# The Range-Energy Relation for Slow Alpha-Particles and Protons in Air

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R ECENT accurate determinations of energies and ranges in nuclear disintegrations give fixed points ranges in nuclear disintegrations give fixed points on the range-energy relation for  $\alpha$ -particles around 1.5 and 2 Mev, and protons below 1 Mev. These, and other evidence, show that the previous range-energy relation of Parkinson et al. for protons gave too high energies, the relation of Holloway and Livingston for  $\alpha$ -particles too low energies for given range. The result of Jesse et al. that the ionization in argon is exactly proportional to the energy, makes it possible to correct the Holloway-Livingston relation. Experiments by Crenshaw give the proton relation at low energies. New relations for both protons and  $\alpha$ -particles are presented which are consistent with all experimental evidence now available.

### 1. INTRODUCTION

The range-energy relation for slow heavy particles ( $\alpha$ -particles below 5 Mev, protons below 1.25 Mev) has long been a very difficult problem. At higher energies, the theory of the stopping power permits rather accurate predictions,1 but at low energies the theory breaks down, particularly because of the capture and loss of electrons by the particle: Even if one knows experimentally what fraction of the time the proton has captured an electron, one cannot calculate the energy loss which the neutral hydrogen atom suffers when traversing matter. Information on the rangeenergy relation at low energies must therefore come entirely from experiment, but experiments in this region are much more difficult that at higher energies. At higher energy, there are natural  $\alpha$ -particles whose energy and range can both be measured very accurately. In this low energy region, one must use either sloweddown  $\alpha$ -particles, or recoils from faster alphas, or artificially accelerated particles. In the first case, the straggling of the energy loss makes the measurement inaccurate, in the second, the determination of energy is very indirect, and with artificially accelerated particles one generally needs windows of solid material whose stopping power relative to air is poorly known.

In the last two years, however, sufficient new information has come in to establish the range-energy relation in this region with some accuracy. There are two main sources of such information, viz., (a) the accurate measurement of the energy release in certain nuclear reactions, together with measurements of the ranges of the resulting particles, and (b) the study of the ionization of particles of various energies in argon and air.<sup>2-4</sup> Mainly on the basis of the latter information, Jesse and Sadauskis3 have already presented a new rangeenergy relation which is essentially identical with ours.

# 2. PREVIOUS INFORMATION

Livingston and Bethe<sup>1</sup> have given a range-energy relation which was, in the region in question, mainly based on the observations of Mano<sup>5</sup> who had used slowed-down  $\alpha$ -particles. For  $\alpha$ -particles below 0.7 Mev, and for protons, the experiments of Blackett and Lees<sup>6</sup> were used who observed the range of recoil particles in the cloud chamber.

Soon after the article by Livingston and Bethe appeared, the range of protons up to 2 Mev was measured by Parkinson, Herb, Bellamy, and Hudson,7 using protons accelerated by the Wisconsin Van de Graaff. They found much higher energies for a given range than previous workers, e.g., for 1-cm range, E=0.66 instead of the 0.56 Mev given by Livingston and Bethe. However, Parkinson et al. used a definition of range different from the customary one: They chose a given thickness of air R, and then observed, for varying initial proton energy E, what fraction of the protons of this energy could penetrate the air laver. This gives a number-energy curve shown schematically in Fig. 1; by drawing the steepest tangent, one obtains an "extrapolated energy"  $E_{\text{ext}}$  as shown in the curve. The curve in Fig. 1 is of course a typical straggling curve; if one takes the energy at which 50 percent of the particles penetrate, one gets a "mean energy"  $E_{\text{mean}}$ corresponding to the given range R. It is reasonable to assume that R is very close to the *mean* range of particles of energy  $E_{\text{mean}}$ . On the basis of this assumption, Livingston and Bethe re-evaluated the experiments of Parkinson et al. and obtained the "Revised 1937 Cornell Range-Energy Curve," which was made available in blueprint form and used in many laboratories. Even after correcting to mean energy, the energies for given range are high, e.g. 0.625 Mev for protons of 1-cm range.

<sup>&</sup>lt;sup>1</sup> M. S. Livingston and H. A. Bethe, Rev. Mod. Phys. 9, 261 (1937).

