Recent Investigation of the Shapes of β-Ray Spectra†

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INTRODUCTION

In the history of β-ray spectroscopy, the year 1949 may be called a year of great prosperity. Not only were many laboratories engaging in active researches in the field of β-ray spectroscopy, but many interesting and significant findings were actually uncovered. Most important of all, some happy agreements have been reached over certain discrepancies which had worried physicists for a long time. Those theoretical physicists who toiled so long on the theory of β-ray decay probably felt for the first time some satisfaction over the good behavior of both the allowed and the forbidden spectra.

FERMI'S THEORY OF BETA-DECAY††

Pauli's Neutrino Hypothesis

The currently accepted picture of the β-process was introduced by E. Fermi[1] in the year 1934. In this process, Pauli's neutrino hypothesis occupies a central position. To explain the apparent non-conservation of energy of the continuous β-spectrum, the neutrino hypothesis postulates that an additional particle, the neutrino, is produced in β decay and carries away the missing energy. In order to account for the fact that this particle is very difficult to detect, it is postulated that it is electrically neutral and of very small mass. The basis of introduction of this new particle was founded rather on a negative ground, but the success of Fermi's theory of β-decay and the results of beta-recoil experiments undoubtedly gave positive support to the neutrino hypothesis.

TRANSFORMATION OF NUCLEONS

The present-day nuclear theory concludes that a nucleus consists of neutrons and protons only. The electron does not exist in nucleus. The β-particles which are emitted by radioactive nuclei must be created in the moment of emission. Thus the β-disintegration can be considered as consisting of the transformation of one neutron into one proton, one electron and one anti-neutrino

$$n^1 = H^1 + e^- + \nu^*.$$

Similarly, a radioactive process in which a positron is emitted is to be considered as

$$H^1 = n^1 + e^+ + \nu.$$

FIVE FORMS OF INTERACTION

In order to calculate the probabilities of these processes, a new interaction between the nucleon and the two light particles (electron and neutrino), in analogy with the interaction between charges and electromagnetic field in quantum electrodynamics must be introduced. Once the interaction term is assumed, the problem of finding the probability of a β-transition is answered by the ordinary non-stationary perturbation theory. The difficult part of the entire problem consists of the choice of the interaction terms. Fermi has proposed the simplest possible interaction satisfying the essential requirements. Actually there are altogether five different interactions which satisfy the invariance requirement. They are

(1) Scalar
$$S = (U_f^*\beta O U_i)(\psi_e^*\beta\varphi_\nu);$$

(2) Polar four vector
$$V = (U_f^* O U_i)(\psi_e^*\varphi_\nu) - (U_f^*\alpha O U_i)(\psi_e^*\alpha\varphi_\nu);$$

(3) Tensor
$$T = (U_f^*\beta\sigma O U_i)(\psi_e^*\beta\sigma\varphi_\nu) - (U_f^*\beta\alpha O U_i)(\psi_e^*\beta\alpha\varphi_\nu);$$

(4) Axial vector
$$A = (U_f^*\sigma O U_i)(\psi_e^*\sigma\varphi_\nu) - (U_f^*\gamma_5 O U_i)(\psi_e^*\gamma_5\varphi_\nu);$$

(5) Pseudoscalar
$$P = (U_f^*\beta\gamma_5 O U_i)(\psi_e^*\beta\gamma_5\varphi_\nu).$$

Where U_i, U_f, ψ_e, and φ_ν are the wave functions of the initial nucleon, final nucleon, electron and neutrino, O is an operator which causes the nucleon to make the transition. β, α, and γ_5 are Dirac operators and σ is the usual spin operator.

TWO APPROXIMATIONS FOR ALLOWED TRANSITIONS

I.

The eigenvalues of the operators α and γ_5 are known to have values of the order of v/c. The velocities of nucleons v in the nucleus are of the order of $C/10$ (where C is the velocity of light). Therefore, the second term in the interaction forms V, T or A has a magnitude only 1/10 as much as that of the first term (only 1/100 on

† The greater part of this article was presented as an invited paper at the Annual Meeting of the American Physical Society in New York, 1950.
†† An excellent review article by E. J. Konopinski, Rev. Mod. Phys. 15, 209 (1943).
[1] E. Fermi, Zeits. f. Physik 88, 161 (1934).

squaring) and can be neglected in calculations for allowed transitions.

II.

Furthermore, the neutrino practically does not interact with matter. Let us temporarily also forget the effects of the Coulomb field on electron for simplicity. Hence plane waves can be assigned to both electron and neutrino

$$\psi_e = A \exp\left[i\frac{p}{\hbar}\cdot r\right] \quad (1) \qquad \varphi_\nu = B \exp\left[-i\frac{q}{\hbar}\cdot r\right]. \quad (2)$$

These are written in expanded forms and substituted into the following functions:

$$(\psi_e^* \overset{\beta}{\underset{\sigma}{1}} \varphi_\nu) = (A \overset{\beta}{\underset{\sigma}{1}} B)\left[1 - i\frac{p+q}{\hbar}\cdot r - \frac{1}{2}\left(\frac{p+q}{\hbar}\cdot r\right)^2 \cdots\right], \quad (3)$$

and

$$(\psi_e \overset{\alpha}{\gamma}_5 \varphi_\nu) = (A \overset{\alpha}{\gamma}_5 B)\left[1 - i\frac{p+q}{\hbar}\cdot r - \frac{1}{2}\left(\frac{p+q}{\hbar}\cdot r\right)^2 \cdots\right], \quad (4)$$

where p and q are the momenta of electron and neutrino. The energy release usual in β-decay limits $(p+q)$ to values of only a few mc units. Although r is of nuclear dimensions and therefore of at most the order of $(1/40)(\hbar/\mathrm{mc})$, the successive terms in the expression of $(\psi_e^* \, {}^\beta 1_\sigma \, \varphi_\nu)$ are smaller by factors of at least 1/10 (only 1/100 on squaring). Therefore all terms except the first one in the expansion (3) can be neglected. In other words, for allowed transitions only the first term of this expansion is kept. The term $(\psi_e^* \alpha \varphi_\nu)$ or $(\psi_e^* \gamma_5 \varphi_\nu)$ is called the relativistic term and is neglected in allowed transitions with reasons explained in approximation (I). With these approximations, the computation of the probability of a β-transformation by the ordinary nonstationary perturbation theory becomes completely straightforward. The most striking feature of the results is that with these approximations the energy distribution of the β-particles for allowed transitions turns out identical whichever of the interaction terms is used.

SELECTION RULES

Nevertheless the various interaction forms do determine the selection rules with respect to spin and parity. For instance, the polar vector form of interaction originally adapted by Fermi led to the nuclear matrix element $|\int U_f^* 1 U_i d\tau|^2$ for allowed transitions. The identity operator 1 certainly does not alter the symmetry properties of U_f and U_i; hence no difference of total angular momentum or parity between U_f and U_i can be allowed. $\Delta I = 0$, no, where "no" indicates no parity change is allowed.

Since the operator β is a scalar just as 1, the same selection rules apply for allowed transitions. $\Delta I = 0$, no.

The pseudoscalar γ_5 changes sign in mirroring transformations; so the parity of $\gamma_5 U_i$ is opposite to the parity of U_i. Consequently $\Delta I = 0$ (yes). "Yes" is equivalent to saying yes, there is a parity change.

Selection rules very different from the original Fermi ones result from tensor and axial vector interactions.

The Pauli spin operator σ causes initial states U_i to overlap on final states U_f in accordance with the selection rules:

$$\Delta I = 0, \ \pm 1 \text{ (no) with } 0 \rightarrow 0 \text{ forbidden.}$$

These are known as *Gamow-Teller* or *G-T selection rules*.

THE ALLOWED SPECTRUM

The spectrum of the allowed β-transition can be given as

$$P(E)dE = \frac{g^2 m_0^5 c^4}{2\pi^3 \hbar^7}\left|\int U_f^* O U_i d\tau\right|^2 \times F(E, Z)(E_0 - E)^2 (E^2 - 1)^{\frac{1}{2}} E dE. \quad (5A)$$

Since in the magnetic β-ray spectrometer the number measured at a given magnetic field divided by momentum (η) is proportional to the number of β-particles per unit momentum, it is more convenient to write the above equation in the following form:

$$P(\eta)d\eta \sim \left|\int U_f^* O U_i d\tau\right|^2 \eta^2 (E_0 - E)^2 F(Z, \eta) d\eta, \quad (5B)$$

where g is a universal constant known as Fermi constant which represents the strength of the coupling giving rise to the transition. $\int U_f^* O U_i d\tau$ is called the nuclear matrix element and is expected to be independent of E and Z. $|\int U_f^* O U_i d\tau|^2$ is actually a measure of the "overlap" of initial and final nuclear states. E_0 is the maximum energy (including rest energy) of the electron. The term $\eta^2 (E_0 - E)^2$ is the statistical distribution of momentum between the electron and the neutrino which can be expected to emerge from almost any conceivable theory involving the sharing of an energy E_0 between a pair of particles. $F(Z, \eta)$ is the Coulomb correction factor to account for the distortion of the electron wave functions due to the nuclear Coulomb field. In Fermi's original paper, this is explicitly expressed by

$$F(Z, \eta) = \eta^{2S} e^{\pi\delta} |\Gamma(1 + S + i\delta)|^2, \quad (6)$$

where $s = (1 - \gamma^2)^{\frac{1}{2}} - 1$; $\gamma = Z/137$; $\delta = \gamma^{(E/\eta)}$.

