The Ferromagnetic Faraday Effect at Microwave Frequencies and its Applications

C. L. HOGAN

Bell Telephone Laboratories, Murray Hill, New Jersey

INTRODUCTION

In his analysis of the ferromagnetic resonance phenomenon, Polder[1] showed that the rf permeability of an infinite saturated ferromagnetic material is not a scalar quantity as it has been ordinarily assumed. He showed, in fact, that the alternating flux density in a saturated ferromagnetic medium is related to the alternating field by a tensor permeability. If the medium is saturated in the z direction, this relation can be written as

$$\mathbf{b}=T_{ij}\mathbf{h}, \tag{1}$$

where $\mathbf{b}$ = alternating component of the flux density, $\mathbf{h}$ = alternating component of the magnetic field, and the tensor T_{ij} has the following form:

$$T_{ij}=\begin{vmatrix} \mu-j\kappa & 0 \\ +j\kappa & \mu & 0 \\ 0 & 0 & 1 \end{vmatrix}. \tag{2}$$

This solution can be derived from the equation of motion of magnetization,

$$\frac{d\mathbf{M}}{dt}=\gamma(\mathbf{M}\times\mathbf{H})-\lambda\left[\frac{(\mathbf{H}\cdot\mathbf{M})\mathbf{M}}{M^2}-\mathbf{H}\right], \tag{3}$$

where $4\pi\mathbf{M}$ = magnetization of medium (gauss); $\mathbf{H}$ = internal magnetic field (oersteds); λ = parameter which measures magnitude of damping force on the precessing dipole moment of ferromagnet; and γ = gyromagnetic ratio of the electron ($\gamma=ge/2mc$ where g is the Landé factor for the electrons concerned).

If a harmonic time dependence, $\exp[j\omega t]$, is assumed for the alternating magnetic field and magnetization, and if it is assumed that the alternating components of the magnetic field are small compared to the dc field applied along the z axis, it can be shown that Eqs. (1) and (2) follow directly from Eq. (3), where

$$\mu=\mu'-j\mu'' \tag{4}$$

$$\kappa=\kappa'-j\kappa'' \tag{5}$$

and

$$\mu'=1+\frac{[\gamma^2H_0{}^2-\omega^2][4\pi M_z\gamma^2H_0\chi_0{}^2]+8\pi\omega^2\lambda^2\chi_0}{\chi_0{}^2[\gamma^2H_0{}^2-\omega^2]^2+4\omega^2\lambda^2} \tag{6}$$

$$\kappa'=\frac{4\pi M_z\gamma\omega\chi_0{}^2[\gamma^2H_0{}^2-\omega^2]}{\chi_0{}^2[\gamma^2H_0{}^2-\omega^2]^2+4\omega^2\lambda^2}, \tag{7}$$

$$\mu''=\frac{4\pi\lambda\omega\chi_0{}^2[\gamma^2H_0{}^2+\omega^2]}{\chi_0{}^2[\gamma^2H_0{}^2-\omega^2]^2+4\omega^2\lambda^2}, \tag{8}$$

$$\kappa''=\frac{8\pi\omega^2\gamma\lambda H_z\chi_0{}^2}{\chi_0{}^2[\gamma^2H_0{}^2-\omega^2]^2+4\omega^2\lambda^2}, \tag{9}$$

where $H_0=H_z[1+(\lambda^2/\gamma^2M^2)]^{\frac{1}{2}}$ (oersteds), H_z = internal magnetic field in z direction (oersteds), and χ_0 = static susceptibility $(M_z)/(H_z)$.

If an infinite plane electromagnetic wave is propagated through a ferromagnetic medium which is saturated in the z direction, it is necessary, in order to describe this wave, to find a solution to Maxwell's equations which is compatible with Eq. (1), and in which b, h, E, and D are all proportional to $\exp[j\omega t-\Gamma(\mathbf{n}\cdot\mathbf{r})]$, where $\mathbf{n}$ is a unit vector in the direction of propagation of the wave and Γ is the propagation constant of the wave. It can be shown that for any given value of the angle θ between $\mathbf{n}$ and the z axis there are two values for the propagation constant which are given by

$$\Gamma_{\pm}=\frac{j\omega(\epsilon)^{\frac{1}{2}}}{c}\left[\frac{(\mu^2-\mu-\kappa^2)\sin^2\theta+2\mu\pm[(\mu^2-\mu-\kappa^2)^2\sin^4\theta+4\kappa^2\cos^2\theta]^{\frac{1}{2}}}{2[(\mu-1)\sin^2\theta+1]}\right]^{\frac{1}{2}}, \tag{10}$$

where $\epsilon=\epsilon'-j\epsilon''$ = complex dielectric constant of medium, c = velocity of light in free space, and ω = angular frequency of wave. The propagation constant of an infinite plane wave through a medium described by a scalar permeability and dielectric constant is

$$\Gamma=j\omega/c(\epsilon\mu)^{\frac{1}{2}}. \tag{11}$$

Hence it is possible to define the quantity which appears in the second radical of Eq. (10) as the effective permeability which the ferromagnetic medium exhibits to the particular wave whose direction of propagation is given by the angle θ and whose polarization is determined by the choice of the signs in Eq. (10). It is convenient to define this quantity as an effective wave permeability since it also occurs in the equations for intrinsic impedance, skin depth, etc., in the place usually occupied by the permeability. It is important at this point to stress that the permeability of a ferromagnetic medium does not have a unique meaning

[1] D. Polder, Phil. Mag. 40, 99–115 (1949).

