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Statistical Factors in Spark Discharge Mechanisms

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As late as 1936, despite considerable study, no proper mechanism had been found describing the process of ordinary spark breakdown, one of the earliest phenomena known to man. The theory of Townsend, based on investigations at low pressures, had by 1923–1928 been proven inadequate since it required *formative* time lags of the order of 10^{-5} second while the observed values at atmospheric pressure were 10^{-7} second or less.[1, 2, 3] The writer, together with W. Leigh, had been investigating the paradox that in positive point corona in air at atmospheric pressure luminosity could be observed at distances from the point where fields were too low to cause excitation. In studying the situation, the writer observed fine electric blue streamers darting outward from the point which indicated a progressive projection of very high field strength regions from the positive point into the gap.[4] In reporting these results, together with interpretation, to the 210th meeting of the American Physical Society at Pasadena, December 1936, the writer pointed out that these streamers constituted a new type of breakdown process and that this process might also prove to be the missing mechanism of spark breakdown at higher pressures. Professor Millikan rose in discussion to state that he believed this observation to be of the utmost significance and that it appeared likely that a new era of investigation and understanding was being ushered in to replace the rather unsatisfactory classical explanations. Encouraged by this enthusiastic endorsement, the writer and his group of young research men engaged in a long sequence of intensive study, of the corona discharge, which by 1939 led to a fairly complete qualitative picture of the positive streamer mechanism and an evaluation of streamer properties.[5] In 1940, together with J. M. Meek, now of Liverpool University, the writer was able to place the streamer mechanism of the ordinary spark on a sound semiquantitative basis.[6] Quite independently, using C.T.R. Wilson cloud-chamber tracks of electron avalanches and interrupted sparks, H. Raether in Germany had arrived at a similar conclusion and by 1940 had succeeded in photographing the breakdown streamer.[7, 8] By 1941 he likewise formulated independently the nearly identical, semi-empirical quantitative criterion for this breakdown mechanism as had been inferred by Meek. Thus Professor Millikan's forecast has been completely substantiated by the fact.

In what follows, it is intended to discuss both Townsend and streamer mechanisms from what should long have been recognized as the obvious and direct approach, but hitherto has been ignored. It has proven very fruitful not only in clarifying many phenomena associated with spark breakdown heretofore obscure, but has also led to a more correct formulation of the criterion for the streamer advance. To present this approach one must begin by defining a spark.

In the most general terms, electrical sparks may be defined as a class of *transient* occurrences in which a given *existing conduction current* in a gas *suddenly and irreversibly changes* to what would be *a current of higher magnitude*, circuit constants permitting, the new current operating more efficiently by different mechanisms under the imposed conditions which rendered the lower

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[1] L. B. Loeb and J. M. Meek, *Mechanism of the Electric Spark* (Stanford University Press, Stanford University, California, 1941).

[2] L. B. Loeb, *Fundamental Processes in Electrical Discharge in Gases* (John Wiley and Sons, Inc., New York, 1939).

[3] Reference 1, p. ix ff., and reference 2, pp. 409, 425, and 449.

[4] L. B. Loeb and W. Leigh, Phys. Rev. **51**, 149(A) (1937).

[5] L. B. Loeb and A. F. Kip, J. App. Phys. **10**, 142 (1939). Also reference 1, p. 514 ff, and 426.

[6] J. M. Meek, Phys. Rev. **55**, 972 (1939); L. B. Loeb and J. M. Meek, J. App. Phys. **11**, 958 (1940), also reference 1, Chapter II.

[7] H. Raether, Zeits. f. Physik **112**, 464 (1939); *ibid.* **117**, 375, 524 (1941).

[8] H. Raether, Archiv. f. Elektrotek. **34**, 49 (1940).

current unstable. The broader definition is needed to cover all sparks. Most sparks commonly observed however, consist in the change from a field intensified ionization current or corona of low current magnitude to either a glow discharge or a power arc. It is sufficient and convenient to limit this discussion to such sparks.

Spark breakdown requires fields X of sufficient intensity and length δ at a given pressure p to insure that a *primary* and a *secondary* process are activated. The primary process consists of a multiplication of electrons in the field by ionization by electron impact, such that a single electron traversing the gap length δ will create a number of electrons $e^{\alpha\delta}$, or in a non-uniform field, $e^{\int_0^\delta \alpha dx}$, of sufficient magnitude as indicated by Townsend.[9] In this expression α is the *first Townsend coefficient*, which gives the average number of *new* electrons created in 1-cm advance of an electron in the field. The quantity α has been evaluated for a number of standard gases as a function of the ratio of the field strength X to pressure p, i.e., $\alpha/p=F(x/p)$. The quantity $1/\alpha$ represents the average distance advance in cm to create a new ion pair.

Alone this process leads to field intensified electron currents which are proportional to the initial number of electrons n_0 starting at the cathode, or $x=0$. The progeny of $e^{\alpha\delta}$ electrons, produced by one electron in traversing the gap δ, is called an *electron avalanche.*

The secondary process which can take one of a number of forms provides a source of new initiating electrons after the electrons in the n_0 avalanches have been drawn to the anode.[10] Three established mechanisms involving the *cathode* are:

1. Electron liberation by positive ion impact on the cathode.
2. Photoelectric liberation of electrons at the cathode by short wave-length photons accompanying the avalanches.
3. In certain gases electron liberation by the action of metastable atoms on the cathode.

