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# Magnetism*

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## CONTENTS

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* From Encyclopaedia Britannica (Encyclopaedia Britannica, Inc., Chicago).

THIS subject may be divided into three parts: (1) *ferromagnetism*, a property possessed by iron and a few other metals, and characterized by strong attraction of one magnetized body to another; (2) *paramagnetism*, a quality of many substances which causes them to be attracted by the poles of an electromagnet or magnetized body; and (3) *diamagnetism*, by virtue of which materials possessing this property are repelled by an electromagnet. Ferromagnetism is an important branch of paramagnetism, and ferromagnetic bodies are differentiated from other paramagnetic bodies by the magnitude of their reaction with a magnetic field, and in other ways described in detail in the following paragraphs.

For practical purposes, ferromagnetism is much the most important kind of magnetism.

After an historical introduction, ferromagnetic phenomena and materials will be described, and the effect of various factors on magnetic properties discussed. Following this, diamagnetism and paramagnetism and the theory of ferromagnetism will be considered, and finally an account of methods used for measuring magnetic quantities

## 1. HISTORY

Centuries before the Christian era the Greeks knew that the mineral lodestone, a magnetic oxide of iron (magnetite, $Fe_3O_4$), had the ability to attract iron and other pieces of the same mineral. This natural magnet was mentioned by Thales of Miletus (ca. 624–548 B.C.); and according to Plato, Socrates states in *Ion* that it will support a chain of iron rings, each held to the one above it by magnetic attraction.[1]

The Roman Lucretius Carus states in his *De Rerum Natura*, in the first century B.C., that ". . . iron can be drawn by that stone which the Greeks call Magnet by its native name, because it has its origin in the hereditary bounds of the Magnetes,"[2] the inhabitants of Magnesia, in Thessaly. The Romans knew also of the repulsive action of magnetism, as shown by a quotation from the same passage: "Sometimes, too, iron draws back from this stone; for it is wont to flee from and follow it in turn." No reference to the directive property of the magnet, as used in the compass, is found in old Greek and Roman literature.

To account for magnetic attraction, some in this period believed that there were hooks on the surface of the magnet and little rings on the surface of the iron to catch them; others that particles were emitted by the magnet and created an empty space toward which the iron would move. Of all the phenomena of electricity and magnetism, frictional electricity and ferromagnetism are the oldest known to man, and perhaps the least understood today.

Beginning in the period A.D. 1000–1200 the history of magnetism is closely connected with the compass and its use in navigation. According to G. Sarton, the first clear mention in any literature of a magnetic needle for indicating direction appears to have been made by Shen Kua (1030–1093), a Chinese mathematician and instrument maker, who, however, mentions only its use on land.

Soon after 1100, the Chinese Chu Yu reported that in the period 1086–1099 the compass was used for navigation by "foreign" sailors going between Canton and Sumatra. Since the trade in this region was practically a Moslem monopoly it is concluded that they were the first to use the compass for directing ships.

The earliest known European reference to the compass is by Alexander Neckam (1157–1217), an English cyclopaedist. In 1269 Petrus Peregrinus de Maricourt, a French crusader, gave the first detailed description as an instrument of navigation. He wrote of the floating compass as an instrument in common use and described a new pivoted compass in some detail. He fashioned a sphere of lodestone and explored its surface with bits of iron, and applied the term pole to the places in which the magnetic power appeared to be concentrated. He believed that the magnet derived its power from the sky.

After his time little progress was made for almost 300 years. In 1600 William Gilbert of Colchester, England, published his *De Magnete* which summarizes the knowledge of magnetism at the time of Queen Elizabeth, whose court physician he was, and reports some of his own experiments. He mentions the dip and declination of the compass and propounds his own great

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[1] *Ion*, 533 ff.

[2] Book vi, lines 906 ff., translated by T. Jackson (Oxford, 1929).

contribution, the discovery that the earth itself is a magnet. He showed that magnetized iron lost its magnetism when heated to a bright red heat and regained it when cooled again to room temperature. His chapter xii, of book v is entitled "Magnetick force is animate, or imitates life; and in many things surpasses human life, while this is bound up in the organick body."[3]

After the time of Gilbert, during the period when Isaac Newton was formulating the law of gravitation and Robert Boyle was investigating the laws of gases, little of importance for magnetism was recorded.

### Magnetostatics and the Nature of Magnetism

In 1785 Charles Augustin Coulomb established with some precision the inverse square law of attraction (and repulsion) between unlike (and like) magnetic poles, a proposition that had been favored or opposed by many before him. This was made the basis of Poisson's mathematical theory giving the forces between magnetized bodies in terms of their intensities of magnetization. Simon Denis Poisson (1781–1840) assumed magnetization to be a molecular phenomenon, but believed that a molecule became magnetic when the two fluids which it contained became separated. In 1852 Wilhelm Weber proposed that each molecule is a permanent magnet, subject to a frictional force that tends to maintain it in its established orientation. This explained the limit of magnetization (saturation) and gave a rather unsatisfactory explanation of residual magnetization, the magnetization remaining after the removal of the magnetizing field. Later James Clerk Maxwell (1831–79) improved upon Weber's explanation of residual magnetization by suggesting that the molecular magnet had no friction but had more than one fixed position of equilibrium.

Before the time of Weber, Hans Christian Oersted made the important discovery (1820) that an electric current would affect a magnetic needle. Quickly following up this lead, André Marie Ampère investigated experimentally and mathematically the mutual forces between currents and showed (1825) that a current in a circuit is equivalent to a "magnetic shell' of calculable strength. He considered a magnet to be due primarily to electric currents circulating within the molecules, and this concept, though not accepted immediately because there was no way either to prove to to disprove it, stood the test of time and was elaborated only by the advent of electron theory and the modern era.

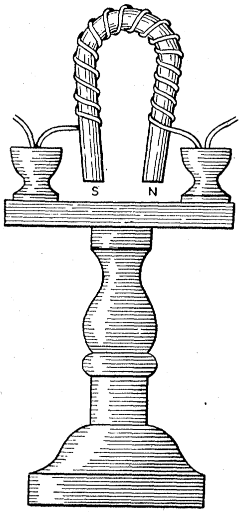

FIG. 1. A drawing of the first electromagnet that could be so called, constructed by William Sturgeon in 1825. The soft iron horse shoe is 5 in. high and is magnetized by the copper wire connected to a battery not shown.

### Electromagnetism

Using Oersted's discovery, both Ampére and Dominique Arago (1786–1853) magnetized steel needles by placing them in a helix of wire carrying an electric current. This was the prelude to the construction in 1825 of the first electromagnet which could be so-called, by William Sturgeon (1783–1850) of Lancashire (Fig. 1). This was made of soft iron bent in the form of a horseshoe and wound with a loose helix of 18 turns of wire, and when excited by a wet battery could lift 20 times its own weight.

Several years later Joseph Henry in the United States built a much improved electromagnet with many turns of wire efficiently applied. While experimenting with this in 1829 he noticed the spark that occurred when the circuit was broken and was led to discovery of self-induction, a treatise on which he published in 1832. The same phenomenon was independently discovered

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[3] Translation by the Gilbert Club. S. P. Thompson, editor (Chiswick Press, London, 1900).

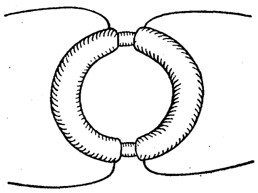


FIG. 2. Faraday's first transformer, with two windings of copper wire on an iron core. Connection of one winding to battery caused momentary flow of electricity in other winding connected to galvanometer.

and published several years later by Michael Faraday (1791–1867). This great English experimentalist also shared with Henry the important discovery of electromagnetic induction, the "conversion" of magnetism into electricity. Although Henry is believed to have performed the experiment first and understood its significance, his results were unknown to Faraday who likewise made the crucial experiment and was the first to publish the results and thereby establish priority. Faraday and Henry made the first transformers (Fig. 2) and prepared the way for the construction of the dynamo and motor and the age of electricity.

The laws of electricity and magnetism were formulated systematically in mathematical language by Maxwell, and his famous treatise was was published in 1873.

## Kinds of Magnetism

The ancients knew the kind of magnetism that we observe in lodestone and iron and call *ferromagnetism*. Nickel and cobalt were discovered in the 18th century and nickel was soon thereafter known to be ferromagnetic while the ferromagnetism of cobalt was proved by Faraday in 1845. In 1846 Faraday observed that a bar of glass was repelled by his electromagnet, and named this property *diamagnetism*; he did not know that S. J. Brugmans (in 1778) and others had previously observed the same phenomenon in bismuth and antimony. Similarly he used the word *paramagnetic* for substances that were attracted by an electromagnet, and showed that this was a property of the substance and not the effect of an impurity of iron. He classified all of the substances that he examined—solids, liquids and gases—as para- or diamagnetic. Ferromagnetism is thus one kind of paramagnetism.

Almost 20 years after Faraday's death Pierre Curie (1895) made careful measurements of the effect of temperature on many materials of these three categories and found that diamagnetism is usually independent of temperature whereas paramagnetism usually decreases as the temperature increases.

A connection between ferromagnetism and light was finally established experimentally by J. Kerr who in 1876 observed the rotation of the plane of polarization of light reflected from a magnetized surface. Previously Faraday (1845) had observed a similar rotation in light transmitted by transparent bodies and later (1896) Pieter Zeeman discovered the broadening of the spectral lines of sodium when the light was emitted in the presence of a magnetic field.

## Modern Theory

One difficulty with Weber's theory was that it gave no satisfactory explanation of residual induction and hysteresis. Maxwell's suggestion of multiple fixed positions for the molecular magnet left unexplained the fact that hysteresis (see below) is found when a magnetizing force, after being once applied, is removed and reapplied. J. A. Ewing proposed (1890) that the constraints on the orientations of the molecular magnets were imposed by the magnetic forces of the surrounding molecules, and showed by means of calculations and models that such forces can account qualitatively for the shape of the magnetization curve and the major and minor hysteresis loops. He first used the word *hysteresis*. J. Swinburne predicted on the basis of Ewing's theory that in a rotating magnetic field the hysteresis should disappear, and F. G. Baily's

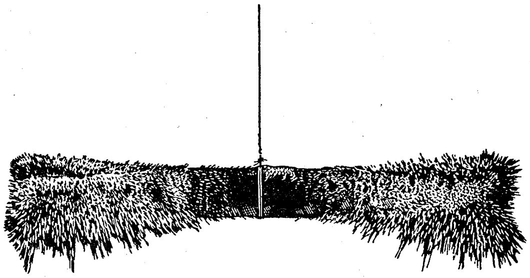


FIG. 3. A magnitized bar attracts iron filings that cling most strongly near the poles of the bar.

experiment (1896) confirmed this in a striking way (see Fig. 29).

The first important influence of modern concepts on the theory of magnetism came when P. Langevin (1905) used statistical theory to explain the variation of paramagnetic susceptibility with temperature. He assumed each molecule to have a definite magnetic moment that tended to be oriented by the applied field and at the same time disturbed by thermal agitation. In 1907 this was seized upon by P. Weiss who postulated that the molecules are exposed both to the applied field and to a so-called molecular field proportional to the intensity of magnetization, and so developed the first important quantitative theory of ferromagnetism. This molecular field is co-operative in nature, in that it depends on the mutual action of many atoms, and is now attributed to the "exchange forces" derived from quantum theory by W. Heisenberg.

It is a consequence of the Weiss theory that ferromagnetic materials must be spontaneously magnetized, that is, small regions or "domains" of the material are magnetized to saturation even when the specimen as a whole has zero intensity of magnetization; in this latter condition the magnetic moments of the regions are oriented at random. Previously such regions were considered identical with the molecules or atoms of the substance but Weiss's deduction was later supported by many experiments (e.g., those on the Barkhausen effect) and is the basis of the domain theory now used in the interpretation of many magnetic phenomena.

The quantum theory by its clarification of atomic structure has had a great influence on magnetic theories. W. Pauli in 1920 showed that there is a natural unit of magnetic moment, the Bohr magneton, and in 1926 S. Goudsmit and G. E. Uhlenbeck proposed the spinning electron as the fundamental magnetic particle having a moment of one Bohr magneton. The quantum theory of paramagnetism and ferromagnetism was developed rapidly by the contributions of W. Heisenberg, J. H. Van Vleck. E. C. Stoner, H. Bethe, F. Bloch, J. C. Slater, and others. The temperature dependence of saturation magnetization, as calculated by P. Debye (1925) was a notable success of the quantum theory of ferromagnetism. Advances in the theory of diamagnetism have also been made after P. Langevin's pioneering work of 1905.

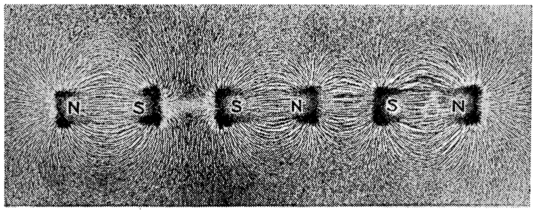


FIG. 4. Iron filings indicate the direction of the magnetic field near a group of bar magnets, show the lines of force emanating from S poles and converging on N poles.

The advances in useful materials, and in methods of measurement of magnetic quantities, are mentioned in appropriate places in the body of the article.

## 2. GENERAL DESCRIPTION OF MAGNETISM

The quality of magnetism first apparent to the ancients, and to us today, is the attractive force that exists between a natural magnet (lodestone) or an artificial magnet (a "permanent magnet") and a piece of soft iron. When a magnet is dipped into iron filings they cling to it, especially in certain places called poles that are usually located near the ends of the magnet (Fig. 3). Such poles always occur in pairs, and in a magnet used as a compass needle the pole that points to the north is the north-seeking pole, or simply the north pole, and the other the south pole. When a magnet is broken, new poles appear near the break so that each piece contains at least one pole of each kind.

### Magnetic Field

A magnet will attract a piece of iron even though the two are not in contact, and this action-at-a-distance is said to be caused by the magnetic field, or field of force around the magnet. This field may be explored by sprinkling iron filings around a magnet, whereupon they form in lines that converge on the poles and indicate also the direction a small compass needle would take if placed at any point (Fig. 4).

Poles exert forces on each other, north and south poles attract each other and like poles repel with a force that varies inversely as the square of the distance between them. A *unit pole* is a convenient concept defined so that two

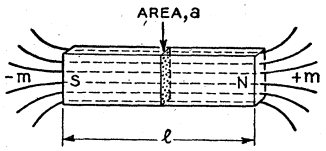


FIG. 5. A pole of strength $m$ and area $a$ corresponds to an intensity of magnetization $I=m/a$, equal also to magnetic moment per unit volume $ml/v$. Lines of induction are endless, pass into the magnet at the S pole and leave at the N pole.

unit poles of like kind, one centimeter apart, repel each other with a force of one dyne. The strength of the field of force, or simply the magnetic field strength, or magnetizing force, may be defined in terms of magnetic poles: one centimeter from a unit pole the field strength is one oersted. A magnetic field may be produced by a current of electricity as well as by a magnet and the unit of field strength can also be defined in terms of current. A magnetic field has direction as well as strength; the direction is that in which a north pole, subjected to it, tends to move, or that indicated by a small compass placed at the point. The field strength is represented by the symbol $H$, and the unit defined above is the oersted.

## Intensity of Magnetization and Magnetic Induction

In order to describe the magnetic properties of materials, one must have a quantitative measure of magnetization. Such a measure is the intensity of magnetization, defined in terms of the number of unit poles in a piece of given cross-sectional area. Suppose that a uniformly magnetized bar, of length $l$ and cross-sectional area $a$, has $m$ unit north poles at one end and $m$ unit south poles at the other (Fig. 5). The intensity of magnetization is then $m/a$, and is represented by the symbol $I$. It may be easily shown that $I$ is also the magnetic moment per unit volume; for the magnetic moment is pole strength times interpolar distance ($M=ml$), and $I=M/v$, if $v$ is the volume $al$.

Maxwell showed that some of the properties of magnetism may be likened to a flow, and conceived endless lines of induction that represent the direction and, by their concentration, the flow at any point. The lines pass from a magnetized material into the air at the north pole, and enter again at a south pole, and pass through the material from the south pole back to the north to form a closed loop.

The total number of lines crossing a given area at right angles is the flux in that area. The flux per unit area is called the flux density, or magnetic induction, and is represented by the symbol, $B$. The lines of induction are caused by both $H$ and $I$, but in magnetic materials they are caused mainly by the magnetization, $I$. The magnetic induction is defined by the relation

$$B=H+4\pi I.$$

The occurrence of the factor $4\pi$ is caused by the fact that a unit pole gives rise to a unit field everywhere on the surface of a sphere of unit radius enclosing the pole, and this sphere has an area of $4\pi$. The lines of induction may be visualized with the aid of Fig. 4 showing the pattern obtained with iron filings.

## Magnetization and Permeability Curves

When a piece of iron is brought near a magnet, or subjected to the magnetic field of an electric current, the magnetization induced in the iron by the field is described by a magnetization curve obtained by plotting the intensity of magnetization, $I$, or the magnetic induction, $B$, against the field strength, $H$. Such curves are of fundamental importance for describing the magnetic properties of materials, and many of them are shown on the following pages. A magnetization curve for iron is shown as the solid line of Fig. 6 (left).

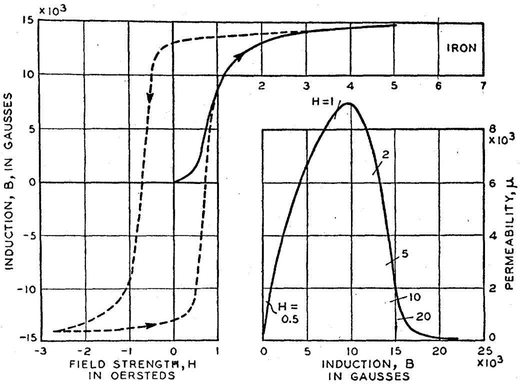


FIG. 6. Properties of materials are described by magnetization curve (solid line) and hysteresis loop (broken line) obtained by plotting $I$ or $B$ against $H$; they may also be described by permeability curve ($\mu$ *vs.* $B$).

The behavior of a material is also described by its permeability curve and hysteresis loop. The ratio $B/H$ is called the permeability, $\mu$, and this represents the relative increase in flux due to the presence of the magnetic material. The permeability curve is obtained by plotting the permeability, $\mu$, against either $I$, $B$, $B$-$H$, or $H$. In any case the curve rises from a point on the $\mu$ axis above the origin (the *initial permeability*) to a maximum (the *maximum permeability*) and falls off rapidly and then more slowly toward a value of one (not zero). When plotted against $B$-$H$, the curve ends at a definite point, i.e., when $B$-$H$ reaches its highest possible value, the saturation (Fig. 6).

If the field strength is first increased from zero to a high value and then decreased again, as indicated by the arrows of Fig. 6, it is observed that the original curve is not retraced; the change of induction "lags behind" the change in field and follows the characteristic curve shown by the broken line. This phenomenon was named *hysteresis* by Ewing, and the characteristic curve is called a hysteresis loop. On this loop, the value of $H$ for which $B=0$ is called the *coercive force*, $H_c$, and this is often used as a measure of quality of the material. Sometimes it is desirable to have $H_c$ large, sometimes small; for different materials $H_c$ may be higher than 1000, or lower than 0.01 oersted. The value of $B$ for which $H=0$ is called the residual induction, $B_r$.

## Ferromagnetism, Paramagnetism, and Diamagnetism

Materials which have magnetic properties similar to iron (e.g., nickel and cobalt and many alloys of these three elements) are ferromagnetic. They have permeabilities of considerable magnitude that vary with the field, they exhibit hysteresis, and they lose these properties rather abruptly when heated to a temperature called the *Curie point* of the material. All ferromagnetic materials known are solids.

In another class of materials, more numerous, the permeabilities are only slightly greater than one, usually between one and 1.001. These do not show hysteresis. The permeability is generally independent of field strength, and as a rule is either independent of temperature or decreases with increasing temperature. These materials, and also the ferromagnetic materials, are paramagnetic. Among the paramagnetic substances that are not ferromagnetic are many of the salts of the iron and the rare earth families and the palladium and platinum metals, the elements sodium, potassium, and oxygen, and the ferromagnetic materials above the Curie points. They may be solids, liquids, or gases.

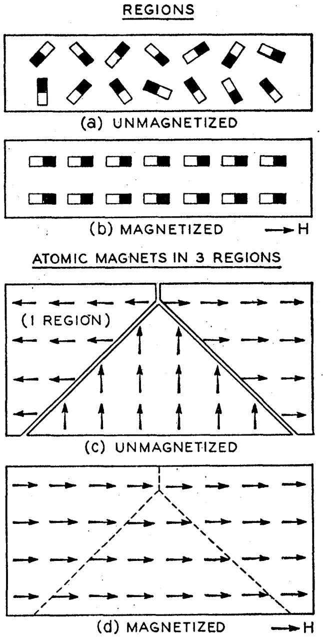


FIG. 7. A magnetic material behaves like a group of small magnets that are oriented in all directions when unmagnetized (a), and turn into parallelism when magnetized (b). Each "magnet" is actually composed of many atoms (c) that turn upon their own axes when the material is magnetized (d).

In diamagnetic substances the magnetization is directed oppositely to the field, i.e., they have permeabilities somewhat less than one. They are, therefore, repelled from the poles of an electromagnet, and move toward a weaker field. Many of the metals and most of the non-metals are diamagnetic.

Paramagnetic and diamagnetic substances are described more conveniently by their susceptibilities than by their permeabilities. The susceptibility is a measure of the increase in magnetic moment caused by the application of a field, and is defined as

$$\kappa = I/H,$$

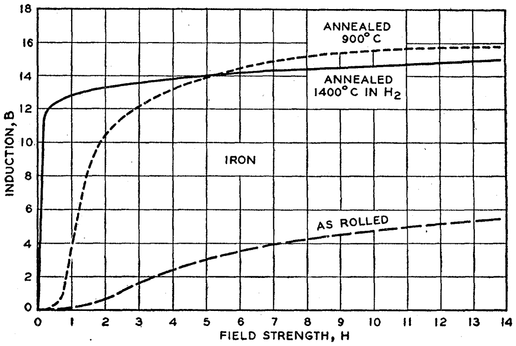


FIG. 8. Magnetization curves of iron in various conditions (P.P. Cioffi).

or the equivalent,

$$\kappa = (\mu - 1)/4\pi.$$

For diamagnetic materials the susceptibility is negative, and for bismuth has a value of −0.000013. For paramagnetic substances like iron the susceptibility may be as high as 500, or higher.

## Nature of Magnetism

A ferromagnetic substance has long been regarded as an assemblage of small permanent magnets (Fig. 7a)—when the material is unmagnetized, the magnets are arranged with haphazard orientations; when it is magnetized, they are lined up with their axes parallel. The nature of this small magnet, or "domain," has been the subject of much consideration over a period of years, and now it is known that the material is composed of many small regions or domains, each of which consists of many atoms (Fig. 7c). Within a domain all of the atoms are aligned parallel, and when the magnetization of the material is changed, the atoms in a domain turn together, while remaining parallel, so that they are aligned more nearly with the magnetic field applied to the material. The domain theory is discussed in more detail later.

The magnetic properties of materials depend mainly on the chemical composition and on the heat treatment they receive after fabrication, and sometimes they are influenced in an important way by the kind of fabrication used. Some ferromagnetic properties, such as saturation induction and Curie point, change but slowly with chemical composition, and are usually quite insensitive to methods of fabrication and heat treatment. On the other hand properties like permeability and hysteresis are highly sensitive to small amounts of impurities and to heat treatment—they are extremely "structure sensitive." In some cases the permeability (e.g., of iron) will increase 1000-fold when 0.1 percent of an impurity is removed by heat treatment, and an equal change in the reverse direction will be caused by bending the material or stretching it so that its form is permanently changed or strained. The susceptibilities of paramagnetic and diamagnetic substances are not usually affected to any considerable extent by fabrication and heat treatment.

## 3. PERMEABLE MATERIALS

Magnetic materials are essential for the operation of many of the devices and machines used in modern industry. Without them the production, distribution, and use of electricity, and also communication by wire and radio, would be seriously limited or even impossible. After the use of iron in the compass became commonplace, the important new uses of magnetic materials were first in the transformer and electric motor and generator, and later in electromagnetic devices such as relays, telephone receivers, loading coils, loud speakers, indicating instruments, and

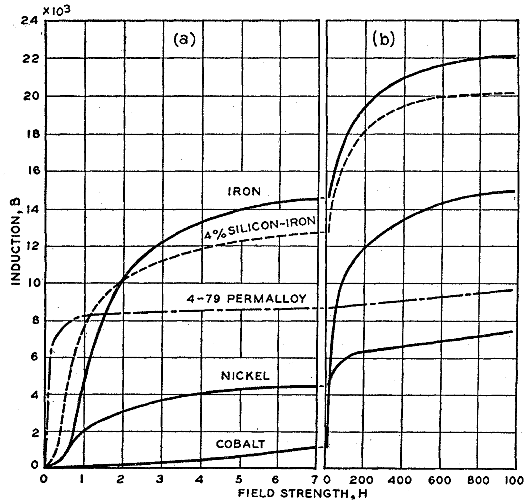


FIG. 9. Magnetization curves of some common materials in the annealed condition in low (a) and high (b) fields.

many other devices upon which all of our industries depend. It is estimated that several million tons of metal are produced annually for use as magnetic material.

