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# Robert Andrews Millikan as Physicist and Teacher

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ROBERT Andrews Millikan was born in the little town of Morrison, Illinois, on March 22, 1868. The second of the six children of the Congregational Minister, Reverend Silas Franklin Millikan and his wife, Mary Jane (Andrews), he grew up in an atmosphere stressing and practising the simple biblical virtues. Under this influence he early acquired the sense of duty, the habit of intellectual honesty, and the power of taking pains which were important factors in his later success.

He received his higher education in Oberlin College, Ohio, where he graduated in 1891, majoring in classical languages. At the time of his graduation the College suddenly lost one of its tutors in physics, and the authorities turned in this emergency to young Millikan, asking whether he could help them out by brushing up on his knowledge of physics sufficiently to fill the tutorship. He undertook the assignment and carried it out with complete success: not only did he act for the next two years as a tutor in physics, but he became engrossed with the subject and decided to devote to it his future academic career.

With this in mind Millikan enrolled in 1893 as a graduate student in Columbia University. Of great benefit to him were the lectures in mathematical physics given by Michael I. Pupin, but recently returned from his studies in Europe, which included a strenuous year of drill in applied mathematics under the great Cambridge coach, Edward J. Routh. Paradoxically, it was the informality of Pupin's lectures that made them remarkable. He used to come to his classes completely unprepared, but, because of his excellent command of mathematical methods, he did not get bogged down in his derivations and eventually came through with the correct result. Thus, every lecture became a demonstration in the improvised solution of difficult problems. The more thoughtful students were greatly impressed and stimulated to work toward acquiring a mathematical facility similar to Pupin's. Although Millikan chose experimental physics as his life work, he picked up a good working knowledge of the mathematical branch which stood him in good stead in analyzing his experimental arrangements and in following the trends of theoretical thought.

The subject for Millikan's doctor's thesis was suggested to him by Professor O. N. Rood of Columbia.[1] It was the old problem "of the polarization of the light emitted by incandescent solid and liquid surfaces" first observed by Arago in 1824. The tentative explanation of Arago himself was to the effect that the surface layers of glowing bodies emit natural light, but that the part of the radiation which has its origin in the deeper incandescent layers is partially polarized because of its refraction in passing through the surface. However, quantitative measurements had never been made (with the exception of an inadequately short series on silver, observed by Violle), and attempts at a theoretical evaluation of its magnitude were completely lacking. Millikan treated the problem exhaustively: he measured the degree of polarization as a function of the angle of emission in glass, platinum, silver, gold, and iron. From the analysis of the results he concluded that Arago's picture of the phenomenon was incorrect. Anticipating the treatment now commonly used in the theory of heat radiation, he introduced instead the assumption that *all the light* coming from the incandescent body may be considered as having originated in its interior and having undergone refraction at the surface. Combined with the Fresnel-Cauchy formulas of refraction, this assumption led to a complete quantitative agreement with the measurements. Moreover, it accounted for a new observation which had escaped detection before Millikan: the two components of an emitted ray, polarized in the plane of emission and normally to it, had different spectral compositions. Thus an old problem of physics was extended and received its final resolution.

Having earned his Ph.D. degree, Millikan decided to round out his education by going for a year to Europe, which was then considered the fountainhead of science. Consequently, he spent the academic year of 1895–6 working in Professor W. Nernst's laboratory in Göttingen. The still youthful Nernst was a scientist of wide scope, with a keen eye for the promising avenues of investigation. Realizing the importance of the dielectric constant $\epsilon$ for the exploration of the structure of matter, he had two years earlier developed an apparatus for the rapid and accurate measure of $\epsilon$ in liquids by means of electromagnetic waves. This method had been extensively used by his students and a good deal of data had been accumulated showing that the dependence of the dielectric constant upon density, in several liquids, was well represented by the Clausius-Mossotti formula. In 1879 this formula had been explained by H. A. Lorentz as being due to the mutual interaction of the electric dipoles produced in each molecule by the electric field. There existed, however, an older picture, going back to Poisson and worked through by Mossotti, also leading to the same formula. According to this view, the dielectric consists of discrete little conducting spheres (molecules) embedded in vacuum (or in an insulating medium). Nernst apparently wondered whether this picture had any validity, at least, for some substances. As a first step, he decided to test whether the Clausius-Mossotti formula was correct for a medium artificially prepared according to Mossotti's specifications. Hence, he suggested that his American student carry out such a test. As the appropriate medium they chose emulsions of water in a mixture of benzene and chloroform having the same specific gravity as water. The globules of water represented the embedded conductors, and the organic mixture the insulating medium. Because of the uniform gravity, the emulsion was sufficiently stable for the lengthy dielectric determinations. Represented as a function of the percentage of water in the emulsion, the measured dielectric constant agreed very well with the Claussius-Mossotti formula. The working up of his data and the writing of his paper was done by Millikan after his return to America. I gathered from occasional conversations with Dr. Millikan, many years ago, that the manuscript sent by him to Nernst had been several times longer than the published paper (2). With his ability to see the wider implications of every problem, he had noticed that the Poisson-Mossotti model of discrete conductors in an insulating medium could account also for the dispersion of the dielectric constant, i.e., for its change with the frequency of the employed electromagnetic waves. Therefore, the larger part of his paper was devoted to working out the theory of dispersion on these lines and to comparing it with some dispersion measurements he had made for the purpose in a suitable medium. Nernst did not wish to sponsor the independent ideas of his student by recommending them for publication. He sent to the Annalen der Physik und Chemie only the part of the manuscript relating to the measurements, leaving in it only the barest hint of a possible explanation of dispersion. The theoretical part he sent back to Millikan advising him to subject it to further experimental tests and to publish it, if at all, on his own responsibility. However, in the meantime P. Drude, the editor of the Annalen der Physik und Chemie had taken up Millikan's hint and had developed it into a theory (3) which followed the same lines of thought as Millikan's own unpublished manuscript, thus superseding it. Later it was shown that the expressions resulting from this theory are formally identical with those obtained under the assumption of a medium containing permanent dipoles capable of rotation.

