MAGNETISM IN DISCONTINUOUS MEDIA

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Introduction

MAGNETS have always interested the inquisitive, perhaps the more so because the peculiar state of things in the neighborhood of magnets is not directly appreciable by any of our sense organs. Since the discovery by Oersted of a connection between magnets and electric currents, and the immediately succeeding researches, by Ampere and by Faraday, which defined this connection and made it possible to use electrical apparatus in the study of magnetism, the problems magnetism presents have been attacked again and again, and the results of experiment have been discussed with great gusto by many writers of books and monographs. The present situation is such that the veriest tyro in physics can easily collect new, and possibly significant, data which the most learned can hardly explain in terms of simple concepts. Under these conditions, which have long obtained, it is not surprizing that experiment has here outstripped theory farther than in any other part of the physicist's universe. It is the aim of this review to call to the attention of those concerned the adequacies and deficiencies of present-day attempts at explanation, and to suggest some promising lines for further advances.

The title of this review has been chosen to emphasize the fact that we are now thoroughly committed to the type of theory which denies the existence of those continuous jelly-like media with which alone the Victorian school of theoretical physicists, and its predecessors, would have anything to do. There are not, as far as we need now to suppose, any media which are absolutely discontinuous in a mathematical sense. All real media, on the other hand, are physically discontinuous in the sense that they naturally subdivide themselves into regions where we can at least imagine ways of experimenting that would disclose relatively slow changes of condition from point to point, or from instant to instant, and regions where changes are so abrupt that no sufficiently delicate probe can be inserted into them, even in imagination, to analyse their peculiarities. The boundaries, necessarily rather hazy, between regions of these two sorts are the discontinuities referred to in our title. We have not, in choosing it, denied ourselves the right to consider any fact of experience, but in the interest of brevity we will take care to pass lightly over those phenomena for which the older continuous fluid theories are still adequate, in order to spend more time with experiments and explanations which stress the atomistic and catastrophic features of our subject.

The Magnetic Field

Magnetic theory made its first great advance when it introduced the notion of the magnetic field. The magnetic field intensity, or magnetic force,

H, is a vector point function which represents in every accessible region the resultant magnetic effect of all the magnets and electric currents with which we may for the moment be concerned. In attempting to define this field intensity in terms of electric charge and its motion—the only undefinable fundamentals which we allow ourselves in this branch of physics—we find it necessary to introduce an electric field intensity, $\boldsymbol{E}$, which is also a vector point function.

The familiar equations stating all that we know about $\boldsymbol{H}$ are:

$$\nabla \cdot \boldsymbol{H} = 0,$$

$$\dot{\boldsymbol{H}} = -c(\nabla \times \boldsymbol{E}),$$

$$\nabla \times \boldsymbol{H} = \frac{1}{c}(\dot{\boldsymbol{E}} + 4\pi\rho \boldsymbol{v})$$

The additional equation necessary for defining $\boldsymbol{E}$ is:

$$\nabla \cdot \boldsymbol{E} = 4\pi\rho.$$

In these equations ρ is the volume density of positive electric charge and v is its velocity at the point considered; c is the velocity of light. The quantity H will be in e.m.u. (gauss) if E is in e.s.u., (which puts ρ in e.s.u. cm^{-3}), v and c in cm $\cdot$ sec.$^{-1}$ The somewhat unusual order of the equations has been chosen to emphasize our pre-occupation with $\boldsymbol{H}$.

Not many years ago those parts of space which were, as the phrase went, occupied by ponderable matter, were regarded as inaccessible to detailed study, and the electromagnetic field equations were revised in such a way that the gross phenomena of magnetism might be expressed by equations almost as simple as those written down in the last paragraph.

This was a bold and clever move, but in the sequel it has made a great deal of trouble, particularly between physicists and electrical engineers, and since all this trouble is due to the fallacious assumption that a magnetic medium is a continuous jelly with any properties we choose to foist upon it, we will do well to consider briefly what went wrong.

Faraday's "tube of force" was undoubtedly the root of all evil in this connection. To preserve these tubes unaltered in number as they were followed through various pieces of matter about the interior of which nothing much was known, it was found necessary to invent for them a vector point function, $\boldsymbol{B}$, called the magnetic induction, which could not be distinguished from $\boldsymbol{H}$ in what was fondly termed "empty space," but which differed markedly from $\boldsymbol{H}$ in "ponderable matter." To make matters worse, it soon appeared desirable to invent two more vector point functions for ponderable matter only, and one of these, in all innocence, was called the magnetic field intensity and given the symbol $\boldsymbol{H}$ in spite of the fact that it did not satisfy the equations—given above—by which alone $\boldsymbol{H}$ had been defined.[1]

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[1] The order in time of these *faux pas* has been altered to make their cumulative absurdity more apparent.

The other new vector point function, the intensity of magnetization, I, did not usurp a place already occupied.

In order to reduce by a little the resultant confusion we will here distinguish this interloping H as H', and call it the *fictitious* magnetic field intensity. It represents, at a point inside a "magnetized body" the magnetic field intensity which would be found there if the body were replaced by a vacuum without changing the state of external magnets or currents in any way. In order at the same time to preserve the values of H, due in part to the thus annihilated body, at external points, it is necessary to distribute over its vacated surface a proper distribution of (fictitious) free magnetic poles. The magnetic field intensity which would then arise at internal points from the presence of these (fictitious) poles is $4\pi I$. In using these definitions for H' and I it should always be remembered that the "body" to be considered is only so much of the real piece of matter as is strictly homogeneous in its magnetic behavior. In many cases the fictitious magnetic poles would have to be spread throughout the volume as well as over the bounding surface of the piece. With all these precautions then, we may use when we like the following equations:

$$B = H' + 4\pi I,$$

$$B = \mu H',$$

$$I = \kappa H'.$$

Here μ and κ are, respectively, the magnetic permeability and the magnetic susceptibility. Both μ and κ may be complex numbers. It is of no interest to us here whether μ and κ are pure numbers or not, nor, whether B and H need separate names for the units in terms of which they are expressed.[2] We can see from these equations, and it is worth emphasizing the point, that B is a space-average of H throughout a small region which is nevertheless large enough to include representative samples of all the various values of H which the most refined analysis could discover.

We must be careful to note that though we may never probe deeply enough into atomic structure to interest ourselves in the intense and strongly curved fields of H which we suppose to exist inside atoms and nuclei, we may still suppose that H has a value at every point in space and use our electromagnetic field equations in more accessible parts of magnetic media without any fear of self-contradiction.

ELEMENTARY MAGNETS

The second great impetus to magnetic theory was afforded to it by Ampere's recognition of the fact that the magnetic effects of permanent and in-

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[2] Just as this goes to press the International Electrotechnical Commission is reported—Nature **126**, 252 (1930)—to have adopted "gauss" as the name for the unit of flux density, *i.e.* of **B** and **H** as used herein, and "oersted" as the name for the unit of magnetic field intensity [inside ferromagnetic media], *i.e.* of **H'** as used herein. The second of these names has not been current in this sense, and unless the confusion between **H** and **H'** gets to be generally understood, it is hardly likely to become so.

duced magnets are attributable to the presence within them of permanently circulating electric currents. Weber added the notion that non-decaying eddy currents can account for diamagnetic behavior, and suggested that both sorts of currents may circulate within the molecule, thus constituting truly elementary magnets.[3] The assumed structure of these Weber elements has changed from time to time as fashion has dictated. They began as resistanceless grooves in which continuous streams of electricity were supposed to flow. At a later period they became spherical or ellipsoidal clouds of negative electricity in rotation about their axes in a more rarefied cloud of positive electricity.[4] Still later they metamorphosed into electrons in orbital motion. At present they are usually conceived to be spinning electrons, the centers of which are found from instant to instant to lie at any distance—but with certain definite distances more highly probable than others—from an atomic nucleus also capable of spin. Concerning the spatial distribution of the electric charge which spins, theories are now rather non-committal, ignoring the possibility of spatial definiteness within sub-atomic regions. This seems rather unkind when we remember that the anomaly in the gyromagnetic ratio—the ratio of the angular momentum to the magnetic moment of the moving electricity—was at one time taken to be strong evidence in favor of the hypothesis that elementary magnets were spinning electrons.

The least magnetic moment of a modernized Weber element is agreed to be about $9.2 \cdot (10)^{-21}$ e.m.u.[5] (The magnetic moment of an atomic nucleus is supposed to be much less than this.[6]) This magnetic moment is also a fundamental unit; a Weber element may have a magnetic moment equal to any integral multiple of these so-called Bohr magnetons. The designation "Bohr" is necessary because of the prior existence of an empirical unit, the "Weiss" magneton, the present agreed value of this being $1.857 \cdot (10)^{-21}$ erg $\cdot$ gauss$^{-1} \cdot$ mol^{-1}. Weber elements with more than a few Bohr magnetons are as yet unknown.

The least magnetic moment associated with matter is that exhibited by an atom, ion or molecule, which may contain from one to several hundred Weber elements with from one to six Bohr magnetons each. It might be supposed that the resultant magnetic moment of structures as complicated as some of these would be a very indefinite quantity with a fair probability of high values in terms of the unit. It appears, however, that the stable configurations are of such a sort that the net magnetic moment is always a small

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[3] W. Peddie gives in his Molecular Magnetism, Longmans Green and Co. 1929, pp. 17–22, a clear account of Weber's contributions to magnetic theory. What Weber called a molecule would now more usually be identified as an atom, especially in crystals of the chemical elements.

[4] R. Gans, Ann. d. Physik [4] **49**, 149–178; **50**, 163–198 (1916).

[5] R. T. Birge—Phys. Rev. Suppl. **1**, 1–73 (1929)—gives $(9.174_{70} \pm 0.013) \cdot (10)^{-21}$ erg$\cdot$gauss^{-1} for the value derived from the best spectroscopic data, $(9.216_{38} \pm 0.016) \cdot (10)^{-21}$ erg$\cdot$gauss^{-1} for the value derived from experiments on the deflection of cathode rays.

[6] E. Fermi, Zeits. f. Physik **60**, 320–333 (1930); J. Dorfmann, Zeits. f. Physik **62**, 90–94 (1930); S. Goudsmit, L. A. Young, Phys. Rev. [2] **35**, 1418–1419 (1930); H. S. W. Massey, Proc. Roy. Soc. [A] **127**, 666–670 (1930).

integral number of Bohr magnetons. In very many cases the net magnetic moment is zero, in no case yet studied is it more than seven Bohr magnetons.[7]

The behavior of isolated atoms, ions and molecules is thus about as simple to handle as that of isolated Weber elements. In what follows we will therefore be justified in discussing such units of matter, when independent of each other's influences, as if they were small rigid magnets of moment $\mathbf{M}$, the scalar part of which, M, is a constant.

