Energy Spectrum of Electrons from Meson Decay*

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THE THREE-PARTICLE DECAY HYPOTHESIS

Observations on the energy of electrons given off in the decay of ordinary sea level or μ-mesons (Table I) are inconsistent with the assumption of a two-particle decay process such as either

$$\mu \text{ (meson)} \rightarrow \nu \text{ (neutrino)} + e \text{ (electron)},$$

or

$$\mu \text{ (meson)} \rightarrow h\nu \text{ (photon)} + e \text{ (electron)},$$

or

$$\mu \text{ (meson)} \rightarrow \mu_0 \text{ (neutral meson)} + e \text{ (electron)}.$$

Consequently, it has been proposed[1] to consider μ-decay as a three-particle process of the type

$$\mu \rightarrow \mu_0 + \nu + e,$$

or, as a special case of this type,

$$\mu \rightarrow 2\nu + e.$$

Triple electron emission—a process which would also conserve charge—is clearly excluded by the cloud-chamber evidence; and emission of even a single photon is inconsistent with recent observations.[2] We wish here to report calculations of the shape of the energy spectrum to be expected on the hypothesis of decay into an electron and two neutral particles.[2a]

In treating the decay process we shall consider the μ and μ_0-mesons to have spin $\frac{1}{2}$:

(a) This spin assignment is certainly correct if the particle μ_0 is to be considered to be a neutrino.

(b) The value 1 for the spin of the μ-meson would lead to a rate of production of bursts by fast mesons which is inconsistent with experiment.[3]

(c) We cannot exclude spin zero. However, the development of usual meson theory—as well as of electromagnetic theory—shows that a $\mu-\mu_0$ field of spin zero, coupled *linearly* with nucleons, will permit reactions in disagreement with experiment. Thus a μ-meson which disappears by reaction with an atomic nucleus should in this case give up all its mass energy to the nucleus and so produce a star comparable in size with those produced by heavier mesons. But production of substantial stars by μ-mesons occurs rarely, if at all.[4] Of course there exist other possibilities—one cannot exclude a spin zero μ-meson so coupled with nucleons that a neutral μ_0-meson has necessarily to come off after reaction, leaving only a small excitation on the nucleus.[4a]

(d) Finally, Klein has noted[5] that there exists on the three-particle decay hypothesis a close phenomenological connection between the process of $e-\nu$-emission by mesons and by nuclei. Thus, not only are the light particles given off the same, but also even the coupling constants required in the two cases are of the same order of magnitude. In a certain sense, the $\mu \rightarrow \mu_0$-transition can be regarded as belonging to the same family as the $n \rightarrow p$ decay, the $H^3 \rightarrow He^3$ transition, etc. In view of this correspondence Klein points out it is quite natural to suppose that μ and μ_0 have the same half-integral spin which characterizes the neutron and the proton.

In accordance with the foregoing considerations we adopt the value $\frac{1}{2}$ for the spin of the μ- and μ_0-mesons, without, however, regarding the value 0 as definitely excluded by present evidence. Thus we treat all the particles involved in the decay process as Dirac particles, satisfying the Dirac wave equation.

* This article forms an extension of consideration presented by J. A. Wheeler under the same heading at the Pasadena Conference.

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[1] C. Møller as reported by C. Marty and J. Prentki, J. de phys. et rad. No. 4, 147 (1948). See also, R. Marshak, Phys. Rev. **73**, 1226 (1948).

[2] E. P. Hincks and B. Pontecorvo, Phys. Rev. **73**, 257 (1948); R. Sard and E. J. Althaus, Phys. Rev. **73**, 1251 (1948); O. Piccioni, Phys. Rev., **74**, 1754 (1948).

[2a] See also the paper of J. J. Horowitz, O. Kofoed-Hansen, and J. Lindhard, Phys. Rev. **74**, 713 (1948), which appeared after the present article had been submitted for publication.

[3] R. F. Christy and S. Kusaka, Phys. Rev. **59**, 414 (1941); R. E. Lapp, Phys. Rev. **64**, 255 (1943); S. Belenky, J. Phys. U.S.S.R. **10**, 144 (1946).

[4] Unpublished observations of the Berkeley group; also Brown, Camerini, Fowler, Muirhead, Powell, and Ritson, Nature (in press); and W. Y. Chang, Rev. Mod. Phys. **21**, 166 (1948).

[4a] A. S. Lodge, Nature **161**, 809 (1948).

[5] O. Klein, Nature **161**, 897 (1948).

COUPLING BETWEEN MESON AND LIGHT PARTICLE FIELD

For form of the coupling between $\mu-\mu_0$, on the one hand, and $\nu-e$ on the other we follow the considerations of beta-decay theory, because of the apparent analogy between meson decay and beta-decay. Specifically, we describe the coupling in terms of the wave functions themselves, and exclude their derivatives. Thus we shall consider nothing in the nature of a Konopinski-Uhlenbeck theory of beta-decay. Moreover, of coupling theories which contain only the wave function itself, we shall consider only the five simple forms—scalar, pseudoscalar, vector, pseudovector, and tensor—although there is nothing in principle to exclude arbitrary linear combinations of these possibilities.

The choice of form of coupling makes a difference in the shape of the spectrum of the decay electrons in the present case, unlike the case of beta-decay. This effect is a relativistic one and arises because the neutral meson has a small or zero mass and a recoil velocity comparable to or equal to the velocity of light. Because of this effect we have to include all the terms in each form of coupling theory and cannot omit all but a single big component as in beta-theory.

In the mathematical treatment of meson decay, as of beta-decay, it is convenient to adopt a description in which the process appears as the disappearance of two particles and the reappearance of two other particles. Specifically, we may represent the process $\mu^+\rightarrow\mu_0+e^++\nu$ in the form $\mu^++\bar{e}$ (electron in negative energy state)$\rightarrow\mu_0+\nu$. Similarly, we represent the process $\mu^-\rightarrow\mu_0+e+\nu$ in the form $\bar{\mu}_0+\bar{\nu}\rightarrow\bar{\mu}^++e$, where a bar represents a particle in a negative energy state.

Thus we consider as the fundamental positive energy particles the entities μ^+ and e, and treat the positon and the particle μ^- as holes. Of course many alternative choices are possible, in which we adopt other conventions as to which particles are fundamental, and which are holes: but all these conventions may be shown to lead ultimately to the same results.

