On Relativistic Cosmogony

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EINSTEIN'S general theory of relativity received brilliant confirmation in the so-called "three tests:" advance of the perihelion of mercury, deflection of the light rays passing near the solar surface, and the gravitational shift of spectral lines in the light emitted by the sun and some other stars.

These tests pertain, however, to one particular solution of the fundamental equations of gravitation describing the local spherically symmetrical field which surrounds a gravitating masspoint.

Another important solution of the same equations corresponds to the case of uniform mass-distribution and is supposed to describe the geometry and time behavior of the universe as a whole. The agreement of the cosmological consequences of general relativity with observational evidence should give us additional and more extensive proof of the correctness of basic ideas of the theory, whereas any serious disagreement would lead to the rather discouraging result that Einstein's theory of gravitation is applicable only to local fields and fails when used for a description of larger regions of space.

During the last decades there have appeared a number of proposals suggesting the modification of the original theory of Einstein for the purposes of cosmological application. These include the fundamental changes in the notion of time (Milne),[1] the assumption of variability of the gravitational constant (Dirac),[2] and the introduction of a new phenomenon of creation of mass (Jordan, Hoyle).[3] In the present article we will intentionally disregard all such proposals, interesting as they are, since our aim here is to see whether or not the problems of cosmology and cosmogony can be understood entirely on the basis of the "old fashioned" general theory of relativity in its original form proposed by Einstein.

The first and the most general cosmological consequence of the Einstein theory is that our universe may be either open and infinite, or finite and closed, and that it may either expand or contract with time.

Hubble's discovery of the red shift in the spectra of distant galaxies, and its interpretation as the result of the uniform expansion of space, represents the first direct confirmation of that basic conclusion of relativistic cosmology. It is true that some physicists and astronomers prefer to believe that the observed red shift is not due at all to mutual recession of galaxies, but must be explained by some as yet unknown physical phenomenon causing the reddening of light which travels over very large distances. However, except for the descriptive statement to the effect that "light quanta may get tired traveling such a long way," no reasonable explanation of such a reddening has as yet been proposed, and, as a matter of fact, can hardly be expected on the basis of present ideas concerning the nature of light. Moreover, abolishing the idea of an expanding universe, one would immediately lose the sound foundation for the interpretation of evolutionary phenomena in astronomy, and it will be very difficult to answer such questions as to why the natural radioactive elements are still in existence, why the stars did not use up all hydrogen an eternity ago, etc.

According to Hubble's measurements, the rate of uniform expansion (at the present time) is given by

$$\left[\frac{1}{l}\frac{dl}{dt}\right]_{\text{present}} = 1.8\cdot 10^{-17}\,\frac{\text{cm}}{\text{sec.}}\Big/\text{cm}. \tag{1a}$$

Assuming that the expansion was linear in the past, we find that it must have started $5.6\cdot 10^{16}$ sec. $=1.8\cdot 10^{9}$ years ago, so that at that epoch the material of the universe must have been in the state of very high density and correspondingly high temperature. Although the figure of $1.8\cdot 10^{9}$ years agrees in order of magnitude with various astronomical and geological estimates of the age of our universe, its exact value is certainly too low. In fact the study of radioactivity of rocks indicates quite definitely that the solid crust of the earth must have existed for at least $2.5\cdot 10^{9}$ years so that the age of the universe is probably closer to $3\cdot 10^{9}$ years. Since this discrepancy is certainly beyond the limits of errors in the astronomical measurements of the red shift, and since taking into account the effect of gravitational attraction between the galaxies during the earlier expansion period, we will still further shorten (though only very little) the calculated age, it appears that we have here the first serious disagreement between the conclusions of relativistic cosmology and the observed facts.

