Rotational Energy Levels of an Alpha-Particle Model for the Beryllium and Carbon Isotopes*†

RICHARD R. HAEFNER
University of Minnesota, Minneapolis, Minnesota

INTRODUCTION

In the past few years numerous experiments have been performed showing the location of energy levels in nuclei, and in many cases, the angular distributions of particles resulting from and partaking in nuclear reactions. With this new evidence, much of it very refined compared to that of a decade or two ago, it is again profitable to investigate the possibility of the existence of alpha-particle clusters in some light nuclei. The nuclei chosen were the beryllium and carbon isotopes, for their simplicity among the alpha-particle nuclei.

Teller and Hafstad[1] investigated the approximate position, angular momentum, and parity of the rotational energy levels of Be^8, Be^9, C^{12}, and C^{13}, using a rigid dumbbell model for the two alpha-clusters in Be^8 and Be^9 and a rigid equilateral triangle model for those of C^{12} and C^{13}, a cluster at each vertex of the triangle. Wheeler[2] had previously shown that for the low energy levels of the beryllium and carbon nuclei the period of existence of an alpha-particle cluster was appreciable compared with the period of the motion of the alpha-particle model. Teller and Hafstad were unable to correlate their findings with experiment for lack of data.

During the 1930's numerous experiments were done on the scattering of alpha-particles by helium nuclei.[3] These experiments will be discussed in the next section. In 1941, Wheeler[4] analyzed these results in terms of the phase shifts of the partial waves for $L=0$, 2, and 4. The partial wave for $L=0$ showed a rapid variation in phase at about 6-Mev energy of the incident alpha-particle, equivalent to 3 Mev in the center-of-mass system, indicating a level in Be^8 at about 3 Mev, of even parity and zero angular momentum. Margenau[5] investigated the interaction of two alpha-particles in general, and then chose a simple model representing the effects found to obtain the *s*-wave phase shift. By use of an infinite repulsive potential for separations of the centers of the alpha-particles from 0 to r_1, an attractive square well from r_1 to 4.5×10^{-13} cm, and the coulomb repulsion for separations greater than 4.5×10^{-13} cm, the *s*-wave phase shifts were obtained as functions of the energy of the incident alpha-particles for various values of r_1. At very low bombarding energies, the results were in agreement with the phase shifts determined by Wheeler, but did not show the resonance effect at higher energies. It was concluded that the two-body interaction was incompatible with the scattering data.

In analyzing the experimental data on the Li^8 beta-decay[6] and the alpha-particle spectrum from the transmutation of B^{11} by protons of 180 kev,[7] Wheeler[8] was forced to the conclusion that the Li^8 data showed an excited state of Be^8 at about 3 Mev having two units of angular momentum, while the proton plus B^{11} reaction showed that Be^8 possessed an excited state at 2.8 Mev of a width compatible with the alpha-alpha scattering analysis. Since the Li^8 beta-decay does not proceed to the ground state of Be^8, it would not go to a 2.8-Mev state of zero angular momentum either.

It will be shown in the following that these results can be explained by the two-alpha-particle model, the excited state in question being a $^1D_2$ state. Existing data will also be examined to determine whether the other beryllium isotopes and the carbon isotopes are describable by an alpha-particle model.

Be^8

There is experimental evidence that the ground state and first excited state of Be^8 have different angular momenta. In the $Li^8(e^-)Be^8$ reaction, approximately 90 percent of the beta-rays leave the Be^8 nucleus in the excited state of about 3 Mev, while less than 2 percent leave the nucleus in the ground state.[9] In the $B^{11}(p, \alpha\alpha')$-Be^8 reaction at the 162-kev proton resonance, the number of transitions to the ground state are 1/50, those to the first excited state.[10] The fact that both states can decay into two alpha-particles requires they both have even parity and even angular momentum, owing to the application of Bose statistics to an interaction involving two alpha-particles.

* Part of a Ph.D. thesis submitted to the Graduate Faculty of the University of Minnesota, 1951. This article arises from one of the last projects sponsored by the Minnesota Technical Research Fund under Dr. Tate's direction.
† Now at Argonne National Laboratory, E. I. duPont Company.
1 L. R. Hafstad and E. Teller, Phys. Rev. **54**, 681 (1938).
2 J A. Wheeler, Phys. Rev. **52**, 1083 (1937).
3 E. Rutherford and J. Chadwick, Phil. Mag. 4, 605 (1927). J. Chadwick, Proc. Roy. Soc. (London) **A128**, 120 (1930). P. Wright, Proc. Roy. Soc. (London) **A137**, 677 (1932). C. B. O. Möhr and G. B. Pringle, Proc. Roy. Soc. (London) **A160**, 193 (1937). S. Devons, Proc. Roy. Soc. (London) **A172**, 564 (1939).
4 J. A. Wheeler, Phys. Rev. **59**, 16 (1941).
5 H. Margenau, Phys. Rev. **59**, 37 (1941).
6 W. A. Fowler and C. C. Lauritsen, Phys. Rev. **51**, 1103 (1937). Rumbaugh, Roberts, and Hafstad, Phys. Rev. **51**, 1106 (1937); **54**, 672 (1938).
7 Oliphant, Kempton, and Rutherford, Proc. Roy. Soc. (London) **A150**, 251 (1935). P. I. Dee and C. W. Gilbert, Proc. Roy. Soc. (London) **A154**, 291 (1936).
8 J. A. Wheeler, Phys. Rev. **59**, 27 (1941).
9 W. F. Hornyak and T. Lauritsen, Phys. Rev. **77**, 160 (1950).
10 H. A. Bethe, Revs. Modern Phys. **9**, 216 (1937).

The following interaction potential was chosen to determine the properties of the Be^8 if it is describable by a two-alpha system:

$$V(r)=\begin{cases}4e^2/r, & r>r_0,\\ -D+q^2\hbar^2/2\mu r^2, & r<r_0,\end{cases} \quad (1)$$

where r is the separation of centers of the alpha-clusters, e is the electronic charge, D is a constant, the well depth parameter, μ is the reduced mass of the two-alpha system, $\hbar$ is Planck's constant divided by 2π, and q^2 is a parameter, a proper choice of which allows convenient use of already tabulated values of wave functions. Since this potential is chosen only to investigate the Be^8 compound nucleus for low excitations, its particular shape can be considered characteristic of potentials showing the following properties: a strongly repulsive potential for very small values of r representing the effect of the Pauli exclusion principle applied to an individual constituent of an alpha-cluster with its quantum-mechanical equivalent in the other cluster; an attractive potential for intermediate ranges and the coulomb potential beyond the range of nuclear forces.