The perturbations in the Hamiltonian in the five coupling cases are:

$$H_S=g_S\beta_H\beta_L(\tau_H\tau_L+\tau_H^*\tau_L^*)\delta(\mathbf{x}_H-\mathbf{x}_L), \quad (1)$$

$$H_V=g_V(1-\boldsymbol{\alpha}_H\cdot\boldsymbol{\alpha}_L)(\tau_H\tau_L+\tau_H^*\tau_L^*)\times\delta(\mathbf{x}_H-\mathbf{x}_L), \quad (2)$$

$$H_T=3^{-\frac{1}{2}}g_T(\beta_H\boldsymbol{\sigma}_H\cdot\beta_L\boldsymbol{\sigma}_L+\beta_H\boldsymbol{\alpha}_H\cdot\beta_L\boldsymbol{\alpha}_L)\times(\tau_H\tau_L+\tau_H^*\tau_L^*)\delta(\mathbf{x}_H-\mathbf{x}_L), \quad (3)$$

$$H_{PV}=3^{-\frac{1}{2}}g_{PV}(\boldsymbol{\sigma}_H\cdot\boldsymbol{\sigma}_L-\gamma_H{}^5\gamma_L{}^5)\times(\tau_H\tau_L+\tau_H^*\tau_L^*)\delta(\mathbf{x}_H-\mathbf{x}_L), \quad (4)$$

$$H_{PS}=g_{PS}(\beta_H\gamma_H{}^5\beta_L\gamma_L{}^5)\times(\tau_H\tau_L+\tau_H^*\tau_L^*)\delta(\mathbf{x}_H-\mathbf{x}_L). \quad (5)$$

Here the subscripts H and L refer to the particles, μ and μ_0, and e and ν, respectively. The operators

TABLE I. Preliminary experimental indications of electrons emitted in specified energy bands. In addition to these results are the following: Conversi, Pancini, and Piccioni, Phys. Rev. **71**, 209 (1947), estimate by absorption measurements an average decay electron energy of about 45 Mev; Retallack, Phys. Rev. **73**, 921 (1948), finds 31 percent of stopped positive mesons eject electrons from his foils, as compared to 52 percent calculated for 50-Mev energy, 16 percent calculated for 25 Mev; B. Rossi, Rev. Mod. Phys. **20**, 537 (1948); Fig. 19 gives data from which it would tentatively follow that the decay electron takes on the average perhaps 30 percent of the rest energy of the meson (or about 30 Mev); M. H. Shamos and A. Russek, Phys. Rev., **74** 1546 (1948), by absorption measurements conclude spectrum consistent with range of energies 20 Mev to 45 Mev; see also E. P. Hinks and B. Pontecorvo, Phys. Rev. **74**, 697 (1948).

Observer	0–20 Mev	20–40 Mev	40 Mev or more
Steinberger, absorption measurements, Phys. Rev., in press, together with unpublished corrections for which we are indebted to him for a personal communication; see also Phys. Rev. **74**, 500 (1948)	~23%	~42%	~35%
R. W. Thompson,* Phys. Rev. **74**, 490 (1948)	0 to 1	0 to 10	0 to 10
Anderson, Adams, Lloyd, and Rau, Phys. Rev. **72**, 724 (1947) and Rev. Mod. Phys. **20**, 334 (1948)		$1\frac{1}{2}$	
Fowler, Cool, and Street, Phys. Rev. **74**, 101 (1948)	1		
Brown, Camerini, Fowler, Muirhead, Powell, and Ritson, Nature (in press)	0	1 to 3	2 to 4
Wilson and Roberts, Nature **145**, 102 (1940)	0	0 or 1	1 or 0
Wang and Jones, Phys. Rev. **74**, 1547 (1948)	0	0	1
Zar, Hershkowitz, and Berezin, Phys. Rev. **74**, 111 (1948)	1 to 2	0 to 1	1
Fletcher and Forster, Phys. Rev. **75**, 204 (1949)	0	1	0

* In each energy interval is listed, not the most probable number of tracks, but the extreme values of the numbers of tracks permitted by Thompson's published uncertainties of measurement.

τ_L and $\tau_L{}^*$ change an electron into a neutrino, and conversely. The operators τ_H and $\tau_H{}^*$ similarly produce the changes $\mu^+\rightarrow\mu_0$ and $\mu_0\rightarrow\mu^+$ in the case of meson decay, while in the case of nucleon decay they bring about the transitions proton→neutron, and the reverse. The symbols α and β denote the usual Dirac operators, out of which may be built up such combinations as $\sigma_x=i\alpha_y\alpha_z$ and $\gamma^5=i\alpha_x\alpha_y\alpha_z$. The quantities g will hereafter be called the coupling constants. They have the dimensionality erg cm^3. The factor $3^{-\frac{1}{2}}$ is introduced in the tensor and pseudovector cases in order to give the same value, g_β, to the coupling constant of ordinary beta-theory in all the four cases: S, V, T, and PV.

TRANSITION PROBABILITIES WHEN μ_0 IS NOT A NEUTRINO

From standard quantum-mechanical perturbation theory we find the elementary transition probability in the form:

$$dW=\left\{\begin{array}{l}\text{probability per second of a } \mu\text{-decay}\\ \text{process in which the electron}\\ \text{emerges in the momentum inter-}\\ \text{val } p_e \text{ to } p_e+dp_e\end{array}\right\}$$

$$=(2\pi/\hbar)(4\pi p_e{}^2dp_e/h^6)(1/dE)\times\int_{dE} Idp_{\nu x}dp_{\nu y}dp_{\nu z}. \quad (6)$$

Here the integral is taken over that shell of volume in the momentum space of the neutrino which is consistent with equation

$$\mu c^2\leqslant E\nu+E_e+E\mu_0\leqslant\mu c^2+dE, \quad (7)$$

where the energy or momentum of μ_0 is calculated from the momentum of the electron and the neutrino by the law of conservation of momentum. The quantity I represents the square of the matrix element for the transition, averaged over the spin of the original meson and summed over all directions of spin of the three products of decay:

$$I_S=g_S{}^2(E_0+\mu_0c^2)(1-cp_e\cos\theta/E_e)/2E_0, \quad (8)$$

$$I_V=g_V{}^2[2-\mu_0c^2(1-cp_e\cos\theta)/E_0-cp_0\cos(0,\nu)/E_0-c^2p_0p_e\cos(0,e)/E_0E_e], \quad (9)$$

$$I_T=g_T{}^2[1+cp_e\cos\theta/3E_e-2cp_0\cos(0,\nu)/3E_0-2c^2p_0p_e\cos(0,e)/3E_0E_e], \quad (10)$$

$$I_{PV}=g_{PV}{}^2[\tfrac{2}{3}+\mu_0c^2(1-cp_e\cos\theta/E_e)/3E_0-(cp_0/3E_0)\{\cos(0,\nu)+cp_e\cos(0,e)/E_e\}], \quad (11)$$

