The Use of Coincidence Counting Methods in Determining Nuclear Disintegration Schemes

ALLAN C. G. MITCHELL

Indiana University, Bloomington, Indiana

I. INTRODUCTION

FOR some years the author and his collaborators[1,*] have been using coincidence counting techniques as an aid in the determination of nuclear disintegration schemes. The information obtained by this method, when taken together with that obtained with the help of a magnetic analyzer—magnetic lens or 180° type magnetic spectrograph—is at present one of the best methods of obtaining nuclear disintegration schemes, or the energy levels of radioactive nuclei. A comparison of the two methods gives a picture of the advantages and disadvantages of each.

Any magnetic analyzer, to be sure, has much greater resolving power than does the coincidence method. In principle, a resolution ($\Delta H\rho/H\rho$) in momentum of the electrons measured of 1–2 percent can be obtained. Measurement of gamma-ray energies can be obtained to an accuracy of 1 to 5 percent by measuring the energy of photoelectrons produced in some radiator, such as lead. It is not difficult to measure the energies of the electrons ejected from the K and L shell by some gamma-ray and thereby obtain the energy of the gamma-ray. In addition, one can measure the momentum distribution of Compton electrons produced in an element of low atomic number, say copper or aluminum, by the gamma-rays under investigation. This method can serve as a check on gamma-ray energies obtained by the photo-effect and is particularly useful in the two cases: (1) when the energy of the gamma-ray is so high that the probability of production of photoelectrons becomes small; and, (2) when two gamma-rays are so situated that the L line of one is superimposed on the K line of the other, for some particular radiator.

The energy of gamma-rays can be measured with considerable accuracy by this method, provided sources of high activity are available. The problem is to determine the energy level scheme from this information which is, at the same time, consistent with the spectrum of the disintegration electrons.

The determination of the momentum distribution of the disintegration electrons (beta-ray spectra) poses a somewhat different problem. The thickness of the source and the material upon which it is mounted influence the shape of the distribution especially at the low energy end of the spectrum. One is therefore forced to use sources of very high specific activity in order to obtain the best results. When the distribution is measured, a "Fermi Plot" is made of the results, and a decision made as to whether the spectrum is simple or complex. If the spectrum consists of two groups, a determination of the two end points can be made with fair accuracy. If more than two groups are present, the accuracy of determining the end points of the lower energy groups is considerably less.

With the help of the information from the gamma-ray spectra and that of the beta-ray analysis, including the occurrence of internal conversion lines, one attempts to draw up a self-consistent energy level diagram. This method does not allow one to be certain that the most energetic beta-ray leads to the ground state of the product or that a beta-ray of a given energy is followed by one or more gamma-rays except in so far as it is consistent with a reasonable energy level scheme. Great accuracy can be obtained, certainly, in the measurement of gamma-rays but, on the other hand, sources of high activity are needed.

Up to the present, in most experiments in which coincidence counting techniques have been used, instruments of high resolving power for measuring beta-ray and gamma-ray energies have not been used. The energy of beta-rays has been determined by absorption methods and that of the gamma-rays by a determination of

*References are to be found in the bibliography at the end of this article.

the range of secondary electrons produced in a radiator. Nevertheless, it has been possible to make a correct determination of energy level schemes by this method. Since no high resolving power apparatus is employed, it follows that sources of low activity will be sufficient for this type of work. The principle of the method is to place two counters near the radioactive source and then measure coincidences between (1) the several gamma-rays which may be emitted, and (2) between the gamma-rays and beta-rays of varying energy. In this manner one can ascertain (*a*) whether there is more than one gamma-ray per disintegration, (*b*) whether the lowest energy beta-ray leads to the ground state of the product, and (*c*) whether there are any low energy groups of electrons associated with additional gamma-ray transitions.

II. ARRANGEMENT OF APPARATUS AND PROCEDURE

Coincidence counting apparatus[1] is usually used in two different modifications for the purposes of (1) measuring coincidences between beta- and gamma-rays; and (2) of determining the energy of gamma-rays by measuring the range of Compton electrons produced by them in aluminum.

The first arrangement is shown in Fig. 1. The two counters are in the same horizontal plane with the source placed midway between them. If coincidences between gamma-rays are to be observed, enough aluminum is placed on either side of the source to prevent any beta-rays from reaching the counters. If beta-gamma coincidences are to be observed, thin sheets of aluminum, whose thickness can be varied, are placed between the source and one of the counters. In this manner the number of beta-gamma coincidences, can be measured as a function of the energy of the beta-rays.

The arrangement for measuring the energy of the gamma-rays is shown in Fig. 2. Gamma-rays from the radioactive source, S, eject Compton electrons from the aluminum radiator R. These electrons then pass through both counters, being recorded as coincidences. Sheets of aluminum are then placed at the point A between the counters to absorb the electrons, and the number of coincidences per minute is measured as a function of absorber thickness.