Another and equally fundamental mechanism ignored until about 1934–1936, and very important at higher pressures, is photoelectric ionization *in the gas* by *very short* wave-length photons accompanying the avalanches.[11] The cathode mechanisms lead to a breakdown of the Townsend type while the ionization in the gas leads to streamer formation.

As regards the various cathode mechanisms leading to a Townsend-type discharge, the liberation of electrons by positive ion impact is known to be active in many cases and quantitatively yields an effective mechanism. The photoelectric effect is known to occur but in spark breakdown it is doubtful if it is very effective. This primarily is due to geometrical dispersion and absorption of photons largely created near the anode. Very few of the active photons from an avalanche can reach the immediate point of emission of the initiating electron so as successively to build up the first avalanche to a spark. It may act in special cases such as in very short gaps and at very low pressures. Metastable atoms in virtue of their diffusive movements suffer a similar disadvantage of dispersion and are limited to a few pure gases only. Thus evidence seems to point primarily to the positive ion impact on the cathode as the only agency in Townsend spark discharge. Aside from this, however, all the agencies yield an expression for the resulting current so similar in form to that for the impact of positive ions that the limited range of data available does not allow us to distinguish quantitatively between the processes. Ionization by impact of *positive* ions *in the gas* cannot be effective under most experimental sparking conditions studied.[12]

It will, therefore, be assumed that a positive ion from an avalanche returning to the cathode from the neighborhood of $2/\alpha$ cm from the anode, beyond which $\frac{3}{4}$ of these ions are formed, will have a *chance* γ of liberating a secondary electron on impact on the cathode. Under these conditions Townsend and J. J. Thomson have shown that the current observed is

$$i=i_0(e^{\alpha\delta}/1-\gamma e^{\alpha\delta}),$$

when expressed in simplified form.[13]

It is now important to discuss this equation

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[9] J. S. Townsend, Nature **62**, 340 (1900); Phil. Mag. **1**, 198 (1901); see also reference 2, Chapter VIII.

[10] Reference 2, Chapter IX, pp. 377, 379, and 403.

[11] Reference 1, p. 37 ff.; also references 3 through 7.

[12] L. B. Loeb, R. N. Varney and W. R. Haseltine, Phil. Mag. **29**, 379 (1940); also reference 2, p. 374 ff.

[13] Reference 2, pp. 377, 380, and 409; reference 1, p. 2.

and draw certain important conclusions which can most fruitfully be extended to the streamer mechanism as well, and constitutes the desired approach.

Returning to the more limited definition of a spark, it will be noted that this usually marks the transition from a condition of a steady-state, dark, field intensified multiplication of an externally imposed ionization current to the luminous glow discharge or even arc currents many orders of magnitude greater. For a given uniform gap of length δ, with an externally conditioned liberation of n_0 electrons from the cathode, following the Townsend mechanism involving a γ with electrons of charge ϵ the dark current i, to a satisfactory order of accuracy is, as stated, given by

$$i = n\epsilon = n_0\epsilon e^{\alpha\delta}/(1-\gamma e^{\alpha\delta}). \qquad (1)$$

Now glows or arcs are self-sustaining and operate economically as a result of secondary emission conditioned largely by a space charge field acting near the cathode directly in the glow or acting indirectly by causing a thermal electron emission from, or near, the cathode in the case of arcs. The exact mechanisms are not important but what is important is the space charge accumulation of positive ions near the cathode that makes such a mechanism possible. Thus, when by means of actions to be discussed the space charge fields of accumulated positive ions reach a given magnitude, we have achieved the transitions called the spark.[14] These transitions take place in a very short time interval in most cases, and the tempo of growth events increases rapidly to breakdown as space charges accumulate.

With this understanding one may regard the expression

$$n = n_0 e^{\alpha\delta}/(1-\gamma e^{\alpha\delta})$$

existing before breakdown, and note that when

$$\gamma e^{\alpha\delta} = 1 \qquad (2)$$

this expression becomes indeterminate mathematically. It was originally used by Townsend as the sparking condition.[15] However, as Holst long ago showed,[16] $\gamma e^{\alpha\delta} = 1$ has another interpretation, as indicated below, which is not indeterminate.

(a) For $\gamma e^{\alpha\delta} < 1$ the current follows Eq. (1) and is not self-sustaining, i.e., it depends on n_0.