$$I_{PS}=g_{PS}{}^2(E_0-\mu_0c^2)(1-cp_e\cos\theta/E_e)/2E_0. \quad (12)$$

Here μ_0 is the mass of the neutral meson and $(0, e)$ and $(0, \nu)$ the angles made by its direction with that of the electron and neutrino, respectively. θ is the electron-neutrino angle. The integration over all values of the neutrino momentum consistent with a given value of the electron momentum is described in the appendix. We find for the elementary transition probability, (6), the result

$$dW=(dp_e/mc)(4\pi^3/6)(g^2/hmc^2)(mc/h)^6f(p_e). \quad (13)$$

Here the dimensionless factor $f(p_e)$ has the form

$$f(p_e)=(p_e{}^2/m^2c^2)(\mu^2c^4-2\mu c^2E_e+m^2c^4-\mu_0{}^2c^4)^2(\mu^2c^4-2\mu c^2E_e+m^2c^4)^{-3} \quad (13a)$$

times the appropriate one of the following expressions:

$$\begin{aligned}K_S=&[(\mu^2c^2-2\mu E_e+m^2c^2)/(2m^2c^2)]\\&\times[3\mu^2c^4-6\mu c^2E_e+2E_e{}^2+2(\mu c^2-E_e)p_ec+m^2c^4]\\&+(\mu_0{}^2/2m^2)[3\mu^2c^4-6\mu c^2E_e+4E_e{}^2+4(\mu c^2-E_e)p_ec+6m^2\mu^2c^4/\mu_0{}^2\\&-m^2c^4+6m^4c^6/\mu_0E_e];\end{aligned} \quad (14)$$

$$\begin{aligned}K_V=&[(\mu^2c^2-2\mu E_e+m^2c^2)/(m^2c^2)]\\&\times[9\mu^2c^4-16\mu c^2E_e+9m^2c^4-2m^2\mu c^6/E_e]+(\mu_0{}^2/m^2)[3\mu^2c^4\\&+2\mu c^2E_e+3m^2c^4(1-2\mu/\mu_0)-(2m^2c^6/E_e)(2\mu-3m^2/\mu_0)];\end{aligned} \quad (15)$$

$$\begin{aligned}K_T=&[(\mu^2c^2-2\mu E_e+m^2c^2)/3m^2c^2]\\&\times[15\mu^2c^4-26\mu c^2E_e-2E_e{}^2-2(\mu c^2-E_e)p_ec+17m^2c^4-4m^2\mu c^6/E_e]\\&+(\mu_0{}^2/3m^2)[3\mu^2c^4+2\mu c^2E_e-4E_e{}^2-4(\mu c^2-E_e)p_ec+7m^2c^4-8m^2\mu c^6/E_e];\end{aligned} \quad (16)$$

$$K_{PV}=\tfrac{1}{3}[K_V+12\mu_0c^4(\mu+m^2c^2/E_e)]; \quad (17)$$

$$K_{PS}=K_S-12\mu_0c^4(\mu+m^2c^2/E_e). \quad (18)$$

DECAY INTO TWO NEUTRINOS AND AN ELECTRON

There are several ways to describe the process $\mu\rightarrow e+2\nu$, even if one limits himself to a specific one of the coupling possibilities listed in (1)–(5). This situation contrasts with the case where the neutral meson had a finite mass, and where there was considered to be no question that μ made a transition into μ_0. In the present case the end products are all light particles, and there is no obvious way to decide whether μ goes to ν or to e. Moreover, it made no difference previously whether the same or different signs were assigned to the particles (μ and e) which for computational purposes were considered to be in positive energy states. Now it does matter. Associated with this difference in the formal description is an important physical difference: in one case meson decay produces a neutrino and an antineutrino. In the other case two neutrinos are created, and the antisymmetry of the wave function with respect to their coordinates has explicitly to be recognized.

When we consider the various alternatives just mentioned, we find that there exist many possible theories. However, each is equivalent to one or other of the possibilities of Table II.

In formulating these theories we use the same coupling terms which are listed in Eqs. (1)–(5). However, the subscript H now refers to the meson and the particle into which it is changed. The operators τ_H and τ_L bring about the transitions listed in the appropriate row of Table II. In the third or "charge retention" theory the operator τ_L is to be replaced by unity because we have only to describe the transition of a neutrino from a negative energy state to a positive energy state.

The calculations give again for the decay probability Eq. (6), where now the quantity I (obtained from the square of the matrix element) has the value:

Antisymmetrical charge exchange,

$$I_S=(g_S^2I_{PS}/g_{PS}^2)=(g_S^2/8)[3-\cos(\nu_1,\nu_2)+mc^2\{1-\cos(\nu_1\nu_2)\}/E_e-cp_e\{\cos(\nu_1e)+\cos(\nu_2e)\}/E_e], \quad (19)$$

$$I_T=(3g_T^2/2)[1-\cos(\nu_1\nu_2)], \quad (20)$$

$$I_V=3I_{PV}g_V/g_{PS}=g_V^2[3-2\cos(\nu_1\nu_2)+mc^2\{1-\cos(\nu_1\nu_2)\}/2E_e-cp_e\{\cos(\nu_1e)+\cos(\nu_2e)\}/2E_e]. \quad (21)$$

Simple charge exchange: change μ_0 to 0 in Eqs. (8)–(12).

TABLE II. The three theories of $\mu\rightarrow e+2\nu$; the symbolism used in the last item of the first column, for example, implies that μ^- goes to e, and simultaneously $\bar{\nu}$ goes to ν.

Description of $\mu\rightarrow e+2\nu$	Exchange of charge	Nature of neutral particles	Charges of positive energy particles	Name here used to describe theory	Spectrum in
$\begin{pmatrix}\mu^+\\ \bar{e}\end{pmatrix}\rightarrow\begin{pmatrix}\nu\\ \nu\end{pmatrix}$	Yes	Identical neutrinos	Opposite	"Antisymmetrical charge exchange"	Fig. 3
$\begin{pmatrix}\mu^-\\ \bar{\nu}\end{pmatrix}\rightarrow\begin{pmatrix}\nu\\ e\end{pmatrix}$	Yes	Neutrino anti-neutrino	Same	"Simple charge exchange"	Fig. 4
$\begin{pmatrix}\mu^-\\ \bar{\nu}\end{pmatrix}\rightarrow\begin{pmatrix}e\\ \nu\end{pmatrix}$	No	Neutrino anti-neutrino	Same	"Charge retention"	Fig. 5

