Monoergic Neutrons from Charged Particle Reactions

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INTRODUCTION

INSTRUMENTS for accelerating elementary particles to high energies are sources of neutrons which are nearly monoergic. These sources are particularly useful in studying the interaction between neutrons and nuclei as a function of neutron energy since the energies can be varied over a considerable range. Electrostatic generators are convenient sources of such neutrons since they are able to accelerate particles up to about 4 Mev with a continuously variable energy and with an energy spread of 3 kev or less.[1] Lower voltage equipment such as Cockcroft-Walton sets have special advantages where thick targets can be used to give strong monoergic neutron sources.

Cyclotrons, and recently, synchrocyclotrons and linear accelerators, have been the only machines capable of accelerating ions to energies greater than 5 Mev and have served as the principal sources of very high energy neutrons.[2] These accelerators will no doubt continue to remain the most important sources of neutrons having energies above 25 Mev, but, in view of the high excitation energies of the residual nuclei involved, it seems unlikely that these neutrons will be strictly monoergic.

The primary emphasis in the following discussion will be placed on the reactions and techniques which are useful for the production of neutrons having accurately known energies between 5 kev and 20 Mev. The majority of the reactions described will be those which have been used a great deal with electrostatic generators in the study of neutron reactions in this energy range. Detailed information on these neutron sources has been obtained only at incident particle energies less than 4 Mev since this has been the upper voltage limit on existing electrostatic generators.

Reactions which may be expected to produce large yields of neutrons at bombarding energies up to 4 Mev are those involving light nuclei. $Li^7(p,n)Be^7$, $C^{12}(d,n)N^{13}$, and $D(d,n)He^3$ have been the most useful sources of monoergic neutrons, and these reactions will be described in some detail. Recently tritium, H^3, which we shall refer to by the symbol T, has become available to research workers not employed by the Atomic Energy Commission, and simultaneously the results of investigations with tritium as a projectile or target material have become declassified. It is, therefore, possible for us to review this information insofar as it bears on the problem of the production of monoergic fast neutrons in the reactions $T(p,n)He^3$ and $T(d,n)He^4$.

The five reactions listed above have proved to be the most useful sources up to the present time. The energies of neutrons obtainable from these reactions as a function of the energy of the incident particle are shown in Fig. 1. In this figure the neutron energy as a function of the bombarding energy is shown for neutrons emitted at 0° and 180° in the laboratory system of coordinates, with respect to the direction of the incident particle. It is obvious that it is of interest to calculate neutron energies for all different angles as a function of incident particle energy. A derivation of the general formulas and the results of such calculations are given in the appendix to this article. Also included in this appendix are nomographs from which one may determine the energy of neutrons produced in each reaction for all angles of emission and energies of incident particles.

Other reactions such as $Li^7(d,n)Be^8$ and $Be^9(d,n)B^{10}$ have been used in many experiments where truly monoergic neutrons are not required. The principal interest in these sources arises from the high intensities and high energies of the neutrons emitted. Such sources have been used to study neutron phenomena as a function of energy by using detecting devices which are sensitive only to neutrons in a restricted energy range. These reactions will probably be superseded in the near future by $T(d,n)He^4$ as the use of tritium becomes more widespread.

[1] A description of some of the existing electrostatic generators and the methods of energy control and measurement are given by: Parkinson, Herb, Bernet, and McKibben, Phys. Rev. **53**, 642 (1938); Wells, Haxby, Stephens, and Shoupp, Phys. Rev. **58**, 162 (1940); Lauritsen, Lauritsen, and Fowler, Phys. Rev. **59**, 241 (1941); Williams, Rumbaugh, and Tate, Rev. Sci. Inst. **13**, 202 (1942); A. O. Hanson, Rev. Sci. Inst. **15**, 57 (1944); McKibben, Frisch, and Hush, Phys. Rev. **70**, 117 (1946); Bender, Shoemaker, and Powell, Phys. Rev. **71**, 905 (1947); Bennett, Bonner, Mandeville, and Watt, Phys. Rev. **70**, 882 (1946).
A description of some of the lower voltage equipment is given by: W. H. Zinn and S. Seeley, Phys. Rev. **52**, 919 (1937); C. M. Slack and L. F. Ehrke, Rev. Sci. Inst. **8**, 193 (1937).

[2] R. Sherr, Phys. Rev. **68**, 240 (1945); Wm. Sleator, Jr., Phys. Rev. **72**, 207 (1947); Hadley, Kelley, Leith, Segre, Wiegand, and York, Phys. Rev. **75**, 351 (1949).

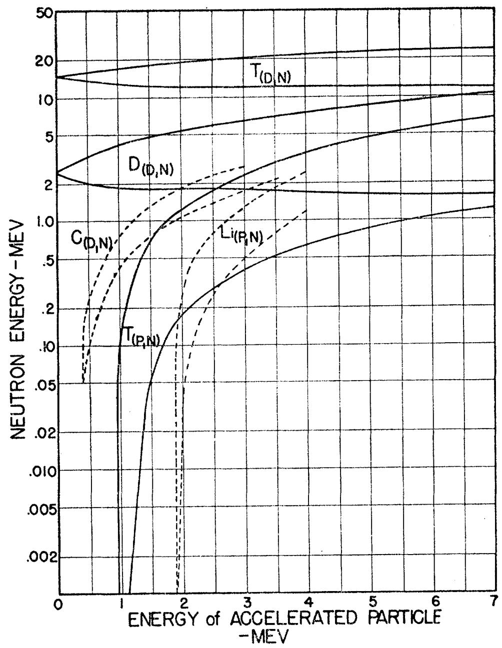

FIG. 1. Range of neutron energies obtainable from various sources. The two curves for each case are for neutrons emitted at 0° and 180° in the laboratory.

1. EXOERGIC REACTIONS

In this section we will discuss the nuclear reactions which occur with the evolution of energy, i.e., exoergic reactions. This attribute makes possible the production of monoergic neutrons with low voltage equipment for the acceleration of ions. In particular, if high current, low voltage accelerators are available, very large fluxes of neutrons may be produced with a minimum of expense. The $D(d,n)He^3$ reaction has been used in this way a great deal since it was one of the first to be recognized as providing a source of monoergic neutrons. It is to be anticipated that the $T(d,n)He^4$ reactions will soon provide most nuclear physics laboratories having low voltage accelerators with a copious source of 14-Mev neutrons.

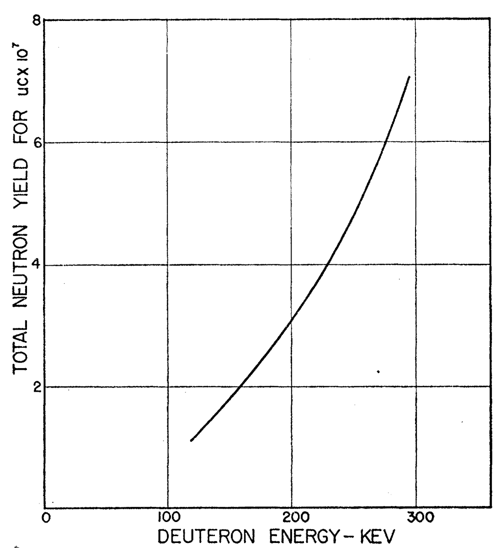

FIG. 2. The total yield of neutrons from a thick ice target of D_2O as a function of incident deuteron energy.

$D(d,n)He^3$

This reaction is described by the equation

$$D^2+D^2\rightarrow He^3+n^1+3.28 \text{ Mev}. \quad (1)$$

The nomograph giving the energy of the neutrons as a function of angle and incident deuteron energy is shown in Fig. 26.

Studies of this reaction have been made in the deuteron energy range below 300 kev.[3] A typical set of observations on the neutron yields from heavy ice (D_2O) targets made by Coon, Davis, Graves, Manley, and Nobles[4] is shown in Fig. 2. The angular distribution of the neutrons from such a source is not isotropic, and the degree of anisotropy varies with deuteron energy. Figure 3 shows an unpublished curve, supplied by E. R. Graves, of the angular distribution of the neutrons from this reaction for an incident deuteron energy of 200 kev. In this energy region the angular distribution in the center-of-mass system of coordinates is well represented by $1+B\cos^2\theta$, where θ is the angle between the incident deuteron and the emitted neutron. A theoretical analysis of the angular distribution as a function of energy in this energy range has been performed by Konopinski and Teller.[5]

With deuteron energies of 400 kev or less, the spectrum of the neutrons at 90° is approximately monoergic with an energy of about 2.5 Mev if a thick target is used, while the neutrons in the forward direction will have a distribution in energy which depends on the bombarding energy used. These energies will, however, be strongly weighted toward the higher values because the yield increases rapidly with energy.

In this low energy range the He^3 nuclei emitted in the neutron producing reaction have such a short range that it is difficult to detect them quantitatively. Consequently, it is difficult to use the He^3 particles as a calibrating monitor for the number of neutrons emitted from the source. However, the companion reaction, $D(d,p)H^3$, can be used as an approximate monitor for the neutrons since the protons have nearly the same yield and angular distribution as the neutrons.[5a] The protons from this reaction have an energy of approximately 3 Mev and can easily be counted quantitatively with a pulse ionization chamber having a thin window.

With deuterons having energies greater than 500 kev it is necessary to use thin targets to obtain even approxi-

[3] R. Ladenberg and M. H. Kanner, Phys. Rev. **52**, 911 (1937); H. Reddemann, Zeits. f. Physik **110**, 373 (1938); Manley, Coon, and Graves, Phys. Rev. **70**, 101 (1946).
[4] Coon, Davis, Graves, Manley, and Nobles, LADC 56 and 75.
[5] E. J. Konopinski and E. Teller, Phys. Rev. **73**, 822 (1948).
[5a] Bretscher, French, and Seidel, Phys Rev. **73**, 815 (1948).

mately monoergic neutrons. Such targets are usually thin gas volumes which are separated from the accelerating tube vacuum by aluminum or nickel foils. Considerable difficulties are met in the use of such thin targets, because of the large number of background neutrons produced in the foils and diaphragms defining the beam. Nickel foils give less background than aluminum foils of the same stopping power, but it is somewhat easier to get good aluminum foils producing less energy loss than nickel. The largest sources of background neutrons seem to be due to deuterons striking the carbon which has been deposited on the apertures and foils during the continued bombardment by the beam. These carbon deposits come from oil vapor in the vacuum system and can be reduced considerably by liquid air traps in the vicinity of the target and by heating the collimating apertures. By exchanging the heavy hydrogen gas in the target chamber for ordinary hydrogen, the contamination sources of neutrons can be measured and one can correct for their contributions to the experimental observations.

