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Tables of Nuclear Shell Structure

P. F. A. KLINKENBERG

Zeeman-Laboratory of the University of Amsterdam, The Netherlands

I. INTRODUCTION AND DESCRIPTION OF THE TABLES

THE idea underlying the tables presented in this paper is the jj -coupling model of nuclear structure proposed by Goeppert Mayer¹ and by Haxel, Jensen, and Suess.² This is applied to the ground states of the odd mass nuclei and the odd-odd nuclei. Even-even nuclei have not been included.

Table I, showing the odd mass nuclei, consists of two parts. On the left page the *odd proton nuclei* are listed and on the right page the *odd neutron nuclei*. For the first only the structure of the system of protons is given and for the latter only the structure of the neutron system. The arrangement is such that the numbers of protons to the left and of neutrons to the right, occurring on one line, are the same. The successive columns in each half of Table I show:

1. N , the number of neutrons;
2. Z , the number of protons;
3. The chemical symbol;
4. A , the atomic mass number; this is followed by an asterisk * for radioactive nuclei;
5. I , the nuclear spin, i.e., the mechanical moment of the nucleus in its normal state, expressed in units \hbar ;
6. μ , the corresponding magnetic dipole moment, expressed in nuclear magnetons $e\hbar/2M_p c$ (where M_p is the proton mass);
7. Q , the corresponding electric quadrupole moment, defined by $\int \rho(3z^2 - r^2)d\tau$ (where ρ stands for the nuclear charge density in the state $m_I = I$), expressed in units $e \times 10^{-24}$ cm²;
8. For odd proton nuclei the occupation of the proton shells and for odd neutron nuclei that of the neutron shells;

9. The ground state of the nucleus according to the shell model.

For the designation of the shells in column 8 the spectroscopic notation is used so that $n-l-1$ is the number of radial nodes of the wave function connected with the state in question. In order to facilitate comparison of protonic and neutronic structures the sequence of the nuclear shells $1s, 2p_{3/2}, 2p_{1/2}, 3d_{5/2}, 2s, 3d_{3/2} \dots$ has been taken the same for both types of odd mass nuclei. Actually there are a few divergences in the order of filling which will be discussed in Sec. II.

It is assumed that the resulting ground state of an odd mass nucleus is essentially the state of the odd nucleon, except for Na²³ and Mn⁵⁵, whose empirical ground levels can be understood only by taking into account *all* protons outside the closed shells.¹ The theoretical explanation of these cases³ suggests that they are the only exceptions of the kind among the odd proton nuclei. An analogous situation might exist in one or two odd neutron nuclei. On the whole, because of the lack of experimental data, the structures given for the odd neutron nuclei are more speculative. In those cases where the state derived from the shell model is definitely at variance with the experimental data concerning I or μ , the sign # is entered in the column headed "Notes."

Table II, comprising the odd-odd nuclei, has been arranged similarly to Table I on the understanding that here, for obvious reasons, both the structures of the proton system (P) and that of the neutron system (N) have been given. The ground state is designated as $(j_1 j_2)_I$, where j_1 refers to the odd proton and j_2 to the odd neutron. The corresponding orbital quantum numbers can be read from the given configurations of protons and neutrons.

Configurations of systems containing an *even* number of like nucleons can be derived easily, and in most cases

¹ Maria Goeppert Mayer, Phys. Rev. **75**, 1969 (1949); **78**, 16 (1950).

² Haxel, Jensen, and Suess, Phys. Rev. **75**, 1766 (1949); Naturwiss. **35**, 376 (1948); **36**, 153, 155 (1949); Z. Physik **128**, 295 (1950).

³ D. Kurath, Phys. Rev. **80**, 98 (1950); I. Talmi, Phys. Rev. **82**, 101 (1950).

unambiguously, by comparing with the configurations belonging to the adjacent odd numbers of nucleons to be found in Table I. However, one should bear in mind that the pairing energy favors the occupation of higher angular momentum.¹

The greater part of the observational material included in the tables has been borrowed from Mack's Table of Nuclear Moments, January, 1950; and for the corresponding literature the reader is referred to the bibliography in that article.⁴ New or improved data which have become available since have been inserted and are designated by a dagger (†) in the column headed "Notes." The corresponding original literature is given in the footnotes.

Unlike Mack's compilation the present tables have not been intended in the first place to give the most accurate values of μ and Q . Accordingly, the author does not want to discuss the degree of accuracy of the various methods of measuring the magnetic moment of the proton in terms of the nuclear magneton⁵ nor the reliability of the various approximations for the diamagnetic correction.⁶ Indeed, the discovery of the influence of the surroundings upon the frequency of a resonating nucleus appears to have reduced the question of diamagnetic shielding in free atoms to a rather academic one. In order to bring the new experimental data in these tables on the same basis as in Mack's table, the author preferred to use the same value of the proton moment (2.79255 nm) and the same values for the diamagnetic correction, which are based on Lamb's formula. With these restrictions the data given for μ are believed to be the most reliable at the moment (November, 1951).

When the μ -value quoted has been obtained from measurements of the hyperfine structure of an electronic state (as contrasted with the methods of direct nuclear resonance or induction), the symbol "Hfs" is entered in the last column, followed by "(opt.*)" if the hyperfine structure measurement is an optical one. As regards the accuracy of the μ -values obtained, the optical method cannot compete with radiofrequency methods, not only because of less accuracy in the optical measurement itself, but still more as a result of the theoretical uncertainties involved in the evaluation of the interaction between the electrons and the nucleus.

Recently, the original formula of Goudsmit-Fermi-Segrè, derived by considering the nucleus as a magnetic point-dipole, has been corrected for two effects due to the spatial extension of the nucleus. The greater of the two is the effect upon the electronic wave functions inside the nucleus exerted by the uniform electric charge distribution. This effect was dealt with by

Crawford and Schawlow⁷ and results in a decrease of the electron probability density at the center. Correspondingly, the magnetic moment as derived by applying the formula of Goudsmit-Fermi-Segrè has to be increased several percent for heavy nuclei. The second effect gives a smaller correction in the same sense. It is due to the (presumed) uniform distribution of the magnetic dipole moment over the nuclear volume.⁸ The application of the corrections for finite nuclear size considerably improves the g -values of heavy nuclei obtained from optical hyperfine structure measurements. This has been shown by comparing them with the values obtained on the method of nuclear induction. Therefore all μ -values in the tables which have been derived from optical hyperfine structure have been corrected for finite nuclear size. The total correction is 2.1 percent for Tc, 5.3 percent for the Nd-isotopes, 5.9 percent for Sm-isotopes, 6.2 percent for the Eu-isotopes, 11 percent for Lu, 12 percent for Ta, 14 percent for Os, and 16 percent for Au.