Charge retention:

$$I_S=g_S^2(E_e+mc^2)[1-\cos(\nu_1\nu_2)]/2E_e, \quad (22)$$

$$I_V=g_V^2[2-mc^2\{1-\cos(\nu_1\nu_2)\}/E_e-cp_e\{\cos(e\nu_1)+\cos(e\nu_2)\}/E_e], \quad (23)$$

$$I_T=g_T^2[1+\cos(\nu_1\nu_2)/3-2cp_e\{\cos(e\nu_1)+\cos(e\nu_2)\}/3E_e], \quad (24)$$

$$I_{PV}=(g_{PV}^2/3)[2+mc^2\{1-\cos(\nu_1\nu_2)\}/E_e-cp_e\{\cos(e\nu_1)+\cos(e\nu_2)\}/E_e, \quad (23a)$$

$$I_{PS}=g_{PS}^2(E_e-mc^2)[1-\cos(\nu_1\nu_2)]/2E_e. \quad (22a)$$

The integration over all values of the momenta of the two neutrinos consistent with a given momentum of the electron goes through just as in the previous calculations, giving for spectral distribution of the decay rate Eq. (13), where now the dimensionless distribution factor $f(p_e)$ has the following values:

Antisymmetrical charge exchange,

$$m^2c^4f_S=m^2c^4f_{PS}=(1/4)[3(E-E_e)^2-c^2p_e^2]\times(2-mc^2/E_e)+(1/2)c^2p_e^2\times[cp_e(E-E_e)-E_e^2+mc^2E_e]/E_e^2, \quad (25)$$

$$m^2c^4f_V=3m^2c^4f_{PV}=[3(E-E_e)^2-c^2p_e^2]\times(5+2mc^2/3E_e)+2c^2p_e[cp_e(E-E_e)-4E_e^2-mc^2E_e/3]/E_e^2, \quad (26)$$

$$m^2c^4f_T=9[(E-E_e)^2-c^2p_e^2]. \quad (27)$$

Simple charge exchange, insert $\mu_0=0$ in Eqs. (13a) and (14) to (18). Charge retention,

$$m^2c^4f_S=3(1+mc^2/E_e)[(E-E_e)^2-c^2p_e^2], \quad (29)$$

$$m^2c^4f_V=6(E-E_e)^2-2c^2p_e^2+4(E-E_e)c^3p_e^3/E_e^2-6mc^2\{(E-E_e)^2-c^2p_e^2\}/E_e, \quad (30)$$

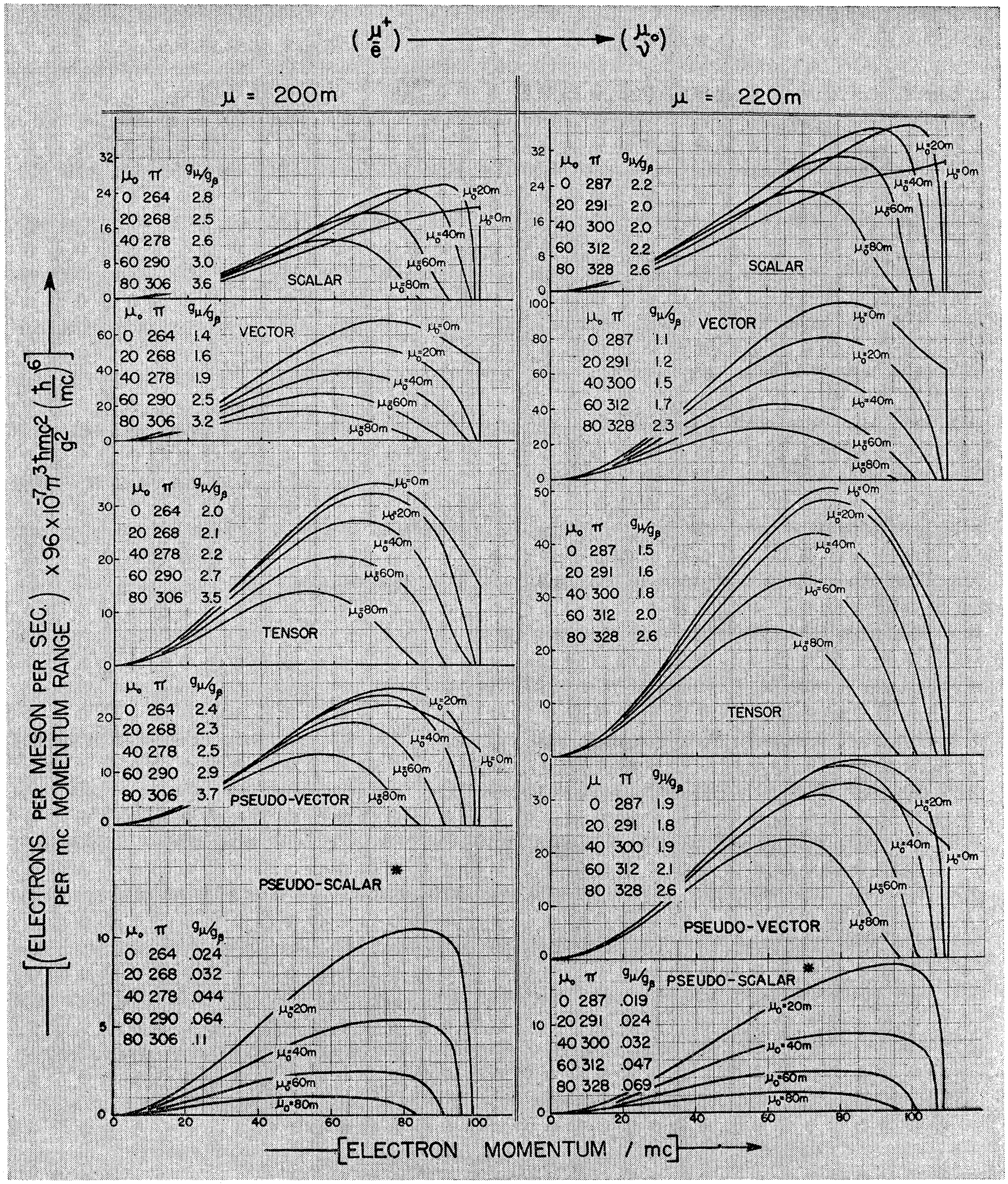


FIG. 1. Energy spectrum of electrons from μ-decay calculated on the three-particle hypothesis for two values of the meson mass (first column, $\mu=200$; second column, $\mu=220$) for several values of the mass of the neutral meson assumed to come off ($\mu_0=80$, 60, 40, 20, 0) and for several forms of the coupling. Note the displacement of the curves to higher energies when μ is increased from 200 to 220—also the substantial decrease in the required value, g_μ, of the coupling constant for μ-decay. Note also the strong influence of the mass of the neutral meson.