<sup>&</sup>lt;sup>2</sup>W. P. Jesse and H. Forstat, Phys. Rev. 73, 926 (1948); 74, 1259(A) (1948).

 <sup>&</sup>lt;sup>3</sup> W. P. Jesse and J. Sadauskis, Phys. Rev. 75, 1110 (1949) and especially Phys. Rev. 78, 1 (1950).
 <sup>4</sup> Jesse, Forstat, and Sadauskis, Phys. Rev. 77, 782 (1950).
 <sup>5</sup> G. Mano, Ann. de physique 1, 407 (1934); J. de phys. et rad.

<sup>5, 628 (1934).</sup> <sup>6</sup> P. A. M. Blackett and D. S. Lees, Proc. Roy. Soc. **134**, 658

<sup>(1932).</sup> <sup>7</sup> Parkinson, Herb, Bellamy, and Hudson, Phys. Rev. 52, 75

<sup>(1937).</sup> 



FIG. 1. Schematic curve of the number of particles emerging from stopping material versus their energy. For further explanation see text.

The range energy relation for slow  $\alpha$ -particles was investigated by Blewett and Blewett (unpublished), using a direct method, and by Holloway and Livingston<sup>8</sup> who studied the ionization along the path and assumed that the energy loss per ion pair in air is independent of the velocity of the  $\alpha$ -particle. Holloway and Livingston found essentially the same relation as adopted by Livingston and Bethe, with slightly *lower* energies for given range, whereas Blewett and Blewett found substantially higher energies (e.g., 1.83 Mev for a range of 0.824 cm where Holloway and Livingston found 1.56 Mev).

Experimentally, the work of Holloway and Livingston is probably very accurate. However, the assumption that the energy of an  $\alpha$ -particle is proportional to the number of ions it forms in air remained doubtful and has turned out to be incorrect (Section 5).

### 3. EVIDENCE THAT THE PROTON ENERGIES WERE TOO HIGH

In 1942, Crenshaw<sup>9</sup> measured the energy loss of deuterons of 60 to 340 kev in air. He used a gas target, thus avoiding errors from windows, and he measured the energy of the emerging deuterons by magnetic deflection, thus avoiding the difficulties inherent in determining the actual end of the range of a particle.<sup>8</sup> In using Crenshaw's results we shall assume, in accord with theory, that the energy loss of protons and deuterons is the same if they have the same velocity, an assumption<sup>10</sup> well-confirmed by Crenshaw's measurements of the energy loss of both kinds of particles in hydrogen and deuterium (his Fig. 7). Integrating Crenshaw's curve, one finds that a thickness of 0.192 cm of standard air is required to reduce the proton energy from 170 to 30 kev. The Parkinson-Cornell curve would give 0.135 cm for this thickness, far outside the experimental error of Crenshaw. It may be objected that the data of Parkinson et al. do not extend below 230 kev; however, they give the absolute range at 230 kev, and this almost uniquely determines the thickness corresponding to an energy reduction from 170 to 30 kev.

We believe that Crenshaw's data should be preferred to Parkinson's on the basis of the experimental methods used. Crenshaw measures the energy loss, i.e., the derivative of the range-energy relation, which will make the latter relation much more accurate than a direct measurement of the range. His measurement is free from the question of defining the end of the range. Further, Crenshaw's experiment was done entirely in the gas, while Parkinson et al. used Al foils of appreciable thickness (about 1 mm of air equivalent) whose stopping power is not well known. We shall therefore accept Crenshaw's data for the low energy region, and conclude that the ranges given by Parkinson et al. for given energy are too small, or their energy for given range too high.