The energy of the maximum energy gamma-ray may be determined by measuring the range of the Compton electrons in aluminum. By comparing the range thus determined with a curve published by Curran, Dee, and Petrzilka,[2] which gives the range of Compton electrons ejected by gamma-rays of known energy, the energy of the gamma-ray in question may be found.

Another method, first devised by Becker and Bothe[3] is applicable in determining the gamma-ray energy when only one gamma-ray is present. This depends on a measurement of the amount of absorber necessary to decrease the number of Compton coincidences to one-half and to one-quarter of their value when no absorber is present. If $D_{1/n}$ is this value, then the energy of the gamma-ray, E, may be calculated from the relation

$$D_{1/n}=C_{1/n}[(E/m_0c^2)^2/(E/m_0c^2+1)]. \quad (1)$$

Here the constant $C_{1/n}$ may be determined by measuring $D_{1/n}$ for a gamma-ray of known energy and computing $C_{1/n}$. The constant $C_{1/n}$ is nearly independent of energy, and has a value 0.0630 g/cm² for $n=2$ and 0.110 g/cm² for $n=4$ for the 2.62 Mev line from ThC″. A measurement of E calculated from $D_{1/2}$, $D_{1/4}$, and the range gives an indication of how many gamma-rays are present. If they differ and the gamma-ray energy, as calculated from $D_{1/2}$, $D_{1/4}$, and the range, appears to increase, there are probably several gamma-rays. Bleuler and Zünti[4] have worked out a method for determining the indi-

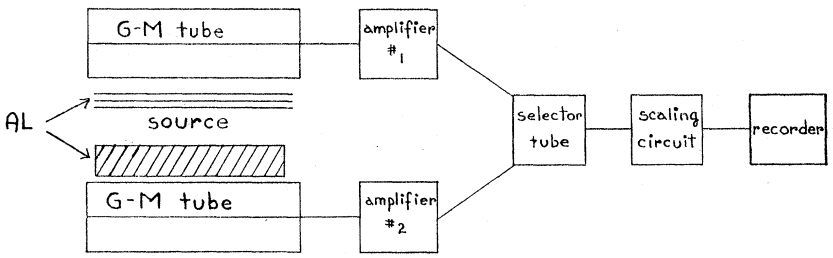

FIG. 1. Arrangement for measuring beta-gamma and gamma-gamma coincidences.

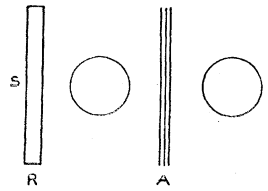

FIG. 2. Arrangement for measuring energy of gamma-rays.

vidual gamma-ray energy, when there are several present, from an analysis of these curves.

Most workers in this field have, in the past, been satisfied to measure the range of the beta-rays in aluminum and to calculate the maximum energy of the beta-ray spectrum from the Feather relation,

$$R(\text{gm/cm}^2) = 0.571E(\text{Mev}) - 0.161, \quad (2)$$

since it was felt that whether the spectrum was complex or not could be determined more easily by measuring beta-gamma coincidences. Feather[5] and Bleuler and Zünti[4] have proposed a method of determining the shape and complexity of a beta-spectrum from an analysis of the absorption curve alone.

III. USE AND ANALYSIS OF THE COINCIDENCE METHOD

(A.) Beta-Ray Spectrum Simple, One Gamma-Ray per Disintegration

In using the coincidence method, the first step is to determine the range of the beta-particles. This gives at once the end point of the beta-ray spectrum and at the same time shows whether there are any gamma-rays present. The source is then investigated for coincidences between gamma-rays by making use of the apparatus as shown in Fig. 1. Enough aluminum is placed between the source and each counter to stop all beta-rays. If there are no gamma-gamma coincidences, it is clear that there is only one gamma-ray per disintegration, and it remains to determine whether the highest energy beta-ray leads to the ground state or whether the spectrum is complex.

The number of beta-gamma-coincidences is now investigated in the same apparatus. The thick aluminum absorber is removed from between the source and one of the counters and the number of beta-gamma coincidences measured as the thickness of the absorber is changed. If the number of beta-gamma-coincidences per disintegration is independent of the beta-ray energy, it is clear that the beta-ray leads to an excited state of the product, from which the gamma-ray follows. If, on the other hand, the number of beta-gamma coincidences decreases as the thickness of absorber increases and reaches a zero value at a range corresponding to an energy less than the end point of the beta ray spectrum, it follows that the spectrum is complex. The energy, beyond which there are no