(b) For $\gamma e^{\alpha\delta} = 1$, *on the average*, each avalanche of $e^{\alpha\delta}$ electrons produced by this primary process is multiplied by a γ large enough to give *one new* secondary electron when the $e^{\alpha\delta}$ electrons return to the cathode. *One initiating electron* at the cathode is on the average then able to *maintain its succession* indefinitely. This then is the *threshold* for a *self-sustaining discharge* independent of n_0. Its marks the *sparking threshold.* Through the effect of the applied potential V across the uniform gap δ on X, from the relation

$$\alpha/p = f(X/p) \qquad (3)$$

and

$$X = V/\delta, \qquad (4)$$

this condition fixes V_s the sparking potential.[15]

For $\gamma e^{\alpha\delta} > 1$ the ionization of successive avalanches is cumulative and more ions are created than start. As electrons have mobilities of the order of a hundred times that of the positive ions, positive ions space charges accumulate in the gap. When these reach a value allowing a glow or arc to operate the spark is complete.[14] The space charge will materialize the faster the more $\gamma e^{\alpha\delta}$ exceeds 1. Townsend early found that at low pressures one can from the conditions $\gamma e^{\alpha\delta} = 1$ and the relations (3) and (4) and the values of γ observed, evaluate V_s in agreement with experiment.[15,17]

The Townsend mechanism of the spark actually proceeds as follows: With $\gamma e^{\alpha\delta} \geqslant 1$, *one* of the initiating electrons crosses the gap forming an avalanche in a time T_e at a speed of about 10^7 cm/sec. in sparking fields at atmospheric pressure in air. Arrived at the anode the $e^{\alpha\delta}$ electrons are absorbed by the anode and 50 percent of the positive ions created are found within $1/\alpha$ cm of the anode with 75 percent within $2/\alpha$ cm of the anode. These start back towards the cathode with a speed about 10^5 cm/sec. and arrive there at T_+ which is about 100 times T_e. On their arrival at the cathode $\gamma e^{\alpha\delta}$ new electrons are

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[14] R. Schade, Zeits. f. Physik **104**, 487 (1937); also reference 1, p. 12.

[15] J. S. Townsend, Phil. Mag. **3**, 557 (1902); see also reference 2, pp. 410 and 421.

[16] G. Holst and Oosterhuis, Phil. Mag. **46**, 1117 (1923); reference 1, p. 6.

[17] Reference 2, p. 416.

created yielding $\gamma e^{\alpha\delta}$ new avalanches following closely along the path of the first one which spread laterally only by diffusion. On arrival at the cathode, the resulting $\gamma e^{2\alpha\delta}$ positive ions form $\gamma^2 e^{2\alpha\delta}$ new electrons after an added interval $T_e + T_+$. When, as a result of η such sequences the $\gamma^{\eta-1}e^{\eta\alpha\delta}$ positive ions have produced a sufficiently heavy space charge, the self-sustaining glow or arc is achieved. It must be noted that as the number of trips increases and the space charge is building up, the distortion produced may of itself materially increase the effectiveness of the ionization, so that the later avalanches will produce many more than $e^{\alpha\delta}$ electrons, thus shortening the time. The time of spark formation, i.e., the *formative time lag* is then $T_f = (T_+ + T_e)\eta$ at the longest, for η is decreased if $e^{\alpha\delta}$ is increased. If the secondary mechanism involved photoelectric ionization *at* the cathode T_f would *not* have involved T_+ as photons travel to the cathode with the speed of light. Thus T_f would have been ηT_e. As stated before, however, what evidence we have to date is that owing to geometric diffusion of photons this faster mechanism does not often occur. The building up of a Townsend spark at low pressures outlined above has been nicely verified experimentally by R. Schade for Ne and H_2.[14] There T_f has been observed to vary from 10^{-1} second to 5×10^{-5} second as the voltage rises from near the sparking potential V_s to $2\,V_s$, thus raising $e^{\alpha\delta}$ considerably. Thus it is noted that near the threshold $\gamma e^{\alpha\delta} = 1$, T_f can be very long because of η and it will decrease to a minimum of $T_f = T_+ + T_e$ as a limit as $\eta e^{\alpha\delta}$ increases above 1 and η approaches unity. With this as a basis, one may reason further as follows.

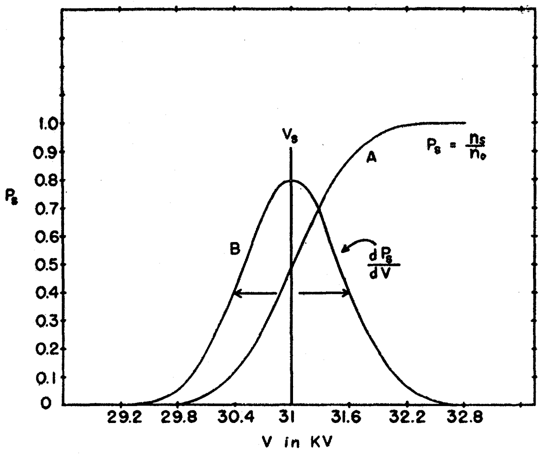

FIG. 1.

Under most sparking conditions $\alpha\delta$ has a relatively small value ranging from 10 to 20. Furthermore, ionization is a chance phenomenon. Thus an individual electron will not ionize regularly every $1/\alpha$ cm of advance, for α is an average value over many ionizing acts. Some electrons will ionize twice in $1/\alpha$ cm, others will ionize only in $2/\alpha$ cm or more. Such fluctuations early in an avalanche sequence can produce large changes in η. Thus individual avalanches will yield electron multiplication fluctuating well above and below the average observed $e^{\alpha\delta}$ in a purely random fashion. Hence if $\alpha\delta$ is greater or less by unity than the average value, $e^{\alpha\delta}$ for the same distance will be 2.7 times or 0.37 as great as the *average* value fixing the threshold. Accordingly, individual electron avalanches yield electron multiplication fluctuating well above and below the average value $e^{\alpha\delta}$.