If one uses spherical polar coordinates, the Schroedinger equation for the probability function $\psi(r, \vartheta, \varphi)$ of the radius vector separating the centers of the two alpha-clusters is separable, and for the potential given by (1) the resulting radial equation is as follows:

$$\frac{1}{r}\frac{d^2}{dr^2}[rR_L(r)]+\left[k^2+\frac{2\mu D}{\hbar^2}-\frac{q^2+L(L+1)}{\hbar^2}\right]R_L(r)=0, \quad r<r_0,$$

$$\frac{1}{r}\frac{d^2}{dr^2}[rR_L(r)]+\left[k^2-\frac{8e^2\mu}{\hbar^2 r}-\frac{L(L+1)}{r^2}\right]R_L(r)=0, \quad r>r_0, \quad (2)$$

where $\psi_{L,M}(r, \vartheta, \varphi)=R_L(r)\Theta_{L,M}(\vartheta, \varphi)$, $\Theta_{L,M}(\vartheta, \varphi)$ is the spherical harmonic of order L and index M, $k=(2\mu E/\hbar^2)^{\frac{1}{2}}=0.3095E^{\frac{1}{2}}\times 10^{13}$ cm^{-1}, and E is the total energy of the system. For the choice of $q^2=30$, the function $R_L(r)$, for values of $L=0$ and 2, and for $r<r_0$, is found to be

$$\begin{aligned} R_0(r)&=A_0 j_5(k_0 r), & r<r_0,\\ R_2(r)&=A_2 J_6(k_0 r)/(k_0 r)^{\frac{1}{2}}, & r<r_0,\end{aligned} \quad (3)$$

where $k_0{}^2=k^2+2\mu D/\hbar^2$ and $j_5(k_0 r)$ is the spherical bessel function of order 5, defined also as $(\pi/2k_0 r)^{\frac{1}{2}}J_{5\frac{1}{2}}(k_0 r)$, the latter function being the bessel function of the first kind of order $5\frac{1}{2}$. The solutions for $r>r_0$ are a linear combination of functions $f_L(kr)$ and $g_L(kr)$, defined as $f_L(kr)=F_L(kr)/kr$ and $g_L(kr)=G_L(kr)/kr$, where $F_L(kr)$ and $G_L(kr)$ are the regular and irregular coulomb wave functions, respectively, defined by Yost, Wheeler, and Breit.[11] The function $R_L(r)$ for $r>r_0$ is written

$$\begin{aligned} R_0(r)&=B_0\{f_0(kr)\cos\delta_0+g_0(kr)\sin\delta_0\}, & r>r_0,\\ R_2(r)&=B_2\{f_2(kr)\cos\delta_2+g_2(kr)\sin\delta_2\}, & r>r_0,\end{aligned} \quad (4)$$

where δ_0 and δ_2 are the phase shifts of the partial waves of zero and two units of angular momentum, respectively, owing to nuclear forces.

In order that the wave function have a finite probability and current at $r=r_0$, we require the continuity of the function and its first derivative for each of the above partial waves when matching the wave functions for $r>r_0$ to those for $r<r_0$. This matching condition enables one to determine the parameters δ_L and A_L/B_L as a function of the energy.

It was found that the criterion for a virtual level of angular momentum L is closely given by the maximum of the ratio A_L/B_L. If this maximum occurs for energies where the irregular wave function is much larger than the regular wave function for $r=r_0$, it follows that $\delta_L\simeq 90°$ at this maximum. This latter statement is equivalent to a Breit-Wigner formulation where there is no potential scattering; and the phase shift of the partial wave is entirely given by the resonant term, giving a value of $\delta_L=90°$ at resonance.

It has been determined that the ground state of Be^8, presumed to be a $^1S_0$ state, is a virtual state at an energy of approximately 90 kev above that for an infinite separation of two alpha-particles.[12] For this energy $g_0(kr_0)\gg f_0(kr_0)$ and the position of the ground state of zero angular momentum is given by $\delta_0=90°$, equivalent to the relation

$$k_0 r_0 j_5'(k_0 r_0)/j_5(k_0 r_0)=kr_0 g_0'(kr_0)/g_0(kr_0) \quad (5)$$

evaluated at $E=90$ kev. This relation determines D as a function of r_0. One can then determine the maximum in the ratio A_2/B_2 as a function of r_0. For values of r_0 from 4.0 to 5.0×10^{-13} cm, this maximum ranges from $E\simeq 2.7$–3.8 Mev. Since the irregular function $g_2(kr_0)$ is of the same order of magnitude as $f_2(kr_0)$ in this energy range, the ratio A_2/B_2 must be used in determining the position of the virtual state. For values $r_0=4.50-4.75\times 10^{-13}$ cm, the maximum of this ratio occurs in the neighborhood of 3 Mev. Consequently, for values $r_0=4.50$ and 4.75×10^{-13} cm, and for the requirement that the $^1S_0$ state be unbound by 90 kev, the variation of A_2/B_2, δ_0, δ_2, and δ_4 were determined as a function of the energy in the center-of-mass energy range 1.5–4.5 Mev, in order that these parameters might be checked by the experiments to be described.

In principle, the best method of procuring evidence for the classification of the first excited state of Be^8 is the scattering of alpha-particles by He^4 nuclei. A measurement of the angular distribution of the scattered particles as a function of energy can give information on the relative strength of the nuclear forces

[11] Yost, Wheeler, and Breit, Phys. Rev. **49**, 174 (1936).
[12] Tollestrup, Fowler, and Lauritsen, Phys. Rev. **76**, 428 (1949).

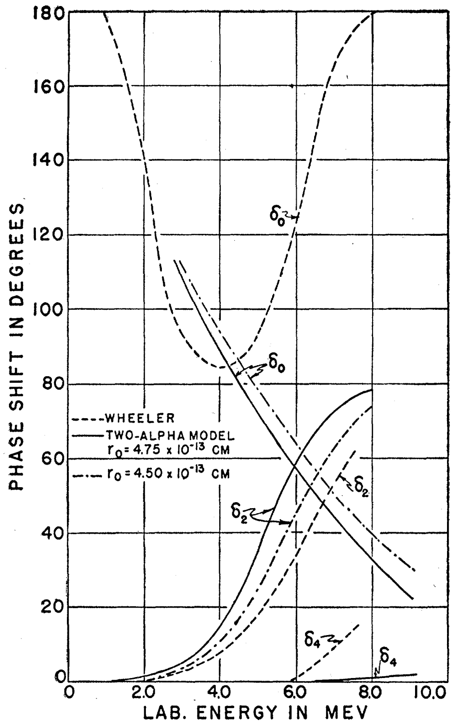

FIG. 1. Phase shifts for alpha-alpha scattering predicted by the alpha-particle model and from Wheeler's analysis of experimental data.

as a function of the angular momentum. In this particular case, since the incident alpha-particle and the struck He^4 nucleus are indistinguishable during and after the collision, the sum of angular distributions of the scattered and recoil particles is the only measurable quantity. One theoretically reproduces the experimental scattering conditions by postulating a plane wave representing the incident beam of particles. In the collision this wave is then refracted in the force field of the target particle, the amount of refraction being, in general, different for the various angular momentum components comprising the incident plane wave. This refraction is represented by waves traveling outward from the collision area. The intensity of these waves at a particular place represents the probability of detecting a scattered particle at this place.

The amount of refraction is expressed as a shift in phase of the wave. If we assume the orbital electrons of He^4 afford complete screening of the nuclear charge at distances greater than $\sim 10^{-8}$ cm, these shifts in phase of the wave will be appreciably different from zero for partial waves of L from zero to $\sim 10^4$. However, it is possible to separate these phase shifts into a coulomb portion and a nuclear portion. The summation of coulomb terms can be obtained in a closed form valid for angles $>10^{-4}$ radian. The remaining terms give the effect of the nuclear forces in the presence of the coulomb force, and the phase shifts in these terms are idenfied with those given in Eq. (4).