An alternative method of obtaining approximately monoergic neutrons from the deuteron-deuteron reaction is described by Bailey *et al.*[6] These workers used a continually replenished thick heavy ice target and bombarded at two voltages which bracketed the resultant neutron energy desired. The difference in the number of neutrons and resultant effect could then be attributed to neutrons of the desired energy. Such a method presents many experimental difficulties and the gas target technique, in general, is more satisfactory.

Information now exists on the total cross section and the differential cross section for the $D(d,n)He^3$ reaction in the energy region between 0.1 and 3.7 Mev. Bennett, Mandeville, and Richards[7] measured the relative yields of neutrons at 0° and 80° in the laboratory system of coordinates as a function of energy of the incident deuterons, from 0.5 to 1.8 Mev. They analyzed these results on the basis of an assumed $1+B\cos^2\theta$ angular distribution, which is now known to be correct only up to approximately 1 Mev. This deviation will be discussed later in this section. The total yield of neutrons on an arbitrary scale was thus calculated over their entire energy region and is probably approximately correct for deuteron energies up to 1 Mev.

The recent work of Blair *et al.*[8] has given values for the differential cross section for the $D(d,n)He^3$ reaction in the deuteron energy region 1 to 3.5 Mev. In their experiments the He^3 nuclei from a gas target were observed with very accurately known geometries, currents and target gas pressures, and an angular definition of the order of two degrees. When these observations

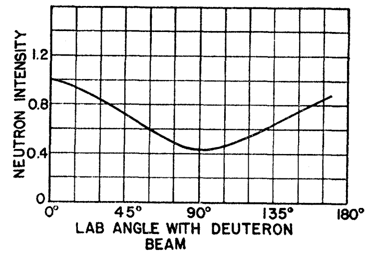

FIG. 3. The angular distribution of the neutrons emitted in the $D(d,n)He^3$ reaction for an incident deuteron energy of 200 kev.

were converted to the center-of-mass system of coordinates, they gave the differential cross section for neutron production directly in this coordinate system.

An analysis of these results shows that the angular distribution in the region above 1-Mev deuteron energy could not be represented by the expression $1+B\cos^2\theta$, but that it was necessary to introduce an additional term to represent the effects of disintegrations produced by incident deuterons making collisions in states of higher angular momentum than $l=1$. The resultant angular distribution could be represented by $A(1+B\cos^2\theta+C\cos^4\theta)$ where θ is the angle of neutron or He^3 emission and where A, B, and C are functions of the incident deuteron energy.

The practical consequence of this angular distribution is that the intensity of neutrons emitted in the forward direction far exceeds that in other directions in the forward hemisphere. The minimum of intensity for 3-Mev deuterons occurs in the neighborhood of sixty degrees in the center-of-mass coordinate system. A slight secondary maximum at ninety degrees is observed.

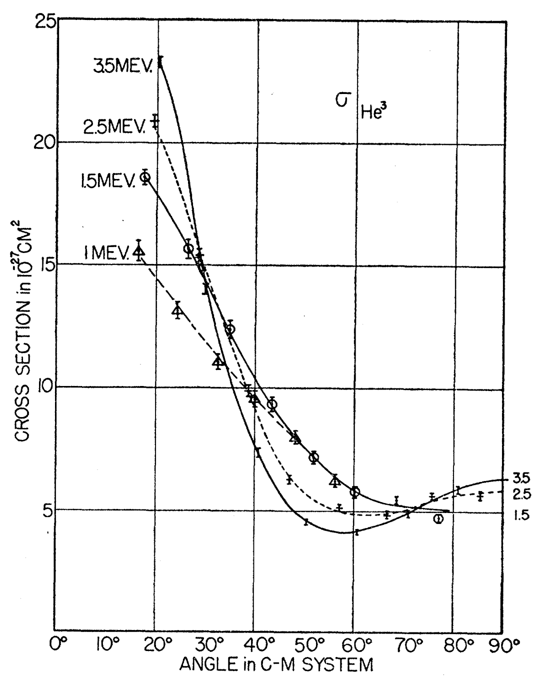

FIG. 4. The differential cross section of the He^3 particle emitted in the $D(d,n)He^3$ reaction as a function of the angle of emission in the center-of-mass coordinates.

[6] Bailey, Bennett, Bergstralh, Nuckolls, Richards, and Williams, Phys. Rev. **70**, 583 (1946).
[7] Bennett, Mandeville, and Richards, Phys. Rev. **69**, 418 (1946).
[8] Blair, Freier, Lampi, Sleator, and Williams, Phys. Rev. **74**, 1599 (1948).

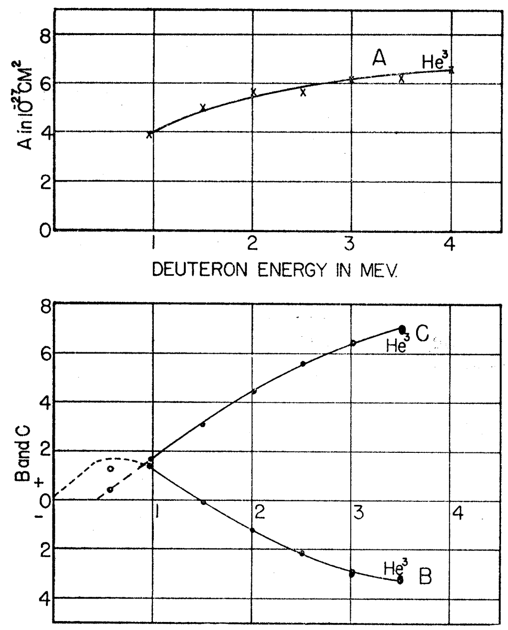

FIG. 5. The values of A, B, and C as a function of E_d which fit the observations on $\sigma(He^3)$ to the expression $A(1+B\cos^2\theta+C\cos^4\theta)$. The dashed curve below 0.5 Mev represents the results collected by Konopinski and Teller (see reference 5), the open circles are the low energy results of Hunter and Richards (see reference 9), and the closed circles are the results of Blair *et al.* (see reference 8).

The results of these observations for the absolute differential cross section are shown in Fig. 4 which represents the differential cross section as a function of θ for different deuteron energies. Figure 5 is a plot of A, B, and C as a function of deuteron energy. In this figure the values of B, as calculated by Konopinski and Teller,[5] are shown by the dotted line in the deuteron energy region up to 400 kev. The values of B in the energy region 0.5 to 1 Mev are adapted from the work

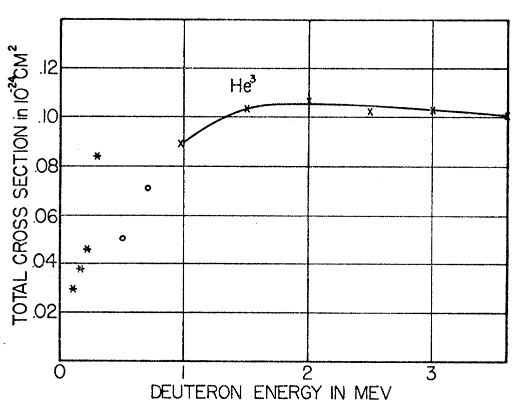

FIG. 6. The $D(d,n)He^3$ total cross section as a function of incident deuteron energy. The asterisks are the results of Graves *et al.* (see reference 4), the open circles are the low energy results of Hunter and Richards (see reference 9), and the closed circles are the results of Blair *et al.* (see reference 8).

of Hunter and Richards.[9] It may be possible to explain the variation of B with energy by the interference of the D and S waves. An analysis of these results is being made by Professor C. L. Critchfield. Figure 6 shows the variation of the total cross section of the $D(d,n)He^3$ cross section as a function of incident energy. The points below 0.5 Mev represent the cross sections obtained by Manley, Coon, and Graves[3] who used a thick heavy-ice target and a $MnSO_4$ bath technique for measuring total neutron yields.

Confirmation of this angular distribution of the particles from the $D(d,n)He^3$ reaction is given by the recent results of Hunter and Richards.[9] These workers measured the angular distribution of the neutrons from a thin deuterium gas target bombarded with deuterons of discrete energies from 0.5 to 3.7 Mev. Their results show that the angular distribution can best be represented by an expression $A+B\cos^2\theta+C\cos^4\theta+D\cos^6\theta$ with a small coefficient D. The agreement between their results and those of Blair *et al.*[8] on the total cross section of the reaction is good. These data also serve to extend the variation of the coefficient B with incident deuteron energy down to 0.5 Mev.

The neutrons from this reaction have been shown to be monoergic at low deuteron energies[10, 11] and are, at least largely, monoergic at deuteron energies of 10 Mev. At these higher energies, however, it is difficult to make neutron spectrum measurements, and conclusive proof of the monoergic nature of the neutrons is lacking. Furthermore, it is extremely difficult to make experimental observations at high deuteron energies with the assurance that the effects observed are due to the D-D neutrons from a thin target of deuterium, since there are always large numbers of neutrons produced by deuterons striking the collimating apertures and foils used to contain the deuterium gas targets.

The present usefulness of the D,D source lies in its power to provide neutrons in the energy region four to seven Mev. Such neutrons are not available from $T(p,n)He^3$ and $Li^7(p,n)Be^7$ sources in existing accelerators with controlled voltages. It is, thus, unfortunate that more data do not exist on the monoenergetic character of these neutrons for deuteron energies above 2 Mev, and that the background difficulties with this source are so formidable. A possible escape from this dilemma has been suggested by the work of Curtis, Fowler, and Rosen.[12] They limited the counting of the D,D neutrons or their secondary effects to those events which occur simultaneously with the emission of a full range He^3 from the primary reaction. This method is particularly useful in determining absolute reaction cross sections since the neutron flux at a given angle can be obtained

[9] G. T. Hunter and H. T. Richards, Phys. Rev. **75**, 335A (1949).
[10] T. W. Bonner, Phys. Rev **59**, 237 (1941).
[11] H. T. Richards, Phys. Rev. **59**, 796 (1947); J. S. Laughlin and P. G. Kruger, Phys. Rev. **71**, 736 (1947).
[12] Curtis, Fowler, and Rosen (LAMS 600), to be submitted for publication in Phys. Rev.

from the He^3 counting rate, and the counts due to neutrons from other parts of the system are eliminated.