The European episode marked the end of Millikan's years of apprenticeship. After his return to America he entered into the functions of a mature and independent academic teacher and research physicist. Toward the end of his stay in Göttingen he had received an offer from Professor A. A. Michelson to join the Physics Department of the University of Chicago in the capacity of an assistant. Although he was in possession of a financially much better offer from Oberlin College, he accepted Michelson's invitation. On the one hand, he was attracted by the opportunity of working side by side with the foremost American physicist of that time, on the other hand, the University of Chicago offered a tempting field of activities to an enthusiastic and energetic young man. It was a very young institution—having been opened only in 1892—with vast resources and ambitions, whose policies with respect to teaching and research had yet to be worked out. It is not surprising that young Millikan eagerly plunged into the work of helping to establish these policies and that his publications for the next ten years were mainly pedagogical. Jointly with S. W. Stratton, J. Mills, and H. G. Gale, as junior authors, he published a number of undergraduate text books which made his name honorably known from one end of America to the other, securing for him an influence on the teaching of physics far beyond the confines of his home university. Since another article in the present issue is devoted to this phase of Millikan's activities,* we shall not enlarge upon it. Suffice it to say that his usefulness was being recognized by regular promotions: he became successively instructor (1899), assistant professor (1902), and associate professor (1907).

It was, perhaps, fortunate that this interruption of Millikan's research work coincided with a period of tremendous enrichment in our physical knowledge. Röntgen's discovery of the x-rays was made at the end of 1895 but became generally known only in 1898, when his papers were reprinted in the Annalen der Physik und Chemie. The discovery of the electron by P. Zeeman and J. J. Thomson followed in 1897, that of radium and polonium (by Pierre and Marie Curie) in 1898, of the quantum of action (by Planck) in 1901. Not being engaged in any piece of research, Millikan must have found himself in a detached frame of mind, possibly more favorable to the absorption and assimilation of the far reaching revolution in our scientific outlook. Of great help in this connection was, without any doubt, his work (joint with C. R. Mann) on the translation of P. Drude's *Theory of Optics*, which appeared in 1902. The English version of this thoroughly up to date book was not only of great service to American graduate teaching but was apt to imbue its translator with the spirit of modern physics. At any event, when the work of organizing undergraduate instruction in Chicago was finished, and Millikan had the leisure to return to research, the choice of his problems stamped him as an independent and farsighted physicist conversant with the needs of his time. It was, certainly, not Michelson's influence that guided him in the selection of his topics: important as Michelson's researches were, they represented the climax and close of old chapters of physics, while Millikan's investigations marked the opening of new chapters.

We intend to give a systematic review of Millikan's scientific work in the second part of this paper. In this place, we only wish to outline the general characteristics of his approach to experimental research. He begins with a thorough study of the work of his predecessors, analyzing their methods with a view of discovering the weak points that could be improved upon. This enables him to start work with an experimental set-up eliminating some of the previous sources of error. Since the problems treated by Millikan are among the most difficult, an easy success in a single paper cannot be expected. But even the first paper usually represents an advance over the preceding work; moreover, it gives him experience and a better understanding of the functioning of his instruments, thus enabling him to devise further improvements in his apparatus, to undertake with it a second piece of research, and to report the further progress in a second paper. He always strives for a complete understanding of all the secondary processes taking place in his set-up, if necessary, trying separate experiments to elucidate some obscure details. In this way, the very sources of error become subjects of research, leading to instructive results, and sometimes to significant discoveries. Thus, by slow degrees Millikan advances to a complete mastery of every aspect of his problem and brings the investigation to a close, in the sense that he obtains final results which could not be improved upon with the experimental resources of the epoch.

All this required much time and hard work. Every subject of research developed into a whole program, often branching out into new subjects. Fortunately, Millikan did not have to accomplish everything singlehanded but could delegate part of the work to his pupils. He

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* "Robert A. Millikan's influence on the undergraduate teaching of physics," by Duane Roller.

always possessed, in a high degree, the ability for teamwork which springs from a friendly and sociable temperament. He is what is colloquially called "a good mixer," that is, he enjoys the social intercourse with his fellow men and likes to do them a good turn. In particular, he delights in paving the way for deserving younger men, who take their first scientific steps under his guidance, and he has much to offer them as the possessor of the vastest research experience and of the soundest experimental technique. Indeed, he has all the qualifications of an exceptional teacher of research physicists, which may be enumerated as follows. (1) He is interested in young scientists and likes to have them about him. Millikan's later relations with his former pupils were always unclouded and, in many cases, they have developed into lifelong friendships. (2) He is able to impart to them a sound technique; especially he teaches them thoroughly to analyze their problem before building the apparatus and never to be satisfied with results that can be improved upon. (3) He sets them a shining example of enthusiasm for research and of hard work in its pursuit. Not many of them will forget how Millikan, after working from morning till six o'clock in the evening as chief executive of the California Institute of Technology, after dinner turned physicist and discussed with them their problems from 7 o'clock till midnight. (4) Because of his farsighted grasp of physics he is able to start his pupils off on important problems which are not readily exhausted but sometimes last them their whole lives.

Every director of research who presides for a number of years over a large laboratory has the right to expect among his personal pupils one or two men of exceptional merit. But in Millikan's case, we find as his close associates, growing to manhood and fame in intimate collaboration with him, far more eminent physicists than can be accounted for by the law of averages. On the strength of this record, Robert Andrews Millikan must be classed as one of the most successful teachers in the history of science.

The public recognition of Millikan's great merits in research and teaching was not long delayed but received an early expression in well deserved academic honors. In 1910 he became full professor at the University of Chicago; in 1911 he received his first honorary decree, D.Sc., from his alma mater, Oberlin College; in 1913 he was awarded the Combstock prize of the National Academy of Sciences, and in 1915 he was elected a member of that body. In 1916–17 Millikan gained prominence in the National organization of science: he played a leading role in the scientific war effort (of the first world war) and in the establishment of the National Research Council, whose Vice President he became in 1917. In 1921 the trustees of the newly founded California Institute of Technology prevailed upon him to sever his connection with Chicago and to assume the duties of Chairman of the Executive Council of the Institute and of Director of the Norman Bridge Laboratory of Physics. It is not too much to say that the Institute as we see it today is essentially his creation. These activities coincided with the international phase of Millikan's influence on scientific organization; from 1922 to 1937 he was Foreign Secretary of the National Academy and concurrently from 1922 to 1932 American Representative on the Council for International Cooperation in Science. In 1946 he relinquished his positions of chief executive and laboratory director, but he continues to serve the Institute as Vice President of the Board of Trustees and to do scientific research. In the meantime he had been the recipient of academic honors too numerous to mention: honorary degrees from scores of universities in many countries; honorary memberships in most of the academies of America, Europe, and Asia; prizes and awards, of which I shall refer by name only to the Nobel prize received by him in 1923.