However, there are no adequate tables of the complex Γ function available.‡ For very light elements, where

‡ Recently, I. Feister published a comparison study of numerical evaluation of the Fermi beta-distribution function with the results obtained from three approximation methods. Phys. Rev. **78**, 375 (1950).

relativistic correction is small, then the non-relativistic Coulomb wave function of electrons will give the non-relativistic Coulomb factor[2]

$$F_N(Z, \eta) = 2\pi\delta/1 - e^{-2\pi\delta}. \quad (7)$$

δ is positive for electrons and negative for positrons; therefore the Coulomb effect on them is very different. One would expect very few low energy e^+, but a lot more of electrons, at the low energy region. A better approximation, especially for large Z, was given by Bethe and Bacher[3]

$$F(Z, \eta) \sim F_N(Z, \eta)\eta^{2S}(\delta^2 + \tfrac{1}{4})^S = F_N(Z, \eta)[E^2(1+4\gamma^2) - 1]^S. \quad (8)$$

This approximation is accurate to about one percent for atomic numbers as large as $Z=84$. Actually, the atomic electrons also have an effect on the shape of the spectrum at low energies. This screening effect was calculated by M. E. Rose and C. Longmire and H. Brown and J. R. Reitz[4] and is positive for both positrons and electrons. It is particularly important for positrons for $E < 300$ kev and $Z > 25$.

KURIE OR FERMI PLOT

Now if one plots

$$\left[\frac{P(\eta)}{\eta^2 F(Z, \eta)}\right]^{\frac{1}{2}} \sim E_0 - E, \quad (9)$$

then the allowed β-spectrum should yield a straight line which cuts the E-axis at E_0. This is known as a "Kurie plot"[5] or "Fermi plot" and is used extensively in the investigation of the distribution of β-spectrum. A great stride[6] was made in 1939 when Lawson and Cork reported their work on In^{114} [7] and Tyler on Cu^{64}.[8] By using comparatively thin sources ($\sim$ a few mg/cm^2), they were able to demonstrate the good agreement between the experimental results and Fermi's original version of β-decay.

INVESTIGATION OF VERY LOW ENERGY ELECTRONS

Nevertheless, the investigation of very low energy electrons involves many difficulties. The most serious of these is the absorption and scattering effect in the finite and non-uniform source thickness and its backing material. The absorption effect of the counter window at very low energy region is also quite troublesome. While the major portion of a spectrum followed the Fermi distribution closely, there were always some deviations present in the very low energy region—below 200 kev. Had these discrepancies at the very low energy region remained real, it would have been necessary to revise Fermi's theory of β-decay.

S^{35} AND Cu^{64}

To clarify this situation, many laboratories concentrated their efforts on the low energy region of the spectra. The outcome from this phase of the work is satisfactory and gratifying. Cook and Langer[9] of Indiana first used their large, high resolution magnetic spectrometer to investigate the positron and electron spectra from Cu^{64}. This is an ideal case for a test of the theory of β-decay from the point of view that both positrons and negatrons are of nearly equal energy and with comparable intensity. The ratio of the positrons to electrons should be particularly free from errors due to elastic scattering. Their results showed that the Kurie plot of the positron spectrum is straight all the way down to 270 kev and that of the electron spectrum down to 190 kev. However, the deviations below these energies were concluded to be real and not instrumental, as it was hard for anyone to understand why a thin source of 100 μg/cm^2 should affect the spectrum at an energy as high as 200 kev. Since it is now known that a source prepared from solution can have local variation in thickness as much as 100 to 1 due to crystallization, the average thickness of a non-uniform source of 100 μg/cm^2 is naturally rather meaningless. The Columbia Group investigated S^{35} [10] and Cu^{64} [11] spectra with the high transmission solenoidal spectrometer (Fig. 1) and particularly directed their attention to uniformity of the sources. They observed a gradual but consistent reduction of deviation *versus* the source thickness in the low energy region. In the case of S^{35}, when a carrier-free source was used, they were able to obtain a Kurie plot straight all the way down to 20 kev where the effect of window absorption sets in (see Fig. 2). In the case of Cu^{64}, in order to avoid the crystallization effect of a solution, a fine colloidal suspension of Cu^{64} was used. The spectra obtained from the thinnest and most uniform source prepared (<100 μg/cm^2) showed (see Fig. 3) much less deviation in the low energy region than that previously reported in other laboratories (about 1 to 4) (Fig. 3). In particular, the experimental value of the ratio of the number of positrons to the number of negatrons *versus* energy agreed beautifully with the theoretical value predicted by the Fermi theory of β-decay. This is in contrast with the large discrepancies observed by Backus[12] and Cook and Langer[9] (Fig. 4).

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[2] Mott and Massey, *Theory of Atomic Collisions* (Oxford University Press, New York, 1933); Kurie, Richardson, and Paxton, Phys. Rev. **49**, 368 (1936).

[3] H. A. Bethe and R. F. Bacher, Rev. Mod. Phys. **8**, 194 (1936).

[4] M. E. Rose, Phys. Rev. **49**, 727 (1936); C. Longmire and H. Brown, Phys. Rev. **75**, 264 (1949). J. R. Reitz, Phys. Rev. **77**, 10 (1950).

[5] Kurie, Richardson, and Paxton, Phys. Rev. **49**, 368 (1936).

[6] The excellent review article by E. J. Konopinski, Rev. Mod. Phys. **15**, 209 (1943).

[7] A. W. Lawson, Phys. Rev. **56**, 131 (1939); A. W. Lawson and J. W. Cork, Phys. Rev. **57**, 982 (1940).

[8] A. W. Tyler, Phys. Rev. **56**, 125 (1939).

[9] C. S. Cook and L. M. Langer, Phys. Rev. **73**, 601 (1948).

[10] R. D. Albert and C. S. Wu, Phys. Rev. **75**, 847 (1948).

[11] C. S. Wu and R. D. Albert, Phys. Rev. **75**, 315 (1949); **75**, 1107 (1949).

[12] John Backus, Phys. Rev. **68**, 59 (1945).

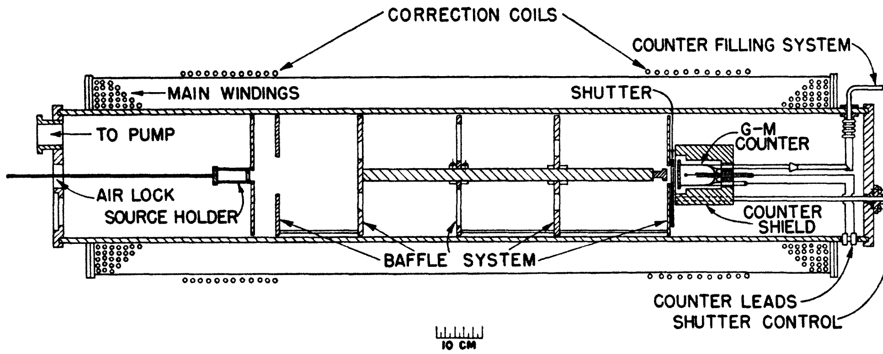

FIG. 1. Schematic diagram of the solenoidal magnetic spectrometer.

On the basis of the good agreement between the theoretical and experimental value of the ratios, Wu and Albert[11] concluded that the Fermi theory of β-decay probably does predict the true distributions for electrons and positrons at very low energies. A confirmation of this conclusion was recently reported by the independent work of Langer, Moffat and Price[13] of Indiana and Owen and Cook[14] of Washington University. In order to obtain a microscopic uniform source, they perfected a method of evaporating the active Cu^{64} metal onto a thin film in vacuum. The autoradiograph of this source showed complete uniformity. The Kurie plots of the data obtained from such thin and uniform sources ranging from a few $\mu g/cm^2$ to 75 $\mu g/cm^2$ show no deviation from the Fermi theory for all energies above 50 kev (Fig. 5). This is indeed a very happy conclusion for all concerned.

MORE EXPERIMENTAL VERIFICATION IN THE LOW ENERGY REGION

More experimental evidence on the validity of Fermi's allowed spectrum in the very low energy region was recently reported by Price, Motz, and Langer[15] of Indiana in that they found that the Kurie plots obtained for Pm^{147} and S^{35} were straight all the way down to 8 kev. The excess of particles at low energies was found to be a function of source thickness. Gross and Hamilton[16] of Princeton reported at the American Physical Society meeting in New York (1950) that they used a new type of electrostatic β-ray spectrograph to investigate the low energy region of the S^{35} spectrum. They also found that the Kurie plot was a straight line from 30 kev down to 7 kev. The deviation below 7 kev is attributed to backscattering and absorption in the source and backing.