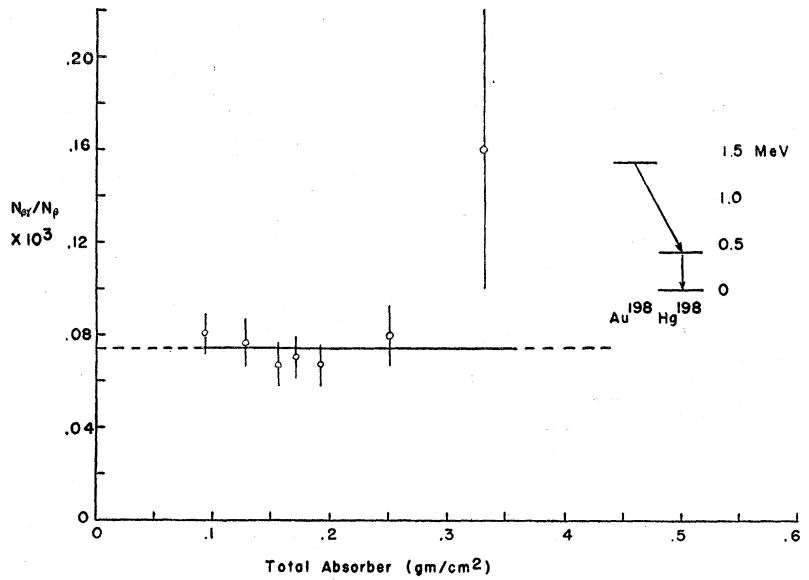

FIG. 3. Coincidences in Au^{198}.

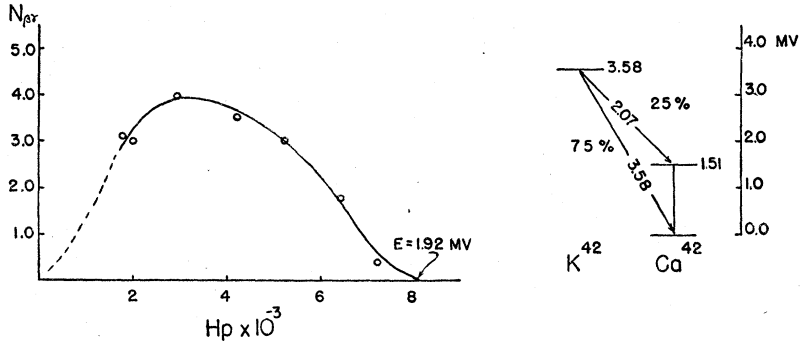

FIG. 4.

beta-gamma coincidences, is the end point of the lower energy group.

In order to get an expression for the number of coincidences and single counts in each counter let N_0 be the number of disintegrations per second, ω_β and ω_γ the solid angle subtended by the beta-counter and gamma-counter, respectively. Let $N_{\beta\gamma}$, N_β, and N_γ be the number of beta-gamma coincidences per second and the number of counts per second recorded in the beta- and gamma-counters, respectively. Finally, let e_γ be the efficiency of the gamma-ray counter and $F_i(x)$ be the fraction of electrons in the ith group entering the counter after passing through an absorber of thickness x.

For the case in which the number of beta-gamma coincidences is independent of the energy, the following relations hold.

$$N_\beta = N_0\omega_\beta F(x), \tag{3}$$

$$N_\gamma = N_0\omega_\gamma e_\gamma, \tag{4}$$

$$N_{\beta\gamma} = N_0\omega_\beta F(x)\cdot e_\gamma\cdot\omega_\gamma. \tag{5}$$

It follows then that

$$\frac{N_{\beta\gamma}}{N_\beta} = e_\gamma\omega_\gamma. \tag{6}$$

It will be seen at once that e_γ, the efficiency of the counter for the gamma-ray in question, can be determined from (6). An example of this type of disintegration is that of Au^{198}, shown in Fig. 3. Coincidence measurements have been made by Norling,[6] Clark,[7] and Jurney.[8] All authors find that the number of beta-gamma coincidences per beta particle is independent of the energy of the beta particles, indicating only one group of beta-rays. The most recent measurements by Jurney failed to find gamma-gamma coincidences, showing that there is only one gamma-ray. Magnetic spectrograph measurements made by Siegbahn[9] and Peacock and Wilkinson[10] show that there is one beta-ray of energy 0.92 Mev and one gamma-ray of energy 0.41 Mev.