The private life of Robert A. Millikan has been uneventful. In 1902 he married a worthy helpmeet, Greta Irvin Blanchard, and has since lived in an atmosphere of quiet but intense happiness which has been a delight and inspiration to all those who were privileged to enjoy the generous hospitality of their home. They are rightly proud of the three sons who sprang from their union and who, in their own right, have attained distinction in the academic world. Although their long and unclouded happiness was darkened, a year ago, by the loss of their second son in a mountaineering accident, Dr. and Mrs.

Millikan bow to the inevitable and bear their great sorrow with fortitude.

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In reviewing Robert A. Millikan's main contributions to the knowledge of physics the chronological method is inconvenient. As a rule, several lines of investigation were conducted concurrently, by him and his collaborators, over considerable periods of time. We prefer, therefore, to follow through the different subjects of his research, one by one, *without attempting to be exhaustive*. To some readers the following account of Millikan's scientific research may appear rather long. But the large volume is unavoidable since it merely reflects the magnitude and diversity of his achievement, the enormous amount of work accomplished in a full and fertile life.

## 1. DETERMINATION OF THE ELECTRONIC CHARGE *e*

The investigation of the electronic charge was started by Millikan in 1907 jointly with his student L. Begeman, with a view of improving the method of H. A. Wilson (4), which seemed to the authors to be the most promising. This method consisted in ionizing the air in a fog chamber and condensing on the ions a cloud by means of a sudden expansion. First, the rate of fall of the cloud under gravity alone was observed, then the rate of fall of a similar cloud when a vertical electric field was superposed upon gravity. Stokes' law of resistance made it possible to obtain the mass of the droplets constituting the cloud from the velocity of their descent under gravity. The additional knowledge of the velocity in the electric field gave information about the ratio of the electric to the gravitational forces and, ultimately, about the ionic charge.

From the start, two details of Wilson's work seemed open to criticism. (a) That author had been producing the ionization in his chamber by means of x-rays, which are known to be subject to large intensity fluctuations. Hence, Wilson's clouds might have been variable as to the number of ions in them and, consequently, as to the size of the droplets condensed on the ions. (b) It was not quite certain that the observed sinking of the upper edge of the cloud was entirely due to its fall and not, partially, to its evaporation. To meet these objections Millikan and Begeman made the following improvements: (a) they used radium as their ionizing agent; (b) they greatly reduced the distances through which the clouds fell, and consequently the periods of observation, so that the conditions in the chamber did not have the time appreciably to change. As a result, they obtained even in their first paper (5), a much smaller spread of the individual determinations than had been observed before them. Their mean value for the electronic charge was $e=4.03\times10^{-10}$ e.s.u., instead of H. A. Wilson's $e=3.1\times10^{-10}$.

While becoming familiar with the working of their apparatus, Millikan and Begeman discovered two other important sources of error. The first of them was a systematic error related to the determination of the coefficient of viscosity of air $\eta$, which knowledge was necessary to calculate the mass of the cloud droplets from Stokes' law. This coefficient greatly depends on the temperature of the air which in the early work was determined indirectly from the expansion ratio applied to the chamber in order to condense the cloud. The theoretical temperature drop caused by expansion was 12°C under their conditions (as also under those of H. A. Wilson). Because of the large heat capacity of the walls and of the condenser plates, Millikan (6) doubted this result and arranged for a direct measurement of the air temperature by means of thermocouples. Far from being 12°C, the drop below room temperature turned out to be less than 0.5°C. This fact led to an important revision of the value accepted for the viscosity and brought the magnitude of the atomic charge to $4.57\times10^{-10}$ (instead of $4.03\times10^{-10}$).

The second source of error became apparent when the authors tried to get away from observing only the upper edge of the cloud and concentrated their attention on the rate of fall of identifiable structures within the cloud itself. They found that different parts of the cloud moved with different velocities. The obvious explanation was that not all droplets of the cloud were singly charged but some doubly, triply, etc. The proof lay in the fact that the charges calculated from the individual velocities were integral multiples of one fundamental value. This discovery blazed the way for further, more accurate investigations. Begeman was assigned the task of measuring the electronic charge separately from the singly, doubly, triply charged parts of the cloud (7). The weighted average of his results was $4.66 \times 10^{-10}$ e.s.u. On the other hand, Millikan himself discovered the method of isolating single charged droplets (8). This was accomplished by balancing the downward gravitational pull by a suitable electric force pulling upward. The balancing could be achieved only for a certain charge and a certain weight of the droplets. Since neighboring droplets have, in general, different (multiple) charges and slightly different masses, most of them were pulled out of the field of vision and only a few discrete globules were so well balanced that they remained. One of these globules served then as the object of further observations on its rate of fall under gravity and electric forces. The result obtained from observing many droplets isolated in this way was $e = 4.65 \times 10^{-10}$ e.s.u.

Millikan was not slow to realize that the method of observing single particles presented almost unlimited possibilities for improving the accuracy. Indeed, making all the observations on a single charged globule eliminated the uncertainties resulting from the averaging over swarms of particles, which were inherent in the older methods. Moreover, he introduced the improvement of preparing the droplets not by expansion and condensation, but by an atomizer. The droplets were produced in this way in the space above the condenser in which the electric fields were applied and the rates of fall observed. Through a tiny hole in the upper condenser plate a few of them entered the field of observation, and after this happened the hole was closed. This procedure offered the following advantages. (a) The temperature of the observation space remained constant. (b) No convection currents were set up in it. (c) As the droplets could be prepared from any liquid, a substance of extremely low vapor pressure could be chosen. This prevented their change of size by evaporation. (d) For the same reason, the surrounding air remained unaffected by the presence of the droplets, and its coefficient of viscosity was that of pure dry air. Excellent results were obtained with oil droplets, and hence this way of measuring electronic charges became known as *Millikan's oil-drop method.*