PROPORTIONAL COUNTER METHOD AND H^3 SPECTRUM

In view of the straggling and reflection effects of a finite source and backing on the low energy electrons and the scattering and absorption effects of a G-M counter window, there is a limit in extending the investigation of the very low energy region below 10 kev by the conventional method. The most appropriate method for this purpose is probably to use proportional counters with the radioactive gas introduced directly inside the counter. Curran, Angus, and Cockcroft[17] of Glasgow and Hanna and Pontecorvo of Chalk River[18] investigated tritium in this manner and found that the Kurie plot is a straight line from its maximum energy 18.6 kev to 0.5 kev (Fig. 6). Since H^3 consists of a simple nucleus, its case should be a good test of the Fermi theory of β-

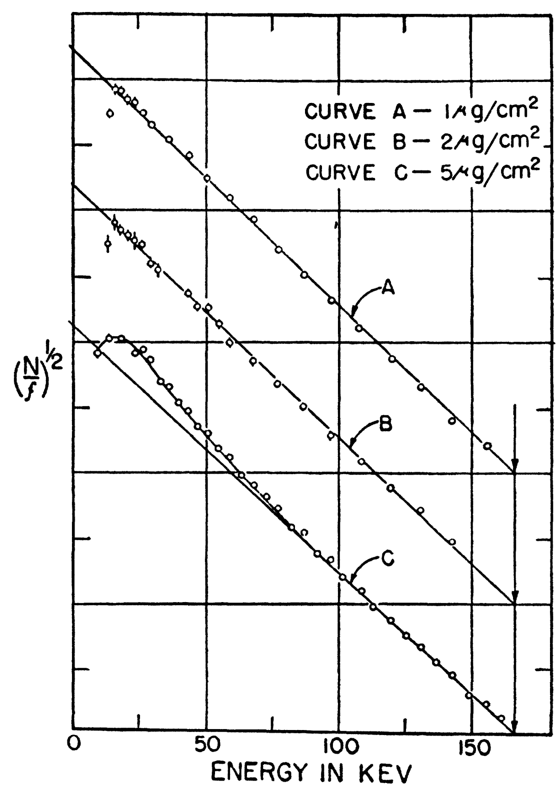

FIG. 2. Fermi plots of the beta-spectrum of S^{35}. (See reference 10.)

[13] Langer, Moffat, and Price, Phys. Rev. **76**, 1725 (1949).
[14] G. E. Owen and C. S. Cook, Phys. Rev. **76**, 1726 (1949).
[15] Price, Motz, and Langer, Phys. Rev. **77**, 743 (1950); Phys. Rev. **77**, 798 (1950).
[16] L. Gross and D. R. Hamilton, Phys. Rev. **78**, 318 (1950).
[17] Curran, Angus, and Cockcroft, Phil. Mag. **50**, 53 (1949); Nature **162**, 302 (1948).
[18] Hanna and Pontecorvo, Phys. Rev. **75**, 983 (1949).

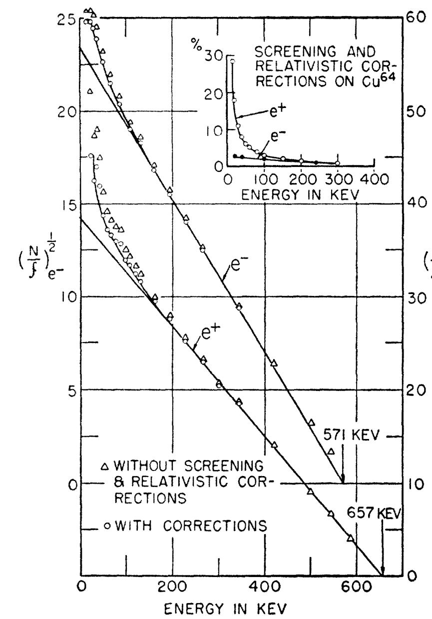

FIG. 3. Kurie plots of Cu^{64} negatron and positron spectra. (See reference 11.)

decay. Furthermore, because of its extremely low upper energy limit, the accurate shape of its spectrum near the upper end is very sensitive to a possible non-vanishing neutrino rest mass. Careful examination[19] for such an effect in the tritium β-ray spectrum has narrowed down the neutrino mass between 0 and 1 kev (Fig. 7). Therefore, the neutrino mass is negligible in most applications of the β-decay theory.

THE CASE OF Cu^{61}

Among the β-spectra of allowed transitions, there was one baffling case which had been hard to understand. This case was that of the Cu^{61} spectrum. Cook and Langer[20] used a Cu^{61} source much thinner than 0.1 mg/cm² mounted on 0.02 mg/cm² zapon. The deviation observed was much larger than in Cu^{64}, beginning at about 500 kev. Owen and Cook[21] reinvestigated the spectrum of Cu^{61} by evaporating carrier-free Cu^{61} onto an aluminum backing and found the same pronounced deviation starting at about 500 kev. However, they repeated the Cu^{64} investigation[14] by using this improved technique of source preparation and were able to eliminate most of the low energy deviations which had appeared in previous work.[9] This strongly suggested to them that the deviation present in the Cu^{61} positron spectrum could very well be due to the complex nature of its spectrum. If such is the case, there must be one or several very weak nuclear gamma-rays associated with this transition. Indeed, F. Boehm's[22] group in Switzerland and Owen and Cook[23] searched for these nuclear gamma-rays and found that they are there. The three gamma-rays are 0.652, 0.279, and 0.070 Mev. The complexity of the spectrum probably arises from a positron transition to a state of Ni^{61} 0.652 Mev above the ground state. The end-point energy of this spectrum would be $1.205-0.652=0.553$ Mev. This end-point energy of this low energy group is in agreement with the deviations reported previously at an energy as high as 511 kev.

Therefore one more riddle is neatly solved.

BETA-SPECTRUM OF He^6

The β-radiation from $He^6(He^6\rightarrow Li^6+\beta^-+\nu^*)$ was one of the historic cases which first suggested the Gamow-Teller selection rules. He^6 emits β-rays of a maximum energy of more than 3 Mev with a half-life of less than one sec. Its ft value, of the order of 10^2–10^3 sec., indicates that it undergoes a transition of superallowed case. On the other hand, He^6 may be considered as consisting of one α-particle and two neutrons. The α-particle has spin equal to zero and the two neutrons in the ground state should have spin zero. The spin of He^6 should therefore be expected to be zero, as is the case for all the known nuclei consisting of even number of neutrons and protons. Li^6 may be considered as made up of an α-particle and a deuteron. The spin of the deuteron is 1; so Li^6 should have $I=1$. The change of spin involved in this transition is therefore $\Delta I=1$. The selection rules of allowed transition as given by the originally proposed interactions (scalar and polar vector form) known as Fermi's selection rules do not permit any change of spin

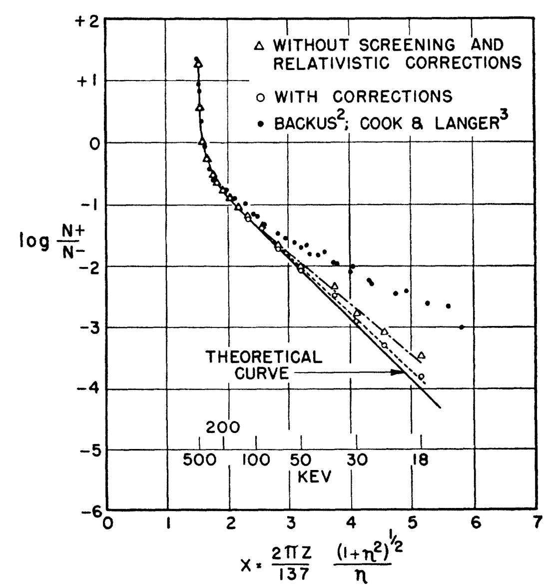

FIG. 4. The ratio of the number of positrons to the number of negatrons as a function of energy. (See reference 11.)

[19] Curran, Angus, and Cockcroft, Phys. Rev. **76**, 853 (1949).
[20] C. S. Cook and L. M. Langer, Phys. Rev. **74**, 227 (1948).
[21] G. E. Owen and C. S. Cook, Phys. Rev. **76**, 1536 (1949).
[22] Boehm, Blaser, Marmier, and Preiswerk, Phys. Rev. **77**, 295 (1950).
[23] G. E. Owen, C. S. Cook, and P. H. Owen, Phys. Rev. **78**, 686 (1950).