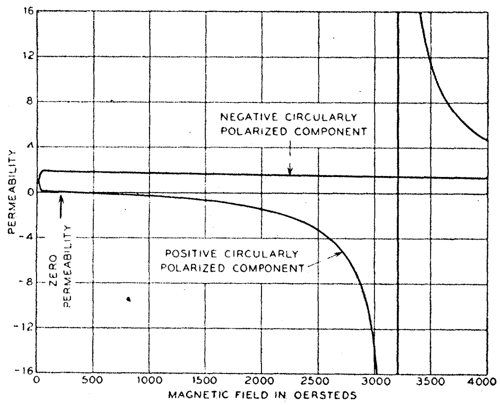

FIG. 1. Real part of the effective permeability of a ferromagnetic medium which is magnetized parallel to the direction of propagation of an infinite plane wave. The curves are computed for a medium whose saturation moment $4\pi M_s = 3000$ gauss and for a wave whose frequency is 9000 megacycles. $\mu_{DC} = 100$.

unless it is carefully defined. When permeabilities of ferromagnetic materials are determined by wave guide or coaxial techniques, the quantity that is determined is an effective wave permeability such as that which occurs in Eq. (10). In a demagnetized sample one actually obtains an average of this quantity over the various domains which constitute the sample in question. Hence at frequencies at which the tensor permeability as defined in Eq. (1) is important, it is obvious that experiments which are designed to measure the tensor components have much more significance than experiments which give some statistical average of the effective wave permeabilities exhibited by the individual domains of a demagnetized sample.

There are two cases of special interest with respect to wave propagation through a saturated ferromagnetic material. The first case is propagation along the z axis. It can be easily shown[2] that for this case the wave decomposes into positive and negative* circularly polarized components. The propagation constants for these two circularly polarized components follow from Eq. (10):

$$\Gamma_{\pm} = j\omega/c[\epsilon(\mu \mp \kappa)]^{\frac{1}{2}}. \tag{12}$$

The effective wave permeabilities for these circularly polarized components can be thus defined as

$$\mu_{\text{eff}}^{+} = \mu - \kappa$$
$$\mu_{\text{eff}}^{-} = \mu + \kappa. \tag{13}$$

The complete expression for these effective permeabilities can be obtained by inserting Eqs. (4), (5), (6), (7), (8) and (9) into Eq. (13). If the effect of damping is neglected in the real part of the permeabilities given by (13), and if the small difference which exists between H_0 and H_z is neglected, these expressions can be written as

$$\mu' \pm \kappa' = 1 + (4\pi M_z \gamma / \gamma H_z \mp \omega), \tag{14}$$

and

$$\mu'' \pm \kappa'' = \frac{4\pi\lambda\omega\chi_0^2[\gamma H_z \pm \omega]^2}{\chi_0^2[\gamma^2 H_z^2 - \omega^2]^2 + 4\omega^2\lambda^2}. \tag{15}$$

Equation (14) describes the real part of the permeability for positive or negative circularly polarized waves, and Eq. (15) gives the imagninary part of the permeability for these two waves. The permeabilities given in Eqs. (14) and (15) are plotted in Figs. 1 and 2 as a function of the magnetic field H_z. The graphs pertain to a wave whose frequency is 9000 Mc, and the constants chosen for the medium are typical of many ferrites. From Figs. 1 and 2 it is quite easy to predict the behavior of an infinite plane wave which is being propagated through a saturated ferromagnetic material in the direction of the applied magnetic field. In the first place, the wave will decompose into circularly polarized components which encounter different permeabilities and hence travel at different velocities. This will result in a rotation of the plane of polarization of the wave which will be directly proportional to the difference between the square roots of the two permeabilities. Thus, the rotation of the plane of polarization as a function of the applied field can be easily predicted from Fig. 1.

Figure 2 indicates that both circular components are only very slightly attenuated by the phenomenon of ferromagnetic resonance except in the vicinity of the absorption line which exists for the positive circularly polarized component only. When the frequency of the wave corresponds to the resonance frequency for the electrons, the positive circularly polarized wave should be very highly attenuated. If the path length through the ferromagnetic material is long enough, this component will be substantially completely absorbed and only the negative circularly polarized component will be propagated. An approximate formula for the absorption

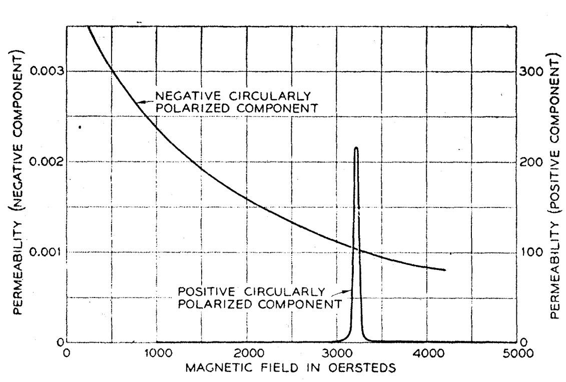

FIG. 2. Imaginary part of the effective permeability of a ferromagnetic medium which is magnetized parallel to the direction of propagation of an infinite plane wave. The constants chosen to compute the curves were the same as those in Fig. 1. In addition the relaxation frequency, λ, was arbitrarily taken as 1.9×10^7 for the curves.

[2] C. L. Hogan, Bell System Tech. J. 31, 1–31 (1952).

* The usual notation is used here, where the positive component is the component which is rotating in the direction of the positive electric current which creates the steady longitudinal field.

of the circularly polarized components by ferromagnetic resonance can be derived which is valid for the negative circularly polarized component at all field strengths and is valid for the positive circularly polarized component only in very weak fields. For a wave whose frequency is 9000 Mc and for a medium described by the constants given in Figs. 1 and 2 this formula reduces to

$$A=8.2(\epsilon')^{\frac{1}{2}}\frac{\mu''\pm\kappa''}{(\mu'\pm\kappa')^{\frac{1}{2}}}\text{db/cm} \quad \text{(dielectric loss neglected).} \quad (16)$$

If it is assumed that the dielectric constant of the medium is 10, then this formula predicts a loss of approximately 0.4 db per cm for the positive component and 0.07 db per cm for the negative component when the applied field is just large enough to saturate the above sample (30 oersteds).

Figures 3 and 4 give experimentally measured values of the rotation of the plane of polarization and the absorption of the positive and negative circularly polarized components as a function of the applied magnetic field for the TE_{11} mode in a circular wave guide when a small cylinder of ferrite was placed along the axis of the wave guide and subjected to a longitudinal magnetic field. It is readily seen that these data agree qualitatively with the behavior predicted by Eqs. (14) and (15).