Useful magnetic materials either have high permeability and are magnetized and demagnetized easily, or are permanent magnets and magnetize with difficulty but retain their magnetism tenaciously. The latter will be discussed in the next section headed PERMANENT MAGNETS.

As stated above, the magnetic properties depend mainly on the chemical composition and heat treatment. Permeable materials are usually processed, or annealed, by heating to some temperature between 800° and 1400°C, often about 1000°C, and cooling at any specified rate to room temperature. The permeability will be lowered if they are deformed beyond the elastic limit, consequently materials are made in final size and shape and are then heat treated and assembled directly in apparatus for use or for test, without bending. For each material there is a specific heat treatment that must be used to develop the desired property to the greatest extent. Magnetization curves of iron as rolled, after a conventional anneal, and after a special anneal in which harmful impurities are removed, are shown in Fig. 8.

### Elements. Iron-Silicon alloys

Iron, cobalt, and nickel are the common ferromagnetic elements and were the only ones known until 1935 when gadolinium was also shown to be ferromagnetic. The magnetism of iron, as well as steel and lodestone, were known to antiquity, while cobalt and nickel were first investigated about the middle of the 18th century. Most of the ferromagnetic substances are alloys containing one or more of the elements iron, cobalt, and nickel; some are non-metallic compounds of iron, cobalt, nickel or (rarely) manganese or chromium; and some are alloys containing no iron, cobalt or nickel but composed of manganese and certain other elements such as aluminum and copper (Heusler alloys).

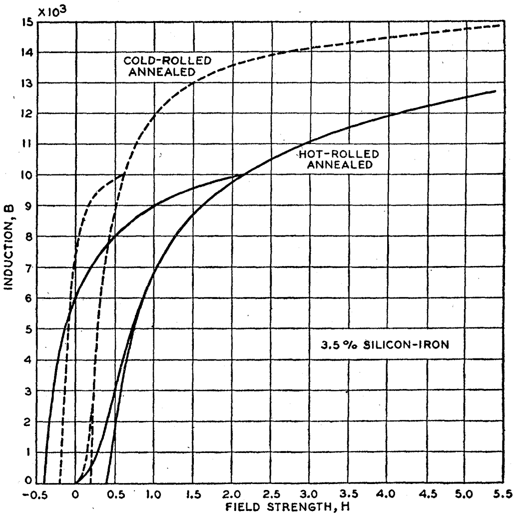


FIG. 10. Permeability of annealed silicon iron is higher and hysteresis lower if sheets have been fabricated by rolling cold.

Magnetization curves for iron, cobalt, and nickel are shown in Fig. 9. In iron one observes the highest magnetization and also the highest permeability. Cobalt has a lower magnetization and the lowest value of maximum permeability. Of these materials iron is by far the most important and is the only one used to any considerable extent in industry as a magnetic material in elementary form. The most used material of all, however, is an alloy of iron and silicon, sometimes called silicon steel or electrical sheet steel even though it is not a steel in the strict sense of the word because it contains no essential carbon. The iron-nickel alloys are used for many purposes when very high permeability is desired and cost is not too important.

Iron silicon alloys containing a few percent of silicon were invented by Sir Robert Hadfield, and came into use in the period 1900–10, mainly through the efforts of E. Gumlich in Germany. At present these alloys contain usually 3 percent to 4 percent silicon, but any amounts up to 6 percent silicon are occasionally used. A magnetization curve for a commercial alloy containing 4 percent silicon is drawn in Fig. 9, and on a different scale magnetization curves and hysteresis loops of two kinds of commercial product, each containing 3.5 percent silicon, are given in Fig. 10.

Iron silicon alloys are usually made from pig iron and ferrosilicon melted and refined in the basic open hearth, or from ingot iron remelted with ferrosilicon in the electric furnace. Ingots are poured and rolled to slabs and the latter

TABLE I. Some properties of commercial silicon-iron alloys in 0.014-in. sheet, annealed. $B_m = 15{,}000$, $f = 60$ cycles/sec.

| Grade | Silicon (percent) | Power loss (watts/lb.) | $\mu_m$ (gausses/oersted) | $\mu$ for $B=100$ | $H_c$ (oersteds) | Resistivity (microhm-cm) | Density (g/cm$^3$) |
|---|---|---|---|---|---|---|---|
| Armature | 0.5 | 4.3 | 5,500 | | 0.9 | 19 | 7.83 |
| Electrical | 1.2 | 3.6 | 6,100 | | 0.85 | 27 | 7.79 |
| Motor | 2.5 | 2.65 | 5,800 | | 0.75 | 42 | 7.70 |
| Dynamo | 3.25 | 2.15 | 5,800 | 1100 | 0.65 | 50 | 7.65 |
| Transformer (1) | 4.0 | 1.65 | 7,000 | 1600 | 0.5 | 59 | 7.60 |
| Transformer (2) | 4.5 | 1.4 | 9,000 | 2000 | 0.3 | 65 | 7.57 |
| Oriented Grain* | 3.0 | 0.9 | 30,000 | 3000 | 0.15 | 47 | 7.65 |

* Properties as measured in direction of rolling.

rolled either hot or cold to sheets of the required thickness, usually 0.014 in. These are then cut to the appropriate size for use in transformers or other apparatus and are preferably annealed by heating to some temperature lying between 800° and 1200°C, usually nearer 800°C; or, they may be annealed before cutting. Iron-silicon sheet or strip that has been fabricated by cold rolling is finding increased use as compared with the hot-rolled product (Fig. 10) because in the former the crystals composing the material are so oriented that the sheets have an especially high permeability when magnetized in one direction—that in which the material passed through the rolls during fabrication.

Some magnetic and physical characteristics of commerical silicon iron alloys are given in Table I. The watt loss there referred to is the power dissipated in the alloy when it is magnetized to a maximum induction of $B = 15{,}000$ by an alternating field of frequency 60 cycles per second. Such losses objectionably raise the temperature of a transformer core in operation. They are partly caused by hysteresis and partly to eddy currents, the nature of which latter will be discussed further below under MAGNETIZATION IN ALTERNATING FIELDS.

## Iron-Nickel Alloys (Permalloys)

These alloys have properties that are unusual and interesting from both a scientific and a commerical point of view. It became generally recognized about 1920 as a result of the work of G. W. Elmen that the alloys containing from 35 percent to 90 percent nickel have permeabilities very much greater than those of iron. These alloys were named "Permalloys." A magnetization curve and hysteresis loop for 78 Permalloy are shown in Fig. 11, and curves for iron are given for comparison.

An iron-nickel alloy containing 50 percent nickel was investigated by T. D. Yensen and named Hipernik. As distinguished from 78 Permalloy, suitable for transformers transmitting the weak signals of communication apparatus, this alloy has high permeabilities in the higher fields used in the transmission of power.

These commercial alloys, 78 Permalloy and Hipernik, have the compositions that are scientifically the most interesting of the binary iron-nickel series. This is illustrated in part (a) of Fig. 12 showing the induction of the alloys at low, intermediate, and high field strengths.

The induction in high fields has a minimum at 30 percent nickel and a maximum at 45 percent to 50 percent nickel. The maximum occurs not only when the field strengths are large but persists and is even more marked in low fields. A

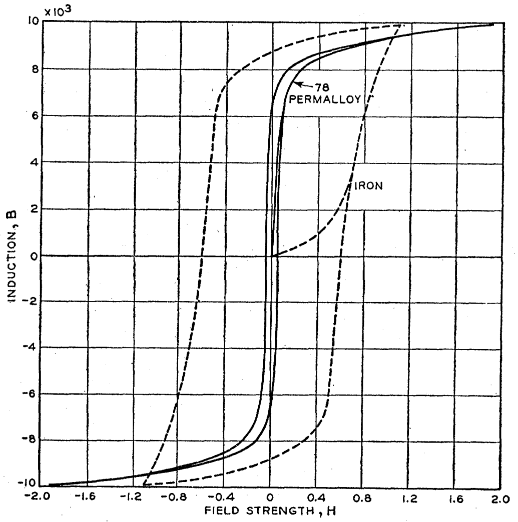


FIG. 11. Comparison of properties of Permalloy and iron.

second maximum appears for low fields when the nickel content is about 80 percent and this corresponds to the commercially useful alloy, 78 Permalloy. The high initial permeability (permeability for very low values of $B$) of 78 Permalloy (Fig. 12b) is associated with its very low magnetostriction (see STRESS AND MAGNETOSTRICTION below).

Even more remarkable magnetic properties are exhibited by iron-nickel alloys given a still different kind of heat treatment. After heating at a higher temperature the material is maintained for a short time at 400° to 600°C and at the same time a magnetic field of several oersteds is allowed to act on it. After cooling to room temperature the maximum permeability is found to be greatly increased and the coercive force diminished. This effect, often called the "magnetic anneal," depends on the composition and is most pronounced when the nickel content is 65 percent to 70 percent, as illustrated in Fig. 12c. The hysteresis loops of material given the magnetic anneal have the characteristic rectangular shape illustrated in Fig. 13. The sides are vertical and the corners often perfectly square, and the coercive force much smaller than that of the same material heat treated in the absence of a field.

### Additions to the Permalloys

Molybdenum, chromium, and copper have been added to Permalloy to advantage. The most used alloys of this kind are "4-79 Permalloy," containing 4 percent molybdenum, 79 percent nickel, and 17 percent iron, and "Mumetal" containing usually 5 percent copper, 2 percent chromium, 75 percent nickel, and 18 percent iron. These are both used in transformers operating at high frequencies and are meritorious primarily on account of their high initial permeabilities, but also because they have higher resistivities and can be prepared with a simple heat treatment. Their permeabilities at various inductions are indicated by the curves of Fig. 14. Initial permeabilities of 10,000 to 30,000 are usually attained.

During World War II a great improvement was made in the Permalloys by heat treating them in pure hydrogen at 1200° to 1300°C. The most useful of these alloys, called "Supermalloy" (Su-perm'-al-loy) contains 5 percent molybdenum and 79 percent nickel, the balance mainly iron. Initial permeabilities as high as 125,000 and maximum permeabilities of more than 1,000,000 can be attained commercially even in thin sheet (Fig. 14). The technical progress in magnetic materials may be measured by comparing these magnitudes with the permeabilities of the best materials available in the first decade of this century—silicon steel made at that time had initial and maximum permeabilities about 1/300 those of Supermalloy (Fig. 15).

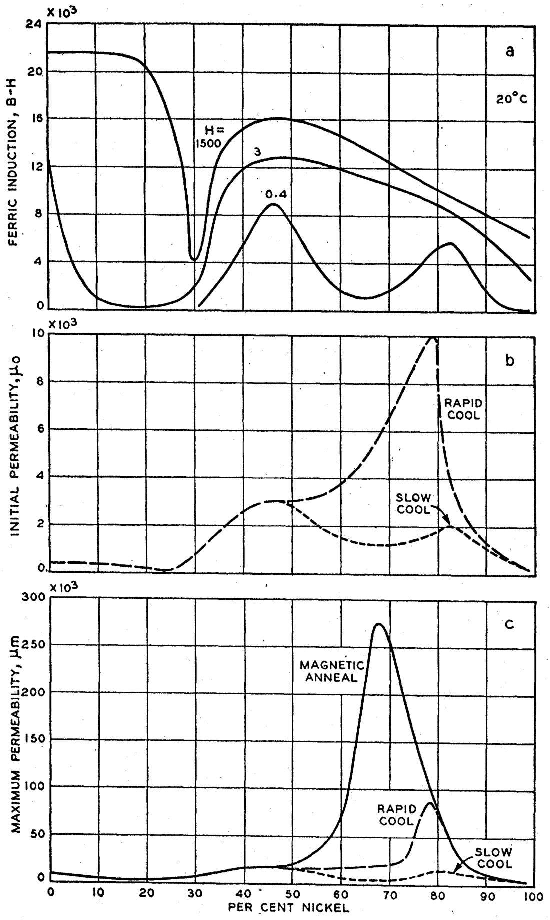


FIG. 12. Some magnetic properties of iron-nickel alloys (G. W. Elmen, R. M. Bozorth).

### Iron-Cobalt Alloys

It is a surprising fact, for which there is no satisfactory explanation, that some alloys of iron

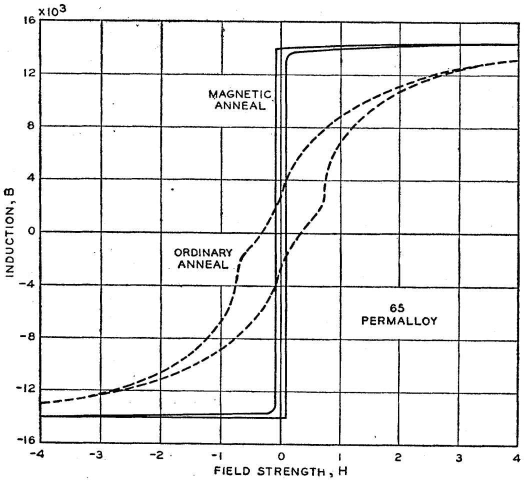


FIG. 13. Effect of annealing in presence of magnetic field (R. M. Bozorth and J. F. Dillinger).

and cobalt can attain greater intensities of magnetization than either of these elements alone. The saturation magnetization is highest for the alloys containing 35 percent to 40 percent cobalt, and then is 10 percent to 15 percent higher than for pure iron. When the field strength is 1000 or below, the highest inductions are attained in the alloy containing equal proportions of the two elements (Fig. 16), except that in fields weaker than two oersteds pure iron has the highest permeability.

As a material the 50 percent alloy, under the name of "Permendur," is a useful component of the magnetic circuits of electromagnets and permanent magnets, where it is used for yokes and pole pieces. Since it is rather brittle and cannot be rolled to thin sheets, 2 percent vanadium is sometimes added and this permits the fabrication of sheets a few thousandths of an inch thick. The latter material is used in quantity in the thin diaphragms of telephone receivers. Some alloys contain 35 percent cobalt and other elements added to increase ductility.

## Iron-Cobalt-Nickel Alloys

Some of these alloys are interesting because they have permeabilities that are independent of field strength over a relatively large range. The same alloys have hysteresis loops of peculiar shape with low residual induction and a constricted middle portion. A representative alloy, called Perminvar, contains 30 percent iron, 25 percent cobalt, and 45 percent nickel. The constancy of permeability with field strength at low fields is brought out by annealing in the usual way at about 1000°C and then heat treating for several hours at a relatively low temperature, 350° to 500°C. The hysteresis loss is then extremely small. A $\mu$ *vs*. $H$ curve and a hysteresis loop for an alloy of the above composition, heat treated for 24 hours at 400°C, are shown in Fig. 17. This material responds to heat treatment in a magnetic field, and a loop for a specimen so treated is also shown in the figure.

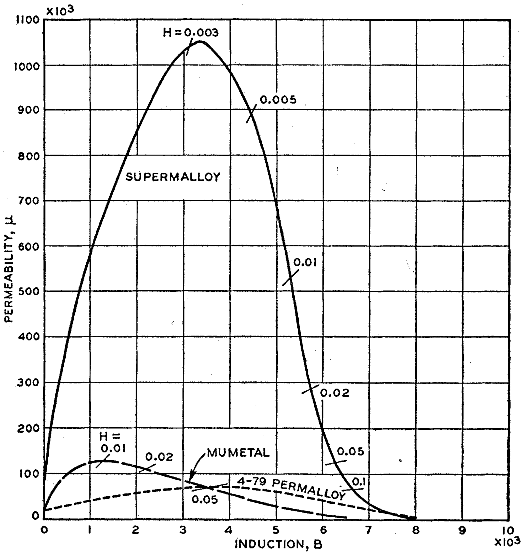


FIG. 14. Comparison of Supermalloy with other high-permeability alloys (O. L. Boothby and R. M. Bozorth).

## Heusler Alloys

These alloys, discovered in 1903, are of unusual scientific interest because they contain no ferromagnetic element. A typical Heusler alloy has the composition corresponding approximately to the formula $Cu_2MnAl$ (61 percent copper, 26 percent manganese, 13 percent aluminum). This has a saturation induction ($B_s$) of about 5500, almost as high as that of nickel, but its permeability is much lower—about 250 at maximum. In the alloy containing tin ($Cu_2MnSn$) a saturation of more than 8000 has been attained. The coercive force is rather high for a permeable material and varies with composition and heat treatment, but may be as low as 4 or 5.

To bring out their best magnetic properties these must be quenched from temperatures up to 800°C and then aged or heated for some time at a relatively low temperature, often 200°C. This last treatment is necessary for effecting a highly ordered arrangement of the atoms in the crystals, a situation necessary for the development of the magnetic properties.

The atom primarily responsible for the magnetism of the Heusler alloys is manganese, but copper also may contribute to the magnetic moment. The distribution of the other atoms around the manganese is also important, as the effect of atomic ordering shows. Ferromagnetism still occurs when the aluminum or tin is replaced by arsenic, antimony, bismuth, or boron, or when copper is replaced by silver.

A number of the more important alloys of high permeability are described in Table II.

## 4. PERMANENT MAGNETS

Permanent magnets, like electromagnets, are used to produce magnetic fields of considerable strength and constancy. Many devices depend on them for operation, e.g., loudspeakers, magnetos, meters of many kinds, magnetrons, telephone receivers, and a host of others. In all of these the magnet necessarily contains an air gap; consequently it always operates under the influence of a demagnetizing field (see section on MEASUREMENT OF MAGNETIC QUANTITIES), such as that existing at the middle of a short bar of magnetized material. Thus the magnet operates not at residual induction but at some lower value of the induction, $B$.

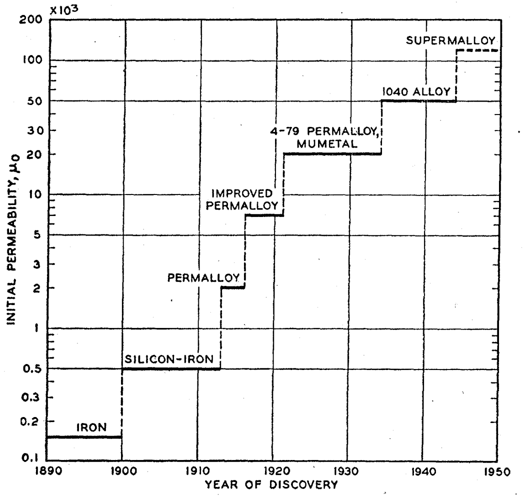


FIG. 15. Initial permeabilities of some useful materials plotted against the year in which they were discovered. Note logarithmic scale.

The important curve for evaluating the quality and behavior of material for permanent magnets is that portion of the hysteresis loop that lies in the second quadrant, between residual induction, $B_r$, and coercive force, $H_c$, and is called the demagnetization curve. Magnetization and demagnetization curves for Alnico 5 (Alcomax) are shown in Fig. 18. The quality of a permanent magnet is characterized not only by $B_r$ and $H_c$, but also by the so-called energy product, the product of $B$ and $H$ for various points on the demagnetization curve. The energy product curve (Fig. 19), showing this product as dependent on $B$, has a characteristic shape, and its greatest product, the maximum energy product, is the best single criterion of magnetic quality for these materials. The reason for this, and its importance in the design of permanent magnets, will be discussed after the most-used materials have been described.

### Magnet Steels

From ancient times until about 1930, materials used as magnets were carbon steels. After

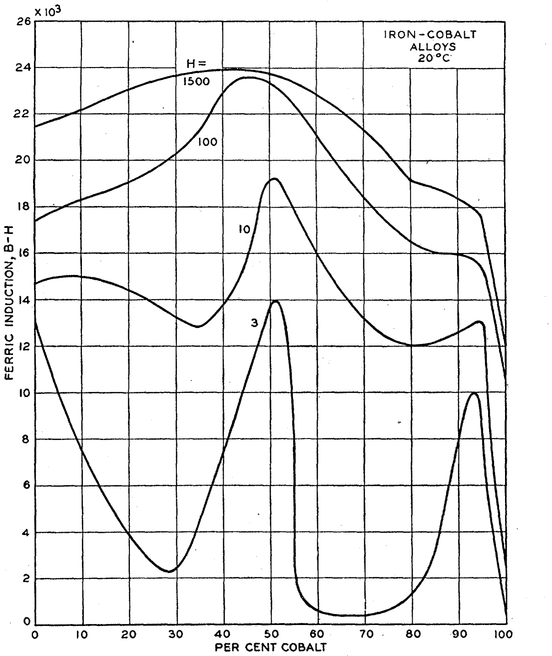


FIG. 16. Magnetic properties of iron-cobalt alloys. Note high magnetization of 50 percent alloy in intermediate and high fields (G. W. Elmen).

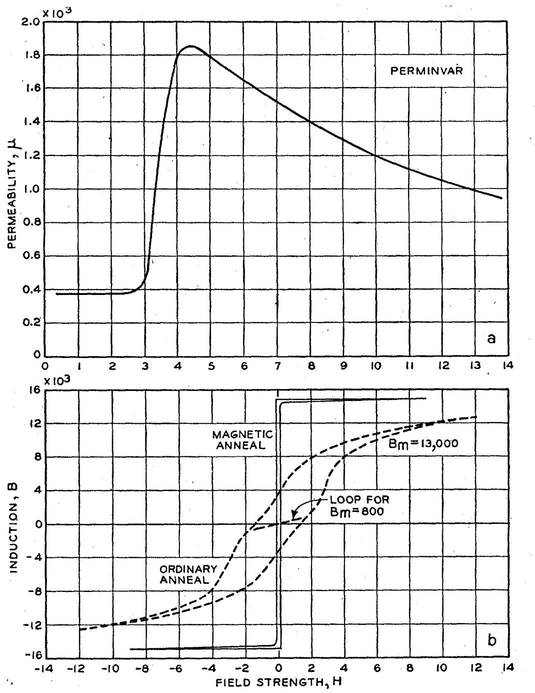


FIG. 17. Perminvar (45 percent Ni, 25 percent Co, 30 percent Fe) has remarkably constant permeability in low fields (a). It has extremely low hysteresis at low inductions (b) and a characteristic wasp-waisted hysteresis loop at high inductions, and is affected strongly by magnetic anneal (G. W. Elmen and G. A. Kelsall).

1855 alloying elements were added, especially tungsten, to improve the quality. In 1916–17 chromium began to displace tungsten, and cobalt was added up to 40 percent with very beneficial effect.

A typical chrome steel contains 3.5 percent chromium and 1.0 percent carbon. In manufacture, after the material has been melted, cast and hot-rolled, it is cut to final size, heated for a short time at 800° to 850°C and then quenched in oil. Before assembling in apparatus it is often aged by maintaining at 100°C or above for 24 hours; this improves the constancy of the field it maintains in operation.

### Dispersion-Hardened Alloys

From the standpoint of magnetic quality the best materials available in 1946 for use in permanent magnets were not steels but dispersion-hardened (precipitation-hardened) alloys containing no essential carbon. The first materials of this kind to find commercial use were the iron-cobalt-molybdenum and iron-nickel-aluminum alloys (Table III), and others are coming into use constantly. The hardness is caused by the presence of very small particles of one kind highly dispersed in a matrix of another composition.

