

The Second Coefficient of Viscosity of Liquids and Gases

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The classical theory of the dynamics of viscous fluids is based on the assumption that there is only one fundamental coefficient of viscosity, μ , the coefficient of shear viscosity. The other quantity, μ' , the second coefficient of viscosity, is assumed to be equal to $-2\mu/3$ in order that $\kappa(=(2\mu+3\mu')/3)$, the coefficient of bulk viscosity, should be zero. In making this assumption classical hydrodynamics parts company from classical elasticity, in which two fundamental quantities, the Lamé constants λ and μ , are introduced.

The above assumption seems to have some basis in theory only in the case of ideal monatomic gases; it has, however, been carried over implicitly to both liquids and gases of all degrees of complexity. One might therefore expect some differences to exist between theoretical predictions and experimental results due to this oversimplification. The present review has been undertaken with a view to exploring the existence and the magnitude of any such differences.

Up to the present, discrepancies have been noticed in the field of investigation which deals with the transmission of sound energy through liquid and gases.

As far as liquids are concerned no work has been done on the absorption of energy associated with vibrations in the sonic range (20 to 20,000 vibrations per second). In the ultrasonic range experimental values of absorption differ from the theoretical ones by factors ranging from 3 to 1000. In gases, both in the sonic and supersonic ranges, the values differ by a factor whose magnitude lies in the range of 4 to 100.

Most of the work which has been done in this field has been concerned with the passage through liquids of sound energy in the ultrasonic range of frequencies. Here it is clear that the value of μ'/μ deduced from experiment is in good agreement with the value of the ratio deduced from information on excess absorption of energy. The viscosity ratio is never negative; its value ranges from about 1 to about 120 in the cases so far investigated.

I. INTRODUCTION

THE passage of sound energy, the energy associated with longitudinal vibrations, both in the audible range (20 to 20,000 vibrations per second) as well as in the supersonic or ultrasonic range (20,000 vps and upwards) through any medium, is always accompanied by an absorption of some of the energy. Until recently, this absorption has been attributed to three main causes:—

(1) Viscosity: When different layers of a medium are in relative motion, the internal forces opposing this motion must be overcome, and this means that mechanical energy must be converted into heat energy.

(2) Heat conduction: Dissipation of energy takes place when heat is conducted from places of compression to those of rarefaction.

(3) Heat radiation: Dissipation of energy takes place

TABLE I. Gases at ultrasonic frequencies. Measured and calculated values of coefficient of sound absorption in different gases. (Bergmann (2)).

Gas	Author	Frequency kc/sec f	$(\alpha/f^2) \times 10^{-13} \text{ cm}^{-1} \text{ sec}^2$		Refer- ence
			Measured	Calcu- lated	
Air	Neklepajev	132 to 415	2.94 to 3.99	1.45	(38)
	Pielemeier	1158 to 1408	1.67 to 1.99	1.45	(40)
	Grossman	178	2.72	1.45	(20)
Oxygen	Pielemeier	655 to 1219	3.47 to 1.90	1.78	(40)
	Abello	612	46.5	1.60	(8)
Carbon dioxide	Grossman	64	277	1.60	(20)
		99	540	1.60	
		178	240	1.60	
Argon	Abello	612	0	2.0	(8)

because of radiation of heat from compressions to rarefactions.

Additional possible causes put forward in recent times are referred to later in this paper.

It has been found experimentally (Table II), in agreement with theory, that the absorption resulting from cause (2) is very small; Parthasarthy and others (94)* have been able to show by actual experimental determination that it is only approximately 1 part in 20 of the total absorption; consequently the effect of cause (3) is smaller still. Thus, only the effects of viscosity need be considered.

II. ABSORPTION OF ENERGY IN GASES

(a) Sonic Frequencies

In 1845 Stokes (54) deduced a formula for the coefficient of absorption of acoustic energy in viscous fluids due to the effect of viscosity, namely,

$$\alpha = 2\omega^2\mu/3\rho a^3, \quad (1)$$

where α = absorption coefficient as defined in Eq. (2) below; $\omega = 2\pi$ times the frequency, f ; ρ = the density of the medium; a = the velocity of propagation of the waves, and μ = the coefficient of shear viscosity. The quantity α arises in the formula,

$$J_x = J_0 e^{-2\alpha x}, \quad (2)$$

where the intensity of a plane sound wave decreases from J_0 to J_x while traversing a distance x .