The angle of rotation of the plane of polarization is given by

$$\theta=\frac{l\omega}{2}\left[\frac{1}{v^-}-\frac{1}{v^+}\right], \quad (17)$$

where l=path length through ferromagnetic medium (cm); and $v_\pm$=velocity of propagation of positive and negative circularly polarized components, respectively. For the ferromagnetic medium discussed above this equation becomes

$$\frac{\theta}{l}=\frac{\omega}{2c}\left(\frac{|\epsilon|+\epsilon'}{2}\right)^{\frac{1}{2}} \times\left[\left(1+\frac{4\pi M_z\gamma}{\gamma H_z+\omega}\right)^{\frac{1}{2}}-\left(1+\frac{4\pi M_z\gamma}{\gamma H_z-\omega}\right)^{\frac{1}{2}}\right], \quad (18)$$

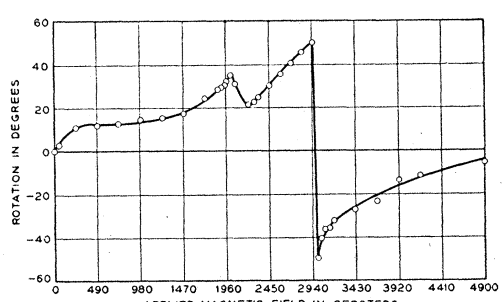

FIG. 3. Faraday rotation *versus* applied magnetic field for a thin "pencil" of nickel zinc ferrite supported along the axis of the wave guide. The minor resonance is believed to be a shape effect. The sample was a cylinder 0.123 in. diameter and 0.815 in. long. The data were taken at 9455 Mc.

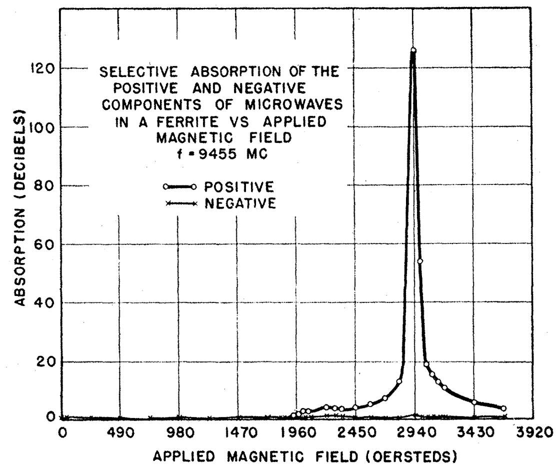

FIG. 4. Selective absorption of positive and negative circularly polarized waves when propagated over same sample as that of Fig. 3.

where $|\epsilon|=(|\epsilon'|^2+|\epsilon''|^2)^{\frac{1}{2}}$=absolute value of dielectric constant. Equation (18) will be valid only when the damping of the electrons in precessional motion can be neglected since it was derived from Eq. (14) where the damping parameter was set to zero. For ferromagnetic materials with absorption lines as narrow as that shown in Fig. 4 it is to be expected that Eq. 18 would be quantitatively valid everywhere except in the near vicinity of the absorption line.

Equation (18) can be reduced to a particularly simple form when the following approximations are valid:

$$\gamma H_z \ll \omega, \quad (19)$$

$$\frac{4\pi M_z\gamma}{\omega}\ll 1. \quad (20)$$

Under these conditions Eq. (18) can be written as

$$\frac{\theta}{l}=\frac{1}{2c}\left(\frac{|\epsilon|+\epsilon'}{2}\right)^{\frac{1}{2}}[4\pi M_z\gamma]. \quad (21)$$

Equation (21) is quite remarkable. It predicts large rotations in the presence of very small applied magnetic fields ($\gamma H_z \ll \omega$); it indicates that the rotation is directly proportional to the magnetization of the medium; and, when Eqs. (19) and (20) are valid, it predicts that the rotation will not depend upon the frequency of the incident radiation.

For the assumed values, $\epsilon'=10$, $\epsilon''=0$, $4\pi M_z=3000$ gauss, $\gamma=17.6\times10^6$ radians/sec/oersted.

Equation (21) predicts rotations of

$$\theta/l=160°/\text{cm}.$$

DESCRIPTION OF EQUIPMENT AND MEASURING TECHNIQUES

The Faraday rotation has been measured in a large number of ferrites in order to verify the above theory.

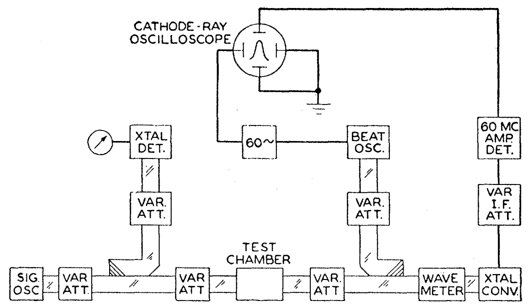

FIG. 5. Set-up of experimental equipment used to measure Faraday rotations.

A diagram of the experimental equipment is given in Fig. 5, and a diagram of the test chamber in which the rotations were measured is given in Fig. 6. In the test chamber, two rectangular wave guides are separated by a circular wave guide, the proper nonreflective transitions being made at each end of the circular section, which is about twelve inches long. One rectangular guide is supported so that it can be rotated about the longitudinal axis of the system. The dominant TE_{10} mode is excited in one rectangular guide, and by means of the smooth transition this goes over into the dominant TE_{11} mode in the circular guide. The rectangular guide on the opposite end will accept only that component of the polarization which coincides with the TE_{10} mode in that guide, the other component being reflected at the transition. Absorbing vanes, inserted in the circular section, absorb this reflected component. The circular guide is placed in a solenoid to establish an axial magnetic field along its length.

The ferrite cylinders to be measured were placed at the mid-section of the circular guide. When a cylinder was used which did not fill the cross section of the guide, it was supported along the axis of the guide by means of a hollow polystyrene cylinder which did fill the guide.

