Some Consequences of the Electromagnetic Interaction between μ^--Mesons and Nuclei*

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IGNORANCE of the deeper relation between nucleons and mesons makes it especially appropriate at this time to investigate those features of the behavior of mesons which are largely independent of uncertainties about the nature of elementary particles. Fortunately, a number of conclusions may be drawn about the interaction between the meson and the nucleus when we assume little more than the laws of electrodynamics, elementary notions of nuclear structure, the principle of microscopic reversibility, and the simplest ideas of quantum theory. Thus, it has become clear not only that the meson possesses characteristic Bohr orbits of its own around the nucleus, but also that trapping into these orbits via ordinary atomic interactions is the precursor of any specific reaction with the nucleus.[1] So far as concerns competition between such a nuclear reaction and normal decay of the meson, the critical point is the time spent in the outer orbits, a question to which much attention has already been given.[2] About the lower levels the only point relevant in this connection is the observation that the normal order of $2s$ and $2p$ levels is inverted in the mesonic case, so that there is no metastability of the $2s$ level to delay the arrival of mesons into the K-orbit.[3] From the cited analyses it follows that entry into the K-orbit can be treated as effectively instantaneous in comparing the probabilities of reaction and of decay.

The force of attraction between the meson and the nucleus, quite apart from its function in bringing the two entities together, is of interest in its own right. Also interesting are the energy levels and interlevel transitions of the meson, the first experimental evidence for which is given by W. Y. Chang in a following paper.[4] Among the possible transitions are not only those in which a photon is emitted or an atomic electron is ejected, but also processes of pair creation and meson induced fission.[5] Finally, the state of motion of the meson in its ground state is all important in determining the dependence upon atomic number of the probability of a specific interaction between the meson and the nucleus. An attempt is made to analyze these questions in the present paper.

POTENTIAL OF INTERACTION OF MESON AND NUCLEUS

The interaction of the meson with the nucleus will consist of two parts. Of these the first is the purely electrical potential determined by the quantum-mechanical average charge distribution of the protons in the ground state of the nucleus. In view of the results of Feenberg, it is a sufficient approximation for our purposes to regard this charge as spread uniformly over a sphere of radius

$$R=(e^2/2mc^2)(\text{mass number})^{\frac{1}{3}}. \qquad (1)$$

The corresponding potential energy function—taking a negative meson to fix ideas—will be

$$V(r)\begin{cases} =-(Ze^2/R)(1.5-0.5r^2/R^2) \text{ for } r<R \\ =-(Ze^2/r) \text{ for } r>R. \end{cases} \qquad (2)$$

* Presented by the author at the Pasadena Conference along with the data in the preceding paper of Sigurgeirsson and Yamakawa, as part of an account which included considerations from the following two papers with J. Tiomno and from the subsequent paper by W. Y. Chang. The present work was reported in preliminary form at the meeting of the Commission on Cosmic Rays of the International Union of Pure and Applied Physics at the Jagellonian University of Cracow, October, 8, 1947, as reported in IUPAP Publication No. RC-48-1, Paris, 1948. Some results on level spacings, transition probabilities, and the Z^4 dependence of the meson charge-exchange reaction have been given in Phys. Rev. **71**, 320 (1947), and on meson-induced fission in Phys. Rev. **73**, 1252 (1948).

[1] J. A. Wheeler, Phys. Rev. **71**, 462 (1947). See also F. E. Prieto Calderon, Phys. Rev. **73**, 650 (1948) for case of hydrogen.

[2] E. Fermi, E. Teller, and V. Weisskopf, Phys. Rev. **72**, 314 (1947); E. Fermi and E. Teller, Phys. Rev. **72**, 399 (1947); H. Fröhlich, Nature **160**, 255 (1947); Fröhlich, Huby, Kolodziejski, and Rosenberg, Nature **162**, 450 (1948).

[3] See reference 1, p. 320.

[4] W. Y. Chang, Rev. Mod. Phys. **21**, 166 (1949).

[5] J. A. Wheeler, Phys. Rev. **73**, 1252 (1948).

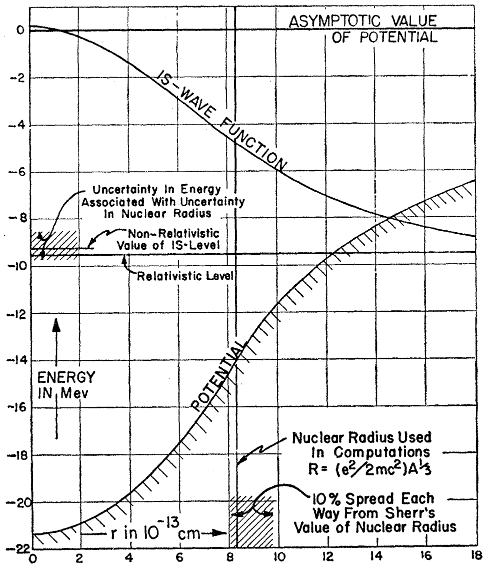

FIG. 1. Potential presented to meson by a lead nucleus.

In the case of Pb, for example, the potential so calculated rises from -21.3 Mev at the center of the nucleus to -14.2 Mev at $r=R=8.3 \times 10^{-13}$ cm (Fig. 1).

The other part of the field acting on the particle will be expected to arise from that specific interaction between meson and nucleon which is also responsible for the disappearance of μ-mesons. There is no satisfactory picture available on the origin of this interaction, but there is one consideration based on scattering experiments and another based on the observed rate of disappearance of μ-mesons to indicate that the supplementary force is small in comparison with the electrostatic forces discussed above.