Already the first investigation with the oil-drop method showed that Stokes' formula was not accurate enough for determining the particle radii. However, the more accurate Stokes-Cunningham formula, which takes into account the dependence of the rate of fall on the mean free path, led to very consistent results (see below) and gave for the electron charge the slightly too high value of $e = 4.891 \times 10^{-10}$ e.s.u. The two final papers of the series were published in 1913 and 1917, respectively (10, 11). They represent two complete and independent determinations of the electronic charge; although the method of the second investigation was the same, it was carried out with a new apparatus, and all the auxiliary constants were re-evaluated. The results were identical, namely, $(4.774 \times 0.009) \times 10^{-10}$, in the first, and $(4.774 \times 0.004) \times 10^{-10}$, in the second—a figure which essentially remained the standard value for over twenty years, though the new more accurate data for the velocity of light and the value of the absolute ohm brought it down (12) to $4.770 \times 10^{-10}$. These papers definitely settled the question of the uniqueness of the electronic charge which was until then open. Even today they are definitive for the oil-drop method in the close consistency and small spread of the individual determinations. Although the investigation has been repeated by several other authors, the accuracy of Millikan's relative results has never been equaled. It was for this work that he received the Nobel prize in 1923. With respect to the absolute result, a small uncertainty lay in the value of the coefficient of viscosity of air accepted by him (Harrington's $\eta_{23} = 1822.7 \times 10^{-7}$, in the second paper). In the nineteen thirties the evidence of indirect determinations of the electronic charge began to accumulate and to point to the conclusion that Harrington's value was slightly too low. Hence, redeterminations were undertaken in the Norman Bridge Laboratory and elsewhere (see below) which led to results clustering about $\eta_{23} = 1830.0 \times 10^{7}$. With this correction Millikan's determinations of 1913 and 1917 would give $e = 4.799 \times 10^{-10}$ e.s.u., a value of the electronic charge which must be considered the most accurate directly obtainable by the oil-drop method. Millikan estimates its accuracy (13) as $\frac{1}{3}$ of 1 percent, owing, primarily, to the uncertainties in the determination of the viscosity of air.

### 2. VISCOSITY OF AIR AND STOKES' LAW

Millikan's interest in the viscosity of air grew out of the fact that an accurate knowledge of it was needed for the evaluation of his experiments on the electronic charge. The then available data were concerned not so much with the absolute value needed by Millikan, but with its relative changes in dependence on temperature and pressure. Hence, he caused several determinations to be carried out under his supervision in his Chicago laboratory. The Poiseuille method of flow through capillary tubes was used by I. M. Rapp (14) and E. Markwell (15), and the method of rotating cylinders by L. Gilchrist (16) and E. L. Harrington (17). It was found that the second method was capable of a higher accuracy. Especially, as perfected by Millikan and Harrington, the rotating cylinder apparatus was superior to any that had been used before that time. (Their result was given above.)

Later Millikan returned to the work with rotating cylinders for the purpose of determining the viscosity of organic vapors. Jointly with R. K. Day (18), he developed in the Norman Bridge Laboratory a modified apparatus which differed from Harrington's in that it could be used at high temperatures. A significant feature of this instrument was a constant speed motor for driving the movable outer cylinder. The motor had been originally designed by H. Benioff for use in driving the chronograph drums of seismographs, and was constant to better than 1 part in 10,000. This apparatus was used by Day to measure the viscosities of several substances in their dependence on pressure. R. K. Day worked with normal and isopentane and W. M. Bleakney (19) with 2-pentene, trimethylethylene, and carbontetrachloride. In all these cases the pressure coefficient was negative.

The circumstances which led to the reawakening of the interest in the absolute viscosity of air were mentioned above. After the importance of its redetermination was pointed out by K. Shiba (20), the work was undertaken independently by several investigators. The method of rotating cylinders was employed by G. Kellström (21), W. V. Houston (22), and J. A. Bearden (23), while W. N. Bond, P. J. Rigden (24), and G. B. Banerjea and B. Plattanaik (25) used improved capillary flow methods. As far as rotating cylinders are concerned, the improvements were not primarily due to instrumental changes because Houston worked with the identical apparatus built by Millikan and Day, while Kellström and Bearden used instruments of a very similar construction. The advances lay rather in a more thorough discussion of the sources of error and in more elaborate corrections for them. The results for $\eta_{23}$ were as follows: Kellström $(1834.9 \pm 2.7) \times 10^{-7}$, Houston $(1829.2 \pm 4.5) \times 10^{-7}$, Rigden $(1830.34 \pm 0.6g) \times 10^{-7}$, Bearden $(1834.02 \pm 0.06) \times 10^{-7}$; Banerjea and Plattanaik $(1833.3 \pm 2.2) \times 10^{-7}$, which are all above Harrington's. The measurements of Day and Bleakney were not appreciably affected by the new absolute values since they were primarily concerned with relative changes.

We have already mentioned that the ordinary form of Stokes' law proved to be insufficiently accurate for the purposes of the oil-drop method. This law connects the force $X$ acting on a spherical particle with its velocity $v$ of motion, its radius $a$, and the viscosity of the gas in which it falls as follows: $X = 6\pi\eta av$. Millikan found that he had to use a more accurate formula, taking into consideration the mean free path $l$ of the gas, namely,

$$X = 6\pi\eta av(1 + Al/a)^{-1},$$

where $A$ is a numerical coefficient. This expression is usually called the Stokes-Cunningham law, although its derivation by Cunningham (26) was spurious. On the other hand, Millikan correctly interpreted it from the very start (9) as the result of "slipping" or "sliding friction" at the surface of the moving particle.** The phenomenon of sliding friction at a solid wall past which a gas flows was predicted by Maxwell who used the following picture: the larger part of the gas molecules hitting the wall (fraction $f$) is reflected in a diffuse way, without preference of

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** Technically, the priority of this explanation belongs to Max Reinganum (26), whose paper appeared when Millikan's was in print.

direction, and only a small fraction, $1-f$, is reflected specularly. Under this assumption the theoretical expression for the constant $A$ is

$$A=0.7004\left(\frac{2}{f}-1\right).$$

A technique was developed by Millikan for changing the pressure of the gas (and consequently the mean free path $l$) while keeping a particle in the field of observation. Thus, the oil-drop method offered a convenient means of testing the Stokes-Cunningham formula and of measuring the coefficient $A$. The most extensive series of measurements (9–11, 27) referred to oil droplets in air and covered the enormous range from $l/a=0.05$ to $l/a=134$. It was found that the Stokes-Cunningham law holds accurately up to $l/a=0.5$, giving an experimental value of the constant $A=0.842$. According to Maxwell's expression, this means $1-f=0.092$, i.e., roughly 10 percent of the molecules undergo specular reflection. The law does not apply to very small particles ($l/a$ large), but the observations with them can be theoretically evaluated in a different way (28) and are consistent with the assumption of 10 percent specular reflection. Of the several other materials investigated by Millikan we mention here little spheres of mercury and of shellac in air for which the coefficient $A$ was 0.708 and 1.078, respectively, corresponding to specular reflections of 0.0 percent and 21 percent. The determinations by the oil-drop method fit well into the data obtained with other methods, especially Millikan's own (27) and Timiriazeff's (29), with rotating cylinders, and M. Knudsen's (30), with capillary tubes, and with oscillating spheres.