(B.) Beta-Ray Spectrum Complex, One Gamma-Ray per Disintegration

If the number of beta-gamma coincidences is dependent on the energy, the equations become

$$N_\beta = N_0\omega_\beta\cdot[F_1(x)+F_2(x)], \tag{7}$$

$$N_\gamma = N_0\omega_\gamma e_\gamma, \tag{8}$$

$$N_{\beta\gamma} = N_0F_1(x)\omega_\beta\omega_\gamma e_\gamma. \tag{9}$$

Hence,

$$\frac{N_{\beta\gamma}}{N_\beta} = \frac{F_1(x)}{F_1(x)+F_2(x)}\omega_\gamma e_\gamma \tag{10}$$

and

$$\frac{N_{\beta\gamma}}{N_\gamma} = F_1(x)\omega_\beta. \tag{11}$$

$F_1(x)$ is zero for a value of x equal to or greater than the range of the first group, so that the end point of the lower energy group is determined from measuring $N_{\beta\gamma}/N_\beta$ as a function of x. A measurement of $N_{\beta\gamma}/N_\gamma$ gives the absorption curve of the lower energy group. Extrapolation of these measurements to $x=0$, and a comparison with N_β at $x=0$ gives the relative intensity of the two electron groups, $F_1/(F_1+F_2)$. Several cases of disintegrations of this type have been found. Among them are Sb^{122}, investigated by Mitchell, Langer, and McDaniel,[1] K^{42} investigated by Siegbahn and Johansson,[11] and

O^{19}, Na^{25} by Bleuler and Zünti;[12] however, the coincidence method was not used to establish the latter two cases. Siegbahn and Johansson used coincidence counting in conjunction with a magnetic lens spectrometer and Fig. 4 shows the results of their investigation. Here the number of beta-gamma coincidences is plotted as a function of $H\rho$, and it will be seen that the interior end point is well established at 1.92 Mev, while the end point of the most energetic group is 3.58 Mev.

Since the energy of the gamma-ray can be determined by measuring the range of secondary electrons emitted from aluminum, the end point of the spectrum for the most energetic group of beta-rays by absorption, and that of the lower energy group by one of the methods discussed above, the disintegration scheme can be determined.

(C.) Beta-Ray Spectrum Simple, More Than One Gamma-Ray per Disintegration

If gamma-gamma coincidences are present, the analysis becomes more complicated. One wishes to determine the number and energy of the gamma-rays and whether they are in cascade or in parallel. The simplest case to discuss is that in which the number of beta-gamma coincidences per disintegration is independent of absorber thickness. As an example, consider the case of Na^{24}. This isotope was investigated first by Langer, Mitchell, and McDaniel[13] and more recently by Wiedenbeck[14] with substantially the same results.

The beta-ray end point of the Na^{24} spectrum is 1.39 Mev.[15] The coincidence experiments showed that there were both beta-gamma and gamma-gamma coincidences. The number of beta-gamma coincidences per recorded beta-count was found to be independent of the energy. Figure 5 shows the results of the experiment of Langer, Mitchell, and McDaniel, in which $N_{\beta\gamma}/N_\beta$ is plotted as a function of the thickness of absorber, in grams/cm^2 between source and counter. This shows at once that all beta-ray transitions lead to the same excited state of the product, and the beta-spectrum is simple. The careful work of Lawson[16] and Siegbahn[15] on the beta-spectrum indicates that there is only one group.

The gamma-ray spectrum of Na^{24} has been investigated by various observers with conflicting reports. What appear to be the most reliable measurements give two gamma-rays—one at 1.38 Mev and one at 2.76 Mev, of practically equal intensity. The inference to be drawn from the gamma-gamma coincidence measurements is that these two gamma-rays are in cascade. The level system which results requires that the energy of Na^{24} be 5.53 Mev above the ground state.

Recently Sachs[17] pointed out that the high energy of Na^{24} was in disagreement with a value originally predicted by Barkas,[18] but since revised.[19] He proposed an alternative scheme whereby the 1.39-Mev beta-ray was followed by two gamma-rays of 1.38 Mev in cascade and also a 2.76-Mev gamma in parallel with these. In order to obtain equal intensities for the

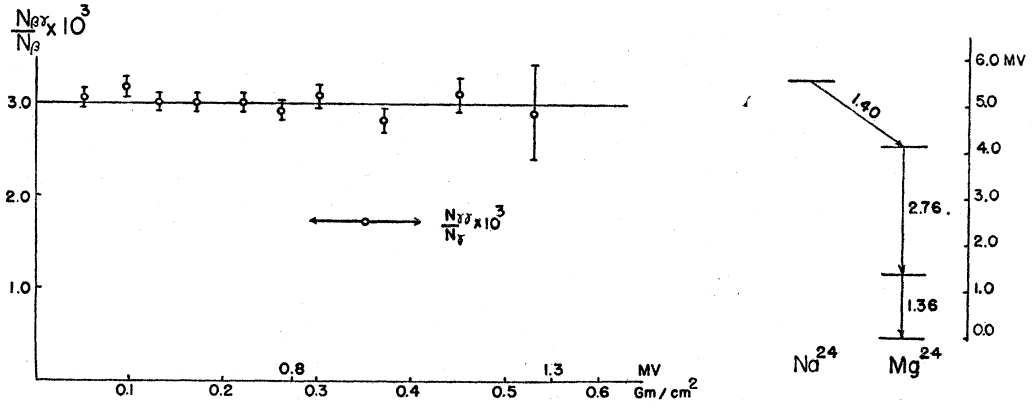

FIG. 5. Coincidence measurements and disintegration scheme of Na^{24}.