The simplest possible ion, an isolated electron, ought, unless the phenomenon of spin can in some way be denied to it, to have a magnetic moment of one Bohr magneton, and this may modify its motion in a predictable way under assigned conditions. Several experiments to settle this point have been imagined,[8] and the results of experiments on the scattering of beta-particles[9] have been taken to throw some light on the subject. No conclusion can as yet be drawn.

Wherever the units of matter have any freedom of rotation, that is under all known conditions, we must expect diamagnetism to exist. If any of the units have values of M different from zero paramagnetism must exist. If any of these units are so constrained that $\mathbf{M}$ can maintain its direction against disturbing agents, ferromagnetism is present. It will be noticed that diamagnetic, paramagnetic, and ferromagnetic are not exclusive categories. It is none the less convenient to consider such properties separately and in the order just given because the conditions become increasingly complex as we thus proceed from the general to the particular. Naturally enough we find more and more convincing evidence for the atomicity of magnetism as we proceed in this direction.

A cross-classification, which is useful in separating the experimental and theoretical work into more homogeneous groups, is that afforded by the physical state of the system considered. We will follow this plan, first dealing with isolated atomic magnets, then with fluid media and finally with solids, that is to say, with crystals. In the treatment of crystals we will take up, not quite in chronological order, the more important theories so far proposed, prefixing the name of its author to the section in which each theory is treated.

ISOLATED ATOMIC MAGNETS

If an atom or molecule be shot through a non-uniform magnetic field its direction of motion will change in a manner and to an amount which depends upon its magnetic moment, $\mathbf{M}$. This is an abbreviated description of the famous Stern-Gerlach experiment, which may be said to have demonstrated the existence of atomic magnets. In the hands of its originators and their

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[7] J. Becquerel and W. J. de Haas—J. de phys. [7] **1** 7S–9S (1930)—assign this magnetic moment to the ions of gadolinium in the mineral xenotime.

[8] L. Brillouin, Comptes rendus, **184**, 82–84 (1927); Proc. Nat. Acad. Sci. **14**, 755–763 (1928); V. M. Albers, T. E. Phipps, Phys. Rev. [2] **31**, 149 (1928); T. Sexl, Zeits. f. Physik **48**, 729–730 (1928); F. Wolf, Zeits. f. Physik **52**, 314–317 (1928); F. E. Myers, R. T. Cox, Phys. Rev. [2] **34**, 1067–1068 (1929); F. Knauer, Zeits. f. Physik **59**, 807–811 (1930).

[9] M. C. Henderson, Phil. Mag. [7] **8**, 847–857 (1929).

followers it has already supplied much evidence confirming the spectroscopist's assignment of electrons to the various groups within the atomic structure.[10] Changing the type of non-uniform field, and the way of entering or leaving its boundaries, should yield additional information.[11]

The Stern-Gerlach experiment is a brilliant example of success where success was hardly to be expected. It is easy to prove that a magnet fired from field-free space into a magnetic field of any sort can never align itself with respect to the field, but must continually precess so as to maintain invariant that component, M_p, of its original magnetic moment, $\boldsymbol{M}$, which lies parallel to the field. The proof being due to Larmor such precession is referred to under his name. All that classical theory can predict under the circumstances is a symmetrical broadening of the beam in the direction of the gradient of $\boldsymbol{H}$, here roughly parallel to $\boldsymbol{H}$ in the strongest part of the field. The actual effect is a splitting of the beam into several beams in each of which M_p is an integral number of Bohr magnetons, including $+M$ and $-M$ among its values.

The logical discontinuity here presented has been encountered in other places, but seldom so patently. To escape from the dilemma it is necessary to jettison classical electro-dynamics.

Consider a Weber element entering a magnetic field and beginning to precess. The system formed by the field and the magnet is characterized by two frequencies, one being that of the rotation which fixes M, the other being that of the Larmor precession. Since the frequency of precession is low in the weak magnetic field first traversed, the corresponding energy states are, on quantum theory, close together and the probability of transition from one state to another is high. The magnetic energy of each state is $W_i = \boldsymbol{M}_i \cdot \boldsymbol{H}$, where the vectors $\boldsymbol{M}_i$ differ only in direction. We may also consider the fact that the states are ill-defined—the quantization weak—because the magnet does not stay long enough in any one part of the field for the precession to complete many cycles at substantially constant frequency. Under these conditions the system radiates energy in small amounts so as continually to adjust its energy to one of the states in which the value of M_p is integral in Bohr magnetons. In the stronger parts of the field the quantization is stronger and the stability of any permissible state is correspondingly greater. Here the resolution of the beam into its components of different M_p has time to occur. Finally, in leaving the field, the quantization again weakens and the individual beams become more diffuse. The greater the number of permissible states—the greater the value M—the less is the chance for clear resolution. Thus Gerlach[10] now states that the results for iron atoms do not, as at first supposed, indicate $M=0$, but suggest that $M>6$. (In the case of cobalt a value $M_p=6$ can just be established.) The experimental resolving

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[10] W. Gerlach—J. de phys. [6] **10,** 273–282 (1929)—has recently summarized the situation and given some further information regarding the first experimental results.

[11] I. I. Rabi, Zeits. f. Physik **54,** 190–197 (1929); D. E. Olshevsky—Phys. Rev. [2] **35,** 659 (1930)—is preparing a critical summary of all possible methods.

power is always inferior to the theoretical limit because all the atoms entering the field do not, as has been tacitly assumed, have equal velocities.

The behavior of a diamagnetic atom or molecule under the conditions of the Stern-Gerlach experiment is quite different from that of the paramagnetic unit so far considered. Since the magnetic moment of the diamagnet is, except for a small correction term,[12] directly proportional to H and antiparallel to $\boldsymbol{H}$, all the units of one kind should follow the same path and no resolution into discrete beams can be expected. It is still possible to imagine that a gas may permanently contain molecules with different moments of inertia or with different distributions of magnetically balanced Weber elements.[13] If so, the experiment should give pertinent data regarding these differences.

FLUIDS

The molecules of a gas under ordinary conditions of experiment may be regarded as magnetically independent. The magnetization, $\boldsymbol{I}$, of a gas-filled space is therefore dependent upon the distribution of the unitary magnets among the same permissible energy states as those of interest in the Stern-Gerlach experiment and its derivatives. Explicitly,

$$I = \frac{1}{V}\sum_{V} M_p,$$

where V is a small volume containing many unitary magnets. This additive property of the units has a very important consequence. It permits us to study the magnetic moment of many molecules and ions in solution, by making proper correction for the magnetic characteristics of the solvent.

Even in complex molecules, and particularly in organic compounds, it is found that the atoms are often sufficiently free to behave independently as regards magnetic matters, so that Pascal[14] and others have been able to assign to each of several important constituent atoms its proper contribution to the magnetic result and to show that the mode of combination has a characteristic effect upon this contribution.[15] In heteropolar compounds, and in what are called coordination compounds, the magnetic contribution of a given chemical element may vary not only in magnitude but even in sign.

In a diamagnetic fluid the parallelism of the induced magnetic moments of the units makes it difficult to find out anything about the individuals from a study of the magnetic susceptibility of the medium they constitute. For a time it seemed that an anomalous variation of susceptibility with pres-

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[12] J. H. Van Vleck, A. Frank, Proc. Nat. Acad. Sci. **15**, 539–544 (1929); F. Bitter, Phys. Zeits. **30**, 497–504 (1929); K. F. Niessen, Phys. Rev. [2] **34**, 253–278 (1929).

[13] E. C. Stoner—Proc. Leeds Phil. Lit. Soc. [Sci. Sect.] **1**, 484–490 (1929)—discusses the effect of spatial distribution of electric charge upon diamagnetic susceptibility.

[14] P. Pascal, Ann. de chim. et de phys. [8] **16**, 359–402, 520–574 (1909); **19**, 5–70 (1910); **25**, 289–377 (1912); **29**, 218–243 (1913).

[15] The comparison of electronic isomers is particularly interesting: S. Bhatnagar, R. N. Mathur, R. S. Mal, Phil. Mag. [7] **10**, 101–109 (1930).

sure—observed by Glaser—would furnish such detailed information. Other experimenters[16] attribute the anomaly to accident, but the discoverer still[17] thinks it significant. The wave mechanical theory of diamagnetism[12] offers some hope to the experimenter by predicting the presence of a paramagnetic term in the susceptibility if the electron distribution is not of spherical symmetry, but this term must generally be small even in comparison with the difficult measurable diamagnetic term.

In paramagnetic solutions the observed magnetic moment per dissolved atom may give very valuable indications of its electronic structure. Weiss[18] and Cabrera[19] have furnished most of the data, and attempt to show that the results are compatible with the existence of a unit of magnetic moment, the so-called Weiss magneton, which has been mentioned above. There are several puzzling cases, however, in which it has been necessary to suppose that ions of one sort may, under one set of conditions, have either of two different magnetic moments, each an integral multiple of the preferred unit. By this means non-integral multiples can be explained away. In the interests of fair play it should be remarked that the same artifice is even more often necessary in explaining experimental results in terms of the five times larger Bohr magneton. In this case, however, the argument for a variety of integral contributions, including zero, is part of the original theory and not an *ad hoc* addition. Much ingenuity has been displayed in bringing the quantum theory into agreement with the curious succession of M-values found for ions having from 18 to 28 extra-nuclear electrons—the iron family—and those having from 54 to 68—the rare-earth group. It will be sufficient here to remark that the agreement now appears remarkably exact.[20]

The value of M is obtained by what amounts to an extrapolation to infinite field intensity or to zero absolute temperature. The law of variation of paramagnetic susceptibility with H and T lies therefore at the bottom of all the experimental and theoretical agreements just mentioned. This fundamental relation was derived by Langevin on the assumption that the molecules of a gas were small permanent magnets without transverse dimensions, so that the principal moment of inertia about the magnetic axis of each was zero. The treatment was classical so that it was assumed that all magnetic

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[16] G. W. Hammar, Proc. Nat. Acad. Sci. **12**, 594–596, 597–598 (1926); V. I. Vaidyanathan, Indian J. Physics **1**, 183–198 (1926); **2**, 135–150 (1928); Phil. Mag. [7] **5**, 380–386 (1928); H. Buchner, Ann. d. Physik [5] **1**, 40–48 (1929); F. Bitter, Phys. Rev. [2] **35**, 1572–1575 (1930).

[17] A. Glaser, Ann. d. Physik [5] **2**, 233–248; **3**, 1119–1135 (1929); **4**, 82–90 (1930).

[18] P. Weiss, Arch. des sci. [4] **31**, 401–438 (1911); **35**, 406–408 (1913); [5] **2**, 417–419 (1920); J. de phys. [6] **5**, 129–152 (1924); Helv. Phys. Acta **2**, 283–284 (1929); Trans. Amer. Electrochem. Soc. **55**, 75–78 (1929); J. de phys. [7] **1**, 185–193 (1930); F. Birch, J. de phys. [6] **9**, 137–141 (1928).