Again the liberation of secondary electrons by positive ion impact on the cathode, given on the average by the *probability* γ, is also subject to fluctuation in individual avalanches. Thus while on the *average* for 100 positive ions γ may take on values of the order 2 to 10 electrons under different conditions, sometimes there will be no electrons for 100 ions. At other times there will be more than the average. Thus in *individual* avalanche sequences $(\gamma e^{\alpha\delta})_1$, as observed, will take on values above and below the *average* given by $\gamma e^{\alpha\delta} = 1$. This will have the result that at values of the applied potential V well below V_s, the sparking threshold given by an *average* $\gamma e^{\alpha\delta} = 1$, an occasional spark will pass by a fortunate combination $(\gamma e^{\alpha\delta})_1$. Likewise, at values of V above V_s as given by the *average* $\gamma e^{\alpha\delta} = 1$, unlucky sequences will fail to give a spark. Thus if one counts the number of electrons n, out of n_0 initiating electrons which cause a spark at any given field strength, and therefore applied potential V, the quantity $P_s = n/n_0$, the *probability if a spark*, plotted against applied potential will have the form shown in Fig. 1A. This curve is the integral from $V = 0$ to $V = V$ of the fluctuations of the value of $(\gamma e^{\alpha\delta})_1$ for individual intiating electrons about the average value

$\gamma e^{\alpha\delta}=1$, fixing V. Differentiation of the curve of Fig. 1A gives the chance of a variation of $(\gamma e^{\alpha\delta})_1$ in the sequence of avalanches about a mean value expressed in terms of V_s. Such a curve is shown in Fig. 1B. If the derivative gives a symmetrical distribution, V_s will be at the peak and in the integrated curve it will represent the point of inflection at $P_s=0.5$, where half the initiatory electrons cause a spark.

Hence for the first time we can define the *actual sparking threshold* as the point at which 50 percent of the initiating electrons lead to a spark. This was first noted by R. R. Wilson.[18] Heretofore the experimental sparking threshold criteria have been varied and vague.[19] A few comments about this curve are in order. While the value of $(\gamma e^{\alpha\delta})_1$ can undergo very wide fluctuations about the mean value, the value of α varies so rapidly with X and thus with V for a fixed δ over the range of sparking fields used that it takes only a rather narrow range of V about V_s to compensate for it. It is very difficult correctly to calculate the width of the curve of Fig. 1B at half-amplitude from a combination of fluctuations in γ and $e^{\alpha\delta}$, although a calculation should be possible.* R. A. Wijsman has rigorously calculated the fluctuation $P(n)$ about a value $\bar{n}=e^{\alpha\delta}$ and finds $P(n)=(1/\bar{n})e^{-n/\bar{n}}$ for $\bar{n}\gg 1$. Thus for $\overline{\alpha\delta}=17$, the chance of $\alpha\delta=18$, is e^{-e} or 0.067 and of $\alpha\delta=16$ is $e^{-(e^{-1})}$ or 0.69. With the rapid variation of α with X the variation of P_s with V will be very rapid. Thus the curve will in general be very narrow about V_x if we consider one multiplying sequence only, i.e., $\eta=1$ in Eq. (5). If η is larger, the variation will be somewhat greater. It will not be much larger since it is in the first avalanche with an average of about only one secondary electron per avalanche that the effect is critical. That such an effect actually exists was shown for the first time by R. R. Wilson.[18] Using sparks occurring in a given time lag interval with constant i_0 he observed the fraction of the times that a spark appeared when a given voltage was applied. The curves he obtained for percentage sparks against voltage were analogous to Fig. 1A. While the cause for the fluctuation observed was at that time suspected, the nature of the measurements did not permit of the simple analysis in terms of the theory outlined above since individual electrons could not be observed. Using these curves, he fixed V_s as the point of inflection at 50 percent sparking and thus for the first time was able to fix the sparking threshold and relate time lag of sparking to voltage. It must finally be noted that data of Wilson actually applied to sparks passing by streamer mechanisms and not the Townsend mechanism under discussion. As will later be seen, this circumstance does not alter the discussion in principle. From this it is clear that in an accurate study of sparking potential thresholds measurements must first be conducted giving P_s as a function of V and thus evaluating V_s. This had not clearly been recognized before Wilson's time and would not have been discovered but for the spread of sparking potentials revealed by his technique.

It must next be noted that under most circumstances the spark can be initiated by a single electron and, except at very low pressures, sparks are usually initiated by single electrons from the cathode. Thus it is not surprising to note that when a potential near V_s or above is applied to a gap, a spark does not materialize at once. The time delay observed was called the *time lag* of sparking. We shall call it the *observational time lag*, T_0. In 1925, Laue and Zuber[20] showed that this lag was composed of a statistical time ele-

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[18] R. R. Wilson, Phys. Rev. **50**, 1082 (1936).

[19] Reference 2, p. 463.

