of Λ to get different types of gravitational behavior has, of course, often been employed in the past, and is a possibility which will remain open until we have some independent way of determining that quantity. It has, however, an arbitrary character that is not pleasing. It is my feeling that the importance of Omer's work lies not in providing a necessarily correct description of the recession of the nebulae, but in showing that the abandonment of the assumption of homogeneity introduces sufficient flexibility so that we do not need to expect trouble as to time scale when we apply the relativistic theory of gravitation to treat the motions of the nebulae.

I hope that you have been able to read this far. If so, I will end by saying that I look for great things from the 200-inch which will have a big effect on theory. I think that our special interest should now lie, not in the approximate linearity of red shift with distance and the approximate uniformity of nebular distribution which have been found, but in the deviations therefrom which we shall find. Perhaps we shall even be able to see out to places in the universe where contraction rather than expansion is taking place. I hope so. Cheerio.

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Postulate *versus* Observation in the Special Theory of Relativity

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INTRODUCTION

In 1905 Einstein[1] published his theory of electrodynamics of moving bodies, which has long since been incorporated in the accepted body of physical science under the less descriptive name of the special theory of relativity. The kinematical background for this theory, an operational interpretation of the Lorentz transformation, was obtained deductively by Einstein from a general postulate concerning the relativity of motion and a more specific postulate concerning the velocity of light. At the time this work was done an inductive approach could not have led unambiguously to the theory proposed, for the principal relevant observations then available, notably the "ether-drift" experiment of Michelson and Morley[2] (1886), could be accounted for in other, although less appealing, ways. Because of the revolutionary character of the postulates and consequences of this theory, there is discernible in the subsequent decades a certain reluctance wholeheartedly to accept its necessity, a reluctance shared at times even by scientists whose own work paved the way to, or confirmed the predictions of, the theory.

It may therefore be appropriate on this occasion to review the present status of the theory, with special reference to the question of the degree to which postulate can now be replaced by observation in deriving the kinematics on which the theory is based. This reexamination, from a unified point of view closely allied to Einstein's original program, will emphasize the decisive nature of the two great optical experiments of Kennedy and Thorndike[3] (1932) and of Ives and Stilwell[4] (1938) which have been performed in the interim, experiments which were designed and carried out for the explicit purpose of testing aspects of the Lorentz transformations which are insensitive to the Michelson-Morley experiment. We shall find, in confirmation of conclusions drawn by Kennedy and by Ives, that these three second-order experiments do in fact enable us to replace the greater part of Einstein's postulates with findings drawn inductively from the observations.

KINEMATICAL PRELIMINARIES

We *postulate* that there exists a reference frame Σ—Einstein's "rest-system"—in which light is propagated rectilinearly and isotropically in free space with constant speed c. In elucidation of this postulate, we have here presupposed that any observer P at rest with respect to this frame may be supplied with two independent kinds of instruments, called *rods* and *clocks*, with which he can measure space and time intervals, respectively. By independent we here mean that the fundamental measurement of one kind of interval is not to be reduced to that of the other with the aid of the postulated constancy of the velocity of light, as would, for example, be the case if the "clock" consisted of a beam of light reflected back and forth between two mirrors on the

[1] A. Einstein, Ann. d. Phys. **17**, 891 (1905).

[2] A. A. Michelson and E. H. Morley, Am. J. Sci. 34, 333 (1887).

[3] R. J. Kennedy and E. M. Thorndike, Phys. Rev. **42**, 400 (1932).

[4] H. E. Ives and G. R. Stilwell, J. Opt. Soc. Am. **28**, 215 (1938); **31**, 369 (1941).

ends of a rod. We assume that the physical geometry of the 3-dimensional space, as revealed by the measuring rods, is Euclidean. The postulate also implies the synchronization of all clocks at rest in Σ, and that the velocity of light in free space is independent of the motion of its source.

