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Charge-Exchange Reaction of the μ-Meson with the Nucleus*

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THE present paper examines the reaction of negatively charged μ-mesons with atomic nuclei on the hypothesis that the elementary process involved is the transition

$$\mu^{-}(\text{meson})+P(\text{proton})\rightarrow \mu_0(\text{neutral meson})+N(\text{neutron}).$$

The coupling constant required to make the rate of reaction agree with observations is determined approximately and is found to be of the same order of magnitude as that required (a) to account for β-decay and (b) to account for the disintegration of μ-mesons on the three-particle hypothesis along the lines discussed in the preceding paper.[1] The theoretical probability distribution of excitation of a nucleus which has reacted with a μ-meson is calculated approximately and correlated with the observations of W. Y. Chang.[2]

The remarkable feature of the $\sim$200-mass meson brought to rest in matter is its failure to produce nuclear disruptions with any appreciable probability, as first shown by the Bristol group,[3] then tentatively confirmed by Lattes and Gardner[4] at Berkeley, and now further demonstrated by the experiments of Chang.[2] This behavior is entirely contrary to that observed for 300-mass negative mesons, which on being stopped in a solid nearly always, and perhaps always, provoke a nuclear disintegration. The difference can certainly not be attributed to the difference between the two cases with respect to the rest energy which would be set free on annihilation of the two particles. The dependence of average star size on energy in the region around 100 Mev and 200 Mev is known from cyclotron experiments[5] to be only slight. Moreover, the meson cannot be supposed to remain in a permanently trapped state, for its life is only 2.15×10^{-6} sec.; nor can it have emitted its energy in the form of a normal decay electron, for such a process would have been observed. Chang's observations in particular emphasize the two conclusions (a) that the whole rest energy of the meson is certainly not imparted to the nucleus, and must therefore go off in some other entity, and (b) this entity must be electrically neutral. Consequently, it will be appropriate to refer to the process in question as a charge-transfer reaction, rather than a capture reaction, the term first used in describing the phenomenon of disappearance.

That some, at least, of the rest energy of the μ-meson is given to the nucleus is strongly suggested by the recent observations of Sard and collaborators.[6] They conclude that roughly one or two neutrons come off on the average per meson stopped in lead. This result could not be understood if nucleons alone were left at the end of the meson reaction. In that case the energy

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* This paper, together with the preceding and following paper, constitutes an elaboration of remarks made by J. A. Wheeler at the Pasadena Conference. The section below on comparison of the Hartree model for O^{16} with the free particle model and the photoelectric model was added after R. F. Christy reported in discussions at the time of the meeting his calculations on the Hartree model for oxygen. We are indebted to Professor Christy and Professor Schiff for discussions of this and other questions of meson physics.

** Buenos Aires Convention—U. S. State Department Fellow from the University of São Paulo.

[1] J. Tiomno and J. A. Wheeler, Rev. Mod. Phys. **21**, 144 (1949). *Note added in proof:* T. D. Lee, M. Rosenbluth, and C. N. Yang have kindly sent us a copy of a Letter to the Editor of the Physical Review dealing with similar questions.

[2] W. Y. Chang, Rev. Mod. Phys. **21**, 166 (1949).

[3] C. M. G. Lattes, H. Muirhead, G. P. S. Occhialini, and C. F. Powell, Nature **159**, 694 (1947); C. M. G. Lattes, G. P. S. Occhialini, and C. F. Powell, Nature **160**, 453 and 486 (1947). See also R. Brown, U. Camerini, P. H. Fowler, H. Muirhead, C. F. Powell, and D. M. Ritson, Nature (in press).

[4] E. Gardner and C. M. G. Lattes, Science **107**, 270 (1948), and later, as yet unpublished, observations kindly communicated to us.

[5] E. Gardner (alpha-particle-initiated stars) and E. Gardner and V. Peterson (deuteron-initiated stars), Phys. Rev. (in press).

[6] Sard, Ittner, Conforto, and Crouch, Phys. Rev. **74**, 97 (1948). See also G. Groetzinger and G. W. McClure, Phys. Rev. **74**, 341 (1948).