[19] B. Cabrera, J. de phys. [6] **3**, 443–460 (1922); Anal. fis. quim. **21**, 505–526 (1923); **23**, 151–157 (1925); J. de phys. [6] **6**, 241–258, 273–286 (1925); **8**, 257–275 (1927); Atti congr. internat. dei fisici, Como 1927, **1**, 95–115 (1928); Anal. fis. quim. **26**, 50–66 (1928).

[20] D. M. Bose, Atti congr. internat. dei fisici, Como 1927, **1**, 119–125 (1928); Phil. Mag. [7] **5**, 1048–1067 (1928); D. M. Bose, H. G. Bhar, Zeits. f. Physik **48**, 716–721 (1928); E. C. Stoner, Phil. Mag. [7] **8**, 250–266 (1929); A. Sommerfeld, Wien. Ber. **139** [2a] 11–17 (1930).

states—all relative orientations of a magnet and the applied magnetic field—were permissible. The temperature dependence of paramagnetic susceptibility arises on this theory from the increasing number of collisions with increasing temperature at constant volume.

It is apparent at a glance that the magnets of Langevin's theory differ in one important respect from the magnetically unbalanced assemblages of Weber elements to which we have committed ourselves in the present discussion. The magnets in our picture must possess angular momenta, those in Langevin's picture must not, for if they did the applied magnetic field—on classical dynamics—could never align them. This contradiction, which we found it necessary to circumvent in the Stern-Gerlach experiment has most recently been emphasized by Honda,[21] but has not escaped previous remark by others.[22] The fact that Langevin's theory works so well for extrapolation is another of those pleasant surprizes which make experimental physics so exciting.

The substitutes for Langevin's theory which are acceptable in quantum mechanics differ from it principally under extreme conditions.[23] The new expression for relative intensity of magnetization in terms of the saturation value,

$$\frac{I}{I_s} = \tanh \frac{MH}{kT},$$

can hardly be distinguished for wide ranges of H and T from Langevin's expression

$$\frac{I}{I_s} = \coth \frac{MH}{kT} - \frac{kT}{MH}.$$

Schidlof[24] has recently discussed the range of validity of the quantum-theoretical result and makes it clear that pushing the equation too close to $T=0$ is not justifiable. The practice of extrapolating Langevin's equation into the same region is still more unwise.

MESO-PHASES

A few solids do not melt to form clear liquids but have a turbid phase which is stable over a definite range in temperature. These meso-phases are classified as nematic—stringy—or smectic—smeary—depending upon the structures they exhibit when subjected to the action of magnetic fields, adjacent surfaces, or other orientating agent. The control of their structure by magnetic fields is formally attributable to their anisotropy of magnetic susceptibility. The axis of the strings in nematic fluids and the normal to

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[21] K. Honda, Zeits. f. Physik **63**, 141–148 (1930). In an earlier paper—Sci. Rep. Tohoku Imp. Univ. [I] **17**, 997–1009 (1928); Zeits. f. Physik **47**, 691–701 (1928)—he avoided this difficulty, in ferromagnetic media only, by inventing a nuclear magnet without angular momentum.

[22] *e.g.* by J. Kroo, Ann. d. Physik [4] **42**, 1354–1396 (1913).

[23] W. Lenz, Phys. Zeits. **21**, 613–615 (1920); W. Schütz, Zeits. f. Physik **54**, 731–735 (1929).

[24] A. Schidlof, Helv. Phys. Acta **1**, 578–600, 601–612 (1928).

the layers in smectic fluids are unique axes of magnetization. An attempt has been made[25] to compute the value of M for a magnetic unit which would be orientated in the observed manner in spite of heat motions, but the assumption that the only controlling force is magnetic in origin is probably not allowable.[26] Besides more recent studies on the magnetic properties of these interesting substances themselves,[27] an attempt has been made to modify the properties of the crystals which freeze out from them by applying a strong magnetic field during their formation.[28] The result of this test was negative in the case of a smectic substance, positive in the case of a nematic substance.

DIAMAGNETIC CRYSTALS[29]

The first attempt to construct a theory for the magnetic behavior of crystals was made by William Thomson (later Lord Kelvin) in 1850.[30] His theory took no account of the atomicity of magnetism, and is of no interest to us here except for the fact that it predicts that crystals of the cubic system are magnetically isotropic. Certain cubic crystals have been reported to be anisotropic by Forrest[31] but contrary results for one of the simplest of the materials he tested—copper—have more recently been obtained by Montgomery[32] in this laboratory.

In non-cubic diamagnetic crystals the anisotropic susceptibility makes it necessary to suppose that the non-decaying eddy currents are constrained to move in particular planes with respect to the crystal axes. This means, in turn, that the atom, or larger structure, through which these currents circulate is prevented by its neighbors in the crystal from rotating freely about a random axis.

Recent experiments by Raman[33] and his colleagues[34] make it appear probable that in bismuth, antimony and graphite the abnormally high diamagnetic susceptibility along the three-fold or six-fold axis is due to circuital currents which encircle not one but many atoms. Subdivision of the crystals,

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[25] W. Kast, Ann. d. Physik [4] **83,** 391–417 (1927).

[26] H. Zocher, Phys. Zeits. **28,** 790–796 (1927); M. Jezewski, Zeits. f. Physik **52,** 268–274, 878–881 (1929).

[27] V. Freederiksz, V. Zolina, Trans. Amer. Electrochem. Soc. **55,** 85–96 (1929); A. v. Wyk, Ann. d. Physik [5] **3,** 879–893 (1929); H. Zocher, V. Birstein, Zeits. f. phys. Chem. [A] **142,** 186–194 (1929).

[28] G. Foëx, J. de phys. [6] **10,** 421–438 (1929).

[29] Non-crystalline solids may be regarded as hyper-viscous fluids and demand no special treatment here. Disorderly aggregates of crystals are considered later. Unless otherwise stated the crystals considered by theory are ideally perfect.

[30] Rep. Brit. Assoc. [II], 23 (1850); Phil. Mag. [4] **1,** 177–186 (1851).

[31] J. Forrest, Trans. Roy. Soc. Edinb. **54,** 601–701 (1926); Phil. Mag. [7] **3,** 464–476 (1927).

[32] C. G. Montgomery, Phys. Rev. [2] **36,** 498–505 (1930).

[33] C. V. Raman, Nature **124,** 412 (1929); Proc. Phys. Soc. London **42,** 309–320 (1930); *cf.* P. Ehrenfest, Physica **5,** 388–391 (1925); Zeits. f. Physik **58,** 719–721 (1929).

[34] V. I. Vaidyanathan, Nature **124,** 762 (1929); **125,** 672 (1930); S. Paramasivan, Indian J. Phys. **4,** 139–145 (1929).

or heating, makes the abnormality less. The effect of heating is signfiicant because, unless inter-atomic supraconduction of the postulated sort is broken up by heat motions temperature changes should not affect the atomic susceptibility.

The alkali metals, for which theory can make some prediction of atomic susceptibility, have been studied with particular care by Lane[35] and by McLennan and his co-workers,[36] who have also made measurements on zinc and cadmium.[37] Bismuth in massive crystals has been reexamined by Fock.[38] Several investigators[39] have reported on crystals of organic compounds or on other classes of compounds where related structures may be compared. Bose[40] has found a simple salt of nickel which is diamagnetic. The effect of alloying has been examined by Davies and Keeping.[41] All these data will find their place in the final theory for the dependence of diamagnetism on crystal structure. Other effects which are as yet outside any theory are changes in the thermoelectric behavior of bismuth when crystallized in a strong magnetic field,[42] the effect of cold work upon the susceptibility of copper and silver,[44] and the changes in the magnetic behavior of solid solutions of copper and gold with heat treatment which favors a regular distribution of each constituent.[45]

Paramagnetic Crystals

In many paramagnetic crystals the susceptibility, independent of direction, may be interpreted in terms of Langevin's theory, or its quantum substitute, as well as if the medium were a fluid containing the same average concentration of unitary magnets. Most of the experimental work has been done at very low temperatures, where the dependence of susceptibility upon

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[35] C. T. Lane, E. S. Bieler, Trans. Roy. Soc. Canada [3] **22** [III], 117–123 (1928); C. T. Lane, Diss. McGill (1929); Phil. Mag. [7] **8**, 354–362 (1929); Phys. Rev. [2] **35**, 977–981 (1930); for the theory see W. Pauli, Zeits. f. Physik **41**, 81–102 (1927); L. Rosenfeld, Naturwiss. **17**, 49–50 (1929).

[36] J. C. McLennan, R. Ruedy, E. Cohen, Proc. Roy. Soc. [A] **116**, 468–483 (1927); J. C. McLennan, E. Cohen, Trans. Roy. Soc. Canada [3] **23**, [III], 159–168 (1929).

[37] J. C. McLennan, R. Ruedy, E. Cohen, Proc. Roy. Soc. [A] **121**, 9–21 (1928).

[38] A. B. Focke, Phys. Rev. [2] **36**, 319–325 (1930).

[39] S. Bhagavantam, Proc. Roy. Soc. [A] **124**, 545–554; **126**, 143–154 (1929); Indian J. Phys. **4**, 1–14 (1929); S. S. Bhatnagar, S. L. Luther, J. Indian Chem. Soc. **6**, 303–307 (1929); S. S. Bhatnagar, R. N. Mathur, Phil. Mag. [7] **8**, 1041–1055 (1929); R. Hocart, Comptes rendus, **188**, 1151–1153 (1929); C. V. Raman, Nature **123**, 605, 945 (1929); F. W. Gray, J. Farquharson, Phil. Mag. [7] **10**, 191–216 (1930).

[40] D. M. Bose, Nature **125**, 708 (1930).

[41] W. G. Davies, E. S. Keeping, Phil. Mag. [7] **7**, 145–153 (1929).

[42] A. Goetz, R. C. Hergenrother, A. B. Focke, Phys. Rev. [2] **34**, 546–547 (1929); A. Goetz, M. Hassler, Phys. Rev. [2] **34**, 549 (1929); A. Goetz, Phys. Rev. [2] **35**, 193–207 (1930). In other crystals so far tested this procedure is without effect: L. A. Welo, Phys. Rev. [2] **34**, 296–299 (1929).

[43] P. L. Kapitza, Nature **124**, 53 (1929).

[44] F. Bitter, Phys. Rev. [2] **36**, 978–983 (1930).

[45] H. J. Seemann, E. Vogt. Ann. d. Physik [5] **2**, 976–990 (1929).

magnetic field intensity becomes important. The Cryogenic Laboratory of the University of Leiden has led in these investigations.

Becquerel and de Haas[46] have recently discovered that the Faraday effect—the rotation of the plane of polarization of light transmitted in the direction of an applied magnetic field—may be used as a measure of the magnetization in transparent crystals. This permits quantitative studies with specimens too small for examination by electrical methods.