In addition to the Faraday rotation, measurements of insertion loss were made by determining the power transmitted under identical conditions with the ferrite cylinder removed, and the ellipticity of the transmitted wave was determined by measuring the power transmitted when the rectangular guide on the detector side was rotated both to positions of maximum and minimum transmission. Power transmission measurements could be repeated within 0.2 db. Measurements of the angle of rotation of the plane of polarization could be repeated within 0.5° except in the region close to the gyromagnetic resonance where rotations were large and ellipticity so great that it was difficult to determine the positions of maximum and minimum transmission. These errors increased up to the point where the transmitted wave was circularly polarized where it was impossible to measure the angle of rotation.

EXPERIMENTAL RESULTS

Equation (21) indicates that, in weak fields at least, the rotation per unit path length through the ferromagnetic material is proportional to the magnetization of the sample and is independent of the frequency of the wave. It is experimentally very difficult to obtain data to check the predictions of Eq. (21). In the first place, Eq. (21) was derived for an infinite plane wave and it is not to be expected that it will hold quantitatively for a guided wave. In fact, Suhl and Walker[3] have shown that in the case of small rotations (i.e., small rotation per wavelength) the rotation in a circular wave guide would be only about 86 percent of the rotation which would occur for an infinite plane wave. More recent work by Suhl and Walker,[4] has, however, indicated that large rotations in a circular wave guide would probably more nearly approach the rotation predicted by Eq. (21) for an infinite plane wave. In reality however, if the rotations are measured in a circular wave guide, the rotation to be expected will lie somewhere between the value predicted by Eq. (21) and 86 percent of that value. Hence, it is meaningless to claim a quantitative verification of Eq. (21) closer than these limits until a more complete analysis has been made.

The analysis of Suhl and Walker applies only to a circular wave guide whose entire cross section is filled with ferrite. If a cylinder of ferrite is used which completely fills the cross section of the wave guide, however, internal reflections within the sample produce an effective rotation, and an ellipticity which is different from that which would result if the wave passed through the ferrite only once. This effect is obviously frequency dependent since the electrical length of the sample changes with frequency. The effect of internal reflections can be reduced by using a ferrite which has a large dielectric loss. Some ferrites have such an extremely high dielectric loss that attenuations of the order of 50 db per centimeter path length can be obtained. Hence, in this type of ferrite, multiple reflections within the sample can be completely neglected even though the cross section of the wave guide is filled with the

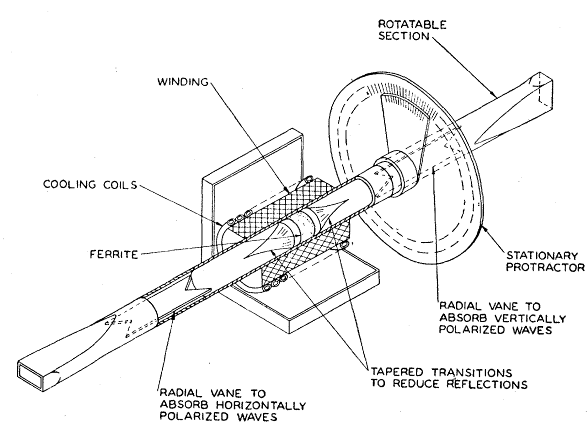

FIG. 6. Detail of test chamber in which rotations were measured.

[3] H. Suhl and L. R. Walker, Phys. Rev. **86**, 122–3 (1952).
[4] H. Suhl and L. R. Walker, unpublished memoranda, Bell Telephone Laboratories.

ferrite and tapered transitions at the faces of the sample are eliminated.

Thus in order to obtain a quantitative check of Eq. (21), it is necessary that a ferrite have a high dielectric loss so that internal reflections can be avoided; it must have a relatively low magnetic anisotropy since anisotropy has been neglected in this analysis; and it must have a relatively low saturation moment so that the approximation expressed in Eq. (20) will be valid. It has been found possible to satisfy all these requirements in a particular manganese zinc ferrite which contained a small excess of iron over that required for perfect stoichiometry. This particular sample had the following measured properties:

$$\epsilon' = 17, \quad \epsilon'' = 24, \quad 4\pi M_{\text{sat}} = 1500 \text{ gauss}.$$

The rotation induced by this sample as a function of the applied magnetic field was measured, and the results are shown graphically in Fig. 7. The physical dimensions of the sample are given in Table I (sample No. 1). Figure 7 quite clearly shows the dependence of rotation upon the magnetization of the sample and indicates that, after the sample is saturated the rotation is sensibly independent of the applied magnetic field. This result verifies the work of Roberts[5] who has previously shown that the Faraday rotation induced in a round wave guide is proportional to the magnetization of the sample. In addition for this particular sample, Eq. (21) predicts

$$\theta/l = 121 \text{ degrees/cm}$$

for an infinite plane wave. As previously stated, Suhl and Walker have shown that the measured rotation in a round wave guide should be less than this and should probably lie somewhere between 104 degrees per centimeter and 121 degrees per centimeter for this particular sample. It is seen from Fig. 7 that the measured rotation at saturation is approximately 123 degrees per centimeter. It is felt that this agreement is good.

The above data seem to bear out Polder's theory of ferromagnetic resonance. A more complete check of his theory could, however, be obtained by measuring the rotation of the plane of polarization as a function of field up through the field required for resonance. Again, however, the wave guide introduces complications which are difficult to unravel. In the vicinity of ferromagnetic resonance the mode which exists in a wave guide completely filled with ferrite differs quite radically from the ordinary dominant mode in an empty guide. In addition, even if a wave guide is only partially filled with ferrite, it is possible for higher modes to exist to further complicate the picture. In order to overcome these difficulties it is necessary to place the ferrite along the axis of the wave guide in the form of an extremely thin pencil. If pencils of ferrite which are approximately 0.125 inch in diameter are placed in an ordinary X-band round wave guide (approximately one inch in diameter) results are obtained which are qualitatively in agreement with the theory developed for an infinite plane wave. If the diameter of the ferrite is increased to 0.250 inch the results are often not in accord with the above theory. Sometimes such samples show sudden increases in rotation soon after the magnetic field is raised above that required for saturation, as in the data Roberts has published. It has even been observed that a magnetized ferrite will not propagate the negative circularly polarized component although this ferrite might cause a relatively low loss to this particular wave when it is in a demagnetized state. There certainly is nothing in the above developed theory which will explain this phenomenon and this behavior must arise from the characteristics of the guided wave.