* See bibliography at end of paper.

TABLE II. Liquids at ultrasonic frequencies. Absorption coefficients ($\times 10^{17}$) of sound, as calculated for different liquids. (Biquard) Values from Bergmann (2). (α_r/f^2 represents absorption resulting from viscosity, α_1/f^2 represents absorption resulting from heat conduction, and α/f^2 represents total absorption.)

Liquid	α_r/f^2	$\times 10^{17}$ α_1/f^2	α/f^2
Acetone	6.54	0.5	7.04
Ether	8.48	0.49	8.97
Benzol	8.36	0.3	8.66
Chloroform	10.045	0.057	10.1
Ethyl acetate	7.95	0.31	8.26
Methyl acetate	6.34	0.44	6.78
Meta xylol	8.13	0.24	8.37
Toluol	7.56	0.28	7.84
Water	8.5	0.0064	8.5

In 1898, Duff (15) found that the absorption coefficient of sound in air was $4.2 \times 10^{-5} \text{ cm}^{-1}$ when the wave frequency was 6800 vps. The calculated value was found to be $0.62 \times 10^{-5} \text{ cm}^{-1}$. That is, the experimental value was larger than the calculated one and it was therefore suggested that the difference between the two was due to heat radiation. Rayleigh (44) however, in calculating the effect of heat radiation arrived at a value of $1.2 \times 10^{-7} \text{ cm}^{-1}$, which is much less than the difference $3.6 \times 10^{-5} \text{ cm}^{-1}$. In discussing this discrepancy he remarked that some other factor must be operating and that Duff's result was very high. Duff, however, later confirmed his value when repeating the experiment (15).

Hart (21) obtained a value of $1.4 \times 10^{-3} \text{ cm}^{-1}$, which shows a still higher divergence; part of the reason for this was that his determinations were carried out rather near to the source of sound, where a steady state had not been established.

The earlier work of Duff and Hart was substantiated by later workers, who confirmed that the calculated values were much smaller than those obtained experimentally. Rich and Pielemeier (45) were able to show that some of the discrepancy was probably due to the presence of impurities in the air. They showed that the addition of carbon dioxide increased the absorption considerably for all frequencies. They did not find a similar divergence when small amounts of air were introduced as impurities into pure carbon dioxide.

In 1931, Knudsen (31a) was able to show that the presence of small amounts of water vapor in air affected the absorption considerably. Gemant (18) explained this fact on Kneser's (30) theory of relaxation time by saying that the absorption in air is reduced by humidity; the water molecules reduce the relaxation time of the air molecules and shift the maximum absorption to higher frequencies.

However, the effect of impurities has not been properly clarified. Some impurities affect the absorption, while others do not. For example, water vapor in carbon dioxide, to the extent of only 0.2 percent, affects the absorption considerably, while the presence of H_2S as an impurity in air seems to have little or no effect. Detailed

measurements have been done by Van Itterbeck and Mariens (56). Theoretical work on the subject has been published by Kellmann and London (28).

Up to the present no work seems to have been done on the absorption of sound in liquids at sonic frequencies.

(b) Ultrasonic Frequencies

Turning to sound energy of higher frequencies, Lebedew (35) derived another relationship for the absorption of energy by taking into account all three causes of dissipation mentioned in section I above. He obtained an expression which was effective only at high frequencies and which contained a term involving dissipation within the molecule. Experiments were performed by Neklepajev (38) at a wavelength of about 1 mm, corresponding to a frequency of about 340 kc/sec; the experimental value for the absorption coefficient which he obtained was twice the value calculated on the basis of the Lebedew formula. This indicates that Stokes' formula would lead to an even greater discrepancy.