This evidence alone would not be convincing, but it is supported by very accurate data from nuclear disintegrations which provide fixed points on the rangeenergy curve above 200 kev. The most important reaction for this purpose is

$$N^{14} + n = C^{14} + H + Q_1, \tag{1}$$

whose usefulness in this connection was first pointed out by Cornog, Franzen, and Stevens.<sup>11</sup> The reaction energy  $Q_1$  can be deduced in two ways, either (a) from the threshold of the inverse reaction,

$$C^{14} + H = N^{14} + n,$$
 (2)

or (b) from the energy of the  $\beta$ -rays from C<sup>14</sup> which is very accurately measured<sup>12</sup> to be  $156.3 \pm 1$  kev, together with the difference in mass between neutron and hydrogen atom which has been determined by Taschek et al.13 as  $782\pm2$  kev, and by Tollestrup *et al.*<sup>14</sup> as  $789\pm6$  kev. Combining these data, one finds from the indirect method (b)

$$Q_1 = 626 \pm 3 \text{ kev.}$$
 (3)

The direct measurement of the threshold of reaction (2) by Shoupp, Jennings, and Sun<sup>15</sup> gives

$$Q_1 = 620 \pm 9 \text{ kev}$$
 (3a)

in good agreement with (3). In reaction (1), the proton

- 1947 (1949).
- <sup>15</sup> Shoupp, Jennings, and Sun, Phys. Rev. 75, 1 (1949).

<sup>&</sup>lt;sup>8</sup> M. G. Holloway and M. S. Livingston, Phys. Rev. 54, 18 (1938).

<sup>&</sup>lt;sup>(1935)</sup>.
<sup>9</sup> C. M. Crenshaw, Phys. Rev. 62, 54 (1942).
<sup>10</sup> The contrary result of H. A. Wilcox (Phys. Rev. 74, 1743 (1948)) was found to be erroneous by T. A. Hall and S. D. Warshaw (Phys. Rev. 75, 891 (1949)) and by T. Huus and C. B. Madsen (Phys. Rev. 76, 323 (1949)).

<sup>&</sup>lt;sup>11</sup> Cornog, Franzen, and Stephens, Phys. Rev. 74, 1 (1948).

<sup>&</sup>lt;sup>12</sup> Cook, Langer, and Price, Phys. Rev. 74, 548 (1948), also others. <sup>13</sup> Taschek, Jarvis, Argo and Hemmendinger, Phys. Rev. 75,

<sup>1268 (1949)</sup> <sup>14</sup> Tollestrup, Jenkins, Fowler and Lauritsen, Phys. Rev. 75,

receives 14/15 of the energy, i.e., according to (3)

$$E_p = 584 \pm 3 \text{ kev.}$$
 (4)

The range of the protons has been measured accurately by Bøggild<sup>16</sup> and by Hughes and Eggler,<sup>17</sup> in cloud chambers operating at reduced pressure. When reduced to standard air, Bøggild finds a range of 1.00 cm, and Hughes and Eggler of 0.991 cm, with an uncertainty of about  $\pm 0.01$ .

Very similar evidence comes from the reaction

$$He^{3} + n = H^{3} + H + Q_{2},$$
 (5)

whose energy release is<sup>13</sup>  $Q_2 = 764 \pm 2$  kev, giving for the proton energy (if the neutron is slow)  $3Q_2/4=573$  $\pm 1.5$  kev, very nearly the same as Eq. (4). The range, according to Hughes and Eggler,<sup>17</sup> is 0.980 cm. Now near 580 kev, the range changes by 0.024 cm for 10-kev energy change; hence, extrapolation from reaction (5) gives a range of 1.006 cm at 584 kev.

Averaging all the information, we find that an energy of 584 kev corresponds to a range of 1.00 cm. The old curve of Livingston and Bethe gives 0.56 Mev for this range, which is too small; the experiments of Parkinson et al. after correction give 0.625 Mev, which is considerably too large. This is in agreement with the conclusion reached from Crenshaw's experiments that Parkinson's relation gives too high energies for a given range.

Another reaction useful for our purposes is

$$Li^6 + n = He^4 + H^3 + Q_2,$$
 (6)

which was studied by Bøggild and Minnhagen.<sup>18</sup> Their result is  $1.04 \pm 0.02$  cm for the range of the  $\alpha$ -particle and  $6.00 \pm 0.06$  cm for the triton. According to accurate recent measurements by Tollestrup, Fowler, and Lauritsen,<sup>19</sup> the energy release in reaction (6) is  $O_2 = 4.788$  $\pm 0.023$  Mev, giving for the energy of the  $\alpha$ -particle  $3/7^{19a}$  of this value, i.e.,  $2.057 \pm 0.010$  MeV, and for that of the triton  $2.731 \pm 0.013$  Mev. (The energy values used in reference 18, based on Q = 4.65 or 4.69, are definitely too low.) From the triton result, we conclude that the energy of a proton of  $2.00\pm0.02$ -cm range is 0.910  $\pm 0.004$  Mev. The relation of Livingston and Bethe gives for this energy a range of 2.00 cm, that of Parkinson et al. (Cornell 1937 revised) gives 1.93 cm. So we find again that the Parkinson-Cornell curve gives too short range; the Livingston-Bethe curve is in this case correct.

