Hall Effect in Ferromagnetic Materials

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Experimental results are reviewed for Fe, Co, Ni, the Co-Ni and Ni-Cu alloy series. Most of the measurements were made at room temperature. In the case of Ni, data are available from about 10°K to 800°K.

In every case studied, the Hall emf consists of a sum of two terms. The first term is proportional to the magnetizing field and has been called the ordinary Hall effect. Its order of magnitude and sensitivity to variations in temperature and in composition are comparable with the Hall effect in nonferromagnetic metals. The second term is proportional to the magnetization and has been called the extraordinary Hall effect.

At room temperature, the extraordinary effect is usually much larger than the ordinary effect. Large changes in the extraordinary effect may be produced by variations in temperature or composition.

In the theory of the ordinary effect, a representation is employed in which conduction may take place in both the s- and d-bands. This theory appears to be moderately successful in interpreting the experimental data.

Various proposals that have been advanced to account for the extraordinary effect are considered briefly. We consider that a satisfactory physical basis for this phenomenon has not yet been established.

1. INTRODUCTION

S early as 1893,¹ it was known that the Hall effect A in the ferromagnetic elements Fe, Co, and Ni is determined mainly by the magnetization. In subsequent experiments by one of us,² it was established that the Hall voltage is proportional to the magnetization at fields up to magnetic saturation and has all of the inherent irreversibilities of magnetization curves. These experiments were made with polycrystalline materials. Webster³ measured the Hall effect in single crystals of iron and obtained similar results. Webster's measurements were made with the primary current and Hall potential along the simple crystallographic axes [100], [110], and [111]. His results showed that the Hall effect due to magnetization is practically isotropic.

Most of the early measurements were made for fields below saturation. The Hall constant was defined as it is in nonferromagnetic materials. That is

$$R = E_y / H_0 j_x, \tag{1}$$

where R=Hall constant, E_y =Hall electric field, j_x = primary current density, and H_0 = applied magnetic field in the z direction. Usually a sample was measured that was sufficiently thin in the z direction relative to the dimensions in the x and y directions so that the demagnetizing factor would be very nearly 4π and $H_0 \cong B$, the magnetic induction of the sample.

In Fig. 1 the measurements made by Smith⁴ for Ni are reproduced. We have plotted the Hall electric field per unit current density $e_y = E_y/j_x$, against the applied field or magnetic induction of the sample. The Hall constant according to Eq. (1) would be the slope of the initial steep portion of each curve. The temperature dependence of R, as shown in Fig. 2, is particularly

striking because the Hall constant does not usually vary significantly with temperature in metals. More recent measurements for Ni by Kikoin⁵ confirm the essential features of Smith's data. Smith⁴ also made measurements for Co and Fe which show similarly anomalous behavior.

The Hall effect in ferromagnetic metals appears to be a fundamentally different phenomenon from the Hall effect in other metals according to the experimental data for fields below saturation. When magnetic saturation is achieved, or the domains have been oriented in the direction of the applied field, the magnetization is almost constant. The Hall effect continues to increase with increasing applied field, but much more slowly as can be seen in Fig. 1. Accordingly the following empirical formula has been suggested for the Hall effect in ferromagnetic metals^{2,6}:

$$e_y = R_0 H + R_1 M, \qquad (2)$$

where e_y = Hall electric field per unit current density, H = magnetizing force in the z direction, and M = magnetization in the z direction. R_0 and R_1 have been designated the ordinary and extraordinary Hall constants, respectively. The data of Smith for Ni have been



FIG. 1. Hall effect in nickel from the data of A. W. Smith.

⁵ I. K. Kikoin, Physik. Z. Sowjetunion 9, 1 (1936). ⁶ A. W. Smith and R. W. Sears, Phys. Rev. 34, 1466 (1929).