* In the case of pseudoscalar coupling the curve $\mu_0=0$ has been omitted through lack of space; it is identical with the curve for $\mu_0=0$ in the case of scalar coupling.

$$m^2c^4f_T=(2/3)[(E-E_e)^2+p_e^2+4(E-E_e)c^3p_e^3/E_e^2], \quad (31)$$

$$3m^2c^4f_{PV}=6(E-E_e)^2-2c^2p_e^2+4(E-E_e)c^3p_e^3/E_e+6mc^2\{(E-E_e)^2-c^2p_e^2\}/E_e, \quad (32)$$

$$m^2c^4f_{PS}=3(1-mc^2/E_e)[(E-E_e)^2-c^2p_e^2]. \quad (33)$$

EVALUATION OF DECAY SPECTRUM

The most recent determinations of the mass of the μ-meson[6] give values in the neighborhood of 210 times the electronic mass, with a considerable degree of accuracy. Consequently, in order to see the possible effect of changes in this mass value we have considered the two extremal values 200 and 220.

On the mass of the μ_0-meson an upper limit of the order of $100m$ appears to be set by the consistent observation of electrons with energies as high as 40 or 50 Mev. No lower limit is put on the mass by the experiments. Consequently, we have considered the cases $\mu_0=0$, 20, 40, 60, 80, (Fig. 1).

These values of the μ_0 mass were chosen quite independently of the phenomenon of decay of the π-meson to the μ-meson, where it is known that a single neutral particle is emitted.[7] However, it is quite conceivable that this particle is to be identified with the μ_0-meson.[1] In this case the mass of the μ_0 can be determined from the mass of π and μ and the value of the characteristic energy imparted to the μ-meson.[7] Earlier mass determinations spoke for a value of μ_0 of the order of 100,[8] but more recent observations[9] are not inconsistent with a value of 0 for the μ_0-mass.

Because of the possible correlation between π-decay and μ-decay, we have presented in all the diagrams the mass of the π-meson which would follow from the assumed masses of μ_0 and μ, taking $8mc^2$ for the characteristic energy of the μ- from π-decay.

[6] W. B. Fretter, Phys. Rev. **70**, 625 (1946); J. G. Retallack, Phys. Rev. **72**, 742 (1947); and especially R. Brode, report at Pasadena conference.

[7] Lattes, Muirhead, Occhialini, and Powell, Nature **159**, 456 and 694 (1947); C. M. G. Lattes, G. P. S. Occhialini, and C. F. Powell, Nature **160**, 486 (1947).

[8] C. M. G. Lattes, G. P. S. Occhialini, and C. F. Powell, Proc. Phys. Soc. **61**, 173 (1948).

[9] Goldschmidt-Clermont, King, Muirhead, and Ritson, Proc. Phys. Soc. **61**, 183 (1948). C. M. G. Lattes, paper presented at the Centenary meeting of the A. A. A. S., September 15, 1948. Powell and collaborators, reference 4.

DETERMINATION OF COUPLING CONSTANT

The lifetime of the μ-meson has been measured to be close to 2.15 μsec. On the other hand, the theoretical value of the lifetime for any given set of masses and any given form of theory depends uniquely upon the value of the appropriate coupling constant, g_μ. We have used this principle to determine in each of the cases shown in the diagrams the value of g_μ required to agree with experiment. For this purpose it was necessary to obtain graphically the area under each curve.

The values of the g_μ's so obtained are presented most meaningfully, not in absolute units, but relative to the constant, g_β, in the corresponding theory of β-decay, especially in view of the possible correlation of these two phenomena.

The absolute values of the β-decay constants themselves for each form of coupling theory (1)–(5) were determined by comparing the experimental value, around 30 minutes, for the half-life of the free neutron[10] with the value as computed from the following formula:

$$0.693/\text{half-life}=1/\tau_\beta=(32\pi^3g^2/\hbar mc^2)(mc/h)^6J. \quad (34)$$

Here the dimensionless quantity J has in the case of scalar, vector, pseudovector, and tensor theories the value[11]

$$J=(\omega^2-1)^{\frac{1}{2}}(2\omega^4-9\omega^2-8)/60+(\omega^2/4)\ln[\omega+(\omega^2-1)^{\frac{1}{2}}], \quad (35)$$

while in the pseudoscalar case, where the rate of decay depends upon the speed of recoil of the proton,

$$J_{PS}=(m/2M)[(5\omega^2+1)(\omega^2-1)^2/12-(6\omega^6-10\omega^4+4\omega)/15+\omega(\omega^2-1)^{\frac{1}{2}}\{7(\omega^2-1)^2/30-(\omega^2+1/6)(\omega^2-5/2)/4-(3/8)(\omega^2+1/6)\ln\{\omega+(\omega^2-1)^{\frac{1}{2}}\}]. \quad (36)$$

Here ω is equal to the difference in mass between neutron and electron-free proton, expressed in units of the electronic mass. Taking the mass difference as 1.257 ± 0.016 Mev,[12] corresponding to $\omega=2.46\pm0.03$, and using for the half-life of neutron the of course uncertain value of exactly

[10] A. Snell, Science **108**, 167 (1948).

[11] E. P. Wigner, Phys. Rev. **56**, 519 (1939).

[12] W. E. Stephens, Rev. Mod. Phys. **19**, 19 (1947).

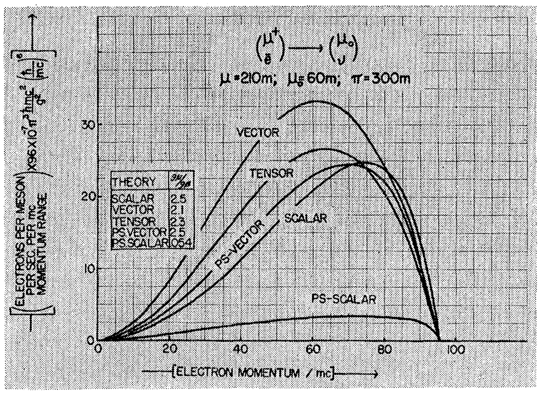


FIG. 2. Influence of form of coupling on shape of spectrum for fixed values of masses of μ- and μ_0-meson. Contrast this result with case of ordinary beta-decay, where the atomic nucleus has negligible velocity and the decay curves have the same shape in all five cases.