For the present case of identical particles, in the center-of-mass system the total wave function is composed of a wave representing the incident particles described above, and the wave representing the target particles traveling in the opposite direction. Because Bose statistics are applicable here, the superposition is such that in an angular momentum decomposition of the total wave function, only waves of even angular momentum occur. If the wave representing the incident particles is normalized to unit current, and likewise for the wave of the target nuclei, the center-of-mass cross section per unit solid angle, $\sigma_{\text{c.m.}}(\vartheta)$ will be the intensity of the superposition of the scattered waves of both the incident and target particles. This is given by:

$$\sigma_{\text{c.m.}}(\vartheta)=\frac{1}{k^2}\bigg|\frac{\eta\exp[i\eta\ln(2/1-\cos\vartheta)]}{1-\cos\vartheta}-\frac{\eta\exp[i\eta\ln(2/1+\cos\vartheta)]}{1+\cos\vartheta}+2\exp(i\delta_0)\sin\delta_0+5(3\cos^2\vartheta-1)\exp[i(\delta_2+\zeta_2)]\sin\delta_2+(9/4)(35\cos^4\vartheta-30\cos^2\vartheta+3)\times\exp[i(\delta_4+\zeta_4)]\sin\delta_4+\ldots\bigg|^2, \quad (6)$$

where ϑ is the center-of-mass angle with respect to the incident beam, v is the relative velocity of the two particles, δ_L is the phase shift of the Lth partial wave, $k=\mu v/\hbar=0.3095[E_{\text{c.m.}}(\text{Mev})]^{\frac{1}{2}}\times 10^{13}$ cm^{-1}, $\eta=4e^2/\hbar v=0.8941/[E_{\text{c.m.}}(\text{Mev})]^{\frac{1}{2}}$, and

$$\zeta_L=2\sum_{N=1}^{L}\tan^{-1}(\eta/N).$$

The center-of-mass energy $E_{\text{c.m.}}$ is one-half the laboratory energy. The first term in Eq. (6) represents the scattered wave amplitude and phase for the incident particle for pure coulomb scattering, the second that for the target particle. The last three terms represent the effect of the nuclear forces on the partial waves fo angular momentum 0, 2, and 4, respectively.

If one includes the factor $1/k^2$ in Eq. (6), he finds the absolute square of the first term is the Rutherford cross section for the incident particle; the absolute square of the second, is that for the target particle, for indistinguishable particles. The sum of these two squared terms is the classical scattering cross section. Since the early alpha-alpha scattering experiments were done in the infancy of wave mechanics, experimental data were given as a ratio R of the actual cross section to the classically expected cross section, simply to show wave mechanical interference effects:

$$R = \frac{\sigma_{\text{c.m.}}(\vartheta)}{\eta^2/k^2\{[1/(1-\cos\vartheta)^2]+1/(1+\cos\vartheta)^2\}} = \frac{k^2\sigma_{\text{c.m.}}(\vartheta)\sin\vartheta^4}{2\eta^2(1+\cos^2\vartheta)}. \quad (7)$$

The only experiments on the scattering of alpha-particles by He^4 nuclei were done in England,[3] with alpha-particles from the naturally radioactive nuclei radium, thorium, and polonium. A very small chamber with an annular ring scattering volume was used in all experiments. In order to obtain a good counting rate with the weak sources available, the geometry of the apparatus was of necessity not very precise. Gas pressures used were in general higher than are used in present-day precision scattering experiments. The energy of the alpha-particles from the radioactive nuclei at the scattering volume was determined by range measurements. Then, with argon in the chamber at a pressure such that the alpha-energy at the scattering volume was the same as desired for alpha-alpha scattering, the geometry factor of the apparatus was determined, assuming Rutherford scattering by the argon. Mica sheets were placed before the source to obtain different energies, the energies were measured by scattering in argon, a Rutherford cross section was assumed, and the geometry factor determined as described was used.

The data were analyzed by Wheeler[4] to obtain the phase shifts δ_0, δ_2, and δ_4. The only consistent set of solutions for these quantities is shown in Fig. 1. The

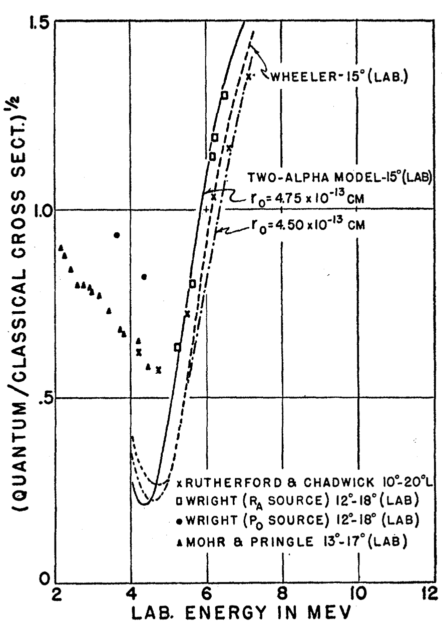

FIG. 2. Comparison of theory and experiment for the alpha-alpha scattering cross section at a laboratory angle of 15°.

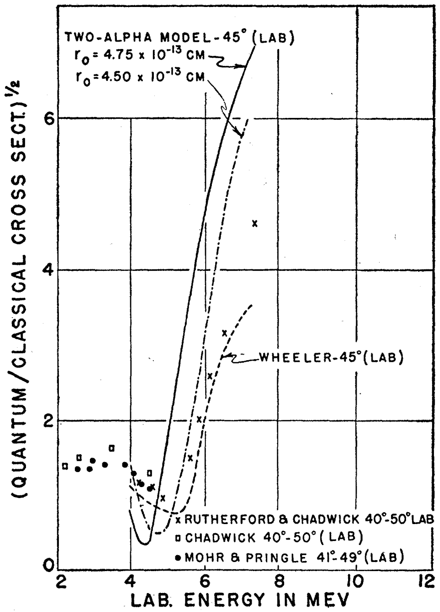

FIG. 3. Comparison of theory and experiment for the alpha-alpha scattering cross section at a laboratory angle of 45°.

s-wave phase shift δ_0 can be decomposed into a part due to potential scattering and a part due to resonant scattering. This latter reveals an s-wave resonance at approximately 6 Mev in the laboratory, or 3 Mev in the center-of-mass system. It is impossible to obtain only the two levels, both with zero angular momentum, from any two-alpha model.

Shown in Fig. 1 are the phase shifts for the two-alpha model described earlier, for $r_0=4.50$ and 4.75×10^{-13} cm. The variation of R in Eq. (7) with energy obtained by using these phase shifts is shown in Figs. 2 and 3 for two different laboratory angles of scattering. For the case of identical particles, the laboratory angle is one-half the center-of-mass angle. The experimental points are included in the figures also.