Rich and Pielemeier (45) obtained experimental values lying between those of Neklepajev and the theoretical one of Lebedew.

High absorption was noticed in carbon dioxide by Pierce (41) and by Abello (8). Table I gives the results of various investigators.

III. ABSORPTION OF ENERGY IN LIQUIDS

If we consider the absorption of sound energy in liquids, we find that the case is not very different from that of gases; the discrepancy between theory and experiment is, however, greater.

Biquard (62) calculated the coefficient of sound absorption for different liquids on the basis of the Stokes formula and also carried out experimental determinations at ultrasonic frequencies. Results are given in Tables II and III.

It is very difficult to measure the attenuation of sound in water and other liquids at low or audible frequencies, and no experimental work in this region seems to have been done. This is because the coefficient of absorption in liquids is likely to be much smaller than in gases if only as a result of density factor in Eq. (1) above.

TABLE III. Liquids at ultrasonic frequencies. Calculated and measured absorption coefficients of sound for different liquids. (Biquard) Values from Bergmann (2).

Liquid	Temp. °C	$\alpha \times 10^2$ calculated	Temp. °C	$\alpha \times 10^2$ measured
Acetone	20	0.44	17	2
Benzol	20	0.54	17	58
Chloroform	20	0.66	18	30
Ether	20	0.57	18	3.5
Ethyl acetate	20	0.67	15	4.9
Meta xylol	20	0.78	20	4.7
Methyl acetate	20	0.53	17	6.9
Toluol	20	0.49	20	5.4
Water	20	0.538	20	1.6

Therefore, in the audible or low frequencies it is difficult to measure the absorption by the techniques at present available. However, an experimental verification in this region is desirable. In this connection it may be noted that extrapolated values of absorption of sound in water, in the range 10^4 to 10^5 vps, obtained from the theoretical values at high frequencies, are greater than the theoretical ones roughly by a factor of two (1).

From the above brief review, we find that the experimental results do not agree with the calculated values either in gases or in liquids at ultrasonic frequencies, nor in air at sonic frequencies. The only exceptions in the case of liquids are mercury and liquid argon (Kittel 78b).

IV. RECENT THEORIES TO EXPLAIN THE DISAGREEMENT

The disagreement between theory and experiment has been investigated by different workers, but no satisfactory explanation has yet emerged. To account for the discrepancy arguments have been put forward to include factors other than the three mentioned in section I. Three new approaches have been made; (i) The 'Relaxation Theory,' (ii) The 'Molecular Structure Theory,' and (iii) The 'Second-Viscosity Coefficient Theory.'

V. THE RELAXATION THEORY

The relaxation theory has, to a great extent, explained away the previous disagreement between calculated and experimental values of absorption in gases. It has done so by introducing two empirical constants. The theory was proposed by Herzfeld and Rice (23) in 1928; in it they showed that, besides internal friction and heat conduction, another factor must be taken into account in the form of a slow rate of exchange of energy between the translational movements of a molecule and its internal degrees of freedom. A mathematical formula has been derived for the absorption coefficient of sound energy in gases, based upon considerations of viscosity, heat conduction, and the new factor just mentioned. Comparison with available data shows that the new effect is either of considerable influence or even predominates. This idea was then developed by many others, who have put forward modified forms of the theory for liquids. (See W. T. Richards, *Revs. Modern Phys.* **11**, 36 (1939)).

This theory is based on the hypothesis that a much larger number of collisions is required to achieve thermal equilibrium for molecular vibrations than for the translational and rotational vibrations. That is, the relaxation time, τ , for the vibrational degrees of motion is much greater than the relaxation time for the rotational degrees of freedom. The latter, under ordinary conditions, is of the order of 10^{-9} second. This hypothesis leads to a complex specific heat dependent on frequency, making it possible to account for the absorption of sound by the introduction of two empirical constants.

The persistence of the excited vibrational states has

been proved by Dwyer (16) by spectroscopic means. On the other hand, it can be calculated theoretically from molecular data given by Landau and Teller (34). It seems certain, therefore, that there is a time-lag in the establishment of equilibrium for the internal degrees of freedom, and hence, there must be dissipation of energy.