30 minutes, we have

$$g_\beta = 2.24\times 10^{-49} \text{ erg cm}^3 \tag{37}$$

for all except the pseudoscalar theory, and there

$$(g_\beta)_{PS} = 2.59\times 10^{-47} \text{ erg cm}^3. \tag{38}$$

Expressed in terms of the quantities usual in the analysis of beta-ray spectra, the values adopted here for the neutron (except in the pseudoscalar case) correspond to ft. $=$ (J as defined in 35) $\cdot$ (half-life in sec.) $= 1.35\times 1800 = 2440$.

These values of g_β were used with the values of g_μ obtained as previously described to determine the values of the coupling ratios, g_μ/g_β, listed in the diagrams. It is unlikely that these ratios will be significantly affected by new determinations of the life of the meson. However, a more accurate value for the neutron half-life is certainly to be expected. If it exceeds by the factor F, the 30-minute value assumed here, then all the ratios g_μ/g_β listed in the diagrams should be increased by the factor $F^{\frac{1}{2}}$.

We have chosen here neutron decay as the basis for determining g_β because it is free of the uncertainties of nuclear wave function computation which affect the analysis of beta-decay of heavier nuclei and because the measurement of the neutron life appears to be ultimately susceptible of reasonable precision.

DISCUSSION OF RESULTS AND COMPARISON WITH EXPERIMENTS

From the comparison of the curves shown in Fig. 1 we conclude: (a) For given μ_0-mass, an increase of the μ-meson mass raises both the upper limit of the spectrum and the energy of maximum intensity. The increase decreases the value of the coupling constant, g_μ, required to account for the 2.15-μsec. meson mean life. (b) For a given value of the μ-mass, and a given theory, a decrease in the μ_0-mass raises the upper limit of the spectrum and is especially effective in moving the maximum to higher energies. On the coupling constant there is not a large influence of the μ_0-mass over the range 0 to 20 m. For larger masses the coupling constant increases with increasing mass.

In Fig. 2 we considered the case $\mu = 210$ m, $\mu_0 = 60$ m, in order to compare the several theories. We observe the center of gravity of the spectrum shifts to higher energies in the progression: vector, tensor, pseudovector, pseudoscalar, and scalar.

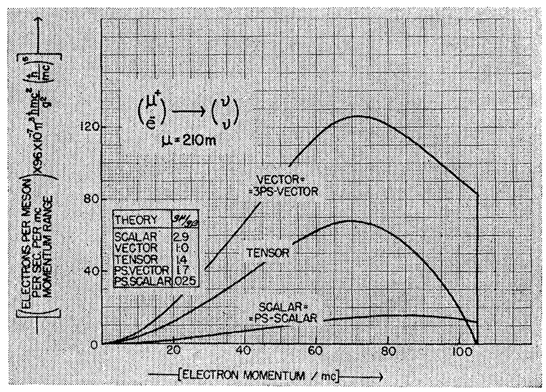


FIG. 3. The first of three simple forms of theory (see Figs. 4 and 5 for the other two) in which $\mu \rightarrow e + 2\nu$. This is the only one of the three in which there is recognized the antisymmetry of the final state with respect to the two neutrinos ("antisymmetric theory with charge exchange").

In Figs. 3–5 we compare the theories in which two neutrinos come off. Out of the nine curves shown on these three diagrams it is interesting to note that all but two remain finite at the upper energy limit. If it shall be determined that the μ_0 mass is zero, the determination of the behavior at the upper limit of the electron spectrum will therefore provide an important means to distinguish between two classes of theories.

Against the "charge retention" theory presented in Fig. 5 a possible objection can be raised. If the transition $\mu^- \rightarrow e^-$ can take place—via coupling with the neutrino field—simultaneously with the production of a neutrino pair, should not a similar process take place with comparable probability via coupling with the electron field to produce an electron pair? But such an interaction would result in a substantial fraction of decay processes of the form $\mu^+ \rightarrow e^- + 2e^+$, in contradiction with the available experimental evidence. Of course, in answer to this objection, the constant of coupling with the electron field can be made quite independent of the coupling with the neutrino field, and given a value small enough to agree with experiment on this point.

It is a remarkable feature of the spectra considered in all five figures, except for the pseudoscalar form of coupling, that they give coupling constants of the same order of magnitude as those of the corresponding beta-theory. (The difference

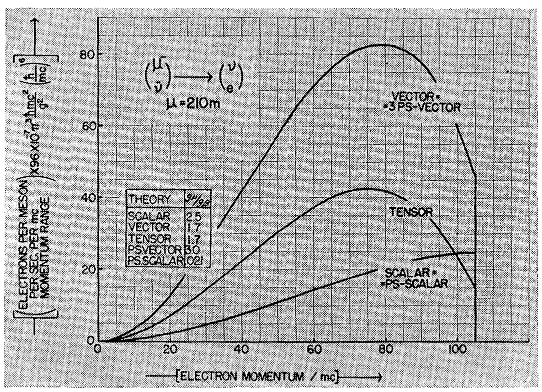


FIG. 4. Spectrum of $\mu \rightarrow e + 2\nu$ for theory in which meson goes to neutrino and gives its charge to a neutrino in a negative energy state, transforming that particle into an electron ("simple charge exchange theory"). There, as in Fig. 5, a meson of negative charge has been considered, solely for convenience of notation; the spectrum is the same for a positive meson. The coupling ratios listed in this table should be changed to read as follows: 2.5, *1.2*, 1.7, *2.2*, 0.021 (exactly as in Fig. 5).

in the pseudoscalar case comes, not from an abnormal value of g_μ, but from a value of g_β about 100 times that required for other forms of beta-theory.) This agreement in order of magnitude between the two g's is certainly a matter of more than chance, and possibly indicates that we have to deal, not with two different theories, but with one and the same theory. If we adopt this view—already advanced by Klein—as a working hypothesis, then we have a criterion to distinguish among the many otherwise acceptable spectra in the diagrams.

The criterion of identical coupling—*plus Snell's value for the neutron half-life*—speak for a zero or small mass (neutrino?) for the neutral meson. Of all the listed types of coupling, the only one generally consistent with a neutron life of 30 ± 15 min. is the vector theory. Only exception is the tensor form of the antisymmetrical neutrino theory with charge exchange, which is also allowed by these criteria. In this connection it should be recalled that the vector and tensor theories are the ones which give the most nearly satisfactory account of the main features of nuclear beta-decay.[13]

To discriminate between the various theories by way of measurements on the electron spectrum appears impossible at the present time, in view of the inadequate experimental evidence. Such evidence as there is, however (Table I), appears to be consistent within the errors of observation with *any* of the curves we have presented.