Important materials used in 1946 for magnets are listed in Table III. This includes data on steels and indicates the mechanical operations required in preparation and the heat treatment

TABLE II. Some properties of high permeability materials (properties important in use are indicated by italics).

| Name | Composition % | Heat treatment* | Initial permeability $\mu_0$ | Maximum permeability $\mu_m$ | Coercive force $H_c$ (oersteds) | Saturation induction $B_s$ (gausses) | Electrical resistivity $\rho$ (microhm-cm) | Curie point $\theta$ (°C) | Density $\delta$ (g/cm³) | Uses |
|---|---|---|---|---|---|---|---|---|---|---|
| Iron | 99.9 Fe | 900°C | 200 | 5,000 | 1.0 | 21,500 | 10 | 770 | 7.87 | Relays |
| Silicon-iron | 4 Si, 96 Fe | 800°C | 400† | *7,000* | .6 | 19,700 | 59 | 690 | 7.60 | Power transformers |
| Silicon-iron | 3.3 Si, 97 Fe | Cold-rolled 800°C | 600† | *10,000* | .2 | 20,000 | 50 | 700 | 7.65 | Power transformers |
| Hipersil‡ | 3.3 Si, 97 Fe | Cold-rolled 1200°C, $H_2$ | 1,500† | *40,000* | .1 | 20,000 | 48 | 700 | 7.65 | Power transformers |
| Sendust | 9 Si, 5 Al, 85 Fe | Cast | 30,000 | 120,000 | .05 | 10,000 | 80 | 500 | 7.1 | |
| 45 Permalloy | 45 Ni, 54 Fe | 1050°C | *2,500* | *25,000* | .3 | *16,000* | 50 | 440 | 8.17 | Transformers, relays |
| Hipernik | 50 Ni, 50 Fe | 1200°C, $H_2$ | 4,000 | *80,000* | .05 | 16,000 | 35 | 500 | 8.25 | Transformers |
| Radiometal | 47 Ni, 3 Cu, 49 Fe | 1050°C | *2,500* | 25,000 | .3 | *15,600* | 55 | — | 8.3 | Transformers |
| 78 Permalloy | 78 Ni, 21 Fe | 1050°C 600°C | *8,000* | 100,000 | *.05* | 10,000 | 16 | 580 | 8.60 | Sensitive relays |
| 4-79 Permalloy | 79 Ni, 4 Mo, 16 Fe | 1100°C | *20,000* | 80,000 | .05 | 8,700 | 57 | 420 | 8.72 | Transformers |
| Mumetal | 75 Ni, 2 Cr, 5 Cu, 18 Fe | 1100°C, $H_2$ | *20,000* | 110,000 | .03 | 7,200 | 60 | 430 | 8.58 | Transformers |
| 1040 Alloy | 72 Ni, 14 Cu, 3 Mo, 11 Fe | 1100°C, $H_2$ | 40,000 | 100,000 | .02 | 6,000 | 56 | 290 | 8.76 | |
| Supermalloy | 79 Ni, 5 Mo, 15 Fe | 1300°C, $H_2$ | *100,000* | 800,000 | .004 | 8,000 | 60 | 400 | 8.87 | Transformers |
| Permendur | 50 Co, 50 Fe | 800°C | 800 | 5,000 | 2.0 | *24,500* | 7 | 980 | 8.3 | Electromagnets |
| V-Permendur | 49 Co, 2 V, 49 Fe | 800°C | 800 | 4,500 | 2.0 | *24,000* | 26 | 980 | 8.2 | Telephone diaphragms |
| Powdered Permalloy | 81 Ni, 2 Mo, 17 Fe | Pressed 650°C | *125* | 130 | — | — | $10^6$ | 480 | 7.8 | Loading coils |

* "$H_2$" indicates heat treatment in pure hydrogen. † Measured at $B=20$ rather than $B=0$. ‡ Also called Corosil.

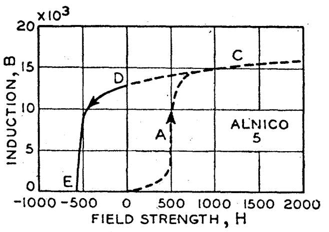


FIG. 18. Magnetization curve (*OAC*) and demagnetization curve (*DE*) of Alnico 5 (Alcomax).

used to bring out the best magnetic properties. Demagnetization curves for some of these alloys are reproduced in Fig. 20. Special attention is called to the alloy called Alcomax in England and Alnico 5 in the United States. In order to develop its remarkable properties it must be cooled at a definite rate in a strong magnetic field (*ca.*, 1000 oersteds) so directed in the specimen that it is parallel to the field required in use. At right angles to the direction of the field applied during treatment the properties are not so good, the energy product being then only about one-third as high. Although Alnico 5 is not forgeable and must be cast and ground to final form, its magnetic properties are outstanding and its uses numerous.

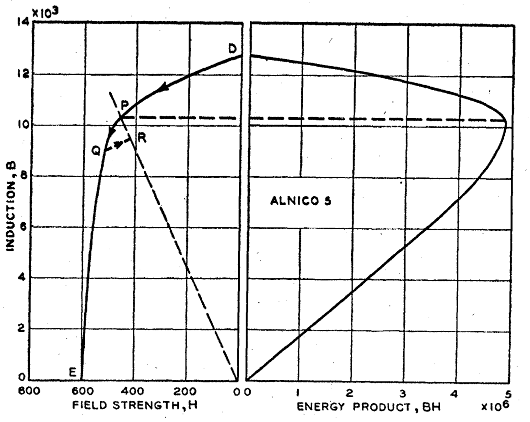


FIG. 19. Demagnetization curve (*DE*) and corresponding energy product curve (*BH vs. B*), showing desirable point of operation, *P*, where energy product is maximum.

### Alloys with Special Properties

In contrast to their usual brittle character, some materials used as permanent magnets have sufficient ductility to permit cold forming. In 1935 it was discovered that an alloy of iron, nickel, and copper could be rolled to thin tape and then heat treated to have high coercive force and residual induction (Table III). Later the iron-cobalt-vanadium alloy known as Vicalloy was found to have superior quality for the purpose for which these alloys are used—the magnetic recording of speech.

A material of quite different kind is made of powdered oxides of iron and cobalt ($Fe_2CoO_4$), pressed and heated to sinter, and cooled from 850°C in a magnetic field of 4000 oersteds. This usually has a coercive force of about 600 and residual induction of about 4000. Its properties

TABLE III. Some properties of permanent magnet materials.

| Name | When used | Typical composition (%)* | Coercive force $H_c$ (oersteds) | Retentivity $B_r$ (gausses) | Energy product $(BH)_m \times 10^{-6}$ | Fabrication | Heat treatment |
|---|---|---|---|---|---|---|---|
| Tungsten steel | 1855 | 6 W, 0.7 C, 0.3 Mn | 65 | 10,500 | 0.3 | Forged | Q850°C |
| Low chrome steel | 1916 | 0.9 Cr, 0.6 C, 0.4 Mn | 50 | 10,000 | 0.2 | Forged | Q800 |
| High chrome steel | 1916 | 3.5 Cr, 1 C, 0.4 Mn | 65 | 9,500 | 0.3 | Forged | Q825 |
| KS magnet steel† | 1917 | 36 Co, 7 W, 3.5 Cr, 0.9 C | 230 | 10,000 | 0.9 | Forged | Q950 |
| Cobalt chrome steel‡ | 1921 | 16 Co, 9 Cr, 1 C, 0.3 Mn | 180 | 8,000 | 0.6 | Forged | Q975 |
| Remalloy§ | 1931 | 12 Co, 17 Mo (or W) | 250 | 10,500 | 1,1 | Forged | Q1200, B675 |
| Mishima alloys‖ | 1931 | 25 Ni, 12 Al | 500 | 7,000 | 1.4 | Cast | Q1200, B650 |
| Oxide magnet | 1933 | 15 Co, 28 O | 900 | 2,000 | 0.6 | Sintered | A1000‡‡ |
| Alnico 2¶ | 1934 | 12 Co, 17 Ni, 10 Al, 6 Cu | 560 | 7,300 | 1.7 | Cast | A1200, B650 |
| Magniflex** | 1935 | 20 Ni, 60 Cu | 450 | 5,300 | 1.1 | Cold-rolled | B600 |
| New KS‖‖ | 1935 | 35 Co, 18 Ni, 6 Al, 8 Ti | 900 | 5,700 | 1.8 | Cast | B650 |
| Platinum cobalt alloy | 1936 | 77 Pt, 23 Co | 2650†† | 4,530 | 3.8 | Forged | Q1200, B650 |
| Vicalloy | 1938 | 52 Co, 11 V | 200 | 11,500 | 1.5 | Cold-rolled§§ | B600 |
| Alnico 5 (Alcomax) | 1940 | 24 Co, 14 Ni, 8 Al, 3 Cu | 550 | 12,500 | 4.5 | Cast | A1300, B600‡‡ |

* Balance iron.

† Called Koerzit in Germany.

‡ Also alloys containing 3, 6, 9 cobalt with variable chromium.

§ Also called Comol.

‖ Also called Alnico 3, Alnico 1 and 4 contain cobalt.

¶ Called Alnico in England and Oerstit in Germany.

** Also called Cunife 1. Cunife 2 and Cunico contain cobalt.

†† *H* for $B=O$.

‡‡ Must be cooled in strong magnetic field.

§§ Cold-rolled in grooved rolls. Higher values in drawn wire, lower in rolled sheet.

‖‖ Composition is given for Alnico 12. Nipermag and Alnico 7 contain smaller amounts of titanium.

Q—quenched from centigrade temperature indicated, in oil (or water).
A—cooled in air from indicated temperature.
B—aged or baked, cooling rate not important.
Forged alloys may also be hot rolled or cast to size.
Cast alloys (unless forgeable) must be ground to size.
Cold-rolled alloys are usually hot rolled before cold rolling.

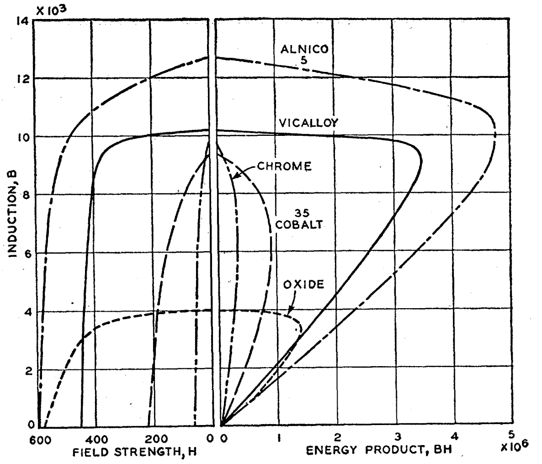


FIG. 20. Demagnetization and energy product curves for several materials important for use in permanent magnets.

differ from those of most other materials in that the density is small and the resistivity high.

### Design of Permanent Magnets

In the design of permanent magnets it is important to use the appropriate material and to shape it so that the minimum amount is used to produce the desired field strength, $H_g$, in a given gap. As already pointed out, this result is accomplished when the product of $B$ and $H$ everywhere in the material has its maximum product, $(BH)_m$. The magnet of Fig. 21 is designed with that in mind. The material is Alnico 5. The length of the material along the curved path, in relation to the length of the gap, determines the position of the line $OP$ of Fig. 19, and this should be placed so that it cuts the demagnetization curve at $B=10{,}000$ corresponding to $(BH)_m$. The cross-sectional area of the magnet is adjusted to take account of leakage; a heavier section is necessary at the lower part because all of the leakage flux must pass through this, and $B$ must at the same time be maintained at its proper value.

After a permanent magnet is designed and constructed it is magnetized with a high field. After removal of this magnetizing field it is usually desirable to apply momentarily a relatively weak field in the opposite direction, whereupon the magnetic state of the material changes as indicated by the points $PQR$ of Fig. 19. This stabilizes the magnet against the effect of stray fields or mechanical shocks or changes in temperature, so that the field in the gap is less likely to change.

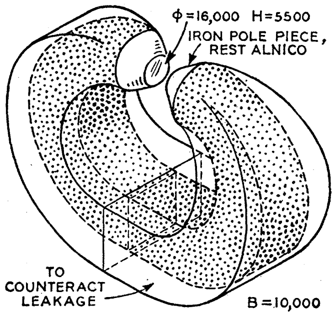


FIG. 21. Practical design of magnet operating at $H_g = 5500$, $B=10{,}000$ almost everywhere in material. Unshaded portion is part of magnet necessary to carry "leakage flux" (P. P. Cioffi).

## 5. THE MAGNETIZATION CURVE

### Three Portions of Curve

A typical magnetization curve, showing the relation between $B$ and $H$ in a specimen initially unmagnetized, may be divided into three main parts separated from each other by the "knee" and the "instep" (Fig. 22). These three regions are identified with different physical mechanisms of change of magnetization, and also with different engineering uses of the materials.

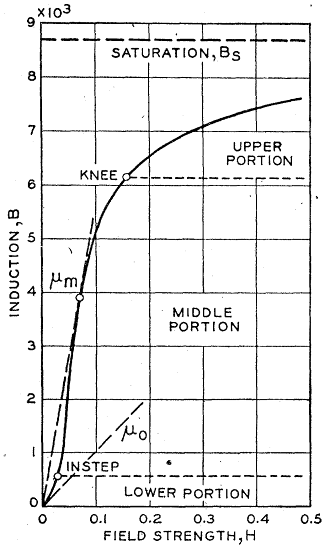


FIG. 22. Three sections of normal magnetization curve separated by "knee" and "instep." $\mu_m$ is largest value of $B/H$, $\mu_0$ is initial slope of curve.

The initial portion of the curve, the "toe," is not horizontal at the origin but has a definite slope called the initial permeability, $\mu_o$, equal to $B/H$ for $H=O$. Fôr low inductions Lord Rayleigh found that the relation

$$B=\mu_o H+aH^2$$

was obeyed, and when the permeability $\mu$ is plotted against the field strength $H$ the points usually lie on a straight line corresponding to the equation

$$\mu=\mu_o+aH.$$

Data for several common materials are shown in FIG. 23. The value of $H$ at which the line begins to bend upward, varies over wide limits from one material to another. In some abnormal materials such as the iron-silicon alloy the curve first becomes convex upward before it begins to rise rapidly at higher values of $H$. Curves of the kind shown in the figure are used to determine the value of $\mu_o$ by extrapolating them to $H=O$.

The upper portion of the magnetization curve (Fig. 22) bends over and approaches the horizontal line marked $B_s$. In high fields, however, it is found (Fig. 24) that $B$ approaches no definite limit but $B$-$H$ does approach a limit, called the saturation induction or simply saturation, designated $B_s$. Since $B-H=4\pi I$, $I$ approaches the limit $I_s$, the saturation magnetization.

The upper part of the curve may be represented fairly well by O. Fröhlich's equation:

$$B-H=H/(a+bH)$$

in which $b=1/B_s$ and $a$ is a constant which measures the magnetic hardness and is larger the stronger the field necessary to attain any given fraction of saturation. When $H$ is small enough to be neglected in comparison with $B$, this equation may be expressed in the equivalent form used by A. E. Kennelly:

$$1/\mu=a+bH$$

and so a linear relation is found when $1/\mu$ is plotted against $H$. From a graph of this kind (Fig. 25) one can easily determine the constants $a$ and $b$, and interpolate to find the value of $\mu$ for intermediate values of $H$. The slope of the line, $b$, may be used to estimate the saturation induction ($b=1/B_s$), but this method cannot be depended on for accuracy for in some materials such as iron the slope changes markedly at inductions close to saturation. The ratio $1/\mu$ is termed the reluctivity.

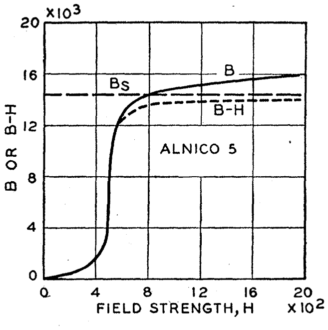


FIG. 24. In high fields $B$–$H$ approaches asymptotically to limit, $B_s$ (saturation).

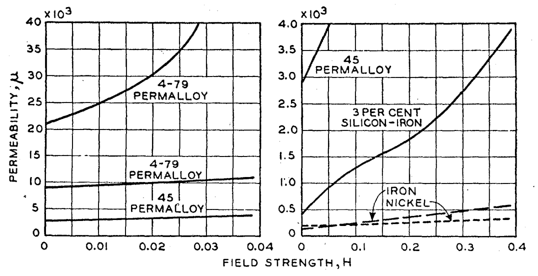


FIG. 23. Curves showing $\mu$ *vs.* $H$ for initial portions of magnetization curves for various materials. Lines are usually straight near axis.

In its middle portion the magnetization curve is the steepest, and the permeability has its greatest value. It is here that irreversible changes in magnetization are a maximum, as indicated by the fact that curves for increasing and decreasing field strength differ markedly. These irreversible changes and the consequent losses in energy will now be discussed.

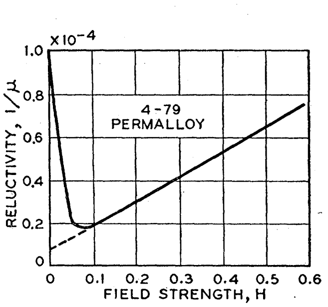


FIG. 25. Reluctivity, reciprocal of permeability, is linear at high field strengths (Frölich-Kennelly).

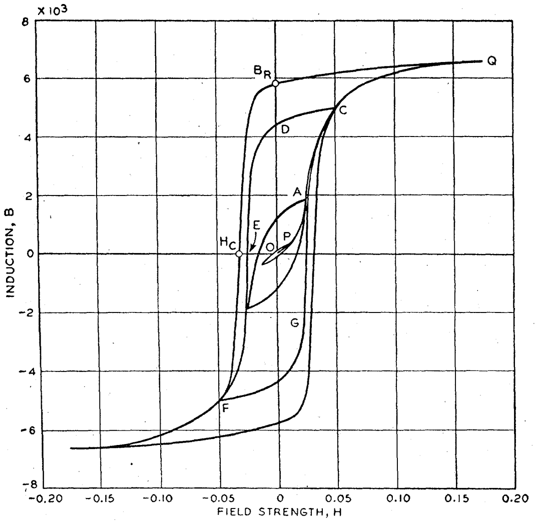


FIG. 26. Normal hysteresis loops for low, intermediate and high inductions. Residual induction, $B_R$, and coercive force, $H_C$, are values of $B$ and $H$ where curves cross axes.

## Hysteresis Loops

A set of normal hysteresis loops is shown in Fig. 26. In forming one of these the field strength is first increased from zero and the induction follows the magnetization curve *OAC*. $H$ is then decreased, and the induction decreases slowly so that the hysteresis loop passes successively through the points $D$, $E$, and $F$, and the field is reversed and the loop passes through $G$ and back to $C$. At $D$ the induction is called residual induction ($B_R$) and at $E$, when $B=0$, the field strength is the coercive force ($H_C$). The induction at the top of a loop as at $C$, is commonly designated $B_m$.

When the tip of the hysteresis loop lies on the first portion of the magnetization curve (*OP* in Fig. 26), the loop is relatively narrow and the slope of the upper part of the loop after it has just receded from the tip, is not greatly different from the slope of the magnetization curve at that point. If the tip lies in the middle portion of the magnetization curve, however, the slope of the loop (e.g., at $C$, going toward $D$), is markedly different from the slope of the magnetization curve. These loops are relatively wide and the energy losses large. In the upper portion of the curve, above the knee, the losses are again small and the process of magnetization reversible.

The area enclosed by a hysteresis loop is proportional to the energy liberated as heat during traversal of the loop. When the area is in the units gausses×oersteds the energy loss due to hysteresis (Warburg's law) is

$$W_h = \frac{\text{area}}{4\pi} = \frac{1}{4\pi} \oint H dB$$

ergs per cm³ for each transversal of the loop, or cycle. In high fields the hysteresis loss in iron, about 10,000 ergs/cm³ cycle, is sufficient to raise its temperature about 0.0003°C for each cycle, and therefore if the field is alternated 50 or 60 times a second about one minute is required for the iron to heat one degree.

The hysteresis loss rises rapidly as the maximum induction of the loop, $B_m$, increases. The curve of Fig. 27 shows the relation between $W_h$ and $B_m$ for ordinary iron having a maximum permeability of 4500. When $B_m$ is very small $W_h$ varies as $B^3$, and in the range $B_m = 1000$ to 15,000, of special interest for work on power transformers, it varies approximately as the 1.6 power of $B$, in accordance with the empirical

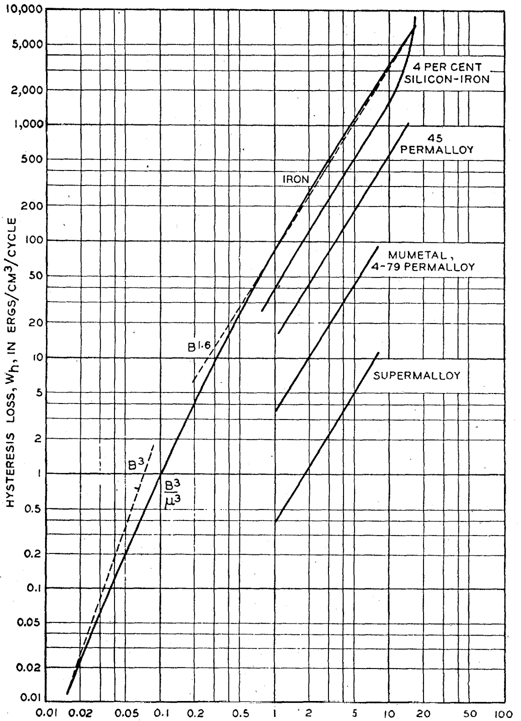


FIG. 27. Hysteresis loss as dependent on maximum induction for some important materials.

law of Steinmetz:

$$W_h = \eta B_m^{1.6}$$

in which the Steinmetz coefficient, $\eta$, is approximately constant for any one material.

The $B^3$ relation at low inductions was derived by Rayleigh. He showed, as noted above, that the magnetization curve near the toe followed the equation

$$B = \mu_o H + aH^2,$$

and also that the hysteresis loop with tips at $B_m$, $H_m$, and $-B_m$, $-H_m$ was described by parabolic equations

$$B = \mu H + (a/2)(H_m^2 - H^2)$$

and

$$B = \mu H - (a/2)(H_m^2 - H^2)$$

for the upper and lower halves of the loop, respectively. (Here $\mu = B_m/H_m$). Such a loop is shown with the corresponding magnetization curve in Fig. 28. Measurements have been made for values of $B_m$ as low as 2 gausses, and only slight deviations from the Rayleigh form observed. The area of a Rayleigh loop may be calculated from the above equations, and the corresponding hysteresis loss, expressed in ergs/cm$^3$/cycle, is

$$W_h = \frac{aH_m^3}{3\pi} = \frac{aB_m^3}{3\pi\mu^3}.$$

This equation corresponds to the lower part of the long curve of Fig. 27, and is valid in iron when $B_m$ is below about 500 gausses.

The hysteresis losses of various materials vary over a wide range. Losses of some of the common permeable materials are indicated by curves in Fig. 27, and one of these has about 1/200 the loss of iron at the same value of $B_m$. Materials used for permanent magnets, on the other hand, have hysteresis losses enormously larger than iron, sometimes by a factor of 500. Thus the overall range in loss in various materials is approximately 100,000 times; i.e., the coefficient $\eta$ of Steinmetz' equation varies from $0.6 \times 10^{-5}$ to 0.6.

### Distribution of Hysteresis Loss over Cycle

The energy loss and the attendant rise in temperature can be calculated satisfactorily as described above for a complete hysteresis loop. But in what part of the loop does most of the rise in temperature take place? This question has been answered by direct experiment, measurements having been made of the very small rise in temperature that occurs when the induction is changed by only a small fraction of the total. Changes of about one-millionth of a degree centigrade have been measured in these experiments. The results of such investigations show that the energy losses are largest where the hysteresis loop is steepest. In addition to this irreversible loss, there is a heating of the specimen in strong fields as the field is increased, and an equal cooling as the field is decreased. This reversible heat does not, therefore, contribute to the hysteresis loss over a complete cycle.

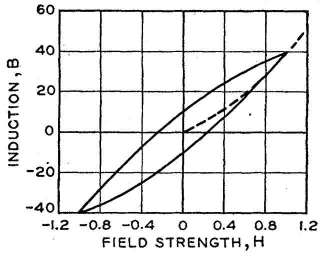


FIG. 28. Parabolic form of hysteresis loop for low inductions (Lord Rayleigh).

### Hysteresis in Rotating Fields

The hysteresis loops mentioned so far have been measured with $B$ and $H$ either parallel or antiparallel to each other. A different result is obtained if the field acting on a specimen is maintained constant in strength and varied continually in direction. This is usually called simply rotational hysteresis as compared with the usual alternating hysteresis. Measurement of rotational hysteresis is accomplished by placing a disk of the material to be examined between the poles of an electromagnet and rotating the disk so that $H$ always lies in its plane. The force necessary to turn the disk is then a measure of the rotational hysteresis loss.

As the field strength increases from zero the rotational hysteresis first increases rapidly (Fig. 29), more rapidly than the alternating hysteresis, but when the induction is half to three-quarters of saturation the alternating hysteresis goes through a maximum and decreases rapidly, ap-

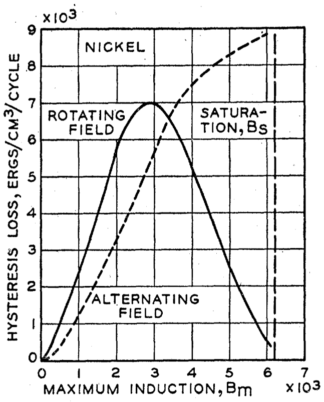


FIG. 29. Rotational hysteresis loss goes through maximum, approaches zero at saturation (A. Perrier).