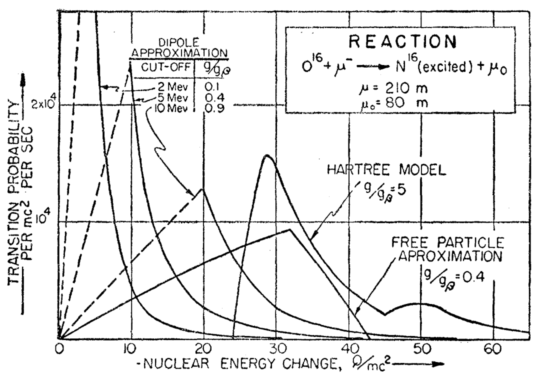


FIG. 1. Determination of coupling constant, g, for reaction $\mu^-+P\rightarrow N+\mu_0$ from the rate of the charge exchange reaction between μ^--mesons and nuclei. In the diagram g is expressed—purely for convenience—relative to the value, $g_\beta=2.2\times10^{-49}$ erg cm^3, attributed to the coupling constant of the quite different process of nuclear beta-decay. (*Note:* The figures for g/g_β given under the heading "Dipole Approximation" should all be divided by 2.) The rate constant of the charge-exchange reaction for oxygen was estimated as $A=2\times10^5$/sec. from the Z^4 law. Nuclear reaction and free decay ($A=1/2.15\times10^{-6}$ sec.) were assumed as equally probable for an atomic number $Z=Z_0\sim10$. To account for this rate of reaction one requires a coupling constant $g=5g_\beta$ in the Hartree model (arbitrarily corrected near the threshold, as described in the text). Use of this model makes it appear that a neutron always emerges from the nucleus as a consequence of the charge-exchange reaction. A correct calculation would undoubtedly describe most of the reactions as causing excitation of the nucleus as a whole. The same remark applies to the calculations made on the free particle approximation and the dipole transition approximation. The latter treatment gives a transition probability which diverges at low energies, because we replaced the discrete spectrum of states in momentum space by a continuum, in order to simplify the calculations. Allowance for this discreteness would give in the case of oxygen a lower limit to the permissible energy transfer of the order of 5 Mev. Hence the cut-off of the dipole-approximation curves at 2, 5, and 10 Mev, as indicated by the dotted lines (see text). All the curves in the present figures were calculated with $\mu=210$ m, $\mu_0=80$ m. A decrease of μ_0 to 0 (neutrino) would increase the transition rates in the Hartree model by about 30 percent (or decrease g about 15 percent). See Figs. 4 and 5 for the equally small effect of changes in the μ- and μ_0-masses in the free particle and dipole approximations. All the treatments considered here are, of course, inadequate for the accurate description of any actual nucleus. But the several approximations are so different that the similarity of the values of the coupling constant required in the three cases must be considered to establish the order of magnitude of g. The very rough *value of g* obtained on this purely phenomenological basis *agrees* within the limits of error of experiment and theory *with the* quite distinct *coupling constants* (a) *for beta-decay* and (b) for *μ-meson decay*. For dependence of charge-exchange reaction probability upon atomic number the free particle model predicts a Z_{eff}^4 law. The Hartree model will be expected to give a similar general trend with Z, with local fluctuations from element to element—the magnitude of these fluctuations probably being overemphasized in the Hartree picture. In the dipole transition model the reaction probability will go up faster than $Z_{\text{eff}}^{3.66}$ (Eq. (30)) because of the dependence of cut-off energy on atomic number. Relative to the excitation of the nucleus in the charge-exchange reaction, all three treatments predict *a change of energy in general less than 20 Mev.* Part of this energy disappears as the difference of mass between Z^A and $(Z-1)^A$ (ordinarily a few Mev; 5.3 Mev to 9.5 Mev for the case of O^{16}, depending upon the mass assumed for N^{16}). Consequently, it will often happen that only a single nucleon is evaporated before the nucleus becomes energetically unable to do anything more than to radiate. In the case of oxygen it is likely that neutrons alone can emerge (heat of evaporation 6.8 Mev or 2.8 Mev for neutrons from N^{16}, depending upon N^{16} mass; 16.8 Mev or 12.7 Mev for protons, which are evidently energetically excluded when at most 20 Mev are available). An additional obstacle to the emission of protons occurs in heavy nuclei, where the Coulomb potential barrier is high. For example, following the charge-exchange reaction in ${}_{82}Pb^{208}$, there is formed ${}_{81}Tl^{208}$, from which the heat of evaporation of a neutron is about 5 Mev, of a proton, slightly larger; but to the proton must be supplied an additional energy of about 10 Mev to surmount the barrier; furthermore, about 5 Mev are needed to transform ${}_{82}Pb^{208}$ into ${}_{81}Tl^{208}$. Similar arguments indicate that *proton emission will compete unfavorably with neutron emission in most nuclei.* The calculated low excitation of the nucleus in the μ-charge-exchange reaction, and the observed high excitation in the reaction of π-mesons with the nucleus (several pronged stars generally formed, with energies of order 100 Mev) argue that the π-capture process is not a charge-exchange reaction. Thus, if the elementary transition were $\pi^-+P\rightarrow N+\pi_0$ (say a neutrino), then application of the free particle model along the lines indicated above would permit a transfer of energy to the nucleus of at most about 30 Mev, in disagreement with experiment. This argument *excludes a value of $\frac{1}{2}$ for the spin of the π-meson,* for the emission of the particle of spin $\frac{1}{2}$-symbolized above as π_0—would then be required to conserve angular momentum.

left over for the nucleons would release protons as well as neutrons, contrary to the findings of Chang. Consequently, it is reasonable to believe that some neutral, non-nucleonic entity carries away a large part of the rest energy of the μ-meson. Since the experiments of Sard and Althaus and Piccioni[7] rule out a photonic character for this radiation, we are led to describe the entity as a neutral meson—without thereby ruling out the possibility that it is a neutrino.[8] In this picture the elementary act in the charge-exchange reaction is the process

$$\mu^-+P\rightarrow N+\mu_0. \quad (1)$$

The possibility of correlating reaction (1) with the well established process of $\pi-\mu$-decay,

$$\pi\rightarrow\mu+\mu_0, \quad (2)$$

and with the artificially induced reaction,

$$N\rightarrow P+\pi, \quad (3)$$

[7] R. D. Sard and E. J. Althaus, Phys. Rev. **73**, 1251 (1948); O. Piccioni, Phys. Rev. **74**, 1754 (1948).

[8] See in this connection B. Pontecorvo, Phys. Rev. **72**, 246 (1947).

need hardly be recalled. But quite apart from the deeper theoretical questions raised by any such proposed correlation, it is evident that one can discuss on purely phenomenological grounds the consequences of a reaction such as (1). This is what we shall attempt to do here.

We shall adopt a mathematical expression for the form of the coupling of the μ, μ_0, and nucleon fields implied by (1), and determine in this expression the constant of proportionality so as to account for the absolute rate of the charge-exchange reaction as determined experimentally.[9] In our choice of interaction Hamiltonian we shall exclude derivatives of the wave function of the character considered by Konopinski and Uhlenbeck in the analogous problem of the interaction of the electron-neutrino field with the nucleon field (beta-decay). Guided by the analogy between these two processes, we attribute the spin $\frac{1}{2}$ to the μ- and μ_0-mesons,[10] and consider specifically an interaction Hamiltonian of the form

$$H = gO_{HL}(\tau_H\tau_L + \tau_H^*\tau_L^*)\delta(\mathbf{x}_H - \mathbf{x}_L). \quad (4)$$

Here the subscripts H and L refer to the particles, P and N, and μ^- and μ_0, respectively. The operators τ_L and τ_L^* change a charged meson into a neutral one and conversely. The operators τ_H and τ_H^* similarly produce the changes $P\rightarrow N$ and $N\rightarrow P$. The operator O_{HL} is a relativistically invariant combination of the Dirac spin operators for the nucleon and meson fields, the simplest possibilities for which are more precisely defined in the preceding paper. The coupling constant g has the dimensions erg cm^3. It is in the beginning assumed to be an independent constant to be determined from experiment, and only later conceived to have any relation to the analogous constant for beta-decay and μ-decay.

In the simplest problem, that of the charge-exchange reaction between a meson and a proton in whose field of force it circulates, we find from a simple application of perturbation theory[11] the decay rate

$$A_H(\text{sec.}^{-1}) = 190(g/10^{-49}\text{ erg cm}^3)^2. \quad (5)$$

On the other hand, the Z^4 law of meson reaction probability,[12] together with the observation[9] that A has the value $1/2.15\times10^{-6}$ for a nucleus of charge number $Z_0\sim10$, allows one to obtain by extrapolation to hydrogen the rough estimate of charge-exchange reaction rate,

$$A_H \sim 1/(10)^4 2.15\times10^{-6} = 47/\text{sec.} \quad (6)$$

By comparison of (5) and (6) we arrive at a tentative estimate for the order of magnitude of the coupling constant,

$$g\sim 0.5\times10^{-49}\text{ erg cm}^3 \sim 1\text{ ev}(e^2/mc^2)^3. \quad (7)$$

As the relevant measurements of rate of the charge transfer reaction were made for intermediate elements, not for hydrogen, we have to investigate the consequences of the charge-exchange coupling for complex nuclei. Not only is such a study important for the question of the coupling, but also it permits a discussion of the expected nuclear excitation following the reaction, and hence gives some idea of the possibility of star formation.