Determinations of the decay spectrum are difficult. Cloud-chamber observations are ordinarily affected with an experimental bias (a) in the selection of acceptable tracks and (b) in discriminating via counters against electron of high or low energy, both of which effects are hard to estimate. Determination of the spectrum by absorption measurements is likewise complicated—by the necessity of uncertain corrections for

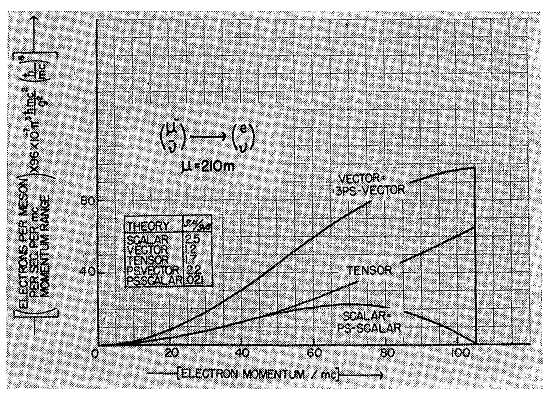


FIG. 5. Spectrum of $\mu \rightarrow e + 2\nu$ for theory in which meson goes to electron, simultaneously raising neutrino from negative energy state to positive energy state ("charge retention theory").

[13] E. J. Konopinski, Rev. Mod. Phys. **15**, 209 (1943); D. R. Hamilton, Phys. Rev. **71**, 545 (1947); C. W. Sherwin, Phys. Rev. **73**, 1173 (1948); J. C. Jacobsen and O. Kofoed-Hansen, Phys. Rev. **73**, 657 (1948).

geometry, for scattering, and for radiative deceleration. Probably it will be some time before results are obtained sufficiently reliable to discriminate between the above theories, and conceivably it will be necessary to develop new experimental techniques.

CONCLUSIONS

(a) We conclude that the three-particle hypothesis is consistent with present experimental evidence on the energy spectrum of electrons from μ-decay.

(b) Experimental evidence on the lifetime of the meson is consistent with the hypothesis that μ-decay and β-decay are separate instances of the same phenomenon, with the same coupling constant, $g\sim 2\times 10^{-49}$ erg cm^3.

(c) The *hypothesis of identical coupling constants and* use of 30 *min.* for the *neutron half-life* is consistent with the vector form of coupling if the second neutral particle has small mass, or is a neutrino; and with tensor coupling for the antisymmetrical two-neutrino theory.

(d) These considerations are purely phenomenological and as they stand are independent of deeper questions of meson theory.

This work was done in association with the cosmic-ray research program at Princeton University supported jointly by the Office of Naval Research and the United States Atomic Energy Commission. We are also indebted to the Friends of Elementary Particle Research for assistance.

APPENDIX ON INTEGRATION OVER MOMENTUM SPACE

R. Ronald Rau

We have to evaluate the integral

$$(1/dE)\int_{dE} I dp_{\nu_x} dp_{\nu_y} dp_{\nu_z} \tag{39}$$

over the thin shell of volume in momentum space consistent with

$$\mu c^2 \leqslant E_\nu + E_e + E_{\mu_0} \leqslant \mu c^2 + dE. \tag{40}$$

Here the energy—or momentum—of the μ_0-meson is determined by the law of conservation of momentum:

$$p_0^2 = p_e^2 + p_\nu^2 + 2p_e p_\nu \cos\theta, \tag{41}$$

where θ is the angle between $\mathbf{p}_e$ and $\mathbf{p}_\nu$. The surfaces of the shell are given by the equations

$$cp_\nu + (p_0^2c^2 + \mu_0^2c^4)^{\frac{1}{2}} = \begin{cases} \mu c^2 - E_e & (42) \\ \mu c^2 + dE - E_e. & (43) \end{cases}$$

For the present calculations we have to consider the electron momentum to point in a fixed direction and to measure a fixed length from an origin 0 in momentum space. Consider the neutrino momentum vector to be measured off from this same origin to a point A. Then (42) and (43) confine A to move on one or other of two ellipsoids with one common focus at 0. Taking polar coordinates p_ν and θ with 0 as origin, we can then write the expression (39) in the form

$$\int_0^\pi p_\nu^2(\vartheta)(\partial p_\nu/\partial E)_\theta I\, 2\pi \sin\theta d\theta. \tag{44}$$

Here

$$2cp_\nu = [(\mu c^2 - E_e)^2 - p_e^2c^2 - \mu_0^2c^4]/[\mu c^2 - E_e + cp_e\cos\theta] \tag{45}$$

and

$$2c(\partial p_\nu/\partial E)_\theta = [(\mu c^2-E_e)^2 + c^2p_e^2 + \mu_0^2c^4 + 2(\mu c^2 - E_e)cp_e\cos\theta]/[\mu c^2 - E_e + cp_e\cos\theta]^2. \tag{46}$$

We also note the expressions

$$E_0 = c(\partial p_\nu/\partial E)_\theta[\mu c^2 - E_e + cp_e\cos\theta], \tag{47}$$

$$p_0\cos(0,\nu) = -p_\nu - p_e\cos\theta, \tag{48}$$

$$p_0\cos(0,e) = -p_e - p_\nu\cos\theta. \tag{49}$$

Finally, we observe that the expression I is always a linear combination of the quantities

$$1;\ (cp_e/E_e)\cos\theta;\quad (\mu_0c^2/E_0)[1-(cp_e/E_e)\cos\theta];\quad (cp_0/E_0)\cos(0,\nu);$$

and

$$(cp_0/E_0)\cos(0,e).$$

All of these quantities are expressible as functions of θ from (45)–(49). Thus the requisite integrals reduce to standard forms and lead to the results given in the body of the text.