## 3. PHOTOELECTRIC EFFECT AND PLANCK'S CONSTANT

The phenomena of photoelectricity were partially elucidated by the work of P. Lenard (31), who showed that short-waved light falling on metal makes it emit electrons. The loss of negative charge in the process causes the metal to assume a positive potential which increases to the point where it is sufficient to make the electrons return and thus prevent the escape of even the fastest of them. This mechanism completely defied any explanation on classical lines and remained mysterious until Einstein (32) introduced in 1905 the assumption of the photon constitution of light. The conception of the photon easily explained the effect and immediately led to Einstein's famous photoelectric equation,

$$V=(h/e)\nu+V_0,$$

connecting the limiting potential $V$ with the frequency $\nu$ of the incident light and with the fundamental constants $h$ and $e$; the potential $V_0$ is the contact-electromotive force of the same metal when it is not illuminated.

It is true that most physicists of the time were not willing to accept this explanation since they regarded the existence of photons even as more of a mystery than the photoelectric effect itself. But just because of the highly controversial character of Einstein's law, its experimental test was attempted by a number of independent investigators. The problem proved, however, to be technically extremely difficult. In 1913 Pohl and Pringsheim (33) published a careful critical review of the numerous investigations and found them all inconclusive, and a similar opinion was expressed in the following year by J. J. Thomson (34). The prevailing degree of uncertainty may be inferred from the fact that such a sound experimenter as C. Ramsauer (35) came in 1914 to the conclusion that the photoelectrons have no limiting velocity at all, but are liberated with a Gaussian velocity distribution.

Millikan's ultimate success was due not only to great experimental skill but in equal measure to long experience acquired by persistent work in this sphere of phenomena. His first publications on photoelectricity (36) were made jointly with G. Winchester and appeared as early as 1907. The purpose was to investigate whether the photoelectric current and the limiting potential depend on the temperature of the emitting metal. Such a dependence was not found—as we know today—because of the degenerate state of metal electrons. Other photoelectric work (37) belonged to the years 1909 and 1912. All these investigations taught him the importance of using very clean metal surfaces and the danger of using sparks as the source of short-wave light, since the spark discharges are liable to falsify the measured potentials by inducing in the apparatus electric oscillations. Indeed, this source of error temporarily led him astray until he corrected for it (38) in 1913.

It does not seem that in the early period of his photoelectric work Millikan was familiar with Einstein's equation. However, when he became aware of it and directed his efforts towards testing it, his progress was rapid. The test of Einstein's equation involves the following measurements. (1) In the first place, it is necessary to determine the photo-potential $V$, i.e., the potential difference which is able to stop the fastest electrons emitted by the surface under the action of light of the frequency $\nu$. For this purpose Millikan used what is known today as the *method of isochromates.* A retarding potential is applied which stops only part of the electrons, and the current carried by the escaping electrons is measured. As the retarding potential is gradually increased, the current becomes weaker. Plotting the current against the potential, it is possible to determine by graphic extrapolation the point at which the current vanishes altogether, corresponding to the photo-potential $V$. The whole curve is determined point by point while the metal is illuminated with light of the constant frequency $\nu$. (2) After the photo-potential $V$ has been measured for a number of different frequencies, $\nu$, it is possible to plot $V$ against $\nu$. According to Einstein's relation the dependence must be rectilinear: the slope of the straight line must be $(h/e)$ and its intercept must be $V_0$, corresponding to the condition $V=(h/e)\nu_0+V_0=O$, whence $V_0=-(h/e)\nu_0$. As we have mentioned above, Einstein's theory implies that $V_0$ is the contact-electromotive force (c.e.f.) of the non-illuminated metal. The second measurement which is necessary to test the formula is therefore the determination of the c.e.f. of the same metal surface. This determination was accomplished electrometrically. To carry out these operations on several metallic surfaces required a very elaborate piece of apparatus, in Millikan's own words (39): "As new operations have been called for, the tubes have by degrees become more and more complicated until it has become not inappropriate to describe the . . . experimental arrangement as a machine shop in vacuo."

The reason for Millikan's success where his predecessors failed lay in carefully choosing the conditions so as to minimize all the sources of error of which the main were as follows: (1) The range of frequencies over which Einstein's formula had been tested in the previous work was too narrow. To extend the range Millikan used alkali metals which are photosensitive up to about $\lambda=6000$A. (2) The reference bodies with respect to which the photo-potential was measured were also photosensitive, complicating the conditions by their own photo-emission due to reflected light. Millikan used as his reference body a Faraday cage of well oxydized copper netting. The photosensitivity of this material extends only to $\lambda=2688$A. Thus the interval from 2688A to 6000A was free of this source of error. The plate against which the c.e.f. was measured (after moving in vacuum the photosensitive surface into the proper position against it) was also made of oxidized copper. (3) For retarding potentials approaching the photo-potential the photo-currents became so weak that their measurement could not be made very accurate. It was found that the photo-current was many times stronger when the emitting surfaces were fresh. Hence the alkali metals were inserted into the tubes in the form of thick cylindric blocks. The "vacuum workshop" contained a rotating knife blade by means of which a thin layer of metal could be shaved off the plane surface of the block. (4) The term $V_0$ of Einstein's equation corresponding to the c.e.f. was found to be different for fresh and for old surfaces, the change sometimes taking place in a short time. The conditions of pressure were investigated and found under which both the freshly shaved surface and the copper oxide surface (40) retained a constant $V_0$ for a time sufficient to carry out a complete series of measurements. (5) Very troublesome were traces of stray light of higher frequency than $\nu$. Their contribution to the photo-current became important when this current was very small, falsifying the apparent point of the isochromate where electronic emission stops altogether. The measures for meeting this source of error were twofold. On the one hand, the illumination was produced by a high pressure mercury-quartz lamp monochromized with a quartz monochromator. The only mercury lines selected were those which had no near companions on the short-wave side. On the other hand, the effect of stray light was studied by cutting it out with the help of proper light filters. If all the refracting surfaces were very clean and the absorbing surfaces very black, it could be reduced to a degree where it was innocuous.