P may assign to an event E four coordinates $(\xi^\mu) = (\tau; \xi, \eta, \zeta)$, consisting of a temporal coordinate $\xi^0 = \tau$ and 3 spatial Cartesian coordinates $\xi^1 = \xi$, $\xi^2 = \eta$, $\xi^3 = \zeta$. He may also define in Σ the "metric"

$$d\sigma^2 = \Sigma\gamma_{\mu\nu}d\xi^\mu d\xi^\nu = d\tau^2 - (d\xi^2 + d\eta^2 + d\zeta^2)/c^2, \tag{1}$$

with the aid of which he can

(a) measure time intervals $d\tau = d\sigma$ at any fixed point ξ^α ($\alpha = 1, 2, 3$) in his space;
(b) measure space intervals $d\lambda = c(-d\sigma^2)^{\frac{1}{2}}$ at any fixed time τ;
(c) characterize all beams of light passing through an event E as the generators of the cone $d\sigma = 0$ with E as vertex.

We next *postulate* the existence of a reference frame S—Einstein's "moving system"—which is moving with any given constant velocity $d\xi^\alpha/d\tau = v^\alpha$ of magnitude $v < c$, with respect to Σ. Any observer R in S may be supplied with rods and clocks of the same physical constitution as those supplied P, with the aid of which he can introduce coordinates $(x^i) = (t; x, y, z)$, consisting of a temporal coordinate $x^0 = t$ and 3 spatial coordinates $x^1 = x$, $x^2 = y$, $x^3 = z$. We further postulate that the physical geometry of the xyz-space, as revealed by the measurement technique, be Euclidean, where the x^a ($a = 1, 2, 3$) are Cartesian coordinates. No assumption is here made concerning the velocity of light or other physical law in S; these are to be inferred from observation and from the laws postulated in the reference system Σ.

The problem of physical kinematics is that of determining the transformation T: $(t, xyz) \rightarrow (\tau, \xi\eta\zeta)$ relating the measurements of an observer R in S with those of P in Σ. This transformation should, on appropriate choice of the spatial axes in S, involve as its only essential parameters the velocity v^α of S with respect to Σ, and should reduce to the identity for $v^\alpha = 0$.

As we shall in the first instance be concerned with the determination of the transformation T with the aid of laboratory experiments, we assume that we may for this purpose confine outselves to the consideration of events E in a space-time neighborhood of a given event E_0 which is so small that we may linearize T, i.e., replace it by the transformation $dx^i \rightarrow d\xi^\mu$ which it induces on the differentials. On choosing E_0 as the common origin of coordinates, T may be taken as

$$T:\quad \xi^\mu = \sum_{i=0}^{3} a_i{}^\mu x^i. \tag{2}$$

The 16 coefficients $a_i{}^\mu$ may now be reduced to 13 by an appropriate synchronization of clocks at various points in the space, to 7 by an appropriate choice of spatial axes, and finally to 4 (including the magnitude v of the velocity of S with respect to Σ) on applying the *a priori* symmetry condition imposed by the requirement that the only vector of intrinsic significance to the kinematics is the given velocity vector v^α. To this reduction we now turn.

First, it will be convenient to replace three of the coefficients of T by the three components v^α ($\alpha = 1, 2, 3$) of the velocity vector. The equation of the path of the spatial origin $x^a = 0$ ($a = 1, 2, 3$) of S is

$$\tau = a_0{}^0 t, \quad \xi^\alpha = a_0{}^\alpha t,$$

and in order that $d\xi^\alpha/d\tau = v^\alpha$ we must therefore have

$$a_0{}^\alpha = a_0{}^0 v^\alpha \quad (a_0{}^0 \neq 0). \tag{3}$$

Next, we adopt the procedure proposed by Einstein for setting clocks which are carried by observers R which are at rest at various points x^a in the reference frame S. Consider a light signal sent out from the spatial origin O at time $t = 0$, as recorded by the master clock maintained there, reflected from the position $x^a = p^a$ at some event E, and received back at O at clock time t_0. We *agree* to set the auxilliary clock situated at $x^a = p^a$ in such a way that it records the time $t_0/2$ for the event E of reflection. To determine the normalization thereby imposed on the coefficients of the transformation T, we make use of the equation $d\sigma = 0$ of the light cone. Expressed in the coordinates x^i, the "metric"

$$d\sigma^2 = g_{ij}dx^i dx^j, \quad \text{where} \quad g_{ij} = \gamma_{\mu\nu}a_i{}^\mu a_j{}^\nu, \tag{4}$$

repeated indices implying, as usual, summation over their range. The direct beam from the origin to E lies on the cone

$$\gamma_{\mu\nu}(a_0 t + a_a{}^\mu x^a)(a_0{}^\nu t + a_b{}^\nu x^b) = 0,$$

and the reflected beam on the cone

$$\gamma_{\mu\nu}[a_0{}^\mu(t - t_0) + a_a{}^\mu x^a][a_0{}^\nu(t - t_0) + a_b{}^\nu x^b] = 0.$$