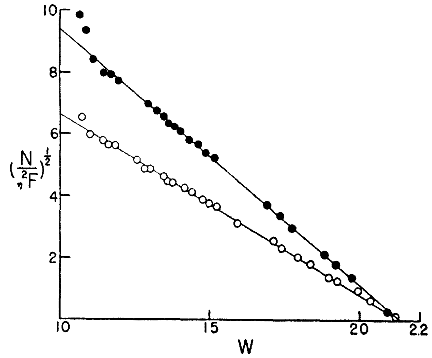

FIG. 5. Fermi plot of the negatron spectrum of Cu^{64}. Solid circles are for data obtained with the 75 $\mu g/cm^2$ source backed by 0.18 mg/cm^2 Al leaf. The open circles are for data obtained with the less than 5 $\mu g/cm^2$ source backed by 15 $\mu g/cm^2$ zapon. (See reference 13.)

units in the allowed transitions. The experimentally indicated allowed transition of He^6 certainly contradicts this class of selection rules. Gamow and Teller, therefore, pointed out that under tensor or axial vector interaction form a change of spin of one unit is permitted even in allowed transitions, owing to the special property of the spin operator σ used in those two interaction forms (see section under Selection Rules).

Much attention has been given to the case of He^6 ever since. But due to its fast decay rate and gaseous form, no accurate information on its upper energy value or its exact spectrum shape was available for a long time. Recently Brown and Perez-Mendez perfected a method of continuously circulating radioactive gas between an activation chamber inside of a cyclotron and a source chamber of a magnetic β-spectrometer. They were able for the first time to determine the upper energy limit of He^6 accurately to 3.215±0.015 Mev. Furthermore the β-spectrum yields a straight allowed Fermi plot from the upper end to the low energy region of 150 kev—a span of 95 percent of the whole energy range (Fig. 8).[23a]

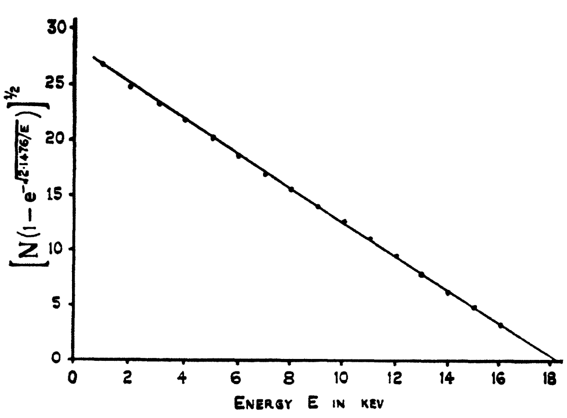

FIG. 6. Fermi plot for H^3. (See reference 17.)

[23a] Brown and Perez-Mendez, Phys. Rev. **75**, 1286 (1949); **77**, 404 (1950).

THE THREE SIMPLEST RADIOACTIVE NUCLEI n, H^3, AND He^6

In the theory of β-decay, the size of the nuclear matrix element involved is generally uncertain. In fact, very little can be said about the extent of overlapping between the initial and final wave functions as the nature of wave functions itself is not well known. However, for the simplest mirror nuclei such as n and H^3 and the simple nucleus He^6 the relative sizes of these nuclear matrix elements have been calculated by Wigner[24] on both Fermi and G-T selection rules. Since the upper energy limit and the half-life of H^3 and He^6 are now much better known, it is interesting to see how close an agreement between the value of M^2ft of H^3 and He^6 can be obtained. Although no exact agreement between them should be expected on account of the fact* that there is a change of spin involved in He^6, nevertheless it is interesting to see what range of half-lives would be predicted for neutrons from the known decay data of H^3 and He^6. Table I, lists the essential data on n, H^3, and He^6 and respective M^2ft values calculated according to both G-T and Fermi selection rules. The acceptable neutron half-life should be somewhere between 10 to 12 minutes as compared with preliminary experimental results of between 9–25 minutes.

SIGNIFICANCE OF FORBIDDEN SPECTRA

However, as it was pointed out at the beginning, the energy distribution of an allowed spectrum is mainly determined by the statistical distribution of momentum between the electron and the neutrino. The good agreement between the theory and the experimental results of allowed spectra does not yield any information regarding the uniqueness of the five possible interaction forms of the Fermi type. On the other hand, it is expected that there are certain forbidden spectra whose distributions should be quite different from that of an allowed transition and the exact shape of the momentum distribution will help in determining the form of the appropriate interaction between the nucleon and the electron-neutrino field.

A FEW BRIEF REMARKS ON THE THEORY OF FORBIDDEN TRANSITIONS

Transitions which violate the allowed selection rules are described as forbidden. In such cases the nuclear matrix elements obtained with the use of the first term of (3), giving the allowed transitions, vanish. Only β-radiation which carried off the difference in angular momenta may be emitted, analogous to electromagnetic radiation of high multipole order. We must, therefore, take the second term of (3) and the first term of (4) together. In the case of first-forbidden transitions for polar vector V interaction, these two terms give the

* The author is indebted to Professor P. Morrisson and Dr. H. Brown for letting her use their results of investigation.

[24] E. P. Wigner, Phys. Rev. **56**, 519 (1939).

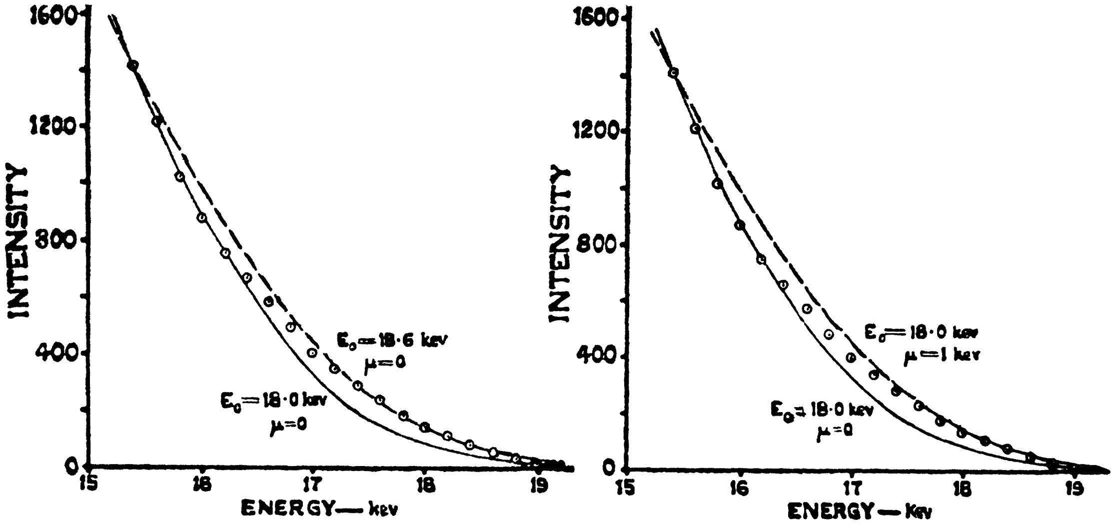

FIG. 7. Comparison of experimental and theoretical Fermi curves for tritium near the end-point. (See reference 17.)

matrix elements

$$\int U_f^* \frac{p+q}{\hbar} \cdot r U_i d\tau \quad \text{and} \quad \int U_f^* \alpha U_i d\tau.$$

The important facts about these matrix elements are that they are cut down in intensity by a factor $[(p+q)/\hbar \cdot r]^2 \sim 1/100$ or $\alpha^2 \sim 1/100$ and that the first matrix element $\int U_f^*[(p+q)/\hbar] \cdot r U_i d\tau$ is now definitely dependent on the energies of the emitted electrons. Furthermore, since $\int U_f^* r U_i d\tau$ and $\int U_f^* \alpha U_i d\tau$ are polar vectors, the selection rules are

$$\Delta I = 0, \pm 1 \text{ (no } o \rightarrow o\text{); yes.}$$

The G-T rules for first-forbidden transitions will be characteristic of quantities formed from σ and γ and α. The selection rules are

$$\Delta I = 0, \pm 1, \pm 2 \text{ (no } o \rightarrow o;\ 1 \leftrightarrow o,\ \tfrac{1}{2} \rightarrow \tfrac{1}{2}\text{); yes.}$$

Of course, in case that neither the allowed nor the first-forbidden selection rules of the above section are obeyed, one must continue introducing higher powers of r from (3) and (4), thus obtaining selection rules characteristic of second-forbidden, third-forbidden, etc. transitions. The first theoretical calculation of the forbidden spectra was done by Konopinski and Uhlenbeck.[25] The final results are given in the form of a "correction factor" C by which the allowed distribution must be multiplied to give a forbidden spectrum. These correction factors C are not only more or less energy dependent but also depend on the type of interaction assumed. Thus it is expected that there should be certain forbidden transitions whose spectra should be markedly different from that of allowed ones, and by carefully investigating the exact shape of its distribution, some information regarding the type of interaction may be obtained.

[25] Konopinski and Uhlenbeck, Phys. Rev. **60**, 308 (1941).