First, the observed scattering of mesons with momenta of the order of 400 Mev/c by atomic nuclei, over and above the scattering to be expected from electric forces, gives an upper limit to the anomalous scattering cross section of less than about 5×10^{-28} cm^2 per nucleon, according to Code, Wilson, and Shutt.[6,7] To express this result in a form more readily visualized, we can idealize the supplementary interaction in the form of a potential well extending over a region small compared to the meson's wave-length. We express by the symbol W the product of the strength of the potential by the volume over which it acts. Then perturbation theory gives for the approximate order of magnitude of the anomalous scattering cross section per nucleon the result

$$\sigma < \sim (2\pi/h) W^2 (4\pi p^2/c^2h^3), \tag{3}$$

where p is the momentum of the meson. Comparing this expression with the 5×10^{-28} cm^2 figure, we find

$$W < \sim 300{,}000 \text{ ev} (4\pi/3)(e^2/mc^2)^3,$$

where we have brought in the so-called classical electron radius $e^2/mc^2 = 2.802 \times 10^{-13}$ cm solely as a means to make the magnitude of W easily visualized. The upper limit for W so determined is a gross overestimate for two reasons: (a) For the lead nucleus and for the meson wave-length used in the experiments, the nucleons will not scatter independently. The true upper limit to the cross section per free nucleon will therefore be much less than 5×10^{-28} cm^2. (b) The latter figure may not even refer to the scattering of mesons. Code's experiments are particularly significant in this connection because he measured the momentum of the particles by their magnetic rigidity rather than by their range, so he could observe for each case the electric polarity. Out of the 359 tracks photographed, 10 had deflections which in Code's units, $\bar{E}\theta$, exceeded 8×10^9. For mesons interacting solely via electric forces, the expected average number is only 1.5. For 7 of the 10 cases details are given in Code's paper, and of these 7 there are 6 positives and only 1 negative. This observation strongly suggests that the apparent anomalous scattering arises from a small admixture of protons with the mesons. Moreover, the available evidence indicates that the proton component at sea level in the relevant range of momenta has an intensity of the order of a percent or percents of the meson component.[8] For heavy particles in the momentum range of interest, where the ionization does not much exceed that associated with mesons, the scattering cross section of the lead nucleus will be expected to be of the order $\pi(8 \times 10^{-13}\text{ cm})^2$,

[6] F. L. Code, Phys. Rev. **59**, 229 (1941); J. G. Wilson, Proc. Roy. Soc. London **A174**, 73 (1940).

[7] R. P. Shutt, Phys. Rev. **69**, 261 (1946). The analysis of the scattering to be expected from electric forces has been given by E. J. Williams, Proc. Roy. Soc. **A169**, 531 (1939).

[8] This evidence is summarized by B. Rossi, Rev. Mod. Phys. **20**, 547 and 562 (1948).

which corresponds to a figure $\sigma \sim 10^{-26}$ cm² when referred to the *per nucleon* basis adopted by Code and Shutt. This cross section is roughly 20 times as much as the upper limit assigned to the anomalous scattering cross section for mesons. Only a few protons scattered with this cross section are required to account for the whole of the apparent effect, whose real existence is therefore made questionable.

The evidently high upper limit given by deflection experiments for the strength of the meson-nucleon interaction may be compared with another source of information. Observations on the rate of disappearance of mesons due to interaction with the nucleus receive in a following paper[9] a reasonable interpretation in terms of the elementary charge-exchange reaction,

$$\mu + P \rightarrow \mu_0 + N. \qquad (4)$$

From the experimentally reasonably well-known absolute rate of the reaction there is made on a purely phenomenological basis a determination of the constant of coupling,

$$g \sim 10^{-49} \text{ erg cm}^3 \sim 10 \text{ ev}(e^2/mc^2)^3, \qquad (5)$$

between the μ, μ_0-field and the nucleon field. Insofar as we can suppose the entities μ and μ_0 to be comparable, it will be reasonable to assume a similar order of magnitude for the strength W of that direct interaction between μ-mesons and nucleons which is said to be of non-electric origin. The estimate so obtained for W is about five orders of magnitude less than the upper limit set by scattering experiments and is thus not inconsistent with observation.

The estimate just made gives a strength for the specifically nuclear interaction between mesons and nuclei which is negligible in comparison with the electric interaction ($\sim$10 ev *vs.* 10^6 or 10^7 ev, in both cases referred to a region of the order $(e^2/mc^2)^3$). Consequently, we are led to consider the hypothesis that for all nuclei the specific interaction between meson and nucleus may be neglected in considering such questions as mesonic energy levels, transition probabilities, and cross scattering sections, where the instability of the meson does not explicitly come into evidence. We now investigate the consequences of this hypothesis.

[9] J. Tiomno and J. A. Wheeler, Rev. Mod. Phys. **21**, 153 (1949).

ENERGY LEVELS OF NEGATIVE MESONS

The motion of a negative meson in the electric field of even the heaviest nucleus is essentially non-relativistic. The calculated potential at the center of the lead nucleus, for example—21 Mev—is small compared to the rest mass, $210mc^2 = 107$ Mev. Looking apart for the moment therefore from relativistic effects, we can discuss the location of the mesonic energy levels in terms of the Schroedinger equation. Two limiting cases are readily considered—very small and very large atomic numbers. In the first case the energy follows from the Bohr formula by replacing the electronic mass by the mesonic mass.

$$E_n = -(\mu c^2/2)(Z/137n)^2, \qquad (6)$$

where n is the total quantum number.

The mesonic level scheme approaches another simple limiting form when the size of the nucleus becomes extremely large. Already in the case of U the meson may be shown to spend about 55 percent of its time inside the nucleus. This feature of the problem suggests consideration of a droplet of nuclear fluid sufficiently large so that the meson spends all its time inside. This particle then moves in a harmonic oscillator potential, the force constant of which is found from Eqs. (1) and (2) to be

$$Ze^2/R^3 \doteqdot (8Ze^2/A)(mc^2/e^2)^3. \qquad (7)$$

The characteristic frequency, ω, of motion in such a potential is evidently independent of the size of the nucleus, and only depends on the composition of the nuclear fluid. The proportion of protons to neutrons we take for definiteness to be that of U^{238}. Then we find for ω a value corresponding to a characteristic energy level spacing

$$\hbar\omega = (8m/\mu)^{\frac{1}{2}}(Z/A)^{\frac{1}{2}} 137mc^2 = 8.7 \text{ Mev}. \qquad (8)$$

Thus the $2p$ level lies 8.7 Mev above the ground state, and the $2s$ level is 8.7 Mev higher yet.

