, and Kittel[11] have shown that it will contribute part of the measured value of β. The equations of Williams, Shockley, and Kittel may be used to correct for this effect when necessary.

IV. DISCUSSION

It is clear that Eq. (3) fits data of the type shown in Fig. 2, and we therefore obtain a value of β in this way. A value for α, and a second, independent, value for β are determined by fitting the data on initial permeability to Eqs. (4) and (6). As mentioned in Sec. II, we use only the frequency dependent part of the permeability for this calculation in the case of nickel ferrite. The consistency of the two values of β provides a useful internal check on the validity of the data of course, but the value obtained from the initial permeability involves an estimate of the cross-sectional area of the winding, and is therefore not as accurate as the other.[5]

The values of α and β are given in Table I. In the case of Fe_3O_4, eddy currents make a small but not negligible contribution. This has been taken account of by means of the low field theory of Willimas, Shockley and Kittel,[11] and the corrected value is given in the last column.

Some remarks on the significance of α and β are in order here. As Eq. (2) shows, α measures the rate of which magnetostatic energy is built up when the position of the wall changes. This energy is built up as the wall passes crystal imperfections, impurity inclusions, and other impediments to its motion. α is thus a measure of the magnetic imperfection of the sample. It is characteristic of a particular sample, and may differ widely among different samples of the same material. As Eqs. (4) and (6) show, the factor $(2M_s^2/\alpha)$ determines the value of the initial permeability at low frequency.

β, on the other hand, is a fundamental property of the material, as Eq. (15) shows. It is determined by the relaxation time of the spin system, and therefore is a measure of the damping of the wall.

It will be noted that these samples are not expected to show the natural ferromagnetic resonance[12] in the

[10] C. Kittel, Revs. Modern Phys. **21**, 541 (1950); see Eq. (3.3.9).

[11] Williams, Shockley, and Kittel, Phys. Rev. **80**, 1090 (1950).

[12] Rado, Wright, and Emerson, Phys. Rev. **80**, 273 (1950).

anisotropy field, since the alternating field and the anisotropy field are parallel. Equations (4) and (6) therefore, are a rather complete description of initial permeability *versus* frequency for these samples over the whole frequency range. Another characteristic of these samples is the freedom of the moving wall from interaction with other domain walls. This removes one impediment to wall motion which must be present ordinarily and tends to give them a lower α than might be expected in other samples.

The second part of our theoretical discussion, and Eq. (15) in particular, makes it possible to determine a value for λ from our measured β. It should be emphasized that this value is determined independently of the value deduced from ferromagnetic resonance line width; it is therefore of interest to compare them. The availability of a second independent determination of λ is especially desirable since the damping mechanism in ferromagnetics is not understood.

In order to evaluate λ using Eq. (15) we proceed as follows. Since the wall is in a 110 plane we have

$$g(\theta)=\tfrac{1}{4}K_1[\cos^4(\theta+35°16')+\sin^2 2(\theta+35°16')], \quad (18)$$

where $\theta=0$ on one side of the wall and π on the other. Then,

$$\int_0^{\pi}[g(\theta)]^{\frac{1}{2}}d\theta=1.38|K_1|^{\frac{1}{2}}. \quad (19)$$

In performing this integration, care must be taken to use the positive value of the square root over the whole interval. A is best evaluated from a fundamental relation recently derived by Herring and Kittel[13] between A and the Bloch constant:

$$A=[S_0/\Omega]^{\frac{1}{3}}[k/13.3C^{\frac{2}{3}}], \quad (20)$$

where k is Boltzmann's constant, C is Bloch's constant as used in the relation $M_s=M_0(1-CT^{3/2})$, S_0 is the atomic spin, and Ω is the atomic volume. (S_0/Ω) is equal to the saturation magnetization at 0°K divided by the Bohr magneton.

For Fe_3O_4, we find $C=4\times10^{-6}$ by fitting the Bloch $T^{\frac{3}{2}}$ law to the saturation magnetization measurements of Weiss and Forrer.[14] From Eq. (20), we then find $A=1.53\times10^{-6}$. $K_1=-1.1\times10^5$ as given by Bickford.[15] $\gamma=1.76\times10^7$. Now from Eq. (15), we find $\lambda=3.5\times10^8$ in Fe_3O_4.

[13] C. Herring and C. Kittel, Phys. Rev. **81**, 869 (1951); see Eq. (5).
[14] P. Weiss and R. Forrer, Ann. Phys., Series 10, **12**, 279 (1929).
[15] L. R. Bickford, Jr., Phys. Rev. **78**, 449 (1950).

TABLE II.

Substance	λ wall motion	λ ferromag. res.
Fe_3O_4	3.5×10^8	9×10^8 (24,000 mc)
$NiFe_2O_4$	2.2×10^7	2.1×10^7 (24,000 mc) 7.2×10^7 (9000 mc)

For nickel ferrite in the same way, using the data of Guillaud and Roux[16] for saturation magnetization, we find $C=7\times10^{-6}$ and $A=9.0\times10^{-7}$. $K_1=-6.2\times10^4$.[17] Using these figures and our measured value for β, we find $\lambda=2.2\times10^7$ in nickel ferrite.

The relation between ferromagnetic resonance line width and λ has been given elsewhere.[18] Sample shape enters this relation, but not in a critical way, and we therefore ignore it except as it affects the value of the dc magnetic field at resonance, H_{res}. The relation is

$$\lambda=\Delta H\gamma M_s/H_{res}, \quad (21)$$

where the line width is $2\Delta H$.

Data taken at 24,000 mc[16] lead by means of Eq. (21) to $\lambda=2.1\times10^7$ for nickel ferrite. Data taken at 9000 mc,[19] however, lead to $\lambda=7.2\times10^7$ for the same material. If we use Bickford's data on Fe_3O_4 taken at 24,000 mc,[15] Eq. (21) leads to $\lambda=9\times10^8$ for Fe_3O_4.

The various values of λ are compared in Table II.

In view of some of the as yet unresolved experimental difficulties in the wall motion experiments and the inaccuracy they may have caused, it seems wise to emphasize the agreement in order of magnitude between the values determined in the two different ways. The value of λ determined from ferromagnetic resonance, in Fe_3O_4, however, is probably somewhat too high because of experimental problems which arise when samples are not small compared to the skin depth.[20] Also, further ferromagnetic resonance work is desirable to determine the variation of λ with frequency.

All the factors in Eq. (17) have now been evaluated. We find for Fe_3O_4, $m=9.5\times10^{-11}$ g/cm^2, and for $NiOFe_2O_3$, $m=9.4\times10^{-11}$. These values are so small that we can neglect the first term in Eq. (2) in both cases. It is possible, however, that with better crystals the value of m in nickel ferrite could be measured from the shape of the signal induced in the secondary winding in the wall velocity *versus* applied field experiment.

[16] C. Guillaud and M. Roux, Compt. rend. **229**, 1133 (1949).
[17] Yager, Galt, Merritt, and Wood, Phys. Rev. **80**, 744 (1950).
[18] See Eq. (A-6) of reference 16. The λ of the present paper is equal to $\gamma\alpha M_s$ in the notation of reference 16.
[19] D. W. Healy, Jr., Phys. Rev. **86**, 1009 (1952).
[20] Yager, Merritt, and Guillaud, Phys. Rev. **81**, 477 (1951).