On requiring that the event $E(t_0/2, p^a)$, at which the beam is reflected, lies on both cones, we find the necessary and sufficient condition

$$\gamma_{\mu\nu}a_0{}^\mu a_a{}^\nu t p^a = g_{0a} t p^a = 0.$$

Since the proposed synchronization is to be made at all points p^a, it follows from the above and Eq. (3) that

$$g_{0a} = a_0{}^0(a_a{}^0 - v^\alpha a_a{}^\alpha/c^2) = 0$$

whence

$$a_a{}^0 = \Sigma_\alpha v^\alpha a_a{}^\alpha/c^2. \tag{5}$$

We choose the ξ-axis in Σ to lie in the plane of events determined by the τ-axis and the t-axis, i.e., by the world lines of P and R. But then the three 4-vectors $\delta_1{}^\mu$, $\delta_0{}^\nu$ and $\xi^\mu = a_i{}^\mu \delta_0{}^i = a_0{}^\mu$ must be linearly dependent, whence

$$a_0{}^2 = a_0{}^3 = 0, \quad \text{or} \quad v^2 = v^3 = 0, \tag{6}$$

and we write $v^1 = v(>0)$; we have here introduced the "Kronecker delta" $\delta_\nu{}^\mu$ to signify 1 if $\mu = \nu$, and 0 otherwise. Similarly, we choose the x-axis in S to lie in the same plane, whence $\xi^\mu = a_i^\mu \delta_1^i = a_1^\mu$ must also be linearly dependent on δ_1^μ, δ_0^μ; therefore

$$a_1^2 = a_1^3 = 0. \tag{7}$$

Now the yz-plane in S, i.e., the plane $t=0$, $x=0$, is given parametrically in Σ by the equations

$$\xi^\mu = a_2^\mu y + a_3^\mu z.$$

We elect to choose the y-axis in such a direction that along it

$$\tau = a_2^0 y + a_3^0 z = 0,$$

although this direction may not (and, as we shall see below, will not) be unique; it follows that with such a choice of y-axis

$$a_2^0 = 0. \tag{8}$$

The η-axis in Σ may now be chosen along this same direction in $\tau=0$, i.e., δ_2^μ and $\xi^\mu = a_i^\mu \delta_2^i = a_2^\mu$ are to be linearly dependent; the only new condition resulting herefrom is that

$$a_2^3 = 0. \tag{9}$$

With this we have fixed *uniquely* the coordinate systems in both Σ and S, except for a possible spatial rotation of the y, z and η, ζ axes (in case both a_2^0 and a_3^0 vanish), and we have set the clocks in S in the same manner as was tacitly adopted in Σ. The conditions (3), (5)–(9) lead to the canonical form

$$\begin{pmatrix} a_0^0 & va_1^1/c^2 & 0 & va_3^1/c^2 \\ va_0^0 & a_1^1 & 0 & a_3^1 \\ 0 & 0 & a_2^2 & a_3^2 \\ 0 & 0 & 0 & a_3^3 \end{pmatrix}$$

for the matrix of the transformation T.

In the problem we have set, in which the kinematics of S are to depend solely on the velocity v^α of S with respect to Σ, the canonical form of T must be independent of the particular choice of the y-axis made above, for otherwise the kinematics in S would depend upon the orientation of the y-axis, and hence upon a direction other than that unique one defined by v^α. (An alternative, pseudo-physical, way of saying the same thing is to require that the one-way velocity of light in S be independent of its azimuth about the x-axis.) From this requirement, which is in a sense only a definition of the problem under consideration, it follows that

$$a_3^1 = 0, \quad a_3^2 = 0, \quad a_3^3 = a_2^2. \tag{10}$$

The canonical form for the matrix thus further reduces to

$$(a_i^\nu) = \begin{pmatrix} a_0^0 & va_1^1/c^2 & 0 & 0 \\ va_0^0 & a_1^1 & 0 & 0 \\ 0 & 0 & a_2^2 & 0 \\ 0 & 0 & 0 & a_2^2 \end{pmatrix} \tag{11}$$

involving the three parameters a_0^0, a_1^1, a_2^2 in addition to the magnitude v of the given velocity.