The wave function for the meson in the ground state of this ideal heavy nucleus varies as $\exp(-\mu\omega r^2/2\hbar)$. From this expression we calculate a 50 percent probability that the meson will lie within the distance

$$1.0915(\hbar/\mu\omega)^{\frac{1}{2}} = 7.2 \times 10^{-13} \text{ cm}. \qquad (9)$$

This quantity defines a critical length for the problem of a meson in the field of a nucleus. For nuclei of substantially greater size—insofar as we are entitled to infer the existence of such nuclei from nuclear theory—the harmonic oscillator model will be a good approximation. Uranium still evidently represents a borderline case, since the calculated $1s-2p$ spacing later obtained for it, 4.9 Mev, is some distance from the limiting figure just given for very heavy nuclei.

To give a more detailed picture of the mesonic levels than that supplied by the two foregoing limiting cases, we have to consider (a) the relativistic fine structure of the levels and (b) the case of intermediate atomic numbers.

The fine structure depends, of course, upon the spin of the meson, which is uncertain. Observations[10] on meson-induced bursts leave open[11] only the spin possibilities 0 and $\frac{1}{2}$. Of these the case of spin $\frac{1}{2}$ is favored by the considerations on meson decay reported in the following paper.[9] Actually, we shall see that it will not make much difference for the levels—whichever possibility is accepted.

To estimate the magnitude of the spin splitting, for example, of the $2p_{1/2}-2p_{3/2}$ doublet, we evaluate the standard quantum-mechanical formula

$$\Delta E=(\hbar^2/4\mu^2c^2)(r^{-1}dV/dr)_{\text{Av}} \times\left\{\begin{array}{l} L \text{ for } J=L+\frac{1}{2} \\ -L-1 \text{ for } J=L-\frac{1}{2}\end{array}\right\} \quad (10)$$

for the separation in the limiting case of very large nuclei. Here taking the gradient of the potential, $V=\frac{1}{2}\mu\omega^2r^2$, leads to the simple value $\mu\omega^2$ for $r^{-1}dV/dr$. For the multiplier of L or $-(L+1)$ in (10) we thus find the coefficient

$$\hbar^2\omega^2/4\mu c^2=(8.7 \text{ Mev})^2/400 \text{ Mev}=0.19 \text{ Mev}. \quad (11)$$

The spin splitting will evidently be small in comparison with the normal level spacing for super heavy nuclei. Moreover, the splitting will decrease with decreasing atomic number much more rapidly than the level spacings themselves. Consequently, we shall look apart from the spin splitting for all actual nuclei, and treat the meson as a particle of zero intrinsic spin.

Also negligible is the hyperfine structure of the p levels caused by interaction of the meson with the magnetic moment of the nucleus. In aluminum, for example, with atomic number 13, nuclear spin $\frac{1}{2}$, magnetic moment 2.2 nuclear magnetons, $2p$ orbit, the calculated hyperfine splitting of the levels with $F=\frac{1}{2}$ and $F=\frac{3}{2}$ is only 9.7 ev.

We shall now evaluate the energy levels of a meson in the field of nuclei of intermediate charge, treating the particle—in accordance with the previous discussion—as if it had spin zero. We adopt the second-order differential equation of Klein and Gordon[12]

$$(i\hbar\partial/\partial t-V)^2\psi=(E-V)^2\psi=\mu^2c^4\psi-\hbar^2c^2\Delta\psi. \quad (12)$$

It would be possible alternatively to adopt the so-called square root equation[13]

$$i\hbar\partial\psi/\partial t-V\psi=(E-V)\psi=(\mu^2c^4-\hbar^2c^2\Delta)^{\frac{1}{2}}\psi. \quad (13)$$

No obvious considerations of principle appear to exclude one or the other of the two alternative relativistic wave equations for the meson. However, we shall find by considering the sample case of the lead nucleus that the uncertainty in the

TABLE I. Solution of the Klein-Gordon equation for the $1s$ state of a negative meson in the field of the lead nucleus, Pb^{207}; mesonic rest mass energy assumed to be $200mc^2$ $=102.2$ Mev, nuclear radius taken as $4.318\hbar/\mu c=4.318$ $\times 1.924\times 10^{-13}$ cm $=8.3\times 10^{-13}$ cm; calculated binding, 9.56 Mev; wave function not normalized; last column gives approximate analytic representation of solution with $A=1.3531$, $B=1.5993$, $C=0.44695$.

	$\rho=\mu cr/\hbar$	r in 10^{-13} cm	V in Mev	ψ	$\rho\psi$	$A\rho^Be^{-C\rho}$
within nucleus	0.0	0.00	−21.3	0.973	0.000	0.000
	0.8	1.54	−21.0	0.947	0.758	0.663
	1.6	3.08	−20.3	0.877	1.403	1.403
	2.4	4.62	−19.1	0.773	1.855	1.868
	3.2	6.16	−17.4	0.650	2.080	2.080
	4.0	7.70	−15.2	0.525	2.098	2.064
outside nucleus	4.8	9.24	−12.7	0.410	1.966	1.934
	5.6	10.8	−10.9	0.313	1.752	1.739
	6.4	12.3	−9.5	0.236	1.508	1.508
	7.2	13.8	−8.5	0.175	1.262	1.272
	8.0	15.4	−7.6	0.129	1.03	1.06
	10.0	19.2	−6.1	0.057	0.57	0.61

[10] R. E. Lapp, Phys. Rev. 64, 129 (1943).
[11] R. F. Christy and S. Kusaka, Phys. Rev. 59, 414 (1941).
[12] See, for example, W. Gordon, Zeits. f. Physik 40, 117 (1926); W. Pauli and V. Weisskopf, Helv. Phys. Acta 7, 709 (1934).
[13] See, for example, G. Wentzel, *Quantentheorie der Wellenfelder* (1943), p. 168.

energy levels due to this ambiguity is less than that due to lack of exact knowledge of the nuclear radius. Consequently, it will be sufficient for the calculation of the level scheme to adopt the Klein-Gordon equation.