However, for first-forbidden transitions, there are particular complications. Since the relativistic term $\int U_f^* \alpha U_i d\tau$ or $\int U_f^* \gamma_5 U_i d\tau$ which occurred in V, T, and A interactions is independent of the energy of the β-particles. If the contribution from these relativistic terms is much larger than the energy dependent term of $\int U_f^* r U_i d\tau$, then the first-forbidden spectrum will exhibit allowed shape. It is also possible, in the cases where the Coulomb potential energy Ze^2/r is large that the kinetic energy available to the electrons E is small; then the term $[(p+q)/\hbar \cdot r]^2$ may be replaced by terms like $(\alpha Z)^2$, again independent of the energy. Therefore, it is theoretically possible for the first-forbidden spectrum to have allowed shape.

BETA-SPECTRUM OF RaE

Until about a year ago, the β-spectrum of RaE[26] (Fig. 9) had been the only one which was agreed upon to have a shape definitely different from that of an allowed transition. Although the shape of the RaE

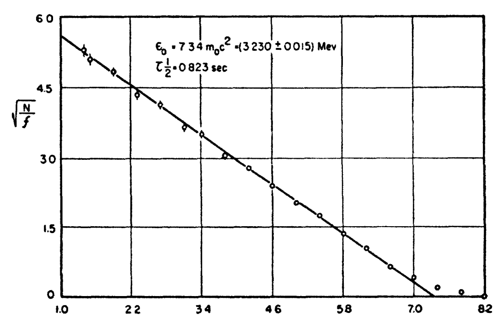

FIG. 8. Fermi plot for He^6 beta-spectrum (See reference 23a).

[26] Flammersfeld, Zeits. f. Physik **112**, 727 (1939); Neary, Proc. Royal Soc. **A175**, 71 (1940); L. M. Langer, Phys. Rev. **75**, 328 (1949); R. Morrissey and C. S. Wu, Phys. Rev. **75**, 1288 (1949).

spectrum can be accounted for fairly satisfactorily,[25] the explanation involves considerable arbitrariness in the evaluation of the several matrix elements that appear in the forbidden factor; therefore one cannot come to any definite conclusions regarding the type of interaction on the basis of RaE results alone.

UNIQUE FORBIDDEN SPECTRA OF TRANSITIONS INVOLVING $\Delta I=2$, YES

Then things started to happen! Langer and Price[27] of Indiana found that Y^{91}, a fission product, exhibits a definitely forbidden shape. Moreover, according to its comparative half-life, ($f_3 \sim 5\times 10^8$ sec.), this transition should be classified as second forbidden. But from the nuclear shell structure analysis,[28] this transition should involve a total momentum change of two units and a parity change, ${}_{39}Y^{91}$ ($p_{\frac{1}{2}}$ odd)$\rightarrow {}_{40}Zr^{91}$ ($d_{\frac{5}{2}}$ even), and from the G-T selection rules,[29] such a transition is theoretically classified as once-forbidden. According to the theory of forbidden spectra, worked out by Konopinski and Uhlenbeck and Greuling,[30] when the disintegration involves a total angular momentum change of one unit higher than the degree of the forbiddenness, it has the special property that only one type of nuclear matrix element fails to vanish for it. This means that a unique energy dependence is predicted, differing from the allowed shape by the factor

$$\alpha \sim p^2+q^2 \sim (E^2-1)+(E_0-E)^2.$$

Here E is the electron energy and E_0 is the total energy release, both in units of mc^2. The factor α emphasizes the relative number of high energy particles and may also emphasize low energy particles if $E_0>2$. Therefore, the uncorrected Kurie plot tends to bulge upward at high energy and may also curve upward at low energy,

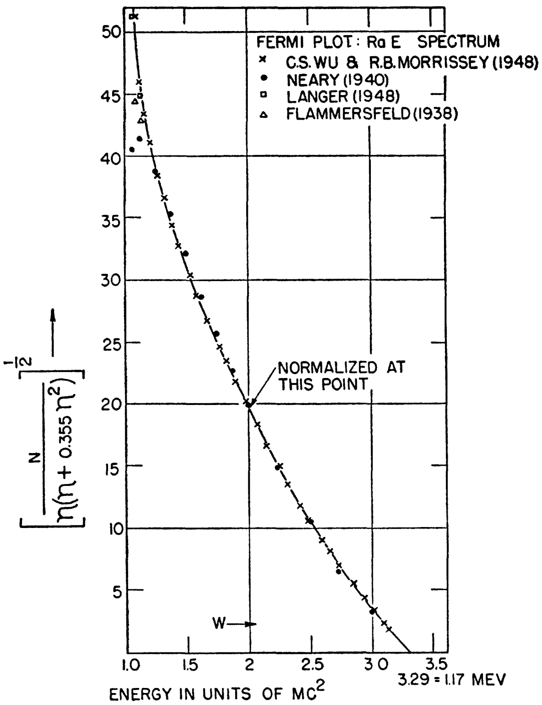

FIG. 9. Fermi plot for RaE beta-spectrum (Wu and Morrissey).

thus producing an inversion point at $E=\frac{1}{2}E_0$ (Fig. 10).

It felt good to see a real forbidden spectrum after seeing so many allowed ones! And the maximum "It never rains but it pours" seems to apply in this case, too. Immediately following the discovery of the Y^{91} spectrum, it was reported by Indiana, Washington University, Iowa State College, and Oak Ridge National Laboratory that Y^{90},[31] Sr^{89},[32] Sr^{90}, Sr^{91},[31] Cs^{137},[33] Rb^{86},[34]

TABLE I. Data on n, H^3 and He^6.

Z following decay		E_0 (Mev)	E_0 (mc^2)	$f=\int_1^{E_0} E\eta(E_0-E)^2 \times F(Z,E)dE$	$t_{\frac{1}{2}}$ sec.	$ft_{\frac{1}{2}}$ sec.	$M^2_{\text{G-T}}$	M^2_F	$M^2 ft_{\text{G-T}}$ sec.	$M^2_F ft$ sec.
1	n^a	0.790	2.545	1.65	540	875	$\frac{3}{4}$	$\frac{1}{4}$	656	219
		0.783	2.532	1.62	1500	2430	$\frac{3}{4}$	$\frac{1}{4}$	1822	607
2	H^{3b}	0.0186	1.0363	2.86×10^{-6}	3.94×10^8	1125	$\frac{3}{4}$	$\frac{1}{4}$	844	271
3	He^{6c}	3.215	7.300	710	0.823	584	$\frac{3}{2}$	0	877	0

[a] The neutron decay energy 0.790 Mev is that calculated by using the new binding energy of the deuteron to determine the neutron mass and may be in error by several percent. The value of 783±1 or 2 kev is given from the Los Alamos work, without use of any mass spectrographic data (Tollestrup Group, in press). The $ft_{\frac{1}{2}}$ value is calculated by using Los Alamos E_0 value. The half-lives given are 9 and 25 min., the extreme values listed by the Chalk River experiments.

[b] The tritium end-point energy used was 18.6 kev, which may be in error by as much as 0.5 kev (2.4 percent introducing an eight percent error in f). The half-life of 12.46 years may also be in error by four percent or so.

[c] The He^6 end-point energy and half-life have both been recently measured in Columbia Laboratory to accuracies of about 0.3 percent and 0.5 percent respectively, so that the error in ft for He^6 should not exceed two percent. However, the value of M^2 of He^6 is probably a little less than that calculated on account of the changing of the nuclear wave function of the two odd nucleons from a singlet to a triplet.

[27] L. M. Langer and H. C. Price, Jr., Phys. Rev. **75**, 1109 (1949).
[28] E. Feenberg and K. C. Hammack, Phys. Rev. **75**, 1877 (1949).
[29] G. Gamow and E. Teller, Phys. Rev. **49**, 895 (1936).
[30] E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. **60**, 308 (1941). E. Greuling, Phys. Rev. **61**, 568 (1942).
[31] E. N. Jensen and L. Laslett, Phys. Rev. **75**, 1949 (1949); Braden, Slack, and Shull, Phys. Rev. **75**, 1964 (1949).
[32] Slack, Braden, and Shull, Phys. Rev. **75**, 1965 (1949).
[33] A. C. G. Mitchell and C. L. Peacock, Phys. Rev. **75**, 197 (1949); C. L. Peacock and A. C. G. Mitchell, Phys. Rev. **75**, 1272 (1949).
[34] Zaffarano, Kern, and Mitchell, Phys. Rev. **74**, 682 (1948).

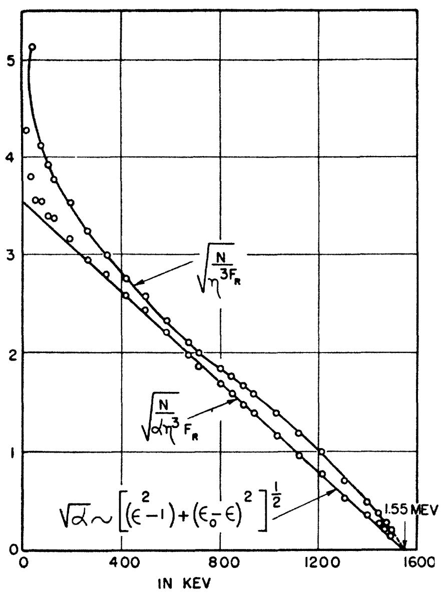

FIG. 10. Fermi plot for Y^{91} beta-spectrum (Wu and Feldman).