$T^3(d,n)He^4$

This reaction is described by the equation

$$T^3+D^2 \rightarrow He^4+n^1+17.6 \text{ Mev}. \quad (2)$$

It is usable either by accelerating the tritons onto thick or thin deuterium targets, by accelerating deuterons through thin foils into tritium gas targets, or by bombarding tritium absorbed in zirconium or other metals.

The reaction is highly exoergic, and if used with an electrostatic generator can produce neutrons having energies of 12 to 20 Mev (as shown in Fig. 7 and the nomograph in Fig. 27; for low bombarding energies and high Q reactions the nomographs lose much of their usefulness). Because of the large energy released, the He^4 nuclei can be rather easily counted. The number of neutrons emitted at a given angle can then be obtained from the He^4 count by the appropriate equations relating the angles of the neutron with those of the He^4 recoils.

This reaction was originally studied by accelerating tritons to about 300 kev to 1 Mev with a cyclotron[13] and observing the alpha-particles. A strong resonance on the low energy side of these measurements led to intensive studies of the reaction from about 20 kev to 125 kev triton energy with a low voltage Cockcroft-Walton accelerator.[14] Measurements of the angular distribution of alpha-particles have recently been made[15] for deuteron energies between 1.0 and 2.5 Mev, giving total reaction cross sections in the equivalent triton energy range of 1.5 to 3.75 Mev. Thin gas targets were used for these measurements.

Figure 8 shows the results of these three different sets of measurements. To fit their data only, Bretscher and French have used a formula composed of a penetration function and a term in the denominator indicating resonance:

$$\sigma(E)=\frac{325\times 10^{-18}\exp[-1.72/(E_{\text{Mev}})^{\frac{1}{2}}]}{E_{\text{kev}}[(124.3-E_{\text{kev}})^2+(71.7)^2]}\text{ cm}^2.$$

This equation gives a rather narrow maximum, and to fit both the Bretscher-French and the Baker *et al.* data, the equation below seems more satisfactory:

$$\sigma(E_{\text{Mev}})=\frac{58}{E}\,\frac{\exp[-1.72/(E)^{\frac{1}{2}}]}{1+(E-0.096)^2/(0.174)^2}\text{ barns}.$$

These two equations are plotted in Fig. 8. The dashed line on the high energy side of the maximum is from the first equation, the latter equation giving the broader

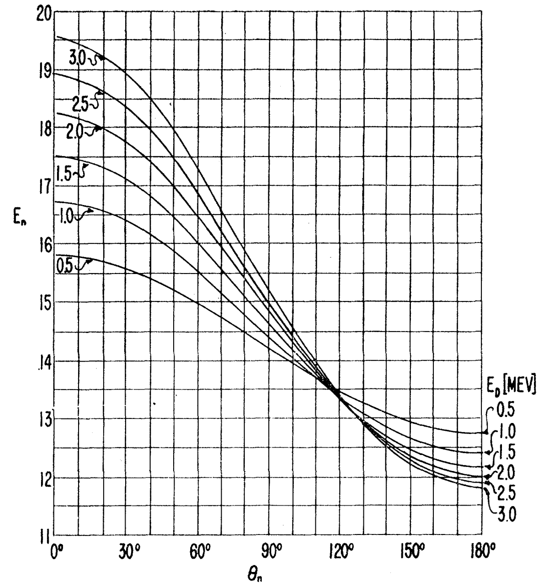

FIG. 7. The energies of the neutrons emitted from the $T(d,n)He^4$ reaction for various E_d as a function of the laboratory angle of emission of the neutrons.

"resonance." Thick target measurements of neutron yield for 600 kev deuteron energy agree more satisfactorily with the latter equation. Angular distribution measurements on the alpha-particles up to 125 kev indicate spherical symmetry in center-of-mass coordinates; the intermediate energy measurements of reference 13 were made at 0° and 90° and indicate a forward asymmetry while the high energy data[15] give a strong asymmetry which is shown in Fig. 9.

Because of the large value of the cross section at a few hundred kilovolts and the Q value of 17.6 Mev, this reaction is an ideal source for the production of 14-Mev neutrons in large quantities with low voltage accelerating equipment utilizing thick targets. The use of T_2O ice targets appears to be impractical. This arises because of rapid contamination of the small amounts of T_2O usually available with ordinary water from various

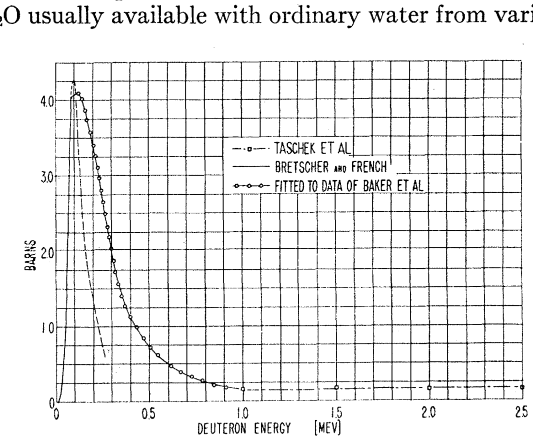

FIG. 8. The total cross section of the $T(d,n)He^4$ reaction as a function of the incident deuteron energy. The meaning of the dashed lines is explained in the text.

[13] Baker, Holloway, Schreiber, and King, LAMS 11.

[14] E. Bretscher and A. P. French, Phys. Rev. **75**, 1154 (1948).

[15] Taschek, Hemmendinger, and Jarvis, Phys. Rev. **75**, 1464 (1949).

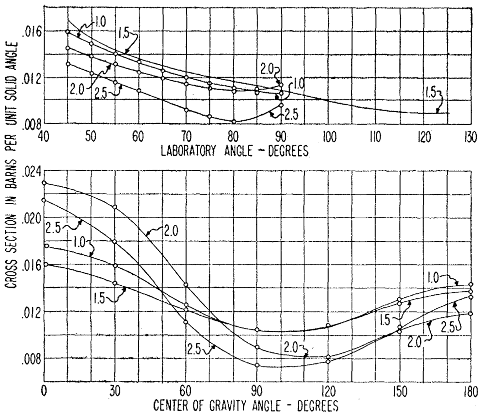

FIG. 9. The angular distribution of the alpha-particles from the $T(d,n)He^4$ reaction for various incident deuteron energies.

"outgassing" sources in an accelerating tube.[14] However, both tantalum and zirconium metals when exceedingly well outgassed absorb large amounts of hydrogen, and, therefore, tritium, and retain them reasonably well under not too intense ion bombardment.[16] The techniques developed by Graves *et al.*[17] have shown that zirconium welded to tungsten after hydrogenation still possesses the good physical properties necessary for a more heavily bombarded target; this appears not to be the case with tantalum which

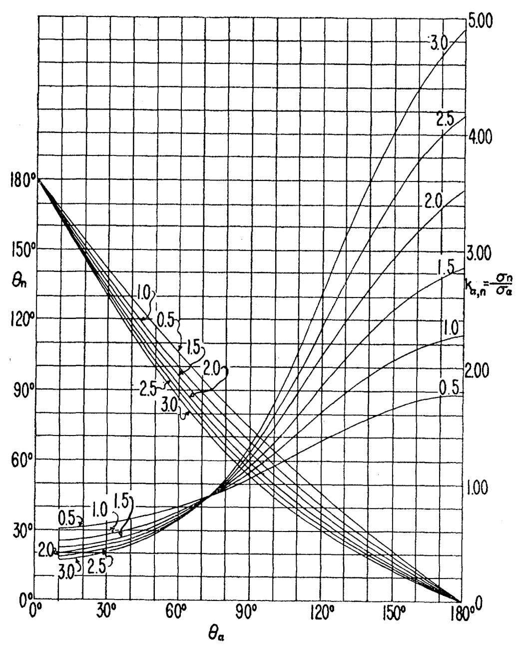

FIG. 10. A solid angle conversion chart for use with the $T(d,n)He^4$ reaction. θ_n is the angle of emission of neutrons in the laboratory system of coordinates. θ_α is the corresponding angle for alpha-particles. $k_{\alpha n}$ is the ratio of the cross section for neutron production to the cross section for alpha-particle production. $k_{\alpha n}$ is shown as a function of θ_α.

[16] F. M. Penning and J. A. H. Moubis, Physica 4, 1190 (1937).

[17] Graves, Rodriguiz, Goldblatt, and Meyer, personal communication, to be submitted for publication in Rev. Sci. Inst.

becomes rather brittle and friable. For equal energy losses, however, gas targets have a great advantage over adsorbed tritium targets, if they can be used, because of the much larger number of tritium atoms present.

Accelerating equipment such as the Cockcroft-Walton sets available in many universities which can provide ordinarily 10 to 50 microamperes of deuterons, if the current density is kept reasonably low, make strong sources of 14-Mev neutrons. Adsorbed gas sources deteriorate only slowly as the tritium escapes.

For accelerators yielding somewhat higher energy, gas targets with thin aluminum windows make even stronger neutron sources when the gas pressure and target length are sufficient to stop the deuteron beam. Thus, deuterons having energies of 500 or 600 kev after passing through an aluminum foil in which the energy loss is about 200 kev, may be stopped in tritium and advantage may be taken of the large cross section for the reaction throughout the whole width of the resonance.

By using thin targets and higher bombarding energies, the neutrons with energy in the range between 13 and 19 Mev are available. A gas target of small volume and incorporating a counter for the detection of the alphas from this reaction has been described.[18] Although not ideally suited for a neutron source because of the massive construction, the variable detection angle for alphas lends itself to absolute neutron flux measurements at a given angle (and, therefore, energy) in terms of the alpha-count at the corresponding angle in the laboratory system. In Fig. 10 are shown the relationships between neutron and alpha-angle and the factor $K_{\alpha n}$ at each alpha-angle needed to multiply the measured alpha-flux to get the corresponding neutron flux. These are plotted for various incident deuteron energies. A gas target with thin walls to minimize neutron scattering and degradation has been used at Los Alamos with currents up to 10 microamperes making a much stronger, and more useful, neutron source which can be used for absolute flux measurements once the differential cross sections for the $T^3(d,n)He^3$ reaction have been well measured. By a combination of varying angle and deuteron energy, the practical range of neutron energies for a 3-Mev accelerator lies between about 13 and 19 Mev. The acceleration of tritons instead of deuterons allows a small reduction of the minimum energy, but not commensurate with the difficulties involved. The energy spreads for a 100-kev thick target are, at most, of the order of one percent of the neutron energies up to about 20 Mev. One feels reasonably assured of monoergic neutrons up to this energy by the apparent absence of excited states in the residual nucleus He^4 below 20 Mev. Both the alpha-particle group and photographic plate spectra of the neutrons indicate no strong excited state.[19]

[18] R. F. Taschek, Rev. Sci. Inst. 19, 591 (1948).