To treat one extreme case with considerable precision, a numerical integration[14] of the second-order equation was carried out for the ground state of a negative meson in the field of a lead nucleus. To satisfy the boundary conditions, the energy, E, had to be taken to be $0.9064\mu c^2$, a value corresponding to a binding energy for the $1s$ state of $0.0936\mu c^2=9.56$ Mev. Sufficient of the numerical results are collected in Table I to give an impression of the large probability for a meson in the state in question to be found within the nuclear interior.

The wave function found by numerical integration can be represented with good approximation by a simple analytical formula with three constants, A, B, C, as seen from the table. Following well-known arguments we may employ this approximate function in a variational principle to obtain a fairly precise estimate of the binding energy, not only for the case of the Klein-Gordon equation, but also for the square root equation, and, finally, for sake of comparison, for the Schroedinger equation. For this purpose we have only to express the several differential equations in the equivalent variational form, $E=$ an extremum, where

$$E=\bar{V}+\{\mu^2c^4-\hbar^2c^2\bar{\Delta}+\bar{V}^2-\langle V^2\rangle_{\mathrm{Av}}\}^{\frac{1}{2}} \qquad \text{(Klein-Gordon)} \quad (12a)$$

$$E=\bar{V}+\langle\{\mu^2c^4-\hbar^2c^2\Delta\}^{\frac{1}{2}}\rangle_{\mathrm{Av}} \qquad \text{(square root)} \quad (13a)$$

$$E=\bar{V}+\mu c^2-(\hbar^2/2\mu)\bar{\Delta}, \qquad \text{(Schroedinger)} \quad (14)$$

in which we define

$$\bar{f}=\int\psi^*f\psi d(\text{volume})\bigg/\int\psi^*\psi d(\text{volume}). \quad (15)$$

The calculations give the results collected in Table II. It will be noted that the non-relativistic value of the energy is substantially different from the two relativistic values, but that their own difference is only as great as the change in energy produced by a 1.5 percent alteration in nuclear radius. In contrast, the best available determinations of nuclear radius[15] appear to fix this quantity within a margin no smaller than 10 percent. In fact, in view of this uncertainty the Jeffrey-Wentzel-Kramers-Brillouin method of approximation to the proper values of the Klein-Gordon equation is seen from Table II to be already sufficiently accurate for our purpose. This method of treatment has therefore been applied to other nuclei, and to excited states as well as ground levels.

The J.W.K.B. semi-classical approximation leads to the connection

$$n=(L+\tfrac{1}{2})+\int_{\rho_{\min}}^{\rho_{\max}}\{[(E-V)/\mu c^2]^2-1-(L+\tfrac{1}{2})^2/\rho^2\}^{\frac{1}{2}}d\rho/\pi \quad (16)$$

between proper energy, E, quantum number, L, of total angular momentum, and total quantum number, n. Inserting for V the expression (2) for the nuclear field, and integrating numerically over the classical range of motion, we find a relationship between E and n which can be represented with good accuracy in the form

$$E-\mu c^2=-\tfrac{1}{2}\mu c^2(Z/137)^2/(n+\delta_0+\delta_1/n)^2. \quad (17)$$

Here $\mu c^2=200mc^2=102$ Mev was adopted in the original computations, but the more recent

TABLE II. Expectation values of energy of $1s$ level determined in the case of lead from an analytic variational function of the form $\rho\psi=A\rho^B\exp(-C\rho)$, with $\rho=\mu cr/\hbar$, $B=1.5993$, $C=0.44695$; the starred figures are comparable.

Quantity	Expectation value
$-\hbar^2c^2\bar{\Delta}$	$0.09086\mu^2c^4$
$\langle V^2\rangle_{\mathrm{Av}}$	$0.02030\mu^2c^4$
$\langle V\rangle^2_{\mathrm{Av}}$	$0.01849\mu^2c^4$
$\bar{V}$	$-0.1360\mu c^2=-13.90$ Mev
E (Klein-Gordon, Eq. 12a)—μc^2	$-0.0924\mu c^2=-9.44$* Mev
E (K.-G. accurate)—μc^2	$-0.0936\mu c^2=-9.56$ Mev
E (K.-G. equation via J.W.K.B. approximation—cf below)—μc^2	$-0.0884\mu c^2=-9.03$ Mev
$\langle\{\mu^2c^4-\hbar^2c^2\Delta\}^{\frac{1}{2}}\rangle_{\mathrm{Av}}$—$\mu c^2$	$0.0428\mu c^2=4.37$ Mev
E (square root, Eq. (13a))—μc^2	$-0.0932\mu c^2=-9.52$* Mev
$-(\hbar^2/2\mu)\Delta$	$0.0454\mu c^2=4.64$ Mev
E (Schroedinger, Eq. (14))—μc^2	$-0.0906\mu c^2=-9.26$* Mev
Change in E for a one percent change in nuclear radius	$0.000516\mu c^2=0.053$ Mev

[14] The integration was carried out by the procedure described by D. R. Hartree, Proc. Manchester Lit. and Phil. Soc. **77**, 91 (1932).

[15] R. Sherr, Phys. Rev. **68**, 240 (1945).

TABLE III. The low energy levels of a negative meson in the nuclear field; to show the relativistic fine structure (not the spin fine structure, which is neglected), the table gives more significant figures than are justified by the absolute accuracy of the calculations; central columns give the values of the "quantum surplus" factors in the corrected Rydberg formula of Eq. (17). Values in parenthesis were obtained by interpolation.

