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Relaxation Processes in Ferromagnetism

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This paper surveys current theoretical work on spin-spin and spin-lattice relaxation in ferromagnetic substances with reference to ferromagnetic resonance. It is shown that it is possible to give a reasonable explanation of the principal features of the experimental results. The calculations, carried out for nickel in particular, make use of a macroscopic spin-lattice interaction determined by the observed magnetostriction and a pseudo-dipolar spin-spin interaction determined by the observed anisotropy. It is not sufficient to use magnetic dipole coupling alone for either of the two relaxation processes. The zero-point fluctuations in the pseudo-dipolar field play a central role in the development of the theory.

THIS paper is concerned with the problem of relaxation processes in ferromagnetic substances. The ferromagnetic phenomena in which relaxation effects are exhibited include the following:

(1) Line width in ferromagnetic spin resonance absorption.

(2) Power saturation with intense rf magnetic fields in ferromagnetic resonance.

(3) Velocity of propagation of the Bloch wall separating domains in ferromagnetic insulators.

(4) Lag and elevation angles of magnetization with respect to the direction of a rotating magnetic field.

(5) Time delay in the establishment of a new equilibrium position for the magnetic moment of a specimen after a sudden change in the external magnetic field.

(6) Time delay in heat transfer between lattice and spin systems.

We shall review below the current experimental and theoretical positions with regard to spin-spin and spin-lattice relaxation times. The review will draw heavily on work recently completed at Berkeley by F. Keffer, and also on unpublished work done in collaboration with J. M. Luttinger at the Institute for Advanced Study.

When a spin-resonance absorption experiment[1] is performed on a ferromagnetic single crystal at the usual microwave frequencies it is observed that the resonance line has a half-width ΔH of the order of 50 to 500 oersteds, corresponding to over-all relaxation times of 10^8 to 10^9 sec. We believe that the principal contribution to the line width at room temperature and below comes from spin-spin relaxation.

By the half-width of the line we mean one-half of the width on a plot of the effective permeability μ_R (or, with ferrites, μ'') against the static magnetic field intensity H between the points at which $\mu_R \doteq \frac{1}{2}\mu_R(\max)$. Typical values of the half-width are given in Table I. Measurements on polycrystalline specimens must be interpreted with considerable caution because the crystalline anisotropy energy causes an apparent line width as a result of the scatter in the resonance frequencies of the individual crystal grains.[2]

It has always been the occasion of some astonishment that the line widths are as broad as they are observed to be, and it is only recently, as discussed below, that we have recognized what seem to be the real mechanisms causing the widths. A number of somewhat superficial mechanisms were explored without success in the early phases of research on ferromagnetic resonance, as discussed in the reviews cited above.

If we were to neglect all mutual interactions among the electron spins in an atomic model of a ferromagnet, other than the isotropic exchange interaction, then the resonance line would be exceedingly sharp, the width being determined by magnetic field inhomogeneities, saturation effects or radiation damping. The energy levels in the static magnetic field H are given by

$$W = g\mu_B HM, \tag{1}$$

where M is the magnetic quantum number associated with the total spin $\mathcal{S}\hbar$ of the entire specimen, and g is the spectroscopic splitting factor. The selection rules for magnetic dipole transitions are $\Delta\mathcal{S}=0$, $\Delta M = \pm 1$, so that only the single frequency

$$\omega = g(e/2mc)H \tag{2}$$

will be observed. We should, strictly, write $H_{\text{effective}}$

TABLE I. Half-width of ferromagnetic resonance lines (near 24 000 Mc/sec and at room temperature).

Substance	Approximate half-width in oersteds
Silicon iron (single crystal)	several hundred
Nickel	400
Supermalloy (annealed)	100
Supermalloy (cold rolled)	300
Nickel ferrite (single crystal)	75
Nickel zinc ferrite	150
Manganese zinc ferrite	50–400[a]
Heusler alloy	100

[a] Appears to vary from sample to sample.

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[1] Ferromagnetic resonance is reviewed by J. H. Van Vleck, Physica **17**, 234–252 (1951); C. Kittel, J. phys. et radium **12**, 291–302 (1951).