¹ A. Kundt, Wied. Ann. 49, 257 (1893). ² E. M. Pugh, Phys. Rev. 36, 1503 (1930); E. M. Pugh and T. W. Lippert, Phys. Rev. 42, 709 (1932). ³ W. L. Webster, Proc. Cambridge Phil. Soc. 23, 800 (1925). ⁴ A. W. Smith, Phys. Rev. 30, 1 (1910).



FIG. 2. Hall constants for nickel determined from the slopes of the initial steep portions of the plots of Hall electric field per unit current density versus magnetic induction.

reconsidered⁷ on the basis of Eq. (2). Although Smith's data above saturation are not very accurate it was possible to determine approximate values of R_1 and R_0 over the entire temperature range studied by Smith.

The constant $R_0 = -0.61 \times 10^{-12}$ volt cm/amp gauss at room temperature, was of the same order of magnitude as the Hall constants for the nearest neighbor nonferromagnetic elements in the periodic table $(R = -0.93 \times 10^{-12} \text{ volt cm/amp gauss for Mn and})$ $R = -0.55 \times 10^{-12}$ volt cm/amp gauss for Cu). The Hall constant defined in the usual way for ferromagnetic metals by Eq. (1) is about a factor of 20 larger. Of particular significance was the fact that the values of R_0 determined from Smith's data were approximately independent of temperature over the entire temperature range.⁸ Recently Jan and Gijsman⁹ have obtained more accurate data at low temperatures which show that R_0 decreases significantly in magnitude between room temperature and 14°K. With the possible exception of this fact, the order of magnitude and temperature dependence of R_0 is about the same as for the Hall constant of nonferromagnetic metals. (Above the Debye characteristic temperature of the lattice the Hall constant is almost independent of temperature in most metals. Low temperature behavior is rather varied. According to Alterthum,¹⁰ when the temperature is decreased from room temperature to very low temperatures, the Hall constant increases to a maximum in diamagnetic metals such as Au, Cu, Cd, and decreases to a minimum in paramagnetic metals such as Al, Pt.)

The constant $R_1 = -74.9 \times 10^{-12}$ volt cm/amp gauss for Ni at room temperature. It increases rapidly with temperature by as much as a factor of about 5 in the neighborhood of the Curie point.⁷ The Hall effect due to magnetization is of a fundamentally different character from the usual Hall effect. Although it has been observed only in ferromagnetic metals, it should be pointed out that if such an effect were present in other metals it would be indistinguishable from the ordinary Hall effect because M would be proportional to H. The

existence of this effect above the ferromagnetic Curie point when Ni is still strongly paramagnetic suggests the possibility that it may occur in nonferromagnetic materials.2,5,8

In most of the earlier measurements found in the literature, investigators have studied the Hall effect below saturation. The main reason for this is that a very high magnetic field is required to study most ferromagnetic materials above saturation. The only data above saturation that has been available is for Ni. The recent acquisition of an A.D.L. magnet at Carnegie Institute of Technology has made fields up to 40 kilogauss available. The group at C.I.T. has undertaken the study of a variety of ferromagnetic metals with the primary objective of measuring the ordinary Hall constant R_0 for which no data have been available. The results to date will be reported herein, and the interpretation of the measured values in terms of the usual theory of the Hall effect will be discussed. These data help to fill an existing gap in the knowledge of the electrical properties of ferromagnetic metals and when completely understood may considerably enhance



parameter α , for nickel.

our presently inadequate understanding of the behavior of the *d*-shell electrons.

The origin of the Hall effect due to magnetization must be of a fairly general character since similar effects due to magnetization are present in an entire class of conduction phenomena. For example, the Ettinghausen, Nernst, and Righi-Leduc effects are also proportional to the magnetization below saturation and increase linearly with H after all of the domains have been oriented.11,12

Considerable thought has been given to the possibility that an effective magnetic field deflects^{7,13} the conduction electrons. That is,

$$\mathbf{H}_{\rm eff} = \mathbf{H} + 4\pi\alpha \mathbf{M}_s, \tag{3}$$

where $\mathbf{H} =$ magnetizing force, $\mathbf{M}_s =$ saturation magnetization in a domain, $\alpha = a$ parameter which would usually be much greater than unity.