		Ele-	*s* states		*p* states		Mev to ionize		
Z	A	ment	δ_0	δ_1	δ_0	δ_1	$1s$	$2p$	$2s$
8	16	O	0.0011	0.0013	−0.0011	0.0013	0.173	0.04358	0.04349
16	32	S	0.0134	0.0051	−0.0046	0.0051	0.672	0.1746	0.1715
26	56	Fe	0.0621	(0.0120)	−0.0121	0.0135	1.59	0.463	0.430
30	66	Zn	0.0824	(0.0156)	−0.0161	0.0181	2.04	0.617	0.562
35	80	Br	0.1085	0.0217	−0.0219	0.0246	2.61	0.842	0.742
47	108	Ag	0.1625	0.0465	−0.0376	0.0414	4.12	1.53	1.26
53	127	I	0.1924	0.0586	−0.0393	0.0399	4.89	1.95	1.55
56	138	Ba	0.2073	0.0650	−0.0373	0.0336	5.28	2.18	1.70
74	184	W	0.2736	0.1031	−0.0172	−0.0091	7.87	3.81	2.76
82	207	Pb	0.3053	0.1182	−0.0025	−0.0103	9.03	4.61	3.28
92	238	U	0.3458	0.1362	+0.0171	+0.0126	10.49	5.63	3.96

value $\mu c^2 = 210mc^2 = 107$ Mev may probably be used with equal validity in the equation. This type of formula is familiar from studies of the energy levels of atomic electrons. In that case the correction to the quantum number in the denominator of (17) arises from the increase of effective nuclear charge towards the inner part of the atomic field. The correction is then such as to diminish the denominator, and is spoken of as "quantum deficit." In the case of the meson the effective nuclear charge decreases towards the center of the field, and we therefore generally have positive values for the corrections δ_0 and δ_1,

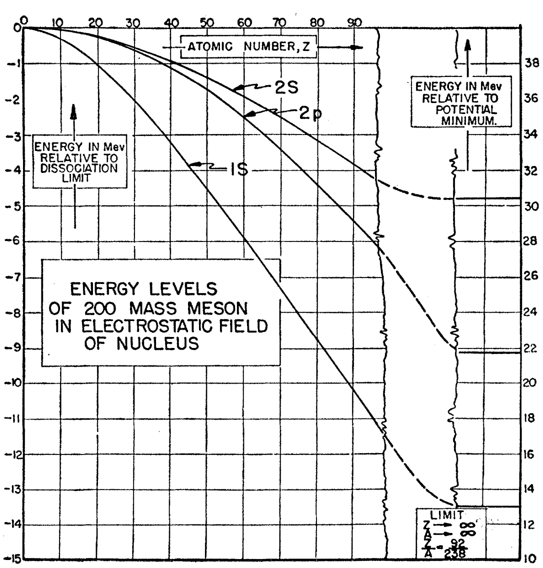

FIG. 2. Energy levels of 200 mass meson in electrostatic field of nucleus.

thus justifying the introduction here of the term "quantum surplus." However, the deviations from the simple Rydberg formula which are described by δ_0 and δ_1 arise not only from the finite extension of the nuclear field, but also from relativistic corrections. Already in the case of a pure Coulomb field, where the proper values of the Klein-Gordon equation are accurately representable in the form

$$E = \mu c^2\{1+(Z/137)^2[n+\{(L+\tfrac{1}{2})^2 - (Z/137)^2\}^{\frac{1}{2}} - (L+\tfrac{1}{2})]^2\}^{-\frac{1}{2}}, \quad (18)$$

a formula of the type (17) gives a good account of the energy level scheme when we put

$$\delta_0 = -(Z/137)^2/(2L+1),$$
$$\delta_1 = (3/8)(Z/137)^2. \quad (19)$$

These relativistic effects dominate only for the very lightest nuclei, while the extension of the nuclear charge is elsewhere more important, as will be seen from Table III. In the calculations for the *s* levels of the two lightest nuclei the J.W.K.B. method could not be employed because the inner turning point in the integral of Eq. (16) lay outside the nuclear radius. Instead, the contribution of charge extension of δ_0 in these cases was derived from the formula

$$\delta_0,\ \text{charge extension} = 0.213(Z/137)^2 A^{\frac{2}{3}}, \quad (20)$$

obtained by a simple application of the first order of quantum-mechanical perturbation theory.

It should be mentioned that the calculations summarized in Table III make no allowance for the screening effect of the atomic electrons. This effect is important only for mesonic orbits of high quantum number which reach well out into the electronic part of the atom. Screening raises the lower levels of Fe, for example, by an amount only of the order of 0.002 Mev and is, proportionately speaking, about equally unimportant for the lower levels of other elements.

Figure 2 shows how the levels, as just determined, go over in the limit of an idealized very heavy nucleus to those of a harmonic oscillator.

A striking feature of the level schemes is the large inversion of the $2p$ and $2s$ levels, due to the finite extension of the nucleus. This effect allows radiative transitions via the $2p$ state to the ground level, and guarantees that the $2s$ state will not be metastable.[3] Equally noteworthy is the large magnitude of the energy released in the transition of a meson from one level to another. This energy may be given off as electromagnetic radiation, or transferred to an atomic electron after the manner of the Auger effect, or used to produce a negaton-positon pair when the energy is sufficient, as for the $2s \to 1s$ transition in Fe and higher elements. It is also conceivable that a nuclear energy level, in one or another isotopic species, lies above the ground state by an amount sufficient very nearly to permit resonance with a mesonic transition, in which case, selection rules allowing, there will be an interesting shift in the interacting levels and a corresponding alteration in the properties of the state in question. Finally, in the case of an element as high as U, the calculated energy release in the $2s-1s$ transition is of an order of magnitude to make meson-induced fission a distinct possibility.

For elements not susceptible to meson-induced fission because not so extreme in mass and charge as U, the transitions between the lowest mesonic levels will occur via the other processes already mentioned. They have in common the feature that they permit in principle an experimental check on the calculated level spacings. Accurate determination of spacing for two elements of quite different charge should permit a good determination both of the mass of the meson and of the nuclear radius. The interesting observations of W. Y. Chang,[4] reported in a following paper, show how one can study the characteristic mesonic radiations. Because of the interest attached to such experiments, the relative probabilities of the radiative and electronic emission processes in question are estimated in the next section.