A revision of the range-energy relation for slow protons is therefore necessary. In addition to indicating this, the nuclear disintegration data give important fixed points on a new relation.

#### 4. EVIDENCE THAT THE ALPHA-ENERGIES WERE TOO LOW

Two nuclear reactions producing He nuclei are valuable for the range-energy relation. The first of these is reaction (6) which gives an energy of  $2.05 \pm 0.01$  MeV for a range of  $1.04 \pm 0.02$  cm. The relation of Holloway and Livingston<sup>8</sup> gives only 1.96 Mev for this range, indicating that the energies given by this relation are too low.

The same conclusion at a lower energy, follows from the often-discussed reaction.

$$B^{10} + n = Li^{7*} + He^4 + Q_3,$$
 (7)

produced by slow neutrons, with Li<sup>7</sup> being left in an excited state. The range of the  $\alpha$ -particles has been determined by Bower, Bretscher, and Gilbert<sup>20</sup> who have succeeded in measuring separately the ranges of  $He^4$  and  $Li^7$  for reaction (7) occurring in the gas of a cloud chamber by investigating the grain density along the track photometrically and observing its discontinuity. They find a range of 0.70 cm for the  $\alpha$ -particle. This must be corrected for the variation of stopping power with energy: The experiments were done in a mixture of He and  $B(COH_3)_3$ , and the stopping power was calibrated by measuring the difference in range between the 2 groups of  $\alpha$ -particles from ThC' of ranges 8.63 and 4.79 cm. According to Gurney's experiments,<sup>21</sup> the stopping power of He is increased by about 3 percent for slow particles; that of  $B(COH_3)_3$  can be estimated to be increased by about 6 percent, using the calculations of stopping power by Hirschfelder and Magee.<sup>22</sup> According to Gilbert,<sup>23</sup> the latter gas contributed about 70 percent of the stopping power; thus the resultant stopping power was increased about 5 percent, and the  $\alpha$ -particle range in standard air comes out to be  $0.73_5$  cm. Substantially the same result  $(0.71_5)$ was found by O'Ceallagh and Davies,<sup>24</sup> measuring the range of the particles from a boron foil, in a cloud chamber filled with a mixture of air and helium, and applying Gurney's correction. Bøggild<sup>25</sup> measured the sum of the ranges of Li<sup>7</sup> and He<sup>4</sup>, and apportioned them to the two particles in the ratio indicated by Bower et al.;<sup>20</sup> the result is 0.709 cm. The average of the three measurements is  $0.72_0$  cm.

The energy release in reaction (7) has been obtained

<sup>&</sup>lt;sup>16</sup> J. K. Bøggild, Kgl. Danske Vid. Sels. Math.-Fys. Medd. 23, <sup>17</sup> D. J. Hughes and C. Eggler, Phys. Rev. 73, 809 (1948).
<sup>18</sup> J. K. Bøggild and L. Minnhagen, Phys. Rev. 75, 782 (1949).
<sup>19</sup> Tollestrup, Fowler, and Lauritsen, Phys. Rev. 76, 428 (1949).

<sup>&</sup>lt;sup>20</sup> Bower, Bretscher, and Gilbert, Proc. Camb. Phil. Soc. 34, 290 (1938).