[19] E. R. Graves and L. Rosen, personal communication.

The thin target yields at energies above 1 Mev are low, but compare favorably with the $D^2(d,n)He^3$ reaction. On the other hand, examples of the thick target yields are: a thick gas target used with a primary deuteron energy of about 600 kev which gives a source strength of 5×10^8 neutrons per microcoulomb of deuterons and a thick zirconium target at $E_d=200$ kev which gives about 10^8 neutrons per microcoulomb of deuterons.

$N^{14}(d,n)O^{15}$

This reaction is described by

$$N^{14}+D^2\rightarrow O^{15}+n^1+5.1 \text{ Mev}. \tag{3}$$

Recent work of Gibson and Livesey[20] with deuterons on nitride targets shows rather definitely that at 1-Mev deuteron energy the reaction has only one group of neutrons, corresponding to $Q=5.1$ Mev. The older work of Stephens, Djanab, and Bonner[21] lists a group of neutrons from an excited state of O^{15} with a Q of 1.1 Mev. This value is not verified by Gibson and Livesey.

The approximate source strengths reported in reference 20 were obtained from a comparison with the $D(d,n)He^3$ cross section. The ratio of D,D to D,N^{14} cross sections observed were 4.4 at 0° and 30°, 1.5 at 90°, and 2.1 at 150°, indicating more nearly symmetric emission for $D+N^{14}$ than for D+D. The 0° yield comparison has been approximately verified at Los Alamos with gas targets and long counter detection.

The great need for monoergic neutron sources to fill the energy gap between D+D and D+T makes $N^{14}(d,n)O^{15}$ worthy of further investigation even though it is a weak source and encourages study of $N^{15}(d,n)O^{16}$ with a Q of about 10 Mev.

2. ENDOERGIC REACTIONS

In contrast to the reactions discussed above in which fast neutrons are produced without requiring very high energy incident particles, for many monoergic neutron-producing reactions Q is negative, and they are called endoergic. The minimum energy, in the laboratory system, of the incident particle of mass M_1 incident on the target nucleus of mass M_2 which will just cause the reaction to proceed is $E_T=-Q(M_1+M_2)/M_2$ and is called the threshold energy. At precisely this threshold bombarding energy, neutrons are emitted in the forward direction only, and these neutrons have a low energy which corresponds to the velocity of the center-of-mass of the system. As the bombarding energy is increased slightly above the threshold energy, neutrons are emitted in a cone of limited angle in the forward direction. At each angle within this cone, except at the edge, there are two groups of neutrons of different energies corresponding to neutrons emitted in two different directions in the center-of-mass system of coordinate.[a] As the incident particle energy is increased, this cone widens until it includes the forward hemisphere. At higher bombarding particle energies, neu-

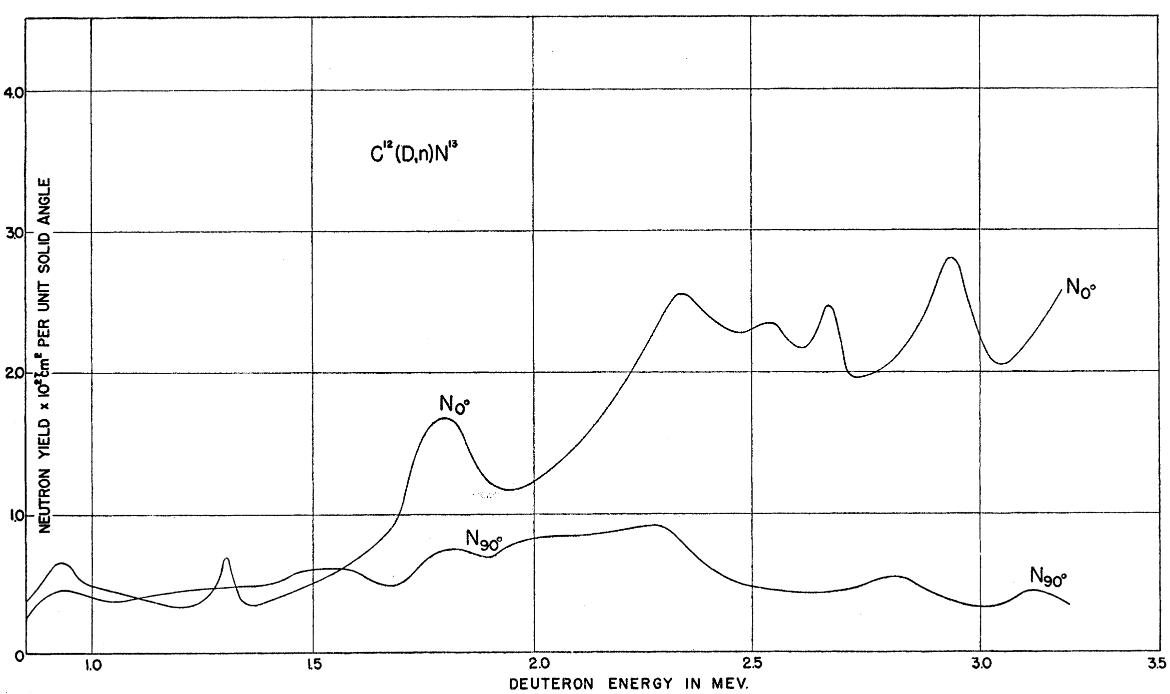

FIG. 11. The yield of neutrons from the $C^{12}(d,n)N^{13}$ reaction as a function of incident deuteron energy for neutrons emitted at 0° and 90°.

[20] W. M. Gibson and D. L. Livesey, Proc. Phys. Soc. **60**, 523 (1948).
[21] Stephens, Djanab, and Bonner, Phys. Rev. **52**, 1079 (1937).
[a] See Appendix.

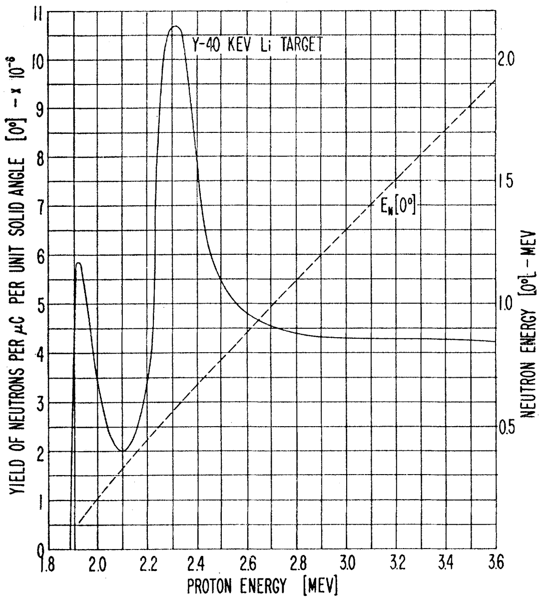

FIG. 12. The yield of neutrons in the forward direction from a 40-kev thick lithium target and the energies of neutrons emitted from a thin lithium target in the forward direction as a function of the incident proton energy.

trons of a discrete energy are emitted at each angle in the complete sphere about the target.

By proper control of the energy of the incident particle it is theoretically possible to produce monoergic neutrons of any energy from endoergic reactions. In practice, it has been difficult to produce useful intensities of neutrons having energies much less than 5 kev

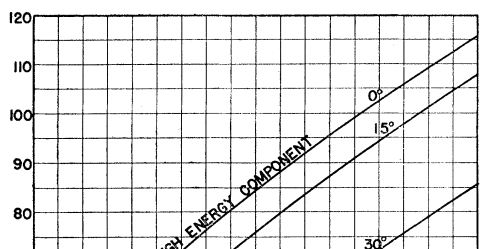

FIG. 13. Neutron energies from a thin lithium target as a function of the difference between the bombarding energy and the threshold energy for various angles of emission in the laboratory.

without excessive energy spread. However, the production of monoergic neutrons of energy greater than 5 kev has proven to be quite practicable. The endoergic reactions which have been used most commonly are $C^{12}(d,n)N^{13}$, $T(p,n)He^3$, $Li^7(p,n)Be^7$. There are also a number of other promising (p,n) reactions such as protons on vanadium and scandium.

$C^{12}(d,n)N^{13}$

This reaction is described by the equation

$$C^{12}+D^2\rightarrow N^{13}+n^1-0.26 \text{ Mev.} \tag{4}$$

The nomograph giving the energy of the neutrons as a function of angle and incident deuteron energy is shown in Fig. 28.

This reaction has been studied by a number of observers[22] and has been found to give rise to monoergic neutrons up to 2.0 Mev deuteron energy.[23] There is, however, a group of high energy neutrons from the reaction[24] $C^{13}+D^2\rightarrow N^{14}+n^1+5.2$ Mev in normal carbon. This group may lead to a certain amount of confusion in some experiments. Since its intensity is only about one percent of the low energy group from C^{12}, it is usually possible to make corrections for its effect. Satisfactory targets have been made with thin soot deposits, evaporated paraffin, or India ink on tantalum backing material. Targets of the separated isotope C^{12} would make this a source of greater usefulness.

The yield of this source has been measured by counting the positron activity of N^{13} and by direct measurements of the neutrons. The angular distribution of the emitted neutrons is exceedingly complex,[25] being relatively small at 90° compared to 0° and rising markedly again in the backwards direction for deuteron energies of approximately 1.5 Mev. Since the main practical use of this source is for accelerators whose maximum energy range lies below the threshold of the $T(p,n)He^3$ reaction, 1019 kev, we are consequently interested principally in the yield of neutrons in the forward direction. Such a yield curve is shown in Fig. 11. The cross section was determined by measuring the neutron flux with a fission counter and is uncertain to approximately twenty-five percent. One can see from these curves that the angular distribution of the neutrons from this source is very anisotropic.

The relatively low yield at low deuteron energies and the difficulties of C^{13} contamination make the use of this neutron source rather unfavorable. Further difficulties arise from the presence of neutrons emitted in the $D(d,n)He^3$ process which is always present in any deuteron accelerator as a result of deuterium contami-

[22] J. D. Cockcroft and W. B. Lewis, Proc. Roy. Soc. **154**, 261 (1936); T. W. Bonner, Phys. Rev. **53**, 496 (1938); Amaldi, Hafstad, and Tuve, Phys. Rev. **51**, 896 (1937).
[23] W. E. Bennett and H. T. Richards, Phys. Rev. **71**, 565 (1947).
[24] T. W. Bonner and W. M. Brubaker, Phys. Rev. **50**, 308 (1936).
[25] T. W. Bonner, ONR Conference, University of Chicago (December 1948).

nation of diaphragms and target. Consequently, it is recommended that the use of this source be avoided unless the voltage limitations of the accelerator prevent one from using the more advantageous $Li^7(p,n)Be^7$ or $T(p,n)He^3$ sources.