This disagreement, however, can be removed by considering the effect of the so-called *cosmological term* in the general equation of the expanding universe. In fact, the most general homogeneous and isotropic solution of the fundamental equatious of gravitation can be written in the form[4]

$$\frac{dl}{dt} = \pm\left(\frac{8\pi G}{3}\rho l^2 - c^2\epsilon^2 + \frac{\Lambda}{3}l^2\right)^{\frac{1}{2}}, \tag{1b}$$

[1] E. A. Milne, *Relativity, Gravitation and World Structure* (Clarendon Press, Oxford, 1935); *Kinematic Relativity* (Clarendon Press, Oxford, 1948).
[2] P. A. M. Dirac, Proc. Roy. Soc. **165**, 198 (1938).
[3] P. Jordan, Naturwiss **26**, 417 (1938); Astr. Nachr. **276**, 193 (1948); F. Hoyle, Nature **163**, 196 (1949).
[4] Cf. R. C. Tolman, *Relativity, Thermodynamics, and Cosmology* (Clarendon Press, Oxford, 1934), pp. 396 and 404.

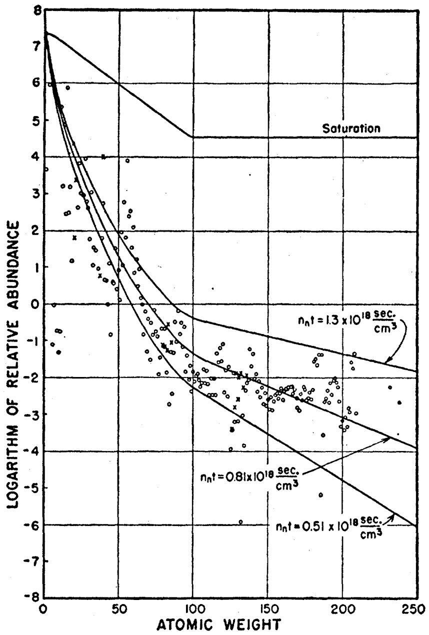

FIG. 1. Calculated values of relative abundance of elements.

where l is an arbitrary length in the expanding or contracting space, ϵ(=const.) is the curvature radius $\mathcal{R}$ expressed in terms of that length, ρ the total mass density (including matter and radiation), and Λ the cosmological constant. It is convenient for future discussion to attach a definite meaning to the length l, and *we define l as the side of a cube containing one gram of matter* (excluding the radiation).

Rewriting Eq. (1) in the form

$$\frac{1}{l}\frac{dl}{dt}=\pm\left(\frac{8\pi G}{3}\rho-\frac{c^2\epsilon^2}{l^2}+\frac{\Lambda}{3}\right)^{\frac{1}{2}}, \tag{2}$$

we notice that the present value of the left side is given by the Hubble's constant: $1.8\cdot 10^{-17}$. Using also the Hubble's value 10^{-30} (g/cm^3) for the present mean density of the universe, we find that the first term under the radical must be negligibly small as compared with the other two.[5] Thus, assuming $\Lambda=0$, we would obtain a linear dependence of l on time for almost the entire past history of the universe, and would come to the erroneous conclusion that the expansion must have started only $1.8\cdot 10^9$ years ago. However, retaining the cosmological term, and ascribing to Λ a definite positive value, we can easily fit the solution of (2) with the observed data. In fact, the above equation (with $\rho=0$) can be integrated in the form

$$t=(3/\Lambda)^{\frac{1}{2}}\ln\{l+[l^2-(3c^2\epsilon^2/\Lambda)]^{\frac{1}{2}}\} \tag{3}$$

so that we can write

$$(3/\Lambda)^{\frac{1}{2}}\ln(l_{\text{pres}}+[l_{\text{pres}}{}^2-(3c^2\epsilon^2/\Lambda)]^{\frac{1}{2}} = 3\cdot 10^9\text{ yr.}=9.3\cdot 10^{16}\text{ sec.} \tag{4}$$

The equation itself gives us

$$\left(-\frac{c^2\epsilon^2}{\Lambda}+\frac{\Lambda}{3}\right)^{\frac{1}{2}}=1.8\times 10^{-17}\text{ sec.}^{-1}. \tag{5}$$

Putting $l_{\text{pres}}=10^{10}$ cm (because $\rho_{\text{pres}}=10^{-30}$ g/cm^3), and solving Eqs. (4) and (5) for ϵ and Λ, we find[6]

$$\epsilon=i\cdot 5\cdot 10^{17}$$

or

$$\mathcal{R}=i\times 5\times 10^{27}\text{ cm}=i\times 5\times 10^9\text{ light years,}$$

and

$$\Lambda=8.6\times 10^{-34}\text{ sec.}^{-2}.$$

This result indicates that the space of the universe has a negative curvature and is therefore infinite in extension, and also that the value of cosmological constant, being different from zero, is still sufficiently small to be of no importance in consideration of planetary motion in local fields.