¹¹ L. L. Campbell, Galvanomagnetic and Thermomagnetic Effects (Longmans Green and Company, New York, 1932). ¹² G. S. Nielson, Phil. Mag. 18, 575 (1934). ¹³ K. Kondo, Rep. Inst. of Sc. and Tech., Tokyo 4, 79 (1950)

⁷ Pugh, Rostoker, and Schindler, Phys. Rev. 80, 688 (1950).
⁸ N. Rostoker and E. M. Pugh, Phys. Rev. 82, 125 (1951).
⁹ J. P. Jan and H. M. Gijsman, Physica 5, 277 (1952).
¹⁰ H. Alterthum, Ann. phys. 39, 933 (1912); 40, 391 (1913).

⁽in Japanese).

Below saturation when the domains are only partially aligned, the effective field given by Eq. (3) would produce a Hall electric field whose magnitude and direction would be different for each domain. However, it has been proved⁷ that the resultant observable Hall emf would depend on the macroscopic magnetization M or the average domain orientation and the Hall constants for the sample would be the same as the Hall constants for the domains. Therefore without loss of generality we can forget about the domains and replace the $4\pi\alpha M_s$ term of Eq. (3) by the uniform field $4\pi\alpha M$. Then Eqs. (2) and (3) imply the following expression for Hall effect:

$$e_y = R_0 H_{\rm eff}, \tag{4}$$

where $H_{\rm eff} = H + 4\pi\alpha M$ is the effective field in the z direction, and $\alpha = R_1/4\pi R_0$.

In the case of Ni it has been possible to estimate the values of α over a wide range of temperatures from Smith's data. In Fig. 3, α has been plotted against temperature. It is not possible to estimate α satis-



FIG. 4. Comparison of Hall effects for a number of materials investigated by S. Foner. (The Hall voltages refer to samples which are 1 mm thick in the direction of the field and carry a current of 25 amperes.)

factorily for low temperatures from these data. This gap has recently been filled by the measurements of Jan and Gijsman⁹ which are also shown in Fig. 3. It appears quite definite that α approaches 2 at very low temperatures.

This is in good agreement with the recent work on the magnetoresistance of Ni carried out by Smit.¹⁴ According to Smit the magnetoresistance produced by the influence of the Lorentz magnetic force on the electron orbits is obscured by the much larger magnetoresistance due to the influence of changes in intrinsic magnetization on the scattering probability of electrons, except at low temperatures. He has studied the "orbital" magnetoresistance at liquid nitrogen temperature and finds that an effective field with $\alpha \cong 2$ is consistent with his data.

Several attempts have been made to give a physical basis for the effective field.^{7,13,15,16} These attempts have



FIG. 5. Ordinary Hall constant versus composition from the measurements of Hall emf above saturation by Schindler and Foner. (n^* is the number of conduction electrons according to elementary theory for the Hall effect. n_s is the number of s-band electrons according to the simple band picture.)

all started from the one-electron approximation in a perfect lattice corresponding to 0°K and might be considered as possible explanations for the low temperature behavior of the Hall effect due to magnetization. However, no success has been achieved in explaining the increase of α from about 2 to almost 100 in Ni when the temperature is increased up to the Curie point. These attempts also fail to clarify the experimental data for Fe, Co, the Co-Ni and Ni-Cu alloy series presented herein.

2. RECENT MEASUREMENTS OF HALL EFFECT AT ROOM TEMPERATURE

Hall effects in the Cu-Ni and Co-Ni systems have recently been measured by Schindler¹⁷ and Foner.¹⁸ In Fig. 4 the Hall voltage is plotted against the applied magnetic field for a number of materials investigated by Foner. From such curves, the values of R_0 and α in Eq. (4) were determined. In Fig. 5 the values of R_0 are shown for all of the elements and alloys measured to date. In Fig. 6 the corresponding values of α are shown.