The auxilliary form (4), the vanishing of which defines the light cones, may now be written

$$d\sigma^2 = g_0^2 dt^2 - [g_1^2 dx^2 + g_2^2(dy^2+dz^2)]/c^2, \tag{12}$$

where

$$\left.\begin{array}{ll} g_0 = (1-v^2/c^2)^{\frac{1}{2}} a_0^0, & \\ g_1 = (1-v^2/c^2)^{\frac{1}{2}} a_1^1, & g_2 = a_2^2. \end{array}\right\} \tag{12'}$$

THE VELOCITY OF LIGHT: THE MICHELSON-MORLEY AND KENNEDY-THORNDIKE EXPERIMENTS

Our problem is now the determination, by recourse to the empirical, of the dependence of the three parameters a_0^0, a_1^1, a_2^2, or alternatively g_0, g_1, g_2, on the magnitude v; the only *a priori* requirement on these functions is that each of them $\to 1$ as $v \to 0$. The fact that the light-paths in S are the generators of the cones $d\sigma^2=0$, Eq. (12), suggests that we may hope to establish the ratios, and only the ratios, of the three parameters by observations involving the velocity of light.

Before proceding to this determination, it is well to review the effect on the velocity of light of the definitions and conventions we have adopted. The linearity of the transformation T insures that light will be propagated rectilinearly in S, and Einstein's synchronization insures *as a matter of definition* the equality of the forward and backward velocity along any given line in S; the magnitude of the velocity of the beam will in general depend on the angle h which it makes with the x-axis. Alternative synchronizations could have been agreed upon; for example, absolute simultaneity (by agreeing that the clocks in S be set to read $t=0$ when $\tau=0$), or in such a way that the velocity of Σ relative to S be equal to $-v$ (whereas under our conventions it is in fact $-va_0^0/a_1^1$). But while such normalizations are theoretically possible, they cannot in practice be carried out as they involve a non-operational appeal to the hypothetical rest-system Σ.

We turn now to an examination of the restrictions imposed on the kinematics by the null-result of the Michelson-Morley experiment. This experiment may be considered as comparing the to-and-fro times taken by two partial beams of light to traverse the two equal and perpendicular arms OM_1, OM_2 of an interferometer, by the behavior of the interference fringes produced on bringing together the two beams after reflection on the mirrors M_1, M_2. No significant difference in times was found, and since the original experiment and its repetitions were carried out at various orientations and at various times of the year, we would seem justified in interpreting this null-result as meaning:

M-M: *The total time required for light to traverse, in free space, a distance l and to return is independent of its direction.*

It follows immediately from $d\sigma=0$, Eq. (12), that the time t required for light to travel a distance l in either sense along a direction making an angle h with the x-axis is

$$t=(l/cg_0)(g_1^2\cos^2h+g_2^2\sin^2h)^{\frac{1}{2}}.$$

If this time is to be independent of h we must have

$$g_2(v)=g_1(v), \quad \text{or} \quad a_2^2=a_1^1(1-v^2/c^2)^{\frac{1}{2}}. \tag{13}$$

This condition is completely equivalent to the Lorentz-Fitzgerald contraction, as it can be shown to imply that the ξ-intercept of a rod at rest along the x-axis in S is contracted in the ratio $(1-v^2/c^2)^{\frac{1}{2}}:1$ in comparison with the η-intercept of a similar rod at rest along the y-axis.