PROBABILITIES OF TRANSITION BETWEEN LOWEST LEVELS

The probability of radiative transitions between the lowest levels of heavy elements is so overwhelmingly greater than that of electronic emission processes that it is sufficient for comparative purposes to limit our attention to the lighter elements. Here it will be a sufficient approximation to treat the nuclear field as that of a point charge. Then we can calculate the probability per second, A, of a transition from the $2p$ state to the $1s$ state from the standard formula[16]

$$A_{\text{rad}}(2p \to 1s) = (\mu c^2/\hbar)(Z^4/137^5)(2/3)^8. \quad (21)$$

In the case of the radiative transition from $2s$ to $2p$, we can take over the value of the matrix element from the corresponding hydrogenic problem,[17] but we have to use the correct value for the energy difference, ΔE, in evaluating the ω^3 factor in the usual formula for transition probability. Thus we find

$$A_{\text{rad}}(2s \to 2p) = (\mu c^2/\hbar)(36 \times 137/Z^2) \times (\Delta E/\mu c^2)^3. \quad (22)$$

For the direct Auger jump from the $2s$ level to the ground state, practically the entire transition probability will be due to interaction of the meson with the two K electrons. The binding energy of one of them is less than the energy released by the jump by the factor $(3/4) \times 200 = 150$. Consequently, the ejected electron can be treated as free. It goes off with relativistic energy in the case of aluminum and heavier elements. However, in those cases the Auger effect becomes less important than radiative transitions out of the $2s$ level, and therefore need not be computed with high precision. Consequently, it will be justified to treat by non-relativistic quantum mechanics not only the meson, but also the electron. Proceeding on this basis, we find for the Auger transition probability the result

$$A_{\text{Auger}}(2s \to 1s) = (me^4/\hbar^3)(m/\mu)^{7/2}(2^{21}/3^{23/2}) = 2.55 \times 10^9/\text{sec}. \quad (23)$$

That this result does not depend upon atomic number may be shown to be intimately connected with the fact that the photoelectric absorption coefficient for the electrons in the K-shell falls off inversely as the 7/2 power of the frequency for high, but non-relativistic, frequencies. The probability of Auger transitions involving the $2p$ and $1s$ or $2s$ levels will be expected to be smaller than the result (23), but not very different in order of magnitude. The escaping electron will be in a p state, the wave function of which will be small in the region where the energy transfer must take place.

[16] Obtained by replacing electron mass by meson mass in the formula given, for example, by H. A. Bethe, *Handbuch der Physik* (1934), Vol. 24, Pt. 1, 2nd ed., p. 440.
[17] Reference 16, p. 441.

TABLE IV. Probabilities per second, A, for transition between lower levels of a negative meson in the nuclear field.

Z	$2s\rightarrow 2p$ radiation	$2s\rightarrow 1s$ Auger	$2s\rightarrow 1s$ pair	$2p\rightarrow 1s$ radiation	Width of $2s$ in ev	Width of $2p$ in ev	$2s-2p$ difference
5	2.4×10^4	2.6×10^9	—	0.8×10^{14}	1.6×10^{-6}	0.52	11 ev
10	2.3×10^8	2.6×10^9	—	1.3×10^{15}	1.8×10^{-6}	0.83	3.2×10^2
15	4×10^{10}	2.6×10^9	—	6.6×10^{15}	2.9×10^{-5}	4.2	2.3×10^3
20	1.4×10^{12}	2.6×10^9	—	2.1×10^{16}	9×10^{-4}	13	9.1×10^3
25	2×10^{13}	2.6×10^9	2×10^8	5.1×10^{16}	1×10^{-2}	32	2.5×10^4
30	1×10^{14}	2.6×10^9	5×10^{10}	1.1×10^{17}	9×10^{-2}	67	5.5×10^4
35	7×10^{14}	2.6×10^9	8×10^{11}	1.9×10^{17}	0.4	120	1.00×10^5
40	2×10^{15}	2.6×10^9	5×10^{12}	3.3×10^{17}	1.4	210	2.7×10^5
92	3.2×10^{17}	—	—	1.9×10^{18}	210	1200	1.7×10^6
∞	1.1×10^{19}	—	—	5.6×10^{18}	7200	3600	8.7×10^6

Negaton-positon pair production in the $2s\rightarrow 1s$ transition may be considered as a special kind of Auger effect, in which the electron to be ejected is initially in a state of negative energy. Following the ideas of the calculation just above, we neglect the influence of the nuclear field on the electron both before and after the transition, and also treat the meson by non-relativistic wave mechanics. However, Dirac's relativistic wave equation is used for the electron. We find

$$A_{\text{pair}}(2s\rightarrow 1s)=(me^4/\hbar^3)(128/3\pi)\times(\mu Z/137m)^8\times\int^{(G^2-4)^{\frac{1}{2}}} dx\, x^8(G^2-4-x^2)^{\frac{1}{2}}(G^2+2-x^2)\times(B^2+x^2)^6(G^2-x^2)^{-7/2}, \quad (24)$$

where x is the resultant of the momenta of negaton and positon expressed in units mc,

$$B=1.5\mu Z/137m=2.19Z,$$
$$Z_1=22.4,$$

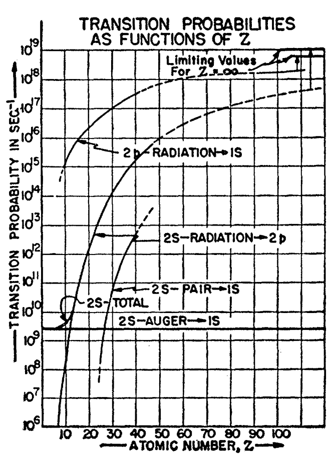

FIG. 3. Transition probabilities **as a** function of Z.

and

$$G=(3\mu/8m)(Z/137)^2=(Z/15.8)^2.$$

The expression (24) has been evaluated numerically. The results obtained for atomic number up to $Z=40$ are well represented by the approximate formula

$$A_{\text{pair}}(2s\rightarrow 1s)=2.51\times 10^{13}\text{ sec.}^{-1}\times[(Z^2/Z_1^2)-1]^{4.98}Z^{-1.49}. \quad (25)$$

Figures for the various transition probabilities are collected in Table IV and shown in Fig. 3.