The complications of the investigation come not from fundamental questions of elementary particle physics, but from the intricacy of nucleonic dynamics. In order to make any progress at all, it proved necessary to make a drastic idealization, treating the total nuclear wave function as the antisymmetrized product of individual particle wave functions, these functions being computed for a simple potential well. Three cases were considered (Fig. 1):

(1) "Free particle model"—conservation of momentum between meson and individual nucleon; result different from that for hydrogen on two accounts:

(a) substantial velocity of nucleons in nucleus, permitting energy changes both significantly greater and significantly less than the amount, $\sim p^2/2M\sim(100\text{ Mev})^2/2\times931\text{ Mev}\sim5$ Mev, imparted by recoil of a proton at rest from a $\sim$100-Mev neutral meson;

(b) a decrease—by a factor of the order of 4—in the accessible volume in phase space, and therefore a corre-

[9] See, for example, T. Sigurgeirsson and A. Yamakawa, Rev. Mod. Phys. **21**, 124 (1949); also J. A. Wheeler, Rev. Mod. Phys. **21**, p. 133 (1949).

[10] The value obtained below for the coupling constant might be little affected by assuming spin zero, but the closeness of the analogy with beta-decay would be destroyed. For this alternative point of view—in which the indirect interaction via π-mesons is the main consideration, see L. I. Schiff, Phys. Rev. **74**, 1556 (1948), A. S. Lodge, Nature **161**, 809 (1948); S. Hayakawa, Prog. Theor. Phys. **3**, 200 (1948), and G. Araki, Phys. Rev. **74**, 985 (1948).

[11] Details of the derivation are given below. In the numerical evaluation we used $\mu=210$ m, $\mu_0=0$.

[12] J. A. Wheeler, Rev. Mod. Phys. **21**, 133 (1949).

sponding decrease in transition probability due to the operation of the Pauli exclusion principle.

As a consequence of this reduction, we have to increase by a factor about 2 the value of the coupling constant required in the free particle model to account for the observed reaction rate of nuclei with Z near 10, as indicated in Fig. 1:

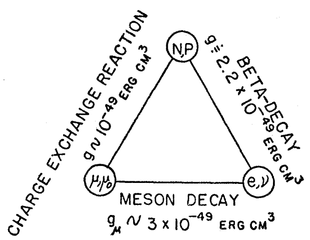
$$g\sim 0.9\times 10^{-49} \text{ erg cm}^3. \qquad (8)$$

Of course this figure is also extremely uncertain, for it is hardly appropriate to apply to a nucleus of $Z\sim 10$ a treatment which assumes a nuclear extension large in comparison with the wave-length of the neutral meson.

(2) The opposite idealized—and in practice unrealizable—case of a nuclear extension small in comparison with the neutral meson wave-length is considered in the "dipole treatment." Here the momentum acquired by the reacting nucleon is supplied mainly by the action on that particle of the specifically nuclear forces. These forces are taken to act exclusively at the nuclear surface in the individual particle treatment. Consequently, the calculated charge-exchange reaction rate, apart from a Z^3 factor which is due to nuclear attraction, increases in proportion to the nuclear surface. Unfortunately, for the application of the dipole treatment the probability distribution of nuclear excitations, Q, increases indefinitely for low values of Q. Such an increase is, of course, physically impossible, due to the finite number of low lying nuclear levels, if to no other cause. The divergence comes about because in our treatment the discrete level spacing of the actual nucleus was replaced by a continuum. To obtain a reasonable result, we must cut off the excitation curve at some not well defined lower limit. This limit is probably bracketed on the upper end by a figure of 10 Mev, of the order of the binding energy of a typical nucleon, and on the lower end by a quantity comparable to the spacing of low lying nuclear levels, ranging from perhaps 0.1 Mev for heavy nuclei to 1 or 2 Mev in the neighborhood of $Z=10$. The two alternative cut-offs shown for oxygen in Fig. 1 lead, however, as seen there, to not too greatly different values of g. Consequently, on the dipole transition model as on the free particle model it is reasonable to accept a value of g of the order of 10^{-49} erg cm^3. A similar conclusion follows from our third model.

(3) The special case of oxygen, described on the free particle model in terms of the closed shell configuration $(1s)_4(2p)_{12}$, is especially appropriate for a more detailed investigation, as pointed out to one of us by R. F. Christy at the time of the Pasadena conference. We have computed the wave functions for neutrons in free states and protons in bound states in the field of force of an appropriate spherical potential well, and determined on this basis the detailed dependence of excitation probability on excitation energy (Fig. 1). An unfortunate anomaly of the computations was the accidental existence in this idealized treatment of a d-wave resonance practically at a zero value for the kinetic energy of the outgoing neutron. This effect, taken at face value, would lead to an infinite value under the appropriate excitation curve in Fig. 1 (see also Fig. 6). As the divergence obviously does not occur in nature, we have, in practice, rounded off at low energies the curve for rate of transitions in which a neutron emerges from the nucleus in a d-wave of low kinetic energy. Specifically, the contribution of the process $2p$-protons→d-neutron plus p-neutral meson was arbitrarily not allowed at low energies to rise above that of the process $2p$-proton→s-neutron plus p-neutral meson (Fig. 6). The curve for total excitation probability so obtained (Fig. 1) has a reasonable dependence on energy. We integrated the area under the theoretical curve and compared with the value of the reaction rate expected for oxygen, $(8/10)^4/2.15\times 10^{-6}$ sec. $=1/5\times 10^{-6}$ sec. on the basis of the Z^4 power law and the existing data on neighboring nuclei (Sigurgeirsson and Yamakawa's Fig. 2). In this way we found for the coupling constant again a value $g\sim 10^{-49}$ erg cm^3.

Several methods of determining the constant g giving similar results, we conclude it is reasonable to assign a value near 10^{-49} erg cm^3 to this quantity (see diagram 1). We compared this result with the coupling constants $g_\beta \doteqdot 2.2\times 10^{-49}$ erg cm^3 for beta-decay and $g_\mu \sim 3\times 10^{-49}$ erg cm^3 for decay of the μ-meson on the hypothesis of three end products.[1] We note that the *three coupling constants determined quite independently agree with one another within the limits of error of* *experiment and theory.* We apparently have to do in all three reaction processes with phenomena having a much closer relationship than we can now visualize.



Of course, the symmetry of the phenomenological coupling scheme between the three kinds of particles—with respect to coupling constants and very possibly with respect to spin—could also be an accident. For instance, the coupling of the μ-meson field and the nucleon field could be an indirect consequence of the direct couplings between π-mesons and nucleons and between π-mesons and μ-mesons.[13]

The dependence of the charge-exchange reaction rate upon μ- and μ_0-masses was investigated for the "free particle" and "dipole transition" models, and in both cases found to be small (Figs. 4 and 5). A similarly complete analysis was not carried out in the case of the Hartree type of treatment of oxygen, because of the difficulty of the computations. The μ-mass was assumed in this case to be $\mu=210$ m, and the value $\mu_0=80$ m was used in the calculations. However, in several sample transition terms the mass $\mu_0=0$ was employed, with results for transition probabilities which were roughly 30 percent higher than those illustrated in the drawings. This uncertainty does not affect our conclusions about the order of magnitude of the coupling constant.