The time t required to travel a distance l in S is now

$$t=lg_1/cg_0, \tag{14}$$

and may, of course, depend on the absolute velocity v of S. To test this dependence is the purpose of the Kennedy-Thorndike experiment. Here the apparatus used is, in principle, similar to that used in the Michelson-Morley experiment, except that the interferometer arms OM_1, OM_2 are as different in length as feasible. The difference Δt of the travel times of the two partial beams is, by application of Eq. (14), related to the difference Δl in the lengths of the two paths by the equation

$$\Delta t=(g_1/cg_0)\Delta l. \tag{15}$$

The velocity v of a point on the earth's surface should undergo a diurnal change because of the earth's rotation, and a much larger annual change because of the revolution of the earth about the sun. Hence some change in the phase difference, i.e., in the equivalent time Δt, should be expected if the kinematical parameter g_1/g_0 does in fact depend on the velocity v with respect to the rest-system Σ. From the null-result of their observations Kennedy and Thorndike conclude "there is no effect . . . unless the velocity of the solar system in space is no more than about half that of the earth in its orbit," and this they judge improbable in view of known stellar motions. We accept this interpretation of their results, and conclude from it that:

K-T: *The total time required for light to traverse a closed path in* S *is independent of the velocity* v *of* S *relative to* Σ.

And since for $v=0$ we must have $g_1/g_0=1$, it follows that this must hold for all v. We may now write, on taking Eqs. (12), (13) into account,

$$\left.\begin{array}{l} g_0(v)=g_1(v)=g_2(v)=(\text{say})g(v), \\ a_0^0=a_1^1=g(v)/(1-v^2/c^2)^{\frac{1}{2}}, \quad a_2^2=g(v). \end{array}\right\} \tag{16}$$

These two optical experiments together thus imply that the velocity of light, as measured in S, is equal to c, independently of its direction and of the velocity v of S with respect to Σ. The light-paths are the minimal geodesics of the "metric"

$$ds^2\equiv d\sigma^2/g^2(v)=dt^2-(dx^2+dy^2+dz^2)/c^2 \tag{17}$$

which performs for S the same measuring duties as $d\sigma^2$ performs for Σ, as described under Eq. (1) above. The transformation T is reduced to the form

$$T:\left\{\begin{array}{ll} \tau=g(t+vx/c^2)/(1-v^2/c^2)^{\frac{1}{2}}, & \eta=gy, \\ \xi=g(vt+x)/(1-v^2/c^2)^{\frac{1}{2}}, & \zeta=gz, \end{array}\right. \tag{18}$$

originally employed by Lorentz. It follows that the velocity of Σ relative to S is in fact $-v$, as was assumed by Kennedy and Thorndike in their derivation of the Lorentz equations.

THE RATE OF A MOVING ATOMIC CLOCK: THE IVES-STILWELL EXPERIMENT

It is clear that no experiment involving only the velocity of light in S can succeed in determining the value of the remaining parameter $g(v)$. Lorentz has shown that it is in fact possible to carry through a consistent electro-dynamics in moving media for arbitrary $g(v)$; the determination of its dependence of v is then left to dynamical experiments, such as those of Kaufmann and Bucherer, involving the mass of a particle. Einstein, on the other hand, determined $g(v)$ to be unity by postulating that the transformations $T(v)$ constitute a group; it is then readily shown from (18) and its inverse that $g(v)g(-v)=1$ and hence, since g is an even function of v which reduces to 1 for $v=0$, it follows that $g(v)=1$.

The last of the three great optical experiments mentioned in the Introduction, that of Ives and Stilwell, was designed and carried out to test this conclusion empirically, by observations on the Doppler shift in light from a moving source. It will suffice for our purpose to restrict ourselves for the moment to the case considered by Ives, in which the source of light is at rest in a system S′ which is moving with a velocity $v'>v$ relative to Σ in the same direction as is the system S; we indicate briefly at the end the modification required for the general case in which the direction and magnitude of motion of S′ are arbitrary.