II. NUCLEAR SHELL SCHEMES

In Fig. 1 we have tried to give account of the ground states of the nuclei by means of level schemes for the proton and the neutron systems.

Up to Z or $N=50$ there is a close analogy between the structures of odd proton and odd neutron nuclei. The differences between the heavier nuclei of both kinds can be described partly in a general way by assuming that in odd neutron nuclei the states corresponding to circular orbits in the classical sense ($4f$, $5g$, $6h$, $7i$) are slightly lifted with respect to the other states as compared with the odd proton nuclei. It might be better to say that the circular orbit states are relatively depressed in the odd proton nuclei, because this can be understood as a result of the Coulomb repulsion pushing the protons into orbits without radial nodes. As a result the order of the levels $5g_{7/2}$, $4d_{5/2}$ and $6h_{9/2}$, $5f_{7/2}$ is different for protons and for neutrons.

The *first* reversal explains the occurrence of the $4d_{5/2}$ -orbit in Zr⁹¹, Mo⁹⁵, and Mo⁹⁷, whereas in the corresponding odd proton nuclei $5g_{7/2}$ is lower, except in I¹²⁷. That the appearance of $4d_{5/2}$ in I¹²⁷ is an accidental phenomenon is demonstrated by the $d_{5/2}$ -character of Eu¹⁵¹ and the spin value 5/2 of Pr¹⁴¹; obviously the $4d_{5/2}$ -state is not occupied in the sequence I¹²⁹—La¹³⁹. It is not impossible that in the odd neutron nuclei the $5g_{7/2}$ -state is filled in *pairs* so that we should have the structures: Mo⁹⁵—2 1; Mo⁹⁷—4 1; Ru⁹⁹—4 1; Ru¹⁰¹—6 1, and Pd¹⁰⁵—6 3 instead of the configurations given in the table. In that case all these nuclei would possess a $d_{5/2}$ ground state. In this connection it would be important to determine the mo-

⁴ J. E. Mack, Revs. Modern Phys. **22**, 64 (1950).

⁵ New determinations since the publication of Mack's article are to be found in C. D. Jeffreys, Phys. Rev. **81**, 1040 (1951); Sommer, Thomas, and Hipple, Phys. Rev. **82**, 697 (1951).

⁶ A table of diamagnetic corrections for all atoms, in accordance with the method of the self-consistent field, is to be found in W. C. Dickinson, Phys. Rev. **80**, 563 (1950).

⁷ M. F. Crawford and A. L. Schawlow, Phys. Rev. **76**, 1310 (1949).

⁸ F. Bitter, Phys. Rev. **76**, 150 (1949); A. Bohr and V. Weisskopf, Phys. Rev. **77**, 94 (1950).

menta of Ru^{101} and Pd^{105} .* The strong competition between the $5g_{7/2}$ - and $4d_{5/2}$ -states in the odd proton structures is observed even after the filling of both. In Ho^{165} , Lu^{175} , and Ta^{181} a $5g_{7/2}$ -proton is taken off the closed orbit to supplement the $6h_{11/2}$ -protons to an even number, whereas the closed $4d_{5/2}$ -state remains unaffected. Immediately after, in $\text{Re}^{185,187}$, the contrary is the case.

The second reversal is demonstrated by the appearance of the $5f_{7/2}$ -orbit in Nd^{143} and Nd^{145} (respectively, 83 and 85 neutrons), as contrasted with Bi^{209} (83 protons) where the $6h_{9/2}$ -orbit is occupied instead. In the odd neutron nuclei the determination of the spin of Er^{167} to $7/2$ makes probable that $5f_{7/2}$ is not directly filled but that first $6h_{9/2}$ is filled in pairs starting from Nd^{145} , and only one neutron occurs in the $5f_{7/2}$ -state up to Gd^{157} . Then the whole sequence from Nd up to Er inclusive will be characterized by a $f_{7/2}$ ground level.

The relative depression of circular orbit states in odd proton nuclei gives also rise to a smaller distance between $5g_{9/2}$ and $3p_{1/2}$ for protons in Fig. 1. This might account for the fact that 3 of the 7 odd proton nuclei between $Z=40$ and 50 have not the anticipated $g_{9/2}$ ground state but $p_{1/2}$. Among the 4 odd neutron nuclei between $N=40$ and 50 there is only one showing the $p_{1/2}$ -state.

Above $N=60$ the odd neutron has the tendency to occupy states of low total momentum. This effect is conspicuous in the "premature" occupation of the $3s$ -state in the odd neutron nuclei Cd^{111} up to Xe^{129} inclusive. Remarkably enough the $3s$ -neutron disappears in Xe^{131} in favor of $4d_{3/2}$. In the odd proton nuclei the $3s$ -orbit occurs prematurely only in Tm^{169} ; after that it occurs at the place where it is expected according to the level scheme, viz., in the two Tl -isotopes. The same tendency of the odd neutron may be regarded responsible for the premature occupation of the $4p_{1/2}$ -orbit in Yb^{171} and heavier odd neutron nuclei. Again it may be better to speak of an upward shift of such levels in odd proton nuclei resulting from the Coulomb repulsion between the protons which disqualifies the states having an appreciable density in the central part of the nucleus. The "central elevation" effect used by various authors here returns in a weaker form.⁹

The values of I and μ for the two Sm isotopes which have been reported recently are at variance with the shell model. There are reasons to suppose that the experimental results are somewhat uncertain so that no attempt is made to reconcile the data given with the predictions of the model.

* Note added in proof, March, 1952: Since this has been written, a tentative determination of the momenta of Pd^{105} has been published by P. Brix and A. Steudel [Naturwiss. 38, 431 (1951)]. This tends to corroborate the alternative configurations given in the text, since the most probable value of I turns out to be $5/2$ and $\mu = -0.6$ mm.

⁹ W. M. Elasser, J. phys. et radium 5, 389 (1934); E. Feenberg and K. C. Hammack, Phys. Rev. 75, 1877 (1949).