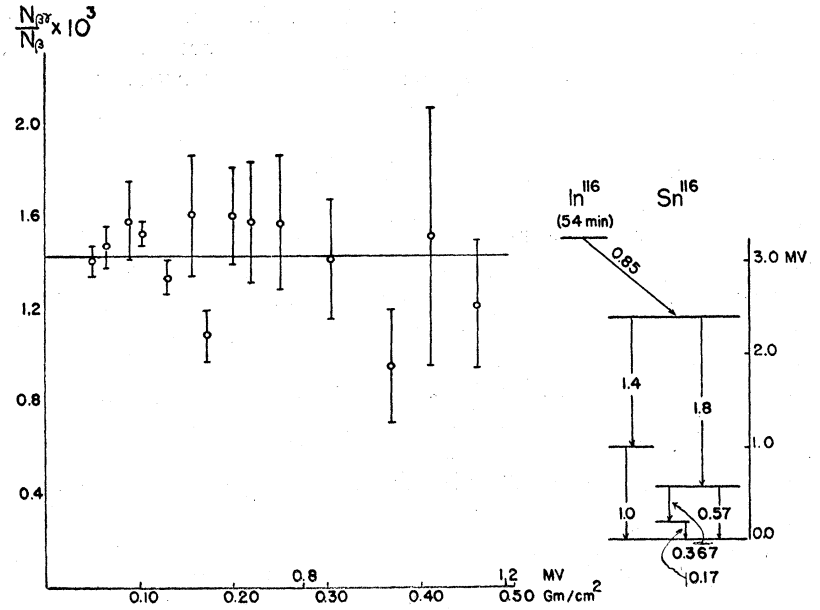

FIG. 6. Coincidence measurements and disintegration scheme of In^{116} (54 min.).

observed 1.38 Mev- and 2.76-Mev gamma-rays a branching ratio of 1:2 has to be assumed.

This question has been resolved by Cook, Jurney, and Langer.[20] Their experiments appear to show that the gamma-ray of 2.76 Mev is in cascade with a lower energy gamma-ray, presumably one of 1.38 Mev. This experiment consisted in showing that an absorption curve in lead, taken with a single counter, had the same shape as an absorption curve of the coincidences, when lead absorbers were placed symmetrically between the source and each counter.

A somewhat different approach to the same problem, but leading to the same result, has been made by Wiedenbeck.[14] His argument is based on the fact that the efficiency of a counter for gamma-rays increases with the energy of the gamma-ray. Consider the energy level scheme in which the 1.38-Mev and 2.76-Mev gamma-rays are in cascade and no other gamma-rays are present. In the usual notation, the various counting rates are given by

$$N_\beta = N_0\omega_\beta F(x), \tag{12}$$

$$N_\gamma = N_0\omega_\gamma(e_{1.4}+e_{2.8}), \tag{13}$$

$$N_{\beta\gamma} = N_0\omega_\beta F(x)\omega_\gamma(e_{1.4}+e_{2.8}), \tag{14}$$

$$N_{\gamma\gamma} = 2N_0\omega_\gamma{}^2 e_{1.4}e_{2.8}. \tag{15}$$

It therefore follows that

$$\frac{N_{\beta\gamma}}{N_\beta} = \omega_\gamma(e_{1.4}+e_{2.8}), \tag{16}$$

and

$$\frac{N_{\gamma\gamma}}{N_\gamma} = 2\frac{e_{1.4}e_{2.8}}{(e_{1.4}+e_{2.8})}\omega_\gamma. \tag{17}$$

The ratio

$$R = \frac{N_{\gamma\gamma}/N_\gamma}{N_{\beta\gamma}/N_\beta} = 2\frac{e_{1.4}e_{2.8}}{(e_{1.4}+e_{2.8})^2}. \tag{18}$$

On the other hand, if there are two 1.4-Mev gamma-rays in cascade, and one 2.8-Mev gamma in parallel with these, the value of R becomes

$$R = 3(e_{1.4})^2/2(e_{1.4}+e_{2.8})^2. \tag{19}$$

If R is plotted as a function of $e_{2.8}/e_{1.4}$, for various values of $e_{2.8}/e_{1.4}$, from approximately 0.5 to 2, then for case (1), $R\sim 0.5$ over the entire range, but for case (2), R decreases very rapidly for $e_{2.8}/e_{1.4}$ going from 0.5 to 2.0. Wiedenbeck measured the values of R for counters of various materials, for which the efficiency ratio $e_{2.8}/e_{1.4}$ should change markedly and found that R was approximately constant and roughly equal to 0.5. He, therefore, concludes that there are only two gamma-rays

(2.76 Mev and 1.38 Mev) and these are in cascade.