* An attempt at an analysis of this problem using Townsend's mechanism was made by Braunbeck in 1926 and amplified by Hertz in 1937. (See reference 35.) It is believed that these analyses, based on considerations at low pressures, are not correct. They fix $e^{\alpha\delta}$ as about 50–500 and assume fluctuations of this quantity unimportant, considering only the more obvious fluctuations of γ. In the rough analysis above γ contributes much less than $e^{\alpha\delta}$, i.e., at most ±1 percent in potential variation. Actually for most sparks $e^{\alpha\delta}$ is very large, of the order of 10^7 at higher pressures and it is *not* $e^{\alpha\delta}$ that fluctuates *directly*. There are a sequence of a few, 10–20, chance determined ionizing events giving $\alpha\delta$. A variation of ± 1 about the average $\alpha\delta$ will increase $e^{\alpha\delta}$, by e or e^{-1} fold. This small fluctuation already is equally potent with γ in influencing the spark. Again it is assumed by Hertz that if $\gamma e^{\alpha\delta}$ is *less* than unity there will be no spark. This is correct as fluctuations *above* and *below* the average value are equally probable. If the *spark* depended on avalanche succession only, Hertz's treatment would be correct. However, it materializes through space charge formation which influences subsequent events. Thus sparks *can* occur below the average value. It is, however, not certain that one can identify V_s, the threshold, as the peak of the curve under these conditions. Obviously careful reconsideration is needed.

[20] M. Laue, Ann. d. Physik **76**, 261 (1925); K. Zuber, Ann. d. Physik **76**, 231 (1925); reference 2, p. 441; reference 1, p. 20.

ment T_s depending on the chance appearance of an initiating electron at the cathode and the chance that this electron could cause a spark, as well as on a formative time lag T_f which is the time taken for a spark to grow. Accordingly, one may write

$$T_0 = T_s + T_f. \qquad (6)$$

In turn, T_s depends on T_a, the average time for an electron to appear near the cathode. This is on the average given by $T_a = 1/n_0$, where n_0 is the number of photoelectrons leaving the cathode, or created in the gap by some outside agency per second. T_s, as noted, also depends on P_s, the chance that an electron will give a spark, which is a function of potential V. Thus one must write

$$T_s = T_a/P_s = 1/n_0 P_s. \qquad (7)$$

Hence

$$T_0 = 1/n_0 P_s + T_f. \qquad (8)$$

On Townsend's theory $T_f = (T_e + T_+)\eta$. With a gap length δ, field strength X, and ion and electron mobilities K_+ and K_e, respectively, we can write $T_+ = \delta/XK_+$, and $T_e = \delta/XK_e$ and, accordingly,

$$T_f = \delta/XK_+ + \delta/XK_e. \qquad (9)$$

Thus for Townsend type sparks we write

$$T_0 = 1/n_0 P_s + (\delta/XK_+ + \delta/XK_e)\eta. \qquad (10)$$

Actually as space charges accumulate, α increases near the cathode, ionization is increased, and the number of trips, η, may be materially reduced. It was noted above that as over-voltage, usually expressed by $(V - V_s)/V$, increases, η approaches unity and T_f approaches a lower limit $T_{f1} = (\delta/XK_+) + (\delta/XK_e)$ with δ/XK_+ the important element as $K_e \gg K_+$.

Hence, for such sparks T_f approaches a lower limit at moderate over-voltages, while T_f may be very long at $V = V_s$ with large η. Thus the Townsend sparking mechanism is characterized by a large T_f which reduces to a minimum value characteristic of δ/XK_+ at moderate over-voltages. In the past in the ignorance of these conditions most measurements of sparking potentials and of time lags have been made with over-voltages since V_s could not be fixed and exigencies of measurement set quite arbitrary instrumentally conditioned limits for the threshold.[19] The value of T_s depends on n_0 and on P_s. P_s also increases and T_s decreases as V increases above V_s. Below V_s, T_s can be very long as P_s becomes very small. Accordingly, to study T_s and P_s, V_s must be determined, T_f must be made small compared to T_s, and n_0 must be made small enough to observe conveniently. If n_0 is too small analysis cannot be pushed far below V_s. An accurate study of P_s is then only possible when a relatively small n_0 and sufficiently large values of V are used so that T_f is not long enough to mask T_s. In all work n_0 must be kept low enough so that the space charge accumulation due to neighboring avalanches does not distort the field, thus causing breakdown materially below V_s. Such lowering was observed by Posin, Varney, Loeb and White, by White, and by Rogowski, Walraff, and Fuchs, and others.[21] It should be indicated that n_0 can now be measured even with much cumulative ionization by evaluating the constants of J. J. Thomson's photoelectric current equation in the presence of a gas at two potentials below ionization and extrapolating n_0 up to V_s.[22]

So far, the mechanism of the spark has been developed in terms of the Townsend sparking mechanism for which to date only the studies of Townsend[15] on V_s and of Schade[14] on time lags apply. Practically all other investigations have been made under conditions of pressure and gap length where Townsend's considerations and a cathode mechanism fail to apply. For, beginning in about 1923, it was observed that T_f for 1-cm sparks near atmospheric pressure was of the order of 10^{-7} sec. which is not compatible with the 10^{-5} sec. to be expected from $T_f = \delta/XK_+$ for the Townsend discharge.[23] Furthermore, such sparks occurred at higher pressures when γ, or its equivalent, was so low that it could not be measured and when the fields at the cathode were so low that such mechanisms could not occur. It was also observed that V_s was inde-

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[21] See reference 2: references 4, 22, 23 (pp. 448, 449); see also reference 1: references 29, 30 (p. 32), 31, 32 (p. 105), and ff. p. 143.