From Fig. 3 it is seen that Auger transitions out of the $2s$ level dominate[17a] for elements of atomic number less than 15, and radiative transitions from this level to $2p$ are the more important for phosphorous and heavier elements. The time required to leave the $2s$ level is in all cases very short compared to the natural life of the meson. This result, together with the semiclassical considerations of Fermi and Teller[2] on transitions between the higher mesonic levels, justify the conclusion that the meson—by whatever route it jumps down—reaches the K level before it has time to decay.

MESON-INDUCED FISSION

In the case of uranium, the calculated energy release in the $2s-1s$ transition, 7 Mev, is—uncorrected—more than sufficient to produce fission. Actually the energy release will be less than this amount if fission occurs, because around a nucleus of the critical dumbbell-shaped form which lies at the saddle point in the nuclear energy surface[18] a $1s$ meson is less tightly bound by 1 or 2 Mev than it is around the normal spherical nucleus. Consequently, it is an open issue whether it will be possible to supply the 5 Mev or 6 Mev required to produce division.[19] But in heavier nuclei with certainty, and possibly in uranium itself, the process will be energetically allowable. The probability of meson-induced fission depends primarily upon the competition of the radiative transition from $2s$ to $2p$, corre-

[17a] Brown, Camerini, Fowler, Muirhead, Powell, and Ritson, *Nature*, in press, report that "ejection of Auger electrons is a rare phenomenon. This result may be associated with a low probability for the meson to pass through the $2s$-level as it drops from state to state (preference for circular orbits—cf. Fermi and Teller, reference 2).

[18] N. Bohr and J. A. Wheeler, Phys. Rev. **56**, 426 (1939).

[19] See reference 22, p. 1065.

sponding to a rate $\sim 3\times 10^{17}$/sec. A crude estimate gives a rate probably not less by an order of magnitude for the $2s-1s$ fission jump. In this estimate no weight is given to processes by which the motion of the meson, via electric coupling, directly excites the capillarity oscillation favorable to fission. The conditions for resonance are not satisfied. Instead, the fluctuating electric field of the meson is considered as equivalent from the point of view of the nuclear matter to the field of a beam of radiation. The cross section of the nucleus for photo-fission being known, an estimate of the probability for meson-induced fission is possible. In this estimate no correction is applied for the different polarity of the two electromagnetic fields. On the contrary, it would seem to be a reasonable approximation to regard a given small portion of the nuclear matter as responding to an oscillating field of a given magnitude in a manner relatively independent of the space distribution of that field. On this basis we conclude that an appreciable fraction of the negative mesons trapped around the heaviest nuclei—uranium possibly included—will induce fission.

When such fission occurs at all, it will occur long before the meson has had an opportunity to undergo natural decay. Moreover, a simple comparison of time of fission with the classical period of motion of the meson in its orbit indicates that the division of the nuclear charge into two will be very nearly adiabatic from the point of view of the meson. Consequently, this particle will be expected to go off in the K-orbit of the heavier fragment. The time of capture into a nucleus of charge 54 may be estimated to be of the order of 10^{-8} sec. This time is sufficient to allow the fragment to be brought to rest, whether moderated in a solid or in air. The meson will then in most cases be expected to produce a nuclear disintegration.[9] However, the degree of excitation favors neutron emission, and a proton track may be expected to be a relatively rare event.[4] When it occurs in a photographic emulsion, we have to expect a remarkable picture: a wavy meson track; from its end two heavy fission particle tracks diverging; from the end of one of these tracks a proton prong projected.

Distinct from this type of *externally* induced fission is the *internally* induced fission, in which the charge exchange reaction of a meson with the nucleus imparts to the system an excitation sufficient to cause division. From general considerations on probability distribution of excitations following the charge exchange reaction[9] we conclude that the *internally* induced fission will occur relatively frequently in nuclei like uranium and heavier. This process, seen in a photographic emulsion, will not be expected to show the proton prong occasionally possible in *externally* induced fission.

The rates of both kinds of meson-induced fission to be expected from the normal sea-level flux of cosmic-ray mesons are much lower than the known rate of spontaneous fission of uranium. Consequently, the study of the new process would appear to demand stronger meson sources.

DEPENDENCE UPON ATOMIC NUMBER OF PROBABILITY OF MESON CHARGE EXCHANGE REACTION

Once in the K-orbit of a heavy nucleus, the meson is found to disappear before it has time to decay. The disappearance reaction is most reasonably understood[9] in terms of a process of the type,

$$P+\mu^{-}\rightarrow N+\mu_0.$$

On such a picture, and using only very general arguments, we can estimate approximately the dependence of reaction probability upon atomic number.

Our present phenomenological arguments are to a considerable extent independent of the deeper going analysis of the reaction mechanism which is presented in a following paper. First, we shall assume that the probability of absorption increases directly with the number of protons available, other factors being assumed constant. Second, we shall assume that the probability of absorption by a single proton is proportional to the probability, $|\psi|^2$, for the meson to be in the neighborhood of this proton. In place of this assumption it would be possible to adopt the pseudoscalar meson theory, according to which the absorption probability is connected, not with the wave function itself, but with its gradient. Then our considerations, instead of leading to a probability of absorption from the K-shell pro-

TABLE V. Mean life, τ, of a meson in a K-orbit, and probability, W_{decay}, that it will give off a decay electron, calculated for two assumed values of the constant Z_0; the last row refers to any nucleus having the same ratio of neutrons and protons as U^{238}.