3. INTERPRETATION OF THE ORDINARY HALL EFFECT IN FERROMAGNETIC MATERIALS

According to the Sommerfeld theory the Hall constant R is related to the number of conduction electrons by the formula¹⁹

$$R = -1/(nNec), \tag{5}$$

where n =number of conduction electrons/atom, N = number of atoms/unit volume, e = magnitude of elec-

¹⁹ A. Sommerfeld and N. H. Frank, Revs. Modern Phys. 3, 1 (1931).

¹⁴ J. Smit, Physica 17, 612 (1951).

¹⁵ V. Rudnitsky, J. Exp. Theoret. Phys. USSR 9, 262 (1939).

 ¹⁶ A. G. Samolovich and B. L. Konkov, J. Exp. Theoret. Phys. USSR 20, 783 (1950).
 ¹⁷ A. I. Schindler and E. M. Pugh, Phys. Rev. 83, 208 (1951).
 ¹⁸ S. Foner and E. M. Pugh, Phys. Rev. 87, 210 (1952).
 References 17 and 18 refer to abstracts of papers presented to the American Divisional Society. The compension of a paper specific barrier of the paper specific barrier of the speci American Physical Society. The experimental results have not yet been published at the time of writing this paper and are repro-duced here with the kind permission of Dr. A. I. Schindler and Dr. Simon Foner.



FIG. 6. Field parameter α , estimated from the measurements of Schindler and Foner.

tronic charge, and c= velocity of light. Equation (5) is applicable to a case where conduction occurs in a single band which is nearly empty. In this case the energy levels are approximately given by

$$\epsilon = (\hbar^2 / 2m_s)k^2, \tag{6}$$

where ϵ =electron energy relative to the lowest energy level of the band, k=magnitude of electron wave vector, and m_s =effective mass of electrons in for example an *s*-band.

An effective number of electrons n^* can be calculated from Eq. (5) and the experimentally determined values for R_0 . Making use of Eq. (5),

$$n^* = -\left(1/R_0 N e c\right) \tag{7}$$

has been calculated and plotted in Fig. 5. The numbers of *s*-band electrons/atom indicated by magnetization measurements, are also shown in Fig. 5 for each material. Excluding the case of Fe, n^* does not differ from n_s by more than a factor of 2 in any case. This is about the extent of agreement usually obtained between the number of conduction electrons expected and the number calculated by means of Eq. (5) and Hall effect measurements for nonferromagnetic metals. For example the agreement in the case of Cu shown in Fig. 5 is within a factor of 1.5.

In the case of nonferromagnetic metals the discrepancies between the measured Hall coefficients and Eq. (5) can often be resolved by replacing Eq. (6) with a more realistic assumption for the band structure. For metals that conduct in a single band such as Cu, a more general formula for R such as Jones and Zener²⁰ have given indicates that the discrepancies may be accounted for by departures of the Fermi surfaces from spheres. For metals that can be expected to conduct in more than one band a representation has been employed where one band is nearly empty and the other nearly filled. In the nearly empty band (*s*-band) the energy levels are assumed to be given by Eq. (6) and in the nearly filled band (*d*-band) by

$$\epsilon = \epsilon_m - (\hbar^2 / 2m_d) l^2, \tag{8}$$

where $\epsilon_m =$ maximum energy of the band, $-m_d =$ effec-

²⁰ H. Jones and C. Zener, Proc. Roy. Soc. (London) A145, 269 (1934).

tive mass of *d*-electrons, \mathbf{l} = wave vector measured from the highest level ϵ_m . With this representation Sondheimer²¹ gives for the Hall constant,

$$R = -\frac{1}{Nec} \left[\frac{1}{n_s} \left(\frac{\sigma_s}{\sigma} \right)^2 - \frac{1}{n_d} \left(\frac{\sigma_d}{\sigma} \right)^2 \right], \tag{9}$$

where n_s =number of s-band electrons, n_d =number of d-band holes, σ_s =s-band conductivity, σ_d =d-band conductivity, $\sigma = \sigma_s + \sigma_d$ = total conductivity. Equation (9) has been moderately successful in explaining departures from the Sommerfeld formula²² and the occurrence of positive Hall coefficients in many metals where conduction can be expected from more than one band.