The theoretical curves show the variation to be expected in R is one could use extremely precise geometric conditions. The experimental points are, however, an integrated cross section over the angular spread indicated, and probably more, since the indicated angular spread was calculated for a point source and detector, conditions which were not met in the experiment. The theoretical cross section shows a minimum in the region of 15° in the laboratory system for laboratory energies around 4 Mev, and should therefore be expected to be less than observed in the experiments. At higher energies the second derivative of the cross section with angle at 15° is not very large, and the cross section should then agree more closely with experiment, as is found.

At 45° laboratory angle the theoretical curves show a maximum in the cross section, and the experimental results should then lie lower because of the integrating

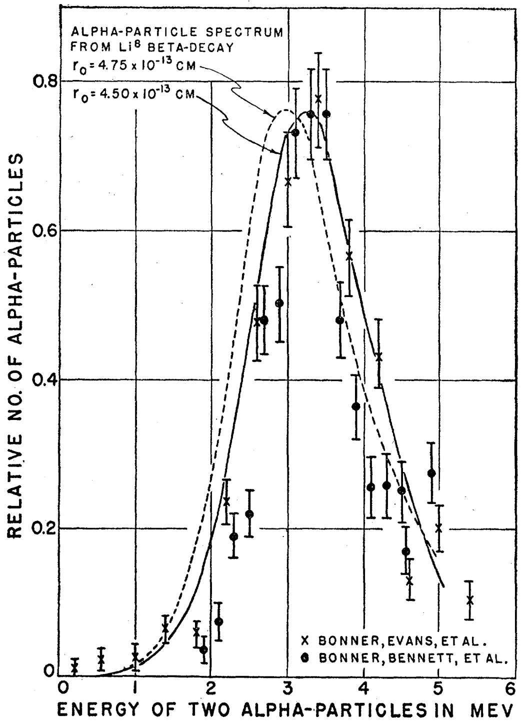

FIG. 4. Theoretical and experimental alpha-particle spectrum from the Li^8 beta-decay.

property of the apparatus. These results are typical of the general situation found. If the theoretical cross section has a strong angular second derivative, the results vary markedly from the experimental data; but in such a direction the variation can be attributed to the geometry of the apparatus. Where the second derivative is small, fair to good agreement with data is obtained.

Further factors in doubting the experimental data when used in a phase shift analysis are the energy spread of the incident beam, the possibility of multiple scattering for the pressures used, and the assumption of Rutherford scattering by argon for calibration purposes. A definite attempt should be made to again perform this scattering experiment in the laboratory energy range 4–7 Mev with refined geometry and a well-defined beam energy. Such experiments could be performed by presently operating Van de Graaff generators if someone could devise a source capable of doubly ionizing helium particles in sufficient quantities, admittedly a difficult undertaking. However, the importance of the results gained therefrom would certainly justify the effort. Since the alpha-particles are spinless, the data can be analyzed in an unambiguous manner, and firm conclusions regarding the validity of the alpha-particle model can be gained.

In the bombardment of Li^7 by deuterons, Li^8, with a half-life of 0.89 second, is formed. The beta-ray has a maximum energy of approximately 16 Mev. The beta-spectrum indicates 90 percent of the transitions leave Be^8 in the 3-Mev excited state, with less than 2 percent in the ground state.[9] This is certainly suggestive if not conclusive that the ground state and excited state at 3 Mev have different angular momenta.

Bonner, *et al.*[13] have conducted an experiment in which the energy spectrum of the alphas from the decay of Be^{8*} has been obtained. The probability of an alpha-particle of energy E_α should be proportional to the transition probability of the beta-process leaving the Be^8 nucleus in an excited state of energy $2E_\alpha$. This probability is $(Q-2E_\alpha)^5 W(E_\alpha)$, where Q is the total energy available in the Li^8 breakup and $W(E_\alpha)$ is the probability for the Be^8 nucleus to have the energy $2E_\alpha$. If there were no resonant interaction and the alphas left in a d-wave, then we have the case that $W(E_\alpha)\sim E_\alpha^{5/2}$, corresponding to the availability of quantum states in phase space for the break-up with two units of angular momentum. For the two-alpha model, however, there does exist a resonant interaction; and a measure of it is given by the square of A_2/B_2. For $E_\alpha\sim 0$, i.e., in the nonresonant region, the square of this ratio is proportional to E_α^2. Therefore, the probability of an energy E_α should be proportional to the square of the ratio A_2/B_2 times the square root of the alpha-energy, over the whole energy range considered. The total probability function, giving the energy spectrum of the alphas, is given with the experimental data in Fig. 4 for two values of the radius for the two-alpha model. The agreement is seen to be quite good for the lesser of the two radii.

A third experiment from which evidence regarding Be^8 has been obtained, and can be obtained in greater detail, is the bombardment of B^{11} by protons of energy $\sim$163 kev. The existence of an excited state of Be^8 at approximately 3 Mev, as well as the ground state, is clearly revealed in this experiment.[14] The energy spectrum of the alpha-particles produced in this reaction is divided essentially into three portions, as shown first by Dee and Gilbert:

(i) A sharp group at $E_\alpha \simeq 5.7$ Mev, due to the reaction $B^{11}(p,\alpha)$-Be^8. The alpha-particles resulting from the breakup of Be^8 in its ground state in this reaction are in number and energy too small to be experimentally identified, since the particles of reaction (iii) to be described fall in the same energy region, but far outnumber those from this reaction.
(ii) A broad group at $E_\alpha=3.85$ Mev, due to $B^{11}(p,\alpha)Be^{8*}$.
(iii) A continuous group resulting from $Be^{8*}\rightarrow 2He^4$. The continuous nature of this group is the result of the addition of the recoil velocity of the Be^{8*} nucleus from reaction (ii) to that of the two alpha-particles in its breakup.

The data cannot give the angular momentum of the excited state; but it can be inferred that its angular momentum is not the same as that of the ground state, since the number of particles leaving Be^8 in its excited

[13] Bonner, Evans, Malich, and Risser, Phys. Rev. **73**, 885 (1948).
[14] Oliphant, Kempton, and Rutherford, Proc. Roy. Soc. (London) **A150**, 251 (1935). P. I. Dee and C. W. Gilbert, Proc. Roy. Soc. (London) **A154**, 291 (1936). McLean, Yound, Whitson, Plain, and Ellett, Phys. Rev. **57**, 1083 (1940).

state is of the order fifty times the number leaving it in the ground state.

Theoretical calculations for the energy spectrum can be made profitably if the angular momentum and parity of the excited state of the compound nucleus C^{12} involved in this reaction were known. It is possible to obtain this information if the angular distribution of the alpha-particles obtained from the reaction (i) above were known. Using a thick target, Williams, Haxby, and Allen[15] determined this distribution to be of the form $1+A(E)\cos^2\vartheta$. However, the function $A(E)$ varied with the proton energy in the same manner as the excitation function for the thick target, namely, the actual resonance yield integrated over a large energy interval. Therefore, the maximum value of $A(E)$, which is ~ 0.7, is to be considered a lower limit of the actual asymmetry. The angle of the alpha-particle with respect to the beam direction is ϑ, and an energy of the order 180 kev was used. Jacobs and Whitson,[16] using BF_3 at a few millimeters pressure found spherical symmetry for the long-range alphas of reaction (i) and in addition also for the continuous group of particles. An experiment should be undertaken to find which of these experiments is in error. When consideration is taken of the companion $B^{11}(p,\gamma)C^{12*}(\gamma)C^{12}$ reaction, which also has a resonance at a proton energy of 163 kev, the results of Jacobs must be placed in doubt. An asymmetry has definitely been shown for the first gamma-ray in this reaction.[17] If this gamma-ray and the long-range alpha-particle from reaction (i) proceed from the same resonant state, the long-range alpha-particles would have spherical symmetry only accidentally.