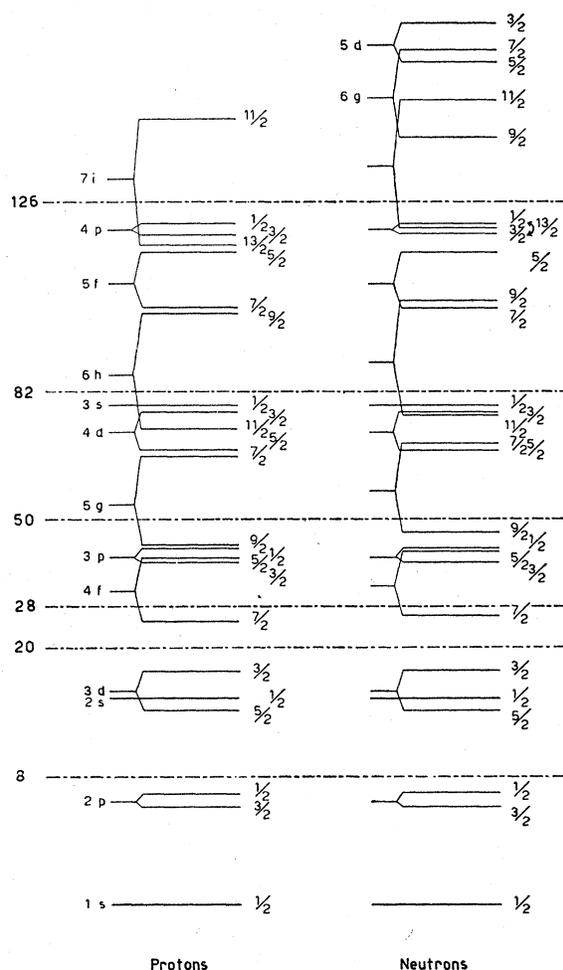


FIG. 1. Nuclear shell schemes.

A strange feature is the occupation of the $4p_{1/2}$ -orbit before that of the $4p_{3/2}$ -state. One could avoid this anomaly only by robbing an inner orbit of 4 neutrons in order to fill the $4p_{3/2}$ -state. But Hg^{201} is characterized by a $4p_{3/2}$ -neutron with all the foregoing states closed and no neutron in the $4p_{1/2}$ -state. This is confirmed by the fact that the $4p_{1/2}$ -neutron reappears in Pb^{207} . Especially the two Hg isotopes are demonstrative of a competition between $4p_{3/2}$ and $4p_{1/2}$. It might be supposed that the splitting of the $4p$ -state is very small and even in the opposite sense in the nuclei Yb^{171} up to Hg^{199} inclusive. In this connection it has been assumed that the two Hf isotopes have the ground level $p_{1/2}$.

Among the odd proton nuclei the interpretation of Ac^{227} and Pa^{231} in terms of the shell model is doubtful, while for Np^{237} the spin $5/2$ given in literature cannot be explained at all. On the contrary an $I=7/2$ would appear quite natural from the level scheme. The empirical evidence is not conclusive so that it cannot be stated that the discrepancy is real.

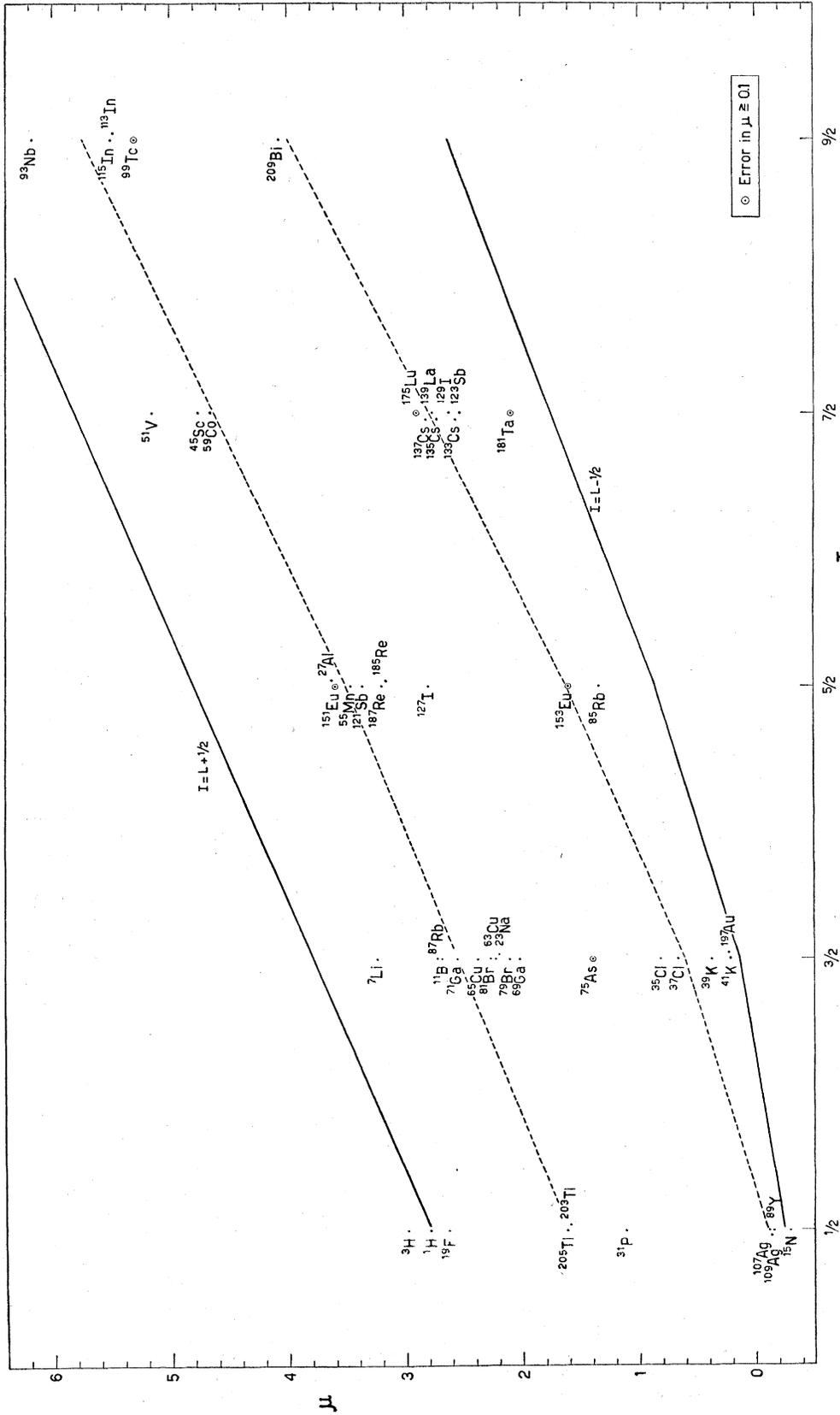


FIG. 2. μ - I -Plot of odd proton nuclei.