Element	Z	Z_{eff}	Case $Z_0=10$ τ(μsec.)	W_{decay}	Case $Z_0=7$ W_{decay}	τ(μsec.)
Be	4	3.925	2.10	0.977	0.910	1.96
B	5	4.855	2.04	0.947	0.812	1.75
C	6	5.78	1.93	0.899	0.682	1.47
N	7	6.68	1.80	0.835	0.548	1.18
O	8	7.56	1.62	0.755	0.425	0.91
F	9	8.40	1.43	0.668	0.326	0.70
Na	11	10.02	1.07	0.498	0.192	0.41
Mg	12	10.83	0.91	0.422	0.149	0.32
Al	13	11.58	0.77	0.358	0.118	0.25
S	16	13.70	0.48	0.221	0.064	0.137
Fe	26	19.40	0.142	0.066	0.017	0.036
Zn	30	21.1	0.103	0.048	0.012	0.026
Br	35	23.0	0.074	0.0345	0.0085	0.018
Ag	47	26.4	0.043	0.0202	0.0049	0.011
I	53	27.67	0.036	0.0168	0.0041	0.0088
Ba	56	28.25	0.033	0.0155	0.0038	0.0081
W	74	30.66	0.024	0.0112	0.0027	0.0058
Pb	82	31.5	0.022	0.0101	0.0024	0.0052
U	92	32.17	0.020	0.0093	0.0023	0.0048
Large		37.25	0.013	0.0061	0.0013	0.0027

portional for the lightest nuclei to the fourth power of the atomic number, would give a result going as Z^6. However, on grounds of simplicity we shall adopt in the following the assumption that it is only the wave function itself which counts. Thus we take for the probability per second of absorption,

$$A_{\text{absn}} = \text{constant} \times \sum_{\substack{\text{all}\\ \text{protons}}} |\psi(\text{at each proton})|^2. \quad (26)$$

In the case of a light nucleus we can use a hydrogenic wave function and take the value of ψ as constant over the nucleus. Then we find

$$A_{\text{absn}} = \text{constant}(Z/\pi)(Ze^2\mu/\hbar^2)^3. \quad (27)$$

The constant in question has the dimensions of a volume divided by a time, and may therefore be written purely for the sake of the eventual simplification which will result in the form

$$\text{constant} = (1/\tau_0)(\hbar^2/\mu e^2)^3(\pi/Z_0^4). \quad (28)$$

Here τ_0 represents the natural mean life of the meson, about 2.15×10^{-6} second, and Z_0 is a pure number, to be found from experiment. Then for light nuclei the probability of absorption from the K-shell takes the form

$$A_{\text{absn}} = (1/\tau_0)(Z^4/Z_0^4). \quad (29)$$

Evidently the undetermined constant Z_0 represents the atomic number of that idealized nucleus which is able to compete for the K-meson on equal terms with the process of natural decay.

For heavier nuclei the hydrogenic approximation will not be justified, but we can still write

$$A_{\text{absn}} = (1/\tau_0)(Z_{\text{eff}}^4/Z_0^4) \quad (30)$$

provided that we define the effective atomic number, Z_{eff}, by the equation (derived from Eq. (26))

$$Z_{\text{eff}} = [(\hbar^2/\mu e^2)^3\pi \sum_{\text{protons}} \psi^2]^{\frac{1}{4}}$$

$$= 47.1\left[Z\int_0^R (r\psi)^2 dr \Big/ A\int_0^\infty (r\psi)^2 dr\right]^{\frac{1}{4}}. \quad (31)$$

This expression for Z_{eff} was evaluated by numerical integration of the wave equation for I and Pb, with the results, respectively, $Z_{\text{eff}}=27.6$ and $Z_{\text{eff}}=31.5$. Evidently the probability of nuclear capture cannot increase indefinitely with atomic number. In a very large system the meson will interact appreciably only with nucleons which lie within a critical distance of the order of magnitude of 7×10^{-13} cm. The calculated value of Z_{eff} for this case is 37.2, obtained by assuming the same nuclear composition as U^{238}, and using in Eq. (31) the harmonic oscillator function already mentioned. Between the values obtained by detailed calculation it is possible to make a good interpolation by means of the empirical formula

$$Z_{\text{eff}} = Z[1+(Z/37.2)^{1.54}]^{-1/1.54}. \quad (32)$$

This formula yields the figures given in Table V for Z_{eff} for selected nuclei.

The probability per second for disappearance of a meson from a K-orbit, allowing for both decay and absorption, will be given by the expression

$$(1/\tau) = (1/\tau_0) + A_{\text{absn}}, \quad (33)$$

corresponding to a mean life

$$\tau = \tau_0/[1+(Z_{\text{eff}}/Z_0)^4]. \quad (34)$$

The chance that a decay electron will be observed to come at all from the negative meson will be

$$W_{\text{decay}} = 1/[1+(Z_{\text{eff}}/Z_0)^4]. \tag{35}$$

Values of lifetime and decay probability are given in Table V. For the unknown constant Z_0 we take the two trial values $Z_0 = 7$ and $Z_0 = 10$.

It is evident from Table V that the calculated lifetime of a meson in the K-orbit of silver, between 1×10^{-8} sec. and 4×10^{-8} sec., is a time of just the order of magnitude for the detection of which AgCl crystal counters have proven themselves suited.

The calculated variation of decay probability, W_{decay}, with atomic number does not show so distinct a line of division between absorbing and non-absorbing nuclei as might have been expected from the available observations. However, the usual delay experiments never give the total number of decay electrons, but only the number given off after a waiting period of the order of one microsecond, introduced to guarantee against spurious events really due to delays in the counters themselves. Thus with $Z_0 = 10$ we might be led to expect that 17.7 percent of the negative mesons stopped in sulfur would give off decay electrons. However, the calculated mean life for this decay process is only 0.38 μsec. With a 1-μsec. waiting period, the stopped negative mesons will have a chance to trigger the delay circuit equal only to $0.177\times\exp(-1/0.38) = 0.177 \times 0.072$ or 1.3 percent. Consequently, the distinction to be expected experimentally between light and heavy nuclei will be considerably sharper than that indicated by the figures for W decay in Table V.

Diagrams correlating the theoretical dependence of electron emitting power upon atomic number with experiment will be found in the paper of Sigurgeirsson and Yamakawa.

In conclusion we can say that the discovery of Conversi, Pancini, and Piccioni has opened up new possibilities for studying the interaction of mesons with nuclei.

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