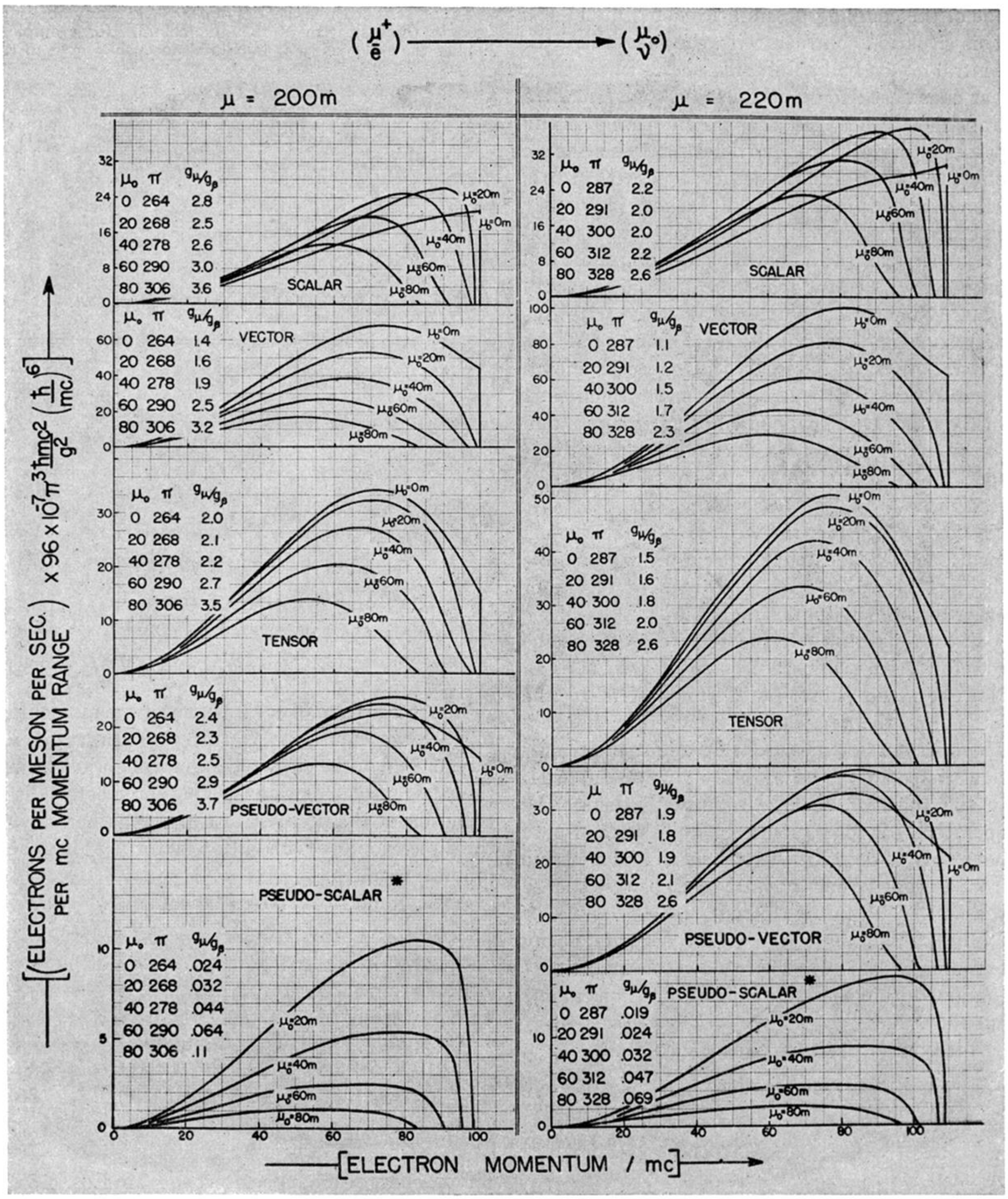
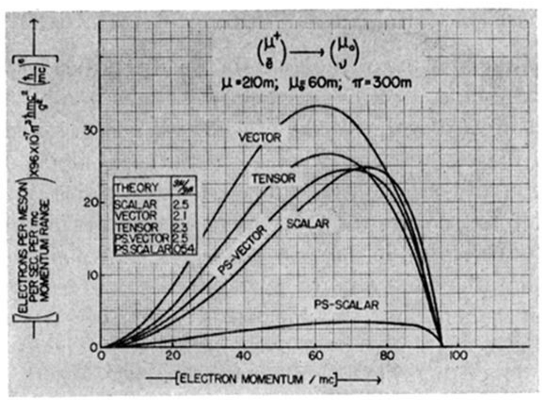


FIG. 1. Energy spectrum of electrons from μ-decay calculated on the three-particle hypothesis for two values of the meson mass (first column, $\mu=200$; second column, $\mu=220$) for several values of the mass of the neutral meson assumed to come off ($\mu_0=80$, 60, 40, 20, 0) and for several forms of the coupling. Note the displacement of the curves to higher energies when μ is increased from 200 to 220—also the substantial decrease in the required value, g_μ, of the coupling constant for μ-decay. Note also the strong influence of the mass of the neutral meson.

* In the case of pseudoscalar coupling the curve $\mu_0=0$ has been omitted through lack of space; it is identical with the curve for $\mu_0=0$ in the case of scalar coupling.



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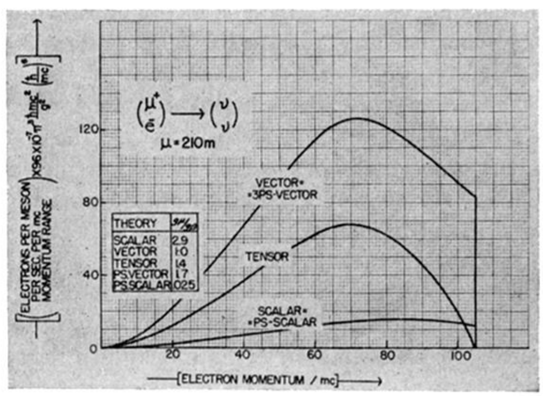

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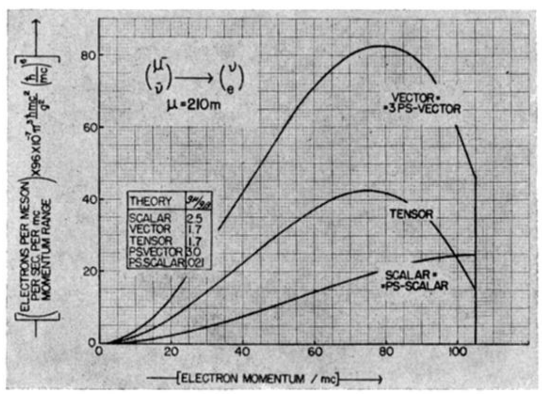

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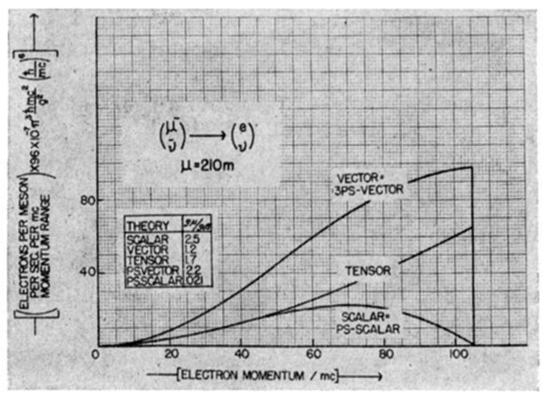

FIG. 5. Spectrum of $\mu \rightarrow e+2\nu$ for theory in which meson goes to electron, simultaneously raising neutrino from negative energy state to positive energy state ("charge retention theory").