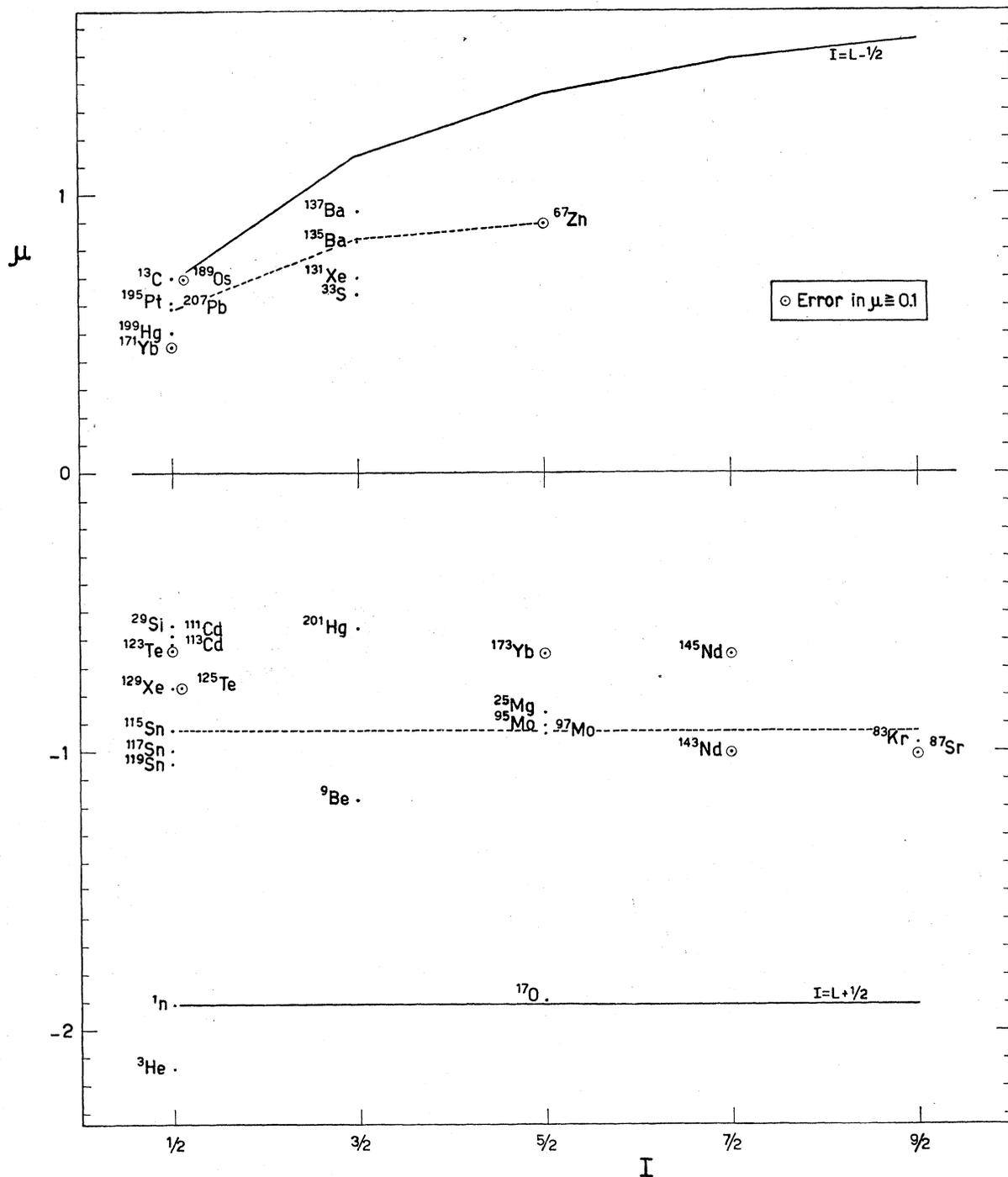


FIG. 3. $\mu-I$ -Plot of odd neutron nuclei.

III. ODD-ODD NUCLEI

No simple rule can be given to predict the ground level of an odd-odd nucleus from the states of the odd nucleons. The configurations of protons and neutrons have been chosen in closest similarity to the configurations in the corresponding odd proton and odd neutron

nuclei. This similarity is an equality in the cases H^2 , Li^6 , B^{10} , N^{14} , Cl^{36} , K^{40} , V^{50} , and La^{138} . In the case of Na^{22} and Na^{24} the protonic configuration of Na^{23} does not give the appropriate information. The simplest solution has been chosen. Then the magnetic moment for Na^{22} calculated on the assumption of jj -coupling between the

TABLE I. Odd nuclei.