[2] The half-width in unoriented polycrystalline specimens from this effect alone is expected to be roughly of the order of $\frac{1}{4}$ of the anisotropy field $H_a = 2K/M_s$, provided that $H_a \ll H_{\text{res}}$.

instead of H, to allow for sample shape and crystal anisotropy. The exchange energy $-2J\sum \mathbf{S}_i\cdot\mathbf{S}_j$ does not change during the transition. The resonance line width in this case will be determined by the secondary effects mentioned above, and we may expect a very narrow line.

The identical model may be considered from the viewpoint of the spin wave approximation[3] to the low-lying states of a ferromagnet. The energy in the external magnetic field is now, for low k,

$$W=\sum_k (n_k+\tfrac{1}{2})(Ck^2+g\mu_B H), \tag{3}$$

where n_k is the excitation quantum number of the spin wave of wave vector $\mathbf{k}$ and C is a constant proportional to the exchange energy. The former selection rules go over to the rule that transitions in a uniform rf field will occur only among the $k=0$ levels, and we have $\Delta n_0=\pm 1$; $\Delta n_k(k\neq 0)=0$. The resonance frequency is thus identical with Eq. (2). We have introduced the spin wave viewpoint because it is used in the recent developments in relaxation problems.

Our model considering only the exchange and Zeeman interactions is thus too idealized to yield a description of the line width. It is known however that there are other interactions among ferromagnetic electrons, including magnetic dipole interactions and also interactions of a spin-orbit-exchange-lattice character, which contribute to the important phenomena of magnetocrystalline anisotropy and magnetostriction. It is essential to include these interactions in the theory. We first consider the calculation of the spin-lattice relaxation time characterizing the exchange of energy between the system of spin waves and the system of lattice vibrations (phonons). What shall we use as the interaction between the spin system and the phonon system? We recall that the observed magnetostriction of ferromagnets results from the magnetoelastic interaction of the magnetization or spin direction and the crystal lattice. We know that we can account approximately for the observed magnetostriction by postulating a coupling term in the free energy density of the form[4]

$$\begin{aligned} f_1=&B_1(\alpha_1^2e_{xx}+\alpha_2^2e_{yy}+\alpha_3^2e_{zz})\\ &+B_2(\alpha_1\alpha_2e_{xy}+\alpha_2\alpha_3e_{yz}+\alpha_3\alpha_1e_{zx}); \end{aligned} \tag{4}$$

here the α's are direction cosines of the magnetization, and the e's are elastic strain components. Terms of this form may be thought of as representing the dependence of the anisotropy energy on the state of strain of the crystal. Values of the constants B_1, B_2 are readily deduced from the observed magnetostriction.

Starting from the macroscopic interaction represented by Eq. (4), we may calculate the spin-lattice relaxation time using some of the standard methods of quantum field theory. We set up a macroscopic spin-wave field and a macroscopic phonon field. The magnetoelastic interaction given by Eq. (4) is treated as a perturbation which causes collisions between the spin waves and the phonons. The kinetic equation for the collision process leads us directly to the spin-lattice relaxation frequency.

The Hamiltonian of the exchange and Zeeman energies is, writing $\mathbf{M}$ as the magnetization,

$$\begin{aligned} \mathcal{H}_{\mathrm{spin}}=\int\{&(A/M_s^2)[(\nabla M_x)^2+(\nabla M_y)^2+(\nabla M_z)^2]\\ &-\mathbf{H}\cdot\mathbf{M}\}dV, \end{aligned} \tag{5}$$

where A is the usual macroscopic exchange constant[5] and on the atomic model is related to the exchange integral J by

$$A=ZS^2R^2J/6\Omega, \tag{6}$$

where Z is the number of nearest neighbors of any given atom, R their distance and Ω the atomic volume. Near saturation we may assume that the magnetization $\mathbf{M}$ is oriented very nearly in the z direction, so that

$$\begin{aligned} \mathcal{H}_{\mathrm{spin}}\cong\int\{&(A/M_s^2)[(\nabla M_x)^2+(\nabla M_g)^2]\\ &+(H/2M_s)(M_x^2+M_y^2)\}dV, \end{aligned} \tag{7}$$

apart from a constant term.