Practically independent of the magnitude of the coupling constant is the nuclear excitation following the meson charge-exchange reaction. This excitation is important in determining the later history of the affected nucleus. The free particle model predicts the highest average energy. The other two models are comparable. But all three predict excitations very much smaller than the rest mass of the lost μ-meson.

[13] See in this connection C. Marty and J. Prentki, J. de phys. et rad. No. 4, 1948; also J. Leite Lopes, Phys. Rev. **74**, 1722 (1948).

The low value of this energy would thus seem to be reasonably well established theoretically. In practical terms, we expect that a nucleus will receive of the order of 10 Mev from a μ-meson via charge exchange.

What will happen to a nucleus with 10 Mev of excitation? If it is a heavy nucleus, where the Coulomb potential barrier against protons is of the order of 10 Mev to 15 Mev, and where the excitation is anyway likely to be distributed over many degrees of freedom, it will be very much less likely for a proton to emerge than for a neutron to get out. When, on the other hand, the excited nucleus is of low atomic number and binding energies are greater, it is quite reasonable to expect that there will be many cases where the nucleus gets too little excitation to emit any kind of nucleon. These conclusions are consistent with the finding of neutrons by Sard and his collaborators, with the photographic plate work of Powell, Leprince-Ringuet, and others, and with Chang's findings, which thus lend support to the present picture of the charge-exchange reaction.

We have looked apart here from the circumstance that the individual particle model, strictly—and therefore incorrectly—employed, would predict no processes of nucleon emission except those in which the former proton comes off as a neutron. In this respect the individual particle model departs widely from the compound nucleus picture of nuclear processes. It is nevertheless not excluded that even on a proper model of the nucleus there should be a slight preference for neutron emission over proton emission in the mesonic charge-exchange reaction. In the following we use the individual particle model to calculate excitation energies, and the compound nucleus picture to analyze the subsequent nuclear reactions.

DETAILS OF CONSIDERATIONS

As first and simplest application of the perturbation theory to the meson-exchange reaction, we consider the reaction of a moving proton with a μ-meson of zero momentum, in the approximation in which we neglect the electrostatic interaction between the two particles. Motion of the proton is assumed because we wish later to apply our result to the nucleus. For convenience

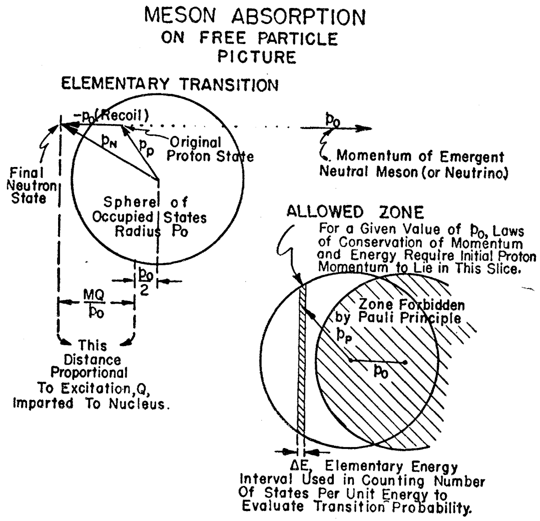


FIG. 2. In the free particle picture, the laws of conservation of momentum and energy are applied directly to the nucleon which is converted from a neutron to a proton. As compared to a single free proton at rest, a nuclear proton may on this model take up a larger amount of energy solely because the recoil momentum imparted to it lies in the same direction in which it is already moving. The figure is drawn for the case where the emergent neutral meson or neutrino has a momentum in the neighborhood of 165 mc.

the wave functions of all the particles involved are considered to be normalized over a cube of side length L, with boundary conditions that each wave function be periodic with respect to a displacement of magnitude L in the x, y, or z direction. Then the number of momentum states of any particle in the range $p^2dpd\Omega$ will be $(L^3/h^3)p^2dpd\Omega$.

The probability per second of an act of absorption will be

$A=(2\pi/h)(1/\Delta E)$(sum of squares of matrix elements of perturbation energy over all states which lie in a small interval of energy, ΔE, about the value required by strict conservation of momentum and energy). (9)

The matrix element here considered will be zero except for those states of the neutron and the neutral meson for which the law of conservation of momentum is exactly fulfilled,

$$\mathbf{p}_n+\mathbf{p}_0=\mathbf{p}_p.$$

Consider a definite (quantized) value of $\mathbf{p}_0$. Then $\mathbf{p}_n$ is completely determined. Thus by counting the states of $\mathbf{p}_0$ we count the states of the whole system. These states will lie in an interval of momentum, $\mathbf{p}_0$, of a magnitude Δp_0 directly proportional in magnitude to the small energy interval ΔE of Eq. (9). The number of relevant states in phase space is, consequently,

$$(L^3/h^3)4\pi p_0^2\Delta p_0.$$

In spin space we have to sum over the two possible orientations of the neutron spin and of the neutral meson, and have to *average* over the two possible directions of the proton and μ-meson spin. The square of the matrix element of the perturbation energy (4), after these operations in spin space, and after the summation over states in phase space, will give a contribution which is represented by the product of the following factors:

g^2, square of coupling constant.

$\frac{1}{2}$, factor from *averaging* over spin, less on this account by a factor 2 than the result given in reference 1 for the decay process $\mu\rightarrow\mu_0+e+\nu$; value obtained by analysis similar to that given in reference 1, except that we go to the limit where nucleons and μ-mesons have non-relativistic velocities.

L^{-6}, factor obtained by multiplying four wave functions with normalization constants $1/L^{\frac{3}{2}}$ and integrating over the volume L^3.

$(L^3/h^3)4\pi P_0^2\Delta P_0$, number of states in phase space.

Inserting the product of these factors in the expression (9), we obtain for the probability per second of a nuclear reaction the figure

$$A=L^{-3}(2\pi/h)(g^2/2)(4\pi p_0^2/h^3)\Delta p_0/\Delta E \quad (10)$$

when we have one proton in a region of space which contains mesons to the number density, L^{-3}.

The value of the momentum interval Δp_0 is fixed by the limits

$$\mu c^2\leqslant(\mathbf{p}_p-\mathbf{p}_0)^2/2M-\mathbf{p}_p^2/2M+[(\mu_0c^2)^2+(cp_0)^2]^{\frac{1}{2}}\leqslant\mu c^2+\Delta E.$$

Differentiating, and treating now the case of immediate interest, where the proton is at rest we have

$$\Delta E/\Delta p_0=p_0\{M^{-1}+(\mu_0^2+p_0^2/c^2)^{-\frac{1}{2}}\},$$

where the expression in curly brackets is the reciprocal of a relativistic analog for the usual reduced mass.

In the case of a meson bound in the K-orbit in the field of a proton, the probability density near the proton, ψ_μ^2, is well known, and we find for the reaction probability

$$A(\text{sec.}^{-1}) = (2\pi^2)^{-1}(g^2/\hbar mc^2)(mc/\hbar)^6(e^2/\hbar c)^3 \times(\mu/m)^3(p_0/mc)^2\{(p_0/Mc) +[(\mu_0 c/p_0)^2+1]^{-\frac{1}{2}}\}^{-1}. \quad (11)$$

This expression was used to obtain the results quoted in the introduction.