N	Z	Atom	A	I (\hbar)	μ (nm)	Q ($e \times 10^{-24} \text{ cm}^2$)	A. Odd proton nuclei											Ground state	Notes	
							1s	2p _{3/2}	2p _{1/2}	3d _{5/2}	2s	3d _{3/2}	4f _{7/2}	3p _{3/2}	4f _{5/2}	3p _{1/2}	5g _{9/2}			5g _{7/2}
0	1	H	1	1/2	+2.79255 ±10		1											s _{1/2}		
2	1	H	3*	1/2	+2.978643 ±28		1											s _{1/2}		
4	3	Li	7	3/2	+3.25586 ±11	(+0.02) ±2	2	1										p _{3/2}		
6	5	B	11	3/2	+2.68858 ±28	+0.03 ±2	2	3										p _{3/2}		
8	7	N	15	1/2	-0.28299 ±3		2	4	1									p _{1/2}		
10	9	F	19	1/2	+2.6285 ±7		2	4	2	1								s _{1/2}		
12	11	Na	23	3/2	+2.21711 ±25						3							(d _{5/2}) _{3/2}		
14	13	Al	27	5/2	+3.6408 ±4	+0.156 ±3					5							d _{5/2}		
16	15	P	31	1/2	+1.31165 ±20						6	1						s _{1/2}		
18	17	Cl	35	3/2	+0.82191 ±22	-0.07894 ±2					6	2	1					d _{3/2}	†	
20	17	Cl	37	3/2	+0.68414 ±24	-0.06213 ±2					6	2	1					d _{3/2}	†	
20	19	K	39	3/2	+0.391 ±1						6	2	3					d _{3/2}	Hfs	
22	19	K	41	3/2	+0.215 ±1						6	2	3					d _{3/2}	Hfs	
24	21	Sc	45	7/2	+4.757 ±2						6	2	4	1				f _{7/2}	†	
28	23	V	51	7/2	+5.1478 ±5									3				f _{7/2}	†	
30	25	Mn	55	5/2	+3.4681 ±4				(20)					5				(f _{7/2}) _{5/2}		
32	27	Co	59	7/2	+4.6484 ±6									7				f _{7/2}		
34	29	Cu	63	3/2	+2.22617 ±36	-0.13 ±1								8	1			p _{3/2}	††	
36	29	Cu	65	3/2	+2.3845 ±4	-0.12 ±1									1			p _{3/2}	††	
38	31	Ga	69	3/2	+2.0167 ±11	+0.2318 ±23				(28)				3				p _{3/2}		
40	31	Ga	71	3/2	+2.5614 ±10	+0.1461 ±15								3				p _{3/2}		
42	33	As	75	3/2	+1.4387 ±3	+0.3 ±2								3	2			p _{3/2}	††	
44	35	Br	79	3/2	+2.10576 ±37	+0.26 ±8								3	4			p _{3/2}		
46	35	Br	81	3/2	+2.2696 ±5	+0.21 ±7								3	4			p _{3/2}		
48	37	Rb	85	5/2	+1.3532 ±4									4	5			f _{5/2}		
50	37	Rb	87*	3/2	+2.7501 ±5									3	6			p _{3/2}		
50	39	Y	89	1/2	-0.14									4	6	1		p _{1/2}	Hfs (opt.)	
52	41	Nb	93	9/2	+6.1659 ±32	~0								4	6	2	1	g _{9/2}		
56	43	Tc	99*	9/2	+5.3 ±5									4	6	2	3	g _{9/2}	Hfs† (opt.)	
58	45	Rh	103	1/2	(-)0.11									4	6	1	6	p _{1/2}	Hfs† (opt.)	
60	47	Ag	107	1/2	-0.111 ±8									4	6	1	8	p _{1/2}	Hfs† (opt.)	
62	47	Ag	109	1/2	-0.129 ±8									4	6	1	8	p _{1/2}	Hfs† (opt.)	
64	49	In	113	9/2	+5.486 ±3	+1.144								4	6	2	9	g _{9/2}	Hfs	
66	49	In	115*	9/2	+5.500 ±3	+1.161								4	6	2	9	g _{9/2}	Hfs	
70	51	Sb	121	5/2	+3.360	-0.3 ±2								4	6	2	10	d _{5/2}	†	
72	51	Sb	123	7/2	+2.547	-1.2 ±2											1	g _{7/2}	†	
74	53	I	127	5/2	+2.8090 ±4	-0.59 ±20				(50)							2	1	d _{5/2}	†
76	53	I	129*	7/2	(+)2.6181 ±3	-0.43 ±15											3		g _{7/2}	†
78	55	Cs	133	7/2	+2.5771 ±9	≤0.3											5		g _{7/2}	
80	55	Cs	135*	7/2	+2.7271 ±33												5		g _{7/2}	Hfs
82	55	Cs	137*	7/2	+2.8397 ±30												5		g _{7/2}	Hfs
82	57	La	139	7/2	+2.7760 ±28	≠0											7		g _{7/2}	
82	59	Pr	141	5/2													8	1	d _{5/2}	

Proton shells
1s 2p_{3/2} 2p_{1/2} 3d_{5/2} 2s 3d_{3/2} 4f_{7/2} 3p_{3/2} 4f_{5/2} 3p_{1/2} 5g_{9/2} 5g_{7/2} 4d_{5/2}
Ground state Notes

TABLE I. Odd nuclei (continued).

N	Z	Atom	A	I (\hbar)	μ (nm)	Q ($e \times 10^{-24}$ cm ²)	B. Odd neutron nuclei										Ground state	Notes	
							1s	2p _{3/2}	2p _{1/2}	3d _{3/2}	2s	Neutron shells							4d _{5/2}
							3d _{5/2}	4f _{7/2}	3p _{3/2}	4f _{5/2}	3p _{1/2}	5g _{9/2}	5g _{7/2}						
1	0	n	1*	1/2	-1.91280 ±9		1										s _{1/2}		
1	2	He	3	1/2	(-).2.127414 ±3		1										s _{1/2}		
5	4	Be	9	3/2	-1.1774 ±8	(0.02)	2	3									p _{3/2}	†	
7	6	C	13	1/2	+0.70225 ±14		2	4	1								p _{1/2}		
9	8	O	17	5/2	-1.8935 ±2	-0.005 ±2	2	4	2	1							d _{5/2}	†	
11	10	Ne	21	≥3/2	<0		(8)			3							d _{5/2}		
13	12	Mg	25	5/2	-0.8552 ±2					5							d _{5/2}	†	
15	14	Si	29	(1/2)	(-0.55538) ±4					6	1						s _{1/2}	††	
17	16	S	33	3/2	+0.6436 ±2	-0.08				6	2	1					d _{3/2}	†	
19	16	S	35*	3/2		+0.06				6	2	3					d _{3/2}		
23	20	Ca	43							6	2	4	3				f _{7/2}		
25	22	Ti	47				(20)					5					f _{7/2}		
27	22	Ti	49										7				f _{7/2}		
29	24	Cr	53	3/2	(-).0.45								8	1			p _{3/2}	††	
31	26	Fe	57		~0		(28)					3					p _{3/2}		
33	28	Ni	61		~0								3	2			p _{3/2}		
37	30	Zn	67	5/2	+0.9								4	5			f _{5/2}	Hfs (opt.)	
41	32	Ge	73	9/2		-0.2 ±1							4	6	2	1		g _{9/2}	†
43	34	Se	77	1/2									4	6	1	4		p _{1/2}	†
47	36	Kr	83	9/2	-0.9704	+0.15							4	6	2	7		g _{9/2}	
49	38	Sr	87	9/2	-1.1								4	6	2	9		g _{9/2}	Hfs (opt.)
51	40	Zr	91	5/2									4	6	2	10	1	d _{5/2}	
53	42	Mo	95	5/2	-0.9140 ±2		(50)					3					d _{5/2}	†	
55	42	Mo	97	5/2	-0.9332 ±1												5	d _{5/2}	†
55	44	Ru	99														5	d _{5/2}	
57	44	Ru	101													1	6	g _{7/2}	
59	46	Pd	105	(5/2)	(-0.6)											3	6	g _{7/2}	Hfs†† (opt.)