It appears to be both reasonable and necessary to assume for the ferromagnetic materials that the d-electrons are itinerant and contribute to conduction. We can for example account for the measured Hall coefficient of Ni by means of Eq. (9) with $n_s = n_d = 0.6/$ atom and $\sigma_d/\sigma = 0.23.^7$ Similarly the measured Hall coefficient for Fe can be accounted for with $n_s = 0.6$ atom, $n_d = 2.2/\text{atom}$, and $\sigma_d/\sigma = 0.87$. It should be noted that Co, according to the old definition of Eq. (1) for the Hall constant, had a positive Hall constant indicating predominating hole conduction, whereas according to the present definition, R_0 is negative and agrees fairly well in magnitude with the Sommerfeld formula and $n_s = 0.7/\text{atom}$. (The reason for this can be seen in Fig. 4. The slope of the plot of Hall voltage versus magnetic induction is positive below saturation and negative above.)

The main features of the R_0 constants for ferromagnetics can be understood in terms of the usual theory of Hall effect for a uniform applied field. The degree to which measured values of the Hall constants agree with predictions by theory is about the same as for nonferromágnetic metals. Close agreement with formulas based on spherical Fermi surfaces such as Eqs. (5) and (9) usually cannot be expected since most attempts at calculating wave functions and energy levels for ferromagnetics indicate that the Fermi surfaces are not spherical to a good approximation except in limiting cases.²³

4. SPECULATIONS ON THE ORIGIN OF THE HALL EFFECT DUE TO MAGNETIZATION

There are usually transverse voltages present which are not due to the Hall effect. Transverse temperature gradients produced, for example, by the Ettingshausen effect, result in a transverse voltage due to the thermocouple action of the sample and potential leads. Magnetoresistance effects which are particularly large in ferromagnetic materials may also result in a transverse voltage. In the data reported herein, particular

²¹ E. H. Sondheimer, Proc. Roy. Soc. (London) A193, 484 (1948). ²² For example, see E. Justi and M. Kohler, Abh. Braunschweig Wiss. Ges. 3, 44 (1951).

 ²³ For example, see H. M. Krutter, Phys. Rev. 48, 664 (1935);
 G. C. Fletcher, Proc. Phys. Soc. (London) A65, 192 (1952).

care has been exercised to correct for these voltages so that the corrected measurements may be properly regarded as Hall effect.

In order to describe the Hall effect in ferromagnetic materials within the framework of existing theory, an effective Lorentz force has been introduced. We assume effective magnetic fields for the Lorentz force,

$$H_s = H + 4\pi\alpha_s M \tag{10}$$

for electrons that are mainly in s-states and

$$H_d = H + 4\pi \alpha_d M \tag{11}$$

for electrons that are mainly in *d*-states. This leads to the Hall effect expression of Eq. (4) where R_0 is given by Eq. (9) and

$$\alpha = \frac{1}{R_0 Nec} \left[\left(\frac{\sigma_s}{\sigma} \right)^2 \frac{\alpha_s}{n_s} - \left(\frac{\sigma_d}{\sigma} \right)^2 \frac{\alpha_d}{n_d} \right].$$
(12)

In order to explain the observed values of α for Ni, it is necessary to give a physical basis for values of α_s and α_d that are much greater than unity and to account for the rapid increase of α with temperature from about 2 to almost 100 mainly through a temperature dependence of α_s , α_d . A correct explanation must also suggest the changes in α with changes in composition shown in Fig. 6.