We consider $\mathbf{M}_x$, $\mathbf{M}_y$ as amplitudes of a vector field. The field is quantized[6] by use of the commutation relation

$$\mathbf{S}_j\times\mathbf{S}_k=i\mathbf{S}_j\delta_{jk}. \tag{8}$$

We express the magnetization in terms of a spin density

$$\mathbf{M}(\mathbf{r})=(g\mu_B\nu/a^3)\sum\mathbf{S}_j\delta(\mathbf{r}-\mathbf{r}_j), \tag{9}$$

where ν is the number of atoms per unit cell of volume a^3. We find, using Eq. (8),

$$[M_x(\mathbf{r}),M_y(\mathbf{r})]=ig\mu_BM_s\delta(\mathbf{r}-\mathbf{r}'). \tag{10}$$

It is convenient to introduce the usual creation and destruction operations a_k^* and a_k into the Fourier analysis of the transverse components of magnetization:

$$\begin{aligned} M_x(\mathbf{r})&=(g\mu_BM_s/2V)^{\frac{1}{2}}\sum(a_k+a_{-k}^*)e^{i\mathbf{k}\cdot\mathbf{r}};\\ M_y(\mathbf{r})&=i(g\mu_BM_s/2V)^{\frac{1}{2}}\sum(a_{-k}^*-a_k)e^{i\mathbf{k}\cdot\mathbf{r}}, \end{aligned} \tag{11}$$

where V is the volume of the sample. The commutation relation becomes

$$[a_k,\ a_{k'}^*]=\delta_{kk'}. \tag{12}$$

On substituting Eq. (11) back in to Eq. (7) and carrying out the volume integration using Eq. (12), we find for the eigenvalues of the unperturbed Hamiltonian

$$W_{\mathrm{spin}}=\sum_k (n_k+\tfrac{1}{2})[(2g\mu_BA/M_s)k^2+g\mu_BH], \tag{13}$$

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[3] An elementary semiclassical treatment of the Bloch theory of spin waves is given in the forthcoming book, *Introduction to Solid State Physics* by C. Kittel (John Wiley and Sons, Inc. New York, 1953), Appendix J.

[4] See, for example, Sec. 2.3 in C. Kittel, Revs. Modern Phys. **21**, 541 (1949); the equation above is the "two constant" form for cubic crystals; more general forms are given in the standard literature.

[5] Reference 4, Eq. (2.1.11).

[6] The field theory of spin waves is treated by C. Herring and C. Kittel, Phys. Rev. **81**, 869 (1951); a simple introduction to quantum field theory is given in L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1949).

where n_k is the eigenvalue of the operator $a_k{}^*a_k$ and is a positive integer. We gain insight into Eq. (13) on recognizing that n_k is the number of reversed spins in the spin wave of wave vector $\mathbf{k}$; the term in k^2 is the familiar spin wave term arising from exchange energy, while the term in H is the Zeeman energy.

We treat the lattice vibrations as a macroscopic field in a crystal which is elastically isotropic. On expressing the elastic energy in terms of the phonon creation and destruction operators $b_K{}^*$, b_K we find for the unperturbed elastic energy, using $b_K{}^*b_K=N_K$,

$$W=\sum[\hbar c_L K(N_K{}^{(1)}+\tfrac{1}{2}) \\ +\hbar c_T K(N_K{}^{(2)}+N_K{}^{(3)}+1)], \quad (14)$$

where the superscripts 1, 2, 3 refer to orthogonal polarization directions; c_L and c_T are the longitudinal and transverse sound velocities, respectively. This expression becomes more familiar on recognizing that $\omega_L=c_LK$, etc. The operator for the lattice displacement vector used in deriving Eq. (14) is

$$\mathbf{u}=V^{-\frac{1}{2}}\sum\mathbf{q}_K e^{i\mathbf{K}\cdot\mathbf{r}}, \quad (15)$$

where

$$\mathbf{q}_K=\sum_{s=1}^{3}\boldsymbol{\varepsilon}_K{}^{(s)}q_K{}^{(s)}$$

$$=(\hbar/2\rho K)^{\frac{1}{2}}\sum_{s=1}^{3}c_s{}^{-\frac{1}{2}}\boldsymbol{\varepsilon}_K{}^{(s)}[b_K{}^{(s)}+b_{-K}{}^{(s)*}]; \quad (16)$$

here the $\boldsymbol{\varepsilon}_K{}^{(s)}$ are an orthonormal set of unit polarization vectors such that $\boldsymbol{\varepsilon}_K{}^{(1)}$ is parallel to $\mathbf{K}$; ρ is the density; $c_1=c_L$; and c_2, $c_3=c_T$. The strains e_{ij} are readily derived from Eq. (15) by differentiating with respect to the spatial coordinates.