Another element with a disintegration scheme of this type, but somewhat more complicated, is In^{116} (54 min.). Beta-gamma and gamma-gamma coincidences have been investigated by Langer, Mitchell, and McDaniel[21] who established that there was only one beta-ray which goes to an excited state of Sn^{116} followed by transitions to the ground state involving several gamma-rays. Curtis and Richardson[22] have investigated the spectrum of the beta-rays and gamma-rays, with the help of a cloud chamber, and give the disintegration scheme shown in Fig. 6. The disintegration scheme is so complicated that it is clearly not possible to determine the number of gamma-rays from coincidence methods alone.

(D.) More Complicated Types of Disintegrations

If two or more beta-ray groups and several gamma-rays are present, the analysis becomes much more complicated. It is possible to ascertain, however, whether the highest energy beta-ray group leads to the ground state or to an excited state of the product, except in cases when a high energy group leading to the ground state is of low intensity.

As an example, consider the case of Mn^{56}. Coincidence measurements on this element have been made by Dunworth,[23] Langer, Mitchell, and McDaniel,[24] Elliot and Deutsch,[25] and Siegbahn and Johansson[12] with identical results. For electrons of energy greater than about 1.0 Mev, the ratio of the number of beta-gamma coincidences per beta-count is a constant independent of absorber thickness, showing that the highest energy beta-ray does not go to the ground state of the product and that the beta-spectrum is complex. In addition, gamma-gamma coincidences are found, indicating several gamma-rays.

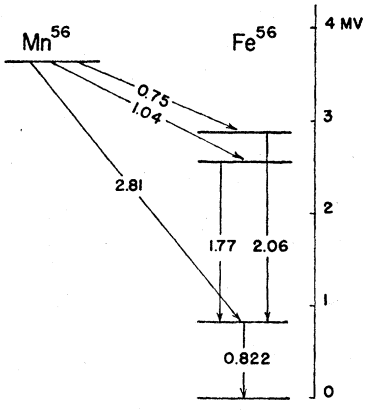

FIG. 7. Energy levels of Mn^{56}.

The actual energy level scheme of Mn^{56} has been worked out by Elliot and Deutsch and also Siegbahn and Johansson, with the use of a magnetic lens. The latter authors also used coincidence counting apparatus in connection with the lens, measuring coincidences between gamma-rays and the resolved beta-ray beam. The spectrum as given by these authors is shown in Fig. 7. It will be seen that the highest energy beta-ray does not lead to the ground state and that the beta-ray spectrum is complex, consisting of three groups. The number of beta-gamma coincidences per beta-ray should be constant beyond 1.04 Mev and should increase below this value, in agreement with the coincidence measurements. In addition, the observed gamma-gamma coincidences are accounted for.

IV. MEASUREMENT OF INTERNAL CONVERSION ELECTRONS

In certain cases the internal conversion coefficient of a partially converted gamma-ray can be determined by coincidence counting. The method was used by the author[26] during the war for the determination of the total number of conversion electrons per disintegration. It is particularly useful for cases in which the specific activity is not large enough to use magnetic analysis of the beta-ray spectrum. The method consists in measuring coincidences between internal conversion electrons and disintegration electrons, called particle-particle coincidences.

Since internal conversion electrons are of low energy, it is necessary to use thin walled counters —window thickness a few mg/cm^2. In addition, care must be taken to eliminate scattered electrons. For this reason it is best to mount the counters side by side, with the source above. It is also necessary to place a shield between the two counters so that an electron which makes a count in one counter cannot produce a secondary which counts in the other.

Consider the situation, for simplicity, of a substance like Au^{198} whose spectrum has been worked out. This substance has one beta-ray and

one internally converted gamma-ray. Let N_p be the number of particles per disintegration, disintegration electrons and internal conversion electrons; let N_{pp} be the number of coincidences per second between disintegration electrons and internally converted electrons; and let α be the internal conversion coefficient. Then

$$N_p = N_0(1+\alpha)\omega_\beta F(x), \tag{20}$$

$$N_{pp} = N_0\omega_\beta{}^2\alpha F^2(x), \tag{21}$$

$$\frac{N_{pp}}{N_p} = \frac{\alpha}{(1+\alpha)}\omega_\beta F(x). \tag{22}$$

Here $F(x)$ is a function denoting the absorption of the electrons as a function of the thickness. In the low energy region it is assumed that $F(x)$ for both disintegration and conversion electrons is the same and experiment shows that $F(x) \sim e^{-\mu x}$.