[22] Reference 2, p. 313; G. W. Johnson, Phys. Rev. **73**, 284 (1948).

[23] Reference 1: references 19, 21, 22 (p. 27).

pendent of the nature of the cathode surface. Again the familiar crooked and branched sparks, as well as midgap spark breakdown observed with Kerr cell shutters above certain pressures and gap lengths, could not be accounted for on Townsend's mechanism. As indicated in the introduction, these difficulties were clarified by the development of the streamer mechanism.[5–8]

In this mechanism an electron avalanche from one appropriately placed initiating electron progresses across the gap to the anode, leaving its positive ions behind in slow motion toward the cathode. As the electrons progress, the positive space charge density left behind begins to reach such dimensions that its own electrostatic field becomes commensurable with the imposed sparking field strength. This seriously distorts the field along the avalanche axis in such a fashion that at the cathode end of the space charge any electrons created near its surface by *photoelectric ionization* of the gas are drawn towards it in fields of very high intensity. In consequence, locally α is so much increased that by the time the electrons of the photoelectron-produced avalanche have been absorbed into the space charge, the tip has been extended towards the cathode. Thus the positive space charge streamer tip advances towards the cathode from anode or midgap, electrons streaming up it, towards the anode, thus further enhancing field distortion and accelerating the process.

The role of photoelectric ionization in spark discharge was anticipated by Cravath in 1935[24] and has recently received strong support from absorption studies for short wave-length photons.[25] Many data have accumulated on streamers including ion densities, diameters, and velocity of propagation through corona studies[5] and through the work of Raether,[7,8] Allibone and Meek, Meek and Craggs, and others,[26] so that there is no question as to the general validity of the concept.

Conditions essential for streamer propagation have been generally qualitatively established although the data and details on photo-ionization are lacking. These are: (1) that the density and character of the gas must be such that sufficient photons of high energy are produced in the avalanches to photo-ionize some atoms or molecules present; (2) that these be absorbed to produce ionization in adequate proximity of the tip;[27] and (3) that the space charge tip field be great enough sufficiently ahead of the streamer tip to give adequate avalanches in the enhanced fields to cause propagation.[6–8,28] In the absence of any quantitative data on (1) and (2), conditions with mixed gases like air were usually assumed adequate when (3) was fulfilled.[27,28] Thus Meek and Raether chose to use condition (3) as the basis for a quantitative criterion for the threshold condition. Independently and arbitrarily they both set the condition that the streamer tip field reach a value of the order of magnitude of the imposed sparking field.[6,7] To calculate the ion density, and hence space charge tip fields, and thus to derive a quantitative theory, Meek and Raether used the diffusion-conditioned radius $\bar{r}$ of the streamer channel as giving a radius within which the positive space charge was confined. The theory derived is thus rudimentary in that it omits the essential items (1) and (2) of photo-ionization and absorption about which only some qualitative facts are known. The writer had analyzed the consequences to be expected when, because of too much diffusion at low pressures and low values of α for very long sparks, photo-ionization and ion density become inadequate, but he could not bring these into the sparking equation.[29] Thus none of the analyses above are really satisfactory, and it remained for the statistical approach used above to indicate a proper solution in the Townsend mechanism. Before proceeding with this analysis it must be indicated, however, that the analogy is largely formal since Townsend's mechanism differs radically from the streamer mechanism in that it depends on the *number of ions* produced while the streamer theory depends on the *ion density*. Furthermore, the character of space charge formation and transition to the

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[24] A. M. Cravath, Phys. Rev. **47**, 254(A) (1935); reference 1, reference 2, p. 31.

[25] E. A. Schneider, J. Opt. Soc. Am. **30**, 128 (1940).

[26] T. E. Allibone and J. M. Meek, Proc. Roy. Soc. **A166**, 97 (1938); **A169**, 245 (1938); J. M. Meek, Inst. El. Eng. London **19** (Feb. 1942); J. D. Craggs and J. M. Meek, Proc. Roy. Soc. **A186**, 241 (1946); and further studies currently being published on sparks and their development.

[27] Reference 1, pp. 37, 50 and 76.

[28] Reference 1, pp. 41 and 50.

[29] Reference 1, p. 71.

new current is very different and much more violent and abrupt than with Townsend's mechanism. In the streamer mechanism the space charge *density* and *photo-ionization* determine *streamer advance*. Breakdown of the gap into a spark is accomplished by the progress of the filamentary conducting streamer channel to the cathode. Its field distortion and photoelectric ionization then approaches so closely to the cathode that the flood of electrons released on junction at this point produce violent potential distortions in the channel. In consequence, a steep potential wave sweeps up the channel, which is already partially conducting, and ionizes almost completely the molecules in the path. This potential wave sweeps up the channel at velocities estimated as high as 10^{10} cm/sec., on the basis of moving lens or film studies of lightning return strokes and long sparks.[26,30] The intense ultra-adiabatic ionization of the streamer channel in the return stroke gives the intense light, heat, and noise of the spark. Whether a glow discharge or an arc ultimately materialize from such a spark, or whether it just goes out depends on external circuit constants such as capacity, induction, and resistance. In any case, the spark materializes in a different form and under radically different conditions from those existing in the Townsend discharge.