Unfortunately, the $C^{12}(d,n)N^{13}$ reaction cannot be entirely avoided in any deuteron accelerator since carbon is always present as a contaminant in vacuum systems employing oil diffusion pumps. An estimate of the magnitude of this background source of neutrons can be obtained by measuring the total yield of neutrons from the accelerator with a heavy target, say Ta, at two bombarding energies, one at a $C^{12}(d,n)N^{13}$ resonance and one removed from such a resonance.

$Li^7(p,n)Be^7$

This reaction is endoergic and is described by the equation

$$Li^7+H^1 \rightarrow Be^7+n^1-1.647 \text{ Mev}. \quad (5)$$

The value of Q is determined from the proton energy at the threshold (1.882 Mev) of the reaction.[26] The energy of the neutrons at any angle for various proton energies can be obtained from the nomograph given by Fig. 29. With the exception of Figs. 12 and 29, the data shown for this reaction are based on the old Q value of -1.63 Mev.

A typical yield curve in the forward direction is shown in Fig. 12. The target in this case was about 40 kev thick for protons of about 1.9 Mev energy. Near the threshold energy, neutrons are emitted from the moving Be^8 nucleus with very low velocities and have an energy of 29 kev corresponding to the velocity of the Be^8 nucleus. At energies below 1.92 Mev there are two groups of neutrons emitted at each angle in the forward direction corresponding to neutrons emitted in different directions in the center-of-gravity system as shown in Fig. 13. The first maximum in the yield curve probably arises entirely from the method of observation since at proton energies just above threshold the neutrons are emitted within a forward cone, the angle of which increases to include all directions at a proton energy of 1.92 Mev. An observation of the number of neutrons emitted in the forward direction at incident proton energies just above the threshold energy gives a total yield measurement and at a slightly higher proton energy the observations can be interpreted as differential yields, if the target is sufficiently thin.

To obtain usable yields of monoergic neutrons below 80 kev it is necessary to use very large proton currents on thin targets and to observe at back angles. A typical neutron yield curve from a 2-kev target at an angle of 120° and 135° is shown in Fig. 14. The energy of the neutrons at various proton energies is shown in the same figure.

Since it seemed quite probable that Be^7 produced in this reaction could be left in an excited state of a few hundred kilovolts a considerable number of experiments have been made to look for a possible low energy group

[26] Haxby, Shoupp, Stephens, and Wells, Phys. Rev. **58**, 1035 (1940); A. O. Hanson and D. L. Benedict, Phys. Rev. **65**, 33 (1944); Herb, Snowden, and Sala, Phys. Rev. **75**, 246 (1949); Shoupp, Jennings, Jones, and Garbuny, Phys. Rev. **75**, 336A (1949).

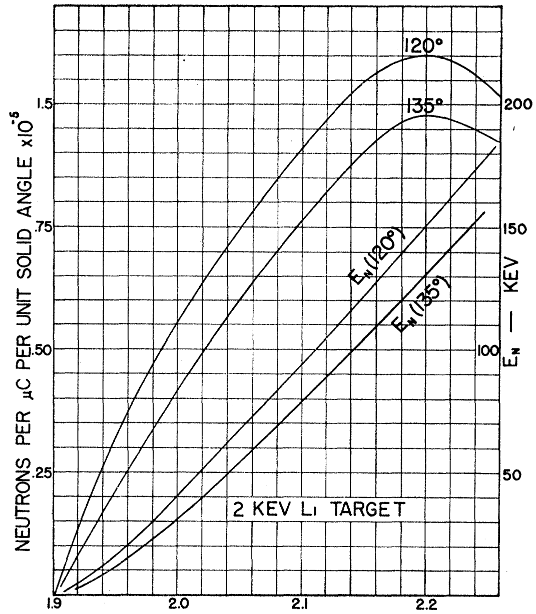

FIG. 14. Neutron yields and neutron energies emitted from a 2-kev thick lithium target as a function of incident proton energy for 120° and 135° in the laboratory. This figure is drawn for an assumed threshold of 1.86 Mev.

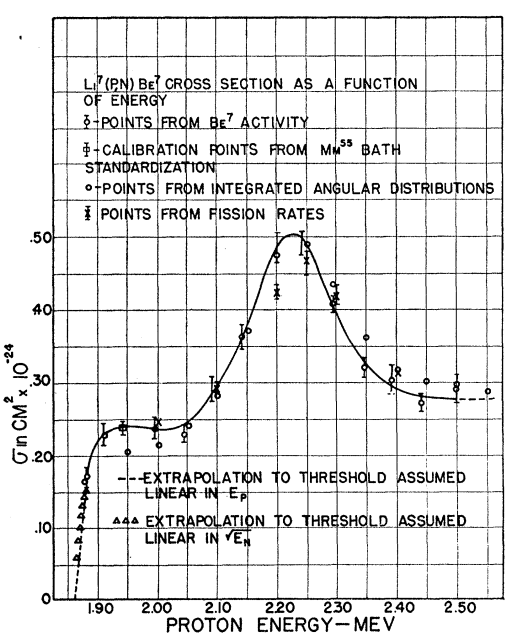

FIG. 15. Total cross section of $Li^7(p,n)Be^7$ as a function of incident proton energy. The threshold energy is assumed to be 1.86 Mev.

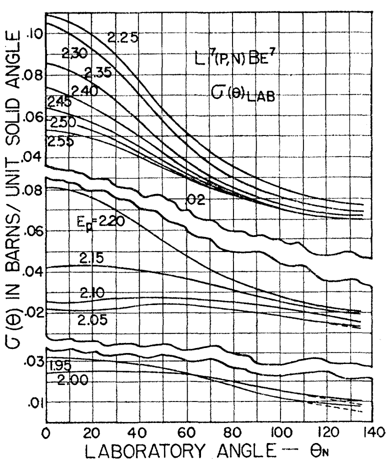

FIG. 16. Differential cross sections for the production of neutrons from the $Li^7(p,n)Be^7$ reaction in the laboratory system at a function of the angle of the neutron θ_n for various values of E_p. E_{th} assumed to be 1.86 Mev.

of neutrons, but all evidence at present seems to exclude such a group with an intensity of greater than about 10 percent of the main group up to proton energies of 3.7 Mev.[27]

The total cross section[28] for the production of neutrons from the $Li^7(p,n)Be^7$ reaction is shown as a function of energy in Fig. 15. The angular distributions of the neutrons at various proton energies are shown in Fig. 16.

The use of thin lithium targets as neutron sources over a considerable period of operation makes the stability of such targets an important consideration. Targets which are mounted to rotate eccentrically about the axis of the proton beam increase the area bombarded and also prevent excessive local heating which would destroy the lithium film. The lithium target may be evaporated on such a rotating thin tantalum target backing from an electrically heated iron crucible moved into place through a Wilson seal or by means of auxiliary evaporation equipment separate from the target which permits the target to be kept in an argon atmosphere during time of transfer. With currents greater than 2 microamperes it is advisable to cool the back side of the target with an air and water spray.

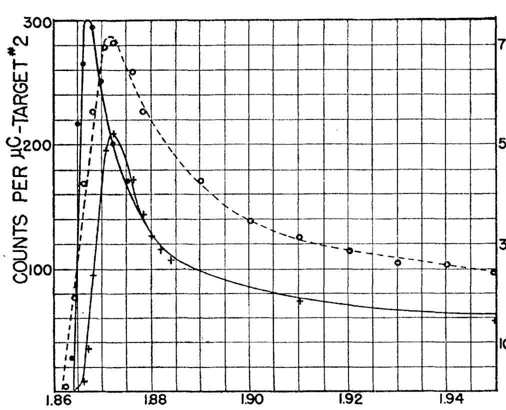

FIG. 17. Relative neutron yields at 0° for various thicknesses and ages of lithium targets. Relative yields at (0°). ○ target 1 fresh—9 kev thick by rise. ● target 2 fresh—2 kev thick by rise. + target 2 after 4 weeks use—6 kev thick by rise.

$Y_1(1.93)/Y_2(1.93) = 4.$

The thickness of the lithium target can be estimated from the difference between the proton energy at threshold and the proton energy corresponding to the first maximum in the neutron yield at 0° as shown in Fig. 17; a simultaneous yield measurement at a proton energy where the cross section is not changing rapidly will then calibrate a counter in terms of counting rate per unit target thickness for rapid use at later times.

The target end of the arrangement is made as light as possible to reduce the scattering of the neutrons from this material. This is particularly important when working with low energy neutrons at back angles, since the yield at these angles is low compared to the yield of higher energy neutrons emitted in the forward direction.

$T^3(p,n)He^3$

This reaction is described by the equation

$$T^3+H^1\rightarrow He^3+n^1-0.764 \text{ Mev.} \quad (6)$$

The nomograph shown in Fig. 30 is calculated on the basis of a former Q value of -0.735 Mev. A recent measurement[29] of the Q value based on an absolute energy scale gives $Q=-0.764$ Mev and the energetic threshold in the laboratory system is 1.019 Mev for proton acceleration.

Large neutron yields are obtained near threshold as in the case of the $Li^7(p,n)Be^7$ reaction, since the Coulomb barrier is even lower than for the latter. The source should be monoergic up to neutron energies of 5 Mev or more since no excited states of He^3 have been observed in the $D(d,n)He^3$ reaction. The momentum relations are similar to those for $Li^7(p,n)Be^7$, except that the neutron energy for threshold protons is about 60 kev instead of 30 kev.

Figure 18 shows the total cross section for this reaction up to 2.4 Mev proton energy as obtained by integration of the differential cross sections[30] shown in Fig. 19. As in the case for $Li^7(p,n)Be^7$ the 0° yield curve shown in Fig. 20 has a pronounced maximum just above threshold which is a result of the center-of-mass solid angle transformation into the laboratory system, and is not to be confused with an excited state

[27] Freier, Lampi, and Williams, Phys. Rev. **75**, 901 (1949).
[28] R. F. Taschek and A. Hemmendinger, Phys. Rev. **74**, 373 (1948).
[29] Taschek, Jarvis, Argo, and Hemmendinger, Phys. Rev. **75**, 1268 (1949).
[30] Jarvis, Hemmendinger, Argo, and Taschek, Phys. Rev. **76**, 168 (1949).

of the compound nucleus. The very large value of total cross section at $E_p = 2.4$ Mev together with the fact that there is no indication of leveling off, do however, at least indicate the presence of an excited state of He^4 at some energy not much higher than the maximum used here. Associated with this reaction is a rather large yield of 20-Mev gamma-radiation[31] coming from proton capture by the tritium and this may interfere with some detectors of the neutrons.