The procedure is to measure *both* N_p and N_{pp}/N_p as a function of the thickness (below that necessary to stop the internal conversion electrons), plot the results on semi-log paper, and extrapolate to zero thickness. In the experiments of the writer, the two curves were parallel, justifying the assumption concerning $F(x)$ made above. Extrapolation of these curves to zero thickness gives

$$\frac{N_{pp}}{N_p} = \frac{\alpha}{1+\alpha}\omega_\beta F(0). \tag{23}$$

In order to determine $\omega_\beta F(0)$, it is necessary to obtain a source which gives a known number of beta-particles/sec., measure the number of counts/sec. from it as a function of absorber thickness in the same geometry, and extrapolate this result to zero thickness. A thin source containing a weighed amount of U_3O_8 is convenient for this purpose. From these two pieces of information it is clear that α can readily be determined.

Since, in the case under consideration, beta-gamma coincidences can also be measured, the following additional equations hold.

$$N_\gamma = N_0(1-\alpha)\omega_\gamma e_\gamma, \tag{24}$$

$$N_{\beta\gamma} = N_0(1-\alpha)\omega_\beta F(x)\omega_\gamma e_\gamma. \tag{25}$$

Hence,

$$\frac{N_{\beta\gamma}}{N_p} = \frac{(1-\alpha)}{(1+\alpha)}\omega_\gamma e_\gamma \tag{26}$$

TABLE I. Disintegration schemes of isotopes worked out with help of coincidence methods.

Element	Radiations	Half-Life	Type	Energy levels product	References
Al^{28}	β^- 2.75 γ 1.80	2.4 min.	I	1.80	(11)
Mn^{54}	K γ 0.835	310*d*	I	0.835	(27)
Co^{58}	e^+ 0.470 K γ 0.805	72*d*	I	0.805	(27)
Au^{198}	β^- 0.92 γ 0.41	2.8*d*	I	0.41	(8), (9), (10)
K^{42}	e^- 3.58, 2.07 γ 1.51	12.4*h*	II	1.51	(12)
Sb^{122}	e^- 1.19 1.77 γ 0.568	2.8*d*	II	0.568	(1), (35), (36), (37)
Na^{24}	e^- 1.39 γ 1.38, 2.76	14.8*h*	III	1.38, 4.14	(13), (14), (15), (16), (20)
Co^{56}	e^+ 1.50 γ 0.845, 1.26	72*d*	III	0.845, 2.11	(25)
Br^{82}	e^- 0.465 γ 1.35, 0.787, 0.547	34*h*	III	1.35, 2.14 2.69	(28)
In^{116}	e^- 0.85 γ 1.8, 1.4 1.0, 0.57 0.36, 0.17	54 min.	III	0.17, 0.57 1.0, 2.4	(21), (22)
Sb^{124}	e^- 2.32, 1.58, 0.98, 0.63, 0.47 γ 2.064, 1.708 0.714, 0.650 0.603, 0.121	60*d*	IV	0.606, 1.33, 1.97 2.32, 2.43	(1), (36), (37)
I^{131}	e^- 0.595 γ 0.080, 0.367	8.0*d*	III	0.080, 0.447	(29)
Sc^{46}	e^- 0.36, 1.49 γ 0.88, 1.12	85*d*	IV	0.88, 2.00	(34)
Mn^{56}	e^- 0.75, 1.04 2.81 γ 0.822, 1.77, 2.06	2.5*h*	IV	0.822, 2.59, 2.88	(12), (24), (25)
Fe^{59}	e^- 0.260, 0.460 γ 1.30, 1.10	47*d*	IV	1.10, 1.30	(30)
Co^{60}	e^- 0.308 γ 1.10, 1.30	5.3*y*	IV	1.10, 2.40	(33)
Ga^{72}	e^- 3.09, 2.2 0.79 and others γ 0.64, 0.71, 0.84, 2.2 and others	14*h*	IV		Unfinished
As^{72}	Not complete	26*h*	IV		Unfinished
As^{76}	e^- 1.29, 2.49 3.06 γ 0.55, 1.20 1.75	26.8*h*	IV	0.55, 1.75	(1), (31)
I^{130}	e^- 0.61, 1.03 γ 0.537, 0.667, 0.744 0.417		IV	0.744, 1.411, 1.948, 2.364	(32)

for x small enough to allow conversion electrons to enter the counter. In addition, the relation

$$N_{\beta\gamma}/N_{\gamma}=N_0\omega_{\beta}F(x) \qquad (27)$$

can be combined with (22) and α obtained directly.

However, a more accurate result is obtained if the first method is employed since, in the second method, errors which arise on account of the use of *both* $N_{\beta\gamma}$ and N_{pp} are inherently larger than if only one of these is used along with the singles count of the U_3O_8 source.