Another extremely important method of comparing the consequences of relativistic cosmology with the observed properties of our universe is presented by Hubble's studies of the red shift and space density in the case of *very distant* galaxies. Since at these distances it becomes impossible to resolve the galaxies into individual stars, distance determinations have to be carried out exclusively on the basis of the observed apparent magnitudes of the galaxies themselves. The study of nearby galaxies, whose distances can be determined by the observation of cepheides or bright stars, has shown that their absolute magnitudes vary only very little, being distributed according to the Gauss law around a certain mean value. Consequently, one can expect that taking the mean apparent magnitude of many galaxies belonging to the same distant cluster, and using the inverse square law for apparent luminosity, one should get a rather exact figure for their distance from us. The results of such a survey carried out by Hubble[7] brought him to the conclusion that: (a) at very large distances the observed red shift begins to increase faster then the distance, (b) at the same distances the mean density of galaxies (i.e., the number of galaxies per unit volume) begins to *increase* with distance. Both results are in apparent contradiction with the above-stated conclusions of relativistic cosmology: in fact, since when observing distant galaxies we are actually looking back into the past, we should expect to find lower rather than higher expansion velocities (the effect of positive Λ in the Eq. (2)). If this particular result of Hubble were actually correct, we would be forced to the conclusion that the age of the universe is even

[5] Since ρ increases rather fast with decreasing l, the first term becomes important during the early stages of expansion, which, however, does not influence the form of the solution for the present epoch.

[6] R. A. Alpher and R. C. Herman, private communication.

[7] E. Hubble, *The Observational Approach to Cosmology* (Clarendon Press, Oxford, 1937).

smaller than the figure of $1.8\cdot 10^9$ years which follows from a linear expansion.

It can also be shown that the density increase which follows from Hubble's observations would indicate a closed space with a rather small radius of curvature in contradiction to the discussion of the previous pages. However, fortunately for the theory, the results of Hubble's survey are not conclusive since they are both based on an arbitrary assumption that at the epoch when light was emitted from these distant galaxies, they possessed exactly the same luminosities as those observed for the nearby galaxies. Such an assumption may not be necessarily true, and it would be indeed rather strange if the integrated luminosity of these giant stellar groups going through the process of complex evolution would remain constant over a period of time comparable with their total age. These doubts were recently confirmed by the observations of Stebbins and Whiteford[8] who have found that elliptic galaxies belonging to distant clusters are considerably brighter in the red part of the spectrum (color excess of 0.30 magnitude at the distance of $7\cdot 10^7$ parsec.), which is most probably due to a larger proportion of red giant stars in the early stages of galactic evolution. Accepting the variability of galactic luminosities with their age, we can remove the discrepancy between the theory and the observational data by a simple assumption that at that distant past (several hundred million years ago) galactic luminosities in the blue part of spectrum (used in Hubble's photographic work) were five to ten percent higher than at the present age. Thus, we come to the conclusion that the present observational evidence is in no way contradictory to the predictions of relativistic cosmology, and that more detailed comparison between the two can be obtained only by further studies which will also involve the problems concerning the evolutionary changes in the individual galaxies.