THE INDIVIDUAL PARTICLE MODEL

In the case of a complex nucleus, the interaction with the μ, μ_0 field will be expressed as the sum of a number of terms of the form (4).

To evaluate the matrix element of this perturbation is, of course, most difficult in the absence of any adequate approximation methods for determining nuclear wave functions. For this reason we are forced to adopt the only generally practicable treatment now available: we represent the nuclear wave function as the product—antisymmetrized with respect to protons and neutrons separately—of normalized orthogonal individual particle wave functions. Such a representation of the wave function would follow from a treatment of nuclear dynamics where one replaces the interaction between the individual particles by a general field—an idealization quite unacceptable on modern views of nuclear structure. However unreasonable the *field of force* itself, the use of the nuclear *wave function* so obtained has led to qualitatively reasonable results on binding energies and on other features of atomic nuclei.[14] Consequently, we shall adopt this approximation in what follows.

When we evaluate the matrix element of the coupling with the (μ, μ_0)-field between antisymmetrized wave functions of the kind just discussed, then, just as in the analogous case of coupling with the electromagnetic field,[15] we obtain always only a single term, corresponding to the transition of an individual nucleon from a definite initial proton state to a definite final neutron state, previously unoccupied.

In order to proceed further with the evaluation of the transition probability, we consider the two idealized cases where the wave-length of the μ_0-meson is small or large compared to the nuclear radius, and the special case of O^{16} where the treatment makes no special assumption on relative magnitude of these two lengths.

FREE PARTICLE MODEL; μ_0—WAVE-LENGTH SMALL COMPARED TO NUCLEAR DIMENSIONS

When the wave-length of the μ_0-meson is small compared to nuclear dimensions, it will be appropriate to use the notion of conservation of momentum in discussing the interaction between the nucleon and the meson field. Thus, the momentum, h/λ, taken away by μ_0 will be large compared to the characteristic order of magnitude, $\hbar/2R$, of the separation in momentum between the various individual particle states

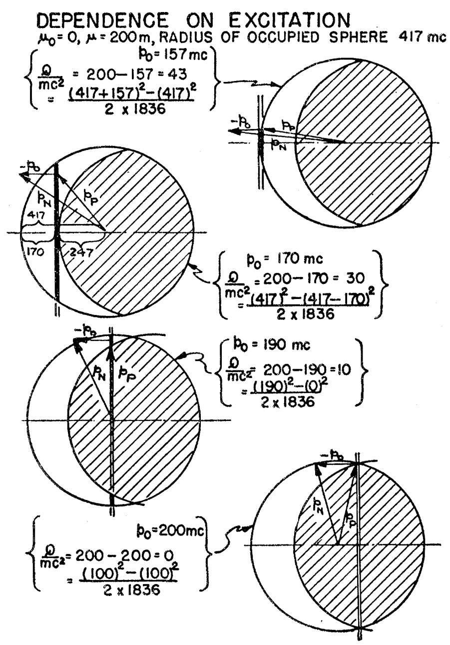


FIG. 3. Protons which undergo the charge-neutralization reaction with negative sea-level mesons cannot go to states already occupied by neutrons. The restriction thus imposed by the Pauli principle, when formulated in the free particle model of the nucleus, is important when the energy transfer, Q, to the nucleon is small. Here p_P and p_N are the momenta of the nucleon before and after charge neutralization, and p_0 is the momentum of the emergent neutral meson.

[14] See, for example, H. A. Bethe and R. F. Bacher. Rev. Mod. Phys. 8, 82 (1936), and H. A. Bethe, Rev. Mod. Phys. 9, 69 (1937).

[15] See, for example, E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (The Macmillan Company, New York, 1935).

under consideration. Under these circumstances and granted that we are to speak about individual particle wave functions at all, it will be reasonable to look apart from any momentum taken up by the nuclear field of force itself. Consequently, we shall treat the nucleons as a collection of free particles, which occupy a sphere of radius $R_0 \doteqdot (e^2/2mc^2)A^{\frac{1}{3}}$ in coordinate space,[16] and which in momentum space occupy all available states up to a critical momentum, P ("Fermi model"). We shall furthermore, for simplicity, assume equal numbers of neutrons and protons, and thus assign to both particles the same critical momentum, such that

$$(4\pi R^3/3)(4\pi P^3/3h^3) = (Z/2) = (A-Z)/2 = A/4 \quad (12)$$

or

$$P = 3\times 137 mc\times(\pi/3)^{\frac{1}{3}} = 417 mc. \quad (13)$$

Moreover, for the time being we shall assign to the wave function, ψ_μ, of the original meson throughout the entire nucleus the value $\pi^{-\frac{1}{2}}(\mu Z e^2/\hbar^2)^{\frac{3}{2}}$, appropriate to the K level in a simple Coulomb field of force.

The reaction probability is obtained by a generalization of the calculation for a single free proton which led to Eq. (11). The changes are simple. Many values are possible now for the energy, Q, imparted to the nucleon. On this account we consider the probability per second, dA, of a reaction which carries the nucleus into an interval of excitation, dQ. The energy left for the meson being $\mu c^2 - Q$, it goes off with a

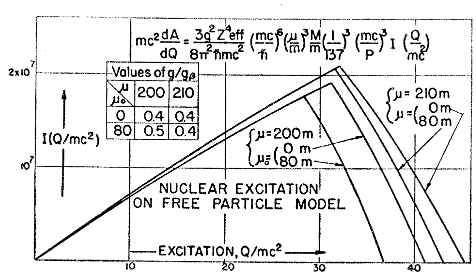


FIG. 4. Distribution of nuclear excitations expected on the free particle model of the nucleus. Here dA represents the probability per second of transition to the interval of energy dQ. The value of the coupling constant, g, required to obtain the observed rate of charge-exchange reaction is expressed for convenience, not in absolute units, but relative to the approximate value, $g_\beta = 2.2\times 10^{-49}$ erg cm³, of the coupling constant of beta-decay theory.

[16] R. Sherr, Phys. Rev. **68**, 240 (1945).

definite momentum, p_0. The reaction rate for the nucleus as a whole will be the sum of the reaction rate as given by (10) for a single proton, taken over all protons which upon reaction will acquire energy between Q and $Q+dQ$. These protons have momenta, $p_{p(||)}$, parallel to $\mathbf{p}_0$ which lie in an interval $dp_{p(||)}$. They occupy a slice in momentum space of the form shown in Fig. 2, with a cross-sectional area which we shall call S. Their number is found by proportion from the number of protons in the whole of the original sphere in momentum space, and is

$$(3/4\pi P^3) Z (S dp_{p(||)}). \quad (14)$$

We multiply this quantity by the elementary transition probability (10), replace the density L^{-3} by ψ_μ^2, rewrite $dp_{p(||)}\Delta p_0$ in the form $\Delta p_{p(||)} dp_0$, and find for the chance per second of a reaction in the relevant excitation interval the expression

$$dA = \pi^{-1}(\mu Z e^2/\hbar^2)^3 (2\pi/\hbar)(g^2/2)(4\pi p_0^2 dp_0/h^3) \times (3/4\pi P^3) Z (S\Delta p_{p(||)}/\Delta E). \quad (15)$$

Here dp_0 is the interval of neutral meson momentum corresponding to the range dQ of neutral meson energy.