Langevin's theory predicts that in low magnetic fields paramagnetic susceptibilities should vary inversely as the absolute temperature. This agreed with the previous experiments of Pierre Curie upon what he regarded as typical paramagnetic substances. As data accumulated it became apparent that many paramagnetic crystals and crystal aggregates show a more complicated dependence upon temperature.[47] Weiss pointed out that the form of Curie's Law could, at least in many cases, be preserved by measuring temperature not from absolute zero but from an origin chosen to give the best fit with the data. In the case of media which, at low temperatures, are ferromagnetic, this arbitrary origin as derived from high-temperature paramagnetism agrees fairly well with the temperature (Curie point, T_c) at which ferromagnetism finally disappears on heating. It is therefore customary to describe the temperature at which the value of the paramagnetic susceptibility, extrapolated from higher temperatures, would become infinite, as the paramagnetic Curie point.[48]

Unfortunately for the simplicity of the Curie-Weiss law, the paramagnetic Curie point is often below absolute zero, and even when it lies high enough to be accessible it is found that the law of variation of susceptibility changes as the critical temperature is approached, most substances remaining merely paramagnetic however much they may be cooled.[49] Weiss interprets such data as indicating allotropic modifications at definite temperatures, the low-temperature modification possessing a paramagnetic Curie point different from that of the high-temperature form—lower if ferromagnetism is successfully evaded. Others prefer to regard these results as proving that the simple inverse proportionality of atomic susceptibility and "corrected" temperature is an approximation at best, which diverges more and more

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[46] J. Becquerel, Proc. Amst. **32,** 523–535, 749–758 (1929); Zeits. f. Physik **58,** 205–216 (1929); J. de phys. [6] **10,** 313–320 (1929): J. Becquerel, W. J. de Haas, Proc. Amst. **32,** 536–549, 578–590, 590–596, 1199–1205 (1929); Zeits. f. Physik **57,** 11–29 (1929); J. de phys. [6] **10,** 283–293 (1929); [7] **1,** 7S–9S (1930); H. A. Kramers Proc. Amst. **32,** 1176–1189 (1929); H. A. Kramers, J. Becquerel, Proc. Amst. **32,** 1190–1198 (1929); J. Becquerel, W. J. de Haas, H. A. Kramers, Proc. Amst. **32,** 1206–1211 (1929); *cf.* M. Ramanadham, Indian J. Phys. **4,** 15–38, 109–129 (1929); L. Rosenfeld, Zeits. f. Physik **57,** 835–854 (1929).

[47] In some cases the paramagnetic susceptibility does not vary with temperature to an appreciable extent: P. Weiss, P. Collet, Comptes rendus **178,** 2146–2149 (1924); P. Collet, Comptes rendus **183,** 1031–1032 (1926); N. Perrakis, J. de phys. [6] **8,** 473–480 (1928); Trans. Amer. Electrochem. Soc. **55,** 79–83 (1929); A. Serres, Comptes rendus **188,** 1239–1241 (1929), S. Holgersson, A. Serres, Comptes rendus **191,** 35–37 (1930).

[48] The discrepancies between ferromagnetic and paramagnetic Curie points, T_f and T_p, have been discussed by R. Forrer, J. de phys. [7] **1,** 49–64 (1930).

[49] H. R. Woltjer, E. C. Wiersma, Proc. Amst. **32,** 735–738 (1929).

from the facts as the system becomes more and more degenerate—less and less "classical"—with decreasing temperature. The most elaborate analysis of this type is that offered by Schidlof.[24] His equations predict that at very low temperatures the paramagnetic susceptibility should increase very slowly, by only one-third of its value—here finite—at the pseudo Curie point—here always positive—in passing from that temperature to absolute zero.

In non-cubic crystals the susceptibility in different crystallographic directions is different, and varies differently with temperature, so that one crystal may have several paramagnetic Curie points and several Curie constants according to its orientation in the magnetic field. This suggests very forcibly that whatever these characteristic quantities really stand for, it depends in some way upon the usual (most stable) orientations of the unitary magnet within the crystal. These, in turn, must be fixed by the mutual effects of adjacent atoms in that systematic arrangement which distinguishes crystals from fluids. As regards paramagnetic, as distinct from ferromagnetic, crystals, theory has made but little progress in the analysis of these effects.

Weiss, who explains the behavior of ferromagnetics by postulating a molecular field proportional to the magnetization, must, in paramagnetics which do not obey Curie's unmodified law, suppose a molecular field of another sort, independent of temperature and magnetization, which takes charge above the Curie point.[50]

Forrest, whose results for diamagnetic copper crystals have already been mentioned,[31] also studied a few paramagnetic crystals, some cubic and some not. He uses his data to determine the space-lattice according to which the unitary magnets must be arranged if the theory proposed for ferromagnetic crystals by Peddie,[51] and extended by himself, is here applicable. This amounts to regarding paramagnetism as a residual effect of ferromagnetism at temperatures where the latter is inappreciable by direct means.

The gyromagnetic ratio for a paramagnetic material—dysprosium oxide—has recently been measured by Sucksmith;[52] all previous values depend upon experiments with ferromagnetic materials. The agreement of the new value with the old indicates that the experiments give information about the unitary magnets upon the existence of which paramagnetic properties depend rather than upon their particular mode of association in ferromagnetics. All the recent data may therefore be discussed with propriety in this section.

The gyromagnetic ratio, *i.e.* the ratio of the angular momentum of a Weber element to its magnetic moment, was predicted by Richardson[53] on the assumption that the Weber element was composed of one or more electrons moving in orbits. If we use $e/mc = 1.761 \cdot (10)^7$, as recommended by Birge[5] for intra-atomic calculations, Richardson's value, $2mc/e = 1.136 \cdot (10)^{-7}$.

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[50] For details of Weiss' theories see especially: Le Magnétisme by P. Weiss and G. Foëx, Colin, Paris, 1926.

[51] W. Peddie, Proc. Roy. Soc. Edinb. **25**, 1025–1059 (1905); **28**, 643–651 (1908); **32**, 216–246 (1912); **47**, 165–176 (1927); Nature **120**, 80–81 (1927).

[52] W. Sucksmith, Proc. Roy. Soc. [A] **128**, 276–293 (1930).

[53] O. W. Richardson, Phys. Rev. **26**, 248–253 (1908).

The best experimental values[54] are all about half as great. Barnett's experiments yield consistently 1.05 *mc/e* with an assigned probable error of about one percent. The English investigators, who put their probable error even lower, get 1.00 *mc/e*. No explanation of this discrepancy can yet be suggested.

As paramagnetic effects must cease to be apparent when all the unitary magnets are co-directed, while diamagnetic effects depend directly upon the applied field intensity up to the maximum intensities yet available, it is apparent that at a sufficiently high field intensity all paramagnetics should become diamagnetic. Such transitions have, indeed, been observed[55] in the case of alloys which are only feebly paramagnetic at best.

FERROMAGNETIC CRYSTALS

A long series of investigators has attempted to explain the stability of magnetization which distinguishes ferromagnetic media from all others, by invoking the mutual action of unitary magnets in crystalline array. This attempt has become especially important since x-ray studies have shown that all ferromagnetic media are predominantly crystalline.

The distinguished leader in the attack upon this fundamental problem of ferromagnetism is Sir [J.] Alfred Ewing. He has had the curious experience of having a theory, which he himself abandoned as inadequate, adopted, extended and vigorously supported by others.[56] It will form a convenient thread of continuity in our discussion of recent experiments and their significance, to follow in detail the advances and retreats along this section of our battle-front.

EWING

In its original form[57] Ewing's theory of ferromagnetism postulated an extended group of identical molecular magnets, not magnetic dipoles but of linear dimensions comparable with intermolecular distances, and made no hypothesis regarding the structure of these magnets. We may now replace "molecule" by "atom" with some gain in precision, since we now think that we know (opine?) that the atoms in—for example—iron crystals are not associated into distinguishable molecules. It then appeared, after a very simple analysis, that such magnets, centered at all the corners of a square-meshed plane net with all their axes parallel and in the plane of the net, are especially stable against disturbance by an applied magnetic field when their

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[54] S. J. Barnett, Phys. Rev. [2] **6,** 239–270 (1915); **10,** 7–21 (1917); **30,** 964–965 (1927); **31,** 1116 (1928); **36,** 789 (1930); S. J. Barnett, L. J. H. Barnett, Proc. Amer. Acad. **60,** 127–216 (1925); A. P. Chattock, L. F. Bates, Phil. Trans. Roy. Soc. [A] **223,** 257–288 (1922); W. Sucksmith, L. F. Bates, Proc. Roy. Soc. [A] **104,** 499–511 (1923); W. Sucksmith, Proc. Roy. Soc. [A] **108,** 638–642 (1925).

[55] K. Overbeck, Ann. d. Physik [4] **46,** 677–697 (1915).

[56] It is interesting to notice that the whole Japanese school of magneticians, some of whom are here referred to, derived their inspiration from Ewing and Knott during the stay of these Scotch physicists at the University of Tokio.

[57] J A. Ewing, Proc. Roy. Soc. **48,** 342–358 (1890); Phil. Mag. [5] **30,** 205–222 (1890).

parallel magnetic axes are directed along one of the two sets of lines forming the sides of the squares.

As a randomly directed magnetic field in the plane of the net is increased from zero to a sufficiently high value—the initially chosen direction of this field remaining ulaltered—reduced to zero and increased in the opposite sense, the predicted behavior of the magnetic moment of the aggregate, or magnetic complex, is strikingly similar to the observed behavior of the magnetic moment of a real piece of iron held stationary and subjected to the same treatment. The more gradual changes of I_p, the component of I parallel to the applied H, H_a, in a real specimen as compared with the abrupt changes in the model were naturally ascribed to the presence in the former of a multitude of magnetic complexes with their easy directions of magnetization pointing in every possible direction with respect to the single direction of H_a.

To build up a three-dimensional model Ewing supposed the square-meshed nets to be superposed as necessary. He did not re-analyze the problem, however, because with the given assumptions it appeared that each atomic magnet is controlled almost wholly by its two nearest neighbors in the row along which it points when H_a is zero.