Another important group of studies based on the general theory of relativity is presented by the work on relativistic cosmogony, which is an attempt to understand the development of various characteristic features of our universe as the result of its expansion from the originally homogeneous state. This includes essentially the theory of the origin of atomic species, which presumably took place during the very early epoch when the material forming the universe was in highly compressed and very hot state, and the theory of the formation of galaxies which must have occurred during the later evolutionary period. The neutron-capture theory of the origin of atomic species recently developed by Alpher, Bethe, Gamow, and Delter[9] suggests that different atomic nuclei were formed by the successive aggregation of neutrons and protons which formed the original hot ylem[10] during the early highly compressed stages in the history of the universe. When, due to the progressing expansion, the temperature of the ylem dropped below 10^9°K, protons and neutrons began to stick together forming the composite nuclei of deuterium. The process continued through the intermittent neutron captures and the adjustments of nuclear electric changes by β-decay until most complex nuclei, such as lead and uranium, were finally formed. The equations governing that process can be written in the form

$$dn_i/dt = f(t)[n_{i-1}\sigma_{i-1} - n_i\sigma_i] \quad (i=1, 2\cdots 238), \qquad (6)$$

where n_i are the concentrations of the nuclei with atomic weight i, σ_i the corresponding capture cross sections for fast neutrons, and $f(t)$ a function describing the time dependence of the collision probability between the growing nucleus and the neutron. The exact form of that function depends on special assumptions concerning the decay rate of free neutrons and the expansion rate of ylem. The measured capture cross sections σ_i show a rather regular variation with atomic weight: they increase exponentially with i up to about $i=100$, and remain nearly constant for larger i's. This variation of capture cross sections is closely correlated with the observed relative abundance of various chemical elements in the universe, since the latter show a rapid exponential decrease up to $i\cong 100$ and also remain nearly constant for larger i's. The integration of Eqs. (6) shows indeed that this correlation is not coincidental, and that it is possible to obtain a close fit with the empirical abundance curve by using the observed values of the capture cross sections. The results of such integration, carried out under the assumption that the neutron density ρ_n remains constant and different from zero for a certain time interval Δt, are shown in Fig. 1. It is seen that the shape of the calculated abundance curve depends crucially on the assumed value of $\rho_n\Delta t$. For very large $\rho_n\Delta t$ the process reaches saturation and the relative abundances become simply inversely proportional to the capture cross sections (curve marked "saturation" in Fig. 1). For smaller

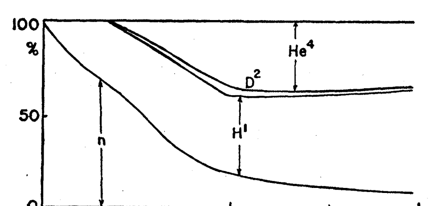

FIG. 2. Concentrations of neutrons, protons, and He⁴ nuclei for first half hour.

[8] J. Stebbins and H. E. Whiteford, Ap. J. **108**, 413 (1948).

[9] G. Gamow, Phys. Rev. **70**, 572 (1946); Alpher, Bethe, and Gamow, Phys. Rev. **73**, 803 (1948); R. A. Alpher, Phys. Rev. **74**, 1577 (1948); R. A. Alpher and R. C. Herman, Phys. Rev. **74**, 1737 (1948).

[10] According to Webster's *New International Dictionary*, second edition, the word *ylem* is an obsolete noun meaning "the primordial substance from which the elements were formed."

value of $\rho_n\Delta t$ one can "overcook" or "undercook" the heavier elements if the value $\rho_n\Delta t$ is chosen too large or too small. The best agreement with the observed curve is obtained for $\rho_n\Delta t = 1.3\cdot 10^{-6}$ g·sec./cm³. This agreement indicates that the origin of atomic species can be considered as *an unfinished building-up process*; the interruption of the process was apparently due to the combined effect of the natural decay of neutrons, and the rapid expansion of the originally dense ylem. For the case of variable neutron density, one can apparently obtain the same close agreement between

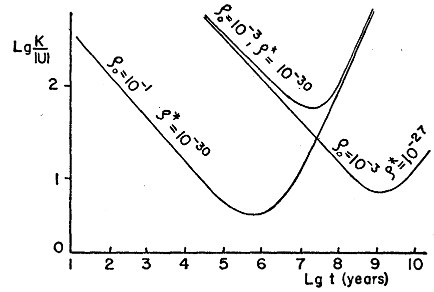

FIG. 3. Variation of the ratio $K/|U|$ for different original densities.

the calculated and observed abundances if one assumes that

$$\int_{t_1}^{t_2} \rho_n(t)dt = 1.3\cdot 10^{-6}\ \mathrm{g\cdot sec./cm^3}, \tag{7}$$

where t_1 and t_2 are the starting and finishing time of the process.