Neutron shells

1s 2p_{3/2} 2p_{1/2} 3d_{3/2} 2s 3d_{5/2} 4f_{7/2} 3p_{3/2} 4f_{5/2} 3p_{1/2} 5g_{9/2} 5g_{7/2} 4d_{5/2}

Ground state Notes

TABLE I. Odd nuclei (*continued*).

N	Z	Atom	A	I (h)	μ (nm)	Q ($e \times 10^{-24}$ cm ²)	A. Odd proton nuclei								Ground state	Notes
							5g _{7/2}	4d _{5/2}	6h _{11/2}	4d _{3/2}	Proton shells					
							3s	6h _{9/2}	5f _{7/2}	5f _{5/2}	7i _{13/2}	4p _{3/2}	4p _{1/2}			
86	61	Pm	147*				8	3							d _{5/2}	
88	63	Eu	151	5/2	+3.6	+1.2	8	5							d _{5/2}	Hfs (opt.)
90	63	Eu	153	5/2	+1.6	+2.5	8	5							d _{5/2}	Hfs (opt.) #
94	65	Tb	159	3/2			8	6	1						d _{3/2}	
98	67	Ho	165	7/2			7	6	4						g _{7/2}	
100	69	Tm	169	1/2			8	6	4	1					s _{1/2}	
104	71	Lu	175	7/2	+2.9 ±5	+5.9	7	6	8						g _{7/2}	
108	73	Ta	181	7/2	+2.1	+6	7	6	10						g _{7/2}	
110	75	Re	185	5/2	+3.1714 ±6	(+2.8)	8	5	12						d _{5/2}	†
112	75	Re	187*	5/2	+3.2039 ±6	+2.6	8	5	12						d _{5/2}	†
114	77	Ir	191	3/2	>0		8	6	12	1					d _{3/2}	†
116	77	Ir	193	3/2	>0		8	6	12	1					d _{3/2}	
118	79	Au	197	3/2	+0.23		8	6	12	3					d _{3/2}	Hfs (opt.)
122	81	Tl	203	1/2	+1.61166 ±14		8	6	12	4	1				s _{1/2}	
124	81	Tl	205	1/2	+1.62750 ±14		8	6	12	4	1				s _{1/2}	
126	83	Bi	209	9/2	+4.082 ±1	-0.4	8	6	12	4	2	1			h _{9/2}	†
(82)																
138	89	Ac	227*	3/2							6				p _{3/2}	†
140	91	Pa	231*	3/2							8				p _{3/2}	
144	93	Np	237*	5/2							10	1			f _{7/2}	#†

† A. Odd proton nuclei.

C^{135,87}: Q from V. Jaccarino and J. G. King, Phys. Rev. **83**, 471 (1951).
 Sc⁴⁵: μ from W. G. Proctor and F. C. Yu, Phys. Rev. **78**, 471 (1950).
 V⁵¹: sign of μ from W. G. Proctor and F. C. Yu, Phys. Rev. **81**, 20 (1951).
 Te⁹⁹: I and μ from K. G. Kessler and W. F. Meggers, Phys. Rev. **80**, 905 (1951) and Phys. Rev. **82**, 341 (1951).
 Rh¹⁰³: I and μ from H. Kuhn and K. G. Woodgate, Nature **166**, 906 (1950).
 Ag^{107,109}: μ from Brix, Kopfermann, Martin, and Walcher, Naturwiss. **38**, 68 (1951) and Z. Physik **130**, 88 (1951).
 Sb^{121,123}: μ from W. G. Proctor and F. C. Yu, Phys. Rev. **78**, 471 (1950).
 I^{127,129}: μ from Walchli, Livingston, and Hebert, Phys. Rev. **82**, 97 (1951).
 Re^{185,187}: μ from F. Alder and F. C. Yu, Phys. Rev. **82**, 105 (1951).
 Ir¹⁹¹: I from Brix, Kopfermann, and v. Siemens, Naturwiss. **37**, 397 (1950).
 Bi²⁰⁹: μ from W. G. Proctor and F. C. Yu, Phys. Rev. **78**, 471 (1950).
 Ac²²⁷: I from Tomkins, Fred, and Meggers, Phys. Rev. **84**, 168 (1951).
 Np²³⁷: I from F. S. Tomkins and M. Fred, J. Opt. Soc. Am. **39**, 357 (1949) ("very probable").
 †† Added in proof, March, 1952:
 Cu^{63,65}: Q from Bleaney, Bowers, and Ingram, Proc. Phys. Soc. (London) **A64**, 758 (1951).
 As⁷⁵: μ from S. S. Dharmatti and H. E. Weaver, Phys. Rev. **84**, 367 (1951).

† Footnotes for "B. Odd Neutron Nuclei" (continued from p. 71).