The results of these investigations were published in part in Millikan's own papers (39–41), in part in those of his pupils (42). They amounted to a complete confirmation of Einstein's equation in all its details. (1) The dependence of $V$ on $\nu$ is rectilinear, since no experimental point was above or below the straight line by more than 1 percent. (2) The slope of the line is equal to $h/e$; indeed, the experimental slopes found by Millikan were $1.376\times10^{-17}$ for sodium, and $1.379\times10^{-17}$ for lithium, while the best modern value is considered to be $1.3793\times10^{-17}$. (3) The intercept $V_0$ of the photoelectric straight line agreed with the electrostatically measured c.e.f to better than 0.5 percent. These beautiful results established beyond any shadow of doubt the role which Planck's quantum of action $h$ plays in the photoelectric effect. Besides, they represented at the time the most accurate numerical determination of that fundamental constant.

Even after the publication of the above described work it was maintained by some physicists—especially of the school of W. Hallwachs (43)—that photoelectricity is not an intrinsic property of metals but is entirely due to the gas impurities occluded and dissolved by them. The last photoelectric investigations suggested by Millikan and carried out under his personal supervision were undertaken to elucidate this question. In Chicago two of his pupils studied the photo-effect of platinum in thin strips which could be degassed by electrical heating (44). But particularly conclusive was the work in the Norman Bridge Laboratory with mercury surfaces cleansed of impurities by means of a continuous overflow in a high vacuum. The long wave-length limit of photosensitivity of pure mercury was found by C. B. Kazda (45) to be $\lambda=2735$A, and this value was confirmed by H. K. Dunn (46).

### 4. EXTREME ULTRAVIOLET SPECTRUM

Investigating the potentials of sparks between metallic electrodes, Millikan made, as early as 1905, the observation that the spark discharge of a large condenser could be maintained in the highest vacuum if the potential difference were sufficiently large. It occurred to him that these "hot sparks" provided a means of investigating the ultraviolet light which, in all likelihood, they were emitting. Indeed, the previous endeavors of extending the knowledge of the short ultraviolet spectrum had been limited by its extreme absorbability, but the use of a hot spark and of a concave reflection grating permitted of completely eliminating all absorption by placing in a vacuum spectrograph the whole path of the rays, from their origin to the recording photographic plate. The first results obtained with a vacuum spectrograph of this type were described by Millikan and Sawyer (47) in 1918. The only part of this early instrument which needed improvement was the grating; through the comparison of several gratings it was found that the best result was given by gratings ruled by "the easy touch method" in which the reflecting strips were parts of the original speculum metal surface and which threw most of the refracted radiation into the first-order spectrum. With this modification the spectrograph afforded at once a very considerable extension of the measurable ultraviolet spectrum; the region up to the line $\lambda=209$A of nickel was explored right away (48), and up to $\lambda=136.6$A of aluminum (49) in the next following paper. All the later work on the extreme ultraviolet was done jointly with I. S. Bowen. It consisted in photographing, measuring, and completely analyzing, as to their spectroscopic terms, the spectra of numerous elements. Not only neutral atoms of the light elements emit lines in this region but also singly or multiply ionized atoms of somewhat heavier elements. Indeed, some of the analyzed spectra were produced by atoms stripped of as many as six (S, Cl) or even of seven (Cl) of their electrons.

The joint work of Millikan and Bowen opened for spectroscopy a new and fruitful region whose exploration came exactly at the right time. By supplying valuable material it influenced and helped the development of theoretical spectroscopy which at this very period was rapidly advancing towards establishing the so-called Russel-Pauli-Heisenberg-Hund rules. The number of publications by Millikan and Bowen is so large that we refrain here from giving their complete bibliography and restrict ourselves to a brief outline of their significance.

(a) Extending the ultraviolet measurements down to $\lambda = 136.6$A helped to close the last unexplored gap in the spectrum of electromagnetic frequencies, because very soon F. Holweck succeeded in reaching the same wave-length region from the side of the x-rays.

(b) More important still, Millikan and Bowen established the essential unity of the optical and the x-ray spectra (49, 50). Already in his presidential address of the year 1917, before the American Physical Society, Millikan pointed out the close analogy which the x-ray spectra (Mosely's formula) bear to the hydrogen spectrum (Balmer's formula) and stressed the importance of studying the extreme ultraviolet spectra of the light elements. Carrying out this program, Millikan and Bowen showed that the $s$, $p_2$, $p_1$ terms of optical series are, respectively, identical with the $L_I$, $L_{II}$, $L_{III}$ x-ray levels.

(c) The spectroscopic terms of the ultraviolet regions, like those of the visible and the x-ray spectra, were classified in terms of four quantum numbers of which three belonged to the translational degrees of freedom of an electron, while the fourth was interpreted as residing in the atomic core. Such an interpretation involved, however, great difficulties from the point of view of the atomic model, which were forcibly stated by Bowen and Millikan (51) in reviewing the material they had accumulated. Precisely these difficulties caused Uhlenbeck and Goudsmit (52) to introduce the concept of a rotational degree of freedom of the electron in order to explain the presence of the fourth quantum number mentioned above. Thus the work of Millikan and Bowen was an essential prerequisite for the discovery of the electronic spin.

(d) The combination of two ultraviolet spectroscopic terms occasionally gives a line in the visible spectrum. Some of these possible radiations have been identified by Bowen with unexplained lines from stellar sources and from the terrestrial atmosphere. In particular, Bowen succeeded in elucidating a series of lines observed in nebular spectra and heretofore ascribed to the hypothetical element "nebulium" (53). He showed beyond doubt that they are due to nitrogen and oxygen by explaining the reasons why a line, which is "forbidden" and absent in terrestrial sources, may occur with considerable intensity under the conditions prevailing in a nebula. While Millikan has no direct share in these important discoveries which are entirely due to Bowen, yet they grew out of a line of research initiated by Millikan.