 <sup>&</sup>lt;sup>21</sup> R. W. Gurney, Proc. Roy. Soc. 107, 340 (1925).
 <sup>22</sup> J. O. Hirschfelder and J. L. Magee, Phys. Rev. 73, 207 (1948).
 <sup>23</sup> C. W. Gilbert, Proc. Camb. Phil. Soc. 44 (Part 3), 447 (1948). Gilbert first pointed out the importance of taking into account the change of stopping power with velocity, but probably overestimated it, obtaining 0.77 cm for the range. Experiments by Bøggild on the sum of the ranges of He<sup>4</sup> and Li<sup>7</sup> agree with the result of Bower et al. before application of any correction for stopping power, and thus favor a *small* value for the range. <sup>24</sup> C. O'Ceallagh and W. T. Davies, Proc. Roy. Soc. (A) **167**, 81

<sup>(1938).</sup> <sup>25</sup> J. K. Bøggild, Kgl. Danske Vid. Sels. Math.-Fys. Medd. 23,



FIG. 2. Ratio of the number of ions produced in argon and in air by alpha-particles of different energies.

by Chao, Lauritsen, and Tollestrup<sup>26</sup> by combining the reactions  $B^{10}(p, \alpha)Be^7$  and  $Li^7(p, n)Be^7$  with the excitation energy of the excited state of Li<sup>7</sup>. Their result is

$$Q_3 = 2.316 \pm 0.006 \text{ Mev.}$$
 (8)

The energy of the  $\alpha$ -particle is 7/11 of this, giving  $1.474 \pm 0.004$  Mev; this energy then corresponds to a range of 0.720 cm. Holloway and Livingston give 1.36 Mev for this range which is again too low.

# 5. IONIZATION MEASUREMENTS

As was shown in the last two sections, disintegration measurements provide a few fixed points on the rangeenergy relations for protons and  $\alpha$ -particles. However, these are too widely spaced to permit reliable interpolation. A complete range-energy relation would be provided by the experiments of Holloway and Livingston if the ionization in air were proportional to the particle energy. However, the discrepancies found in the last section between the actual disintegration energies and those deduced from the Holloway-Livingston relation show that this proportionality is not valid. Jesse, Forstat, and Sadauskis<sup>4</sup> have given further evidence for this conclusion (see Table I).

On the other hand, these authors have shown that the ionization in argon is exactly proportional to the particle energy. The evidence for this is the following:

1. Jesse et al. have measured the ionization in argon of natural  $\alpha$ -particles from various emitters and found it proportional to the accurately known energy of the  $\alpha$ -particle. The measurements were made between 5.3 and 8.8 Mev, and proportionality was valid within 0.5 percent, except for one case where an adequate cause was found for a greater (1 percent) discrepancy.

2. Jesse et al. measured the total number of ions produced in argon by the two particles, Li<sup>7</sup> and He<sup>4</sup> emitted in reaction (7). The ratio of this number to the exactly measured total energy release (8) is the same as the corresponding ratio for Po  $\alpha$ -particles, within 0.3 percent.

A similar experiment was done by Franzen, Halpern, and Stephens<sup>27</sup> for the ionization in argon produced by the two particles (triton and proton) emitted in reaction (5), and the two particles ( $C^{14}$  and proton) emitted in reaction (1). In both cases the energy release is accurately known from direct measurements (see Section 3), and the ratio of energy release to total ionization was found to agree with the corresponding ratio for Po  $\alpha$ 's within 0.4 percent.

Barring very strange accidental compensations, we must agree with the authors mentioned, that the ionization in argon is strictly proportional to the energy, independently of the energy and also of the kind of particle used (H, H<sup>3</sup>, He<sup>4</sup>, Li<sup>7</sup>, C<sup>14</sup>). With regard to the latter point, the experiments of Franzen et al. also give direct evidence: The protons from reactions (1) and (5) have very nearly the same energy; the experiments therefore show that the difference between the ionization of a triton of about 200 kev, and a C14 of about 40 kev, is just what is expected from the proportionality law.

3. Gray,<sup>28</sup> analyzing measurements of Gurney<sup>29</sup> made in 1925 with slowed-down natural  $\alpha$ -particles, found that the numbers of ions produced in argon, neon, helium, and hydrogen stood in a constant ratio to each other, for any energy of the  $\alpha$ -particle. Thus the energy required per ion, w, must either vary in the same way with energy for all four gases, which would be a most implausible accident, or be independent of energy which is much more reasonable.