It is worth while pointing out here that for this reaction, as for $Li^7(p,n)Be^7$, the presence of two neutron groups is not a serious handicap since, even for spherical symmetry in the center-of-mass system, the "slow" group appears in the laboratory in an intensity ratio to the "fast" group which is the ratio of laboratory energies, and this ratio, particularly at 0°, is quite favorable.

The low threshold for reaction (6) makes it a convenient source of neutrons in approximately the same range given by the $Li^7(p,n)Be^7$ source, but more easily obtainable with a lower voltage accelerator. The yields over the whole range are generally better than those from $Li^7(p,n)Be^7$, and the trend of the yield at high energies is to still larger values.

In cases where tritium for targets is available, a rapid change from low to high energy source is possible by changing from proton to deuteron beam on the same target. Gas target techniques are essentially those described previously.

$V^{51}(p,n)Cr^{51}$

This reaction is described by the equation

$$H^1 + V^{51} \rightarrow n^1 + Cr^{51} - 1.50 \text{ Mev.} \qquad (7)$$

This reaction has been studied[32] in some detail. There is only one isotope of natural vanadium, and thin targets of the metal are rather readily prepared by evaporation. Figure 21 shows the thin target yield curve in the forward direction; the threshold for the reaction is at 1.53 Mev, but the yield is very low here. At higher energies the $V(p,n)$ reaction is a strong source of neutrons, but little is known of the spectrum. Measurements have been made up to a neutron energy of about 400 kev from this source, and no evidence for a slow group of intensity more than ten percent of the fast group was observed.

The primary interest in a source with a heavy target element, such as vanadium, is the low energy of neutrons near threshold and the approximate independence of energy upon angle; thus, neutrons of a few kev energy are obtained having only a small variation in energy with angle at proton energies slightly above threshold. Although intensities are low, the large currents from Van der Graaff generators and present high detector sensitivities make this a promising reaction

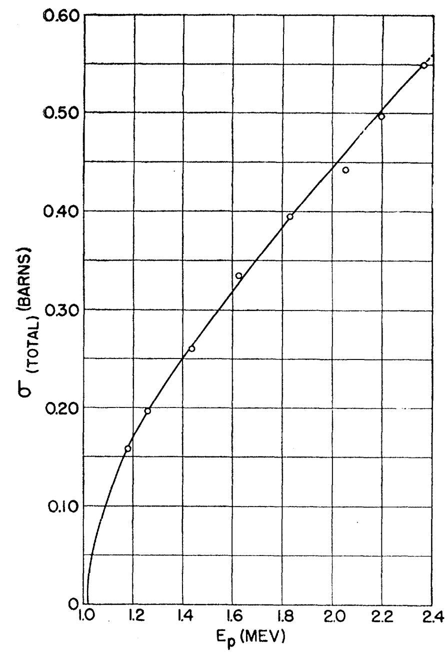

FIG. 18. The total cross section for the $T(p,n)He^3$ reaction as a function of incident proton energy.

for neutrons from, say, 2 kev to 20 kev. Very thin targets and well defined proton energies are necessary to obtain monoergic neutrons in this energy range. The observed yields are roughly the same as those from the $Li^7(p,n)Be^7$ reaction at the backward angles in the same energy range. The primary advantage of the

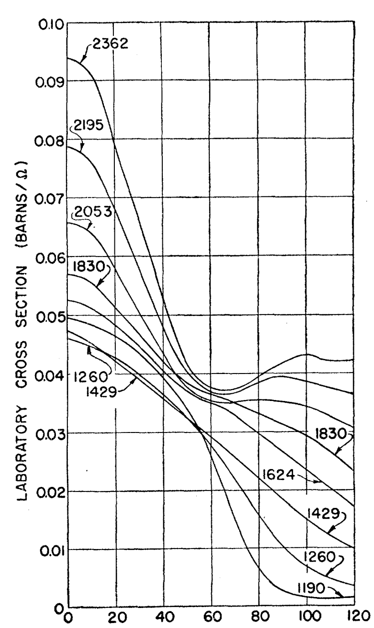

FIG. 19. The differential cross section for the $T(p,n)He^3$ reaction as a function of the angle of neutron emission in the center-of-mass coordinates for various incident proton energies.

[31] Argo, Gittings, Hemmendinger, Jarvis, Mayer, and Taschek, Phys. Rev. 76, 182 (1949).
[32] R. F. Taschek, LADC 154, MDDC312.

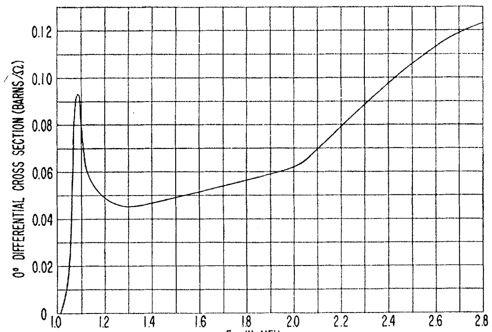

FIG. 20. The yield of neutrons from the $T(p,n)He^3$ reaction in the forward direction as a function of incident proton energy.

$V(p,n)$ source over the $Li(p,n)$ arises from the more symmetrical distribution of intensity from the former. The relative number of neutrons initially emitted in the forward direction, and then scattered into a detector placed in the backward hemisphere, to the number of neutrons which are initially emitted in the direction of the detector is much less for the $V(p,n)$ source than the $Li(p,n)$ source. These scattered neutrons are present in appreciable quantities and make it difficult to interpret observations on low energy neutrons from the latter source.

$Sc^{45}(p,n)Ti^{45}$

This reaction is described by the equation

$$H^1+Sc^{45}\rightarrow n^1+Ti^{45}-2.8 \text{ Mev}. \quad (8)$$

The momentum relations of this reaction are similar to those of $V(p,n)$, and, thus, the advantages as a source

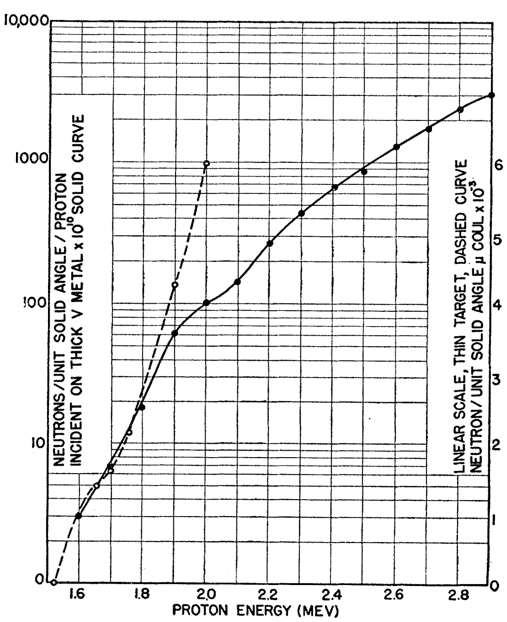

FIG. 21. The $V^{51}(p,n)Cr^{51}$ yield curve in the forward direction as a function of incident proton energy.

of low energy neutrons are essentially the same. However, the threshold for this reaction is at 2.9 Mev, which makes the neutron yield near threshold much larger than for the $V(p,n)$. The reason for this lies in the fact that for these heavy elements, contrary to the $Li(p,n)$ reaction, the penetration of the barrier by the incident protons is the most important factor in determining yield near threshold; thus, the incoming proton has a much better probability of producing a reaction at 2.9 Mev than at 1.6 Mev, if the barriers are roughly equal.

The thick target forward yield curve[32] for $Sc(p,n)$ is shown in Fig. 22; it will be seen that the yield in the forward direction is approximately 40 times that for $V(p,n)$ near threshold, which makes it strong enough for convenient use. The cross section is roughly 3×10^{-26} cm^2. However, the high threshold energy is not easily obtained with most electrostatic generators. This reaction has not been studied in any detail as to the spectrum of the neutrons. In the immediate neighborhood of threshold where the usefulness of the neutrons would be most apparent, the level spacing may be sufficient to give monoergic neutrons of 30 or 40 kev.

SELECTION OF MONOCHROMATIC NEUTRONS FROM COMPLEX SOURCES

The emission of neutrons occurs with appreciable probability in practically all nuclear reactions in which it is energetically possible. In general, the neutrons will not be monoergic, since the residual nucleus may be left in various excited states. These excited states are usually of the order of one Mev apart in the lighter elements and several groups of neutrons may be emitted if the energy released in the reaction is of the order of 5 Mev or greater. At very high bombarding energies corresponding to excess internal energies of the order 20 Mev the statistical theory of nuclei predicts closely spaced energy levels and an approximately continuous neutron spectrum which is relatively weak in the high energy region. Although the neutron group corresponding to the highest Q values for the reaction is, in general, weak, experiments with essentially only this group have been possible by the use of radioactive threshold detectors such as copper (11 Mev) and carbon (21 Mev),[33] the selection of groups of neutrons by means of proton recoils,[34] or by the pulse heights due to neutron reactions in an ion chamber.[35]

In this manner it has been possible to study phenomena due to neutrons of specified energies up to 24 Mev by using the $Li^7(d,n)Be^8$ reaction. The spectrum of neutrons from this reaction for 0.7-Mev deuterons obtained by Richards[11] is shown in Fig. 23. With higher

[33] E. O. Salant and N. F. Ramsay, Phys. Rev. **57**, 1075 (1940); D. C. Grahame and G. T. Seaborg, Phys. Rev. **53**, 795 (1938); R. Sherr, Phys. Rev. **68**, 240 (1945); Amaldi, Bocciarelli, and Trabacchi, Phys. Rev **70**, 103 (1946)

[34] Ageno, Amaldi, Bocciarelli, and Trabacchi, Phys. Rev. **71**, 20 (1947); William Sleator, Jr., Phys. Rev. **72**, 207 (1947).

[35] B. T. Feld, Phys. Rev. **70**, 429 (1946)

deuteron energies the highest energy neutron group is moved to higher energies with decreased relative yields.

The thick target yield from this reaction is greater than that from the $D(d,n)He^3$ reaction at energies of one Mev and is useful in the production of radioactivities produced by either fast or slow neutrons.