Kapitza (26) in 1938, while working on the anomalies of heat conductivity of helium below the λ -point, and also on viscosity determinations, was led to the conclusion that the viscosity must be so small that even in the slowest flow between plates about 0.5×10^{-6} m apart the Reynolds number was above the critical value for turbulence. He uses this fact to explain the above anomalies.

Recent papers on the subject are referred to below in the appropriate section of the bibliography.

VI. THE MOLECULAR STRUCTURE THEORY

This theory is of comparatively recent origin, having been proposed by Hall (122) in 1947. Although this approach comes later than that of the second-viscosity theory, it is convenient to describe it now, and to leave the discussion of the second-viscosity coefficient to a later part of the paper.

Hall searched for the cause of the excess ultrasonic absorption in water, not in the time-lag, but in a lag in the rearrangement of the molecules during an acoustic compression. He derived an expression for such a structural absorption in a liquid and said that the experimental value obtained for water in the temperature range 0°C to 80°C , agreed reasonably well with an approximate calculation made on the lines of the above suggestion. Hall applied a relaxation treatment to the structural component of compression on the assumption that two molecular states of packing exist—a compact one (closed-pack) and an open one (ice-like). The passage of sound waves changes the equilibrium distribution of molecules; (i) in a compression molecules from the open structure may move to the closely packed one and (ii) in a rarefaction from the closely packed structure to the open one. In order to effect a change from one state to the other, some work has to be done. The coefficient of viscosity required for this is almost equal to that required for shear viscous flow. The relaxation time associated with this process has been found to be of the order of 10^{-12} sec for water. This mechanism implies a bulk viscosity coefficient (κ) or a second viscosity coefficient (μ') for water, which has a magnitude several times the shear viscosity of water. Hall's results, taken from his paper, are given in Table IV. (Hall uses the symbol η for the bulk viscosity.) In column 7 are given the values of the excess absorption as found from the experimental values of Pinkerton (97a).

Table IV shows a good agreement in the case of water between the values of the excess absorption coefficient and those calculated from the theory of structural relaxation in the stated range of temperature. Hall, therefore,

suggested that similar effects may be responsible for discrepancies in other liquids of the associated group, i.e., liquids having hydrogen-bonded molecules.

Very recently, Sette (128) working on ethyl alcohol came to the conclusion that the value of α/f^2 computed for structural absorption, based on Hall's treatment, is much smaller than the difference between the experimental value and that obtained by the use of classical theory; he does not however quote a value calculated on the basis of Hall's theory. The values which he does give are

	Calculated from classical theory	Experimental
Ethyl alcohol	$25 \times 10^{-17} \text{ sec}^2 \text{ cm}^{-1}$	$50 \times 10^{-17} \text{ sec}^2 \text{ cm}^{-1}$

Although it seems possible that the relaxation and molecular structure theories may succeed in explaining the disagreement of absorption values between calculation and experiment, a final decision must be postponed. It may, however, be noted that both theories involve, basically, the effect of viscous forces of both types. In relaxation theory there is a time-lag between the rearrangement of the molecules, and this can only be due to the presence of viscous forces; also in the molecular structure theory a relaxation treatment has been applied and a relaxation time determined. As shown by Hall himself, this implies a bulk viscosity coefficient (κ) or a second viscosity coefficient (μ') for a liquid.

VII. SECOND VISCOSITY

The term "second viscosity" is not a familiar one, and we have not been able to find it discussed in detail in accepted text books. Passing references are made, however, in Lamb's *Hydrodynamics* (4) and in Sommerfeld's *Mechanics of Deformable Bodies* (6). The classical theory of viscosity is very similar to the classical theory of elasticity. In elasticity it is necessary to introduce two moduli of elasticity; similarly, it might have been expected that in order to describe the dynamics of fluid motion it would be necessary to introduce a shear viscosity and some second viscosity. The term viscosity suggests internal friction in fluid media, and the coefficients of viscosity generally measured and given in tables are the values of shear viscosity. When a volume of fluid is compressed without change of shape, viscous forces come into play. Some of these may be due to viscous forces of the shear type, but some of them must be forces of the "second viscous" type. It is evident that the amount of work done by the viscous forces must depend upon the rate of compression. No direct determination of the coefficient of second viscosity has yet been made, because no direct method is available for determining it. Recently, however, a determination has been made by evaluating the ratio of the coefficients of second viscosity to shear viscosity when ultrasonic vibrations are passed through a liquid, as described below:

In 1942, Tisza (144) drew attention to the fact that the reduction of the two viscosity coefficients to one

according to Stokes's relationship $2\mu + 3\mu' = 0$ was not justified, except in the special case of a perfect monoatomic gas. By introducing a bulk viscosity coefficient $\kappa = (2\mu + 3\mu')/3$ Tisza developed a quantitative theory of the absorption of sound in liquids. He pointed out that it was possible to show that the relaxation theory is a form of classical hydrodynamics, and that for polyatomic gases κ can be expressed in terms of a characteristic constant of Tisza's theory. In polyatomic gases and liquids generally $\kappa \gg \mu$. He also discussed other hydrodynamical consequences of the introduction of κ .

The equation giving the components of the normal stresses are of the type

$$p_{xx} = -p + 2\mu \frac{\partial u}{\partial x} + \mu' \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial z} \right). \quad (3)$$

For incompressible fluid of constant density the equa-

TABLE IV. Absorption coefficient for water—calculated and experimental values. μ =shear viscosity (values from the International Critical Tables, Vol. V, P. 10, 1928), τ =relaxation time as calculated from the theory, κ =bulk viscosity calculated from the theory, α =absorption coefficient, f =frequency, β_∞ =instantaneous compressibility (the effective value as the frequency ν tends to ∞).

1	2	3	4	5	6	7
Temp. °C	μ millipoise (experimental)	τ $\times 10^{-12}$ sec	κ millipoise (calc-Hall)	$(2\alpha/f^2)10^{-17} \text{ cm}^{-1}$ sec ² (calc-Hall)	$\beta_\infty = 1.5 \times 10^{-12}$	μ' calculated from excess absorption
0	17.94	4.05	80	80	74	79.6
4	15.68	3.49	70	68	63	—
5	15.19	3.37	68	65	59	61.4
10	13.10	2.86	60	54	49	49.5
20	10.09	2.14	47	38	35	34.3
30	8.00	1.64	37	28	25	25.9
40	6.53	1.30	29	22	20	19.5
50	5.49	1.08	24	18	16	16.0
60	4.70	0.89	21	15	14	13.8
70	4.07	0.75	17	12	11	11.6
80	3.57	0.64	14	11	10	10.6

tion of continuity is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial z} = 0, \quad (4)$$

so that even if μ' did exist in liquids, it would not enter into the equations for the normal stresses.

Stokes himself did not treat the relation $\kappa=0$ as a definition, but foresaw the possibility of the necessity of introducing two coefficients of viscosity should experiment demand it. In this connection Tisza gives the following interesting quotation from Stokes's (54) original paper:—" . . . of course we may at once put $\kappa=0$, if we assume that in the case of a uniform motion of dilatation the pressure at any instant depends only on the actual density and temperature at that instant and not on the rate on which the former changes with time. In most cases to which it would be interesting to apply the theory of the friction of fluids, the density of

the fluid is either constant, or without sensible error may be regarded as constant, or else changes slowly with time. In the first two cases, the results would be the same and in the third nearly the same, whether κ were equal to zero or not. Consequently, if the theory and experiment should in such case agree, the experiment must not be regarded as confirming that part of the theory which relates to supposing κ to be equal to zero. . . ."