Methods for more complicated schemes can be worked out along the lines suggested in the discussion of beta-gamma coincidences.

V. SUMMARY OF WORK TO DATE

The examples given in the text above were chosen as being illustrative of each type of disintegration under discussion. Altogether, disintegration schemes have been worked out for at least 21 radioactive isotopes with the help of coincidence counting methods. The results of the measurments can best be summarized in a table. For use in understanding Table I, the disintegration schemes are divided into types as follows: Type I, beta-ray (or positron) spectrum simple, followed by one gamma-ray, no particle emission to ground state (this includes K capture followed by a single gamma-ray); Type II, beta-ray (or positron) spectrum consisting of two groups, one of which leads to the ground state of the product, one gamma-ray; Type III, beta-ray (or positron) spectrum simple followed by more than one gamma-ray; Type IV complex beta- and gamma-spectra. The actual energy levels which have been worked out for these elements can be found in the references cited.

BIBLIOGRAPHY

(1) A. C. G. Mitchell, L. M. Langer, and P. W. McDaniel, Phys. Rev. **57**, 1107 (1940).
(2) S. C. Curran, P. I. Dee, and V. Petrzilka, Proc. Roy. Soc. **169**, 269 (1938).
(3) H. Becker and W. Bothe, Zeits. f. Physik **76**, 421 (1932).
(4) E. Bleuler and W. Zünti, Helv. Phys. Acta **XIX**, 375 (1946).
(5) N. Feather, Proc. Camb. Phil. Soc. **34**, 599 (1938).
(6) F. Norling, Ark. f Mat. Ast. och Fys. **B27(3)**, 9 (1941).
(7) A. F. Clark, Phys. Rev. **61**, 242 (1942).
(8) E. T. Jurney, Phys. Rev., in press.
(9) K. Siegbahn, Proc. Roy. Soc. **A, 189**, 527 (1947).
(10) C. L. Peacock and R. G. Wilkinson, in press.
(11) K. Seigbahn and A. Johansson, Ark. f Mat. Ast. och Fys. **34A**, No. 10 (1946).
(12) E. Bleuler and W. Zünti, Helv. Phys. Acta **XX**, 195 (1947).
(13) L. M. Langer, A. C. G. Mitchell, and P. W. McDaniel, Phys. Rev. **56**, 962 (1939).
(14) M. L. Wiedenbeck, Phys. Rev. **72**, 429 (1947).
(15) K. Siegbahn, Phys. Rev. **70**, 127 (1946).
(16) J. L. Lawson, Phys. Rev. **56**, 131 (1939).
(17) R. G. Sachs, Phys. Rev. **70**, 572 (1946).
(18) W. H. Barkas, Phys. Rev. **55**, 691 (1939).
(19) W. H. Barkas, Phys. Rev. **72**, 346 (1947).
(20) C. S. Cook, E. T. Jurney, and L. M. Langer, Phys. Rev. **70**, 985 (1946).
(21) L. M. Langer, A. C. G. Mitchell, and P. W. McDaniel, Phys. Rev. **56**, 380 (1939).
(22) B. R. Curtis and J. R. Richardson, Phys. Rev. **57**, 1121 (1940).
(23) J. V. Dunworth, Nature **143**, 1056 (1939).
(24) L. M. Langer, A. C. G. Mitchell, and P. W. McDaniel, Phys. Rev. **56**, 422 (1939).
(25) L. G. Elliot and M. Deutsch, Phys. Rev. **64**, 321 (1943).
(26) A. C. G. Mitchell, unpublished.
(27) M. Deutsch and L. G. Elliot, Phys. Rev. **65**, 211 (1944).
(28) A. Roberts, J. R. Downing, and M. Deutsch, Phys. Rev. **60**, 544 (1941).
(29) J. R. Downing, M. Deutsch, and A. Roberts, Phys. Rev. **61**, 686 (1942).
(30) M. Deutsch, J. R. Downing, L. G. Elliot, J. W. Irvine, Jr., and A. Roberts, Phys. Rev. **62**, 3 (1942).
(31) K. Siegbahn, Ark. f Mat. Ast. och Fys. **34A**, No. 7 (1946).
(32) A. Roberts, L. G. Elliot, J. R. Downing, W. C. Peacock, and M. Deutsch, Phys. Rev. **69**, 268 (1943).
(33) M. Deutsch, L. G. Elliot, and A. Roberts, Phys. Rev. **68**, 193 (1945).
(34) C. Peacock and R. G. Wilkinson, Phys. Rev. **72**, 251 (1947).
(35) C. E. Mandeville and M. V. Scherb, Bull. Am. Phys. Soc. **22**, 17 (1947).
(36) B. D. Kern, D. J. Zattarano and A. C. G. Mitchell (in press).
(37) C. S. Cook and L. M. Langer (in press).