S³³: μ from S. S. Dharmatti and H. E. Weaver, Phys. Rev. **83**, 845 (1951).
 Ge⁷³: Q from J. M. Mays and C. H. Townes, Phys. Rev. **81**, 940 (1951).
 Se⁷⁷: I from S. P. Davis and F. A. Jenkins, Phys. Rev. **83**, 1269 (1951). This is in accordance with predictions from studies of radioactive decay (Jensen, Nichols, and Clement, Phys. Rev. **81**, 143 (1951); R. Canada and A. C. G. Mitchell, Phys. Rev. **81**, 485 (1951); and Phys. Rev. **83**, 955 (1951)). Herewith the confusion around this spin value (see Mack's article) finally is removed.
 Mo^{95,97}: μ from W. G. Proctor and F. C. Yu, Phys. Rev. **81**, 20 (1951).
 Xe¹²⁹: μ from W. G. Proctor and F. C. Yu, Phys. Rev. **78**, 471 (1950).
 Xe¹³¹: μ from μ (Xe¹²⁹) and the optical value of μ^{129}/μ^{131} ; Q from J. Koch and E. Rasmussen, Phys. Rev. **77**, 722 (1950).
 Nd^{143,145}: I from B. S. Bleaney, Physica **17**, 175 (1951); μ from K. Murakawa and J. S. Ross, Phys. Rev. **82**, 967 (1951).
 Sm^{147,149}: I and μ from K. Murakawa and J. S. Ross, Phys. Rev. **82**, 967 (1951).
 Er¹⁶⁷: I from B. S. Bleaney and H. E. D. Scovill, Proc. Phys. Soc. (London) **A64**, 204 (1951).
 W¹⁸³: I from H. Kopfermann and D. Meyer, Z. Physik **124**, 685 (1948) and G. R. Fowles, Phys. Rev. **78**, 744 (1950).
 Os¹⁸⁹: μ from S. Suwa, Phys. Rev. **83**, 1258 (1951).
 Hg²⁰¹: μ from μ (Hg¹⁹⁹) and the optical value of μ^{199}/μ^{201} .
 U²³⁵: I from G. L. Stukenbroeker and J. R. McNally, J. Opt. Soc. Am. **40**, 336 (1950); 5/2 is the more probable value; there is a large Q.
 †† Added in proof, March, 1952:
 Si²⁹: (ρ) measured = -1.10984 according to S. S. Dharmatti and H. E. Weaver, Phys. Rev. **84**, 843 (1951).
 Cr⁵³: I and μ from B. S. Bleaney and K. D. Bowers, Proc. Phys. Soc. (London) **A64**, 1135 (1951).
 Pd¹⁰⁵: I and μ from P. Brix and A. Steudel, Naturwiss. **38**, 431 (1951).
 Te^{123,125}: μ from S. S. Dharmatti and H. E. Weaver, Phys. Rev. **84**, 843 (1951).

TABLES OF NUCLEAR SHELL STRUCTURE

TABLE I. Odd nuclei (continued).

N	Z	Atom	A	I (ħ)	μ (nm)	Q (e × 10 ⁻²⁴ cm ²)	B. Odd neutron nuclei											Ground state	Notes					
							Neutron shells																	
							5g _{7/2}	4d _{5/2}	6h _{11/2}	4d _{3/2}	3s	6h _{9/2}	5f _{7/2}	5f _{5/2}	7i _{13/2}	4p _{3/2}	4p _{1/2}	6g _{9/2}	7i _{11/2}	5d _{5/2}				
63	48	Cd	111	1/2	-0.59492 ±8		6	6			1											s _{1/2}		
65	48	Cd	113	1/2	-0.62238 ±8		8	6			1												s _{1/2}	
65	50	Sn	115	1/2	-0.91779 ±10		8	6			1												s _{1/2}	
67	50	Sn	117	1/2	-0.99982 ±10		8	6	2		1												s _{1/2}	
69	50	Sn	119	1/2	-1.04600 ±10		8	6	4		1												s _{1/2}	
71	52	Te	123	1/2	-0.73579 ±4		8	6	6		1												s _{1/2}	††
73	52	Te	125	1/2	-0.88705 ±4		8	6	8		1												s _{1/2}	††
75	54	Xe	129	1/2	-0.777		8	6	10		1												s _{1/2}	†
77	54	Xe	131	3/2	+0.700	~-0.15	8	6	12	1													d _{3/2}	†
79	56	Ba	135	3/2	+0.8346 ±25		8	6	12	3													d _{3/2}	Hfs
81	56	Ba	137	3/2	+0.9351 ±27		8	6	12	3	2												d _{3/2}	Hfs
83	60	Nd	143	7/2	-1.0 ±2		8	6	12	4	2		1										f _{7/2}	Hfs† (opt.)
85	60	Nd	145	7/2	-0.65 ±9		(82)					2	1										f _{7/2}	Hfs† (opt.)
85	62	Sm	147*	(5/2)	(-0.32) ±5							2	1										f _{7/2}	Hfs† (opt.)#
87	62	Sm	149	(5/2)	(-0.26) ±4							4	1										f _{7/2}	Hfs† (opt.)#
91	64	Gd	155									8	1										f _{7/2}	
93	64	Gd	157									10	1										f _{7/2}	
95	66	Dy	161									10	3										f _{7/2}	
97	66	Dy	163									10	5										f _{7/2}	
99	68	Er	167	7/2								10	7										f _{7/2}	†
101	70	Yb	171	1/2	+0.45							10	8					1					p _{1/2}	Hfs (opt.)
103	70	Yb	173	5/2	-0.65	+3.9						10	8	3									f _{5/2}	Hfs (opt.)#
105	72	Hf	177	(1/2, 3/2)								10	8	4				1					p _{1/2}	
107	72	Hf	179	(1/2, 3/2)								10	8	6				1					p _{1/2}	
109	74	W	183	1/2								10	8	6	2			1					p _{1/2}	†
111	76	Os	187									10	8	6	4			1					p _{1/2}	
113	76	Os	189	1/2	+0.7 ±1							10	8	6	6			1					p _{1/2}	†
117	78	Pt	195	1/2	+0.60592 ±8							10	8	6	10			1					p _{1/2}	
119	80	Hg	199	1/2	+0.50413 ±3							10	8	6	12			1					p _{1/2}	
121	80	Hg	201	3/2	-0.5590 ±1	+0.5						10	8	6	14	1							p _{3/2}	†
125	82	Pb	207	1/2	+0.58950 ±7							10	8	6	14	4		1					p _{1/2}	
143	92	U	235*	5/2,7/2								10	8	6	14	4	2	10	6	1			d _{5/2}	†

(126)

† B. Odd Neutron nuclei.
 Be⁹: I from N. A. Schuster and G. E. Pake, Phys. Rev. 81, 886 (1951) and from Hatton, Rollin, and Seymour, Phys. Rev. 83, 672 (1951); Q from the latter; sign of μ from F. Alder and F. C. Yu, Phys. Rev. 82, 105 (1951).
 O¹⁷: I and μ from F. Alder and F. C. Yu, Phys. Rev. 81, 1067 (1951); Q from Geschwind, Gunther-Mohr, and Townes, Phys. Rev. 83, 209 (1951).
 Mg²⁵: μ from F. Alder and F. C. Yu, Phys. Rev. 82, 105 (1951).
 Si²⁹: (gI)_{measured} = (-)1.11 according to Hatton, Rollin, and Seymour, Phys. Rev. 83, 672 (1951).
 Footnotes continued on opposite page.