It can be seen from the above that Stokes was concerned only with incompressible fluids. So long as one is working at relatively low frequencies in the sonic range, the compressions and rarefactions in the fluid are not likely to be of considerable importance in connection with the absorption of energy, even though some such absorption may in fact occur. But, as soon as one comes to ultrasonic frequencies, even liquids may undergo compressions and expansions which involve a considerable absorption of energy. The previous disagreement between the theory and experiment in this range of frequency now seems to receive some clarification. Accordingly, Tisza (144), Eckart (134), and Gurevich (136) re-investigated the hydrodynamical equations of motion in the light of the above considerations and arrived at results which are given below.

In 1946 Fox and Rock (135) measured $2\alpha/f^2$ for water at various temperatures when the frequency was 17.25 Mc. The values varied from $137 \times 10^{-17} \text{ cm}^{-1} \text{ sec}^2$ at 0°C to $40 \times 10^{-17} \text{ cm}^{-1} \text{ sec}^2$ at 33°C . At 4°C , the value $101 \times 10^{-17} \text{ cm}^{-1} \text{ sec}^2$ was compared with the shear viscosity contribution $30 \times 10^{-17} \text{ cm}^{-1} \text{ sec}^2$. The excess absorption could not be attributed to the translational degrees of freedom of the molecules and was therefore ascribed to a second viscosity. The value so obtained was found to be 0.052 poise at 4°C and 0.026 poise at 20°C . The validity of this assumption can only be tested when some direct determination of the second viscosity is made and the values compared.

In 1948, Gurevich (136) showed that by introducing two coefficients of viscosity he could arrive at a modification of Stokes's law. This he claimed provided, in most cases, a more adequate interpretation of experimental data on absorption. By taking the two coefficients of viscosity into account he arrived at a modification of Stokes's formula in the form of

$$\alpha^2 = \frac{\omega^2 \left(1 + \frac{k^2 \omega^2}{a^4}\right)^{\frac{1}{2}} - 1}{2a^2 \left(1 + \frac{k^2 \omega^2}{a^4}\right)}, \quad (5)$$

where α = absorption coefficient, ω = angular frequency, a = velocity of sound in the medium, and $k = (4\mu + 3\mu')/3\rho$, where μ = coefficient of shear viscosity, μ' = coefficient of second viscosity, and ρ = density of the medium.

Gurevich stated that in his opinion the use of the above formula in place of the one due to Stokes provided

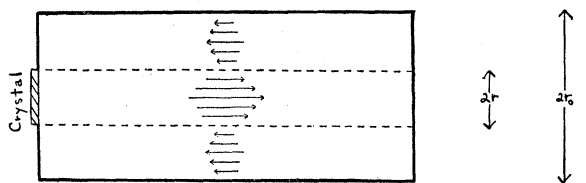


FIG. 1. The arrows indicate the directions of the fluid velocities in the middle portion of the cylinder.

a more adequate interpretation of experimental data, except in cases where the product $k^2\omega^2$ was less than a^4 . He arrived at the conclusion that the absorption coefficient is a function only of the viscosity and frequency and mentioned that he had confirmed this conclusion by experiment in the case of acetic and formic acids.

In 1948 Eckart (134) made an important contribution in this field. He derived a formula, connecting μ and μ' , which was capable of experimental test. He developed a systematic account of the theory of the second-order acoustic effects, which Rayleigh had neglected on the assumption of infinitely small motions. He first considered acoustic radiation pressure and the absorption of acoustic energy. He used this mathematical theory to investigate the second-order viscous forces and finally calculated the steady flow produced in a fluid by the passage of a sound beam of circular cross section. The fact emerged that the steady flow was dependent on the value of both coefficients of viscosity.

The solution of Eckart's equation shows that acoustical streaming consists of a vortex motion, produced at the boundaries of the sound beam. Quantitative considerations of these vortices are much simplified if the fluid is confined to a cylinder whose length is greater than its radius.

Eckart suggested that the sound beam should be directed along the axis of a long tube containing the liquid and should pass through it without reflection. A schematic diagram of such an arrangement is shown in Fig. 1.