We next treat the collisions between the spin waves and phonons caused by the magnetoelastic coupling term given in Eq. (4). The interaction Hamiltonian in the present approximation is

$$\mathcal{H}_{\text{int}}=\int\{(B_1/M_s{}^2)[M_x{}^2(e_{xx}-e_{zz})+M_y{}^2(e_{yy}-e_{zz})] \\ +(B_2/M_s{}^2)[M_xM_ye_{xy}+M_xM_se_{xz} \\ +M_yM_se_{yz}]\}dV. \quad (17)$$

This may be written in terms of phonon and spin wave creation and destruction operators using the transformations in Eqs. (11), (15), and (16). The form resulting from the transformations may be interpreted directly in terms of various types of collision processes. For example, a term $a_{k'}{}^*a_kb_K{}^*$ represents a process in which the spin wave $\mathbf{k}$ decays into a spin wave $\mathbf{k}'$ and a phonon $\mathbf{K}$.

The machinery above was set up in 1950 by Luttinger and Kittel in work which has remained unpublished. They went on from Eq. (17) to calculate the relaxation time characterizing a spin wave state with $k=0$ in the presence of a distribution of spin waves and phonons in thermal equilibrium at room temperature. Under the conditions prevailing in microwave resonance experiments the rf magnetic field excites spin waves with wave numbers in the range 1 to 10^5 cm^{-1}, the higher values occurring in metals where the eddy current skin depth rather than the radiation wavelength is decisive. The thermally excited spin waves have k's for the most part much higher, near 10^7 cm^{-1}, so that for all practical purposes we may regard the spin waves excited by the rf field as having $k=0$.

The calculations of Luttinger and Kittel gave, for typical values of the ferromagnetic constants and using the interaction (17) which is linear in the strains, a spin-lattice relaxation time of the order of 10^{-1} sec. The experimental data suggest a spin-lattice relaxation time of the order of 10^{-7} sec at room temperature, so that the disagreement is very serious. The interpretation of the experiments is not, however, unambiguous. It is also possible to consider an interaction quadratic in the strains: for a cubic crystal one adds to Eq. (4) terms of the form

$$B_3(\alpha_1{}^2e_{xx}{}^2+\alpha_2{}^2e_{yy}{}^2+\alpha_3{}^2e^2{}_{zz}),$$

plus other terms quadratic in the strains. These quadratic terms are responsible for the dependence of the velocity of a given elastic wave on the direction of magnetization in the crystal.[7] For nickel it is found experimentally that $B_3\sim 6\times 10^9$ cgs. By use of only the coupling $(B_3/M_s{}^2)M_x{}^2e_{xx}{}^2$, the relaxation was calculated to be approximately 10^{-2} sec, still much too long.

We are faced then with the problem of accounting for a factor of 10^5 to 10^6 discrepancy between the experimental and calculated values of the spin-lattice relaxation times. On studying the calculation above and comparing it with a related calculation by Akhieser[8] we are able to see where the trouble lies. Akhieser calculated the lifetime averaged over all $\mathbf{k}$ of a spin wave state with respect to interactions with other spin waves (spin-spin relaxation) and with the lattice vibrations (spin-lattice relaxation). Akhieser's spin-lattice relaxation times are shorter by a factor of the order of 10^{-5} than those calculated by Luttinger and Kittel for their problem, even though Akhieser used only the magnetic dipole interaction which is known to be too weak to account for the observed magnetostrictive properties of ferromagnets. The reason Akhieser gets short times is that he is interested in thermal problems and is therefore dealing with an average over all spin wave states, thus weighting states of high $k(\approx 10^7$ cm$^{-\prime})$ heavily, whereas Luttinger and Kittel deal with a state of low $k(\approx 0)$ excited by the microwave field.

Now if the $k=0$ spin waves were to come into equilibrium with other spin-waves in a time short in comparison with the time for equilibrization with the lattice vibrations, then the microwave spin-lattice problem would take on a form very close to Akhieser's problem, and we might expect to come up with reasonable theo-

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[7] W. P. Mason, Phys. Rev. 82, 715 (1951).