TABLE II. Odd-odd nuclei.

N	Z	Atom	A	I (h)	μ (nm)	Q ($e \times 10^{-24}$ cm ²)	Shells										Ground state	Notes						
							1s	2p _{3/2}	2p _{1/2}	3d _{5/2}	2s	3d _{3/2}	4f _{7/2}	3p _{3/2}	4f _{5/2}	3p _{1/2}			5g _{9/2}	5g _{7/2}	4d _{5/2}	6h _{11/2}	4d _{3/2}	3s
1	1	H	2	1	+0.857354 ± 9	+0.00273 ± 5	{ P N	1														(1/2, 1/2) ₁		
3	3	Li	6	1	+0.82189 ± 4	< 9.10 ⁻⁴	{ P N	2	1													(3/2, 3/2) ₁	Hfs	
5	5	B	10	3	+1.8004 ± 7	+0.06 ± 4	{ P N	2	3													(3/2, 3/2) ₃		
7	7	N	14	1	+0.40365 ± 3	+0.02	{ P N	2	4	1												(1/2, 1/2) ₁		
11	11	Na	22*	3	+1.74582 ± 22		{ P N	2	4	2	1	2										(5/2, 5/2) ₃	Hfs	
13	11	Na	24*	4			{ P N	2	4	2	1	2										(5/2, 5/2) ₄	Hfsf	
19	17	Cl	36*	2		-0.0168 ± 1	{ P N	6	2	1												(3/2, 3/2) ₂	†	
21	19	K	40*	4	-1.291 ± 4		{ P N	6	2	3	4	1										(3/2, 7/2) ₄	Hfs	
27	23	V	50				{ P N	(20)														(7/2, 7/2)		
49	37	Rb	86*	2	(-1.68) ± 1		{ P N	8	4	5	2	9										(5/2, 9/2) ₂	Hfsf	
81	57	La	138*				{ P N	(28)														(7/2, 3/2)		
105	71	Lu	176*	≥ 7	+4.2 ± 8	+7 ± 1	{ P N	8	6	12	4	2	10	8	6	12	4	2	10	8	4	1	(11/2, 13/2) ₁₀	Hfs (opt.)

Na*: I from K. F. Smith, Nature 167, 942 (1951).
 Cl*: Q from Johnson, Gordy, and Livingston, Phys. Rev. 83, 1249 (1951).
 Rb*: I and μ from E. H. Bellamy, Nature 168, 556 (1951). He gives $\mu = \pm(1.66 \pm 0.4)$ and in case the negative sign is valid (which is probable) $\mu = -1.68 \pm 0.01$.

odd proton and the odd neutron is in close agreement with the experimental value. Except for the lightest nuclei H^2 and Li^6 the jj -coupling is the best approximation. This is to be expected particularly when the odd proton and the odd neutron are in nonequivalent orbits,¹⁰ i.e., in K^{40} , Rb^{86} , La^{138} , and Lu^{176} .

For Rb^{86} the structure of the proton system has been chosen equal to that of Rb^{85} , and the structure of the neutron system (49 neutrons) is that of Sr^{87} . The observed spin is then obtained by placing j_1 and j_2 in opposite directions. On the assumption of jj -coupling the computed magnetic moment agrees approximately with the observed one, provided the sign is negative, as the observer considers probable. Though this case is not so crucial with regard to the coupling type as is the case of K^{40} , it seems worth mentioning that LS-coupling does not lead to an acceptable value for the magnetic moment of Rb^{86} . The best LS-level is 3P_2 giving $\mu = -0.62$; jj -coupling gives -2.12 , while the observed value is $(-)$ 1.68. A calculation in jj -coupling, using g -factors of proton and neutron which correspond to the empirical Schmidt-curves¹¹ in the $\mu-I$ -plots of Fig. 2 and 3 improves the predicted μ to -2.00 . The proton and neutron states of Rb^{86} given in Table II are the only possible ones with regard to β -radioactivity data which demand odd parity for its ground state.

The case of Lu^{176} (71 protons, 105 neutrons) is particularly interesting because of the exceptionally high spin. This cannot be explained by combining the proton system of Lu^{176} and the neutron structure of Hf^{177} (105 neutrons). By computing the magnetic moments for all combinations of proton and neutron orbits which are allowed in the shell model, it has been shown that the

odd proton is in the $6h_{11/2}$ - and the odd neutron in the $7i_{13/2}$ -state.¹¹ The most probable value of the spin is 10, but 9 or 11 cannot be excluded. The result would not be affected by departures from jj -coupling.

In Table I it has been assumed that the $7i_{13/2}$ -orbit is occupied for the first time at $N=109$ (by one pair of neutrons), after closing of the $5f_{5/2}$ -state. The appearance of the $7i_{13/2}$ -neutron in the ground state of Lu^{176} might mean that $7i_{13/2}$ is lower than $5f_{5/2}$. In that case the filling of $7i_{13/2}$ (in pairs) would start at Yb^{173} . However, this does not throw light on the existing discrepancy in the magnetic moment of that nucleus since the configuration $(i_{13/2})^2f_{5/2}$ would certainly yield a $f_{5/2}$ ground state. Unless the discrepancy is due to observational errors, the anomaly may be associated with the extraordinarily large quadruple moment. So the original interpretation has been maintained.

The unknown magnetic moments of Na^{24} and Cl^{36} can be calculated approximately from the configurations given in Table II. They are much less dependent on the coupling type than those of K^{40} and Rb^{86} . For Na^{24} the predicted value of μ is $+2.2$ and for Cl^{36} $+0.9$ nm.

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¹⁰ E. Feenberg, Phys. Rev. **76**, 1275 (1949).

¹¹ P. F. A. Klinkenberg, Physica **17**, 715 (1951).