The volume $S dp_{p(||)}$ includes all those points in the space of the proton momentum which satisfy the conditions:

(a) $\mathbf{p}_p$ lies inside the occupied proton sphere,
(b) $\mathbf{p}_p - \mathbf{p}_0$ lies outside the occupied neutron sphere,
(c) the energy increase,

$$Q = \mu c^2 - [(\mu_0 c^2)^2 + c^2 p_0^2]^{\frac{1}{2}} = (\mathbf{p}_p - \mathbf{p}_0)^2/2M - \mathbf{p}_p^2/2M, \quad (16)$$

lies in the interval Q to $Q+\Delta E$. From (c) it follows that the component of $\mathbf{p}_p$ parallel to $\mathbf{p}_0$ lies in the interval from

$$p_{p(||)} = -(MQ/p_0) + (p_0/2) \quad (17)$$

to

$$p_{p(||)} + \Delta p_{p(||)} = -(MQ/p_0) + (p_0/2) + (M\Delta E/p_0). \quad (18)$$

The shape of the region allowed by this condition and (a) and (b) is shown by Figs. 2 and 3. We have to deal with a circular ring when the excitation is small, i.e., so long as the absolute value of the component $p_{N(||)} = p_{p(||)} - p_p$ of the momentum of the final state of the nucleon is less than the critical radius, P, of the sphere of occupied states in momentum space. This condition is satisfied for all values of Q from zero up to the limit, $Q=Q^*$, given by the implicit equation

$$(MQ^*/p_0^*)+(p_0^*/2)=P. \quad (19)$$

Then the volume of the ring-shaped slice is

$$\begin{aligned} S\Delta p_{p(\|)} &= \pi(p_{p(\perp)}^2 - p_{N(\perp)}^2)\Delta p_p \\ &= \pi(p_N^2 - p_{p(\|)}^2)\Delta p_{p(\|)} \\ &= \pi p_0(2MQ/p_0)(M\Delta E/p_0). \end{aligned} \quad (20)$$

When the value of the energy imparted to the nucleon exceeds the limit Q^* set by Eq. (19), the Pauli principle does not come into action. The slice in momentum space is a circular disk, of volume

$$\begin{aligned} \pi p_{p(\perp)}^2\Delta p_{p(\|)} &= \pi(P^2 - p_{p(\|)}^2)\Delta p_{p(\|)} \\ &= \pi\{P^2 - [(MQ/p_0)-(p_0/2)]^2\}(M\Delta E/p_0). \end{aligned} \quad (21)$$

The upper limit, $Q=Q_{\max}$ to the possible energy transfer is obtained when the recoil momentum, p_0, is imparted to a nucleon which was already moving with the maximum possible momentum, P, in the direction opposite to that of the emitted neutral meson. At this limit the radius of the circular disk in momentum space goes to zero (top diagram in Fig. 3) and expression (21) vanishes. Thus we have as implicit means to determine $Q_{\max}$ the equation

$$(MQ_{\max}/p_{0\min})-(p_{0\min}/2)=P. \quad (22)$$

Our result for the transition probability can be put into the form

$$\begin{aligned} dA/(dQ/mc^2) &= Z^4(3g^2/8\pi^2\hbar mc^2)(mc/\hbar)^6(\mu/m)^3 \\ &\quad \times(M/m)(1/137)^3(mc/P)^3 I(Q/mc^2), \end{aligned} \quad (23)$$

where we have for the dimensionless quantity I the expression $I(Q/mc^2)=m^{-3}c^{-4}(\mu c^2-Q)$ times

$$\begin{cases} 2MQ \text{ for } 0\leqslant Q\leqslant Q^* \\ P^2-[(MQ/p_0)-(p_0/2)]^2 \quad \text{for } Q^*\leqslant Q\leqslant Q_{\max} \end{cases} \quad (24)$$

The probability distribution of nuclear excitations to be expected on the present model is proportional to the quantity I, plotted as a function of Q in Fig. 4, for $\mu=200$ m and 210 m, and for $\mu_0=0$ m and 80 m.

The absolute value of the calculated transition probability is seen to be proportional to Z^4 for all nuclei. This result depends, of course, upon the assumption that the μ-meson moves in its K-orbit under the influence of a hydrogenic field of force. This assumption will be appreciably in error for atomic numbers greater than $Z\sim 15$. For these and heavier elements Z^4 in Eq. (23) is to be replaced by Z^4_{eff}, where Z_{eff} has been calculated by Wheeler.[12]

DIPOLE TRANSITION MODEL

In the treatment of the mesonic charge-exchange reaction just given, the wave-length of the outgoing meson was treated as small compared to the size of the nucleus. Actually this wave-length—or more significantly, the reduced wave-length, $\lambdabar=\lambda/2\pi$, which represents the classical distance of closest approach of a particle with one quantum unit, $\hbar$, of angular momentum—is of the order of $\hbar/150mc\sim 3\times 10^{-13}$ cm. Thus $\lambdabar$ is comparable with the dimensions of actual nuclei, just the most difficult case about which to attempt to draw general conclusions. In order to have another simple limiting treatment to bracket real nuclei we therefore consider the idealized—and in actuality, of course, unrealizable—case where the nucleus is significantly smaller than the wave-length of the neutral meson, but still large enough to justify a statistical treatment. Then in the evaluation of the matrix element of the fundamental interaction we expand the wave function of the neutral meson in powers of the displacement; we get a series,

$$1+i\hbar^{-1}\mathbf{p}_0\cdot\mathbf{r}-0.5\hbar^{-2}(\mathbf{p}_0\cdot\mathbf{r})^2+\cdots,$$

in which the first term gives no contribution because of the assumed orthogonality of the wave functions of occupied proton states and unoccupied neutron states. The third and higher terms we neglect because they contain the neutral meson wave-length to higher powers in the denominator than does the second term. Only this term, the dipole contribution, do we consider.