As it applies to this discussion, streamer advance and spark breakdown, then, in essence, depend on applied potentials V_s and field X_s, necessary to give a total field ahead of the tip sufficiently great over an adequate small volume, Δx^1 deep, and of solid angular aperture of $\pi/2$, so that at least one photoelectron, produced in this volume by photons from the tip in advancing to the tip, can extend the space charge by making[8]

$$e^{\int_{\Delta x'}^{0}\alpha' dx} = e^{dx}. \tag{11}$$

Here x is the length of path needed to produce a streamer propagating space charge density in the imposed field, X_s, at V_s where the first coefficient is α. The quantity α' is the value of α in the combined vector field $X_1 = X + X^1$, with X^1 the space charge field. If several, n_1, photoelectrons are produced in Δx^1, then

$$n_1 e^{\int_{\Delta x'}^{0}\alpha' dx} = e^{\alpha x}$$

must equal

$$e^{dx},$$

which reduces the fields needed. Now, as stated, the creation of photoelectrons depends on the number of short wave-length photons capable of ionizing the gas which is produced, together with the chance that they give adequate ionization ahead of the streamer tip to produce advance. Thus the streamer advance depends on the number of active photons in the advancing tip, the range $\Delta x'$ of an adequate space charge field strength zone ahead of the tip, and the absorption of the photons to photo-ionization in this zone. Now the active photon production accompanying the ionization $e^{\alpha x}$ in the avalanche is not known. However, since it involves the production of photons on a level of energy in the neighborhood of the ionizing potential, and since the photons are produced by electron impact, it certainly must be proportional to the number of electrons at any point in the avalanche. That is, it can be set as roughly $fe^{\alpha x}$. Here f is a numerical factor characteristic of the gas which can vary slightly with X/p and, hence, with the existing electron energy, and may be greater or less than unity. Again the advance of the streamer will depend on the *chance* ϵ that one or more of these photons ionize sufficiently in the high field volume element characterized by $\Delta x'$. This chance, in turn, depends on the value of $e^{\alpha x}/\bar{r}^3$, defined by Eq. (11), which is a function of the space charge density determining $\Delta x'$, and on the absorption coefficient μ for photo-ionization. Here $\bar{r}$ is the diffusively determined average radius of the avalanche head where streamers form.[6,7,28]

The statistical fluctuation entering into ϵ comes from the probability of photo-ionization as well as fluctuations in $e^{\alpha x}/\bar{r}^3$. Hence, we can again write that the growth of a streamer, and thus a spark, is set by a condition

$$\epsilon f e^{\alpha x} = 1, \tag{12}$$

which marks the sparking threshold. This *new threshold* criterion places the Meek-Raether condition as a factor affecting ϵ and still permits its

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[30] B. F. J. Schonland and H. Collens, Proc. Roy. Soc. **A143**, 654 (1934); also reference 1, p. 106. F. H. Mitchell and L. B. Snoddy, Phys. Rev. **72**, 1202 (1947).

rough application. It goes further, however, in now including photon production which in the past was ignored quantitatively but was assumed to be adequate. Obviously it is incomplete in detail as ϵ must be replaced by a complicated relation defining $\Delta x'$ and including the absorbing cross section defined by μ.

In form, then, the criterion superficially resembles the Townsend criterion. It differs radically from it in that it is through μ critically dependent on pressure p as well as X/p and, further, in that it depends on $e^{\alpha x}/\bar{r}^3$ which is a *concentration* that is also dependent on pressure, and not just a *number* of ions. This will have as a secondary consequence the condition that sparking thresholds dependent on it will *not* conform to the similarity principle and that Paschen's law will not be strictly obeyed, while it is obeyed in Townsend's mechanism.[28]

However, from the formal similarity of the threshold equation $\epsilon f e^{\alpha x}=1$ and Townsend's condition $\gamma e^{\alpha\delta}=1$, it is clear that the streamer mechanism will show the same sort of variation with applied potential about the sparking threshold as is shown by the Townsend mechanism. The spark will depend on the appearance of *one initiating electron* from an appropriate point in the gap and on the chance P_s that at the potential V the quantity $\epsilon f e^{\alpha x}=1$. It can now, however, partake of zigzagged or branched paths and of midgap breakdown because of the nature of streamer advance as has been indicated elsewhere.[31] Sparks will thus sometimes appear at values of V less than that given by the value V_s set by the average $\epsilon f e^{\alpha x}=1$ and will sometimes not appear above V_s. P_s will vary with V as in Fig. 1. The statistical time lag will be determined by $T_s=1/n_0P_s$. P_s will, however, now be of a different character, depending on ϵ, and will vary with pressure as well as X/p and V.

On the streamer mechanism there will also be a formative time lag T_f which will now depend on three factors. First, the avalanche must traverse the appropriate distance x which will be approximately the gap length δ at V_s, but will rapidly take on a smaller value x above V_s as α increases rapidly with X and to form a streamer it is only required that $\epsilon f e^{\alpha x}=1$.