OTHER POSSIBLE (p,n) AND (d,n) REACTIONS

Although the $Be(p,n)$ reaction has a large yield, the properties of the neutrons are similar to those of $Li(p,n)$, and the latter has sufficient advantages, such as ease of target preparation, large yield at a lower threshold, etc., to make it unprofitable to develop this reaction as a source. The low yields and multiple isotopes of Cr and Cu are not encouraging for development as sources. Mn and Co might be worth investigating further, although V has a considerably better yield and essentially the same neutron spectrum.

The reaction $D(p,n)2p$ has been investigated[32, 36] from its energetic threshold of approximately 3.3 Mev up to 3.6 Mev with the hope of developing a source of fast collimated neutrons due to the high velocity of the center of gravity. Neutrons from this reaction were observed, the cross section apparently being approximately 10^{-30} cm^2 for an incident proton energy of 3.4 Mev.

The isotope O^{18} present naturally to 0.2 percent gives sufficient yield to be easily detectable near its threshold of 2.6 Mev, indicating a (p,n) reaction cross section of approximately 10^{-25} cm^2. The energy of the neutrons at threshold should be about 7 kev, which might make its further investigation as a source worth while with O^{18} enriched targets.

Other (d,n) reactions such as $F^{19}(d,n)Ne^{20}$, which results in several groups up to 11 Mev in energy, and $Be^9(d,n)B^{10}$ with several groups up to 4.5 Mev in energy, are probably of little interest as sources. Present indications are that such sources will be superseded in the high energy region by the monoergic neutrons from $T(d,n)He^4$ with the deuterons well controlled in energy. The advent of electrostatic generators capable of accelerating ions to energies of 8 to 10 Mev should close the gap between present maximum $D(d,n)$ neutron energies and the lowest $T(d,n)$ energies of approximately 12 Mev, providing the $D(d,n)$ neutrons remain monoergic at such high deuteron energies. At present, it might be worth while to try to close the energy gap by further investigations of (d,n) reactions with large Q's to find one with a single neutron group or with two well defined groups such as seems to be the case with the $N^{14}(d,n)O^{15}$ reaction.

It is clear from the above discussion that one interesting region, between 1 kev and 15 kev, in the neutron energy spectrum is still in a very unsatisfactory state. This region is of interest because present velocity selector techniques with neutrons of energy less than 1 kev allow one to resolve nuclear energy levels in heavy elements, and it would be desirable to extend this resolving power up to higher energies. The recent investigations of Barschall *et al.*[37] have shown that it is possible to investigate nuclear energy levels by scattering of neutrons of energies greater than 10 kev and an energy spread of less than 20 kev. It is not obvious, at present, which of several methods will bridge this gap in the neutron spectrum most effectively.

[36] R. V. Smith and H. T. Richards, Phys. Rev. **74**, 1871 (1948).

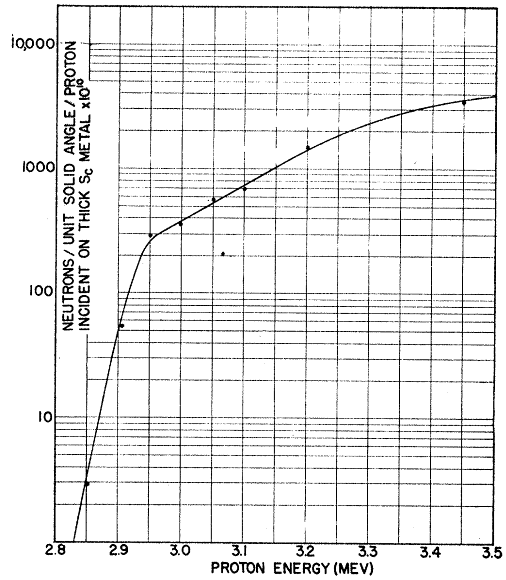

FIG. 22. The $Sc^{45}(p,n)Ti^{45}$ yield curve in the forward direction as a function of incident proton energy.

APPENDIX

The determination of the energy of the neutrons emitted from a given nuclear reaction as a function of the angle of observation and the incident particle energy can be calculated in a very direct manner from the equations for conservation of energy and momentum. In order that one may understand the nomographs for

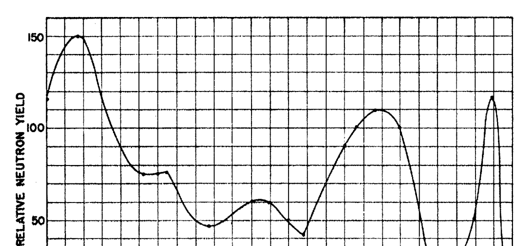

FIG. 23. The energy spectrum of neutrons emitted in the forward direction from a thin lithium target when bombarded with 0.7-Mev deuterons.

[37] Barschall, Bockelmann, and Seagondollar, Phys. Rev. **73**, 659 (1948).

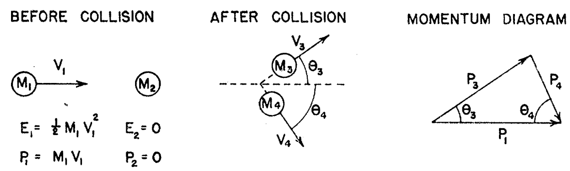

FIG. 24. Representation of a nuclear reaction in the laboratory system of coordinates.

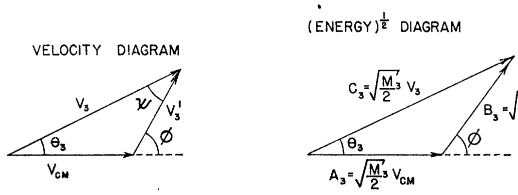

FIG. 25. Relations between velocity vectors and angles in the laboratory and center-of-mass systems of coordinates.

neutron energy given by J. L. McKibben,[38] which are presented here in Figs. 26 to 30, we shall carry through a discussion of the general two-body disintegration mechanics.

A schematic representation of a nuclear disintegration is shown in Fig. 24. An incident particle of mass M_1 and a kinetic energy E_1 collides with a stationary nucleus of mass M_2 producing two particles of mass M_3 and M_4 and with energy E_3 and E_4. If the momenta of these particles are P_1, $P_2(=0)$, P_3, and P_4, respectively, conservation of momentum yields the equations

$$\mathbf{P}_1=\mathbf{P}_3+\mathbf{P}_4 \qquad \text{(A1)}$$

$$P_4^2=P_1^2+P_3^2-2P_1P_3\cos\theta_3. \qquad \text{(A2)}$$

The equation expressing the conservation of energy

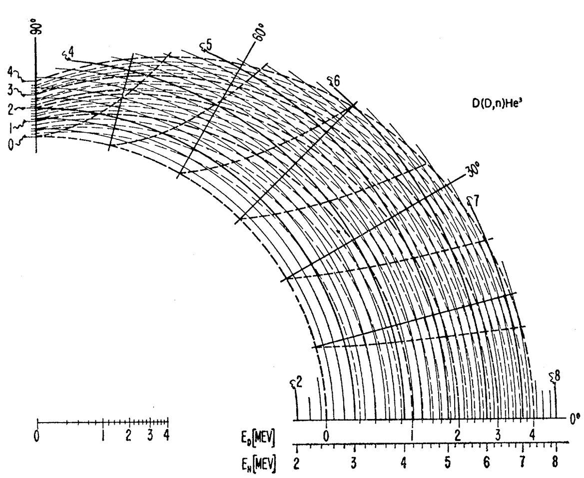

FIG. 26. Energy nomograph for the neutrons from the D(d,n)He^3 reaction. Directions for use are given in the Appendix.

[38] The section on neutron energies and the nomographs in Figs. 26 to 30 are based on the work of J. L. McKibben, Phys. Rev. **70**, 101A (1946).

including the internal energy Q, *released* during the nuclear reaction, is

$$E_3+E_4=E_1+Q, \qquad \text{(A3)}$$

which can be written in terms of the momenta, since $P^2=2ME$, as

$$\frac{P_3^2}{2M_3}+\frac{P_4^2}{2M_4}=\frac{P_1^2}{2M_1}+Q. \qquad \text{(A4)}$$

If the energy of interest is that of particle M_3 (neutron), P_3 can be obtained by eliminating P_4 from Eqs. (A2) and (A4). The expression for E_3 is

$$E_3=E_1\frac{M_1M_3}{(M_3+M_4)^2}\left\{2\cos^2\theta_3+\frac{M_4(M_3+M_4)}{M_1M_3}\times\left[\frac{Q}{E_1}+\left(1-\frac{M_1}{M_4}\right)\right]\pm2\cos\theta_3\times\left[\cos^2\theta_3+\frac{M_4(M_3+M_4)}{M_1M_3}\left[\frac{Q}{E_1}+\left(1-\frac{M_1}{M_4}\right)\right]\right]^{\frac{1}{2}}\right\}. \qquad \text{(A5)}$$

E_4 can be obtained from Eq. (A3). The angle at which M_4 appears can be obtained from the momentum

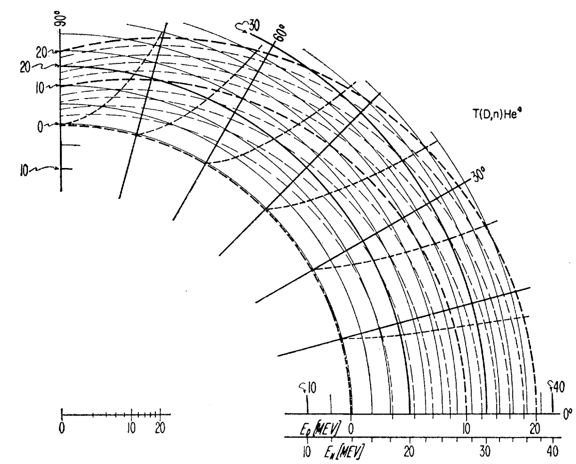

FIG. 27. Energy nomograph for the neutrons from the T(d,n)He^4 reaction. Directions for use are given in the Appendix.

diagram which yields the following relation:

$$\sin\theta_4=\left(\frac{M_3E_3}{M_4E_4}\right)^{\frac{1}{2}}\sin\theta_3. \qquad \text{(A6)}$$

While it is not necessary to concern ourselves with the center-of-mass system in the derivation of Eq. (A5), it is convenient to do so, since the energies of M_3 and M_4 are independent of the angle ϕ in this system. The center-of-mass concept is made evident from the velocity diagram, Fig. 25, in which the velocity V_3' of particle M_3 in the center-of-mass system is related to V_3 and the velocity of the center-of-mass V_{cm}. Since