In order to compare these results with the relativistic expressions for the expanding universe, we will first make the assumption that during the formation process the main contribution to the density was due to the material particles. Remembering that during the very early stages of expansion the first term under the radical in Eq. (1) was of primary importance, we can write

$$dl/dt = (8\pi G/3\cdot 1/l)^{\frac{1}{2}}, \tag{8}$$

where we used our convention concerning the meaning of the length l. This integrates in the form:

$$l = (6\pi G)^{\frac{1}{3}}\cdot t^{\frac{2}{3}}, \tag{9}$$

or

$$\rho_{\mathrm{mat}} = \frac{1}{l^3} = \frac{1}{6\pi G t^2} = \frac{8.3\cdot 10^5}{t^2}\ \mathrm{g/cm^3}. \tag{10}$$

If we assume that the process started at a certain time t_0 (when temperature became sufficiently low to permit deuteron formation), and continued to exhaustion, we must write

$$\int_{t_0}^{\infty} \frac{8.3\cdot 10^5}{t^2} = \frac{8.3\cdot 10^5}{t} = 1.3\cdot 10^{-6}, \tag{11}$$

from which it would follow $t_0 = 6\cdot 10^{11}$ sec. = 20,000 years! This is an apparent contradiction since all the neutrons, which have the mean life of about 20 min., would have decayed long before the process could have started.

It is clear that in order to have any appreciable amount of heavy elements formed by the process of neutron capture, we must assume that the building-up was essentially completed when the age of the universe was comparable with the lifetime of neutrons. But if Δt was of the order of magnitude of 20 min. or $\cong 10^3$ sec., the neutron density must have been only about $10^{-6}/10^3 \cong 10^{-9}$ g/cm³. Since, on the other hand, the temperature at which the process could have started was of the order of 10^9°K, the mass density of radiation aT^4/c^2 must have been comparable to the density of water! Thus, we come to an important conclusion that our earlier assumption concerning the prevailing role of matter density during the growth period of atomic nuclei was incorrect, and that one must assume that *during this early epoch the expansion of the universe was governed entirely by the density of radiation.*

In that case, the relativistic expansion formula must be written as

$$\frac{1}{l}\frac{dl}{dt} = \left(\frac{8\pi G}{3}\frac{aT^4}{c^2}\right)^{\frac{1}{2}}, \tag{12}$$

or, remembering that for the adiabatic expansion of black body radiation $T\sim l^{-1}$, as

$$-\frac{1}{T}\frac{dT}{dt} = \left(\frac{8\pi G}{3}\frac{aT^4}{c^2}\right)^{\frac{1}{2}}. \tag{13}$$

This integrates as

$$T = \left(\frac{3c^2}{32\pi G a}\right)^{\frac{1}{4}}\cdot\frac{1}{t^{\frac{1}{2}}} = \frac{1.5\times 10^{10}}{t^{\frac{1}{2}}}\,^{\circ}\mathrm{K}. \tag{14}$$

The density of matter is not determined by the expansion formula, but, since $\rho_{\mathrm{mat}}\sim l^{-3}\sim T^3\sim t^{-\frac{3}{2}}$, we can write

$$\rho_{\mathrm{mat}} = \rho_0/t^{\frac{3}{2}}. \tag{15}$$

Substituting it into (7), we find

$$2\rho_0/t_0^{\frac{1}{2}} = 1.3\cdot 10^{-6}. \tag{16}$$

Assuming for t_0 the value of 400 sec. (which corresponds to starting temperature of 10^9°K), we find $\rho_0 \cong 10^{-5}$ g/cm³. This value gives, of course, only the lower limit of the density because we have disregarded in our calculations the decay of neutrons. A more careful estimate suggests that the value of ρ_0 is actually closer to 10^{-3} g/cm³.