Since all ferrites which have been tested to date give data which are in complete agreement with the above theory when the ferrite is placed along the axis of the guide in the form of an extremely thin pencil and since quantitative agreement has been obtained for one carefully selected ferrite, it is believed that Polder's theory of ferromagnetic resonance is adequate to describe this phenomenon but considerably more work is necessary in order to understand the properties of wave guide modes in magnetized ferrites. A typical set of

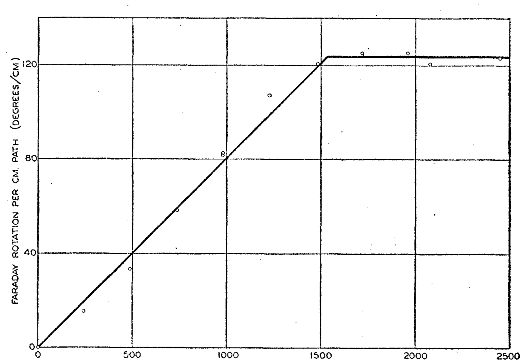

FIG. 7. Faraday rotation versus applied magnetic field for a thin disk of manganese zinc ferrite which completely filled cross section of wave guide.

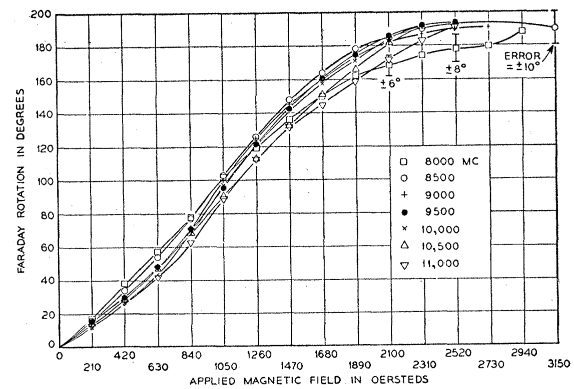

FIG. 8. Dependence of Faraday rotation upon frequency.

[5] F. F. Roberts, J. phys. et radium 12, 305 (1951).

TABLE I. Faraday effect for ferrites. Frequency = 9000 Mc.

Sample No.	Dimension (cm)	Applied magnetic field (oersteds)	Rotation/cm path	Insertion loss (db)	Ellipticity[a] db
No. 1	0.447×2.28	0	0	10.0	>50
	(length×diam.)	245	15.6	10.3	>50
		490	33.5	10.0	23.2
		735	58.2	9.2	15.0
		980	81.6	9.1	12.1
		1225	107	9.2	10.9
		1470	120	10.0	10.4
		1715	125	11.0	9.3
		1960	123	11.2	9.0
		2205	121	11.3	7.7
		2450	123	11.4	6.6
		2695	...	12.4	5.0
		2940	...	13.0	3.7
		3185	...	...	3.0
		3675	...	...	1.4
2	1.77×2.28	0	0	23.2	≫30.
		245	38	21.4	23.0
		490	77	16.7	7.6
		735	124	12.4	2.1
		980	157	9.9	1.4
		1225	170	7.7	0.7
		1470	180	6.0	0.7
		3430	183	7.1	0.0
3	5.08×0.32	0	0	<0.1	>50.
	(ends of cylinder	215	4.0	0.1	50.
	tapered to a	430	5.0	0.1	50.
	point)	645	5.1	0.1	50.
		860	5.6	0.1	50.
		1075	6.0	0.1	50.
		1290	7.0	0.1	50.
		1505	8.6		50.
		1720	10.8		50.
		1935	14.0		50.
		2150	20.4		27
		2365	16.8		10
		2580	4.0		9
		2795	−7.6		10
		3010	−12.0		
		3115	−12.8		
		3225	−13.2		26
		3440	−11.2		49
		3655	−9.2		>50
		3870	−7.6		>50
		4085	−6.0		>50
		4300	−5.0		>50
		4515	−4.2		>50
		4730	−3.8		>50

[a] Data given show the difference in db between the major and minor components of the elliptically polarized transmitted wave.

data taken on a thin pencil of ferrite is given in Figs. 3 and 4. Another set is given in Table I for a magnesium ferrite sample.

Figure 8 shows the rotation of the plane of polarization *versus* magnetic field for a cylinder of ferrite measured at several frequencies from 8000 to 11 000 megacycles per second. The rotation does not change appreciably for this change in frequency and it is believed that a large portion of this variation is associated with the properties of the guided wave and hence this particular data should not be applied too strictly as a check of Eq. (21) which was derived for an infinite plane wave.

The loss characteristics of different ferrites as a function of the applied magnetic field varied widely. Some, such as commercially available manganese zinc ferrite, showed extremely high loss which was associated with the imaginary part of the dielectric constant. This loss was not affected by the application of a magnetic field, but remained substantially constant as the field was applied. However, as the field approached that necessary for ferromagnetic resonance the total power absorbed increased, since the positive circularly polarized component was almost completely absorbed by the sample. In fact, by measuring the ellipticity of the transmitted wave, it is possible to compute the difference between the absorption of the positive and negative circularly polarized components. This has been done for the sample of nickel zinc ferrite which was discussed in connection with Figs. 3 and 4.

Figure 9 gives the ellipticity of the wave propagated through this sample as a function of the applied field and indicates how the wave becomes circularly polarized when the applied field is equal to that necessary for resonance. Figure 10 shows the total power absorbed by this material as a function of the applied field. It is to be noticed that the loss decreases as the sample is magnetized. For this particular sample this original loss is believed to be due to domain walls. Experiments are under way to verify this.