For these conditions Eckart's equations lead to a velocity of streaming on the axis of the tube given by the relationship,

$$v = (\omega^2 r^2 G I / \rho a^4) (2 + \mu' / \mu), \quad (6)$$

where r = radius of the sound beam, I = intensity of the sound beam, G = a constant which takes into account the relative size of the sound beam and the tube. (If the sound beam fills the tube $G=0$, and the streaming stalls. An increased axial velocity results as the tube diameter increases.) ω = frequency of the sound beam = $2\pi f$, ρ = density of the medium through which the sound beam passes, a = velocity of sound in the medium, μ'/μ = ratio of the two viscosities, and v = velocity of streaming.

For a fluid confined in a cylindrical tube having its length greater than the radius and the sound beam passing along its axis without reflection G is given by the

relation

$$G = \frac{1}{2}(r^2/r_0^2 - 1) - \log r/r_0 \quad (7)$$

where r_0 is the radius of the tube.

Several papers on this subject have already appeared; a list is given in the bibliography.

VIII. EXPERIMENTAL DETERMINATION OF THE SECOND VISCOSITY IN LIQUIDS IN THE ULTRASONIC RANGE

In 1949 Liebermann (138) published a paper giving results of experiments carried out along the lines suggested by Eckart. He gave values for the ratio of the viscosities for water and a number of organic liquids. His results are given in Table V.

Column 3 gives the values of μ'/μ resulting from Liebermann's experiments. Column 5 gives the values resulting from Liebermann's analysis of experimental results obtained by earlier experimenters and the following are the conclusions at which he arrived:

- (i) No correlation appears to exist between the shear and second viscosities.
- (ii) The effect of variation of temperature, in the case of water, showed that the effects are the same for both viscosities.
- (iii) If the values of the second viscosity are substituted in acoustic calculations, most of the discrepancy between the theoretical and observed values of the absorption of sound in liquids at very high frequencies disappears.
- (iv) Columns 3 and 5 in Table V show some agreement between the direct determination of μ'/μ and the values of this ratio obtained from measurements of absorption. However, the agreement in some cases, is not very good and further work is needed.
- (v) The viscosity ratio for all liquids investigated has a positive value, rather than the value $(-\frac{2}{3})$ for the ideal monatomic gas.
- (vi) The value of the ratio differs widely.
- (vii) The value of the second viscosity for most liquids appears to be much greater than that of the shear viscosity.

Hence it can be said that there is no justification for the extension of the assumption $\kappa=0$ from the case of the ideal monatomic gas to that of liquids.

BIBLIOGRAPHY

Many of the references listed below are given in the review "Supersonic phenomena" by W. T. Richards (Revs. Modern Phys. **XI**, 36 (1939)).

References in the present paper are listed under the following headings:

- I. Text books (containing brief references).
- II. Absorption of sound in gases.
- III. Absorption of sound in liquids.
- IV. The relaxation theory.
- V. The molecular structure theory.
- VI. Shear viscosity.
- VII. Second viscosity.

I. Text Books

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- (2) L. Bergmann, *Ultrasonics* (G. Bell and Sons, London, 1938).
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TABLE V. Summary of viscosity ratio for a number of liquids. (Liebermann (138)).

Liquid	2 Fre- quency (Mc)	3 Viscosity ratio μ'/μ	4 Coeff. of shear vis. (centi- poise) μ	5 Vis. ratio from sound absorption μ'/μ
Water	5	2.4	1	2.1
Methyl alcohol	5	1.3	0.60	1.1
Ethyl alcohol	5	3.8	1.2	3.6, 2.6, 2.3
Acetone	5	3.1	0.30	3.8, 3.1, 2.9
Propyl alcohol (iso)	5	5.1	2.2	—
Amyl acetate	5	9.9	0.89	10.2, 12.3
	2	9.6	0.89	—
Xylol (m)	5	11	0.62	7.7, 6.5
	2	11	0.62	—
Ethyl formate	5	15	0.40	21, 10
	4	22	0.40	—
	3	31	0.40	—
	2	85	0.40	—
Chloroform	2	24	0.57	27, 34, 35
Carbon tetrachloride	2	28	2.0	27, 21, 22
Benzene	2	107	0.65	140, 130, 150
Carbon disulphide	2	200	0.37	—

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