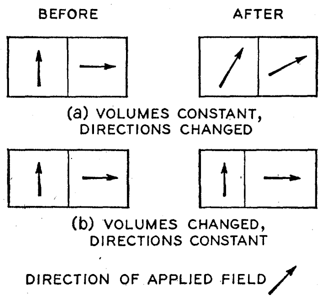


FIG. 30. Change in magnetization results from (a) change in direction (rotation) of magnetization in small regions of material, called *domains*, or (b) increase in size of some domains (boundary displacement) at expense of neighbors.

proaching zero at saturation. The disappearance of hysteresis in a high rotating field was predicted as a result of Ewing's theory, and its confirmation in 1896 indicated the fundamental soundness of the theory. Rotational hysteresis is of practical importance in the magnetic material forming the segments or teeth of rotating dynamo-electric machinery.

### Domain Theory of Magnetization

Many of the phenomena of the magnetization curve and hysteresis loop can be described to advantage in terms of the domain theory. The theory that a ferromagnetic material is composed of many small regions, each magnetized to saturation in some direction, was first stated by P. Weiss in 1907, and the description he gave at that time is accepted with but slight modification. The existence of such domains is evident from H. Barkhausen's experiments and even more definitely from the study of powder patterns (see below). In the unmagnetized state the directions in which the domains are saturated (the directions of easy magnetization) are distributed either at random or in some other way such that the resultant magnetization of the specimen as a whole is zero. Application of a field changes only the direction of magnetization in a given volume. The three main parts of the magnetization curve are distinguished from each other by the nature of this change in direction, which may take place in any of several ways.

The magnetic moment of any one domain is specified by the magnitude and direction of its magnetization and by its volume. Ordinarily at constant temperature the moment of a domain and therefore the magnetization of the ferromagnetic body of which it is a part, is changed by

(a) a change in the direction of magnetization of the domain—"rotation"— or
(b) a change in the volume of the domain—"moving boundary."

These processes are indicated graphically in Fig. 30. It has been shown by R. Becker that the moving boundary is particularly important for changes occurring in low and medium fields. Processes may be classified also as reversible or irreversible accordingly as the energy dissipated in heat is a relatively small or large fraction of the potential energy.

In each of these portions of the magnetization curve, one type of process preponderates as indicated here:

(1) Initial portion—reversible boundary displacement.
(2) Middle portion—irreversible boundary displacement.
(3) Upper portion—reversible rotation.

Figure 31 shows the three important processes in relation to the magnetization curve and hysteresis loops, and the corresponding energy losses due to hysteresis, for a specimen of a high permeability alloy. Figure 32a is a highly schematic representation of the domains in a part of an unmagnetized single crystal that may itself be a portion of an ordinary polycrystalline specimen of iron. The arrows, circles, and crosses indicate the directions of local magnetization, and the sides of the squares the directions of the crystal axes. Figures 32b and 32c represent the states of magnetization corresponding to the instep and knee of the curve, attained by reversible and irreversible boundary displacement, respectively, and 32d represents saturation attained in high fields by the process of rotation. Although the domains have been represented here by squares, implying that the actual domains are cubes, actually no such conclusions should be drawn regarding the shape of domains. There is some evidence to be considered below to show that they are rod-like or plate-like in form.

The factors which determine the direction of magnetization in a single domain of a specimen are the crystal structure, the state of strain, and, of course, the direction and strength of the magnetic field. The effect of crystal structure and of strain will be discussed in some detail in later sections of this article. It may be mentioned here in a preliminary way that the nature of the crystal structure affects mainly the upper part of the magnetization curve of a permeable material. In iron, for example, the crystal structure is such that the magnetic properties of a single crystal vary considerably with direction, and as a result of this variation exceptionally high fields are necessary to approach saturation in ordinary polycrystalline iron. The rotation of the domain by the field must be made against relatively powerful forces, and so the magnetization curve above the knee rises slowly over a considerable distance. This behavior may be contrasted with that of Permalloy containing about 70 percent nickel and 30 percent iron—here the magnetic properties vary but little with the direction in the crystal, the domains therefore rotate easily, and the magnetization curve flattens out quickly above the knee, attaining saturation in relatively weak fields (Fig. 9).

Just as strain affects magnetic properties, so magnetization changes the dimensions of a ferromagnetic body, or the body is said to exhibit magnetostriction. If the length of a material changes by a relatively large amount when magnetized to saturation, this material will be relatively sensitive to strain, i.e., magnetostriction and strain-sensitivity are reciprocal properties. Strain affects mainly the lower and middle portions of the magnetization curve, and the strain and magnetostriction together determine the ease with which the interdomain boundary is moved by a change in the magnetic field. Thus the highest initial permeabilities are found in materials having very small magnetostriction,

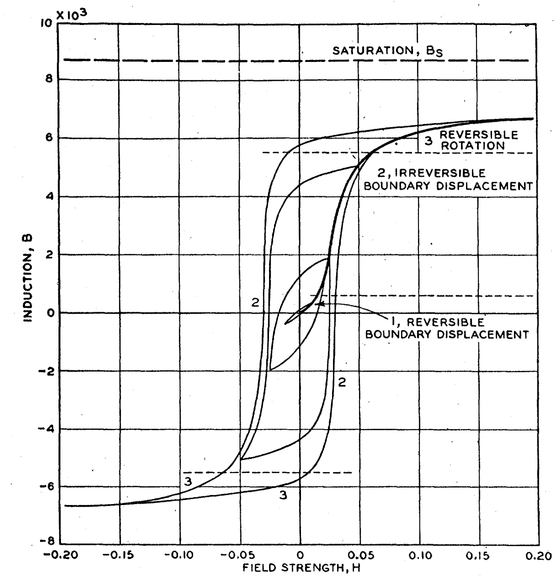
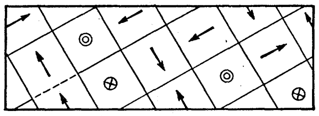
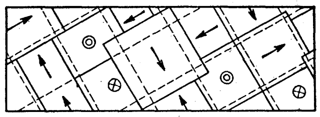


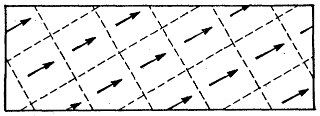
FIG. 31. Nature of change in magnetization in relation to position on magnetization curve and hysteresis loop.



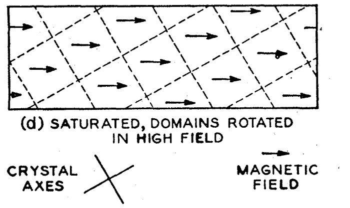
(a) DEMAGNETIZED



(b) PARTIAL MAGNETIZATION



(C) SUDDEN REVERSALS COMPLETE (KNEE OF MAGNETIZATION CURVE)



(d) SATURATED, DOMAINS ROTATED IN HIGH FIELD

FIG. 32. Schematic of changes in domains at various stages of magnetization.

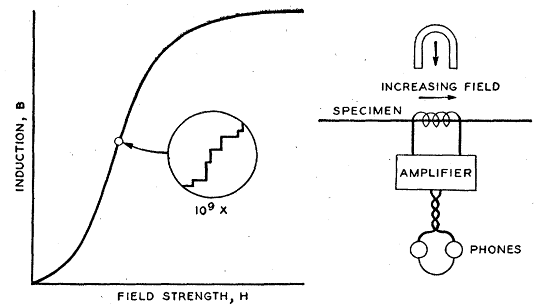


FIG. 33. Barkhausen effect shows that magnetization changes in discrete steps. Movement of magnet changes magnetization and causes noise to be heard in phones.

e.g., Permalloy containing about 80 percent nickel.

## Barkhausen Effect

In 1919 H. Barkhausen discovered the effect known by his name, and interpreted it as demonstrating that the magnetization of iron proceeds by steps and not in a continuous manner. This effect may be demonstrated simply as shown by the right side of Fig. 33, which is self-explanatory. As the field is changed slowly the succession of clicks heard in the phones, or in a loudspeaker, persists only when the magnetization is changing along the steep part of the magnetization curve or hysteresis loop. The clicks are identified with irreversible changes in magnetization (process (2) above) and are direct evidence of the existence of domains.

An oscillographic record of the Barkhausen effect is shown in Fig. 34. Here each impulse is caused by the sudden change of magnetization in some part of the specimen. The area under each impulse is a measure of the volume of the domain, the magnetization of which changed suddenly in direction at the time. The largest domains reverse on the steep portions of the magnetization curve and hysteresis loop, where the average volume of the domains is about $10^{-6}$ mm$^3$, equal to the volume of a cube 0.01 mm on an edge, and the farther one goes along the magnetization curve beyond the knee the smaller the impulses become. This agrees with the domain theory, according to which the domains in the upper portion change in direction slowly and reversibly and not by impulses. A domain size of the same order of magnitude has been derived from experiments on neutron scattering.

## Powder Patterns

For many decades iron filings have been used to portray the directions of lines of magnetic force in air and to detect flaws or inhomogeneities in magnetic materials. In 1931 it occurred to L. von Hàmos and P. A. Thiessen to use fine magnetic powder to detect the local inhomogeneities in magnetization that the domain theory predicts. Independently F. Bitter applied a suspension of siderac ($Fe_2O_3$), having particles about $10^{-4}$ cm in diameter, to a polished magnetized surface and observed under the microscope that the powder formed parallel lines regularly spaced about 0.1 mm apart and aligned approximately perpendicularly to the direction of magnetization.

The technique and interpretation of such patterns were then the subjects of study by a number of workers. A notable advance was made by W. L. McKeehan and W. C. Elmore who first observed a well-defined pattern on a demagnetized single crystal. Figure 35 shows such a pattern (middle) and also the patterns observed when the magnetization is directed into (left), or out of (right) the same portion of the surface. These striking pictures are probably the most direct evidence of the existence of magnetic domains. The suspension used for the experiments was a true colloid of $Fe_2O_3$ particles small enough to show Brownian movement, and a

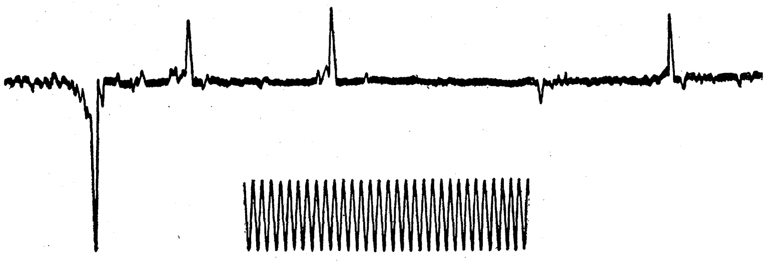


FIG. 34. Oscillographic records of impulses detected in experiments on Barkhausen effect. Each impulse corresponds to change of magnetism in a domain. Timing line has frequency of 60 cycles per second (R. M. Bozorth).

change in magnetization of the magnetic specimen was accompanied by a movement of the lines immediately visible to the eye.

Elmore simulated a magnetic crystal composed of domains by putting together small bars of permanent magnets, and found that the stray fields produced at the surface were similar to those observed on real crystals by the powder method.

Experiments on cobalt have been particularly instructive. Bitter observed two types of patterns on polycrystalline material and Elmore, working with single crystals, found the hexagonal lacelike patterns on surfaces perpendicular to the crystal axis and the straight line patterns on planes parallel to the axis (Fig. 36). These patterns are in accord with the magnetic properties of cobalt, known to have a direction of easy magnetization parallel to the crystal axis. The domains are then expected to be long in the direction of the axis and packed together like a bundle of needles. The boundaries of such domains are outlined by the concentrations of powder shown in the patterns. Moving pictures of the patterns of cobalt taken with slowly changing field show sudden displacements much larger than those usually attributed to the Barkhausen effect.

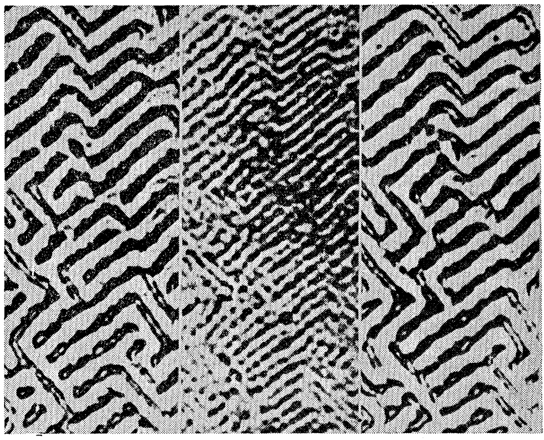


FIG. 35. Pattern of fine powder on surface of iron crystal is caused by domain structure beneath. In middle section magnetization of specimen is zero (McKeehan and Elmore). Magnification about 200 times.

### 6. STRESS AND MAGNETOSTRICTION

The magnetic properties of many ferromagnetic materials are sensitive to the application of stress, to such an extent that stress may be ranked with field strength and temperature as one of the primary factors affecting magnetic change. Stresses of two kinds should be distinguished: large stresses, that cause a permanent deformation of the material, almost invariably cause a decrease in permeability; stresses within the elastic limit may cause either an increase or a decrease in the permeability, depending on the nature of the material.

When a moderate tension is applied to a specimen of nickel, its permeability is decreased. The magnetization curve taken with tension applied always lies below that for the unstrained material (Fig. 37), and the curve is always lower the greater the tension. The effect of stress on the magnetization curve is greatest in the middle portion of the curve, and approaches zero when the specimen is either unmagnetized or saturated.

With some materials, on the other hand, the effect of tension is to increase the permeability. For example, in Permalloy containing 68 percent nickel and 32 percent iron the maximum permeability is higher by a factor of 8 when the tension is 2 kg/mm$^2$ and by a factor of about 30 when the tension is 11 kg/mm$^2$ (15,600 lb./in.$^2$), the elastic limit of this material. As the stress exceeds the elastic limit the permeability of Permalloy, and indeed that of any material, decreases rapidly.

The effect of stress on magnetization is closely related to the change of dimensions that occurs when a magnetic body is magnetized (magnetostriction); that is, the effect of strain on magnetization and the effect of magnetization on strain are interdependent. When nickel is magnetized it contracts a small fraction of its original length, at most 40 parts per million. The fractional change of length, $\Delta l/l$, with field strength, $H$, (Fig. 38) is first slow, then rapid, then again slow as it approaches a finite limit called "saturation magnetostriction," $(\Delta l/l)_s$. When 68 Permalloy is magnetized it increases in length as shown.

Nickel and 68 Permalloy are representative of materials with negative and positive magnetostriction, respectively. Tension increases the magnetization of a body having positive mag-

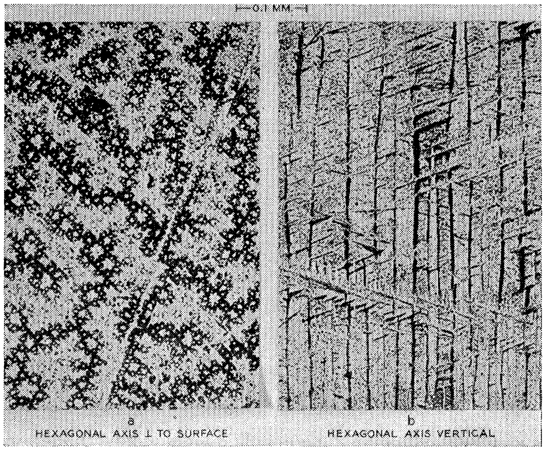


FIG. 36. Powder patterns of cobalt on surfaces perpendicular (a) and parallel (b) to hexagonal axis of crystal (H. J. Williams).

netostriction, decreases the magnetization if it has negative magnetostriction. The effect of compression is opposite to that of tension, consequently the magnetization of magnetostrictively positive materials is decreased by pressure. In referring to tension and compression it has been assumed that they are applied along a line parallel to the direction of the magnetic field; if pressure is applied in a direction at right angles to the field the effect on magnetization is opposite to the effect of compression applied parallel to the field, i.e., compression applied to nickel at right angles to the field will increase the magnetization measured parallel to the field.

Iron and some other materials have magnetostriction that is sometimes positive and sometimes negative. The magnetostriction of iron is positive in weak fields, negative in strong fields; and the effect of tension is, as expected, to increase magnetization in low fields, decrease it in high fields.

Generally the magnetostriction begins to increase rapidly with field strength at about the knee of the magnetization curve. The magnetization reaches to one-half to two-thirds of its saturation value while the magnetostriction is still only one-fourth to one-third of its final value, $(\Delta l/l)_s$. For example when nickel is subjected to a field of $H=3.6$, $B/B_s=0.64$ while $(\Delta l/l)/(\Delta l/l)_s=0.33$ only (Fig. 38).

## Iron-Nickel and Iron-Cobalt Alloys

Both the strain-sensitivity and magnetostriction of the iron-nickel alloys are of special interest. The saturation magnetostriction (the greatest change in length) goes through zero at about 81 percent nickel, a composition close to that of the Permalloys having the highest permeability, and McKeehan has pointed out the close theoretical connection between the low magnetostriction and high permeability of these alloys. The strain-sensitivity of the permalloys is also zero at about 81 percent nickel, as would be expected. The greatest changes in length of any materials are observed in iron-cobalt alloys having 40 percent to 60 percent cobalt—here the expansion is as large as 60 to $80\times10^{-6}$, about twice the change in nickel.

## Changes in Volume

When iron is magnetized in not too strong a field its length increases, and its cross-sectional area decreases in about the same proportion, so that the volume remains approximately constant. However, there is a small change in volume, usually an expansion, that can be detected with sensitive apparatus. When the field is about 2000 oersteds, the fractional change in volume in iron is about $10^{-6}$, and in nickel about one-third of this amount. It is especially large in the iron-nickel alloys containing about 30 percent nickel, and here amounts to about $40\times10^{-6}$ when $H=2000$.

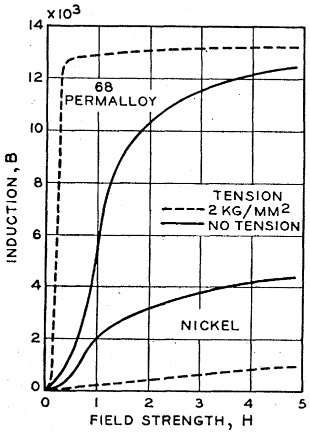


FIG. 37. Effect of tension on magnetization of 68 Permalloy and nickel is in opposite directions. Tension is held constant while field is increased from zero.

Closely connected with the change in volume with field strength is the change of magnetization with hydrostatic pressure. As indicated by the magnitude of the volume magnetostriction, this change is small.

### Domain Interpretation of Stress

According to the domain theory, when tension is applied to a material with positive magnetostriction, the magnetization of a single domain will tend to be aligned parallel to the direction of the tension. In an unmagnetized material with domains oriented initially at random the domains are rotated by a sufficiently large tension as shown in Fig. 39 (a) and (b), because the directions of easy magnetization in the various domains have been changed by the stress. If a field is then applied, half of the domains reverse in direction and the material is saturated. (No magnetostrictive change in length accompanies such domain reversals.) Compression causes the domains to orient transversely (Fig. 39d). If a material has negative magnetostriction, tension will effect the same result, and a field subsequently applied will rotate the domains toward saturation.

The domain theory has been successful in predicting the highest value of the initial permeability that can be attained when impurities have been reduced to a minimum and internal strains relieved as much as possible by annealing. The small strain then remaining is due to the magnetostriction of the separate domains and is proportional to the saturation magnetostriction; and the smaller the magnetostriction the larger the initial permeability according to the relation

$$\mu_o = \frac{4I_s^2}{(\Delta l/l)_s^2 E},$$

in which $E$ is Young's modulus of elasticity and $I_s$ the saturation magnetization. This equation, although by no means exact, states again the fact that $\mu_o$ is large when $(\Delta l/l)_s$ is small, as it is in the iron nickel alloys containing about 80 percent nickel; and the equation holds with fair accuracy for a series of iron-nickel alloys. It was derived by M. Kersten.

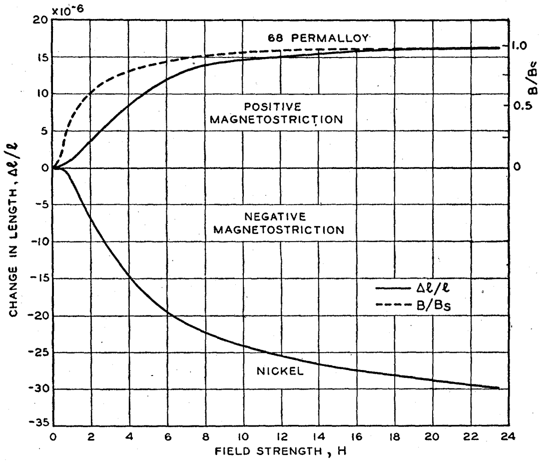


Fig. 38. Application of field causes increase in length of 68 Permalloy (positive magnetostriction), and a decrease in nickel (negative magnetostriction). Compare with reciprocal effect of Fig. 37.

## 7. PROPERTIES OF SINGLE CRYSTALS

Improvements in the technique of growing large metallic crystals have opened to investigation the magnetic properties of single crystals of iron, cobalt, nickel, and many alloys of these elements. Crystals of some metals such as nickel and nickel-iron alloys, are grown by slow freezing of a melt of pure material. This method is not adapted to iron, because of its phase transformation at about 900°C; single crystals of this element are grown by stretching a pure specimen a definite amount and then heating it for a long time just below the transformation temperature.

In the cubic crystals of iron and nickel the magnetic properties depend on the direction, with respect to the crystal axes, in which these properties are measured. Figure 40 shows the magnetization curves for iron and for nickel, taken with $B$ and $H$ measured in the three principal directions, namely: (1) parallel to an edge of the cube—the [100] direction; (2) parallel to a diagonal of a cube face—the [110] direction; and (3) parallel to the space diagonal—the [111] direction. The direction for which the magnetization curve lies highest is the direction in which magnetization is most easily acquired, and is called the direction of easy magnetization. In iron this direction is parallel to a cube edge; in nickel it is parallel to a cube diagonal.

According to the domain theory the direction of easy magnetization in a crystal determines the

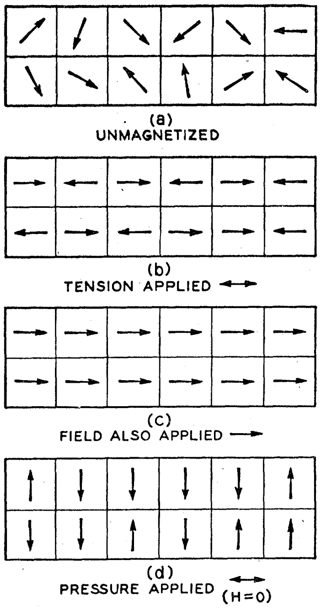


Fig. 39. Schematic domain interpretation of effect of stress and magnetic field applied to material with positive magnetostriction.

actual direction of magnetization in a single domain not subjected to magnetic field or strain, and in unmagnetized iron the domains are oriented in each of the six directions parallel to a cube edge so that the net magnetization is zero. In nickel there are eight such directions.

Magnetization curves of single crystals of a silicon-iron alloy (3.8 percent silicon) have a special interest because this is the first case in which measurements were extended to very low inductions, and because the high permeabilities obtained in these crystals are enormous compared with the permeabilities found in commercial silicon-iron sheet having the same silicon content. These data were made available by cutting a single crystal in the form of a hollow parallelogram so that all sides were parallel to equivalent directions in the crystal, e.g., parallel to diagonals of cube faces, and so forming a closed magnetic circuit and measuring in the usual way. The highest permeability observed is about 100 times that of commercial transformer sheet (Fig. 41).

## Calculation of Magnetization Curves

A theoretical study of the properties of crystals has made it possible to calculate the magnetization curve for any direction in a single cubic crystal like iron or nickel, provided a single constant of the crystal, the anisotropy constant, $K$, is known. This constant is a measure of the difference between the magnetization curves in different directions, and is numerically equal to four times the area between the $I$ *vs.* $H$ curves for the [100] and [110] directions. The constant is positive for iron and negative for nickel, and is approximately zero for iron-nickel alloys containing 60 percent to 70 percent nickel.

Comparison of theory and experiment is made in Fig. 42 using H. J. Williams' data for a single crystal containing 3.8 percent silicon, for which $K = 280{,}000$ ergs/cm$^3$. The theory does not take account of the energy associated with the lower and middle portions of the magnetization curve; therefore, the calculated curves do not correspond to the true magnetization curve below its knee. In a polycrystalline material the magnetization curve is a combination of the separate curves for the many small single crystals of which it is composed. The calculation of such a composite curve has been made to show in what manner the magnetization curve approaches saturation, and fair agreement with observation has been found.