[8] A. Akhieser, J. Phys. (U.S.S.R.) 10, 217 (1946).

retical values of the spin-lattice relaxation time in ferromagnetic resonance. Fortunately there are in fact good reasons for believing that the spin-spin relaxation time is usually shorter than the spin-lattice relaxation time, at least at not too high temperatures.

We note first that if we neglect spin-lattice relaxation the width of the resonance line is a measure of the spin-spin relaxation time. Our earlier result that resonance occurs only at a discrete field strength resulted directly from the selection rules $\Delta n_0=\pm 1$, $\Delta n_k(k\neq 0)=0$ applied to Eq. (3). However, in the presence of magnetic dipolar and pseudo-dipolar interactions between the spin waves the states of a single k will no longer be eigenstates. The correct eigenstates are mixtures of the unperturbed states of various k's. The mixing may be thought of as caused by the action of the fluctuating demagnetizing or pseudo-dipolar fields of the spin waves on each other. For temperatures of the order of half the Curie temperature or less the zero-point demagnetizing fields associated with the zero-point motion of the transverse components of the spin waves are expected to be more important than the magnetic fields associated with thermally excited spin waves. The transverse magnetization components go as $S_t=(S_x^2+S_y^2)^{\frac{1}{2}}=[S(S+1)-S^2]^{\frac{1}{2}}=S^{\frac{1}{2}}$. In the classical limit $S\rightarrow\infty$, the zero-point transverse components of the magnetization become relatively insignificant, and the mixing effect vanishes: The interaction energy per spin is of the order $\mu H_t\sim\mu M_s/S^{\frac{1}{2}}$, where M_s is the saturation magnetization,[9] or $\mu H_t\sim C/S^{\frac{1}{2}}$, where C is the pseudo-dipolar interaction energy of two adjacent spins.

As a result of the mixing of the originally unperturbed states caused by the dipolar-type interaction, admixtures of the $k=0$ state will be found in other states, especially in those within μH_t of the original state. This means that magnetic dipole transitions may now be expected to take place over a broad group of states, instead of only between the equi-spaced $k=0$ levels. For T less than, say, half of the Curie temperature the mixing arises almost entirely from the zero point motion and in this way we can account for a temperature independent line width (within an order of magnitude) from room temperature to absolute zero, although the experimental evidence for such an extrapolation is at present incomplete. Yager in preliminary unpublished measurements finds that the line width in Supermalloy is roughly constant between room temperature and liquid air temperature. The constancy of the line widths measured by Bloembergen[10] in nickel below 100°C and in Supermalloy below 300°C is very suggestive, although he did not continue the measurements below room temperature. Healy[11] finds an increase of width in a nickel ferrite single crystal by a factor of two on going from 600°C to -200°C.

We now suggest that ordinary ferromagnetic substances are characterized, at temperatures substantially below the Curie point, by a spin-spin relaxation time of the order of 10^{-8} to 10^{-9} second. We have not made a direct calculation of the spin-spin time, but our estimate follows plausibly on introducing the pseudo-dipolar interaction into Akhieser's results; it also follows from the argument below. An apparent objection to our point of view is based on Van Vleck's calculations[12] of the second and fourth moments of magnetic resonance frequencies in paramagnetic and ferromagnetic crystals. The result of Van Vleck which is apparently the strongest contradiction of our arguments is that at 0°K in a ferromagnetic crystal

$$\langle\Delta\omega^2\rangle_{\mathrm{Av}}=0. \tag{18}$$

This would require unambiguously that the line be of zero width. However, we believe that the argument by which Van Vleck derived Eq. (18) is incomplete, as shown by Keffer.[13] The essential point is that Van Vleck has worked throughout with a truncated Hamiltonian, omitting entirely those parts of the dipolar Hamiltonian corresponding to transitions with $\Delta M=0$, ± 2, ± 3, while the part he retains allows only $\Delta M=\pm 1$. The other transitions are omitted in paramagnetic problems because they are associated with weak satellite lines distinct from the principal line. However, when the dipolar energy is comparable with the Zeeman energy the satellites can spread out over the main line and the classification of transitions by ΔM changes is no longer applicable. For the $\Delta M=2$ transition Keffer finds that the second moment about its center contains terms of the order J^2, indicating that the satellite has spread out. The part of the Hamiltonian Van Vleck retains gives no mixing of spin wave states, while the part he drops does give mixing. Keffer's argument is that in the ferromagnetic problem $(J,C\gg\mu H)$, because of the overlap of the satellites with the main line, we should work with the whole Hamiltonian. The second moment at absolute zero as calculated by Keffer comes out to be, for a fcc lattice,

$$\hbar^2\langle\Delta\omega^2\rangle_{\mathrm{Av}}\cong(3g\mu_B M_s/S^{\frac{1}{2}})^2. \tag{19}$$