If, instead of propagating along the z axis, an infinite plane wave is propagated along the y axis, the propagation constant can be obtained by setting θ equal to $\pi/2$ in Eq. (10). This again gives two propagation constants:

$$\Gamma_{-}=j\omega(\epsilon)^{\frac{1}{2}}/c, \tag{22}$$

$$\Gamma_{+}=\frac{j\omega}{c}\left[\epsilon\left(\frac{\mu^2-\kappa^2}{\mu}\right)\right]^{\frac{1}{2}}. \tag{23}$$

The first propagation constant refers to a wave which is polarized with its magnetic vector along the z axis and the other propagation constant is for a wave which is polarized with its magnetic vector along the x axis. Two very interesting results are apparent with respect to this second wave. In the first place, it is no longer completely transverse since there is a magnetic field along the y axis. The other interesting result is that the medium presents an effective permeability to this wave which is dependent upon the applied magnetic field in the z direction. The effective permeability is given by

$$\mu_{\text{eff}}=\frac{\mu^2-\kappa^2}{\mu}=\mu_{\text{eff}}'-j\mu_{\text{eff}}''. \tag{24}$$

Again, if damping is neglected, the real part of the above equation becomes

$$\mu_{\text{eff}}'=\frac{\gamma^2B_z^2-\omega^2}{\gamma^2H_zB_z-\omega^2}, \tag{25}$$

where B_z= flux density in z direction ($B_z=H_z+4\pi M_z$).

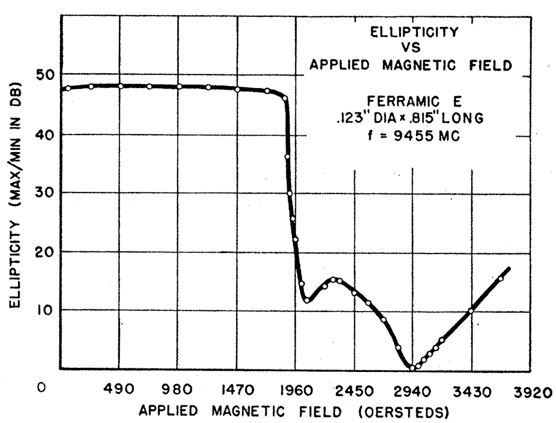

FIG. 9. Ellipticity of wave propagated through wave guide containing sample discussed in Figs. 3 and 4.

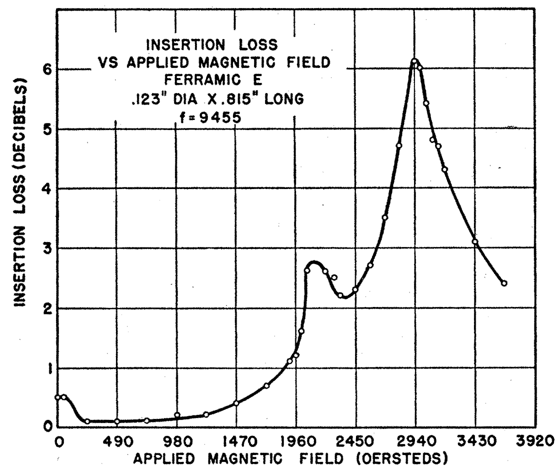

FIG. 10. Total insertion loss of wave under conditions referred to in Fig. 9.

Equation (25) is plotted as a function of the magnetic field, H_z, in Fig. 11 for a ferrite with the particular properties given. The curve applies to a wave whose frequency is 9000 Mc.

It will be noticed that at zero field the permeability is unity that it decreases rather rapidly toward zero in small fields. This part of the curve is actually a mirror image of the magnetization curve of the ferromagnet. The shape of this portion depends upon the dc permeability of the material. After the material saturates, the effective permeability curve levels out somewhat and is much less field dependent until the region of the resonance is approached. As indicated in Eq. (25) above, the effective permeability has a zero when B_z=3210 gauss; this point may or may not come before saturation of the sample, depending upon whether the saturation magnetization is more or less than 3210 gauss. For the example taken, the zero occurs after the sample is saturated.

It is quite easy to preduct, from Fig. 11, the behavior of the dominant mode being propagated in a wave guide through a ferromagnetic medium under the above conditions.

At zero applied field, the wave sees a dielectric material with a permeability of unity and with an ordinary dielectric constant (ϵ=6 to 20 for ferrites at 9000 mc). As the field is gradually increased, the permeability of the medium approaches zero very rapidly. As the permeability approaches zero, the wavelength in the medium becomes longer and longer and obviously before a zero permeability is reached a phenomenon analogous to cutoff appears. However, since the propagation is in a lossy dielectric, the cutoff is not as sharp as in a lossless guide. Nevertheless, there should be a region when the permeability is close to zero where large reflections take place and very little power is propagated through a slug of ferrite which fills the guide.

The intrinsic impedance of a dielectric material is given by

$$Z=(\mu/\epsilon)^{\frac{1}{2}}, \tag{26}$$

where both the permeability, μ, and the dielectric constant, ϵ, are usually complex quantities. Some ferrites exist in which the imaginary part of both these quantities is negligibly small at 9000 Mc except in the region of ferromagnetic resonance when obviously the imaginary part of the permeability is large. Other ferrites exist in which the imaginary part of the permeability or the dielectric constant or both cannot be ignored regardless of the strength of the applied magnetic field. The behavior of all these ferrites is qualitatively the same and the differences between them are only matters of degree.

In particular, if the imaginary part of the permeability is negligible when the real part is zero, then the impedance of the ferromagnetic medium will be truly zero at this point regardless of the dielectric loss; and the medium will then have properties analogous to a material with infinite conductivity. A slug of ferrite should, under these conditions, completely reflect an incident wave. Ferramic I is a ferrite which satisfies the condition of having a negligible magnetic loss component at the point where the real part of the permeability is zero and this material can be made to almost completely reflect a microwave at this point. Other ferrites, such as Ferramic G, which have appreciable magnetic loss at all magnetic field strengths up to and beyond the field required for resonance, can never have a truly zero impedance and this should be evidenced by the fact that Ferramic G never can be made to be a perfect reflector. These predictions are substantiated by experiment.

When the field is increased beyond the resonance, it is possible to produce high effective positive permeabilities in the medium, and under some conditions it has been possible to make the effective permeability of the medium equal to the dielectric constant. Thus the intrinsic impedance of the ferrite was made equal to the intrinsic impedance of air. Under these conditions, no reflection from the ferrite slugs occurred and in fact some ferrite slugs have been made essentially completely transparent by this technique.