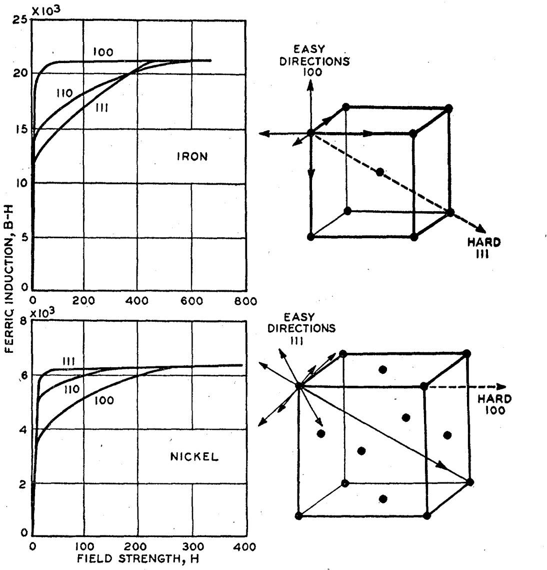


Fig. 40. Magnetic properties and crystal structures of single crystals of iron and nickel. The groups of arrows at the corners of the cubes—6 for iron and 8 for nickel—indicate the directions of easy magnetization (K. Honda and S. Kaya, W. L. Webster).

In iron and nickel the anisotropy constants decrease as the temperature increases, so that these metals are isotropic over a considerable range of temperature before they become nonmagnetic at the Curie point. At very low temperature, 20°K, the constant for nickel is 10 to 15 times what it is at room temperature, while for iron the corresponding ratio is only 1.5.

### Anisotropy of Iron-Nickel Alloys

Unusual magnetic properties are found in alloys having small values of the anisotropy constant. Such alloys are the Permalloys having nickel contents between 60 percent and 80 percent nickel, in which range of composition the constant goes from positive to negative (Fig. 43). When the anisotropy is small, and the orientation of the domain therefore controlled only weakly by the crystal structure, the material responds easily to either magnetic field or to mechanical stress and therefore saturates in relatively low fields and is unusually strain sensitive.

The small anisotropy of the Permalloys containing 60 percent to 80 percent nickel is also responsible for the fact that these alloys show unusual properties when heat treated in a magnetic field. A hysteresis loop of material so treated, and the high values of maximum permeability associated with the Permalloys (Fig. 12), have already been mentioned.

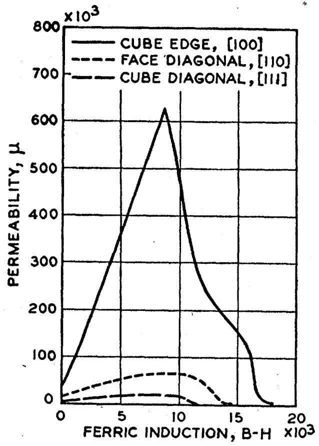


Fig. 41. Magnetic properties of single crystals of iron-silicon alloy (3.8 percent Si), measured in different directions in the crystal (H. J. Williams).

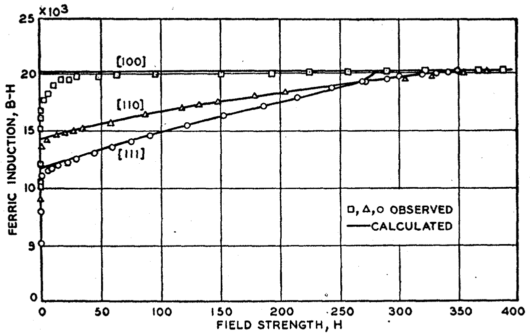


Fig. 42. Comparison of theory and experiment for magnetization of iron-silicon crystal in three principal directions (H. J. Williams).

### Anisotropy of Cobalt

Cobalt differs from the other ferromagnetic elements, iron and nickel, by having a hexagonal crystal structure instead of a cubic one. At room temperature the direction of easy magnetization is parallel to the hexagonal axis of the crystal, and when unmagnetized the domains therefore lie in one of the two orientations parallel to this axis. This small number of easy directions, compared with the six or eight in a cubic crystal, means that the domains are needle-like and that the whole magnetic structure may be regarded as a bundle of needles magnetized initially in either direction at random. This picture is supported by powder patterns (see above) and by the measurements made by L. H. Germer of magnetic fields close to the surfaces of a single crystal of cobalt.

The crystal anisotropy of cobalt is large, consequently a strong field is necessary to saturate a crystal in the direction of most difficult magnetization. More than 10,000 oersteds are required for this purpose at room temperature (Fig. 44), compared with 600 oersteds for iron and 300 for nickel.

At higher temperatures cobalt becomes isotropic, and above 300°C the hexagonal axis is the direction of most difficult magnetization, all directions at right angles to the axis being equally easy. Magnetization curves for the principal directions at room temperature and at 390°C are shown in Fig. 44.

### Magnetostriction in Single Crystals

The magnetostrictive change in length of a single crystal is not the same in all crystallo-

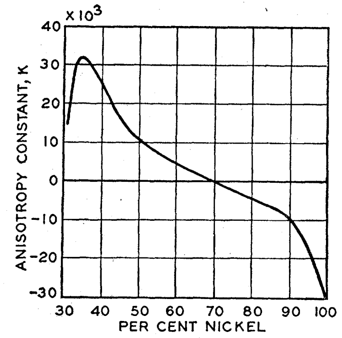


FIG. 43. Dependence of crystal anisotropy of iron-nickel alloys on composition. Note that anisotropy constant is zero near 70 percent nickel (H. J. Williams and R. M. Bozorth).

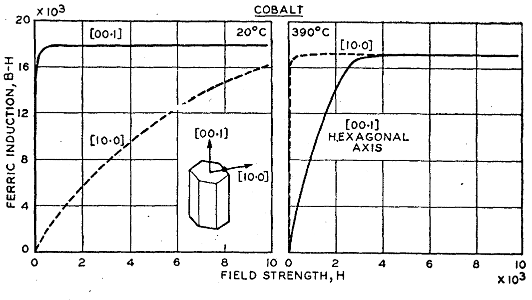


FIG. 44. Crystal anisotropy of cobalt reverses sign at about 270°C, direction of easy magnetization changing from hexagonal axis (00·1) by 90° to (10·0) (K. Honda and H. Masumoto).

graphic directions. Especially is this true in iron which expands when the field is applied parallel to a crystal axis, [100], contracts when it is applied parallel to a cube diagonal, [111] (Fig. 45). In the direction of the face diagonal, [110], the crystal expands in low fields and contracts in high fields. Nickel contracts in all field strengths in all directions.

## 8. TEMPERATURE AND THE CURIE POINT

It has been known for many years that when iron is heated to a high temperature it loses its ferromagnetism and is no longer strongly attracted by a magnet. The temperature at which any ferromagnetic material loses its magnetism is known as the Curie point; it is 770°C for iron and 358°C for nickel. The highest known Curie point is 1120°C, for cobalt. For some materials the Curie point lies near the absolute zero of temperature.

When a magnetic material is subjected to a high constant field, an increase in temperature brings about a continuously accelerating decrease in induction. The induction comes down abruptly, almost to zero, at the Curie point (Fig. 46), and the curve is retraced when the temperature is lowered again. On the other hand when the iron is subjected to a weak field the induction will first increase with increase in temperature and after passing through a maximum will drop to a low value at the Curie point as before.

The way in which other magnetic quantities change with temperature is indicated in Fig. 47 for iron. In any material which may be called normal the curves are likely to have this same general character; e.g., the initial and maximum permeabilities first increase and then decrease with increasing temperature, and the coercive force and hysteresis loss continually decrease. The characteristic maximum in the initial and maximum permeabilities, just below the Curie point, is associated with the low magnetic anisotropy at this temperature. The way in which the size and shape of the hysteresis loop of iron change as the temperature approaches the Curie point is shown in Fig. 48.

### Effect of Phase Changes

Iron and nickel are normal as to their magnetic behavior with change of temperature. No changes

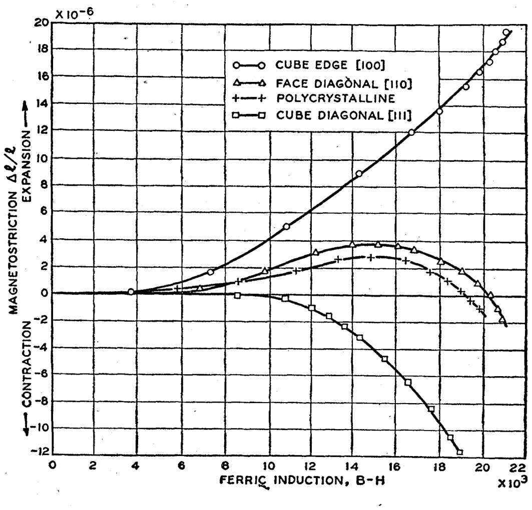


FIG. 45. Dependence of magnetostrictive change in length on crystal direction in iron (W. L. Webster).

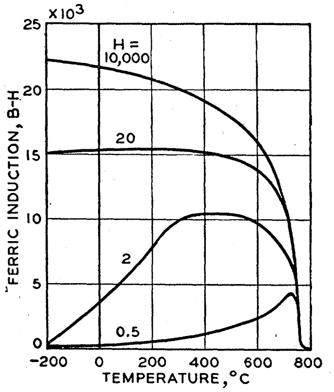


FIG. 46. Effect of temperature on magnetization of iron subjected to fields of various strength. Magnetization drops to low value at Curie temperature, 770°C (E. M. Terry).

in their crystal structures occur between room temperature and the Curie point. On the other hand such changes in structure do occur in many alloys, among which the so-called irreversible alloys of iron and nickel (5 percent to 30 percent nickel) investigated by J. Hopkinson, and some of the iron-cobalt alloys may be considered as examples. In the 60 percent cobalt-iron alloy a change in phase, due to a rearrangement of the atoms in the crystal, from a body-centered cubic ($\alpha$) to a face-centered cubic ($\gamma$) form, occurs when the alloy is heated above 980°C. The $\alpha$-phase is ferromagnetic, the $\gamma$-phase is non-magnetic (i.e., paramagnetic), consequently the material loses its ferromagnetism when heated through the temperature of change of phase, 980°C, and the magnetization (measured in a high field) drops toward zero precipitously (Fig. 49) instead of in the normal fashion as it does for iron (Fig. 46).

## 9. EFFECT OF MAGNETIZATION ON OTHER PROPERTIES

Preceding paragraphs have described the magnetostriction that occurs during magnetization. Magnetization also effects changes in many other physical properties; and a ferromagnetic material, even though unmagnetized, possesses certain physical properties that are characteristic of its ferromagnetism. The more important of these properties are the following: (1) electrical resistance; (2) thermal expansion; (3) Young's modulus and other elastic constants; (4) magnetocaloric effect; (5) specific heat. The resistivity of magnetic materials is affected also by stress—tension may either increase or decrease the resistivity, depending on whether the material has positive or negative magnetostriction.

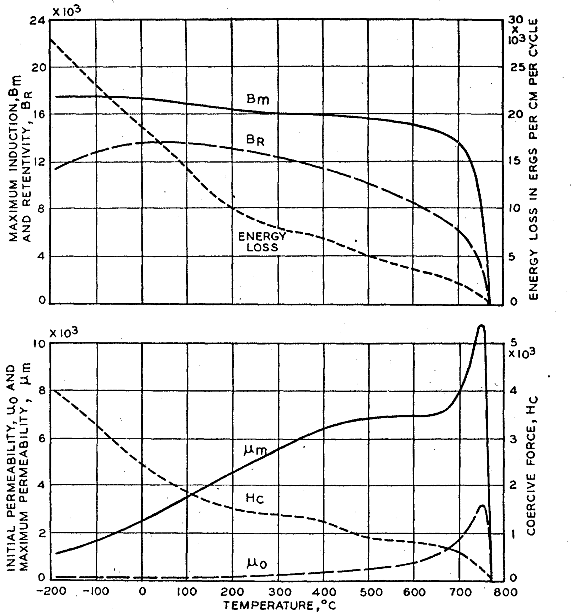


FIG. 47. Variation of some magnetic properties with temperature. Material is electrolytic iron annealed at 800°C (E. M. Terry, 1910).

### Magnetism and Resistivity

When nickel is magnetized in a high field, its electrical resistivity is increased by about 2 percent and a similar but smaller effect is found in iron. Such changes were first reported by W. Thomson (Lord Kelvin) in 1851. A larger change in resistivity, as high as 4 percent to 5 percent, is shown by the iron-nickel alloys having 70 percent to 85 percent nickel. The increase in resistivity with field strength for the 84 percent Permalloy (Fig. 50) illustrates the general rule that most of the change occurs when the magnetization is well beyond the knee of the magnetization curve, and suggests that the change in resistivity like the magnetostrictive change in length, is associated with domain rotations and not with domain reversals. There is a close relation between changes in length and changes in resistivity, and these quantities usually have a constant ratio.

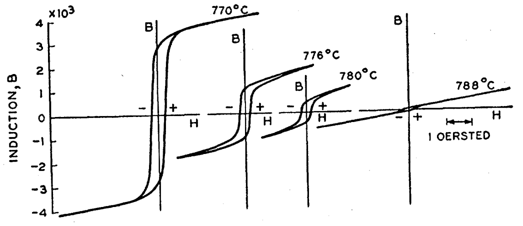


Fig. 48. Change in form of hysteresis loops of iron near the Curie point (H. Kühlewein).

The resistivity of a ferromagnetic material is abnormally low by virtue of the fact that it is ferromagnetic. This may be illustrated by comparison of the resistivities of nickel and palladium, elements occupying similar positions in the periodic table. The resistivity-temperature curves (Fig. 51) are drawn so that they coincide at a temperature just above the Curie point of nickel, when both metals are non-magnetic, and the separation of the curves below this temperature indicates the lowering of the resistivity due to the ferromagnetism. The ferromagnetism becomes progressively weaker as the Curie point is approached, and the separation of the curves in the figure correspondingly less.

## Magnetism and Thermal Expansion

The thermal expansion of magnetic materials is abnormal to such an extent that occasionally such a material will contract instead of expand as the temperature is raised in the vicinity of the Curie point. In some alloys, such as invar, the expansion is very small over a range of temperatures that includes room temperature, and this property is of great technical importance.

The origin of the abnormal expansion lies in the change in the forces between atoms that occurs when the material becomes ferromagnetic. The iron nickel alloys show these changes in a striking way (Fig. 52). As nickel and the alloys containing more than 70 percent nickel cool through the Curie point, the expansion coefficients first increase and then decrease rapidly. In contrary fashion, the coefficients of alloys with 30 percent to 70 percent nickel first decrease sharply and then increase. These changes may be regarded as the result of the superposition of a "magnetic" expansion or contraction on the normal; in invar, containing about 36 percent nickel, the anomalous and normal expansion neutralize each other at room temperature. When the nickel content is about 70 percent, there is no anomalous magnetic expansion or contraction at any temperature; this is associated with the fact that at this composition the Curie point of these iron-nickel alloys is a maximum and independent of composition.

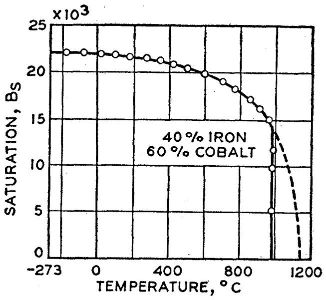


Fig. 49. Curie point determined by change in phase, at 980°C, of iron-cobalt alloy (60 percent cobalt) (A. Preuss).

## Abnormal Stress-Strain Relation

In almost all substances a small mechanical stress, $\sigma$, (tension) will produce a strain, $\epsilon$, (expansion) that is proportional to the stress, as required by Hooke's law

$$\epsilon = \sigma / E,$$

in which Young's modulus, $E$, is constant. In

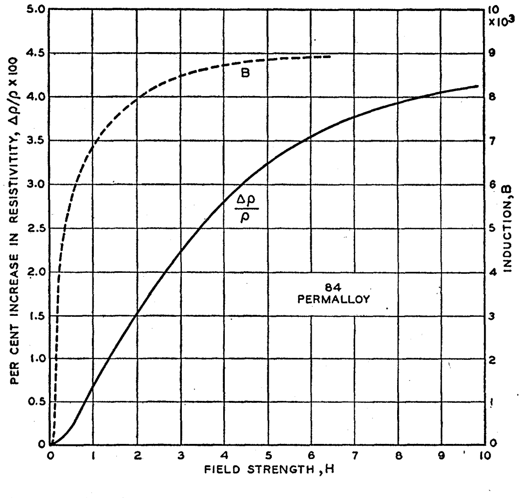


Fig. 50. Resistivity ($\rho$) of magnetic material (Permalloy) increases slightly with magnetization. Most of increase occurs after magnetization is well beyond knee of magnetization curve (L. W. McKeehan).

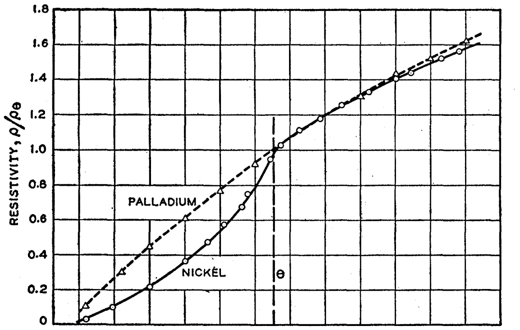


FIG. 51. Resistivity of magnetic material is abnormally low, as long as it is ferromagnetic (H. H. Potter and J. G. G. Conybeare).

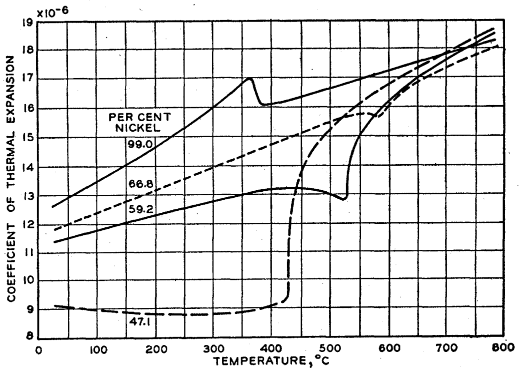


FIG. 52. Thermal expansion curves of magnetic material show anomalies near and below Curie point (P. Chevenard).

an unmagnetized magnetic material, however, a tension will change the direction of local magnetization (Fig. 39), and with such changes is bound a change in length equal to the magnetostriction of the material. Therefore when tension is applied to a magnetic material, the whole expansion will be the sum of the normal expansion and the magnetostrictive expansion, consequently Young's modulus will vary with stress and deviations from Hooke's law occur.

In a magnetic material Young's modulus will therefore vary both with the amount of strain and with the intensity of magnetization. The change of $E$ with magnetization has been studied in a variety of materials and may be 10 percent or larger.

### Specific Heat and Magnetocaloric Effect

In comparing the specific heats of nickel and copper at various temperatures (Fig. 53) it is apparent that there is an unusual effect in nickel in the neighborhood of the Curie point. Such a rapid rise and still more rapid drop in the curve, occurring with rising temperature, is characteristic of a ferromagnetic material and is accounted for qualitatively and to a considerable extent quantitatively by theory. The heat necessary to raise the temperature of a magnetic material like nickel is equal to that necessary for a similar but non-magnetic material (e.g., copper), plus the heat necessary to overcome the mutual interatomic forces that hold the magnetic moments of neighboring atoms parallel.

This characteristic specific heat of ferromagnetic bodies occurs whether or not the substance under examination is magnetized in the usual sense of the word. The existence of this phenomenon is strong evidence in favor of the existence of spontaneous magnetization in certain regions or domains in ferromagnetic materials.

Similar evidence for spontaneous magnetization is supplied by the magnetocaloric effect. According to theory, at temperatures near the Curie point high fields should increase the spontaneous magnetization, and with this increase there should be a rise in temperature of the material. Experimentally, P. Weiss and R. Forrer observed increases in temperature of more than 1°C when a field of 18,000 oersteds was applied to nickel just below its Curie point. The change of spontaneous magnetization with temperature, derived from the data, is in good accord with the theory.

## 10. MAGNETIZATION IN ALTERNATING FIELDS

In many of their uses magnetic materials are subjected to an alternating field, produced by an alternating current in the wire associated with the material. The effects of the frequency of alternation on the magnetic properties of the material are primarily of three kinds: (1) the effective permeability of the material is reduced; (2) the energy loss in the material is increased; and (3) there is a time lag between the field strength and the corresponding induction. There also is some evidence that (4) at ultra-high frequencies the true permeability is less than it is at low frequencies.

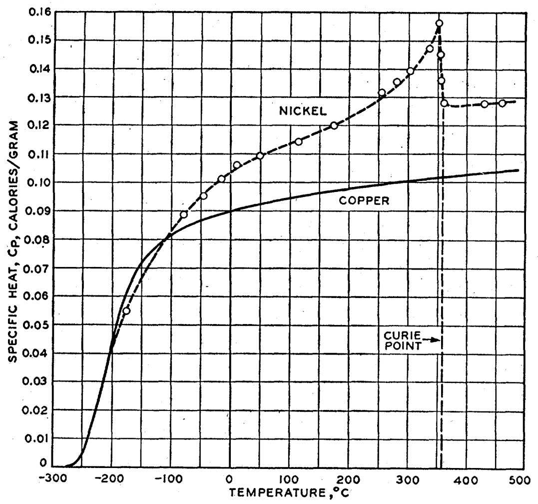


FIG. 53. Specific heat of a magnetic substance is unusually high as long as it is ferromagnetic. Curves for nickel (E. Lapp) and copper (H. Klinkhart) are compared.

### Effect of Eddy Currents

When the magnetic flux in a conducting medium changes with time, an electromotive force is generated, and there is a resulting flow of currents within the material. These Foucault currents, or eddy currents, depend on the geometry of the material specimen and on its resistivity and permeability; their direction is always such as to counteract the change in field that produced them (Fig. 54). The effect of eddy currents is to prevent the field from penetrating immediately to the interior of the material, and when the applied field is varying continually, as it is in much of the apparatus in which magnetic material is used, the field strength in the interior of the material may never be more than a small fraction of the field strength at the surface. The magnetic induction, therefore, decreases from the surface toward the interior.

To avoid this reduction in field strength, and resultant decrease in alternating flux, the magnetic cores of transformers and other apparatus are laminated, each lamination being insulated electrically from its neighbor. The flux-carrying capacity of such a lamination depends on the thickness ($t$ in cm), permeability ($\mu$), and resistivity ($\rho$ in ohm-cm$\times 10^9$) of the material and on the frequency ($f$ in cycles/sec.) of alternation of the magnetic field. It can be calculated accurately provided the permeability does not vary with field strength, and can be expressed in terms of the effective permeability, $\bar{\mu}$, defined as the ratio of the amplitude of the induction, $\bar{B}$, (averaged over the section of the lamination) to the amplitude of the applied field strength, $H_a$. When $H_a$ varies sinusoidally with time, $\bar{\mu}/\mu$ varies with frequency is shown graphically in Fig. 55.

In laminations as commonly used in power transformers, the reduction in permeability is only a fraction of 1 percent, and therefore negligible. When the frequency is high, $\bar{\mu}/\mu$ is inversely proportional to $f^{\frac{1}{2}}$, or

$$\bar{\mu}=\frac{(\mu\rho)^{\frac{1}{2}}}{2\pi t}\cdot\frac{1}{f^{\frac{1}{2}}}.$$

In a transformer working at radiofrequencies the constants of the materials used may be such that 95 percent of the true permeability is lost. At these high frequencies the flux carrying power or value of a material of given resistivity and thickness is directly proportional to the square root of its permeability.

### Energy Losses in Alternating Fields

Eddy currents in a magnetic material consume power and raise the temperature of the material. The power loss, $W_e$, may be calculated with some precision provided $\mu$ does not vary with $B$. At low frequencies it is proportional to $f^2$. At high frequencies it is proportional to $f^{\frac{3}{2}}$, if the material is always magnetized to the same value of $\bar{B}$:

$$W_e=\frac{\pi t\bar{B}^2}{(\mu\rho)^{\frac{1}{2}}}\cdot f^{\frac{3}{2}}.$$

If the amplitude of the magnetizing current is held constant, the loss is proportional to $f^{\frac{1}{2}}$. The hysteresis loss of a magnetic material at intermediate and high inductions is given approxi-

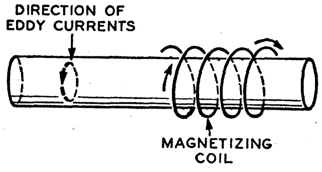


FIG. 54. Direction of eddy currents in solid bar is opposite to that of applied magnetizing current giving rise to them.

mately by Steinmetz' relation (see above).

$$W_h=\eta B^{1\cdot 6}f.$$

The total loss due to hysteresis and eddy currents at low frequencies may then be written

$$W=W_h+W_e=\eta B^{1\cdot 6}f+AB^2f^2,$$

where $A$ can be calculated theoretically under certain conditions, as shown above. This relation is sometimes used for the separation of the total loss (as measured with a wattmeter at a given value of $B$) into its hysteresis and eddy components.