We note that the second moment vanishes if we let $S\rightarrow\infty$ while keeping M_s constant. The nonvanishing of the second moment is therefore a quantum effect, as

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[9] In reality we have to deal with a pseudo-dipolar interaction C (arising from spin-orbit effects) which may be 10 to 100 times stronger than the magnetic dipolar interaction; the mixing is increased in the same ratio. For estimates of the magnitude of the pseudo-dipolar interaction, see J. H. Van Vleck, Ann. inst. Henri Poincaré **10**, 57 (1947); see also reference 4, Appendix B. For nickel Van Vleck finds that the pseudo-dipolar interaction must be about 50 times greater than the magnetic dipolar interaction in order to account for the anisotropy. For the super-exchange interaction a relation due to Van Vleck and Kittel gives $C\approx J(g-2)^2$.

[10] N. Bloembergen, Phys. Rev. **78**, 572 (1950).

[11] D. W. Healy, Jr., Phys. Rev. **86**, 1009 (1952).

[12] J. H. Van Vleck, Phys. Rev. **74**, 1168 (1948); **78**, 266 (1950).

[13] F. Keffer, thesis, University of California, Berkeley, 1952, pp. 40–59; Phys. Rev. **88**, 686 (1952). Keffer has also shown that the width in the paramagnetic region $(T>T_c)$ under the same conditions is increased by the factor 10/3, a result later obtained by P. W. Anderson independently.

expected from our earlier discussion of the fluctuating transverse demagnetizing fields associated with the zero-point spin waves.

Except for the numerical constant and the dependence on the spin quantum number S, Eq. (19) has the same form as the usual Van Vleck expression for the second moment of the resonance line in a paramagnetic salt, and we may expect just as in the paramagnetic case to find that Eq. (19) is not by itself a good index to the apparent width of the resonance, as the fourth moments in both the paramagnetic and the absolute-zero ferromagnetic calculations involve J^2. The general effect of the exchange interaction may be interpreted qualitatively following Van Vleck and Gorter as leading to a motional narrowing of the resonance line. The concept is useful, but probably not rigorous. The apparent line width due to spin-spin relaxation is given qualitatively on this empirical analogy by

$$\hbar\Delta\omega \approx \hbar(\Delta\omega)_d[\hbar(\Delta\omega)_d/J], \tag{20}$$

where $(\Delta\omega)_d$ is the dipolar width which would obtain in the absence of the exchange interaction. The effect of exchange is to reduce the width in the ratio $\hbar(\Delta\omega)_d/J$. Writing the pseudo-dipolar interaction energy between two atoms as C we have

$$\hbar\Delta\omega \approx C^2/JS. \tag{21}$$

Estimating $C \approx 3\text{cm}^{-1}$ and $J \approx 100\text{ cm}^{-1}$, we see that the line width may very reasonably be of the order of 0.1 cm^{-1}, corresponding to a spin-spin relaxation time of the order of 10^{-9} second at 0°K. This is the proper order of magnitude.

We have thus made our basic assumption $\tau_{\text{spin-spin}} < \tau_{\text{spin-lattice}}$ for $T/T_c < \frac{1}{2}$ seem plausible. We can account for a spin-spin relaxation time of the order of 10^{-9} second, which is shorter than the room temperature spin-lattice time ($\approx 10^{-7}$ sec). We suppose then in what follows that for the purpose of calculating spin-lattice relaxation times we may assume the spin system to be effectively in thermal equilibrium.

The calculation of spin-lattice relaxation times at various temperatures under the assumption that the spins are in thermal equilibrium has carried out by Abrahams.[14] The calculation is not too unlike one which Akhieser carried out for another purpose, but Abrahams uses the realistic interaction given in Eqs. (4) and (17), while Akhieser used the magnetic dipolar interaction which is known to be 10 to 100 times too small. Abrahams applied the general method discussed above as it lends itself easily to the use of the phenomenological interaction, while Akhieser used an extension of the Holstein-Primakoff method.[15] It would be quite possible, however, to apply the Holstein-Primakoff method with a pseudo-dipolar interaction.