a. Lorentz Force due to Internal Magnetic Fields

The magnetization arises from the electron spin which produces dipoles that are very small even on an atomic scale. The magnetic field produced by these dipoles varies over a wide range of values in extremely short distances. The resultant effect on an electron which traverses many interatomic distances can be described in terms of an effective magnetic field such that

$$\langle \mathbf{f} \rangle_{AV} = -(e/c) \langle \mathbf{v} \times \mathbf{h} \rangle_{AV} = -(e/c) (\langle \mathbf{v} \rangle_{AV} \times \mathbf{H}_{eff}), \quad (13)$$

where $\mathbf{f} = \text{Lorentz}$ force; the $\langle \rangle_{Av}$ denotes an average for the electron being deflected, $\mathbf{h} = \text{actual magnetic field}$ at the location of the electron under consideration, $\mathbf{v} =$ electron velocity, $\mathbf{H}_{eff} =$ effective magnetic field. For a beam of Dirac electrons with extremely high velocity, so that v may be considered a constant of the motion, Weizsäcker²⁴ has shown that $H_{eff} = H + 4\pi M$. According to Lorentz the spatial average of **h** is $\langle \mathbf{h} \rangle_{Av} = \mathbf{H} + 4\pi \mathbf{M}$ so that in this case the beam electrons simply report the average field. Wannier's²⁵ calculations for high energy cosmic ray particles consider the effect of the beam particle on the local distribution of magnetization. He finds small deviations from the limiting case treated by Weizsäcker which are produced by Coulomb interaction between the beam particle and the *d*-electrons.

conduction electron for which the crystalline field cannot be neglected and for which v could not be considered a constant of the motion. Nevertheless, it has been considered by Webster²⁶ that on the basis of Wannier's calculations the effective field for conduction electrons would be less than $\langle \mathbf{h} \rangle_{Av}$ because the Coulomb repulsion between conduction and d-electrons would prevent conduction electrons from reaching the high field regions near the *d*-electrons.

Kondo²⁷ has recently made a calculation appropriate to the conduction electrons of Ni and finds that $H_{\rm eff} = H + 4\pi\alpha M$ where $\alpha_s = 0.34$ for s-electrons and $\alpha_d = 2.5$ for d-electrons. He has also found a linear increase with temperature of α_s , α_d due to the influence of the lattice vibrations on the microscopic distribution of magnetization, but it is much too small to explain the Hall effect.

There does not seem to be any possibility of accounting for values of α very different from unity on the basis of the magnetic field due to the polarized *d*-electrons.

b. Effective Field due to Spin-Orbit Interaction

Rudnitsky¹⁵ first suggested the possibility of a Hall effect due to the action of the inhomogeneous field from the primary current on the spins of the *d*-electrons. The magnetic field due to the current is illustrated in Fig. 7. For simplicity we consider a model where there are N electrons per unit volume of magnetic moment μ and charge -e. These electrons are assumed to produce the primary current and to have their spins aligned in the z direction so as to produce a magnetization $M = N\mu$. Due to the inhomogeneous field from the primary current, the force on an electron in the y



FIG. 7. A typical Hall effect sample. The directions of magnetization, primary current, and Hall voltage are shown. The magnetic field due to the primary current density is indicated roughly by field lines.

These calculations do not apply to the relatively slow

 ²⁴ C. F. v. Weizsäcker, Ann. Physik 17, 869 (1933).
 ²⁵ G. H. Wannier, Phys. Rev. 72, 304 (1947).

D. L. Webster, Am. J. Phys. 14, 360 (1946).
 K. Kondo, Saitama University, Tokiwacho, Urawa, Japan. We wish to thank Professor Kondo for showing us his calculations prior to publication.