For the probability per second, dA, for a mesonic charge-exchange reaction which imparts to the nucleus an excitation between Q and $Q+dQ$, or—equivalently—gives to the outgoing meson a momentum between p_0 and p_0+dp_0,

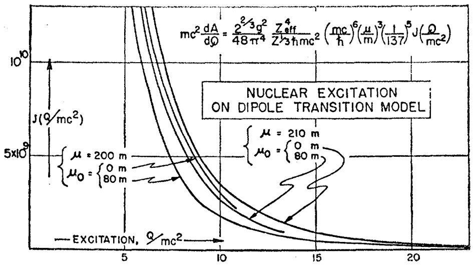


FIG. 5. Distribution of excitation energies expected on the model in which the only term considered in the power series development of the neutral meson wave function is that which is linear in the displacement. See discussion in text. In the formula at the top of the diagram a factor 4 should be inserted on the right-hand side as in Eq. (30).

we now have the expression

$$dA = (2\pi/\hbar)(4\pi p_0^2 dp_0/h^3)\times$$

$$\pi^{-1}(\text{Bohr radius for original meson})^{-3}(p_0/\hbar)^2\times$$

$$(g^2/2)2(\Delta E)^{-1}\sum{}^{(\Delta E)}\{|x_{PN}|^2+|y_{PN}|^2+|z_{PN}|^2\}/3. \quad (25)$$

Here the factor one-half in $g^2/2$ comes about for a given proton by averaging over its spin and the spin of the μ^--meson, and summing over the spins of neutron and neutral meson. The factor 2 following $(g^2/2)$ arises from the existence of two protons in each state assignable to a point particle, for only over such states do we propose to sum in performing the operations $\sum{}^{(\Delta E)}$.

The increase in energy of the nucleon in the act of absorption will be associated with a change in momentum. This momentum will be provided, not by the neutral meson with its long wavelength, but by the rest of the nucleus. In terms of the individual particle model, the momentum of the nucleon is provided by the wall of the potential well. Thus we expect that the rate of reaction—as evaluated in our idealization of pure dipole transitions—will be directly related to the surface area of the nucleus, not, for example, to the $\frac{2}{3}$ power of the nuclear volume. To bring this point into evidence and to simplify the computations, we treat the otherwise hardly reasonable case of a potential well which has the form of a rectangular prism, with side lengths A, B, C, thus

$$\psi_P = 2^{\frac{3}{2}}(ABC)^{-\frac{1}{2}}\sin(j\pi x/A)\sin(k\pi y/B)\sin(l\pi z/c),$$

where j, k, and l are integers.

The matrix element of x between two such standing waves, $\psi(jkl)$ and $\psi(j'k'l')$, will vanish unless $k=k'$ and $l=l'$. In this case the matrix element is

$$-(2A/\pi^2)\{(j-j')^{-2}-(j+j')^{-2}\} \quad (26)$$

if $j-j'$ is odd, and is otherwise zero. In order to satisfy the principle of energy conservation, we must have

$$Q=\mu c^2-E(\mu_0)\leqslant(\hbar^2\pi^2/2MA^2)(j'^2-j^2) \ll \mu c^2-E(\mu_0)+\Delta E=Q+\Delta E. \quad (27)$$

Moreover, the initial and final momenta must lie, respectively, inside and outside the sphere of occupied states in momentum space:

$$\hbar^2\pi^2[(j'^2/A^2)+(k'^2/B^2)+(l'^2/C^2)]\geqslant P^2 \geqslant \hbar^2\pi^2[(j^2/A^2)+(k^2/B^2)+(l^2/C^2)]. \quad (28)$$

For a given value of j, of j', and therefore of Q, the number of values of k and l permitted by these conditions is

$$(MBC/2\pi\hbar^2)\{Q \text{ or } P^2/2M-\pi^2k^2j^2/A^2, \text{ whichever is smaller}\}. \quad (29)$$

We sum over the values of j and j' consistent with condition (28), using as variable of integration the quantity θ defined by:

$$j^2\pi^2\hbar^2/2MA^2=Q\sinh^2\theta,$$
$$j'^2\pi^2\hbar^2/2MA^2=Q\cosh^2\theta.$$

We let the summation run over both odd and even values of the two integers, j and j', consequently using for average square of the matrix element only $\frac{1}{2}$ the square of (26), and then replace the summation by an integration. Proceeding in this way, we obtain finally for probability per second of a transition to the interval of excitation dQ the result:

$$dA/(dQ/mc^2)=4(2^{\frac{2}{3}}Z^4{}_{\text{eff}}/48\pi^4Z^{\frac{1}{3}})(g^2/\hbar mc^2) \times(mc/\hbar)^6(\mu/m)^3(e^2/\hbar c)^5 J(Q/mc^2), \quad (30)$$

where the dimensionless quantity J has the expression:

$$J=\begin{cases}\varphi(\theta_1)-\varphi(\theta_0) & \text{if} \quad Q\leqslant P^2/2M \\ \varphi(\theta_1) & \text{if} \quad Q\geqslant P^2/2M\end{cases} \quad (31)$$

where

$$\varphi(\theta)=(3/32)\sinh2\theta+(1/96)\sinh6\theta-(\theta/4)\cosh2\theta$$

and

$$\sinh^2\theta_1=\cosh^2\theta_0=P^2/2MQ.$$

The results given by formula (31) are plotted in Fig. 5 for two values of the μ-mass and two values of the μ_0-mass. Naturally the rise of the transition probability at low excitations cannot continue indefinitely. It is easy to derive a sum rule to prove this point. About the cut-off of the theoretical curves, and the conclusions to be drawn from them, we refer to the discussion in the beginning of the paper and in the caption of Fig. 1. Here we only note that the probability diverges at low energies because there the integration fails to take properly into account the finite level spacing.

FREE PARTICLE TREATMENT FOR OXYGEN WITHOUT LIMITATIONS ON WAVE-LENGTH OF NEUTRAL MESON

In contrast to the two idealized limits so far considered of a neutral meson wave-length large or small compared to nuclear dimensions, we have for actual nuclei the much more complicated case of intermediate wave-lengths. A general treatment being apparently infeasible, we consider here—in line with a suggestion made by Professor R. F. Christy at the Pasadena meeting—the case of oxygen. This element is especially important because it lies in the region of atomic numbers where the probability of mesonic reaction with the nucleus is relatively well known, and it is especially convenient to treat because on the model of free particles in a potential well the 1*s* and 2*p* shells are completely filled and all other states are empty. Moreover, with a reasonable choice of the radius and depth of the potential well there will be no more bound states, so that in this idealization all charge-exchange reactions must lead to the continuous spectrum. Of course under these circumstances we correlate the nuclear excitation Q with the difference between the final energy in the continuum and the original energy in the bound state.

The elementary contribution to the transition probability comes from a jump of a nucleon between a proton state, P, of definite angular momentum quantum numbers L_P and m_P, and a neutron state, N, with quantum numbers L_N and m_N. Of course the given proton state will contain two protons. Summing, therefore, over both spin directions for proton as well as over both orientations of neutron and neutral meson

TABLE I. Values of factor T_{PNL} defined in (37). Derived from related quantities in Condon and Shortley's *The Theory of Atomic Spectra*, p. 175. T vanishes when $L+L_N+L_P$ is odd.