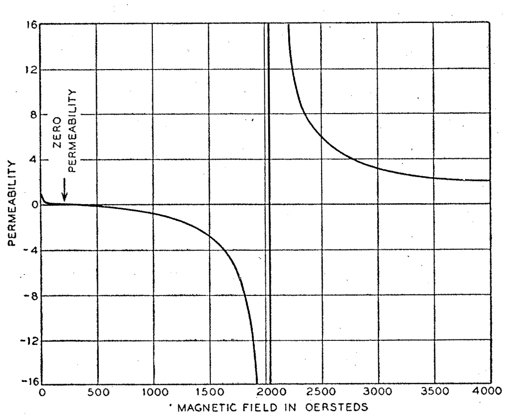

FIG. 11. Real part of the effective permeability of a ferromagnetic medium in which a magnetic field is applied parallel to the electric vector of an infinite plane wave. The curve applies to a wave frequency is 9000 megacycles being propagated through a medium whose saturation moment $4\pi M_s=3000$ gauss. $\mu_{DC}=100$.

Data taken on a sample of Ferramic I are given in Table II. The data were taken at 9000 Mc by placing a cylinder of ferrite in a round wave guide.

It is interesting to compare this effect with that which leads to the Faraday rotation (longitudinal field). In the case of the longitudinal field, the wave splits into two circularly polarized components. The permeability of the negative circularly polarized component has no interesting behavior as a function of field. The permeability of the positive circularly polarized component, however, is qualitatively the same as the permeability shown in Fig. 11. Hence any effect which can be obtained in the transverse magnetic field can also be obtained with a longitudinal field if a positive circularly polarized wave is propagated rather than a linearly polarized wave. Thus it should be possible to produce circularly polarized waves by reflecting the positive component at the region of zero permeability rather than absorbing it in the region of ferromagnetic resonance. This prediction has been experimentally verified.

TABLE II. Data from sample of Ferramic I.

Applied magnetic field (oersteds)	Reflection coefficient E_r/E_i	Insertion loss (db)
Ferramic I (powdered cylinder—0.895-in. diam. ×0.38 in. long)		
0	...	7.4
1600	0.418	3.9
1700	0.335	4.1
2100	0.90	31.4—cutoff
2400	0.57	51.1—resonance absorption
3000	0.496	29.4
3800	0.571	7.7
4400	0.183	0.3—impedance match
Empty guide looking into matched load	0.170	...

APPLICATIONS

Applications of the above phenomena are numerous but the most interesting and potentially most important applications are derived from the Faraday effect. It has long been known that the Faraday rotation of the plane of polarization in optics is antireciprocal. Its antireciprocal property distinguishes the Faraday effect from optical rotations caused by birefringent crystals, or by the Cotton-Mouton effect, which are reciprocal. That is, if a plane polarized light wave is incident upon a birefringent crystal in such a manner that the plane of polarization is rotated through an angle θ in passing through the crystal then this rotation will be cancelled if the wave is reflected back through the crystal to its source. In the Faraday rotation, however, the angle of rotation is doubled if the wave is reflected back along its path. Hence, if the length of path through the "active" material is adjusted so as to give a 90° original rotation, the beam on being reflected will have its plane of polarization rotated a total of 180° in passing in both directions through the material.

Lord Rayleigh[5] described a one-way transmission system in optics which makes use of the Faraday rotation. Lord Rayleigh's "one-way" system consisted of two polarizing Nicol prisms (oriented so that their planes of acceptance made an angle of 45° with each other), with the material causing the Faraday rotation placed between them. Thus, light which was passed by the first crystal and whose plane of polarization was rotated 45° would be passed by the second crystal also. But, in the reverse direction, the rotation would be in such a sense that light which was admitted to the system by the second crystal would not be passed by the first.

The antireciprocal property of the Faraday effect affords a means of realizing a microwave circuit element which is analogous to Tellegen's[6] gyrator. Such a gyrator is illustrated in Fig. 12 along with diagrams which help explain its action. Beneath the gyrator are construction lines which indicate the plane of polarization of a wave as it travels through the gyrator in either direction. On each diagram is a dotted sine wave for reference only which indicates the constant plane of polarization of an unrotated wave. It is noticed that for propagation from left to right in Fig. 12, the screw rotation introduced by the twisted rectangular guide adds to the 90° rotation given to the wave by the ferrite element making a total rotation of 180°. For a wave traveling in the reverse direction these two rotations cancel each other, producing a net zero rotation through the complete element. The unique property of the Faraday rotation becomes immediately apparent from this diagram. In the case of the rotation induced by the twisted rectangular guide, the wave rotates in one direction in going from left to right through the twisted section, and rotates in the opposite direction when it transverses the section from right to left. For the case of the rotation induced by the ferrite element, the direction of rotation is indicated by the arrow in the upper figure for either direction of propagation. The important characteristic of the element is the time phase relation between two points such as A and B in the upper diagram. It is seen with the help of the diagrams illustrating the rotating waves that the field variations are in phase at points A and B for propagation from left to right and they are 180° out of phase for propagation from right to left. In other words the transmission line is an integral number of wavelengths long between A and B for propagation from left to right and is an odd integral number of half-wavelengths long for propagation from right to left.

[5] Lord Rayleigh, Nature **64**, 577 (1901).
[6] B. D. H. Tellegen, Philips Research Reports, **3**, 81–101 (1948).