When apparatus operates at low inductions, as it often does for transmission of the feeble signals used in communication, the energy losses incurred are usually measured with a Maxwell bridge, as described below under MEASUREMENT OF MAGNETIC QUANTITIES. The data obtained are the a.c. resistance, $R$, in ohms, and the inductance, $L$, in henries, of a coil surrounding the material under test. The ratio $R/L$ is characteristic of the magnetic material and is related to the energy losses in ergs/cm³ by the equation

$$\frac{R}{2\pi fL}=\frac{8\pi\bar{\mu}W}{B^2}.$$

The reciprocal of $R/(2\pi fL)$ is often called the quality factor, $Q$. When the frequency is low, the losses due to eddy currents and hysteresis may now be separated by plotting $R/(2\pi fL)$ against frequency. $W_h$ varies as $B^3f$ (see "Hysteresis Loops" above) and $W_e$ as $B^2f^2$, consequently

$$\frac{R}{2\pi fL}=h_c+hB+ef,$$

a relation in which $h$ and $e$ are called the hysteresis and eddy current constants, respectively, and $h_c$ is a constant representing an energy loss of unknown origin, important only at very low inductions.

At high frequencies and low inductions the losses in a material of high permeability are usually due predominantly to eddy currents. These can be calculated as described above and for them $R/(2\pi fL)$, or $Q$, is shown by the curve of Fig. 55.

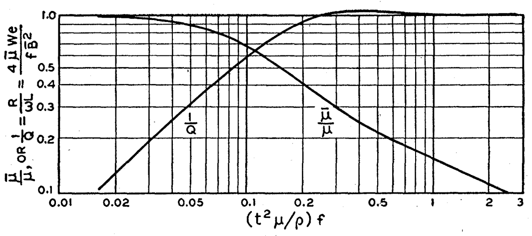


FIG. 55. Theoretical change of apparent permeability $\bar{\mu}$ and energy loss $W_e$ due to eddy currents, with frequency, $f$, for sheets of given thickness $t$ (cm), permeability $\mu$, and resistivity $\rho$ (ohm-cm$\times 10^9$).

### Permeability at Very High Frequencies

In considering the effect of frequency on permeability it has been assumed that the true permeability, $\mu$, is independent of frequency and that only the apparent permeability, $\bar{\mu}$, diminishes as the frequency increases. There is some reason to believe, however, that at ultra-high frequencies the magnetic moment of a domain is not capable of changing with the required rapidity. R. Becker has calculated the order of magnitude of the frequency at which the magnetization should fail to respond. By considering the magnetic energy required to move the surface separating one domain from its neighbor, assuming a reasonable value of the thickness of a domain, he estimates that the permeability will begin to decrease seriously when the frequency is about $2\times 10^9$ cycles/sec., corresponding to a wave-length in air of 15 cm. The existence of decrease in permeability at wave-lengths under 50 cm is indicated by the work of K. F. Lindmann.[4]

### Time Lag in Magnetization

As already mentioned, eddy currents cause a delay in magnetization so that after a sudden change in $H$ from one value to another, a perceptible time elapses before $B$ acquires its final corresponding value. There are other reasons, too, why $B$ continues to change with time after $H$ has become constant. One well-known cause of such change is metallurgical aging. This is usually observed in permanent magnets, the induction in which can be observed to change over a period of years, and this change is attributed to the slow precipitation of metallic

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[4] K. F. Lindmann, Zeits. f. tech. Physik. **19**, 158 (1938).

phases from supersaturated solution at room temperature. Similar changes occur also in ordinary annealed iron to such an extent that the coercive force may eventually be doubled even though the material has not been heated above 100°C. Materials that have been severely cold worked sometimes become magnetically softer on account of the slow relief of the internal strains caused by the working.

A more rapid change of induction with time has been observed repeatedly since Ewing's experiments of 1885. One method of experimenting is to change $H$ suddenly, then observe the change in $B$ as dependent on time. Observable changes may last as long as 15 or 20 minutes, or as short a time as $\frac{1}{10}$ second, depending on the nature of the specimen and the temperature. In some materials the only lag observed can be accounted for by eddy currents. A second way of observing a change of magnetic state with time is to measure the permeability at very low inductions, at various times after demagnetizing and after the measuring current has been applied. It is particularly noticeable that in commercial silicon iron, subjected to a small alternating field of fixed amplitude, the permeability will continue to decrease measurably with time for more than a day. Such changes are associated in some way with the impurities present, for they disappear when these are removed by heat treatment at a high temperature in hydrogen. It has been suggested that the diffusion of small traces of nitrogen or carbon in iron will affect the movement of domain boundaries in such a way as to account for the observed changes of permeability with time.

### 11. DIAMAGNETISM

Faraday showed in 1845 that all substances may be classified as diamagnetic and paramagnetic. He distinguished between the paramagnetism and diamagnetism of feebly magnetic substances by suspending them in a strong inhomogeneous field and noting whether they were drawn into or repelled from the strongest part of the field. The force with which a diamagnetic material is repelled by a field is proportional to the strength $H$ and gradient $dH/dx$ of the field and to the susceptibility $\kappa$ and volume, $v$, of the material:

$$f=\kappa vHdH/dx.$$

The susceptibilities of solid diamagnetic substances are usually about $-1\times10^{-6}$ or $-2\times10^{-6}$, and are normally independent of field strength. Bismuth has the exceptionally large value of $-13\times10^{-6}$, but even in this substance the numerical value of the susceptibility is millions of times less than that of the ferromagnetic metals. Bismuth is also exceptional in that its susceptibility varies somewhat with the field at low temperatures (50°K).

While diamagnetic and paramagnetic materials are not often of technical use, as such, a study of their behavior is very important for our scientific knowledge of the nature of matter. Diamagnetism is an atomic property, and usually occurs when the atom has a symmetrical electronic structure and no permanent magnetic moment. In discussing diamagnetism it is appropriate to use the term atomic susceptibility defined as $\chi_A=\kappa A/d$, $A$ being the atomic weight and $d$ the density. Then $\chi_A H$ is the magnetic moment of one gram atomic weight, and when divided by Avogadro's number is the average magnetic moment per atom resolved parallel to the magnetic field. Similarly $\chi_M=\kappa M/d$ is the molecular susceptibility, $M$ being the molecular weight.

As P. Curie showed in his classical researches (1895), diamagnetism is usually independent of temperature, and so is not disturbed markedly by atomic collisions or the positions of nearby atoms. However, some change in the diamagnetic susceptibility is generally noted when a substance changes from the solid to the liquid state.

Following the work of Curie, K. Honda and M. Owen in 1910 and 1912 reported measurements of the susceptibilities of many elements. Figure 56 shows the atomic susceptibilities as dependent on atomic number according to more recent data summarized by P. W. Selwood. Here it is apparent that the rare earth metals are strongly paramagnetic, that the metals of the "transition groups" containing palladium and platinum are rather strongly paramagnetic, and that most good metallic conductors of electricity are weakly paramagnetic. This implies what is in fact the case, that in a metal the conducting electrons are paramagnetic. The other metals, and the non-metals, are weakly paramagnetic or diamagnetic, except that antimony and bismuth have moderately strong diamagnetism.

Most ionic and molecular compounds are also diamagnetic because their electrons are paired and their permanent moments tend to cancel each other; however, gaseous oxygen, $O_2$, and nitric oxide, NO, are paramagnetic. Figure 57 shows the atomic or molecular susceptibilities of some typical materials and the way in which they change with temperature.

## Theory

Our understanding of the origin of diamagnetism is due largely to P. Langevin who published his celebrated paper on magnetism and electron theory in 1905. He considered an atom with a single electron of charge, $e$, and mass, $m$, traveling with velocity, $v$, in a circular orbit of radius, $r$ (Fig. 58). This "current" gives rise to a magnetic moment, $M$, proportional to the current and the area of the orbit:

$$M=\frac{ve/c}{2\pi r}\cdot\pi r^2=\frac{ver}{2c}.$$

As pointed out by Sir Joseph Larmor, application of a magnetic field $H$ will create an e.m.f. in the orbit, as a result of the flux threading it, and this will cause the electron to change its velocity by an amount

$$\Delta v=-\frac{eHr}{2mc}$$

in such a direction as to change the moment, $M$, by the amount

$$\Delta M=\frac{er\Delta v}{2c}=\frac{e^2Hr^2}{4mc^2}.$$

The susceptibility of this "atom" is therefore $-\Delta M/H$, and the susceptibility of one gram atomic weight is

$$\chi_A=-\frac{Ne^2r^2}{4mc^2}.$$

If an atom contains many electron orbits, oriented in all directions with respect to the field, the expression for atomic susceptibility becomes

$$\chi_A=-\frac{Ne^2\Sigma\langle r^2\rangle_{\text{Av}}}{6mc^2}=-2.83\times10^{10}\,\Sigma\langle r^2\rangle_{\text{Av}}$$

in which the mean square orbital radius is summed over all the orbits in the atom.

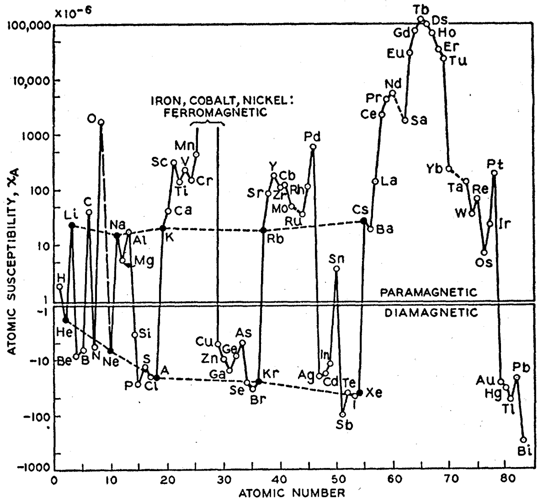

FIG. 56. Atomic susceptibilities of the elements at room temperature. Dotted lines connect alkali metals (paramagnetic) and rare gases (diamagnetic).

The equation above, known as Langevin's equation of diamagnetism, is applicable to every substance, whether it be diamagnetic or paramagnetic. If an atom has a permanent moment and the material is consequently paramagnetic, it is still true that its susceptibility has the diamagnetic component given by this equation.

Knowing the number of electrons in the atom, one may calculate from the diamagnetic susceptibility the average (r.m.s.) radius of an atom and compare this with the radius determined by other means. Values so obtained are consistent with our other knowledge of atomic structure.

In quantum mechanics the Langevin formula still holds when the field of the nucleus is spherically symmetrical, as it is in atoms or ions. The meaning of the orbital radius, $r$, however, is somewhat different, since the electrons are assumed to be distributed in less localized orbits. Calculation of diamagnetic susceptibilities by wave mechanics has been attempted for a number of the simple atoms and molecules and has been moderately successful for the rare

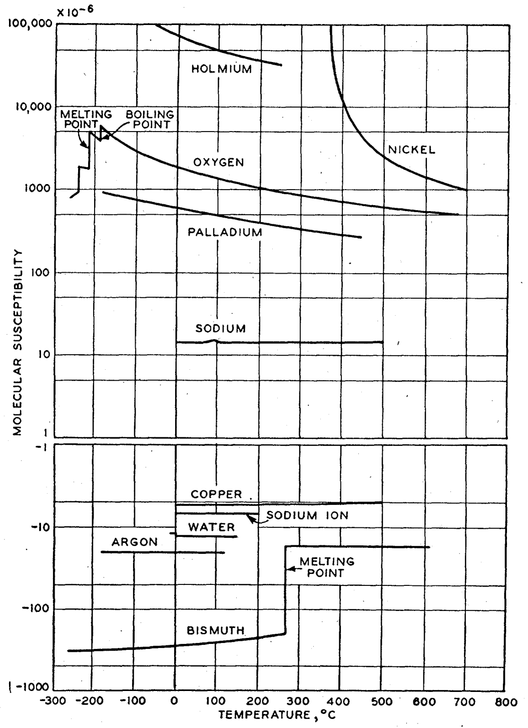

FIG. 57. Variation of atomic or molecular susceptibility with temperature, for some representative materials. Susceptibilities of diamagnetic and weakly paramagnetic substances are normally independent of temperature, those of strongly paramagnetic substances decrease with increasing temperature.

monotomic gases He, Ne, A, Kr, and Xe, and molecular hydrogen, $H_2$. For helium, theory indicates an atomic susceptibility ($\times 10^6$) of 1.65 to 1.90, depending on the method of calculation; G. G. Havens' measured value is 1.91. For hydrogen the calculated and observed values are 2.10 and 2.01. For the other elements listed both theoretical and measured susceptibilities vary over a greater range.

Positive ions of the alkali and alkaline earth metals, and of the halogens, have the rare gas structure and can also be treated theoretically. The lack of precision in the calculation arises in the approximate nature of wave functions and the effective molecular charge assumed.

## Simple Salts and Their Solutions

Most salts are diamagnetic because the ions of which they are composed (e.g., $Na^+$ and $Cl^-$) have the completed electron shells characteristic of the rare gases and consequently have no permanent magnetic moment. Measurements are made either on solid salts or on solutions, and the susceptibilities of the separate ions can be deduced from a series of such measurements. There is some ambiguity in separating the susceptibility of the salt into its component parts, and there is also some change with concentration and state of aggregation, but each ion has a value of $\chi_A$ that is approximately constant. Values of the separate ions, deduced by Miss V. C. G. Trew and others, are plotted in Fig. 59 against the number of extra-nuclear electrons in the ion. The increase of $\chi_A$ with the number of electrons, $n$, is to be expected according to simple theory; the relative large increase between $n=10$ and $n=18$ is shown also by ionic radii deduced quite independently from investigations of crystal structure and of the refraction of light. In ions having the same number of electrons but a different nuclear charge, as in the series $I^-$, Xe, $Cs^+$, and $Ba^{++}$, the diamagnetic susceptibility decreases numerically with increasing charge because the larger charge draws the electrons closer to the nucleus and so diminishes the value of $\Sigma\langle r^2\rangle_{Av}$ of Langevin's equation.

The principle of additivity of the diamagnetism of ions may be extended to include also the more complex ions $NO_3^-$, $SO_4^{--}$, $PO_4^{---}$, $NH_4^+$ and others. Values for these are less certain than those for the simple ions already considered.

## Organic Compounds

In his early comprehensive experiments (1908–13) P. Pascal found that the molecular susceptibility of many organic compounds is the sum of the susceptibilities of the atoms composing them, plus additional terms characteristic of the vari-

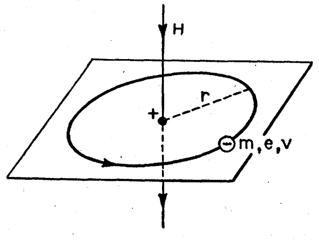

FIG. 58. Velocity, $v$, of electron is changed by application of field $H$, so that induced magnetic moment is in direction opposite to that of field.

ous bonds occurring in the molecule. The atomic susceptibilities and constants of some elements and bonds are given in Table IV.

The method of using these numbers may be illustrated for ethyl bromide, $C_2H_5Br$. The calculated value of $-\chi_M \times 10^6$ is $(2\times 6.00)+(5\times 2.93)+30.6-4.1=53.1$, and is close to the observed value of 53.3. Pascal's constants have been extended to many more complicated organic structures and groups of atoms. His work was also the basis of the additivity relations of inorganic ionic compounds discussed in previous paragraphs.

### Diatomic Molecules

While the Langevin theory is not very illuminating when applied to a simple diatomic molecule like $H_2$, quantum mechanics can be called upon provided the wave functions are known with sufficient accuracy. In the expression for $\chi_M$ there is a term, always positive, added to the negative term of the Langevin equation.

J. H. Van Vleck and N. H. Frank have used the wave functions of molecular hydrogen to calculate both terms of the expression, and found them to be $-4.66\times 10^{-6}$ and $+0.51\times 10^{-6}$. The net result of $-4.15\times 10^{-6}$ may be compared favorably with the experimental values ranging from about $-3.9$ to $-4.0\times 10^{-6}$.

Calculation of heavier diatomic molecules cannot be carried out so rigorously, and the approximate wave functions generally used for these substances give too high a value of $\chi_M$. Our knowledge of gases is also limited by the fact that measurements are difficult to make with accuracy.

### Susceptibility of Water

The magnetic properties of water are interesting for three reasons: (1) it is often used for calibration of apparatus; (2) its susceptibility must be subtracted from that of a solution to obtain the value for the solute; and (3) the variation of its susceptibility with temperature has been measured accurately and this gives us a conception of the amount and nature of the temperature variation of susceptibility of diamagnetic substances generally. The value of $\chi_M\times 10^6$ is 0.7218 at 20°C, and increases about 0.013 percent per °C. The opinion of various investigators is that this change is caused by depolymerization of the water with increase in temperature. Experience with water supports the general conclusion that diamagnetism is independent of temperature except for the slight change in the effect of surrounding molecules on the electron distribution of the atom or molecule under consideration. Such changes occur, of course, during melting and vaporization (Fig. 57) and when molecules combine to form groups, as in polymerization.

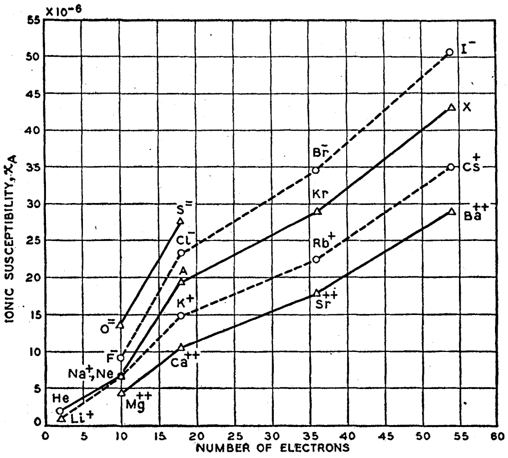

FIG. 59. Ionic susceptibilities of similar structures increase with number of electrons in ion.

### 12. PARAMAGNETISM

The paramagnetism of the materials examined by Curie was dependent on temperature, the susceptibility decreasing with increasing temperature. After that time it was found that there was a large class of paramagnetic substances for which the susceptibility was practically independent of temperature (Fig. 57); this class includes many of the metallic elements when, and only when, they are in the solid conducting state, and the paramagnetism of these materials is caused by the conduction electrons. This kind of paramagnetism is referred to as weak paramagnetism, whereas the former class has strong paramagnetism and is caused by a permanent magnetic moment of the component atoms or molecules.

The origin of the two kinds of paramagnetism is shown schematically in Fig. 60, where the

TABLE IV. Contribution of atoms and bonds to susceptibility of organic molecules (Pascal).

| Element | $-\chi_A \times 10^6$ | Element | $-\chi_A \times 10^6$ | Bond | $-\chi_B \times 10^6$ |
|---|---|---|---|---|---|
| H | 2.93 | Cl | 20.1 | C—C | 0 |
| | | | | C═C | −5.5 |
| C | 6.00 | Br | 30.6 | C═C | −0.8 |
| N (chain) | 5.57 | I | 44.6 | N═N | −1.8 |
| N (amine) | 2.11 | Na | 9.2 | C═Cl | −3.1 |
| O (alcohol) | 4.61 | K | 18.5 | C—Br | −4.1 |
| S | 15.0 | Zn | 13.5 | Benzene ring | 1.4 |
| P | 26.3 | Si | 20 | | |

curved lines represent electron shells in an atom. The outer line is broken to indicate that the electrons in this shell are loosely bound and become conduction electrons in a solid metal or valence electrons in a compound. In a metal the spins of a portion of these electrons can be changed by an applied field in a way that can be explained by quantum mechanics, and therefore give rise to weak paramagnetism as shown, for example, by metallic sodium. Most of the other shells have their full complement of electrons, and the magnetic moments due to their orbital and spin motions are self-neutralizing so that they contribute only diamagnetism to the atom. On the other hand some of the shells in some atoms are incomplete and therefore have a resultant moment that is large compared with the spin of the conduction electrons or the diamagnetic moment of the closed shells. Such incomplete shells occur notably in the iron group and rare earth group of elements, also in the palladium and platinum groups, and all of these show strong paramagnetism. In the extreme cases of the rare earth elements dysprosium and holmium, the fourth shell (Fig. 60) contains electrons whose spin and orbital moments combine to give the largest magnetic moment of any atoms.

Before discussing the experimental results further it is desirable to outline the classical and quantum theories of strong paramagnetism, and the quantum theory of weak paramagnetism.

### Langevin Theory

Curie found that his measurements at various temperatures could be expressed by the law now known by his name:

$$\chi_M = C_M/T,$$

in which $T$ is the absolute temperature and $C_M$ the Curie constant characteristic of the substance. The classical theory of this kind of paramagnetism was developed by Langevin who investigated mathematically the behavior of an ensemble of magnetic dipoles, each of moment $\mu_M$, in a field of strength $H$. The effect of the field is to cause alignment, the thermal agitation to destroy this alignment. Assuming that the "molecules" are far enough apart so that their mutual forces can be neglected, the energy due to the field is

$$W = -\mu_M H \cos\theta$$

for each dipole oriented so that $\mu_M$ makes the angle $\theta$ with $H$. If the ensemble is subject to thermal agitation, the principles of statistical mechanics show that at any temperature, $T$, the number of dipoles oriented in the solid angle $d\omega$ about the direction $\theta$, is proportional to

$$e^{-W/kT}d\omega = e^{(\mu_M H \cos\theta)/kT}d\omega$$

(Boltzmann's equation). For all the molecules the average value of the moment in the direction of the field, $\bar{\mu}_M$, is calculated and the ratio of this to the total moment, or the moment when all magnets lie parallel, is found to be

$$\frac{\bar{\mu}_M}{\mu_M} = \operatorname{ctnh}\frac{\mu_M H}{kT} - \frac{kT}{\mu_M H}.$$

This is Langevin's equation of paramagnetism. At ordinary temperatures $\mu H \gg kT$, and this equation may be expanded in series form:

$$\frac{\bar{\mu}_M}{\mu_M} = \frac{\mu_M H}{3kT} - \frac{1}{45}\left(\frac{\mu_M H}{kT}\right)^3 + \cdots.$$

Now the moment of a gram molecule is $N\bar{\mu}_M$ and the molecular susceptibility is then $\chi_M = N\mu_M/H$, so that at ordinary temperatures, where the first term of this equation is sufficient, we have

$$\chi_M = \frac{N\mu_M}{H} = \frac{N\mu_M^2}{3kT} = \frac{C_M}{T}.$$

This is Curie's law, with the Curie constant given by

$$C_M = N\mu_M^2/3k.$$

Thus Langevin's work explained Curie's law and showed how the moment per molecule could be determined from the results of experiment.

### Quantum Theory

The advent of quantum theory has changed our notions of atomic structure in fundamental ways, and our interpretation of paramagnetic phenomena in several particular ways now to be discussed under four headings.

(1) Langevin assumed that any orientation of magnetic moment with respect to an applied field is possible; quantum theory requires that only discrete changes of angular momentum are possible, and this imposes restrictions on the orientations of magnetic moment. As a result the Langevin equation of paramagnetism is replaced by a more complicated expression. In the simplest case, when an atom contains a single electron free to change its direction of spin, there are just two possible orientations—those parallel and antiparallel to the field—and the function becomes

$$\bar{\mu}_M/\mu_M = \tanh(\mu_M H/kT) = \mu_M H/kT + \cdots,$$

the last part being applicable when $H/T$ is not too large. When many orientations are possible the function approaches Langevin's as a limit.

(2) There is a natural unit for magnetic moment, the Bohr magneton, equal to the moment of one electron spinning about its own center. Its value is

$$\beta = eh/4\pi mc = 9.27 \times 10^{-21} \text{ erg/gauss.}$$

Here $e$ is the charge and $m$ the mass of the electron, $h$ is Planck's constant, and $c$ the velocity of light. It is convenient to convert susceptibility data to the apparent number of Bohr magnetons per atom, by dividing the moment per atom, determined from Langevin's expression for the Curie constant, by $\beta$. This number is called the effective Bohr magneton number: $\mu_{eff} = \mu_M/\beta$. In terms of the Langevin theory it may be expressed

$$\mu_{eff} = \frac{\mu_M}{\beta} = \left(\frac{3k\chi_M T}{N\beta^2}\right)^{\frac{1}{2}} = 2.83(\chi_M T)^{\frac{1}{2}} = 2.83\sqrt{C_M}$$

and can be calculated provided enough is known of the structure of the atom.

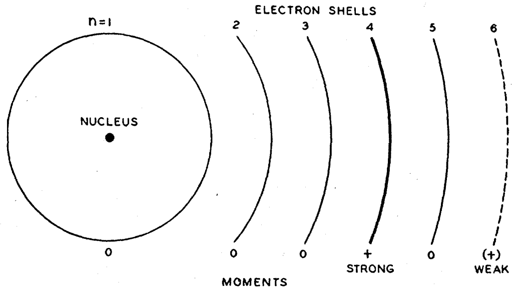

FIG. 60. Paramagnetism occurs when inner shell is incomplete (heavy line) and therefore has permanent moment (strong paramagnetism), or when electrons of outer shell (broken line) become conduction electrons in metal (weak paramagnetism).