In order to get a somewhat deeper insight into the details of element formation, it is desirable to study more carefully the process of the growth of light nuclei by using the actual capture cross sections instead of the smoothed out values. The calculations of that kind were first carried out by Gamow[11] who integrated numerically the exact equations for neutron-proton capture, taking into the account the decay of neutrons as well as the expansion of ylem. Assuming that the original ylem was formed entirely by neutrons, he found that in order to turn one-half of that material into deuterons and heavier nuclei leaving another half in the form of pure hydrogen,[12] one must assume $\rho_0 = 5\cdot 10^{-4}$ g/cm^3 in reasonable agreement with the results of previous calculations. A considerably more detailed study of the formation of light elements was carried out by Fermi and Turkevitch[13] who extended their integrations all the way up to helium, taking into consideration twenty-eight different nuclear reactions which may take place in this region of atomic weights. The results of these calculations, carried out for $\rho_0 = 1.7\cdot 10^{-3}$ g/cm^3, are shown graphically in Fig. 2, which gives the concentrations of neutrons, protons, and He^4 nuclei for the first half hour. The amount of other light nuclei remains always very small, and toward the end of the process is equal to $1.6\cdot 10^{-3}$, $5\cdot 10^{-4}$, and $9\cdot 10^{-6}$ for deuterium, tritium, and He^3, respectively.

A serious difficulty arises, however, when one attempts to pass a "crevasse" caused by the absence of atomic nuclei with mass 5. Fermi and Turkevitch suggested that the best way of jumping over that crevasse is given by the thermonuclear reaction

$$He^4 + T^3 \rightarrow Li^7 + h\nu. \qquad (17)$$

However, using the ordinary penetration formula, one finds that the amount of heavier nuclei which can be formed by that process is still too small by a factor of a million. This difficulty may be avoided if there exists a strong resonance for the reaction (17) in the region near 400 kev. The existing information on the excited states of Li^7 nucleus does not seem to indicate a resonance level in this particular region, but, on the other hand, the reaction (17) was never studied directly and the possibility of such a resonance is not completely excluded.[14] Another possible way of avoiding the difficulty of the "mass-5-crevasse" lies in a rather radical change in the initial conditions chosen for the process of element formation. It must be remembered that all the calculations described were carried out with the assumption that original ylem was formed entirely by neutrons. A more reasonable assumption, which follows from the considerations of equilibrium between neutrons, protons, electrons, and neutrinos during the first seconds of expansion, would be that at the start of element formation one-half or even a larger fraction of ylem was already formed by protons. Making this assumption, we can raise the value of ρ_0 by a few orders of magnitude without the danger of complete extinction of hydrogen since, indeed, there will be not enough neutrons to capture all protons. On the other hand, the increased value of ρ_0 will considerably accelerate the rate of the reaction (17) which will help to build a larger number of heavier nuclei. Whether or not this is the correct way of jumping the "mass-5-crevasse" can be shown, however, only by further calculations in this direction. Summing up, we may say that, although the problem of the origin of atomic species during the early stages of the expanding universe is still far from being completely settled, it gives very promising results and can be, in this sense, considered as confirming the general conclusions of relativistic cosmogony.

We turn now to the later stages of expansion during which the originally homogeneous material filling the entire space of the universe was broken up into the separate gaseous clouds which have later evolved into the stellar galaxies. This process was first studied by Gamow and Teller[15] who generalized the classical Jeans' condition of *gravitational instability*[16] for the case of the expanding space. They were able to show that the conditions necessary for such condensations are definitely not fulfilled at the present epoch, but could have been closely satisfied when the age of the universe was about 10^7 years, and all the distances were several hundred times smaller then now. Since the present distances between the galaxies exceed their mean diameters by about the same factor, this result represents a valuable confirmation of the assumption that the formation of galaxies was due essentially to gravitational instability of the expanding masses of the universe. However, these early studies could not lead to any definite conclusions about the actual sizes and masses of the condensations since they depend essentially on the temperature of the gas at the time of the breaking-up process. The estimate of the mass of gravitational condensations became, however, possible when the temperature conditions in the expanding universe were fixed by the above described theory of the origin of atomic species. It was, in fact, shown by Gamow[17] that in this case the application of classical

[11] G. Gamow, Nature **162**, 680 (1948); compare also, R. A. Alpher and R. C. Herman, Nature **162**, 774 (1948) where some numerical errors in the original calculations are being corrected.