The models which Ewing[58] and others[59] have constructed for demonstration purposes exhibit the principal phenomena of ferromagnetism in a highly satisfactory manner. Attempts at fixing the length of an atomic magnet lead, on the other hand, to serious difficulties. We now know accurately enough the atomic spacing and arrangement in every crystal known to be ferromagnetic. We also know from saturation data the average magnetic moment per atom, and except for a few speculations,[60] it is universal practice to assume that each atom has this magnetic moment, rather than that some have more and some less. This leaves us only the ratio of the length of the magnet to the lattice constant at disposal in fixing the transverse field intensity at which the initial alignment is completely upset, and in fixing the corresponding degree of measurable magnetization, I_p. Ewing himself was the first to recognize that no single value of this ratio would satisfy the experimental data. If the ratio is made nearly as great as unity the maximum stable deflection may be made as small as the narrow range of "elastic" magnetization requires. (The refinement of measurement continually narrows this range.) But then the intensity of applied field necessary to upset the initial arrangement becomes much too great. Ewing finally[61] avoided the difficulty by shortening the magnet responsible for I and introducing for its proper control a symmetrical group of magnets fixed in the atom, and presumably also fixed with respect to the crystal space-lattice. The analogy of

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[58] J. A. Ewing, Proc. Roy. Soc. Edinb. **47**, 141 (1927).

[59] Most recently: W. M. Roberds, Rev. Scient. Instr. [2] **1**, 400–403 (1930).

[60] P. Weiss, J. de phys. [6] **5**, 129–152 (1924); E. C. Stoner, Magnetism and Atomic Structure, Dutton, New York, 1926, pp. 171–172, 339; R. Forrer, Comptes rendus **182**, 1272–1275, 1530–1532; **183**, 121–123, 559–560, 738–741 (1926).

[61] J. A. Ewing, Proc. Roy. Soc. Edinb. **42**, 97–128 (1922); less completely: Proc. Roy. Scc. [A] **100**, 449–460 (1922); Phil. Mag. [6] **43**, 493–503 (1922).

this new model with the atomic models of even date was properly emphasized and it was shown that Weber elements could be so arranged as to meet the requirements, thus reducing the artificiality of the picture in terms of bar magnets.

Hysteresis energy losses, on Ewing's theory in either of its forms, must be accounted for by dissipation of magnetic potential energy whenever sudden changes of the orientation of atomic magnets occur. Transient eddy currents and magnetostrictive shocks are two possible modes of transfer of this energy from an atom to its environment during and after such changes.

HONDA AND OKUBO

Between the first and second editions of Ewing's theory, Honda and Okubo[62] had developed his first ideas in somewhat greater detail, and still claim[63] to avoid the objection later raised by their originator. They make the atomic magnets much shorter than the least distance between atom centers and thus relatively enhance the influence of non-adjacent atoms. They also consider, as Ewing does not, the effect of neighboring complexes with their elements in various orientations, and conclude that the intensity and direction of the resultant applied field within a single complex may be very different from $\boldsymbol{H}_a$ due to external agencies. With this proviso it is easy to see that even a very small $\boldsymbol{H}_a$ may combine with the local field in such a way as to make some of the complexes give way. Hysteresis may thus become apparent at low values of H_a, and of *average* I_p, as experimental data require. Honda and Okubo, like Ewing, confine their analysis to a regular arrangement of atoms lying in a single plane, and suffering deflections only therein.

PEDDIE

The earliest attack upon the general problem of ferromagnetism in three dimensions was launched by Peddie.[51] He did not, however, consider a step-by-step rotation of atomic magnets in oblique or transverse fields but only the magnetic stability of co-directed magnetic doublets arranged on a space-lattice. His method was to calculate the magnetic field at a lattice-point due to equal doublets at all the other lattice-points of an infinite lattice. If this field coincides in direction with the direction chosen for the doublet axes the arrangement is inherently stable and the degree of stability is measured by the computed field intensity. If the computed field direction is not that of the doublet axes an applied field with a suitable component perpendicular to the doublet axes is necessary and sufficient for stability. Under the assumptions all changes of orientation of the atomic magnets are continuous—this is a consequence of the assumption of doublets—so that Peddie's theory makes no provision for that magnetic hysteresis which is the distinguishing feature of ferromagnetism. What it is well able to deal with is the difference

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[62] K. Honda, J. Okubo, Sci. Rep. Tohoku Imp. Univ. [I] **5**, 153–214 (1916); **6**, 183–196 (1917); Phys. Rev. [2] **10**, 705–742 (1917); **13**, 6–26 (1919).

[63] K. Honda, Dictionary of Applied Physics **2**, 516–526 (1922); K. Honda, J. Okubo, Sci. Rep. Tohoku Imp. Univ. [I] **12**, 27–32 (1923).

in direction of $\boldsymbol{I}$ and $\boldsymbol{H}_a$ which is observed in all ferromagnetic crystals except for a few directions of $\boldsymbol{H}_a$. Since the theoretical $\boldsymbol{I}$ is constant in magnitude, and always has the value corresponding to magnetic saturation, Peddie's theory is also inapplicable to low values of $\boldsymbol{H}_a$, for which it is observed that $\boldsymbol{I}$ has smaller values.

Forrest[31] extended this theory to crystals of the orthorhombic system and applied it, as has already been stated, to both diamagnetic and paramagnetic crystals. He can only obtain qualitative results because, as he admits, thermal agitation must, at any instant, effectively free many atoms from the magnetic control which would be due to a perfectly orderly array. That is, there is no approach to magnetic saturation. His results are peculiar in that the space-lattice indicated by this partial application of theory to his measurements, or to those of others, is often not the space-lattice disclosed by x-rays.

Gans

A strikingly different point of view is that taken by Gans.[64] Instead of assuming rigid parallelism of the unitary magnets within the region considered he assumes their axes arranged in space according to a general distribution function which can be specialized to suit various initial conditions. The direction of that part of the local field intensity at any magnet which depends upon the positions and orientations of the others—what Gans calls the magnetic molecular field—is itself dependent upon the magnetic molecular field values at these other magnets, so that the general solution for $\boldsymbol{I}$ in terms of $\boldsymbol{H}_a$ proves unmanageable.[65] Special solutions are accordingly worked out for vanishing H_a and correspondingly random orientation of the magnets, and for infinitely great H_a and corresponding parallelism of all the magnets. Surprizingly enough these special solutions extrapolated respectively to high and low H_a agree so well over a wide range of I_p that Gans concludes that either may be used without sensible error. If this be admitted the study of magnetization curves can give little or no information concerning the initial distribution of unitary magnets having only magnetic constraints. It should be emphasized in this connection that this first theory of Gans ignores the crystalline character of the medium by postulating that the magnetization shall always be parallel to the applied field, and that it equally ignores hysteresis.

Much more recently, Gans[66] has proposed a theory for the magnetization of polycrystalline metal which includes more of the essential phenomena. Each crystallite is supposed to have a single direction of easiest magnetization and a sensibly rectangular hysteresis loop. The magnetization curves and hysteresis loops computed on assuming further that the distribution of

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[64] R. Gans, Gött. Nachr., 118–164 (1910); 197–273 (1911); Ann. d. Physik [4] **63,** 382–396 (1920).

[65] The magnetic molecular field for a paramagnetic gas has also been considered by Gans—Ann. d. Physik [4] **50,** 163–198 (1916)—with somewhat better success.

[66] R. Gans, Schriften d. Königsb. gelehrten Ges. Naturwiss. Kl. **6.** 2, 79 pp. (1929). A long abstract is given by W. Steinhaus, Phys. Ber. **11,** 768–769 (1930).

unique directions of magnetization in space is at random agree very well with curves regarded as typical of well annealed metal. In this computation the effective field within each crystallite is divided into three parts: the applied field, the field due to the combined action of remote parts of the specimen, and the field due to adjacent crystallites. The direction and magnitude of the third component alone varies from point to point so as to account for the different values of applied field and of average magnetization at which discontinuities occur.

On the experimental side Gans[67] has succeeded in demonstrating the underlying diamagnetism of a ferromagnetic alloy—permalloy—in a field exceeding 8000 gauss, say 150000 times greater than H_c in this case.

MAHAJANI

A new start at a general theory of ferromagnetism has been made by Mahajani.[68] Like all the theorists yet mentioned in this section, he makes the stabilizing mechanism purely magnetic. Like Ewing, he considers the unitary magnets to have linear dimensions comparable with inter-atomic distances. In accordance with previous practice, too, he defers consideration of temperature effects. In the quantitative part of his work Mahajani uses the experimental results of Webster[69] on iron and of Weiss[70] on pyrrhotite.

The only improbable assumption in this theory—aside from the perfection of the crystal—is that the atomic magnets associated with one of the two simple cubic space-lattices into which the body-centered cubic space-lattice of iron may be decomposed, are always all parallel to one line, those associated with the other simple cubic lattice, to another. Since the points of these two lattices ought to be crystallographically equivalent there is no excuse for this postulate on physical or chemical grounds. Fortunately the quantitative successes of the theory, which belong exclusively to nearly saturated conditions, do not depend upon this assumption, the energies being computed in a straightforward way for the complete space-lattice.

Since Mahajani compares his theory with experiment by computing a "molecular field" and observing its agreement in magnitude with the quantity of this name computed from experimental data it will be necessary to explain what is meant by the term, which is due to Weiss.[71] The molecular field, $\boldsymbol{H}_m$, is a vector, having the dimensions of magnetic field intensity but regarded by Weiss as not arising from magnets, which is, in an isotropic medium, parallel to the magnetization, $\boldsymbol{I}$, and proportional to its magnitude, I, i.e. $H_m = m \cdot I$. In a crystal it is not in general parallel to $\boldsymbol{I}$, but its components $(H_m)_x$, $(H_m)_y$, $(H_m)_z$, referred to principal axes of magnetization—

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[67] R. Gans, Naturwiss. **18,** 184–185 (1930).

[68] G. S. Mahajani, Proc. Camb. Phil. Soc. **23,** 136–143 (1926); Phil. Trans. Roy. Soc. [A] **228,** 63–115 (1929).

[69] W. L. Webster, Proc. Roy. Soc. [A] **107,** 496–509 (1925).

[70] P. Weiss, Arch. des sci. [4] **19,** 537–558; **20,** 213–230 (1905); J. de phys. [4] **4,** 469–508, 829–846 (1905).

[71] P. Weiss, J. de phys. [4] **6,** 661–690 (1907); Arch. des sci. [4] **37,** 105–116, 201–213 (1914).

whereon $\boldsymbol{H}$ and $\boldsymbol{I}$ are parallel—are put by Weiss proportional to the components of I along these axes:

$$(H_m)_x = m_x \cdot I_x,$$
$$(H_m)_y = m_y \cdot I_y,$$
$$(H_m)_z = m_z \cdot I_z.$$

It is of course the torque $\boldsymbol{H}_m \times \boldsymbol{I}$ which balances the torque $\boldsymbol{H}_a \times \boldsymbol{I}$, and thus accounts for the angular separation of $\boldsymbol{I}$ from $\boldsymbol{H}_a$. In a cubic crystal we would expect $m_x = m_y = m_z$, but this brings us back to Thomson's prediction[30] of magnetic isotropy in such media. Ferromagnetic cubic crystals are certainly not isotropic so we must, as Webster[69] pointed out, abandon the simple equations for $(H_m)_x$, $(H_m)_y$, $(H_m)_z$, replacing them by more complicated forms. Mahajani shows that the requirements of symmetry are met by the expressions:

$$(H_m)_x = m_x(a_1 I_x + a_3 I_x^3 + a_5 I_x^5 + \cdots),$$
$$(H_m)_y = m_y(a_1 I_y + a_3 I_y^3 + a_5 I_y^5 + \cdots),$$
$$(H_m)_z = m_z(a_1 I_z + a_3 I_z^3 + a_5 I_z^5 + \cdots).$$

For cubic crystals two terms must be considered; for non-cubic crystals one term is enough. The quantities derived from experiment are the maxima of the molecular field, i.e. $(H_m)_x$ for $\boldsymbol{I}$ along the x-axis, etc. In Webster's experiments these are of the order of 500 gauss. To fit this Mahajani must assume the radius of his Weber element to be about $2 \cdot (10)^{-9}$ cm, a not unreasonable value for an electronic orbit.