## 5. COLD EMMISSION OF METALS

The phenomenon of negatively charged cold metallic surfaces giving off an electric current, when the potential gradient at their surface is very large, was first investigated by R. F. Earhart (59) in 1901. Millikan became interested in this problem still in his early Chicago days, and it was studied in his laboratory by G. M. Hobbs (55) in 1905. In the following years, however, his time was fully occupied with work on the electronic charge and on the photo-effect, so that he was able to return to the questions of cold emission only in the nineteen twenties. After some preliminary exploration (56), a thorough study was undertaken by Millikan and Carl F. Eyring (57) working with very thin tungsten wires in an extreme vacuum. They introduced for the phenomenon the term *field current* and arrived with respect to it to the following conclusions. (a) Although the emission characteristics of a wire depended on its previous heat treatment, a very strong field current brought the wire into a steady state. In this state the field currents were reproducible as long as they were weaker than the field current that had been used for conditioning. The following statements refer to the reproducible field currents of conditioned wires. (b) The field current set in at a certain minimum potential gradient which had the order of magnitude of a few hundred thousand volt $\text{cm}^{-1}$. (c) The minimum potential gradients as well as the field current were entirely independent of temperature in the interval from 300° to 1000° abs. (d) The field current $I$ seemed to be a function only of the potential gradient $F$ at the point of emission and not to depend on the total potential difference applied to the wire. It had been claimed by other authors that $\log I$ plotted against $F^{\frac{1}{2}}$ gave a straight line (58); this was definitely untrue for the data found by Millikan and Eyring.

The $F^{\frac{1}{2}}$ law follows from the classical theory on the assumption that the field current is in its essence nothing but a thermionic current modified by the presence of a very strong electric field. Thus, the observations (d) of Millikan and Eyring set the cold emission apart from the thermionic emission as an independent phenomenon and their result (c) pointed in the same direction. Further work by Millikan and co-workers (59) established that the field currents $I$ are, indeed, quite independent of the potential difference and (within the stated limits) of the temperature and are a function of the potential gradient $F$ only, being accurately represented by an empirical formula due to Charles C. Lauritsen,

$$I = I_0 \exp(-b/F),$$

where $I_0$ and $b$ are constant.

Millikan's uncanny ability for choosing the most timely problems asserted itself also with respect to the work on cold emission. Its theoretical explanation given in 1928 independently by J. R. Oppenheimer and by R. H. Fowler and co-workers (60) showed it to be due to the quantum-mechanical phenomenon of electrons leaking through a potential barrier. It was the first example of a previously unknown mechanism which has since received important applications in the theories of atomic and of nuclear structure. As given by Fowler and Nordheim, the theoretical law of field currents is

$$I = CF^2 \exp(-b/F),$$

which is experimentally indistinguishable from Lauritsen's formula.

### 6. COSMIC RAYS

The first reports about a penetrating radiation in the atmosphere were read before the Washington, D. C. meeting of the American Physical Society (Dec. 31, 1902) by two independent teams of investigators: E. Rutherford and H. L. Cooke (61), of Montreal, and J. C. McLennan and E. F. Burton (62) of Toronto. At first it was believed that the origin of the penetrating radioactive rays lay in the top layers of the solid earth. However, in the years 1909 to 1911 the Swiss meteorologist, A. Gockel, (63) took an ionization chamber (with 2-mm brass walls) on several balloon ascents and found that the intensity of the penetrating radiation did not materially decrease up to heights 2.8 km. This result was confirmed and extended by V. Hess (64) and W. Kolhörster (65), who made numerous balloon ascents up to heights of 5 km and 9 km, respectively. With rising elevation the atmospheric ionization first decreased, reaching a minimum at about 700 m. From then on it increased, first slowly, then more rapidly. This pointed to a component of the penetrating radiation coming downward from high altitudes, possibly from outside the terrestrial atmosphere, whence the name "cosmic rays." Though this result was in principle well established, the quantitative side of the measurements was by no means accurate. Indeed, estimates of the coefficient of absorption of the downward radiation, made on the basis of Kolhörster's data, lay in the vicinity of $\mu = 0.57 \times 10^{-2}$ cm$^{-1}$ of water (66), a value which later proved to be greatly in error.

Millikan became actively interested in cosmic rays following his removal to Pasadena when he conceived the idea of sending self-recording instruments high up into the atmosphere with the help of sounding balloons. The first recording electroscopes and barometers were constructed by him jointly with I. S. Bowen, and in the spring of 1922 sent up to heights of 15.5 km (67). The same year he caused his student, R. M. Otis, to make cosmic-ray measurements on Mt. Whitney (4130 m), and the following summer he went with Otis to the top of Pike's Peak (4300 m) for an elaborate series of measurements (68). Although this early work did not compare in accuracy with his later standards, yet it proved conclusively that the coefficient of absorption derived from Kolhörster's data was far too high. This discrepancy disappeared when Kolhörster, after measuring cosmic-ray intensities in ice caves of the Jungfrau glacier, scaled down his absorption coefficient to less than one-half, namely, to $\mu = 0.25 \times 10^{-2}$ cm$^{-1}$ of water. Thus it became evident that the cosmic rays are many times more penetrating than any known radioactive rays, a fact which clearly demonstrated the importance of their further investigation. While heretofore cosmic rays had been studied only by meteorologists and specialists in radioactivity, Millikan recognized in them a subject capable of yielding information of wider importance for the whole of physics.

From then on Millikan brought to bear on this problem the whole of his vast experience as an experimental physicist. His next piece of work was carried out jointly with G. H. Cameron (69), and consisted in sinking electroscopes to various depths of mountain lakes and in measuring the cosmic-ray intensity as a function of depth (69). Chosen were Muir Lake (3540 m), near the top of Mt. Whitney, and Lake Arrowhead (1530 m) in Southern California; the greatest depth to which the instruments were lowered was 27 m. The very neat absorption curves which their measurements yielded justify the statement that it marked the beginning of modern accuracy in cosmic-ray work. Two important results were inferred from the analysis of these curves. (1) It was concluded that all the penetrating radiation came from above the upper lake and, within the precision of the analysis, no part of it had its origin in the air between the levels of the two lakes. (2) The radiation was found to have a band structure consisting of harder and softer components whose coefficients of absorption ranged from $\mu=0.30$ to $\mu=0.18$. Later, as the measurements were extended to higher elevations and to greater depths in water, new, softer, and harder components were found (70, 71), results which were confirmed by other observers. Millikan realized fairly early that the radiation measured in the atmosphere is not necessarily the primary radiation coming from outside but may consist of secondary and tertiary rays. Hence, it is not safe to make inferences about the nature of the primary cosmic radiation from the band structure observed in one single geographical location.