K^{42},[35] Cl^{38},[36] Sb^{124},[37] Sn^{123}, Sn^{125},[38] all exhibit the "α" type forbidden spectrum. The existence of this type of forbidden spectrum gives strong support to the G-T selection rules and to the theory of nuclear shell structure. Therefore, it seems clear that part of the true interaction must either be tensor or axial vector interaction.

It was also pointed out by Shull and Feenberg[39] that when the ft values of this class of transition multiplied by its corresponding value of (E_0^2-1) to account for the forbidden correction factor α, they are all of the order of 10^{10}. A table (Table II) of ft and $(E_0^2-1)ft$ values of these nuclei is prepared and given here for reference use. Furthermore, from the nuclear shell structure of the spin-orbit coupling scheme, the odd nuclei involved in this type of transition all undergo a change of orbit angular momentum of $\Delta L=1$ and total angular momentum $\Delta I=2$.[40] For example:

$_{38}Sr^{89}$ ($d_{5/2}$ even) $_{39}Y^{89}$ ($p_{1/2}$ odd)
$_{38}Sr^{91}$ ($d_{5/2}$ even) $_{39}Y^{91}$($p_{1/2}$ odd)
$_{39}Y^{91}$ ($p_{1/2}$ odd) $_{40}Zr^{91}$ ($d_{5/2}$ even)
$_{18}A^{41}$ ($f_{7/2}$ odd) $_{19}K^{41}$ ($d_{3/2}$ even).

TABLE II. $(E_0^2-1)ft$ values for beta-transitions with $\Delta I=2$. Yes.

Radio-active parent	$t_{1/2}$ (sec.)	E_0 (mc^2)	f	ft (sec.)	$(E_0^2-1)ft$
$_{17}Cl^{38}$	$2.2\times\frac{2}{1}\times10^3$	11	8.5×10^3	3.7×10^7	0.45×10^{10}
$_{19}K^{42}$	$4.5\times\frac{4}{3}\times10^4$	8	1.78×10^3	1.1×10^8	0.7×10^{10}
$_{37}Rb^{86}$	$1.7\times\frac{4}{5}\times10^6$	4.6	180	3.8×10^8	0.8×10^{10}
$_{38}Sr^{89}$	4.75×10^6	3.93	0.83×10^2	4×10^8	0.6×10^{10}
$_{38}Sr^{90}$	8×10^8	2.04	1.7	1.4×10^9	0.4×10^{10}
$_{38}Sr^{91}$	$3.6\times\frac{5}{3}\times10^4$	7.3	2.0×10^3	1.2×10^8	0.6×10^{10}
$_{39}Y^{90}$	2.25×10^5	5.40	4.7×10^2	1×10^8	0.3×10^{10}
$_{39}Y^{91}$	5×10^6	4.0	0.85×10^2	4.3×10^8	0.65×10^{10}
$_{55}Cs^{137}$	1×10^9	2.08	5.6	5.6×10^9	1.7×10^{10}
$_{18}A^{41}$	$6.6\times\frac{100}{0.7}\times10^3$	6	4×10^2	3.8×10^8	1.3×10^{10}
$_{35}Br^{84}$	1.8×10^3	11	1.4×10^4	2.5×10^7	0.3×10^{10}

BETA-SPECTRUM OF Cl^{36}

About the same time, the Columbia group (Wu and Feldman) received some radioactive Cl^{36} with a specific activity of only 0.05 μc/mg. Meantime, the new quadratic focusing baffle of the solenoidal spectrometer, designed according to Frankel's[41] and Persico's[42] theoretical investigations, had just been completed and tested. It was found to have increased the transmission about four times as compared with the old baffle (Fig. 11). A source of LiCl 0.1 mg/cm^2 was made and its spectrum[43] was investigated with the new baffle system. Even a glance at the momentum distribution curve (Fig. 12) shows a pronounced shift to the high energy end which indicates that it is a highly forbidden spectrum. The Kurie plot exhibits pronounced curvature toward the energy axis down to the very low energy region. This is the first shape of this kind ever observed among the forbidden spectra.

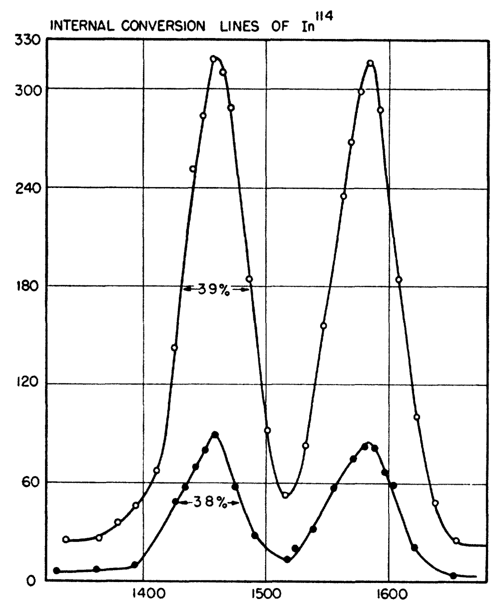

FIG. 11. Increase in transmission in a ring-focused baffle (Feldman and Wu).

[35] F. B. Shull and E. Feenberg, Phys. Rev. **75**, 1768 (1949).
[36] L. M. Langer, Phys. Rev. **77**, 50 (1950).
[37] L. M. Langer, private communication.
[38] C. A. Helmholz, private communication; Ketelle, Nelson, and Boyd, Phys. Rev. **79**, 242 (1950).
[39] Shull and Feenberg, Phys. Rev. **75**, 1768 (1949).
[40] L. W. Nordheim, Phys. Rev. **78**, 294 (1950).
[41] S. F. Frankel, and E. C. Nelson, ONR report NP-1120 June 1948, S. F. Frankel, Phys. Rev. **73**, 804 (1948).
[42] E. Persico, Rev. Sci. Inst. **20**, 191 (1949).
[43] C. S. Wu and L. Feldman, Phys. Rev. **76**, 693 (1949).

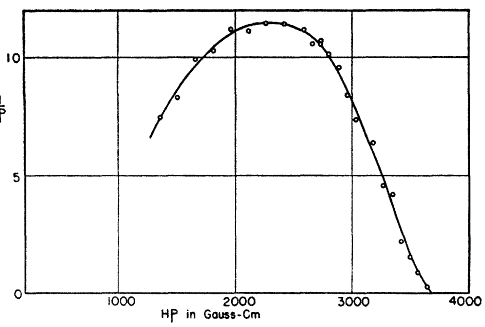

FIG. 12. Momentum distribution curve of Cl^{36} (Wu and Feldman).[43]

The spin of Cl^{36} was not known when its β-spectrum was first observed. If one follows the usual procedure of interpreting a forbidden shaped spectrum, a perfect fit (Fig. 13) is found between the observed spectrum and the so-called D_2 correction factor (using Marshak's[44] notation)

$$D_2 \sim \frac{1}{30}(E_0-E)^4+\frac{1}{9}(E^2-1)(E_0-E)^2+\frac{1}{30}(E^2-1)^2,$$

which is the unique forbidden correction factor predicted by Marshak for Be^{10}, a transition in which the spin

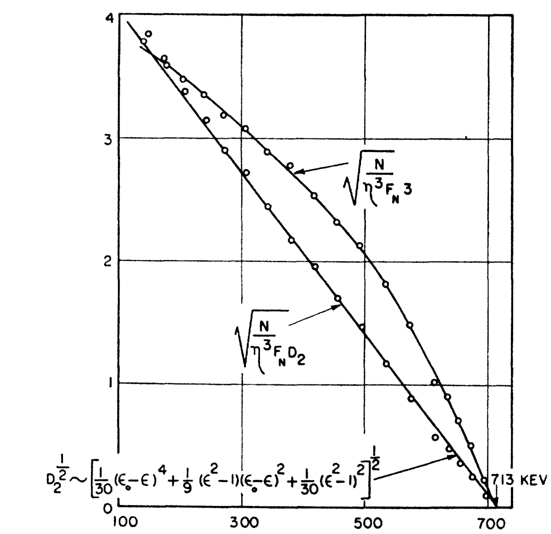

FIG. 13. Fermi plots of Cl^{36} beta-spectrum (Wu and Feldman).[43]

change is 3. Moreover, Cl^{36} has approximately the same half-life[45] and upper energy limit as Be^{10}. (See Table III.) Theoretically, it would have been considered a parallel case to that of Be^{10} if the spin change had remained unknown. When C. H. Townes determined the spin of Cl^{36} by comparing the microwave spectrum of Cl^{36} CN with the theoretical patterns of several values of nuclear spins of Cl^{36}, it turned out that the spin of Cl^{36} is 2.[46]

[44] R. E. Marshak, Phys. Rev. **75**, 513 (1949).
[45] Wu, Townes, and Feldman, Phys. Rev. **76**, 692 (1949).
[46] C. H. Townes and L. G. Aamodt, Phys. Rev. **76**, 691 (1949).