It turns out from the calculations of Abrahams that the terms of greatest importance in Eq. (17) are those describing the scattering of a spin-wave with the emission or absorption of a phonon. The effective part of the interaction Hamiltonian then consists of a phonon emission term

$$\mathcal{H}_{\text{int}}^{e} \propto \sum_{K,k} b_K^{*} a_k a_{k'}^{*} f(\mathbf{K}), \tag{22}$$

plus a phonon absorption term

$$\mathcal{H}_{\text{int}}^{a} \propto \sum_{K,k} b_K a_k a_{k'}^{*} f(\mathbf{K}), \tag{23}$$

with the condition

$$\mathbf{k}' = \mathbf{k} + \mathbf{K} \tag{24}$$

applying in both cases. Here $f(\mathbf{K})$ denotes a function of $\mathbf{K}$.

The net number of collisions for unit time which transfer energy to the lattice is given by the kinetic equation, where ϵ denotes the energy,

$$w_{\text{coll}} = \frac{2\pi}{\hbar} \sum_{K,k} [|H_{\text{int}}^{e}|^2 - |H_{\text{int}}^{a}|^2] \delta(\epsilon_K + \epsilon_k - \epsilon_{k'}). \tag{25}$$

Now the matrix elements of the creation and destruction operators are simply expressible in terms of the occupation numbers N_K, n_k, $n_{k'}$, so that Eq. (25) assumes the form

$$w_{\text{coll}} = \frac{2\pi}{\hbar} \sum_{K,k} |f(K)|^2 [N_k n_k (n_{k'}+1) - (N_K+1)(n_k+1) n_{k'}] \delta(\epsilon_K + \epsilon_k - \epsilon_{k'}). \tag{26}$$

We now define a spin temperature T_s and a lattice temperature T_l. This permits us to express the occupation numbers in terms of the Bose-Einstein factors appropriate to the two temperatures. We proceed from Eq. (26) to calculate the net rate of energy transfer $\dot{Q}$ between the two systems. The result of the calculation is that

$$\dot{Q} \cong \frac{V\Delta T}{T^2} \cdot \frac{1}{(2\pi)^3} \cdot \frac{\hbar B_1^2}{2\rho k} \cdot \left(\frac{\hbar c}{2A}\right)^8 \cdot \left(\frac{M_s}{g\mu_B}\right)^6 \cdot \frac{1}{2\gamma} \cdot I(\gamma, \xi), \tag{27}$$

where, writing c for the sound velocity,

$$\gamma = (\hbar c)^2 M_s / 2g\mu_B A k T;$$
$$\xi = 2(g\mu_B/\hbar c)^2 (AH/M_s), \tag{28}$$

and

$$I(\gamma, \xi) = \int_0^\infty \frac{x^4 dx}{\cosh\gamma x - 1} \left[\log \frac{e^{\alpha+\gamma\xi} - e^{-\gamma x}}{e^{\alpha+\gamma\xi} - 1}\right],$$

where $\alpha = \gamma(x-1)^2/4$. The integral must be evaluated numerically.

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[14] Elihu Abrahams, thesis, University of California, Berkeley, 1952; E. Abrahams and C. Kittel, Phys. Rev. **88**, 1200 (1952).

[15] T. Holstein and H. Primakoff, Phys. Rev. **58**, 12 (1940).

Assuming that the temperature difference $\Delta T = T_s - T_l$ decays exponentially, the spin-lattice relaxation time τ is defined by

$$\frac{d}{dt}\Delta T = -\frac{1}{\tau}\Delta T. \tag{29}$$

In terms of the rate of energy transfer $\dot{Q}$ and the heat capacities C_s, C_l of the spin and lattice system, we find

$$\tau = \frac{\Delta T}{\dot{Q}[(1/C_s)+(1/C_l)]}. \tag{30}$$

We can estimate both C_s and C_l as a function of temperature, so that with Eq. (27) for $\dot{Q}$ we can determine τ.