4. It would be virtually impossible to develop a complete theory of the energy per ion, w. However, Fano<sup>30</sup> has given very good arguments to show that wshould be *approximately* independent of the particle energy E. The energy lost by the incident particle is ultimately used for excitation of discrete states, for ionization, and for giving to secondary electrons kinetic energies below the first excitation level of the stopping atoms. Any faster secondary electron would distribute its energy further among excitation and ionization. Since many intermediate steps take place, it is plausible that the ultimate distribution of energy between excitation, ionization and kinetic energy of secondary elec-

TABLE I. Comparison of observed ranges of particles from nuclear reactions with ranges calculated from the energy.

		Observed	Range	
Reaction	Particle	energy	From energy	Observed
	α α Η <sup>3</sup> Η Η	$\begin{array}{c} 1.474 \pm 0.004 \\ 2.057 \pm 0.010 \\ 0.910 \pm 0.005 \\ 0.584 \pm 0.003 \\ 0.573 \pm 0.0015 \end{array}$	$\begin{array}{c} 0.725 \pm 0.003 \\ 1.037 \pm 0.006 \\ 1.97_0 \pm 0.017 \\ 1.007 \pm 0.008 \\ 0.980 \pm 0.004 \end{array}$	$\begin{array}{rrrr} 0.72 & \pm 0.015 \\ 1.04 & \pm 0.02 \\ 2.00_{5} \pm 0.02 \\ 0.995 \pm 0.01 \\ 0.980 \pm 0.01 \end{array}$

Franzen, Halpern, and Stephens, Phys. Rev. 77, 641 (1950).
 L. H. Gray, Proc. Camb. Phil. Soc. 40, 95 (1944).
 R. W. Gurney, Proc. Roy. Soc. 107, 332 (1925).
 U. Fano, Phys. Rev. 70, 44 (1946).

<sup>&</sup>lt;sup>26</sup> Chao, Lauritsen, and Tollestrup, Phys. Rev. 76, 586(A) (1949).

RANGE-ENERGY RELATION



FIG. 3. Range-energy relation of slow alpha-particles in air of 15°C and 760 mm.

trons is roughly independent of both the energy and the nature of the incident particle, and that therefore w is likewise independent. Moreover, if w has been shown to be independent of energy, this gives a strong pre-

sumption that it is also the same for all kinds of incident particles (see argument 2).

For the rare gases, w is only slightly greater than the ionization potential, I. This must be interpreted as



FIG. 4. Range-energy relation for slow protons.

showing that excitation of discrete levels is very improbable, and that the kinetic energy given to secondary electrons is small; Fano has given plausible reasons for this. In these conditions, w cannot vary much: even if the excitation probability of discrete levels were to

change by say, 20 percent, and if the average kinetic energy of the secondary electrons were to vary by a similar amount, the change of w would only be a few percent. It is thus plausible that w for rare gases is more nearly constant than for other materials. This is not a proof, but in conjunction with the results of Gray reported under 3, it makes it very plausible that w is very nearly independent of energy for all noble gases.<sup>31</sup>

This result may now be used to obtain an absolute range energy relation from the data of Holloway and Livingston. It will be remembered that these authors measured the number of ions in air as a function of the mean residual range of slowed-down Po  $\alpha$ -particles. Similar measurements were made by Stetter,<sup>32</sup> with the only difference that he used extrapolated ranges, and slowed-down ThC'  $\alpha$ -particles. These experiments, especially those of H. and L., will give a range-energy relation if we can correct from ionization in air to energy. This can now be done since we know that the ionization in argon is "exactly" proportional to the energy. We need only measure the ratio of the ionization in argon to that in air as a function of the residual range in air,

$$C = E_{\rm argon} / E_{\rm air}, \tag{9}$$

and to multiply the "energies" of Holloway and Livingston by this correction factor C.

The ratio C has been measured by Jesse and Sadauskis<sup>3</sup> for Po  $\alpha$ -particles which were partially stopped in air. Their results are reproduced in Fig. 2. This figure also contains the result of the earlier measurements by Gurney<sup>29</sup> which were fitted to agree with Jesse and Sadauskis at 2-cm residual range, the highest value to which Gurney's measurements extended. Finally, the figure contains a value for energy zero, viz.,  $1.1\pm0.1$ , which was obtained by Gerthsen<sup>33</sup> using hydrogen canal rays. In view of arguments 2 and 4 above, we consider it safe to assume that the energy per ion pair is the same for protons as for  $\alpha$ -particles of the same velocity (and, indeed, the same for all particles heavy compared to the electron).