The fact that the excited state of the compound nucleus C^{12} can break up into two particles of intrinsic spin zero, the alpha-particle and Be^8 in its ground state, limits the possible values of total angular momentum and parity for this excited state. For states of odd (even) parity, only odd (even) total angular momentum can occur. Using an $L-S$ coupling scheme, one can calculate the angular distributions of the possible reaction products for a second-order transition process involving a resonant state of the compound nucleus formed in the reaction. Details of this type of calculation are given in a paper by Chao.[18] For a spin of 3/2 and odd parity of the ground state of B^{11}, there are two total spin states, $S=2$ and $S=1$, formed in the collision of a proton with the B^{11} nucleus. If it is assumed that only one state in C^{12*} is contributing appreciably to the reaction and that only the lowest angular momentum in the formation and decomposition of the compound nucleus consistent with a given total angular momentum and parity of C^{12*} need be considered, the angular distribution of the alpha-particles from reaction (i) will have the form $(1/3)(1-\cos^2\vartheta)$ for $S=2$ and $(1/9)(1+3\cos^2\vartheta)$ for $S=1$, for incident p-wave protons, and $J=2^+$ for C^{12*}. (J is the total angular momentum, all angular momenta being given in units of $\hbar$. The superscript + and − refer to even and odd parity, respectively.) The C^{12*} property $J=0^+$ or $J=1^-$, with formation by p- and s-wave protons, respectively, leads to no asymmetry; and for $J=3^-$, formed by d-wave protons, one obtains a greater asymmetry that is observed experimentally. If the transition is three times more probable for the spin state $S=1$ than that for $S=2$, approximate agreement with the data of Williams *et al.* is obtained. The transition involving the $S=2$ state requires the turning over of an intrinsic spin, since the final reaction products have no spin. For a tentative choice of $J=2^+$ for the C^{12*} compound nucleus, reaction (i), giving long-range alpha-particles, will proceed with particles emitted in a d-wave, while those of reaction (ii) will be emitted in an s-wave, if the 3-Mev state of Be^8 is a $^1D_2$ state as predicted by the alpha-particle model. This could be the explanation of the intensity factor of fifty for reaction (ii) over (i), since the long-range alphas will have their wave amplitude reduced more by the barrier penetration than will the shorter range alphas having less angular momentum. The assignment given must be considered tentative, and further conclusions must await results which clear up the contradiction in experimental data.

Agreement of the theoretical results, based on the alpha-particle model, with experiment for Be^8, can be found when account of the experimental conditions is taken. However, conclusive evidence for the validity of this model can only be obtained from the results of the experiments suggested.

C^{12}

The angular momentum and parity of the levels of C^{12} described by an alpha-particle model can be obtained from a rigid rotator model, with alpha-clusters situated at the vertices of an equilateral triangle. This model further gives a rough indication of the energy level spacing. When used for low-lying energy levels, this model is considered adequate to indicate properties of C^{12} which can be checked experimentally.

If the identical particle aspect of the model is neglected, the rotational energy level spectrum and the wave functions for this model are those of the symmetric top rotator, with the moment of inertia about the figure axis twice as large as that about either of the two perpendicular axes in the plane of the alpha-particles.[19] The figure axis is that axis perpendicular to the plane of the alphas and passing through their center of mass. For a given set of cartesian coordinates in space, the total angular momentum is quantized to values of $[J(J+1)]^{\frac{1}{2}}\hbar$ for positive integral J, the z component of this angular momentum is spatially quantized to $M\hbar$,

[15] Williams, Haxby, and Allen, Phys. Rev. **55**, 140 (1939).
[16] J. A. Jacobs and W. L. Whitson, Phys. Rev. **59**, 108(A) (1941).
[17] Kern, Moak, Good, and Robinson, Phys. Rev. **83**, 211(A) (1951).
[18] C. Y. Chao, Phys. Rev. **80**, 1035 (1950).
[19] G. Herzberg, *Infrared and Raman Spectra* (D. Van Nostrand Company, Inc., New York, 1945), p. 24.

TABLE I. Results of the analysis of the $Be^9(\alpha, n)C^{12}$ and $Be^9(\alpha, n)C^{12*}$ reaction.

C^{13*}	C^{12*}	Spin state	L-wave	Long-range neutrons	Short-range neutrons	$R(90°)$	$R(0°)$
$3/2^+$	2^+	$3/2^+$	s	$7-6\cos^2\alpha$	Isotropic	1.6	11.2
$3/2^+$	2^+	$5/2^+$	d	$7-6\cos^2\alpha$	$1+1.6\cos^2\alpha$	...	...
$5/2^+$	2^+	$3/2^+$	d	$1+2\cos^2\alpha$	$2+\cos^2\alpha$	...	...
$5/2^+$	2^+	$5/2^+$	s	$1+2\cos^2\alpha$	Isotropic	1.6	0.53
$3/2^+$	1^-	$3/2^-$	p	$7-6\cos^2\alpha$	$1+1.4\cos^2\alpha$	1.6	>1.7
$3/2^+$	1^-	$1/2^-$	p	$7-6\cos^2\alpha$	$1-0.85\cos^2\alpha$		<26.9
$5/2^+$	1^-	$1/2^-$	f	$1+2\cos^2\alpha$	$1+2\cos^2\alpha$	...	...
$5/2^+$	1^-	$3/2^-$	p	$1+2\cos^2\alpha$	$6+7\cos^2\alpha$	1.6	2.0
$3/2^+$	1^+	$1/2^+$	d	$7-6\cos^2\alpha$	$1-0.85\cos^2\alpha$	...	...
$3/2^+$	1^+	$3/2^+$	s	$7-6\cos^2\alpha$	Isotropic	1.6	11.2
$5/2^+$	1^+	$1/2^+$	d	$1+2\cos^2\alpha$	$1+2\cos^2\alpha$	1.6	>0.8
$5/2^+$	1^+	$3/2^+$	d	$1+2\cos^2\alpha$	$2+\cos^2\alpha$		<1.6
$3/2^+$	2^-	$3/2^-$	p	$7-6\cos^2\alpha$	$1+1.4\cos^2\alpha$	1.6	>8.7
$3/2^+$	2^-	$5/2^-$	p	$7-6\cos^2\alpha$	$9-2.0\cos^2\alpha$		<26.9
$5/2^+$	2^-	$3/2^-$	p	$1+2\cos^2\alpha$	$6+7\cos^2\alpha$	1.6	>0.14
$5/2^+$	2^-	$5/2^-$	p	$1+2\cos^2\alpha$	$1-0.73\cos^2\alpha$		<1.16

and the component of angular momentum about the figure axis to $K\hbar$, with $|M|$ and $|K|$ integral but less than or equal J.