[12] Because hydrogen is known to form about 50 percent of all the matter in the universe.

[13] As yet unpublished. The author is grateful to Drs. E. Fermi and A. Turkevitch for communication of their interesting results.

[14] It was recently proposed by E. Wigner (private communication) that the building-up process can be brought over the "crevasse 5" by building a "chain-reaction bridge" across it. Thus, for example, one can think about the nucleus of C^{10} which captures a triton and undergoes the reaction $C^{10} + T^3 = Li^6 + Be^7 + 2$ Mev. The nuclei Li^6 and Be^7 would build up into two C^{10} nuclei which, in turn, will bring a pair of tritons across the "crevasse." Thus, after a few steps there will be established a "bridge" bringing across a large number of tritons into the heavier region. It seems that this particular reaction may be not quite suitable since the C^{10} nucleus has a proton excess, and is not very likely to be built up by neutron capture process. But there may exist other possible reactions of that type which involve nuclei with neutron-excess.

[15] G. Gamow and E. Teller, Phys. Rev. **55**, 654 (1939).

[16] J. Jeans, *Astronomy and Cosmology* (Cambridge University Press, London, 1928).

[17] G. Gamow, Phys. Rev. **74**, 505 (1948); Nature **162**, 680 (1948).

formula of Jeans leads to condensation masses comparable to the observed masses of stellar galaxies. According to the Jeans' formula, the diameter of gravitational condensations in a gas of density ρ and temperature T is given by

$$D^2 \cong (10\pi/9mG_\rho)\cdot\tfrac{3}{2}kT, \qquad (18)$$

which gives for its mass

$$M \cong (5\pi k/3mG)^{\frac{3}{2}}(T^3/\rho)^{\frac{1}{2}}. \qquad (19)$$

Substituting T and ρ from the formulas (14) and (15), we find that M is independent of time, and is of the order of

$$M \cong 10^{40}\ \mathrm{g} \cong 10^7\ \text{sun-masses},$$

which is comparable to the observed masses of stellar galaxies, though smaller by one or two order of magnitudes. If, according to earlier studies of Gamow and Teller we assume that galaxies were formed when the age of the universe was about 10^7 years, we find that $D \cong 10^3$ light years which is a correct order of magnitude for galactic diameters.

It should be stated, however, that the classical Jeans' formula is not directly applicable for our purposes since it was derived for the case of non-expanding gas. The revision of that formula, and the general study of instability conditions in a gas filling an expanding universe, was undertaken recently by Gamow, Metropolis, Teller, and Ulam[18] who came to a surprising result that in the case of an expanding space described by Eq. (1), the conditions for gravitational instability cannot be satisfied for any time. In fact, if one considers a rudimentary condensation as a sphere within which the density of matter is slightly higher then the density elsewhere, one can prove that the kinetic energy K of its residual expansion is *always* larger than the absolute value of its total gravitational energy $|U|$. Thus, independent of its size or the epoch of its formation, each rudimentary condensation of that type is bound to expand and mix with the rest of the gas. The behavior of $K/|U|$ ratio as the function of time for different assumed values of original density ρ_0 (at $t=1$ sec.), and present density ρ^* is shown graphically in Fig. 3, and we can notice that although this quantity always reaches a minimum for a certain epoch, it never becomes smaller than unity. It must be remarked, however, that the above statement is only true for rudimentary condensations (small density excess), and that gravitational forces become quite effective once the condensation reaches a high degree of development.