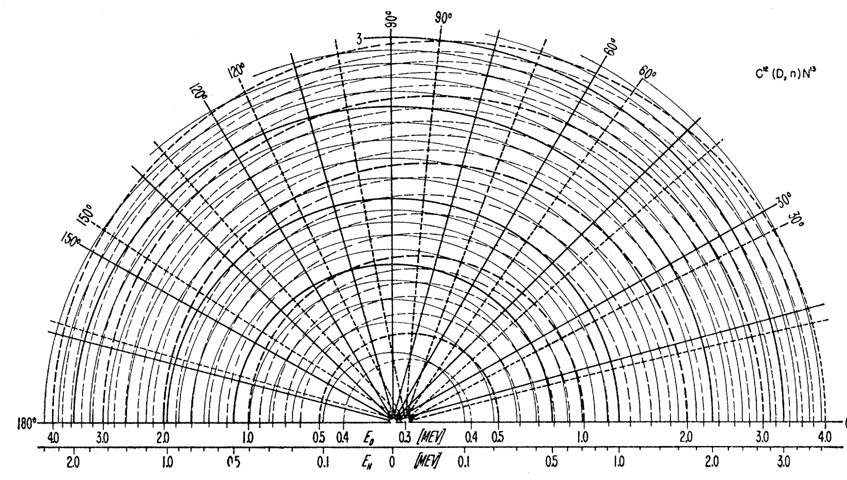

FIG. 28. Energy nomograph for the neutrons from the $C^{12}(d,n)N^{13}$ reaction. Directions for use are given in the Appendix.

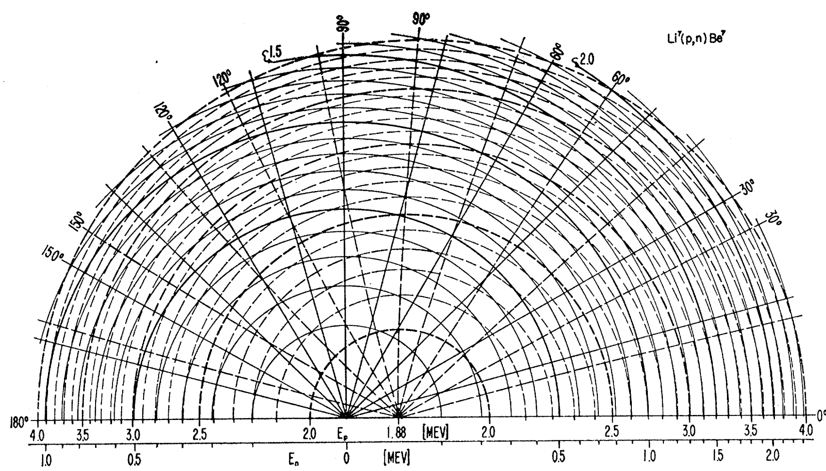

FIG. 29. Energy nomograph for the neutrons from the $Li^7(p,n)Be^7$ reaction. Directions for use are given in the Appendix.

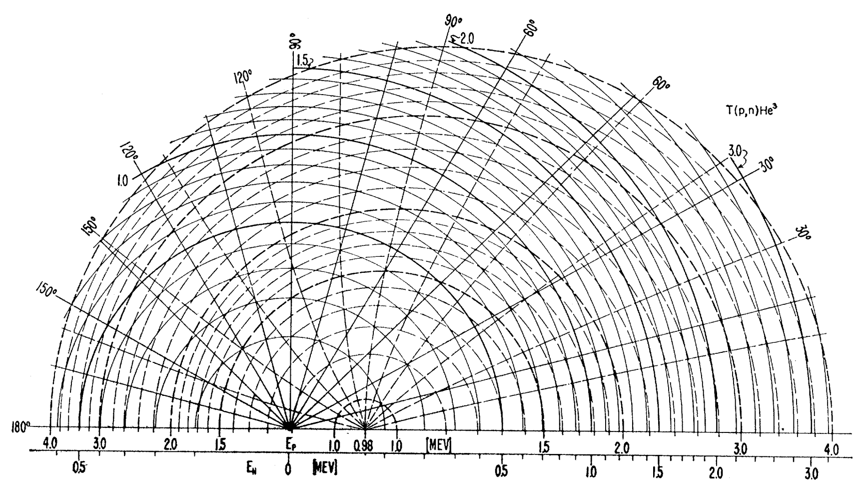

FIG. 30. Energy nomograph for the neutrons from the $T^3(p,n)He^3$ reaction. Directions for use are given in the Appendix.

we are primarily interested in the energy relations, we can follow the method of McKibben[38] in changing this velocity diagram into an $(\text{energy})^{\frac{1}{2}}$ diagram by multiplying each side of the triangle by $(M_3/2)^{\frac{1}{2}}$ as shown in Fig. 25(b). The sides of this triangle are then of length

$$A_3=(M_3/2)^{\frac{1}{2}}V_{\text{cm}}=(M_3/2)^{\frac{1}{2}}\frac{M_1}{M_1+M_2}V_1=\frac{(M_1M_3)^{\frac{1}{2}}}{M_1+M_2}(E_1)^{\frac{1}{2}} \qquad \text{(A7)}$$

$$B_3=(M_3/2)^{\frac{1}{2}}V_3'$$

$$C=(M_3/2)^{\frac{1}{2}}V_3.$$

To eliminate V_3' from the expression for B_3 we consider that in the center-of-mass system of coordinates

$$M_3V_3'=M_4V_4'$$

and

$$E_1-\frac{M_1}{M_1+M_2}E_1=\left(\frac{M_3}{2}\right)(V_3')^2+\left(\frac{M_4}{2}\right)(V_4')^2-Q$$

from which

$$V_3'=\left\{\frac{2M_2M_4}{M_3(M_1+M_2)^2}\left[E_1+\frac{M_1+M_2}{M_2}Q\right]\right\}^{\frac{1}{2}}.$$

The corresponding equation for V_4' can be obtained from this last expression by interchanging the subscripts 3 and 4. The physical meaning of the expression $-Q(M_1+M_2)/M_2$ is the minimum energy E_{th} of the incident particle which is necessary to make a reaction proceed.

With these substitutions the value of B_3 becomes

$$B_3=\frac{(M_2M_4)^{\frac{1}{2}}}{M_1+M_2}(E_1-E_{\text{th}})^{\frac{1}{2}} \qquad \text{(A8)}$$

and finally,

$$C_3=(E_3)^{\frac{1}{2}}. \qquad \text{(A9)}$$

The value of ψ indicated in Fig. 25 is seen to be

$$\psi=\phi-\theta_3=\sin^{-1}\left(\frac{\sin\theta_3}{B_3/A_3}\right). \qquad \text{(A10)}$$

Figures 26 to 30 are nomographs for the energy of various reactions which have been made by generalizing Fig. 25(b). In a velocity diagram for a particle of definite mass the locus of points of equal energy is a circle. In these nomographs the solid semicircles drawn concentrically about the origin represent a series of values for E_3, the energy of particle 3 (neutron) in the laboratory system, at the laboratory angle θ_3. The laboratory angles are indicated by the *solid* radial lines from the origin. A series of *broken* semicircles which are *not concentric* are drawn with their centers to the right of the origin. These circles intersect the horizontal axis at values appropriate to the value of E_1 and points on these semicircles represent a value of E_3 for that bombarding energy E_1. The angular coordinate of a point on a broken semicircle as measured from the origin of that broken semicircle gives the value of the center-of-mass angle ϕ. The broken lines represent the locus of values of ϕ. The displacement of the centers of the broken semicircles to the right of the origin is given by the value of A_3.

Since a point on one set of semicircles is also a point on the other set of semicircles (provided the threshold is exceeded), the nomograph connects the four quantities E_1, E_3, θ_3, and ϕ. Interpolation is required in reading the points which do not happen to be on two intersection semicircles. Nomographs printed on a larger scale have been distributed by J. L. McKibben and can be purchased as document MDDC 223 from the Document Division of the AEC, Oak Ridge, Tennessee.

The theoretical interpretation of nuclear reactions usually requires that the observed laboratory intensities be transformed into the center-of-mass system of coordinates. The observed intensities (number of particles per unit solid angle) and the differential solid angles satisfy the following relations:

$$I_{\text{cm}}d\omega_{\text{cm}}=I_{\text{lab}}d\omega_{\text{lab}}$$

$$\frac{d\omega_{\text{lab}}}{d\omega_{\text{cm}}}=\frac{\sin\theta_3 d\theta_3}{\sin\phi d\phi}=\frac{\sin^2\theta_3\cos(\phi-\theta_3)}{\sin^2\phi}. \qquad \text{(A11)}$$

In terms of the velocity diagram,

$$\frac{d\omega_{\text{lab}}}{d\omega_{\text{cm}}}=\frac{B_3^2\cos\psi}{C_3^2}=\frac{M_2M_4}{(M_1+M_2)^2}\frac{E_1-E_{\text{th}}}{E_3}\cos(\phi-\theta_3). \qquad \text{(A12)}$$

In those cases where the neutron intensity is determined from counting the number of associated particles, $I_4(\theta_4)$, the transformation connecting these intensities is given by

$$\frac{I_4(\theta_4)}{I_3(\theta_3)}=\frac{\sin\theta_3 d\theta_3}{\sin\theta_4 d\theta_4}=\frac{\cos(\phi-\theta_3)}{\cos(\pi-\phi-\theta_4)}\cdot\frac{\sin^2\theta_3}{\sin^2\theta_4}. \qquad \text{(A13)}$$

In all endoergic reactions there exists a range of bombarding energies E_1 which exceed E_{th} but are still small enough to result in having $B_3<A_3$. In this region the particles are confined to a cone whose half-angle is given by $\sin^{-1}B/A$. Then $\cos(\phi-\theta_3)$ is zero at the edge of the cone and $(d\omega_{\text{lab}}/d\omega_{\text{cm}})$ is zero just at this edge. If the center-of-mass distributions is uniform, the laboratory distribution must be very large at this limiting angle. It is also interesting to note that there are two values of $C_3=(E_3)^{\frac{1}{2}}$ which correspond to two different values of ϕ, for each θ_3 within the limiting angle. The implications of this double-valued character for the neutron energy are discussed in the section on endoergic reactions.

The layout and drawings of the energy nomographs shown in Figs. 26 to 30 were done by Mr. Charles Lehman of the computing group of the Los Alamos Scientific Laboratory.