The subsequent cosmic-ray work of Millikan and his collaborators was too comprehensive to be surveyed paper by paper. We shall restrict ourselves to enumerating its most significant features.

(a) For many years the electroscopes developed by Millikan and his school were more accurate than those employed by other workers. Marked improvement in accuracy was achieved by the use of high pressure ionization chambers, with 8 atmos. pressure of air in 1928 and 30 atmos. in 1931. (Subsequently the air was replaced by argon.) Two years later H. V. Neher developed his self-recording instrument with a very sensitive, temperature-independent, and vibration-free quartz system which even today satisfies all requirements of precision.

(b) Because of the superior accuracy of his measurements, Millikan was able to disprove claims put forward, at different times, by other observers with respect to large daily variations of cosmic-ray intensity in dependence on the positions of the sun and of the stars. His school contended from 1923 on (70, 71) that the diurnal variations are either very small or nonexistent.***

(c) For the same reason Millikan and co-workers obtained, comparatively early, good curves of cosmic-ray intensities at high elevations in the atmosphere. Airplane flights yielded accurate results from 1933 on (71); sounding balloons, whose data were at first less reproducible, gradually also became highly reliable. It was found that the intensity curves have at a certain height an inversion point (74), then reach a maximum, after which they turn back. In as much as the response of the ionization chamber stands in a simple relation to the energy of the radiation, it is possible to derive from an intensity curve the total energy penetrating from outside into the atmosphere in the form of cosmic rays. Thus Millikan's energy determinations are independent of any hypothesis about the nature or mechanism of the radiation phenomena.

(d) After the geomagnetic effect had been discovered by J. Clay and co-workers (75), Millikan and Neher found that it was more strongly marked at high altitudes. On the one hand, it was possible to calculate the velocity of the primaries eliminated by the geomagnetic effect between two locations of different latitudes. On the other hand, the comparison of the atmospheric cosmic-ray-intensity curves for these two locations yielded the coefficients of absorp-

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*** At present it is generally agreed that the amplitude of the daily solar wave (73) is of the order 0.25 of one percent of the mean intensity. The question of the existence of small sidereal variations is as yet undecided.

tion of the radiation components weakened or removed in passing from the first location to the second. The results showed that the primitive view of cosmic-ray absorption as entirely caused by ionization was quite untenable. Since the existence of the maximum in the curves, mentioned under (c), pointed in the same direction (as also the existence of some unusually penetrating cosmic-ray corpuscles, ascertained by other observers (76) with Geiger counters), these data formed a strong incentive for the development of modern theories of the stopping of fast particles.

(e) In 1929 D. Skobelzyn (77) presented strong evidence that some fog tracks he had observed in a Wilson expansion chamber were caused by cosmic rays. Sensing with his characteristic intuition the opening of a new research province, Millikan realized at once that here was a new and promising approach to the problems of the nature of cosmic rays and of the mechanism of their absorption. He constructed in 1931, jointly with Carl D. Anderson, a large vertical expansion chamber in a homogeneous magnetic field of 20,000 Gauss. Even the early photographs showed the presence of electrons with kinetic energies of over $10^9$ electron volts (78). It is common knowledge how in Anderson's hands (seconded by S. H. Neddermeyer) this method led to the discovery of the positron and the mesotron and how it continues to yield insight into atomic phenomena of fundamental importance in nuclear physics.

(f) In recent years Millikan, in collaboration with Professors H. V. Neher and W. H. Pickering, was engaged in a refined analysis of the primary cosmic-rays spectrum by means of studying the geomagnetic effect (79). This is accomplished by measuring at each of the different geographic locations two atmospheric intensity curves. The first is the curve of the ionization chamber intensities, as a function of altitude, already mentioned under (c) and (d). These measurements refer to the total intensity coming in from all sides. The second curve is that of the intensities of cosmic rays coming down in the vertical direction. They are obtained by sending up sounding balloons equipped with a vertical arrangement of Geiger counters. When the observer moves from a northerly geographic location southward, the positively charged primary cosmic rays of a given velocity are affected by the terrestrial magnetic field as follows. The first rays to be cut out are those coming from the eastern horizon. From then on the remaining rays form a gradually contracting cone with a roughly westerly axis. That cone reaches at a certain latitude the vertical, but in going further south it continues ot contract; the rays from the western horizon are the last to disappear. In this way, the latitude at which a certain portion of the vertical intensity is cut out furnishes a sharp criterion for the primary velocity of this portion. The method of measuring both the vertical and the all-sided intensities permits of a much better resolution of the primary cosmic-ray spectrum than the method of measuring the all-sided intensity alone. As a result of extensive investigations it was found that the energy spectrum of the primaries possesses a band structure. To explain both the origin of cosmic rays and the existence of the bands Millikan proposed the theory of annihilation of atoms in the interstellar space. Atoms get annihilated in a single elementary process, converting their whole intrinsic energy into the kinetic energy of a positive-negative particle pair created in the process. The rays of each observed band are then due to the annihilation of atoms of one particular element especially abundant in the interstellar space. In this way he associates the several bands, respectively, with helium, carbon, nitrogen-oxygen, silicon and finds a reasonable agreement between the particle energies of the bands and the intrinsic energies of the atoms, as also between the total energies of the bands and the abundances of the atoms. If the theory of atom annihilation is not generally accepted, its value as a working hypothesis cannot be denied. In Millikan's hands it served time and again for planning new observations which produced valuable data independently of any theory.

At this writing Dr. Millikan's interest in cosmic rays is as active as ever. He has but recently returned from an extended field trip during which he sent up sounding balloons in numerous locations ranging from Texas to Saskatchewan. Now he is engaged in evaluating the new results and is giving, besides, a course of lectures on "History of Modern Physics."

He is doing a good deal of administrative work for the Institute largely created through his efforts, when he is not kept busy preparing new editions of his books or doing other literary work. The staff members of the California Institute hope to enjoy for many years the benefit of his keen mind, kind heart, and vast experience.

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