In this way Abrahams calculates for nickel, at 300°K, $\tau = 6\times10^{-7}$ sec; at 3 K, $\tau = 5\times10^{-4}$ sec; at 1°K, $\tau \approx 1$ sec. The spin-lattice estimates reported here do not include the effect of relaxation by coupling with the conduction electrons. At low temperatures in metals we might expect this process to be important, and the relevant calculations are underway. In ferrites, however, the present estimates should be valid. There are at present no direct experimental measurements available for the spin-lattice relaxation time, although the line width measurements by Bloembergen[10] may be interpreted as suggesting that at high temperatures spin-lattice relaxation makes an appreciable contribution to the line width, so that the spin-lattice time well above room temperature may perhaps drop to 10^{-8} to 10^{-9} sec.

An important question not yet fully solved is the interpretation of several reorientation experiments which it had originally been thought measured the spin-lattice relaxation time. Galt[16] has deduced a spin-lattice relaxation frequency of 3×10^{8} sec^{-1} in magnetite at room temperature from measurements of the velocity of propagation of a domain wall.[17] Bloembergen and Damon[18] deduce a spin-lattice time of 3×10^{-8} sec for nickel ferrite at room temperature, while a similar value is found in manganese zinc ferrite. These values are obtained from ingenious microwave saturation measurements using intense rf fields, of the order of 40 oersteds. It has been suggested to the authors by M. H. Cohen and independently by P. W. Anderson that, as in Waller's calculations, experiments of these types measure essentially (to within a numerical factor) the same relaxation time as determines the line width. In brief, one argues that when the spins have come into equilibrium with each other in the time τ_s they will also have come into equilibrium in the external field. The processes which mix the spin wave states must give rise to angular momentum exchange with the specimen as a whole, thereby allowing the spin system to change its orientation with respect to an external field in a time of the order of τ_s. It may be expected that this question will shortly be resolved experimentally by low temperature measurements. At room temperature the reorientation time appears experimentally to be quite close to the time associated with the line width, in approximate agreement with the "one time" viewpoint.

The over-all picture given here of relaxation processes in common ferromagnetic substances suggests that, to within an order of magnitude at least, line widths (in single crystals) at 0°K will be roughly equal to line widths at temperatures as high as one-half the Curie temperature,[19] the spin-spin relaxation process being dominant and having values in the range 10^{-8} to 10^{-10} sec. At higher temperatures the spin-spin time may decrease, and the spin-lattice time will also decrease, perhaps in some cases sufficiently to dominate the line width near the Curie point, although calculations valid in this region have not been carried out. The reorientation time is thought to be approximately equivalent to the time associated with the line width. The spin-lattice time in ferrites is expected to become very long at low temperatures, of the order of one sec at 1°K, but will appear directly only in thermal measurements.

In conclusion we wish to express our thanks to Professor J. M. Luttinger and Professor F. Keffer for their collaboration on this problem, and for permission to refer to work of theirs unpublished at the time of writing. We are indebted to Professor J. H. Van Vleck for discussions of various aspects of the problem. This work has been supported in part by the U. S. Office of Naval Research.

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[16] J. K. Galt, Phys. Rev. **85**, 664 (1952).

[17] The connection is derived by L. Landau and E. Lifshitz, Physik. Z. Sowjetunion **8**, 153 (1935); C. Kittel, Phys. Rev. **80**, 918 (1950).

[18] N. Bloembergen and R. W. Damon, Phys. Rev. **85**, 699 (1952); N. Bloembergen and Shyh Wang, Phys. Rev. **87**, 392 (1952). According to a private communication from Bloembergen, this time should be reduced by a facter of 4π.

[19] There is no reason to exclude width changes by a factor of two or so such as are reported in a nickel ferrite single crystal by D. W. Healy, Jr., Phys. Rev. **86**, 1009 (1952).

DISCUSSION

G. T. RADO, *Naval Research Laboratory, Washington, D. C.:* Did you consider in your calculations the effect of crystalline imperfections on the line width?

C. KITTEL, *University of California, Berkeley, California:* One can produce situations in which imperfections control the line width. Yager has, for example, broadened the line in Supermalloy by deliberate cold working. In the rather carefully selected specimens usually employed in resonance experiments I do not think that the width is caused by imperfections. The general agreement of the order of magnitude of the line widths over a wide range of substances is perhaps evidence for this statement.