(16)

direction would be

$$f_y = \mu(\partial H_y / \partial z), \tag{14}$$

where H_y = magnetic field due to the primary current density j_x . According to Maxwell's theory,

$$\partial H_y/\partial z = (\partial H_z/\partial y) - (4\pi/c)j_x.$$
 (15)

 H_{y} , H_z may be calculated from the Biot-Savart law with the assumption that j_x is constant. For a sample of the dimensions ordinarily employed in Hall effect studies

 $\partial H_y/\partial z \cong -(4\pi/c) j_x$

and

$$f_y \cong -(4\pi\mu/c)j_x$$
.

In order to satisfy the condition that $j_y=0$, there must be a transverse electric field such that $f_y-eE_y=0$. Making use of Eq. (16) and writing $M=N\mu$, we obtain the following rough estimate of the Hall effect,

$$E_y = -\left(4\pi M/Nec\right)j_x.\tag{17}$$

Equation (17) has the required symmetry and form of a Hall effect expression. Aside from the questionable formulation, the main objection is that the estimated effect is too small to explain the Hall effect except perhaps at very low temperatures. If we take into consideration the fact that the *d*-electrons carry only a small fraction of the primary current in most ferromagnetic materials, the predicted Hall effect would be still smaller than the right-hand side of Eq. (17) by a factor of (σ_d/σ) . Furthermore, there is no suggestion of the observed temperature dependence in this theory.

Rudnitsky's theory can be interpreted as a classical calculation of the interaction of the spin of an electron with the orbits of other electrons. Presumably the coupling of the spin of an electron with its own orbit should be a much greater effect. Several investigators have attempted to calculate the Hall effect due to this.^{14,17}

The spin-orbit interaction energy of an electron can be written in the form²⁸

$$U_{so} = -(1/2\mathrm{mc})[(\mathbf{u} \times \mathbf{E}) \cdot \mathbf{p}], \qquad (18)$$

where \mathbf{y} =magnetic moment of the electron under consideration, \mathbf{p} =momentum of the electron, \mathbf{E} =electric field due to all the other charges. In a one-electron Hartree type approximation U_{so} may be regarded as a perturbation term. It is similar in form to the perturbation term $(e/\text{mc})(\mathbf{A} \cdot \mathbf{p})$ that accounts for the Hall effect in the usual theory²⁹ for a constant field \mathbf{H} =curl \mathbf{A} . For a rough calculation we consider the effective field due to spin-orbit interaction to be

$$\mathbf{H}_{so} = \operatorname{curl} - (\mathbf{y} \times \mathbf{E})/2e = (1/2e) [(\mathbf{y} \cdot \nabla)\mathbf{E} - \mathbf{y}(\nabla \cdot \mathbf{E})]. \quad (19)$$

An order of magnitude estimate is obtained by regarding the magnetic moment and velocity of the electron under consideration as constants of the motion and \mathbf{H}_{so} as an operator for which the average value is

$$\mathbf{H}_{\rm eff} = \int \varphi_k^* \mathbf{H}_{so} \varphi_k d\tau.$$
 (20)

 φ_k is the unperturbed Bloch wave function for the electron under consideration in a crystal lattice. We shall consider a unit volume over which φ_k is normalized.

For a perfect lattice $|\varphi_k|^2$ and **E** should have crystal periodicity making possible the reduction of Eq. (20) to an integral over a single atomic polyhedron. Any polyhedron may be approximated by an inscribed sphere of radius r_0 and volume $\tau_0 = 1/N$ where N is the number of atoms per unit volume. In the atomic polyhedron we assume that φ_k has the symmetry of an *s*- or *d*-function and $\mathbf{E} \cong E(r)\mathbf{r}/r$ is approximately spherically symmetric. With these assumptions Eq. (20) can be reduced to

$$\mathbf{H}_{\rm eff} = -\left(4\pi N \mathbf{y}/3e\right) \int |\varphi_k|^2 \rho d\tau_0, \qquad (21)$$

where ρ is the charge density define by $\nabla \cdot E = 4\pi\rho$, which is due to the nuclei and all other electrons except the one under consideration. Equation (21) is essentially the same as the result obtained by Konkov and Samolovich.¹⁶

In the limiting case where the electron under consideration is free, $|\varphi_k|^2 = 1$, $N \int \rho d\tau_0 = e$, so that the result due only to the missing charge under consideration is $H_{\text{eff}} \simeq -(4\pi/3)\mu$ which is vanishingly small.