AKULOV

The novelty of Akulov's treatment[72] of ferromagnetic crystals lies in the fact that he considers not only the torque exerted upon each atomic magnet by the whole array, but also the anisotropic cohesive forces which arise when the magnets are co-directed in various crystallographic directions. For the computation of these latter it is sufficient to regard the units as magnetic dipoles. Akulov is thus enabled to account qualitatively for the maximal magnetostrictions observed in iron by Webster[73] and by Honda and Mashiyama.[74] These changes in dimensions, unlike the saturation values of magnetization, are entirely different for different crystallographic directions, and therefore afford critical means for testing the theory. The longitudinal strains predicted by Akulov's theory are just about half as great as those observed by Honda and Mashiyama for the various possible directions in $\{100\}$ planes. In other planes the predicted and observed values cannot be made to agree throughout by merely introducing a constant factor, but the variations of strain with direction of applied field are qualitatively the same.

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[72] N. Akulov, Zeits. f. Physik **52**, 389–405 (1928); **54**, 582–587; **57**, 249–256 (1929); **59**, 254–264 (1930).

[73] W. L. Webster, Proc. Roy. Soc. [A] **109**, 570–584 (1925).

[74] K. Honda, Y. Mashiyama, Sci. Rep. Tohoku Imp. Univ. [I] **15**, 755–776 (1926).

In order to explain non-parallelism of $\boldsymbol{I}$ and $\boldsymbol{H}_a$ in non-saturated states, Akulov, like all before him, has to extend the unitary magnet in space so that it acquires a quadrupole moment as well as a dipole moment. He notes, however, that this quadrupole moment may depend upon structural details not determined solely by the Weber element itself. In testing his theory Akulov uses the magnetization curves giving I_p as a function of H_a for single iron crystals as plotted by Honda and Kaya.[75] These curves require correction because the self-demagnetizing field varies in a more complicated way than that assumed by these investigators.[76] Since Akulov's theory ignores hysteresis it can only apply with rigor to relatively intense magnetizations where the width of the hysteresis loop becomes negligible. The mode of comparison between theory and experiment is a little difficult to describe. The area between the I_p-axis and the I_p-*vs.*-H_a curve is measured up to a line drawn parallel to the H_a-axis at a distance I_s therefrom. This area changes as $\boldsymbol{H}_a$ swings about in a chosen crystallographic plane containing two axes for which $\boldsymbol{I}$ is parallel to $\boldsymbol{H}_a$. The curve connecting the area thus measured with the azimuth of $\boldsymbol{H}_a$ is closely reproduced by the theory. The agreement becomes more striking when it is remembered that only one parameter, the quadrupole moment, is at disposal.

The magnitude of the transverse component of magnetization, I_n, furnishes a second value of the quadrupole moment. The two values are indistinguishable and their mean is $4.52 \cdot (10)^{-29}$. The axis of the quadrupole moment coincides with that of the dipole moment. As the value for the dipole moment is about 10 Weiss magnetons, the linear dimensions necessary to explain the quadrupole moment are of the right order of magnitude.

Nickel crystals are considered in the fourth paper of this series. The theoretical predictions are compared with the data of Mashiyama[77] on magnetostriction and with those of Kaya,[78] and of Sucksmith, Potter and Broadway[79] on the relations of I_p and I_n to $\boldsymbol{H}_a$. It is not clear from the statements here made whether or not a numerical factor has been applied to bring theory and experiment into quantitative agreement—as was necessary in the case of iron crystals. The qualitative agreement, at least, is astonishingly good.

In extending his theory to lower field intensities Akulov assumes, contrary to custom in this subject but with fair plausibility, that the atomic magnets point in all possible directions when the crystal is unmagnetized, that is that their orientations are in statistical disorder. He also assumes spatial quantization with respect to $\boldsymbol{H}_a$. The local magnetic field arising from neighboring atoms is thus ignored. Whether this omission vitiates the result is doubtful in view of the much more serious restriction he introduces in supposing that there are only two stable orientations for each magnet, one parallel to $\boldsymbol{H}_a$, the other anti-parallel. The relations which hold in ferromagnetic

[75] K. Honda, S. Kaya, Sci. Rep. Tohoku Imp. Univ. [I] **15**, 721–753 (1926).

[76] D. D. Foster, R. M. Bozorth, Nature **125**, 525 (1930).

[77] Y. Mashiyama, Sci. Rep. Tohoku Imp. Univ. [I] **17**, 945–961 (1928).

[78] S. Kaya, Sci. Rep. Tohoku Imp. Univ. [I] **17**, 639–664 (1928).

[79] W. Sucksmith, H. H. Potter, L. Broadway, Proc. Roy. Soc. [A] **117**, 471–485 (1928).

metals between magneto-resistance, elasto-resistance and magnetostriction seem inconsistent with a simple two-way magnetization in the general case.[80]

Starting from these more or less debatable postulates Akulov arrives at the following results for magnetization. If we take areas between I_p-*vs.*-H_a curves not up to I_s as before, but up to any arbitrary value I_p, we find the curves $\int_0^{I_p} H\,\mathrm{d}I_p$-*vs.*-$\phi$, where ϕ is the azimuth of $\mathbf{H}_a$ are derivable from the curve $\int_0^{I_s} H\,\mathrm{d}I_p$-*vs.*-$\phi$ by a mere change of scale. If we keep $\mathbf{I}$ constant in magnitude by a suitable choice of $\mathbf{H}_a$ as the direction of $\mathbf{I}$ is allowed to vary we find that $|\mathbf{H}_a \times \mathbf{I}|$ is the same function of the direction of $\mathbf{I}$ for all values of I. Both of these predictions are verified by the experiments which are quoted. A third consequence of the theory is a formula by which may be calculated the difference $H_a-(H_a)_0$, where H_a and $(H_a)_0$ are, respectively, the magnitude of the applied field in a random direction, and that in a principal direction of magnetization, corresponding to a given value of I_p. (In the case of $(H_a)_0$, $I_p=I$, in the case of H_a, $I_p<I$.) This relation also appears to be supported by experiment.

As regards magneto-resistance Akulov assumes that the proportional change in resistance for a current flowing along a single-crystal wire when an intense transverse magnetic field is applied can be derived from the corresponding magnetostrictive strain by using a suitable numerical factor—of the order of 10^3—which is the same for all transverse directions of $\mathbf{H}_a$ in a given wire and for wires with axes in all possible crystallographic directions. The data of Kaya[81] support this generalization. Akulov also offers some evidence for supposing that the numerical factor just mentioned is invariant as we pass from saturated to non-saturated states by decreasing H_a, and that a relation of equal simplicity applies to longitudinal magneto-resistance and magnetostriction effects, at least over the range in which the effects do not show hysteresis with respect to the applied field.

All in all, the successes of the theory are impressive and it represents a notable advance in dealing with that considerable body of ferromagnetic phenomena which is most important in magnetic fields of an intensity notably greater than the coercive force, H_c. Here it is not unreasonable, as a first approximation, to suppose that the whole array of atomic magnets wheels together under the action of an increasing magnetic field constant in direction. In much weaker fields the behavior of ferromagnetic media is almost certainly more complicated than can be accounted for by Akulov's theory.

BECKER

Becker[82] has recently re-emphasized the fact that a cubic arrangement of dipoles has no magnetic stability when co-directed.[83] Some further deviation

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[80] For a discussion of these relations, here neglected, see L. W. McKeehan, Phys. Rev. [2] **36**, 948–977 (1930).

[81] S. Kaya, Sci. Rep. Tohoku Imp. Univ. [I] **17**, 1027–1037 (1928).

[82] R. Becker, Zeits. f. Physik **62**, 253–269 (1930).

[83] E. Ising—Zeits. f. Physik **31**, 253–258 (1925)—showed that this is true for even a single row of magnets.

from cubic symmetry—beyond that involved in the assignment of a common direction for the dipole axes—must be introduced. Two possibilities are considered. One is that already laid at the foundation of Ewing's theory and employed in slightly different forms by Mahajani and Akulov. This is the assignment of suitable linear dimensions—with corresponding quadrupole moment—to the unitary magnets. The other possibility lies in a proper distortion of the space-lattice, and this is preferred by Becker for two reasons. It permits a closer correlation of magnetic properties with mechanical strains and it avoids the inconsistency which Mahajani and Akulov fall into when they give up spontaneous magnetization in zero field and still ignore hysteresis losses in single crystals.

The theory of the distorted dipole lattice is of particular interest to those who have been driven to conclude that mechanical constraints are the chief, if not the only, source of ferromagnetic stability and of the resulting hysteresis.

For simplicity, but admittedly only as a tentative approximation, Becker assumes that I lies always in the plane determined by H_a and the strain-determined spontaneous magnetization, I_0. The form and dimensions of the regions within which I_0 may be considered constant in direction are left open for speculation. If the regions contain but a few atoms each we have much the same picture as the one I drew in a previous paper,[84] in which the stable orientation of each atom was fixed mechanically by the distances and orientations of its nearest neighbors. (This hypothesis has the awkward consequence that by making it we lose the convenience of applying the results of lattice theory, which assumes identical elements at every point of an indefinitely extended space-lattice.)

Becker's result for the longitudinal magnetostrictive strain along a four-fold axis in an iron crystal comes out about one-fifth of the observed value, or less than half of that computed by Akulov. This discrepancy must indicate a mistake in one or both of the computations, because their postulates, and the data used for comparison, are all identical. In nickel crystals the case is even worse as the predicted effect is of the wrong sign. (Akulov merely changes the sign of a coefficient to avoid mentioning this difficulty.)