$Argon^{36}$ contains an even number of both neutrons and protons. Its spin is probably zero. Now, with a spin change of 2 and parity change either yes or no, a β-transition can be any one of the many transitions al lowed by the selection rules. Table IV shows the matrix elements which permit the transition $2\rightarrow 0$ with the associated parity changes. The correction factors for these matrix elements have been given by Konopinski and Uhlenbeck,[30] and by Greuling.[30] No single one of these correction factors agrees with the experiment. In other words, no single interaction fits the experimental data (Fig. 14). Longmire[47] tried linear combinations of interactions. Such combinations give cross-terms with new shapes. The results are shown in Fig. 15. The combinations of $(2S, 2V)$ and $(2T, 2A)$ are ruled out on the ground of Fierz's theoretical findings that these combinations seriously modified the shape of even allowed spectra by a term $(1\pm CE)$ where E is the electron energy. The combinations $(2S, 2T)$ and $(2A, 2V)$ are almost identical and fit the data quite well. Moreover, these combinations of interactions also give agreement with the observed spectra with α-correction factor such as Y^{91}, Y^{90}, Sr^{90}, Sr^{89}, Cs^{137}, Rb^{86}, and K^{42} as the cross-terms cancel out in any cases where the spin change involved is one unit higher than the order of forbiddenness. It is still too early to make any final conclusion on the case of Cl^{36}. It could be that the determination of the spin or the distribution of the spectrum is in error. It could also turn out to be a stimulating case, as Marshak recently suggested, in helping pin down the correct linear combination of the five forms of interaction. Recently, M. M. Bouchez and Nataf[48] have pointed out that this case might also be explained with a pure invariant, if an alternation of the ordinary selection rules is taken into account, which might be necessary for light nuclei.

BETA-SPECTRUM OF Be^{10}

Now, let us look at two other interesting cases, Be^{10} and K^{40}. Be^{10} has a comparative half life of 4.4×10^{13} sec. and a spin change of $(0\rightarrow3)$ 3 units. K^{40} has a comparative half-life of $\sim10^{18}$ sec. and a change of spin of 4 units. Their spectra have been theoretically investigated and predicted by Marshak.[44, 49] In the case of Be^{10}, the observed half-life requires the rejection of all matrix elements except the four which are associated with the energy spectrum given by D_2. Of these four, three $(2T$, $2A$, and $3T)$ are G-T type interactions and one

TABLE III. Data on $_4Be^{10}$ and $_{17}Cl^{36}$.

	$t_{\frac{1}{2}}$	E_0	ft
$_4Be^{10}$	$2.7\times10^6 y$	2.1	4.4×10^{13} sec.
$_{17}Cl^{36}$	$0.44\times10^6 y$	2.4	3.3×10^{13} sec.

[47] Longmire, Wu, and Townes, Phys. Rev. **76**, 695 (1949).
[48] S. R. de Groot and H. A. Tolholk, Physica **XVI**, 456 (1950); R. Bouchez, C. R. Acad. Sci. Paris **230**, 440 (1950).
[49] R. E. Marshak, Phys. Rev. **70**, 980 (1946).

TABLE IV.* Matrix elements which permit the transition $2 \to 0$ with the associated parity changes.

Interaction	First forbidden	Second forbidden	Third forbidden
S		R_{ij} no	
V		R_{ij}, A_{ij} no	Yes
T	B_{ij} yes	T_{ij}, A_{ij} no	
A	B_{ij} yes	T_{ij} no	
P		$\gamma^5 R_{ij}$ yes	

* The notations of Konopinski and Uhlenbeck's article (Reference 30) are used here.

($3V$) is a Fermi-type interaction. Because of the extremely long life and small activation cross section, the specific activity of Be^{10} obtained with the ordinary method of preparation is rather poor. Nevertheless Bell and Cassidy[50] of Oak Ridge first used a source a few mg/cm^2 thick with their scintillation spectrometer and noticed the deviation of the Kurie plot from an allowed shape at the high energy end. Subsequently, after finding the D_2 spectrum of Cl^{36}, the Columbia group (Feldman and Wu) carried out an extensive comparative study[51] between several identical sources of Y^{91}, RaE, P^{32}, Cl^{36}, Be^{10}, and Cu^{64} and reached the conclusion[52] that the true distribution of the Be^{10} β-spectrum might well be a D_2 spectrum as predicted theoretically. Right after that, Fulbright and Milton[53] of Princeton reported their work done in a high pressure proportional counter and also concluded that the spectrum of Be^{10} agrees very well with the D_2 spectrum. Most recently, a highly enriched Be^{10} source was successfully prepared by the Y-12 research laboratory of the Carbide and Carbon Chemical Corporation, Oak Ridge, Tennessee, under the supervision of C. P. Keim. By investigating sources of BeO less than 0.5 mg/cm^2 inside magnetic spectrometers, D. Hughes, Alburger and Eggler[54] and the Columbia group (Feldman and Wu)[55] all reported the good agreement between the experimental and theoretical predicted spectrum (Fig. 16). Bell and Cassidy[56] obtained the same result on their scintillation spectrometer. This is certainly a great triumph for the theory of β-decay.

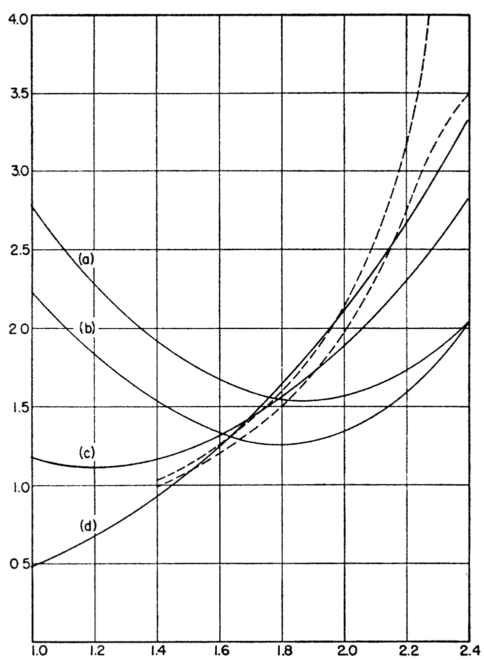

FIG. 14. Correction factors for Cl^{36} electrons. The area between the dashed curves represents the experimental data. Curve (a) is for $2T(T_{ij})$ and (approximately) $\frac{1}{4}\ 2V(R_{ij})$; curve (b) same scale as (a), is for $2A(T_{ij})$ and (approximately) $\frac{1}{4}\ 2S(R_{ij})$; curve (c) is for $2T(A_{ij})$ and $2V(A_{ij})$; curve (d) is best fit with $2T$, or $2V$ (approximately). The correction factor for $3V$ has roughly the same shape as (b). Ordinate scale is arbitrary. (See reference 47.)

[50] P. R. Bell and J. M. Cassidy, Phys. Rev. **76**, 183 (1949).
[51] L. Feldman and C. S. Wu, Phys. Rev. **76**, 697 (1949).
[52] C. S. Wu and L. Feldman, Phys. Rev. **76**, 698 (1949).
[53] H. W. Fulbright and J. C. D. Milton, Phys. Rev. **76**, 1271 (1949).

BETA-SPECTRUM OF K^{40}

In the case of K^{40}, the spin change is 4 units. The theoretical work was done by Marshak[49] and independently by Greuling.[30] If the parities of K^{40} and Ca^{40} are different, then the β-transition requires third forbidden tensor or axial vector interaction. According to the special property of the theory of forbidden spectra, when the disintegration involves a spin change of one unit higher than the degree of the forbiddenness, a unique spectrum is predicted. If the parities of K^{40} and Ca^{40} are the same, the transition must be fourth for-

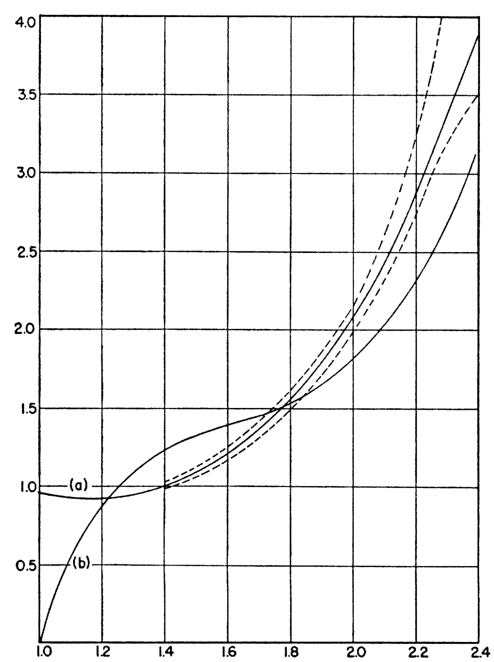

FIG. 15. Correction factors for Cl^{36} electrons. The area between the dashed curves represents the experimental data. Curve (a) is for the combination ($2S$, $2T$), or (approximately) for the combination ($2A$, $2V$). Curve (b) is the closest fit for the combination ($2S$, $2A$). (See reference 47.)