(3) The effective moment of an atom may change with temperature or field strength in a calculable way, and Curie's law will not then be obeyed. Under certain conditions $\chi_M$ will be of the form

$$\chi_M = \frac{N\mu_M^2}{3kT} + N\alpha.$$

Such an expression with the constant $\alpha$ included, accounts quantitatively for the behavior of some of the rare earth ions, and is applicable in principle in other cases where, however, it cannot often be worked out in detail.

(4) The temperature-independent or weak paramagnetism of many metals (Fig. 57) and of some conducting oxides and salts was quite inexplicable on classical theory. Quantum theory, however, provides an explanation by showing that conduction electrons, behaving as an electron gas, have just this observed character, as previously stated. Comparison of theory with experiment is discussed below.

The applications of these items to substances of various types will now be considered.

### Rare Earths

The ions of the rare earth elements cerium (58) to ytterbium (70) are good examples of strongly paramagnetic substances. The permanent magnetic moment is caused by the 4f subshell of the 4th electron shell in the usually trivalent ions. This inner shell is protected by the outer shells from the influence of neighboring atoms and so the assumptions underlying the theory are fairly well satisfied.

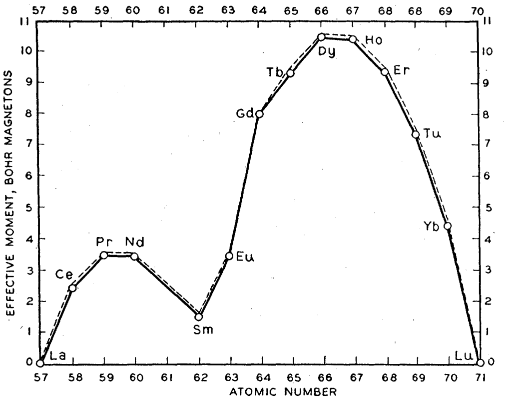

FIG. 61. Effective moment of the trivalent ions of the rare earth elements, at room temperature. Broken line shows values calculated by Van Vleck theory.

Ionic susceptibilities are deduced from measurements on solutions and on solid salts and oxides, by subtracting the contributions of water and other ions (Fig. 61). The susceptibilities decrease with increasing temperature, as expected, except that $\chi_M$ for the samarium ion $Sm^{+++}$ is almost constant above room temperature and that for europium shows also some unusual behavior with temperature.

Calculation of the susceptibilities of the rare earth ions by quantum mechanics is one of the important successes of the Van Vleck theory (Fig. 61), based on the earlier work of F. Hund. Both terms of the last equations above, those dependent on and independent of temperature, were derived quantitatively from theory, and the unusual change of $\chi_M$ with temperature for Sm and Eu satisfactorily explained.

Rare earths in metallic form have large susceptibilities, and gadolinium when cooled below 16°C is ferromagnetic. Values of $\chi_A \times 10^6$ are more than 100,000 for terbium and dysprosium at room temperature. Important deviations from Curie's law are observed, and a law of the form

$$\chi_A = C/(T-\theta)$$

is closely followed. This is the Curie-Weiss law applicable to many solid paramagnetics, particularly those in which the magnetic cores of the atoms are under the influence of magnetic or electric fields of nearby atoms, and the Langevin assumptions, therefore, not satisfied. For gadolinium, as for the other ferromagnetic elements, $\theta$ is approximately the Curie point (see THEORY OF FERROMAGNETISM, below), but the existence of $\theta$ in the Curie-Weiss equation does not mean necessarily that the material will be ferromagnetic below the temperature $T=\theta$; rather, the law breaks down before this point is reached.

### Ions of the Iron Group

Strong paramagnetism occurs also in the ions of the elements lying between (but not including) scandium and copper, and is most marked in the ions of manganese, iron, and cobalt. The moments of the ions are generally less than those of rare earth ions, because the incomplete shell responsible for the paramagnetism has a capacity of 10 instead of 14 electrons.

In calculating the moments from quantum theory the method used for the rare earths is not at all successful, as shown by the dotted line of Fig. 62. Results of calculations made with other assumptions are shown by the other lines, (a) and (b), and it is seen that the observed values lie between these curves. E. C. Stoner has indicated the reason for this. The shell responsible for the moment is the outermost shell of the ion and so is subject to the influence of surrounding ions. These affect the orbital motions in a way difficult to calculate, but if the effect is severe only the spin moments will be influenced by the magnetic field and curve (a) results. On the other hand if the interaction between neighboring ions is weak, the orbital motions will be affected and curve (b) is calculated; the further assumption is here made, that the energy of thermal agitation is capable of changing the distribution of electron spins in the unfilled shell, an assumption consistent with known facts concerning the structures of these ions. When the influence of surrounding ions on the unfilled shell is moderate, the actual ionic moments may be expected to lie between the two theoretical limits, as in fact they do.

### Paramagnetism at Low Temperatures

At ordinary temperatures the Langevin equation may be represented with sufficient accuracy by the first term of the series expansion, as noted above. However, at low temperatures and in strong fields $\mu_M H/kT$ may be so large that the complete equation, or its quantum equivalent, must be used. Under these conditions the susceptibility is no longer independent of the field strength, and the magnetic moment of the material approaches a limiting value corresponding to alignment of the molecular magnets parallel to the field.

The first experimental test of this phenomenon was carried out at temperatures down to 1.3°K and field strengths up to 22,000 oersteds, and the results confirmed the theory in a satisfactory manner. A slightly different relation was predicted by quantum theory, and this agrees with experiment entirely within the limits of accuracy of the experiments.

The magnetic behavior of certain paramagnetic salts is important in the attainment of very low temperatures. A temperature just 0.004 degree above absolute zero was attained by W. S. de Haas and E. C. Wiersma in 1935.[5]

### Paramagnetic Gases

Most monotomic gases, such as those in the eighth column of the periodic table, are diamagnetic because they have complete electron shells and, therefore, no residual magnetic moments. Metal vapors, however, have monatomic molecules in which the single electron in the outer shell should give rise to paramagnetism. Measurements on such vapors are difficult, but potassium and thallium are reported to be paramagnetic and the former obeys Curie's law over a limited range of temperatures and has a susceptibility, according to the rather inaccurate measurements, corresponding to a single uncompensated electron ($\chi_M = 1.06/T$).

Molecules containing more than one atom are generally diamagnetic because they contain an even number of electrons whose moments neutralize each other. There are two well-known exceptions to this rule, oxygen ($O_2$) and nitric oxide (NO). The former has an even number of electrons that remain uncompensated in a way not understood, the latter has an odd number of electrons. The paramagnetism of many other "odd" molecules has been studied to a lesser extent.

Oxygen follows Curie's law ($C_M = 1.00$) except at high pressures when the Curie-Weiss law is obeyed, with the constant $\theta$ small and negative. Liquid oxygen also follows the Curie-Weiss law, and the susceptibility increases with decreasing temperature through the solidification point.

The molecular susceptibility of nitric oxide has been measured from 20 to −160°C (below its normal boiling point) and the effective Bohr magneton number, $\mu_{eff}$, found to vary by about 15 percent over this range of temperature. Accurate calculation of $\mu_{eff}$ and its variation with temperature has been carried out by Van Vleck, who based his work on the electronic structure derived from spectroscopic data. The close agreement found between calculated and observed values is a tribute to the adequacy of the theory.

### Paramagnetism of Free Electrons

The weak temperature-independent paramagnetism of many of the metals (Figs. 56 and 57) was inexplicable on the Langevin theory, according to which all paramagnetism should decrease with increase in temperature. According to modern theory the conduction electrons of a metal have spin moments that can be oriented slightly by a magnetic field. In the simplest case with one free electron per atom quantum theory predicts an atomic susceptibility of $1.88 \times 10^{-6}$ $(A/d)^{\frac{2}{3}}$, where $A$ is the atomic weight and $d$ the density in g/cm$^3$. There is also a smaller diamagnetism predicted by quantum theory, equal

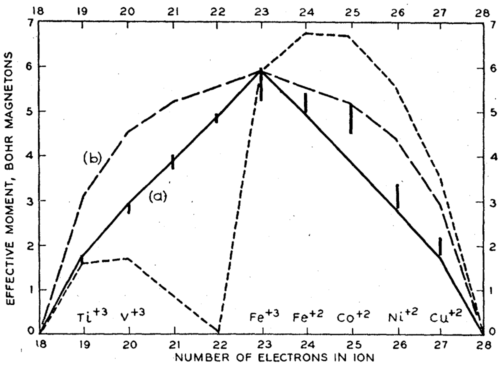

FIG. 62. Effective moment of ions of iron group of elements, plotted against number of electrons in ion. Vertical lines indicate range of observed values, connecting lines theoretical values based on different assumptions.

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[5] W. S. de Haas and E. C. Wiersma, Physica 2, 335 (1935).

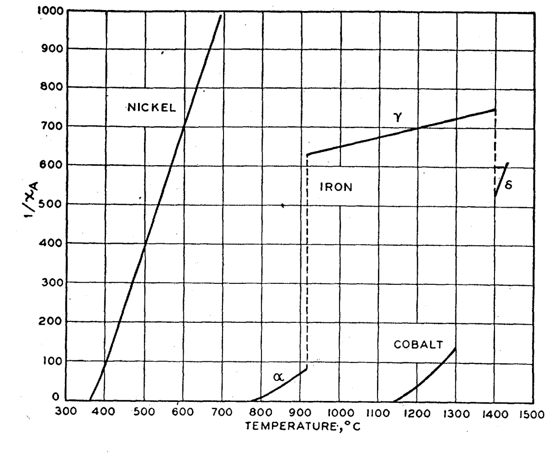

FIG. 63. Reciprocal of atomic susceptibility $\chi_A$ of iron, cobalt, and nickel above their Curie points (P. Weiss and G. Foëx).

to one-third the paramagnetic contribution when the electrons are free.

In actual metals the electrons are not wholly free but are partially bound to the atoms, and are influenced by the so-called correlation and exchange forces, quantum mechanical in nature. Also, the ionic core of the atom has an important diamagnetic contribution to the susceptibility. These various components have been calculated for sodium, and when summed they come within about 10 percent of the observed susceptibility. For other elements the calculations are less satisfactory.

The paramagnetic contributions of the electrons will vary with the nature of the metal, and the diamagnetism of the ionic cores will be different for different atoms, and it is easy to see qualitatively that these and other factors will cause some metals to be weakly paramagnetic, others weakly diamagnetic. The order of magnitude of the susceptibility is explained by theory; more exact calculations must await further quantitative development of atomic theory.

### Ferromagnetic Materials above the Curie Point

All materials that exhibit ferromagnetism are paramagnetic when they are heated above the Curie temperature. As the temperature continues to increase, the susceptibility decreases continually according to the Curie-Weiss law for strongly paramagnetic substances, unless there is a change in the phase structure of the material.

If the Curie-Weiss law were followed exactly, a straight line would be obtained when $1/\chi_A$ is plotted against the temperature. The data for iron, cobalt, and nickel (Fig. 63) show some variations from the expected linear relations, for reasons not well understood. The breaks in the curve for iron are caused by changes in its crystal structure at the indicated temperatures.

One should be able to calculate from the slope of each of these lines the magnetic moment per atom, and compare this with the moment calculated from the saturation magnetization determined at lower temperatures when the material is ferromagnetic. For nickel the ferromagnetic moment is 0.61 Bohr magneton per atom, and the moment calculated from the Curie constant is 1.6 in the same units. Similar discrepancies occur for iron and cobalt. These results show that there is not an integral number of electron spins per atom, therefore the atoms will not all have the same moment at any instant. This means that the calculation of magnetic moment from the Curie constant cannot properly be made by the relation used. Modifications of the theory are necessary.

### Molecular Beams

According to the quantum theory, the magnetic moment of a molecule may have only a limited number of orientations with respect to the direction of a magnetic field in which it is placed. A direct test of this was made successfully by O. Stern and W. Gerlach in 1921. In this experiment silver atoms, issuing from an oven containing melted silver (Fig. 64), are formed into a beam by using small slits, and then pass into the non-uniform field of an electromagnet. In the field the atoms are oriented so that the magnetic moment of each is either parallel or anti-parallel to the direction of the field. Those oriented parallel to the field are deflected in the direction of the stronger field by a force proportional to the gradient (downward in the diagram), those oriented in the opposite direction are deflected toward the weaker field (upward). Thus the beam is split into two components, as detected by the image formed on the screen, and from the separation of the images the atomic moment can be estimated. It is necessary to know the velocity of the beam (calculated from the temperature of the oven) and the gradient of the field (about 200,000 oersteds/cm in some experiments).

Later experiments, designed for more precise determination of the atomic moment, show that for the alkali metals and Cu, Ag, and Au the moment has the theoretical value of one Bohr magneton, within the experimental error of about 1 percent. The elements Zn, Cd, Hg, Sn, Pb, etc., have zero atomic moment, as expected, because the two electrons in the outer shell are mutually compensating. Other elements, e.g., thallium, are found to have the non-integral number of Bohr magnetons derived by quantum theory from spectroscopic data.

### Nuclear Moments

The nucleus of an atom may itself have a magnetic moment. This is always much smaller than a Bohr magneton, and is detected most readily when the outer electronic structure is balanced magnetically so that its net moment is zero. Nuclear moments do not have any important influence on the magnetic properties of materials, but they are very important for our knowledge of nuclear structure, and techniques for their determination have been developed almost continually since 1931. Here the particular method known as "radiofrequency spectroscopy," used first by I. I. Rabi in 1939, will be outlined briefly.

As in other experiments a molecular beam is provided and defined by slits as shown in Fig. 65. When the molecules enter the field, the vector representing the magnetic moment precesses around the direction of the field. Molecules are deflected downward by the field gradient produced by magnet $A$, and upward by the gradient of magnet $B$, and arrive at the point $D$ where they are detected by a suitable device. Molecules of different velocities are focused at the same point after traversing other paths, provided no change occurs in the magnetic moment along the path.

A change in the vertical component of the moment is produced in the uniform field, $H$, of magnet $C$ by superposing upon the field a rather weak alternating field, of frequency, $f$, produced by current in the copper tubes shown. The disturbing action of this field may best be explained by analogy with a spinning top precessing about a vertical axis under the influence of gravity. If a disturbing force is produced, as by oscillating the point of support, the top will become unstable provided the frequency of oscillation is equal to the frequency of precession. In the case of the molecules, instability is produced when their frequency of precession (proportional to the field strength $H$) and the frequency of oscillation of the weaker field are equal, and unstable molecules then deviate from their previous paths as indicated by the dotted line $E$. $H$ is varied slowly until the number of molecules arriving at $D$ is a minimum, and $H$ and $f$ noted. The ratio of the angular momentum, $J$, to the magnetic moment, $\mu$, is then given by

$$\frac{\mu}{J} = 0.001312 f/H.$$

The method derives its name from the fact that $f$ lies in the radiofrequency range, e.g., $f = 5585$ megacycles per second for the $\mathrm{Li}^7$ nucleus when $H = 3420$ oersteds. For this nucleus $J = (\frac{3}{2})(h/2\pi)$ and $\mu = 3.25$ nuclear magnetons or 3.25/1840 Bohr magnetons. The nuclear moments of $\mathrm{Li}^7$ atoms are rather large compared with those of other nuclei, but each is only about 0.2 percent of the moment of an electron.

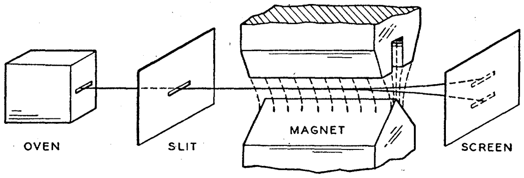

FIG. 64. Diagram of Stern-Gerlach experiment for determining magnetic moments of gaseous atoms (e.g., sodium). Beam of vaporized metal is split by strong inhomogeneous magnetic field.

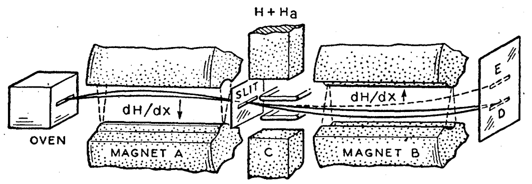

FIG. 65. Rabi method of determining nuclear moments by "radiofrequency spectroscopy." Beam of atoms is bent by magnets as shown, is also acted upon by high frequency field generated in copper tubes shown.

## 13. THEORY OF FERROMAGNETISM

The first requirement of a ferromagnetic material is that the atoms, of which it is composed, have permanent magnetic moments. The magnetic moment of an atom is caused by the electrons which it contains, since the nuclear contribution is negligible. An electron possesses a magnetic moment because of its spin, and may have an additional moment by virtue of its orbital motion.

A second requirement of ferromagnetism is the existence of some kind of interatomic force that maintains the magnetic moments of many atoms parallel to each other. Without such forces the atoms would be disordered by thermal agitation and the moments of neighboring atoms would neutralize each other and the large magnetic moment characteristic of ferromagnetic material would not exist.

Our knowledge of these interatomic forces and of the structure of the atom now permits a general description of the origin of ferromagnetism. The following steps in the theory may be enumerated: (1) the ultimate magnetic particle is the spinning electron, and a change in magnetization is fundamentally a change in the directions of spin of certain electrons, not to any appreciable extent to a change in their orbital motions; (2) in iron, cobalt, and nickel the electrons responsible for ferromagnetism lie in the incomplete third shell of these atoms. In complete shells the electrons are arranged symmetrically so that the spins neutralize one another and the combined moment is zero; (3) in ferromagnetic materials the moments of neighboring atoms are held parallel by strong quantum mechanical forces, whereas in the atoms of other substances the forces usually tend to make the moments of neighboring atoms anti-parallel and self-neutralizing. The circumstances under which these quantum forces effect parallel orientations and form ferromagnetic domains, or anti-parallel orientations that lead to paramagnetism, will be discussed in the following paragraphs. Before considering these steps in modern theory in detail, however, it is appropriate to recall the earlier theories and their development.

### Early Theories

Sir James Ewing was one of the first to attempt to explain ferromagnetic phenomena in terms of the forces between atoms. He assumed with Weber that each atom was a permanent magnet free to turn in any direction about its center. The interatomic forces holding neighboring atoms parallel were assumed to be the usual magnetic forces known to act between large-scale bar magnets. The $I$, $H$ curve and hysteresis loop were calculated for a linear group of such magnets and were determined experimentally using models having as many as 130 magnets arranged at the points of a plane square lattice. The calculations for a linear chain show that as the field is gradually increased in magnitude from zero there is at first a slow continuous rotation of all the magnets together, then sudden change in orientation and finally a further continuous rotation until the magnets lie parallel to the field. The $I$, $H$ curves calculated for such a group of magnets resemble in general form the actual curves of iron: they show a permeability first increasing, then decreasing, and saturation and hysteresis.

G. S. Mahajani has calculated the magnetic potential energy of a group of magnets arranged in space in the same way that the iron atoms are arranged in a crystal. In agreement with Ewing's theory the magnets tend to be parallel, and some directions in the space-array are more stable or "preferred" than others. On the other hand our present knowledge of the structure of atoms and crystals makes it clear that such magnetic forces between atoms are not large enough to maintain the magnetic moments of neighboring atoms parallel, as they must be if ferromagnetism is to exist. Such weak forces would be readily disturbed by the thermal motion of atoms at room temperature and even at temperatures a few degrees above absolute zero.

It is now well established that there are very powerful forces not contemplated when Ewing made his model and proposed his theory, which maintain the dipole moments of neighboring atoms parallel. These are the electrostatic forces of "exchange" which will be discussed below in connection with the quantum theory.

## Molecular Field Theory

In order to understand how atomic forces give rise to ferromagnetism it is desirable to review briefly Weiss's theory of ferromagnetism, which introduces a so-called "molecular field" that presently will be identified with the nature of these forces. Using an approach different from Ewing's, Weiss did not attempt to explain the nature of the interatomic forces but treated them empirically by assuming that they exerted a powerful effect that aided the applied field in lining up the atomic moments. This theory is an extension of Langevin's theory of a paramagnetic gas, which culminated in a formula relating the magnetization, $I$, to the field strength $H$, and the temperature, $T$—the hyperbolic cotangent law,

$$\frac{I}{I_0}=\operatorname{ctnh}\frac{\mu_A H}{kT}-\frac{kT}{\mu_A H}$$

already referred to in the section on PARAMAGNETISM. In deriving this, the assumptions are made that the elementary magnets, each of moment $\mu_A$, are subject to thermal agitation and momentarily may have any orientation with respect to the direction of the field, and that they are too far apart to influence each other. Quantum theory alters the first of those assumptions by stating that, in such an ensemble of atoms, the electrons responsible for the magnetic moments will have only a limited number of possible orientations, in the simplest case only two, one parallel and the other anti-parallel to the direction of the field. In this case the equation corresponding to Langevin's is

$$\frac{I}{I_0}=\tanh\frac{\beta H}{kT},$$

$\beta$ being the spin moment of one electron.

Weiss assumed that there exists in a ferromagnetic material a "molecular field," proportional to the intensity of magnetization, that aids the magnetic field present. He therefore replaced the $H$ of Langevin's equation by $H+NI$, calling $N$ the molecular field constant. The resulting equation,

$$\frac{I}{I_0}=\tanh\frac{\beta(H+NI)}{kT},$$

is perhaps the most important in the theory of ferromagnetism. It indicates that even in zero field there is still a magnetization of considerable magnitude, provided the temperature is not too high. Putting $H=0$ and $\theta=\beta NI_o/k$, this equation reduces to

$$\frac{I}{I_o}=\tanh\frac{I/I_o}{T/\theta}.$$

This purports to specify the magnetization at zero applied field by a function that is the same for all materials, when the magnetization is expressed as a fraction of its value at absolute zero ($I_o$) and the temperature as a fraction of the Curie temperature on the absolute scale. This magnetization *vs.* temperature relation, plotted as the upper solid line of Fig. 66, means that at all temperatures below $\theta$ the intensity of magnetization has a definite value even when no field is applied.

According to this conception the $I$ of the last equation represents the magnetization of a domain, and is determined experimentally by measuring the magnetization of a specimen when all domains are parallel, i.e., at saturation ($I=I_s$). This equation should then be written

$$\frac{I_s}{I_o}=\tanh\frac{I_s/I_o}{T/\theta}.$$

It is well known, however, that a piece of iron may be apparently unmagnetized at room temperature. Weiss explained this by formulating the domain theory, already described above, according to which each part of a ferromagnetic material is magnetized to saturation, the magnetic moments of the separate parts being oriented in different directions so that the over-all effect is zero.

These relations are also important for ferromagnetic materials above the Curie point, when they are paramagnetic. Then $I$ is small compared to $I_o$ and we may write

$$\frac{I}{I_o}=\frac{\beta(H+NI)}{kT}.$$

If we put $\theta=\beta NI_o/k$ as before, and $C=I_o\beta/k$,

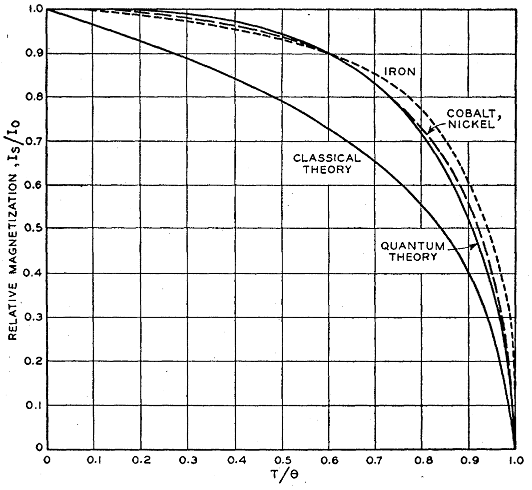

FIG. 66. Saturation magnetization as dependent on absolute temperature, in relative units. Solid curves are theoretical, for classical (Weiss) and quantum theories.

this becomes

$$I = \frac{CH}{T - \theta},$$

the Curie-Weiss law. Thus $\theta$ is explained by the existence of the molecular field, $NI$, that aids the applied field, $H$, in orienting the elementary magnets. However, as pointed out above, the fact that a paramagnetic substance follows this law does not mean necessarily that it will become ferromagnetic at temperatures below $T = \theta$.