Take the light source in S′ at the origin $x'^{a}=0$ of spatial coordinates in S′, and let it pass through the origin $x^a=0$ of S at time $t'=t=0$. It then follows, from Eqs. (18) and the corresponding equations for the transformation $T':x'^{i}\rightarrow\xi^{\mu}$, that the velocity u of S′ relative to S is

$$u=(v'-v)/(1-vv'/c^2), \tag{19}$$

as in the special theory of relativity. The parametric equations of motion of the light source, relative to S, in terms of the proper time t' of the source, are

$$t=pt', \quad x=upt', \tag{20}$$

where

$$p(u,v)\equiv g(v')/g(v)(1-u^2/c^2)^{\frac{1}{2}}, \qquad v'=(u+v)/(1+uv/c^2). \tag{20'}$$

Suppose now that the source in S′ is sending out light signals with a frequency ν', as measured in S′. A signal sent out at a time $t'<0$, while the source is approaching the observer R at the spatial origin of S, will be received by R at a time

$$t_+ = pt' + (-upt')/c = (1-u/c)pt'.$$

These signals will accordingly be received with a frequency ν_+ and wave-length λ_+ given by

$$c/\nu_+ = \lambda_+ = (1-u/c)p\lambda' \tag{21}$$

where $\lambda' = c/\nu'$ is the wave-length in S′. Similarly, signals sent out while the source is receding from R will be received with the frequency ν_- and wave-length λ_-

$$c/\nu_- = \lambda_- = (1+u/c)p\lambda'. \tag{21'}$$

Now under the assumption that the source is a permissible clock, i.e., that its period is independent of the proper time, a similar source which is stationary in S will emit radiation with a frequency $\nu = \nu'$ and wave-length $\lambda = \lambda'$. Light from the approaching (or receding) source should therefore suffer a Doppler displacement

$$\Delta\lambda_\pm = [(1 \mp u/c)p - 1]\lambda, \tag{22}$$

respectively, and the mean of the two lines should be displaced an amount

$$\Delta\bar{\lambda} = \tfrac{1}{2}(\lambda_+ + \lambda_-) - \lambda = (p-1)\lambda \tag{22'}$$

with respect to the wave-length λ of the stationary source in S. For a general radial motion of the source S′ it can be shown that the Doppler displacements are exactly the same as in (22), (22′) above, provided only that the velocity v' appearing in the factor $g(v')$ be determined from

$$1 - v'^2/c^2 = (1-u^2/c^2)(1-v^2/c^2)/(1+uv\cosh/c^2)^2 \tag{23}$$

where h is the angle in S between the direction of motion of the source and the x-axis.

These predictions were tested by Ives and Stilwell on light from high speed canal rays. They found that there was in fact a second-order shift $\Delta\bar{\lambda}/\lambda$ in the mean of the two lines which could, within the observational error, be represented by $\frac{1}{2}(u/c)^2$, where u/c is determined to first order by the observed Doppler shifts. This means that the factor $g(v')/g(v)$ in p, Eq. (20′), may be taken as unity, correct to terms of order higher than the second. Now since there is no correlation to be expected between $u \cosh$ and v, we may conclude that $g(v)$ is itself unity to within terms of higher order than $(v/c)^2$. While it is of course theoretically possible that higher order terms might enter into the parameter $g(v)$, we follow Ives and Stilwell interpreting their results as meaning:

I-S: *The frequency of a moving atomic source is altered by the factor $(1-u^2/c^2)^{\frac{1}{2}}$, where u is the velocity of the source with respect to the observer.*

Our parameter $g(v)$ is then taken rigorously as unity, and the transformation T, Eq. (18), is the Lorentz transformation, in the form deduced and used by Einstein. The "metric" $d\sigma^2$ may now be removed from quotes, for

$$d\sigma^2 = ds^2 = dt^2 - (dx^2 + dy^2 + dz^2)/c^2 \tag{24}$$

is a formal invariant of T, and may therefore serve as a metric in any frame S. Our kinematics becomes the Minkowskian geometry of space-time.

CONCLUSION

We have with this completed the task of replacing, so far as possible, Einstein's relativity postulate by facts drawn from experience. The preliminary analysis revealed that, within the setting of the problem, the relationship between the moving frame S and the rest-frame Σ involves three irreducible parameters g_0, g_1, g_2 which are unknown functions of the velocity v. The three second-order optical experiments of Michelson and Morley, of Kennedy and Thorndike, and of Ives and Stilwell, furnished empirical evidence which, within the limits of the inductive method, enabled us to conclude that the three parameters may be taken as independent of the motion of the observer. The kinematics *im kleinen* of physical space-time is thus found to be governed by the Minkowski metric, whose motions are the Lorentz transformations, the background upon which the special theory of relativity and its later extension to the general theory are based.