A very much larger result can be obtained when the electron under consideration is not free. We shall consider the case of a *d*-electron and assume that near the center of the polyhedron the wave function approaches the atomic wave function which vanishes at the center. This is a characteristic property of the *d*-state. Accordingly, the charge density ρ to be considered in evaluating Eq. (21) is only that due to the Z electrons outside the nucleus. For any polyhedron $\int \rho d\tau_0 = -Ze$, omitting the nuclear charge and neglecting the charge of the electron under consideration. Therefore

$$\int |\varphi_k|^2 \rho d\tau_0 = -Ze |R_k(r_1)|^2,$$

where $R_k(r)$ is the radial part of φ_k and r_1 is some value of r between 0 and r_0 , the radius of the inscribed sphere of the polyhedron. Since $N\mathbf{y}$ is of the order of magnitude of the magnetization \mathbf{M} , the effective field is of the form

$$\mathbf{H}_{\rm eff} = 4\pi \alpha \mathbf{M},$$

with

$$\alpha = \frac{1}{3}Z |R_k(r_1)|^2.$$

(22)

For the ferromagnetic elements Z is about 26, and it is

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²⁸ L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1949), Chapter XII.
²⁹ H. Jones and C. Zener, Proc. Roy. Soc. (London) A144, 101

^{(1934).}

reasonable to suppose that $|R_k(r_1)|^2$ would be greater than the free electron value of unity so that a value of α might be obtained that is much greater than unity. According to this rough calculation, the direction of effective field for an electron depends on the spin orientation. Therefore a net effect would only be expected if there is a net polarization of the conduction electrons.

A Hartree type calculation such as we have outlined would give too large a result for the effective field because it neglects correlations of the electrons. The probability of finding any other electron close to the electron under consideration should be very small. It is especially improbable to find an electron of the same spin close to the electron under consideration. Kondo¹⁴ attempted to correct for correlation of electrons by introducing "Fermi holes" around electrons where it would be improbable to find any other electron. From a numerical calculation he estimated $\alpha_d \cong 14$ and $\alpha_d \cong 28$ for d-electrons in Fe and Ni, respectively. His calculations show that these results are extremely sensitive to the choice of wave functions and therefore should not be taken too seriously except perhaps as order of magnitude estimates.

Calculations to date have only been attempted for a perfect lattice with all spins oriented consistent with the Pauli principle and the observed magnetization. The temperature dependence observed for α has not yet been explained. It does not seem likely that the in-

fluence of lattice vibrations on charge distribution can account for this. A possible explanation of the temperature dependence may be in the spin dependence of the correlations of electrons. The correlation of electrons when there is a net polarization of electron spins at low temperatures should be substantially different from the correlation near the Curie point when the net polarization becomes very small. It will be necessary to improve considerably the semiclassical calculations made to date in order to investigate such effects.

5. CONCLUSIONS

The ordinary process for Hall effect accounts for only a small part of the observed Hall voltage in ferromagnetic materials. A satisfactory physical basis has not yet been established for the large Hall effect produced by the magnetization. Of the proposals advanced to account for this, only the spin-orbit coupling of conduction electrons appears to offer any possibility.

Since the situation is simpler at low temperatures from a theoretical point of view, we are of the opinion that at present, the most profitable approach to the problem is to study Hall effect in ferromagnetic materials at low temperatures where experimental data is nonexistent except in the case of Ni.

We wish to express our appreciation to Dr. A. I. Schindler and Dr. Simon Foner for permission to present their measurements and to Dr. J. E. Goldman for discussions and criticism of this paper.