For the present model with three identical alpha-clusters certain values of J and K cannot occur. When the Bose statistics for the alpha-particles are used, the wave function of the model must remain unchanged when any two alpha-particles are interchanged. A rotation of 120° about the figure axis is equivalent to two interchanges of pairs of alpha-particles. The wave function remains unchanged for this transformation only when $K=3n$ for integral n. Further, a rotation of 180° about an axis coincident with a radius vector from the center of mass to any one of the particles interchanges the other two particles. For $K=0$, the wave functions are the usual spherical harmonics, and the Bose statistics require J even, just as for Be^8, for this interchange. For $K=3n$ and n not zero, symmetric wave functions are obtained for all $J\geq|K|$. As a result of these limitations, the energy levels of the model are:

$$E=(\hbar^2/m_\alpha r^2)[J(J+1)-\tfrac{1}{2}K^2]\text{ergs}$$
$$=(10.75/m_\alpha r^2)[J(J+1)-\tfrac{1}{2}K^2]\times10^{-26}\text{ Mev}, \quad (8)$$

where $K=\pm3n$, $n=1, 2, 3, 4, \ldots, J\geq|K|$; $K=0$, $J=0, 2, 4, 6, \ldots$; r is the separation of two alpha-particles.

The ground state of C^{12} then has $J=0$, $K=0$, the first excited state $J=2$, $K=0$ for this model. Since for $K=0$ these first two levels are exactly similar to those expected of Be^8 when a dumbbell rotator is assumed, the separation of the ground and first excited states of C^{12} should be of the order of that found for Be^8. However, the C^{12} states are bound; and, further, this binding will decrease the alpha-particle separations compared with that for Be^8. The C^{12} first excited state should then lie higher than that for Be^8, both because of the decreased separation of alpha-particles and because of the fact that for a given interaction potential the energy levels are farther apart when bound than when virtual. The first excited state of C^{12} for the alpha-particle model is then tentatively identified with the lowest experimentally observed state 4.47 Mev above the ground state, corresponding to a value $r=3.81\times10^{-13}$ cm. The ground and first excited states of the model have even parity, the parity operation, 180° rotation about the figure axis, showing states are even or odd as K is even or odd.

The results can be checked by existing data of several experiments. In the transmutation of Be^9 by 1.4-Mev alpha-particles, long- and short-range neutrons are produced, leaving C^{12} in its ground and first excited state, respectively. Bradford and Bennett[20] observed the relative yields of the two above reactions at 0° and 90° from the beam. At 90° they found the short-range neutrons 1.6 times more intense than the long-range neutrons. At 0° this ratio was 0.4. The 90° data were based on two thousand tracks in the photographic emulsion, while the 0° data were based on only 90 tracks.

For an alpha-particle energy of 1.4 Mev, a broad resonant state of C^{13} is excited. In order that definite calculations concerning the angular distributions of the neutrons can be made, it is assumed that all neutrons observed emanate from this excited state. Further, only the lowest orbital angular momentum conserving parity will be considered. Since only the ratios at two angles are known, it is unprofitable to carry out the calculations for large angular momenta of the Be^9 and alpha-particle pair. Consequently, the analysis was made only for a p-wave interaction of this pair. When data at more angles and energies become available, more precise and extensive calculations can be made. For the p-wave interaction, the C^{13} excited state will have even parity and a J value of 1/2, 3/2, or 5/2, if odd parity and a spin of 3/2 for Be^9 are assumed. The value 1/2 leads to spherical symmetry of both groups of neutrons and need not be further considered. Table I shows the results of the analysis for $J=3/2$ and 5/2 of the C^{13} compound nucleus for four possible choices of angular momentum and parity of the first excited state of C^{12}. The orbital angular momentum for each spin state of the short-range neutrons and C^{12*} is indicated, as well as the angular distribution of each group of neutrons. The ratio of distributions is normalized to 1.6 at 90°, and the ratio at 0° determined with this normalization. When the orbital angular momentum of the short-range neutrons and C^{12*} is the same for both spin states, only a lower and upper limit of the 0° ratio can be determined. As can be seen, only the C^{13} compound nucleus with angular momentum $(5/2)\hbar$ and $J=2$, even

[20] C. E. Bradford and W. E. Bennett, Phys. Rev. **78**, 302 (1950).

or odd parity, for C^{12*} are consistent with the experimental data. All calculations were made for the center-of-mass system, since the difference of laboratory and center-of-mass systems is within the experimental uncertainty.

Further information on the properties of the first excited state of C^{12} can be obtained from the $N^{15}(p, \alpha)C^{12*}(\gamma)C^{12}$ reaction. In the bombardment of N^{15} by protons, resonant states of the O^{16*} compound nucleus occur at proton energies of 0.9, 1.0, and 1.2 Mev.[21] For the 0.9-Mev resonance, the reaction leading to the excited state of carbon has a cross section of 0.6 barn, while the reaction leading to the ground state of carbon does not appear. The 1.0-Mev resonance is associated with the breakup into an alpha-particle and C^{12} in the ground state with a cross section of 0.5 barn, the reaction leading to the excited state only of the order 2 millibarns. Thirteen-Mev gamma-rays can also be produced in this reaction. For the resonance at 1.2 Mev, both states of C^{12} can result, the cross section being 0.6 and 0.2 barn for the reaction leading to the ground and excited states of C^{12}, respectively.

The only information at present available concerning these reactions, which can be of use in determining the angular momentum and parity of the excited state of C^{12} is the angular distribution of the gamma-rays associated with the 0.9-Mev resonance. Wilkinson found this angular distribution to be of the order $1+0.3\cos^2\alpha$. This information was obtained by Hornyak and Lauritsen for their review article on energy levels of light nuclei,[22] and has not yet appeared in print elsewhere.

Again using a second-order transition process,[18] one can determine the statistical weights of the magnetic substates of the residual nuclei as well as the angular distribution of the light particles given off. The angular distribution of the gamma-rays from the residual nuclei will be determined by the weighting of the substates. The distribution of gamma-quanta for a given change in total angular momentum and a given change in magnetic quantum number has been tabulated by Arnold.[23]

Since there is zero cross section for the reaction leading to the ground state of carbon by alpha-emission at the 0.9-Mev resonance, the O^{16*} state cannot have even (odd) parity if its J value is even (odd). Thus, this excited state must be one of the following: $J=0^-$, 1^+, 2^-, 3^+, ... The first of these would lead to no asymmetry and the latter to an asymmetry of higher powers of $\cos\alpha$ than experimentally observed. The two intermediate values were tested with four possibilities of angular momentum and parity for the excited state of C^{12}. The results are shown in Table II. Assuming a spin of $\frac{1}{2}$ and odd parity for the ground state of N^{15}, one finds spin states of one or zero units of angular momentum, $S=1$ or $S=0$, may be responsible for the formation of the O^{16} compound nucleus. If one uses the lowest L-wave of a definite parity for the formation and breakup of the compound nucleus, he finds the only parameter left undetermined is the ratio of the amounts of each spin state entering the reaction. $S=1$ corresponds to a singlet combination of the intrinsic spins of proton and the odd neutron of N^{15}, while $S=0$ corresponds to the triplet combination of these spins. The column A/B gives the ratio of the weight given to the state $S=1$ to that for $S=0$ necessary to give a gamma ray distribution of $1+0.3\cos^2\alpha$. A value of $A/B\gg1$ indicates little spin-orbit coupling. The final column of the table gives the angular distribution of the alpha-particles in the reaction obtained by using the value of A/B in the previous column, where β is the angle between the alpha-particle direction and that of the incident beam.