A magnetization curve appropriate to polycrystalline metal is derived by Becker on the assumption that the local strains, which fix in each region the direction of its initial magnetization, are large in comparison with magnetostrictive strains—a highly reasonable hypothesis—so that the change in the elastic energy due to rotating the magnets in any region may be negligible. The degree of strain in each region, and the direction of magnetization which the strain stabilizes, together fix a critical field intensity, different for different directions of H_a, at which discontinuous changes in I_p take place for sufficiently wide ranges in H_a. There thus appear a coercive force, a remanent magnetization, a single Barkhausen discontinuity, and a hysteresis energy loss per cycle, for each region. A proper averaging over all the regions leads

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[84] L. W. McKeehan, Phys. Rev. [2] **25**, 274–279 (1925).

to the sort of hysteresis loop usually found. The initial susceptibility can be computed on the same basis.

It is estimated that the maximum H_c explicable in this way by purely elastic strains is 140 gauss, a good approximation to the facts of experience. The coercive force, in Becker's theory, depends directly upon the numerical factor which fixes the magnitude (and sign) of the longitudinal magnetostrictive strain, so that he finds his theory substantiated by the low value of H_c in permalloy.[85] The wide variation of remanent magnetization, I_r, in different materials is accounted for by wide differences in the distribution function for the directions of I_0 in the unmagnetized specimen.

There is, of course, abundant evidence for the intimate relation of ferromagnetic and mechanical properties. Among recent experiments which point to such a relation are those by Kussmann and Scharnow[86] on solid solutions, where hardness and H_c increase together. It may also be noted that in the hydrogenized iron of Cioffi,[87] the low coercive force is accompanied by abnormal mechanical softness.

Elastic stress on the specimen as a whole will be expected on Becker's theory to alter the coercive force and remanence in the general way in which they have been observed to vary. It is interesting to notice in this connection that Ewing[57] stated very clearly the importance of elastic strain in changing the equilibrium orientation of the magnets in his original model. He did not, to be sure, imagine an initial strain, which is the real contribution made by Becker to the general solution.

Raising the temperature should also have a predictable effect upon H_c which should diminish in the same ratio as I_s. Less rapid variation is reported by Forrer.[48]

A prediction of Becker's theory which has not yet been tested is that negative Barkhausen discontinuities may occur in a special case—that of nickel in tension—perhaps accessible to study.

It will be observed that Becker's ideas regarding the uniqueness of the direction of easy magnetization in the homogeneous regions, and regarding the dependence of gross properties upon the distribution function for these directions, are similar to those recently offered by Gans,[66] to which reference has already been made. The superiority of Becker's presentation, as compared with that of Gans, lies in the opportunity it affords for computing the coercive force without recourse to magnetic data.

Experiments on the Barkhausen effect should properly be considered in their relation to Becker's theory. It must suffice, however, to mention some of the more striking results, leaving it to those working in this field to recognize the obvious connections and subject them to quantitative tests. The

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[85] L. W. McKeehan, P. P. Cioffi, Phys. Rev. [2] **28**, 146–157 (1926).

[86] A. Kussmann, B. Scharnow, Zeits. f. Physik **54**, 1–15 (1929); Zeits. f. anorg. Chem. **178**, 317–324 (1929).

[87] P. P. Cioffi, Nature **126**, 200–201 (1930).

most quantitative experiments yet to hand in this field are those of Bozorth,[88] whose analysis of the high-frequency output of search coils surrounding a wire subjected to a smoothly varying magnetic field leads to a value for the mean change in magnetization in a Barkhausen discontinuity at various points on the hysteresis loop, and suggests that discontinuities account for most if not all of the whole change in magnetization where loops are steep. It is not yet clear whether the 10^{15} or so atoms which must cooperate in a discontinuity of average size are all the atoms in a sharply bounded region, or whether they form chains, or network, with unaffected atoms lying between them. The smallness of the effect of grain-size upon ferromagnetic properties,[89] and the fact that Barkhausen discontinuities of ordinary size are found to occur in large mono-crystals, make it clear that the grain or crystallite of the metallographist is not magnetically homogeneous. Only when a substance is finely divided,[90] or made into a very thin film,[91] do its intrinsic magnetic properties suffer any important changes.

Other notable papers on the Barkhausen effect are those by Heaps and his students[92] which deal with the effects in minute specimens, where the discontinuities may be few and relatively very large. The most recent results show that large discontinuities in magnetization are accompanied by discontinuous changes in length, *i.e.* by magnetostrictive jumps. Such changes in length had previously been observed by Arkadiew[93] who simply listened to a telephone receiver diaphragm actuated by the tightly stretched specimen as it was carried through a magnetic cycle, but the simultaneity of length and magnetization changes was not then demonstrated. Studies by Preisach[94] show that the jumps in I_p may be made enormous (or run together so as to be difficult of separate observation) by tension or torsion, and Sixtus and Tonks[95] report a kind of trigger action by which a local increase in H_a can set off a discontinuity which progresses far beyond the region in which it started. The relatively slow propagation of such waves of magnetization—much slower than the velocity of sound in the medium—may be wholly accounted for by eddy currents. It is attractive to speculate, however, that

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[88] R. M. Bozorth, Phys. Rev. [2] **34,** 772–784 (1929); R. M. Bozorth, J. Dillinger, Phys. Rev. [2] **35,** 733–752 (1930).

[89] O. v. Auwers, Wiss. Veröffentl a. d. Siemens-Konz. **4,** 266–273 (1925); **7,** 197–209 (1928); Zeits. f. techn. Physik **6,** 578–582 (1925); **9,** 475–478 (1928); K. Daeves, Zeits. f. techn. Physik **10,** 67–68 (1929); O. v. Auwers, Zeits. f. techn. Physik **10,** 68 (1929); G. J. Sizoo, Zeits. f. Physik **51,** 557–564 (1928); **53,** 449–457; **55,** 144 (1929); O. v. Auwers, G. J. Sizoo, Zeits. f. Physik **60,** 576–580 (1930); T. D. Yensen, Metals & Alloys **1,** 493–495 (1930).

[90] L. A. Welo, O. Baudisch, Phil. Mag. [7] **3,** 396–410 (1927); W. Köster, Zeits. f. anorg. Chem. **179,** 297–298 (1929).

[91] The ferromagnetic properties of thin films are still very puzzling and the conclusions that can as yet be drawn are, with few exceptions, empirical. For a recent contribution to one phase of the subject see J. H. Howey, Phys. Rev. [2] **34,** 1440–1447 (1929).

[92] C. W. Heaps, J. Taylor, Phys. Rev. [2] **34,** 937–944 (1929); C. W. Heaps, A. B. Bryan, Phys. Rev. [2] **36,** 326–332 (1930).

[93] W. Arkadiew, Comptes rendus **184,** 1233–1234 (1927).

[94] F. Preisach, Ann. d. Physik [5] **3,** 737–799 (1929).

[95] K. J. Sixtus, L. Tonks, Phys. Rev. [2] **35,** 1441 (1930).

the delay may be due in part to the necessity for waiting before each advance for a favorable condition as regards energy of thermal agitation, and that we may yet trace a continuous progression of phenomena from the almost instantaneous response of the atomic magnet to the slow secular variation of magnetic moment in well-aged magnet steel. If so, magnetic lag—the reality of which has been a matter of dispute for forty years[96]—may be admitted as an intrinsic ferromagnetic phenomenon.

The enlargement or coalescence of microscopic Barkhausen discontinuities into major catastrophes in magnetization was first observed by Forrer[97] in suitably over-strained nickel wires. The abnormal values of remanence which he observed led him to postulate a rather complicated unitary magnet in this metal, the two parts of which might take up either of two configurations in the absence of applied field. The high remanence corresponded to a preponderance of the usually less numerous configuration. He extended the same reasoning to iron, in which he thinks that the unit must have three parts and in a recent paper[98] offers different types of hysteresis loop in iron as experimental evidence in favor of his hypothesis. This effect may, of course, be explained in other ways. The dependence of remanence upon the structure of the specimen is undoubtedly real and has been discussed in an earlier paragraph.

In this group of phenomena should be included the effects of applied stresses upon magnetization. These have been known from time immemorial. The most recent addition to the list is the effect of quenching stresses on he magnetization of toroids.[99] Here, too, belong any observed effects of magnetization upon mechanical hysteresis. Guye and Dupraz[100] have shown that a longitudinal magnetic field reduces the logarithmic decrement of torsional vibrations of large amplitude in wires of iron, steel and nickel. This must be taken to mean that the orientations of atomic axes are rendered more stable against discontinuous rotation under shearing stresses by the added magnetic control thus supplied.

The rate of heat production at various stages of a hysteresis cycle must be affected by the incidence and relief of the elastic strains involved in changes of magnetization, and by any mechanical hysteresis which occurs in consequence. All observers[101] agree that most of the heat is liberated in that part of the cycle in which I_p increases (or decreases) rapidly from zero. The most elaborate and careful study, by Ellwood, discloses cooling in two regions on each branch of a sufficiently extensive hysteresis loop in specimens of carbon

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[96] The most recent clash: C. Lapp, Ann. de phys. [10] **8,** 278–395 (1927); **12,** 276–277 (1929); R. M. Bozorth, Phys. Rev. [2] **32,** 124–132 (1928).

[97] R. Forrer, J. de phys. [6] **7,** 109–124 (1926); **10,** 247–262 (1929).

[98] R. Forrer, J. Schneider, Comptes rendus **190,** 1391–1393 (1930).

[99] O. Tesche, Zeits. f. techn. Physik **11,** 239–242 (1930).

[100] C. E. Guye, A. Dupraz, Helv. Phys. Acta **2,** 320–342 (1929).

[101] U. Adelsberger, Ann. d. Physik [4] **83,** 184–212 (1927); F. W. Constant, Phys. Rev. [2] **32,** 486–493 (1928); K. Honda, J. Okubo, T. Hirone, Sci. Rep. Tohoku Imp. Univ. [I] **18,** 409–417 (1929); W. B. Ellwood, Phys. Rev. [2] **36,** 1066–1082 (1930).

steel. No explanation of these coolings, one of which was also observed by Adelsberger, has yet been offered.

Besides the data on single crystals made use of by Becker and his immediate predecessors we now have comparable work from other laboratories on iron[102] and nickel,[103] and new information regarding cobalt.[104] None of this material has been examined for its theoretical significance.

HEISENBERG

The electron interchange interaction theory of Heisenberg[105] is a first attempt to lay down a quantum-theoretical basis for discussions of ferromagnetism. Stripped of its mathematics it seems merely to say that an atomic magnet, if it is not to be freely assignable to any of its possible magnetic states —giving rise to paramagnetism only—must have several possible states giving positive contributions to the magnetization of the medium (this requires that the quantum numbers characterizing the magnetic element must not be too small), and must have enough immediate neighbors so that its energy of interaction with these may be considerable. The electron interchange associated with the theory is not an actual transfer of an electron from one atom to another, and *vice versa*, as the word interchange might seem to imply, but only a mutual effect of two electrons, one in atom A and one in atom B, by which the states permissible to each are reduced in number, and, in particular, the state of mutual parallelism is made more probable. Seen from this stand-point we notice that we have been discussing such interchange for some time past.