#### 6. FINAL RANGE-ENERGY RELATION

To obtain the final range-energy relation for  $\alpha$ -particles, we multiply the energies given by Holloway and Livingston by the smoothed correction factors given in Fig. 2. The result is given in Fig. 3. This figure is essentially identical with Table V of Jesse and Sadauskis. The curve in Fig. 3 passes within experimental error, through the fixed points established by nuclear reactions. This can be seen from Table I which gives the observed ranges of the particles from the reactions listed in Sections 3 and 4, and the ranges calculated from the observed particle energies, using the rangeenergy relations<sup>33a</sup> of Figs. 3 and 4. Since particle energies can now be observed with greater accuracy than ranges, we have preferred to calculate the range from the observed energy rather than vice versa. Further comparisons are found in the paper by Jesse and Sadauskis.

For protons we have the well-known relation,<sup>34</sup>

$$R_{\rm H}(E) = 1.0072 R_{\alpha}(3.971E) - 0.20 \,\,{\rm cm},$$
 (10)

to obtain the proton range  $R_{\rm H}$  from the  $\alpha$ -range. This was used in Fig. 4 down to  $E_p = 500$  kev. Since the  $\alpha$ -particle energies, for given range, are higher than those of Livingston and Bethe, the same is true of the proton energies.<sup>35</sup> Thus the correction from Livingston and Bethe's curve is in the direction required by the experiments by Parkinson *et al.*, but it is not as large as those experiments indicated. In fact, the relation of Fig. 3 gives consistently ranges 1 mm longer than those of the Parkinson-Cornell energy relation: This may indicate that the measurements of Parkinson *et al.* were substantially correct, except for the definition of the end of the range.

At energies below 170 kev, we used the experiments of Crenshaw. Between 170 and 500 kev, we tried to obtain the smoothest possible fit.

The comparison with nuclear disintegrations is made again in Table I. The agreement is excellent, with the possible exception of the tritons from  $\text{Li}^6+n$  for which the observed range is slightly too large. It may be that the constant, of 0.20 cm, in Eq. (10) is too large; it is based on the very old measurements of Blackett and Lees. Exact investigations of proton ranges at somewhat higher energies (2-3 Mev) in cases where the energy is accurately known, would be of value in this connection.

We believe the new relation for  $\alpha$ -particles to be correct within about 25 kev. The proton relation we believe to be correct to 15 kev below 0.2 and above 0.5 Mev, and to 20 kev in the intervening region. The range-energy relation in the low energy region is thus much more satisfactory than it was in 1937, or even as late as 1948. However, a more direct measurement of the relation for slow protons and  $\alpha$ -particles, preferably by studying the energy loss when traversing given layers of air, would still be desirable.

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<sup>&</sup>lt;sup>31</sup> It is not easy to understand that w is also nearly constant for H<sub>2</sub>. It is not correct, as Gray (reference 28) has assumed, that for H<sub>2</sub> theory would predict constancy of w. To develop an accurate theory would be as difficult for H<sub>2</sub> as for other substances.

<sup>&</sup>lt;sup>32</sup> G. Stetter, Zeits. f. Physik 120, 639 (1943).

<sup>&</sup>lt;sup>33</sup> C. Gerthsen, Ann. d. Physik 5, 657 (1930).

 $<sup>\</sup>overline{}^{33a}$  To be precise, Tables V and VI of reference 3 were used which differ from Figs. 3 and 4 by 0.01 cm or less.

<sup>&</sup>lt;sup>34</sup> Reference 1, Eq. (760a).

<sup>&</sup>lt;sup>35</sup> This was already pointed out by Jesse and Sadauskis.



FIG. 2. Ratio of the number of ions produced in argon and in air by alpha-particles of different energies.



FIG. 3. Range-energy relation of slow alpha-particles in air of 15°C and 760 mm.



FIG. 4. Range-energy relation for slow protons.