The calculations for Table II were made in the center-of-mass system, but here again the difference of center-of-mass and laboratory systems is very small. As seen in the column A/B, the values of the angular momentum of the excited state of C^{12} fitting the data with the least amount of spin orbit coupling are $J=2^+$ and $J=1^-$. Although it is not conclusive, the analysis of this reaction with that discussed immediately preceding it strongly suggest that the 4.47-Mev excited state of C^{12} has $J=2^+$. The reaction can yield further information or confirmation if one is able to obtain the angular distribution of the alpha-particles, which have an energy approximately 1 Mev. The angular distributions of the products of the reaction at a proton energy of 1.2 Mev could also prove useful in determining the J values of the excited states concerned.

The existing data from the reactions discussed show the properties described by the alpha-particle model for C^{12}, but an analysis of the new data on the gamma-ray distributions from the transmutation of B^{11} by 163-kev protons[17] must still be made and found to be in agreement before one can consider the model valid.

TABLE II. Results of the analysis of the $N^{15}(p, \alpha)O^{16*}$ reaction.

O^{16*}	S	C^{12*}	L-wave of alpha	Ang. dist. of gamma-ray	A/B	Ang. dist. of alpha-particles
1^+	1	2^+	d	$7+3\cos^2\alpha$	10.0	$1+0.67\cos^2\beta$
1^+	0	2^+	d	$10-6\cos^2\alpha$		
1^+	1	2^-	p	$9-3\cos^2\alpha$	0.74	$1+0.11\cos^2\beta$
1^+	0	2^-	p	$6+6\cos^2\alpha$		
1^+	1	1^+	s	$1+\cos^2\alpha$	3.71	Isotropic
1^+	0	1^+	s	$2-2\cos^2\alpha$		
1^+	1	1^-	p	$3-\cos^2\alpha$	0.74	$1-0.45\cos^2\beta$
1^+	0	1^-	p	$1+\cos^2\alpha$		
2^-	1	2^+	p	$3+15\cos^2\alpha-16\cos^4\alpha$	1.5	$1-0.78\cos^2\beta$
2^-	0	2^+	p	$6-18\cos^2\alpha+24\cos^4\alpha$		
2^-	1	2^-	s	$1-3\cos^2\alpha+4\cos^4\alpha$	None	...
2^-	0	2^-	s	$6\cos^2\alpha-6\cos^4\alpha$		
2^-	1	1^+	p	$9-3\cos^2\alpha$	None	...
2^-	0	1^+	p	$10-6\cos^2\alpha$		
2^-	1	1^-	d	$7+3\cos^2\alpha$	*	$3-6\cos^2\beta+7\cos^4\beta$
2^-	0	1^-	d	$6+6\cos^2\alpha$		

* Best possible fit is $1+0.43\cos^2\alpha$.

[21] Schardt, Fowler, and Lauritsen, Phys. Rev. 80, 136 (1950).
[22] Hornyak, Lauritsen, Morrison, and Fowler, Revs. Modern Phys. 22, 291 (1950).
[23] W. R. Arnold, Phys. Rev. 80, 34 (1950).

Be^9

The position of the low energy levels of Be^9, based on the alpha-particle model, can be obtained by analogy with the vibration-rotation energy level scheme of the carbon dioxide molecule and the use of the Pauli exclusion principle. The carbon dioxide molecule was found to be linear, the two oxygen atoms flanking the carbon atom. Of the possible normal vibrations of this molecule, two are degenerate. Let the figure axis be that axis passing through the three atoms when the molecule is in its ground state. Then, in either of two planes perpendicular to each other and passing through the figure axis, there may be a vibration in the plane consisting of the motion of the two oxygen atoms perpendicular to the figure axis, the carbon motion also perpendicular to the figure axis, but in the opposite direction to the motion of the oxygen atoms. Since the energy level for this vibration is the same for motion in either plane, the system is degenerate and the most general motion of this type is a linear superposition of the two degenerate vibrations, a motion which can be regarded as a rotation about the figure axis. Further, when the rotation-vibration spectrum is considered, rotations about an axis perpendicular to the figure axis are possible. For Be^9 the two alpha-particles correspond to the oxygen molecules, the neutron to the carbon atom. If symmetry properties are neglected, the energy levels for this model are given by:[24]

$$E=(\hbar^2/m_\alpha r^2)[J(J+1)-K^2]. \quad (9)$$

The notation described for C^{12} is used. When applying this model to Be^9, one must bring in the effect of the Pauli exclusion principle, requiring the neutron outside the two alpha-particles to be in a p-wave. This requires that one use energy levels of Eq. (9) with $|K|=1$. A rotation of 180° about an axis perpendicular to the figure axis and passing through the center of mass interchanges the two alpha-particles. Wave functions symmetric under this operation can be found for all J greater than or equal to one. All these functions have odd parity, showing a sign change for the transformation in which all particles are reflected through the origin, the center of mass.

If one omits for the moment the intrinsic spin of the neutron, he finds the ground state for this model has $J=1$, $|K|=1$, and the first excited state has $J=2$, $|K|=1$, both states have odd parity. Since $|K|$ is the same for both states, the separation of states is that found for a simple dumbbell rotator. If the separation of the two alpha-particles were the same as found for Be^8, an excited state of Be^9 would appear at about 2.2 Mev above the ground state. If the intrinsic spin of the neutron is included, the ground state of Be^9 is presumed to have a spin of 3/2; and for the alpha-particle model, the first excited state would have a spin of 5/2.

A virtual state of Be^9 at 2.42 Mev was first found by Davis and Hafner[25] in the inelastic scattering of protons by Be^9 at proton energies of 3.5 and 7.1 Mev. The data are insufficient to indicate the angular momentum and parity of this state.

An analysis of the $Be^9(\gamma, n)Be^8$ yield has been made by Guth and Mullin,[26] who found no level at 2.42 Mev, but rather a level at 1.5 Mev, just bound, to which their analysis assigned the character ${}^2S_{\frac{1}{2}}$ based on a ground state ${}^2P_{\frac{3}{2}}$. The experimental yield shows a maximum immediately above threshold ($\sim$1.6 Mev), a minimum at $\sim$2.5 Mev, and a subsequent increase for energies above that. The 2.42-Mev level could have remained undetected, since the energy separation of experimental data is large compared with the width of the level.