As first laid down, and in its more rigorous development,[106] Heisenberg's theory gives necessary, but not necessary and sufficient, conditions for ferromagnetism. As soon as physical details begin to intrude into the mathematics, we notice an improvement in its definiteness. Stoner[107] finds it adequate to consider together in the assignment of magnetic states not all the atoms in the specimen but only a single atom and its immediate neighbors. He has gone a little too far in supposing that only two atomic orientations need to be considered, as this prevents him from considering magnetostriction phenomena[108] and ignores the crystalline symmetry of the atomic environment. Slater[109] has made important progress in discovering a reason for

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[102] W. Wolman, Archiv f. Elektrot. **19**, 385–404 (1928); G. J. Sizoo, Zeits. f. Physik **56**, 649–670; **58**, 718 (1929); Physica **10**, 1–18 (1930); H. Gries, H. Esser, Archiv f. Elektrot. **22**, 145–152 (1929); Archiv f. d. Eisenhüttenwesen **2**, 749–761 (1929).

[103] G. J. Sizoo, Zeits. f. Physik **57**, 106–114 (1929); Physica **10**, 1–18 (1930).

[104] S. Kaya, Sci. Rep. Tohoku Imp. Univ. [I] **17**, 1157–1177 (1928); Z. Nishiyama, Sci. Rep. Tohoku Imp. Univ. [I] **18**, 341–357 (1929).

[105] W. Heisenberg, Zeits. f. Physik **49**, 619–636 (1928).

[106] F. Bloch, Zeits. f. Physik **61**, 206–219 (1930); E. Teller, Zeits. f. Physik **62**, 102–105 (1930).

[107] E. C. Stoner, Proc. Leeds Phil. Lit. Soc. [Sci. Sect.] **2**, 50–55, 56–60 (1930).

[108] This has been emphasized by Heaps and Bryan, *loc. cit.*[92] in criticizing Bozorth's calculation of the volume taking part in a Barkhausen discontinuity on the basis of complete reversals of magnetization.

[109] J. C. Slater, Phys. Rev. [2] **35**, 509–529; **36**, 57–64 (1930).

the unique position of iron, cobalt and nickel as ferromagnetic elements, by importing into the theory the idea that the probability of interaction increases as the ratio of interatomic distance to radius of incompleted electron shell increases. This also gives the long awaited "explanation" of the ferromagnetism of Heusler's alloys.[110] The supposition is that when copper replaced manganese in compounds of the type Mn_3X, where X is any trivalent metal, the copper must be so highly ionized that its incomplete three quantum shell, lying deep within its structure, becomes available for ferromagnetic interchange. The question which frequently arises as to the possibility of a ferromagnetic condition of manganese may receive its answer in like manner. Freese[111] has been unable to get ferromagnetic manganese by any of a variety of methods of preparation, and work by Miss Wheeler in this laboratory, as yet unpublished, has the same negative result.

The curious fact that alloys of iron and cobalt in certain proportions have a higher saturation intensity of magnetization than either element alone, discovered by Weiss,[112] has been confirmed by the systematic studies of Elmen,[113] and Masumoto.[114] The discoverer[115] has given further attention to this matter. It now appears that the maximum value of I_s lies at rather higher cobalt content than at first supposed, when it was attributed to a compound Fe_2Co, no evidence for which has yet been found by x-ray methods. The ternary alloys of iron, cobalt and nickel should offer a proving ground for theories which attribute ferromagnetic behavior to peculiarities of atomic environment. Among these alloys the group—the perminvars—discovered by Elmen[116] to have extremely constant magnetic permeability over a considerable range in H_a, with correspondingly low hysteresis losses, are especially interesting. Cobalt dissolved in a large excess of platinum displays, according to Constant,[117] a notably higher value of I_p than can be accounted for by the cobalt atoms only unless these are in an abnormal state. This may prove to be another instance of environmental influence.

The attempt by Dorfmann and his colleagues[118] to explain ferromagnetism as due to conduction electrons has met with little favor. An error in sign[119] vitiates the quantitative argument, and even qualitatively it rested solely on the fact that magnetic properties and thermoelectric properties

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[110] For recent work on the crystal structures and magnetic properties of these alloys see the following: L. Harang, Zeits. f. Krist. **65**, 261–285 (1927); H. O. Doerum, Avh. Oslo 1929 No. 10, 32 pp.; E. Persson, Zeits. f. Physik **57**, 115–133 (1929); H. H. Potter, Proc. Phys. Soc. London **41**, 135–142 (1929); S. Valentiner, G. Becker, Zeits. f. Physik **57**, 283–291 (1929).

[111] H. Freese, Phys. Zeits. **29**, 191–197 (1928)

[112] P. Weiss, Trans. Faraday Soc. **8**, 149–156 (1912).

[113] G. W. Elmen, J. Franklin Inst. **207**, 583–617 (1929).

[114] H. Masumoto, Sci. Rep. Tohoku Imp. Univ. [I] **18**, 195–229 (1929).

[115] P. Weiss, R. Forrer, Ann. de phys. [10] **12**, 279–374 (1929).

[116] G. W. Elmen, J. Franklin Inst. **206**, 317–338 (1928).

[117] F. W. Constant, Phys. Rev. [2] **34**, 1217–1224 (1929).

[118] J. Dorfmann, R. Jaanus, Zeits. f. Physik **54**, 277–288 (1929); J. Dorfmann, I. Kikoin, Zeits. f. Physik **54**, 289–296 (1929); *cf.* F. Bloch, Zeits. f. Physik **57**, 545–555 (1929).

[119] E. C. Stoner, Nature **125**, 973 (1930).

change rapidly at the same temperature—no direct connection has been established. The work of Smith and Dillinger,[120] to be sure, extends the temperature range over which there is an evident relation, but it seems safer to attribute both effects to changes in structural stability rather than to make the same electrons serve in both cases. For similar reasons the Hall and allied effects seem to lie beyond the scope of this review. Attempts to localize the magnetic electrons by the diffraction of x-rays[121] have so far been fruitless.

Elastically constrained atomic magnets might be expected to show resonance by abnormal values of susceptibility at particular high frequencies. Within the range of experiments so far conducted the results have been negative.[122]

WEISS

The molecular field theory of Weiss,[70] already discussed to some extent in connection with the theory of Mahajani, is not, strictly speaking, an atomistic theory at all. It makes no use of the spatial relations of the unitary magnets. It is only mentioned again in this section because some of the recent experimental work which at present seems chiefly concerned with it will in all probability be needed shortly in a more comprehensive theory of ferromagnetic crystals.

The loss of ferromagnetic properties at a fairly definite temperature—the Curie point—is hardly explicable without resort to an atomistic theory of some sort. Recent theories which focus attention upon thermodynamic reasoning[123] are not therefore much more satisfactory in this respect than the admittedly formal equations of Weiss.[124] On the experimental side Weiss and Forrer,[125] and Tyler,[126] working with nickel, and Bates,[127] working with manganese compounds having low Curie points, have tried to make more precise the relation between I_s and T, which has a very simple form in Weiss' theory. The results are hardly consistent enough to permit any decision for or against the theoretical equation, though Weiss regards his results as conclusive in his favor. The difference between the Curie points for heating and cooling is very large in some cases—unfortunately among these is the case studied by Bates—and this difference has been shown, at least for iron-nickel alloys, to be independent of the rate of heating or cooling.[128] Until these temperature-hysteresis effects are better understood the matters at issue cannot be settled.

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[120] A. W. Smith, J. Dillinger, Phys. Rev. [2] **33**, 398–402 (1929).

[121] T. D. Yensen, Phys. Rev. [2] **32**, 114–123 (1928); J. C. Stearns, Phys. Rev. [2] **35**, 1–7 (1930).

[122] G. R. Wait, F. G. Brickwedde, E. L. Hall, Phys. Rev. [2] **32**, 967–973 (1928); N. N Malov, J. Appl. Phys. Moscow-Leningrad **6**, 26–38 (1929).

[123] R. H. Fowler, P. L. Kapitza, Proc. Roy. Soc. [A] **124**, 1–15 (1929); E. C. Stoner, Phil. Mag. [7] **10**, 27–48 (1930).

[124] P. Weiss, J. de phys. [7] **1**, 1–10, 163–175 (1930).

[125] P. Weiss, R. Forrer, Ann. de phys. [10] **5**, 153–213 (1926).

[126] F. Tyler, Phil. Mag. [7] **9**, 1026–1038 (1930).

[127] L. F. Bates, Phil. Mag. [7] **6**, 593–597 (1928); **8**, 714–732 (1929).

[128] G. Gossels, Zeits. f. anorg. Chem. **182**, 19–27 (1929).

The magneto-caloric effect—the reversible rise in temperature observed when an intense magnetic field is applied—is another facet of the Curie point problem. Here all the data come from Weiss' laboratory.[129] The anomalous specific heat of ferromagnetics[130] is still another. So too is the thermoelastic anomaly.[131]

CONCLUSION

This review has mentioned many as yet unsolved problems, but a glance at the too copious references with which the text has been encumbered will show how wide spread the interest in magnetism has recently become, and will encourage a hope that the present position may soon be consolidated and made a safer starting point for further advances.

It is apparent from what has been said in criticism of the various theories, particularly those which attempt to cover the behavior of ferromagnetic crystals, that a comprehensive explanation must assign the magnetic states of atomic, ionic and electronic magnets with proper regard to the local value of magnetic field intensity $\boldsymbol{H}$—not merely its applied part $\boldsymbol{H}_a$—, and that the assignment must be principally conditioned by the spatial configuration—including here even slight distortions in crystal space-lattice if the medium is crystalline—and chemical characteristics (nuclear and electronic constitution) of neighboring atoms and ions. In nearly perfect crystals at low temperatures many contiguous units may be found, to a necessary approximation, in the same magnetic state and, at least in imperfect crystals, some of the states must be extremely stable, in order to make possible the phenomena of ferromagnetism. Under paramagnetic conditions there must be equilibrium between magnetic and thermal energies. In diamagnetic media the non-decaying eddy currents must be localized and their constraints, if any, determined by their environment. The complexity of the indicated theory is such that its complete presentation may not soon occur. Meanwhile we must be careful to use the partial explanations now at hand with due caution and with frequent reference to experiment for their correction.

Sloane Physics Laboratory,
September 6, 1930.

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[129] P. Weiss, J. de phys. [6] **2,** 161–182 (1921); P. Weiss, R. Forrer, *loc. cit.*[125]

[130] L. F. Bates, Proc. Roy. Soc. [A] **117,** 680–691 (1928); B. G. Whitmore, Phil. Mag [7] **7,** 125–129 (1929); E. Lapp, Ann. de phys. [10] **12,** 442–451 (1929).

[131] E. Bauer, J. de phys. [6] **10,** 345–359 (1929).