Least of L, L_N, and L_P	Median of L, L_N, and L_P	Greatest of L, L_N, and L_P	T_{PNL}
0	0	0	1
0	1	1	3
0	2	2	5
1	1	2	6
0	3	3	7
1	2	3	9
2	2	2	50/7
0	4	4	9
1	3	4	12
2	2	4	90/7
2	3	3	28/3
2	4	4	900/77

spin, and averaging over spin possibilities for the original meson, we have for this elementary transition probability the expression

$$(2\pi/\hbar)(1/\Delta E)(g^2/2)2\sum{}^{(\Delta E)}$$
$$\left|\int \psi_{\mu_0}{}^*\psi_N{}^*\psi_P\psi_\mu d(\text{volume})\right|^2. \quad (32)$$

To reduce this expression to a form convenient for calculation, we introduce the following notations:

(a) We write the wave function for the neutral meson in the form

$$\psi_{\mu_0}=(2/B)^{\frac{1}{2}}r^{-1}f_L(r)Y_{Lm}(\theta,\phi), \quad (33)$$

where B is the radius of the very large spherical container over which ψ_{μ_0} is normalized, $f(r)$ is a function which behaves asymptotically for large r as a sine wave of unit amplitude, and which is tabulated for a reasonable range of arguments,[17] and Y_{Lm} is a spherical harmonic normalized so that $\int|Y_{Lm}|^2\sin\theta d\theta d\phi=1$, and with indices which represent the angular quantum numbers of the emergent neutral meson. The number of neutral meson states of the given L and m per range of energy ΔE is

$$(\Delta E)^{-1}(2B\Delta p_0/h)=2BE_0/c^2p_0h.$$

[17] F. L. Yost, J. A. Wheeler, and G. Breit, Terr. Mag. and Atm. Elec. 443 (December 1935), give the function f directly; the function $f(r)/r$ is given by *Tables of Spherical Bessel Functions* (Columbia University Press, New York, 1947).

TABLE II. Phase shifts in radians (modulo π) for neutron wave functions for idealized model of oxygen nucleus for states of angular momentum up to $L=4$.

Kinetic energy of neutron	δ_0	δ_1	δ_2	δ_3
3.8 Mev	3.30	2.67	2.87	0.11
9.0	3.04	2.20	2.49	0.23
14.3	2.72	1.95	2.18	1.40
19.6	2.42	1.85	1.92	1.83
30.1	1.95	1.89	1.57	1.72
40.7	1.63	1.83	1.39	1.44
51.2	1.46	1.66	1.38	1.30

(b) The proton wave function is written

$$\psi_P=(1/R)^{\frac{1}{2}}r^{-1}g_P(r)\,Y_P(\theta,\phi), \qquad (34)$$

where R represents the radius of the nucleus and g is so normalized that

$$R^{-1}\int_0^\infty g^2(r)dr=1.$$

The subscript P on Y is an abbreviation for the two appropriate angular momentum quantum numbers.

(c) The wave function of the neutron would be written in a similar way if that particle were created in a bound state. Being in the continuum in the present applications, its wave function is written as

$$\psi_N=(2/B)^{\frac{1}{2}}r^{-1}h_N(r)\,Y_N(\theta,\phi). \qquad (35)$$

Here the symbols have their previous meanings.

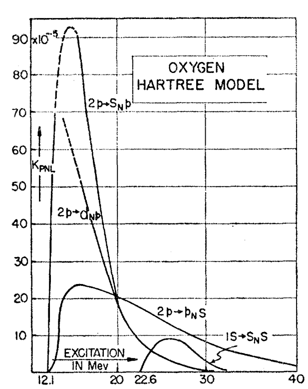


FIG. 6. Distribution of nuclear excitation expected on Hartree individual particle type of treatment for oxygen. Transition probabilities dA/dQ plotted in dimensionless units K. The triplet of labels on each curve refer, respectively, to the bound state of the proton before the reaction and the continuous energy wave functions of neutron and neutral meson afterwards.

The function $h_N(r)$ is so normalized that it behaves at infinity as a sine wave of unit amplitude. The number of neutron states of given L_P and m_P in the interval of excitation energy dQ is $2BMdQ/hP_N$.

(d) For the wave function of the original meson we shall take the usual hydrogenic expression for the K level,

$$\psi_\mu=\pi^{-\frac{1}{2}}(\mu Ze^2/\hbar^2)^{\frac{3}{2}}\exp(-\mu Ze^2r/\hbar^2). \qquad (36)$$

(e) In the integration over angles we have to deal with an expression which we may write in the form

$$T_{PNL}=4\pi\sum_{m_P,\,m_N,\,m}\left|\int Y_P Y_N Y_{Lm}d\Omega\right|^2. \qquad (37)$$

Here the factor 4π has been introduced purely for convenience of definition. The sum over magnetic quantum numbers will allow us to obtain the transition probability summed over all protons in the shell in question (Table I).

Combining factors in (32), we find the probability per second, dA, of a reaction process in which the nucleonic system receives an increment in energy between Q and $Q+dQ$ via disappearance of a proton from a given shell and appearance of a neutron of angular momentum L_N and a neutral meson of angular momentum L:

$$dA_{PNL}/d(Q/mc^2)=(2/\pi^3)(g^2/\hbar mc^2)(mc/\hbar)^6 \times(Ze^2/\hbar c)^3(\mu/m)^3\cdot(\hbar/mcR)(M/m) \times K_{PNL}(Q/mc^2). \qquad (38)$$

Here the dimensionless factor, K_{PNL}, has the value

$$(E_0/cp_0)(mc/p_N)\left\{\int f_L g_P h_N \times\exp(-\mu Ze^2r/\hbar^2)r^{-1}dr\right\}^2 T_{PNL}. \qquad (39)$$

In the numerical calculations we took for the nuclear radius $(e^2/2mc^2)(16)^{\frac{1}{3}}=3.53\times10^{-13}$ cm. We determined the depth of the well in such a way that a $3d$ level just failed to come into existence. The depth of this well, $65mc^2$, or 33 Mev, is such that the calculated energy to remove a proton from the $2p$ level is 12.05 Mev, as compared to the value 12.11 Mev for the $O^{16}-N^{15}$ energy difference. The corresponding energy calculated for the $1s$ level is 22.6 Mev.

The wave functions for the neutron were obtained by fitting the usual oscillatory solution inside the potential well onto an oscillatory solution of longer wave-length outside. The shift of the asymptotic phase of the external wave compared to its value for zero field of force is listed in Table II.

The proton wave functions were obtained in a similar way. The necessary matrix elements were computed by numerical integration.

The elementary portions of the transition probability are plotted in Fig. 6. The totalized transition probability is shown in Fig. 1, for comparison with the results of the free particle model and the dipole transition model. The two thresholds evident in the picture are due to transitions out of the $2p$ and $1s$ levels, respectively. The curve in Fig. 6 for the transition

$$2p(\text{proton})\rightarrow d(\text{neutron}) + p(\text{neutral meson})$$

was corrected at low energies, as discussed earlier, before being combined with the other curves to give the totalized curve presented in Fig. 1.

ACKNOWLEDGMENT

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