However, the alpha-particle model can also explain its nonappearance. On the assumption that the 2.42-Mev state is the result of a rotational state of the alpha-particles, as given by the model, the photodisintegration would have to occur by a magnetic dipole or electric quadrupole transition, if this state were to appear in such a process. Guth and Mullin have calculated the cross section for the magnetic dipole transition and found it to be $\sim$0.01 that for the electric dipole. If the electric quadrupole transition were as weak as that for magnetic dipole, either of these transitions would not be observable in the yield due to electric dipole transitions.

One must conclude that the alpha-particle model for Be^9 is in accord with the existing experimental data both from energy considerations and from properties of the levels such as angular momentum and parity.

C^{13}

The alpha-particle model for the C^{13} nucleus has the three alpha-particles situated at the vertices of an equilateral triangle, as with C^{12}, with the additional neutron having a p-wave function with a node in the plane of the alpha-particles. In so far as rotational motion of the model as a whole is concerned, the properties of the symmetrical top described this motion. Aside from symmetry properties and the change in moment of inertia caused by the additional neutron, C^{13} is similar to C^{12} in its rotational properties.

In determining the symmetry properties for this model, rotations about the figure axis have the same symmetry as that for C^{12}, since the neutron lies on the figure axis. The wave function describing the complete system is to a first approximation a product of the eigenfunction of the neutron and the rotational eigenfunction of the complete system. A rotation of 120° about the figure axis is equivalent to interchanging pairs of alpha-particles twice, and must therefore leave the wave function symmetric. This can be obtained for $K=3n$ for integral n. A rotation of 180° about an axis in the plane of the three alpha-particles and passing through the center of mass and one of the alpha-particles, followed by a reflection of the neutron in the plane of the alpha-particles, will interchange two of the alpha-

[24] D. M. Dennison, Revs. Modern Phys. 3, 280 (1931).
[25] K. E. Davis and E. M. Hafner, Phys. Rev. 73, 1473 (1948).
[26] E. Guth and C. J. Mullin, Phys. Rev. 76, 234 (1949).

particles. Since the neutron reflection will always produce a change of sign in its wave function, only states in which the rotational eigenfunction changes sign for the 180° rotation described will be allowed by the Bose statistics. For n integral and non-zero, rotational eigenfunctions odd under this rotation can be found for all $J \geq |K|$. However, for $K=0$, only rotational eigenfunctions for odd J will be odd for the rotation described, since these eigenfunctions are the spherical harmonics.

The parity operator is equivalent to 180° rotation about the figure axis, followed by a reflection of the neutron wave in the plane of the alpha-particles. The latter always produces a change in sign, the former being even or odd as K is even or odd. Therefore, the parity of a state is odd for even K and even for odd K.

With these symmetry restrictions, the ground state has $J=1$, $K=0$, the first excited state $J=3$, $|K|=3$, if the neutron intrinsic spin is neglected. Neglecting the inertial effect of the neutron, and assuming the same separation of alpha-particles as used for C^{12}, one places the first excited state of the model at 4.2 Mev. Since an alpha-particle is more tightly bound in C^{13} than C^{12}, the separation of the alphas of the former might be smaller than for the latter nucleus. However, the neutron will tend to increase the moment of inertia, so the total effect is difficult to estimate. A search of the available data on the excited states of C^{13} brought forth no evidence concerning the parity or angular momentum of any of these states, with the exception of the level at 3.1 Mev.

The level at 3.1 Mev has been shown by Thomas[27] to emit electric dipole radiation to the ground state. On this account alone the level is not identified with the first excited state of the alpha-particle model. Further, on the basis of the C^{12} data, the effect of the added neutron on the moment of inertia, and the necessary increase of separation of the alpha-particles over that for C^{12} seem a bit out of reason in order to have the first excited state of the model at 3.1 Mev. It is this excited state that is concerned in an anomaly of the reaction products in the bombardment of C^{12} by deuterons.[28] It had been hoped that a large angular momentum for this state might explain the predominance of short-range protons and gamma-rays produced over the production of neutrons and long-range protons. A large angular momentum for this state would allow the short-range protons to be emitted with less angular momentum than the other products and with a consequently greater yield owing to less barrier penetration. During the investigation, a possible factor in explaining the anomaly was found. For the reaction products C^{13} plus long-range proton or for N^{13} plus neutron, spin states of 0 or 1 unit of angular momentum are formed, $S=0^-$ or $S=1^-$, with odd parity. If the N^{14*} compound nucleus has the following angular momentum and parity, 0^+, 1^-, 2^+, 3^-, $\ldots$, it is impossible to have the reaction proceed to the spin state $S=0^-$. In this state the intrinsic spins of C^{13} and the proton are aligned and opposite the 1 unit of orbital angular momentum of C^{13}. For the bombardment of C^{12} by deuterons then, the $S=0^-$ spin state is reached with conservation of intrinsic spin. The reaction leading to the spin state $S=1^-$ requires a spin-orbit effect and may consequently be less intense than a reaction leading to an excited state of C^{13}, for which the above selection rule would not appear. An analysis of all data available for these transmutations of C^{12} by deuterons, taking account of the effects just described, is planned in the near future.

An association of the first excited state of the alpha-particle model is made with the C^{13} excited state at 3.9 Mev, found by Gove,[29] since it appears most reasonable from the moment of inertia expected. However, nothing more definite can be said at present.

[27] R. G. Thomas, Phys. Rev. **80**, 138 (1950).

[28] Bonner, Evans, Harris, and Phillips, Phys. Rev. **75**, 1401 (1949).

Be^{10} AND C^{14}

The ground-state spin of Be^{10} is presumed to be zero, and that for C^{14} is known to be zero. The alpha-particle model then has the angular momentum of each of the external neutrons in opposite directions. The characteristics of Be^{10} and C^{14} then are the same as found for Be^8 and C^{12}, the ground and first excited states of both having zero and two units of angular momentum, respectively, with even parity.

Little experimental information other than position of excited states is known for either Be^{10} or C^{14}. The first excited state of the alpha-particle model for Be^{10} is tentatively assigned to the experimentally observed level at 3.37 Mev from the $Be^9(d, p)Be^{10*}$ reaction.[30] Since an alpha-particle is more tightly bound in Be^{10} than in Be^8, the value 3.37 Mev for the first excited state is reasonable when comparison is made with Be^8.

The lowest experimentally observed excited state of C^{14} is $\sim$5.7 Mev above its ground state.[31] The constants necessary for the fitting of the model to this level, while not completely unreasonable, are somewhat out of line with those of the other carbon isotopes. No definite conclusions concerning the applicability of the alpha-particle model can be reached for this nucleus.

ACKNOWLEDGMENTS

The writer would like to express his thanks to Professor C. L. Critchfield for suggesting this problem, and for his continual encouragement and advice throughout its progress. Appreciation is also due Mr. W. R. Stratton for discussions concerning the experimental aspect of the problem.

This research was assisted by the University of Minnesota Technical Research Fund subscribed to by General Mills, Inc., Minneapolis Star Journal and Tribune Company, Minnesota Mining and Manufacturing Company, and the Northern States Power Company.

[29] H. E. Gove, Phys. Rev. **81**, 364 (1951).

[30] W. W. Buechner and E. N. Strait, Phys. Rev. **76**, 1547 (1949).

[31] R. G. Thomas and T. Lauritsen, Phys. Rev. **78**, 88 (1950).