The streamer formation will take a time given by x/XK_e. Then the streamer must move back to the cathode. The velocity of streamer advance owing to field distortion and photoelectric ionization in advance to the tip, will be greater than for avalanche advance, as Raether had indicated by theory and direct observation.[8] This time interval can be set, following Raether,[8] by

$$x/bXK_e. \tag{13}$$

Here b is a factor given by $(X_1/X)^{\frac{1}{2}}$, the square root of the ratio of the enhanced tip field to the undistorted field, multiplied by $(\Delta x'+\bar{r})/\Delta x'$, where $\Delta x'$ is the depth of the sensitive zone and $\bar{r}$ is the radius of the streamer tip, so that

$$b=(X_1/X)^{\frac{1}{2}}(\Delta x'+\bar{r})/\Delta x'. \tag{14}$$

This, according to Raether,[8] may reach values lying between 2 and perhaps 10. Finally it will depend on the time taken for the return stroke T_p, i.e., for ionization of the streamer channel by the potential wave. This is shorter by one order of magnitude or more than the other intervals and can be neglected. Hence, for streamers

$$T_f=\frac{x}{XK_e}+\frac{x}{bXK_e}+T_p. \tag{15}$$

Thus, sensibly, T_f will be of the order of $T_f=x/XK_e$, which is the order of the 10^{-7} second or less, experimentally observed. With increasing values of V above V_s, at which point $x=\delta$, the time T_f will then *decrease* indefinitely proportionately to the ratio of x/δ, since with midgap breakdown the time is set by the streamer advance to the cathode, the remaining region, $\delta-x$, toward the anode breaking down simultaneously by further anode streamer action in the distorted field. Hence, with streamers breakdown will occasionally occur at values of V below V_s and 50 percent of the initiating electrons may cause breakdown at V_s. The time involved will primarily be given by $T_f=\delta/XK_e$ and will thus decrease with increasing X and V, at fixed δ in the narrow region of observed sparks below V_s, as XK_e increases. Above V_s the time of breakdown will *rapidly* decrease as x decreases below δ and as X increases. Experimentally, these anticipated results had been observed at values

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[31] Reference 1, p. 58 ff.

of V above V_s, the latest and most complete of these coming from the studies of R. R. Wilson[18] and H. Raether.[7]

In conclusion, it can be seen that, starting with a proper definition of the spark and defining its significance in terms of space charge accumulations of one form or another resulting from increased potential, we arrive at a general condition for a sparking threshold in terms of the product of two quantities expressing a primary and a secondary process. One secondary process involving *cathode* phenomena leads to a Townsend-type mechanism, the other occurring *in the gas* leads to a streamer mechanism. Both primary and secondary expressions, being subject to statistical fluctuation, make the sparking threshold indefinite. The analysis given, however, leads to a proper evaluation of the threshold potential in terms of experimentally, or potentially experimentally, determinable data. With sparking threshold defined, the reasoning at once leads to the understanding and analysis of observed time lags allowing analysis of statistical and formative lags. It follows from these considerations that these time lags will vary as potential is varied above and below the threshold in both Townsend and streamer sparks. This at once makes possible an understanding of previous observations and indicates how future measurements must be conducted. It also indicates, despite formal similarities, a fundamental difference between the Townsend and streamer mechanism and for the first time shows how profitably to apply future data to a proper threshold for streamer sparks. Having gone this far, the character of further studies based on the procedures in this analysis is in part indicated. The fundamental difference between the Townsend and streamer theory lies in the circumstance that the streamer theory involving ion concentrations and not numbers, and photoelectric absorption in the gas will not follow Paschen's[32, 36] law. This classical law says that V_s is a $f(p\delta)$. Thus with the *streamer* mechanism sparking potentials V_s plotted as functions of $p\delta$ will follow different curves, depending on whether p or δ are varied starting from any one point, V_s, $(p\delta)$.[33] For a long time there were not sufficient data over an extended range of p and δ to test this point. Lately, Howell and Trump, Stafford and Cloud,[34] using a large Van de Graaff generator and carrying measurements up to 60 atmospheres have shown deviations of considerable magnitude, such as those predicted by Meek's and Raether's theories.[35, 36] It is now urgent that with the aid of time lag and threshold investigations as clarified by this discussion, the values of p and δ for transition from the Townsend mechanism to the streamer mechanism be determined. This will give important information on the still missing data needed to complete the streamer theory. With this summary it is hoped that the value of this new method of analysis will be sufficiently established.

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[32] Reference 1, p. 58.

[33] L. Fisher, Phys. Rev. **69**, 530 (1946); L. B. Loeb, *Certain Aspects of Mechanism of Spark Discharge*, Meeting of Physical Society, London (April 23, 1947), in press.

[34] A. H. Howell, Trans. Am. Inst. Elec. Eng. **58**, 193 (1939); Trump, Stafford, and Cloud, Trans. Am. Inst. Elec. Eng. **60**, 132 (1941).

[35] W. Braunbeck, Zeits. f. Physik **39**, 6 (1926); **105**, 180 (1937); G. Hertz, Zeits. f. Physik **106**, 102 (1937).

[36] C. G. Miller and L. B. Loeb (in press).