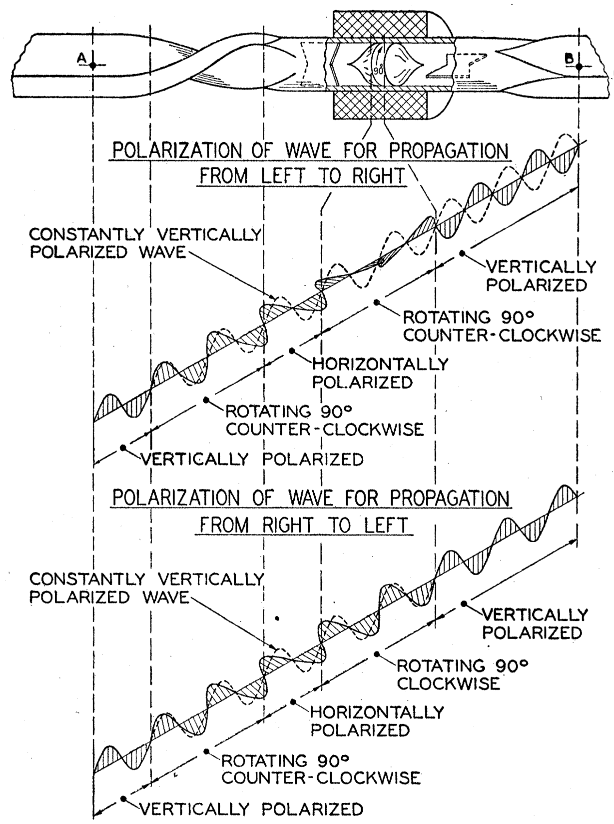

FIG. 12. The microwave gyrator with diagrams which help to explain its action.

If the rectangular wave guides on each side of the ferrite are rotated about their common axis so as to make an angle of 45° with each other, then a one-way transmission system can be created which is similar to Lord Rayleigh's one-way transmission system of optics. It is, however, probably more useful, for within the validity of the above analysis, this one-way transmission system does not depend upon frequency but is broad band. This one-way transmission system can be used, for example, to isolate the generator or detector from the wave guide in microwave systems. In this application it has the great advantage over the attenuators which are presently used for this purpose in that it can be made practically lossless for the direction of propagation which is desired but the reflected wave will be completely absorbed and hence more complete isolation can be effected.

It is suggested that this device which makes use of the 45° rotation be called a *polarization circulator* since it is essentially a more complex and more useful circuit element than its simple one-way transmission property would at first indicate. The polarization circulator actually has four output branches corresponding to the two different polarizations at each end of the device. The polarizations of the four output branches are indicated schematically in Fig. 13. It is noticed that

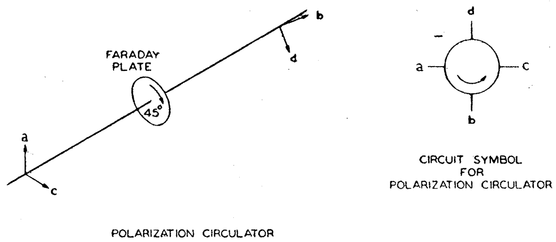

FIG. 13. Schematic diagram of polarization circulator.

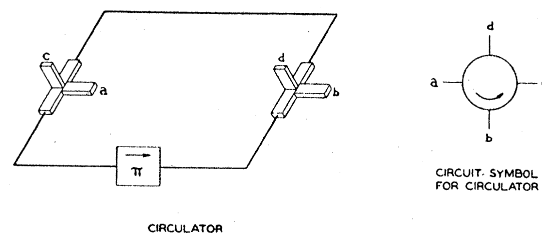

FIG. 14. Schematic diagram of circulator.

power sent into the polarization circulator with polarization a is turned into polarization b, also b is turned into c, c is turned into d, and d is turned into *minus* a. This property is indicated very clearly by the circuit symbol suggested in Fig. 13, the phase inversion between arms d and a being indicated by the minus sign between the d and a arms.

Another one-way transmission system can be created by combining the gyrator with two magic tees. This combination is indicated in Fig. 14. Since this device has all of the fundamental properties of the polarization circulator with the exception of the phase inversion between arms d and a it is suggested that it be called a *circulator* and the circuit symbol suggested which indicates its properties is also given in Fig. 14.

This list of applications is obviously not complete since it includes only the fundamental elements from which innumerable specific applications can be made.

In addition to the applications discussed above, which depend upon the antireciprocal property of the element for their operation there are several simple applications which are based only upon the fact that the amount of rotation can be controlled externally by adjusting the magnetic field. Among these uses are electrically controlled attenuators, modulators, and microwave switches.

If a circularly polarized wave is propagated in a circular guide or if the dominant mode is propagated in a rectangular wave guide with the magnetic field being applied along the electric vector of the wave, the above phenomenon allows one to realize externally controlled microwave phase shifters, attenuators, modulators, and microwave windows.

ACKNOWLEDGMENTS

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DISCUSSION

A. G. FOX AND M. T. WEISS, *Bell Telephone Laboratories, Holmdel, New Jersey:* As Dr. Hogan has stated, one can apply the plane wave analysis of Faraday rotation, ferromagnetic resonance, and other phenomena in ferrites, only with very great caution to the interpretation of measurements made in ferrite filled wave guides. We would like to describe some of our own experiences in this matter.

1. Mode conversion problem.—Because of the relatively high index of refraction of most ferrites, a dominant mode hollow metal guide, when completely or even partially loaded with a ferrite rod, may be capable of propagating higher order modes. Furthermore, experiments have shown that a pencil of high index of refraction placed in the center of a circular wave guide will have a tendency to convert the dominant TE_{11} mode to the TM_{11} mode.

This mode conversion problem became apparent to us while making measurements in circular guide of loss and ellipticity of large diameter ferrite rods, properly tapered, as a function of magnetic field. The results showed peculiar large variations of these quantitites. We now believe that these variations can be attributed to a partial conversion of the dominant TE_{11} mode to the TM_{11} mode by the ferrite pencil which was of sufficient diameter to permit the propagation of both modes. Thus, as the magnetic field is varied, the permeability changes, causing the electrical length of the line to change, so that the interference between the TM_{11} and TE_{11} modes can vary also.

In order to check the foregoing hypothesis, experiments were performed using circularly polarized waves. As is well known, theory predicts that, for a positively rotating circularly polarized wave, the permeability, μ, of the ferrite starts at unity for zero magnetic field, and then decreases, passes through zero, and goes through resonance as the applied longitudinal magnetic field is increased. For a negatively rotating circularly polarized wave, on the other hand, μ starts at unity, increases until saturation is reached, and finally approaches unity again at very high magnetic fields.