[54] Alburger, Hughes, and Eggler, Phys. Rev. **78**, 318 (1950).
[55] L. Feldman and C. S. Wu, Phys. Rev. **78**, 318 (1950).
[56] P. R. Bell and J. M. Cassidy, Phys. Rev. **77**, 301 (1950).

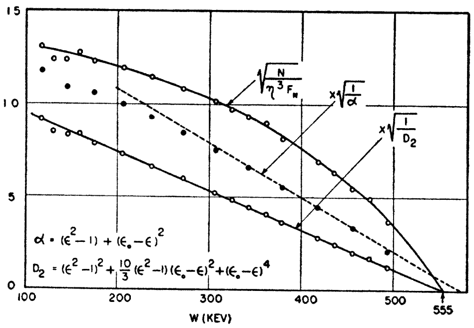

FIG. 16. Fermi plots of Be^{10} beta-spectrum (Wu and Feldman).

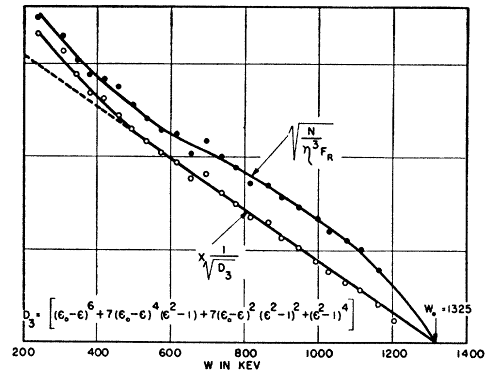

FIG. 17. Fermi plots of K^{40} beta-spectrum (Feldman and Wu).

bidden. Among them, the C_{4S}, C_{4P}, and C_{4A} lead to unique energy spectra; C_{4V} and C_{2T} are more flexible. Unfortunately K^{40} has a half-life of 2.7×10^9 years and the natural abundance of K^{40} is only 0.016 percent. To obtain a true spectrum of K^{40}, a highly enriched K^{40} source must be used. Feldman and Wu[51] of Columbia investigated the effects of thick sources on the high energy end of the spectra and concluded that the true distribution of the K^{40} β-radiation is most likely not allowed but exhibits a concave Kurie-plot at the high energy region. Recently, Bell, Weaver, and Cassidy[57] used an enriched source of KCl 2.5 mg/cm² thick with their scintillation spectrometer and obtained a Kurie plot quite concave to the energy axis above 700 kev (Fig. 17). The end-point is 1.36±0.05 Mev. When this concave curve is corrected by the correction term for the third forbidden axial or tensor interaction, it yields a straight line down to 0.7 Mev (half of the energy region). The strong turn up at low energies is found to be characteristic with the scintillation spectrometer as the effect of scattering of electrons out of the crystal is unavoidable. Recently, Alburger[58] investigated the K^{40} (seven percent enrichment) β-spectrum of a source of 2.4 mg/cm² in his thin lens spectrometer of a resolution of 17 percent. The Kurie plot corrected for third forbidden tensor or axial interaction is straight all the way down to 500 kev. The deviation below 500 kev is interpreted as due to the unfavorable source thickness used and it is confirmed by auxiliary experiment with P^{32}. The latest investigation by Feldman and Wu§ of the β-spectrum from an enriched K^{40} source (∼2.5 mg/cm²) on the Columbia solenoidal spectrometer (resolution about 10 percent) also confirms the third-forbidden axial or tensor interaction and rules out fourth-forbidden scaler, pseudoscaler or axial interaction. Further theoretical investigation¶ of the correction factor C_{4V} and C_{4T} in a method similar to that used in the case of Be^{10} by estimating the magnitude of the matrix element by Greuling's method shows that these two correction factors give the same unique spectrum as that from third-forbidden axial or tensor interaction. Therefore, the fourth-forbidden vector or tensor interaction can explain the observed forbidden shape equally as well. Nevertheless, the nuclear shell structure predicts a change of parity involved in this transition. If this is the case, the fourth-forbidden transition is naturally ruled out on account of the parity change from either Fermi or G-T selection rules. The maximum energy of the spectrum as extrapolated from the corrected Kurie curve is 1325±15 kev, somewhat lower than the previous determinations reported.

[57] Bell, Weaver, and Cassidy, Phys. Rev. **77**, 399 (1950).
[58] D. E. Alburger, Phys. Rev. **79**, 236 (1950).
§ To be published.
¶ L. Feldman, private communication.

CONCLUSION

For some time past, people have been puzzled and discouraged over the fact that even apparently forbidden transitions yield allowed distributions. Theoretically, it is quite possible for the first forbidden transitions to have the allowed shape. But it is only under very special conditions that the second forbidden transitions exhibit an allowed shape. On the other hand, the strikingly forbidden shapes do come from the very definitely higher order transitions of $\Delta I=2$, $\Delta I=3$, and $\Delta I=4$ as theoretically expected. Therefore, reexamining carefully the large volume of recently accumulated information concerning the β-spectra and its interpretation with the guidance of nuclear shell structure, it is quite clear now that the empirical classification of the degree of forbiddenness on the basis of ft values alone is too high. The very lowest $\log_{10}ft\sim3$ to 4 are allowed transitions between nuclei having similar nuclear wave functions. These transitions are called the superallowed transitions, while allowed transitions between nuclei not having very similar wave functions have $\log ft$ ranging from 4 to 6. If the transition probability of successive orders of forbiddenness diminishes by factors of 1/100, one would expect to have $\log_{10}ft$ for first forbidden transition from 6 to 8 or higher. Forbidden transitions[40] of second order or higher should generally have $\log_{10}ft>9$. Therefore, it is no surprise that most of the spectra investigated exhibited allowed shape.

If this is the case, it may also help to explain why so few cases of beta-gamma angular correlation[59] have

[59] D. L. Falkoff, Phys. Rev. **79**, 323, 334 (1950); Ph.D. Thesis; R. L. Garwin, Phys. Rev. **76**, 1876 (1949).

been observed. According to the theory not only for allowed β-transition but also for any forbidden β-transition having an allowed spectrum shape, no angular correlation between $\beta-\gamma$ can be expected. The only three definitely anisotropic distributions reported[60–62] so far are Rb^{86}, Tm^{170}, and Sb^{124}. The shape of the β spectrum[37] of Sb^{124} was recently investigated, and it was found that it indeed exhibits an α-type forbidden distribution. It is therefore highly desirable to have the shape of the responsible β-spectrum of Rb^{86} and Tm^{170} investigated. Additional information on the $\beta-\gamma$ angular correlation will undoubtedly help further in the interpreting of the theory of β-decay.

The experimental observation of the three unique forbidden spectra of Y^{91}, Be^{10}, and K^{40} as theoretically predicted is a triumphant test for the theory of forbidden β-decay. The so-called α-type spectrum exhibited by Y^{91} group is typical of first forbidden transition for spin change $\Delta I=2$ (yes). It gives strong evidence that part of the true interaction must be either tensor or axial vector. The spectra of $Be^{10}(\Delta I=3)$ and $K^{40}(\Delta I=4)$ can be interpreted by the tensor or axial vector interaction if the degree of forbiddenness (2nd and 3rd) is one less than the spin change ΔI and by the tensor or polar vector interaction if the degree of forbiddenness (3rd and 4th) is the same as the spin change ΔI. But if one considers the parity change involved in the decay of Be^{10} (no) and K^{40} (yes) as predicted by shell structure models, then the spectrum of Be^{10} and K^{40} can be classified with the α-type spectra into a special group where spin change ΔI is one unit greater than the degree of forbiddenness and permitted only by G-T selection rules, tensor or axial vector interaction. Nevertheless, in order to decide between tensor and axial vector interaction, further information is needed. The spectrum of Cl^{36} cannot be explained by single interaction but by linear combinations of $(2S, 2T)$, $(2V, 2A)$ and possibly $(2V, 2T)$. These combinations of interactions also give agreement with the three unique forbidden spectra of Y^{91}, Be^{10}, and K^{40}. In order to obtain further information on the true linear combinations, more cases of beta-decay with its degree of forbiddenness equal to its spin change ΔI must be found and investigated. The only known case of this class is Rb^{87}. It involves a spin change $\Delta I=3$ and most likely undergoes a third order forbidden transition. However, its extremely long half-life and very low upper energy limit make the investigation of its beta-spectrum a most difficult task.

This is a brief summary of the present status of β-spectra. The fruitful year which has just passed was made possible only through the concerted efforts of all those who are interested in this field. It certainly provided a stimulating and pleasant time for us all!

[60] D. T. Stevenson and M. Deutsch, Phys. Rev. **78**, 640 (1950); R. Stump and S. Frankel, Phys. Rev. **79**, 243 (1950).
[61] T. B. Novey, Phys. Rev. **78**, 66 (1950).
[62] S. L. Ridgway, Phys. Rev. **79**, 243 (1950).