Thus, it becomes necessary to find some new agent which could be made responsible for *the beginning of the condensation process*. According to the above-mentioned authors, the beginning of the condensation process must be ascribed to the interaction between gas particles and the black body radiation filling up the expanding space. In fact, it can be shown that in the case of the expanding universe the temperature of the gas drops faster than the temperature of radiation (because of different ratios of specific heat), so that there must be a constant energy transfer from the black body radiation into the gas. This results in a certain temperature difference between the radiation—and gas—temperatures, and one can prove that this temperature lag changes with the age of the universe according to the formula

$$\frac{T_{\mathrm{rad}} - T_{\mathrm{gas}}}{T_{\mathrm{rad}}} \cong \frac{t(\text{years})}{10^{12}}. \qquad (20)$$

Since the galaxies were presumably formed at the age of about 10^7 years, when, according to (14), radiation temperature was about $10^{\circ 3}$K, we find that at that epoch $T_{\mathrm{rad}} - T_{\mathrm{gas}} \cong 0.01\,^\circ\mathrm{C}$.

This, at the first sight insignificant, temperature difference produces, however, rather strong interaction forces between the particles of gas which can be considered under these conditions as completely ionized hydrogen. One can show, in fact, that in the case of an electron gas of temperature T_{gas} mixed with radiation of the temperature T_{rad}, each pair of particles will be subject to a mutual attractive force

$$F = \frac{4aT_{\mathrm{rad}}^3(T_{\mathrm{rad}} - T_{\mathrm{gas}})\sigma^2}{r^2}\left(\frac{v}{c}\right)^2, \qquad (21)$$

which arises from the so-called "shadow casting" effect in the phenomenon of radiation pressure.[19] In the above expression $\sigma = 0.6\times 10^{-24}$ cm^2 is the Thompson cross section, and v is the thermal velocity of electron gas. The cumulative effect of these forces appears to be comparable with the forces of Newtonian gravity acting on large masses of gas. Since these forces vary with distance in the same way as the ordinary forces of gravity (and can be called in this sense "*mock-gravity*" forces), they will effectively increase the attraction between various parts of the expanding gas and could be made responsible for the formation of rudimentary condensations, even though $K/|U|$ is larger than unity. It is important to notice that the forces of "mock-gravity" have the nature of range forces since the above-described mechanism of interaction between two particles will operate only at distances at which "the particles can see one another." This fact will favor the formation of condensations with linear dimensions comparable to the free path of light quanta in the material in question. Remembering that the mean density of main galactic bodies is of the order of magnitude 10^{-21} g/cm^3, we find that during the prestellar epoch when the galaxies were made of ionized hydrogen the

[18] Gamow, Metropolis, Teller, and Ulam, Phys. Rev. (to be submitted).

[19] The importance of such radiation forces acting on the dust particles in the interstellar space for the process of star formation was recently emphasized by Spitzer (Ap. J. **95**, 329 (1942)), and Whipple (Ap. J. **104**, 1 (1946)).

free path of light quanta was

$$\lambda = \frac{m_n}{\rho\sigma} \cong \frac{10^{-24}}{10^{-21} \cdot 10^{-24}} \cong 10^{21} \text{ cm} \cong 10^3 \text{ light years.} \quad (22)$$

The agreement of that figure with the measured values of galactic diameters gives an additional weight to the above-described hypothesis. We may also notice that, using the "free path"—criterium for the sizes of condensations, we obtain for their masses the values which exceed the minimum values given by Jeans' formula and stand in better agreement with the observed masses of stellar galaxies. It must be stated, in conclusion, that the introduction of the "mock-gravity" forces can be helpful for the formation of rudimentary condensations only when the $K/|U|$ ratio is *not much larger* than unity. We can see from the curves given in Fig. 3 that for ordinary assumption ($\rho_0 \cong 10^{-3}$; $\rho^* \cong 10^{-30}$), the minimum value of $K/|U|$ is still rather large. This suggests that either the value of ρ_0 or the value of ρ^* (or both) may have been assumed too low. We have seen in the previous discussion that it may become necessary to increase the value of ρ_0 to avoid the difficulties of the "mass-5-crevasse." On the other hand, present astronomical information would also not contradict a ρ^*-value higher by one or even two orders of magnitude. It may also be noticed that the introduction of cosmological constant, as described earlier in this article (and not taken into account in the calculation of the curves shown in Fig. 3), will act in the direction of reducing the value of $K/|U|$ at minimum. Thus, it seems that also in this particular field of cosmogony, the use of standard equations of the general theory of relativity shows us a reasonable way toward the understanding of the evolutionary features of our universe.