## Quantum Theory

The nature of the molecular field was unknown to Weiss when he developed his theory; he knew only that its power in orienting the elementary magnets was equal to that of a magnetic field of millions of oersteds (in iron, NI is as high as $7 \times 10^6$ oersteds). A rational explanation of this was first proposed in 1928 by W. Heisenberg who showed that it can be explained in terms of the quantum-mechanical forces of exchange acting between electrons in neighboring atoms. Imagine two atoms some distance apart, each atom having a magnetic moment of one Bohr magneton due to the spin moment of one electron. A force of interaction, the exchange force already referred to, has been shown to exist between them in addition to the better-known electrostatic and (much weaker) magnetic forces. It is known that such quantum forces are negligible, as one would expect, when the atoms are two or three times as far apart as they are in crystals. It is supposed also, on the basis of discussions by H. Bethe, and by J. C. Slater, that as two atoms are brought near to each other from a distance, these forces at first cause the electron spins in the two atoms to become parallel (positive interaction). As the atoms are brought nearer, the spin moments are held parallel more firmly until at a certain distance the force diminishes and then becomes zero, and with still closer approach the spins set themselves antiparallel with relatively strong forces (negative interaction). In the curve of Fig. 67 the energies corresponding to these forces, as estimated by Bethe, are shown as a function of the distances between atoms.

Bethe's curve was drawn originally for atoms with definite shell radii and varying distances from atom to atom. It may be used equally well for a series of elements if we take account of the different radii of the shell in which the magnetic moment resides. The criterion of interaction for the metals of the iron group is the radius, $R$, of the atom (half the internuclear distance in the crystal) divided by the radius, $r$, of the 3d shell, in which the magnetic moment resides. In Fig. 67 this ratio $R/r$ has been used as abscissa and the elements iron, cobalt, and nickel have been given appropriate positions on the curve. Slater has shown that the ratio $R/r$ is larger in the ferromagnetic elements than in other elements having incomplete inner shells, and that the point at which the curve crosses from the non-ferromagnetic to the ferromagnetic region is near $R/r = 1.5$. This also explains why

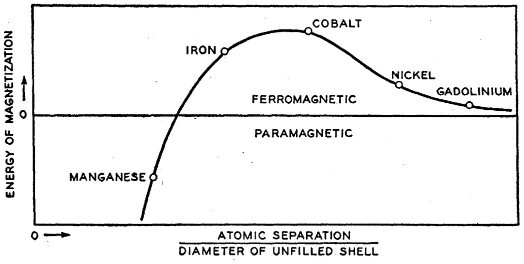

FIG. 67. Bethe's curve showing that ferromagnetic substances must have favorable ratio of distance between atoms to diameter of unfilled inner electron shell responsible for permanent moment.

manganese becomes ferromagnetic when the atoms are separated by abnormally large distances as they are in the Heusler alloys and some compounds.

## Atomic Structure of Ferromagnetic Substances

It has already been pointed out that the fundamental magnetic particle is the spinning electron. One might think that the orbital motions of the electrons in the atom would also contribute to ferromagnetism, owing to their magnetic moments, but it has now been established that when the magnetization is altered all that changes is the direction of the spin of certain of the electrons in the atoms—the orbital motions remain practically unchanged. The experimental basis for this conclusion is the determination of the ratio of angular momentum to magnetic moment; this ratio is found, in the experiments of S. J. Barnett and others, to be appropriate to the electron spin and definitely not to the orbital moment.

The electrons that are responsible for the magnetic properties of iron, cobalt, nickel, and their alloys lie in a definite "shell" in the atom. As shown in Fig. 68, there are four shells or regions, more or less well defined, into which all the electrons circulating about the nuclei of these atoms may be divided, when the atom is separated from its neighboring atoms, as it is, for example, in a gas. Two of these shells are subdivided as shown. When the atoms come closer together to form the solid metal, the fourth or outermost shell of each becomes disrupted, and the two electrons which comprised it wander easily from atom to atom and are the "free" electrons responsible for electrical conduction. The electrons in the outer part of the third shell are those responsible for the ferromagnetic moment. Some of these electrons spin in one direction and some in the opposite, as indicated, so that their magnetic moments neutralize each other partially but not wholly, and the excess of those spinning in one direction (+) over those spinning in the other (−) causes each atom as a whole to behave as a small permanent magnet.

The average moment of the atom in a ferromagnetic material may be determined from the saturation magnetization, $I_o$, at the absolute zero of temperature by dividing $I_o$ by the number of atoms in a cubic centimeter. Expressed in Bohr magnetons, $\beta$, the average or "effective" atomic moment is

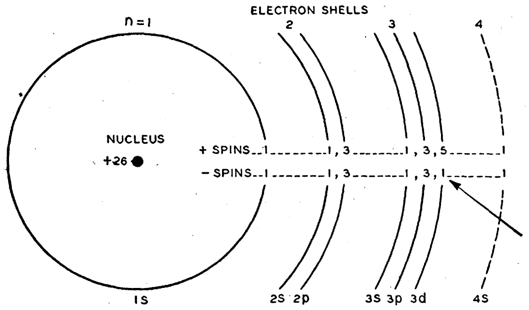

FIG. 68. Electron shells in an iron atom, showing location of inner unfilled shell (3d) responsible for magnetic moment. Electrons of outer shell (4s) of isolated atom become "free" electrons of metal.

$$\mu_{eff} = \frac{I_o A}{N\beta d},$$

$A$ being the atomic weight, $N$ Avogadro's number, and $d$ the density. Moments of iron, cobalt, and nickel atoms, so calculated, are

| | |
|---|---|
| Fe | 2.22 Bohr magnetons |
| Co | 1.70 |
| Ni | 0.61 |

The meaning of these non-integral values may be discussed in either of two different ways used for many problems of the solid state. In the Heisenberg method of approach the atoms have their own electrons (some atoms necessarily more than others), and the disturbing effect of the interaction between neighbors is estimated as well as possible. The other approach followed by F. Bloch, E. C. Stoner, and others, is to consider the metal as composed initially of ions and free electrons, and to calculate the way in which the ionic fields modify the behavior of the electrons, making them less free. According to the latter or "collective electron" description both the 3d and 4s shells of iron, cobalt, and nickel atoms are incomplete and there is a distribution of electrons between them determined by the specific nature of the atomic fields. The calculations indicate that in iron the electrons in these shells are distributed (on the average) about the nuclei as indicated in Fig. 69. The 3d shell is a rather dense

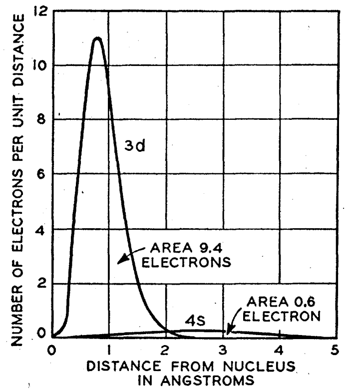

FIG. 69. Distribution of electrons in 3d and 4s shells of iron according to the Heisenberg model.

ring of electrons, as contrasted with the 4s shell which extends farther from the nucleus, so far that in the solid the shells of neighboring atoms overlap considerably.

The above description of electron distributions in their relation to ferromagnetism is supported beautifully by experiments on the alloying of nickel with copper, zinc, and other elements. The substitution of one copper for one nickel atom in the lattice is equivalent to adding one electron to the alloy. This electron seeks the place of lowest energy in the alloy and finds it in the 3d− shell of a nickel atom rather than in the copper atom to which it originally belonged. This lowers the magnetic saturation of the alloy by one Bohr unit, since the added electron in the 3d− shell just neutralizes the moment of one in the 3d+ shell. Addition of more copper to nickel decreases the average moment until the empty spaces in the 3d− shell are just full; this should occur when 60 percent of the atoms are copper, and then the magnetic saturation at 0°K should be just zero. The theory is well confirmed by the data (Fig. 70). When divalent zinc, having two available electrons per atom, is added to nickel, the 3d shell is filled twice as fast as when monovalent copper is added, again in accordance with theory, and metals of higher valences behave accordingly.

## 14. MEASUREMENT OF MAGNETIC QUANTITIES

### Basic Relations

As a prelude to a discussion of the production and measurement of magnetic fields, and the measurement of magnetization, it is desirable to consider quantitatively the fields and forces excited by currents and magnets. These may be discussed according to the following list:

(1) The fields produced by magnets; (2) the fields produced by currents; (3) the force on a magnet in a field; (4) the force on a current in a field; (5) the electromotive forces produced in coils by changes in induction within them. It is on these relations that most of the measurements of induction and field strength are based.

### *Field of a Magnet*

Consider a bar magnet *NS* (Fig. 71) of pole strength $m$ and interpolar distance $l$ and therefore magnetic moment $M = lm$. Using the fundamental relation according to which the field strength varies inversely as the square of the distance, $d$, from a pole,[6]

$$H = m/d^2, \tag{1}$$

and adding vectorially the fields produced by both poles, it may be shown that the field strength at distant $r$ from the center of the magnet is given by

$$H = (M/r^3)(1+3\cos^2\theta)^{\frac{1}{2}}$$

provided $r$ is large compared with $l$. Here $\theta$ is the angle between the axis *SN* of the magnet and the line *OP* drawn from its center to the point, *P*, at which the field is observed. The direction

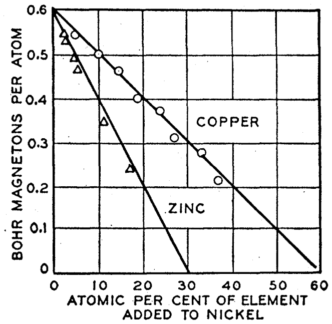

FIG. 70. Reduction in magnetic moment per atom of nickel, caused by adding copper (with one extra electron per atom) or zinc (two per atom). Extra electrons go into unfilled shell and therefore decrease its moment.

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[6] This relation applies exactly when the magnet is surrounded by a vacuum; when the surrounding medium is air a very small modification may be necessary. In general, when surrounded by a medium of permeability $\mu$ the law is $H = m/\mu d^2$.

of the field at this point is defined by $\varphi$, the angle it makes with $OP$ prolonged, and is determined by

$$2\tan\varphi=\tan\theta.$$

A graphical construction based on this relation may be made easily by trisecting $OP$ so that $OC=2\,CP$, drawing $CD$ at right angles to $OP$ to cut $SN$ produced at $D$. Then $DP$ is the direction of the field at $P$. Here, as elsewhere in this article, $H$ is expressed in oersteds, $B$ in gausses, dimensions in centimeters, forces in dynes, and currents in amperes.

Important special cases are the end-on position (first Gaussian position) for $\theta=0$, and the broadside position (second Gaussian) for $\theta=90°$. For these the fields are, respectively,

$$H_1=(2M/r^3)(1+l^2/2r^2),$$
$$H_2=(M/r^3)(1-3l^2/4r^2).$$

When two magnets are placed very close together end-to-end, with opposite poles $m_1$ and $m_2$ separated by a distance, $d$, small compared with the extent of the surfaces, the two magnets are attracted with a force that may be very strong. For each portion of one of the nearby surfaces the corresponding part of the other has an attraction given by the fundamental relation

$$f=\frac{m_1m_2}{d^2}.$$

Summing for the effect of all parts of one surface on all parts of the other one can obtain

$$f=AH^2/8\pi$$

proportional to the pole area, $A$, but independent of the distance between as long as this is small.

### *Fields Produced by Currents*

The simplest way of producing a magnetic field of known strength and direction is by the use of a long coil or solenoid of wire. The field is parallel to the axis of the coil, and when a current of $i$ amperes flows in the windings (having $n$ turns per cm) the field strength within the coil is

$$H=4\pi ni/10. \qquad (2)$$

When many layers of wire are used in a solenoid that is not very long in comparison with its diameter, the field strength varies with the position, and can be calculated with the help of the appropriate formula.[7]

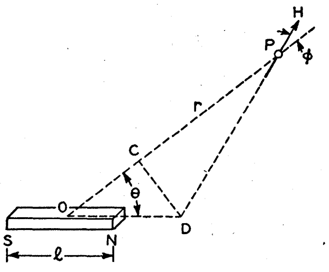

FIG. 71. Field produced at any point $P$ by presence of bar magnet $NS$ depends on distance $r$ and angle $\theta$.

When a field is produced by the current in a long straight wire of radius $a$, the direction of the field is everywhere at right angles to the wire axis, and at distance $x$ from the axis the field strength is

$$H=2i/10x$$

outside the wire, while inside it the field strength is

$$H=2ix/10a^2.$$

To create a relatively uniform field in a large volume Hermann Helmholtz proposed the arrangement of two thin circular coaxial coils of diameter $2a$ and axial separation $a$. At the center of symmetry, $O$, the field strength is

$$H=0.899\,Ni/a.$$

$N$ is the total number of turns of wire in each coil.

Other forms of coils may be used, and the strength and direction of the field calculated by summing the fields due to each current element. The magnitude of the field $dH$ caused by a current $i$ flowing in an element of wire $dl$ cm long, at a point $r$ cm from the current, is given by the equation

$$dH=(i/10r^2)dl\,\sin\theta,$$

$\theta$ being the angle between the direction of the current and that of the line, $r$, connecting $dl$ to the point. The direction of the field is at right angles to both the current and $r$.

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[7] G. C. Foster and A. W. Porter, *Electricity and Magnetism* (London, 1913), third edition, p. 364.

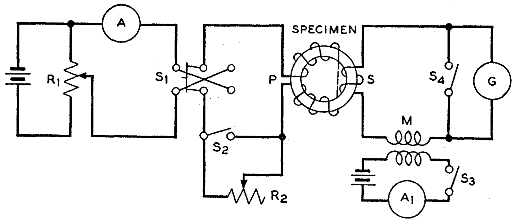

FIG. 72. Diagram of method of measurement of $B$ and $H$ in ring specimen. Current in primary winding $P$ is controlled by resistances $R_1$ and $R_2$ and switches $S_1$ and $S_2$, and is read on meter $A$. Secondary winding $S$ is connected through mutual inductance $M$ to ballistic galvanometer $G$. $A_1$ and $S_3$ are used for calibration.

### *Force on Magnet in Field*

The force on a single magnetic pole of strength $m$ in a field of strength $H$ is

$$f = Hm$$

dynes acting parallel to the field. In a uniform field a magnet of moment $M'$ is acted on by a torque tending to turn the magnet upon an axis at right angles to its length and to the field so that it will lie parallel to the field. The magnitude of this torque is

$$L = M'H \sin\alpha$$

where $\alpha$ is the angle between the field and the length of the magnet. If the field is produced at $P$ by a magnet of moment $M$ placed at $O$ (Fig. 71), the torque acting on $M'$ is

$$L = MM'(1+3\cos^2\theta)^{\frac{1}{2}}(\sin\alpha)/r^3.$$

If a pivoted magnet is suspended in a field it will have a natural period of oscillation equal to

$$T = 2\pi(K/MH)^{\frac{1}{2}}$$

seconds, if $K$ is the moment of inertia about its point of suspension. If an additional known moment of inertia, $k$, is added to that already present, without changing the magnet moment, and this increases the period to $T_1$, the expression

$$MH = 4\pi^2 k/(T_1^2 - T^2)$$

can be used to determine either $H$ or $M$ provided the other is known.

In a non-uniform field a magnet will experience a translational force as well as a torque, because the force pulling one pole parallel to the field will be opposed by a larger force caused by a larger field acting on the other pole. If the gradient of the field is $dH/dx$, the force on a magnet of moment $M$ is

$$f = MdH/dx.$$

When the moment is induced by the field and is therefore

$$M = Iv = \kappa Hv,$$

the force acting on the volume, $v$, is

$$f = \kappa vHdH/dx. \qquad (3)$$

### *Force on Current in Field*

When a current $i$ flows in an element of wire of length $dl$ in a field of strength $H$, a force acts on the wire of magnitude

$$f = Hidl(\sin\theta)/10. \qquad (4)$$

Here $\theta$ is the angle between $dl$ and $H$, and the direction of the force is at right angles to both. In two long parallel straight wires carrying currents the field of each acts on the current in the other to produce an attractive force if the currents are in the same direction, a repulsive force if they are anti-parallel, of magnitude

$$f = 2ii'/(100a)$$

dynes per cm length of wire, $a$ being the distance between wire axes.

### *Electromotive Force Due to Changing Flux*

When an electric circuit encloses a changing magnetic flux, $\varphi$, an electromotive force is generated in the circuit proportional to the time rate of change of flux, $d\varphi/dt$, and to the number of times, $N$, the circuit threads the flux. Thus if $N$ turns of wire are wrapped around a piece of iron of cross-sectional area $A$, and a change in the induction, $B$, is produced in the iron, a voltage is produced at the ends of the wire equal to $10^{-8}$ $NA$ times the rate of change of induction in gausses per second:

$$E = 10^{-8}NAdB/dt.$$

The total change in $B$ that has occurred in a specified time may be evaluated by integrating this equation with the result

$$B = 10^8 \int Edt/NA. \qquad (5)$$

## Common Methods

### *Ballistic Method with Ring*

In determining magnetization curves and hysteresis loops the ballistic method, employing a ring specimen, is generally the most satisfactory. H. A. Rowland used this method in 1873, and was the first to express the results of measurement in an absolute system. A ring is cut with a radial thickness small compared with its diameter and two windings of wire are applied (Fig. 72); a secondary winding (*S*), usually consisting of many turns of fine wire, is wound close to the specimen and connected to a ballistic galvanometer or fluxmeter (*G*), and a primary winding (*P*) of turns evenly spaced is applied and connected to a current source. In the primary circuit means are provided for adjusting the current and for changing it rapidly from one value to another in the same or opposite direction. In the secondary circuit a mutual inductance (*M*) with an air core is provided for calibration and a switch for short circuiting the galvanometer. The field strength, *H*, is calculated according to Eq. (2) from the number of turns and the dimensions of the coil and the current indicated by the ammeter (*A*).

When *H* is changed suddenly from one value to another, the resulting change in *B* induces a voltage in the coil *S* and causes a deflection of the galvanometer that is proportional to the change in *B*, according to Eq. (5) above (the ballistic galvanometer has a deflection proportional to $\int E dt$ provided the voltage impulse occurs in an interval of time short compared with the natural period of oscillation of the glavanometer coil and mirror). Since ballistic measurements always involve differences one must start with a known value of *B*. The most common procedure for accomplishing this is first to reduce the magnetization to zero, or demagnetize the core, by subjecting it to an alternating field of high strength and gradually reducing the amplitude to zero. To determine a point such as *M* in Fig. 73, current corresponding to $H_1$ is applied and reversed several times with the galvanometer short-circuited, to establish a steady cyclic state, and then the galvanometer is connected in and the deflection for reversal of $H_1$ noted. This deflection then corresponds to a change in *B* of $2B_1$, and $B_1$ can easily be calculated from the constant obtained by calibration. The point *M* on the normal magnetization curve is thus established and other points on the curve are determined similarly.

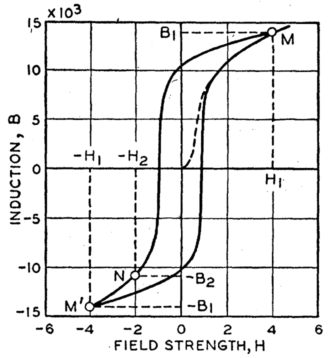

FIG. 73. Hysteresis loop illustrating ballistic method of measurement of *B*. When field strength is changed from $H_1$ to $-H_1$, *B* changes from $B_1$ to $-B_1$, and galvanometer deflection measures this change in *B*.

To determine a point, *N*, on a hysteresis loop the field is alternated several times between the points of the loop (e.g., *MM′*), then $H_1$ is changed suddenly to $-H_2$ and the deflection noted and the corresponding change in *B* is subtracted from $B_1$, already determined, to give the value of $-B_2$ at *N*. By varying $H_2$, a sufficient number of points on the whole loop can be obtained.

The ring specimen is not adapted to rapid testing, for some time is consumed in machining the ring and in winding it with wire. However, several methods are available for the rapid application of turns. Stiff wires may be held in fixed positions and arranged so that the ring will slip over many of them at once, the wires being connected together by a plug-and-socket arrangement or with mercury contacts to make a single circuit. Also, toroidal winding machines are made that apply more than 25 turns a second to a closed ring.

Ordinarily the ballistic galvanometer and ring specimen are used for fields of intermediate strength. When the field strength is high, e.g., 1000 oersteds, the number of turns in the primary winding, or the current through it, must be large and the consequent heat generated interferes with the measurements. If the field strength is too low, many turns must be wound on the secondary in order to obtain a sufficiently large

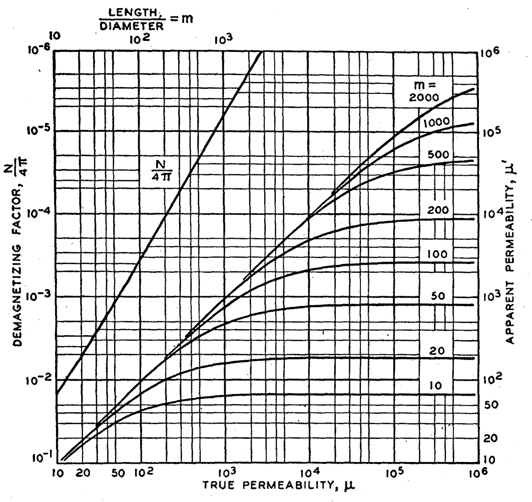

FIG. 74. Chart for converting apparent to true permeability, $\mu'$ to $\mu$, of cylinders of given ratio $m$ of length to diameter. Also demagnetizing factors, $N/4\pi$.

galvanometer deflection, and this process is tedious and under some circumstances there may not be enough space available inside the ring. In very low fields a.c. methods should be considered.

### *Rod Specimens; Demagnetizing Factors*

The ballistic method may be used with a straight specimen in the form of a rod, this being magnetized with the same procedure as that used for a ring. The specimen may easily be slipped into a long solenoid previously prepared, and the secondary winding or search coil should be placed around a small central portion of the specimen. Under these circumstances the more difficult quantity to determine is the field strength, because the field created by the solenoid will be disturbed by the magnetic poles of the specimen. The field created by the specimen itself is called the demagnetizing field, $\Delta H$, and is proportional to the intensity of magnetization. It is usually specified by the demagnetizing factor, $N$, which depends on the ratio length/diameter of the rod:

$$\Delta H = NI.$$

The field acting on the middle of the rod is then the resultant of the field in the solenoid, $H_o$, and the demagnetizing field, $\Delta H$, and (when $I$ is replaced by $(B-H)/4\pi$) is

$$H = H_o - \frac{N}{4\pi}(B-H).$$

Values of $N/4\pi$ have been determined by a number of experiments and selected values are given in Table V.

The term apparent permeability, $\mu'$, is often applied to the ratio $B/H_o$, and its relation to the (true) permeability, $\mu$, can be derived from the above equation:

$$1/\mu = 1/\mu' - N/4\pi$$

by considering $N/4\pi$ to be negligibly small compared with one. The relation between $\mu$ and $\mu'$ for rods of various dimensional ratios $m$ = length/diameter is shown graphically[8] in Fig. 74. It is apparent that rods having dimensional ratios as large as 1000 must be used to determine accurately the value of the permeability when it is more than 100,000.

In rods the induction varies from place to place in the bar, decreasing from the middle toward the ends. It is only when the specimen is in the form of an ellipsoid that the induction is uniform throughout. In specimens of such form the demagnetizing factor may be calculated accurately from the lengths of the three axes of the ellipsoid, and the direction of the induction determined. Equations are given below for the two most useful cases: (1) the prolate ellipsoid having the major axis $m$ times the two equal minor axes, and magnetized parallel to its major

TABLE V. Demagnetizing factors, $N/4\pi$, for rods and ellipsoids magnetized parallel to long axis.

| Dimensional ratio (length/diameter) | Rod | Prolate ellipsoid | Oblate ellipsoid |
|---|---|---|---|
| 0 | 1.0 | 1.0 | 1.0 |
| 1 | .27 | .3333 | .3333 |
| 2 | .14 | .1735 | .2364 |
| 5 | .040 | .0558 | .1248 |
| 10 | .0172 | .0203 | .0696 |
| 20 | .00617 | .00675 | .0369 |
| 50 | .00129 | .00144 | .01472 |
| 100 | .00036 | .000430 | .00776 |
| 200 | .000090 | .000125 | .00390 |
| 500 | .000014 | .0000236 | .001567 |
| 1000 | .0000036 | .0000066 | .000784 |
| 2000 | .0000009 | .0000019 | .000392 |

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[8] R. M. Bozorth and D. M. Chapin, J. App. Phys. **13**, 320–326 (1942).

axis and (2) the oblate ellipsoid or ellipsoidal disk having two long axes each $m$ times the short axis, the specimen being magnetized parallel to a long axis.

Prolate:

$$\frac{N}{4\pi}=\frac{1}{m^2-1}\left[\frac{m}{(m^2-1)^{\frac{1}{2}}}\log_e(m+(m^2-1)^{\frac{1}{2}})-1\right],$$

Oblate:

$$\frac{N}{4\pi}=\frac{1}{2}\left[\frac{m^2}{(m^2-1)^{\frac{3}{2}}}\text{arc sin}\frac{(m^2-1)^{\frac{1}{2}}}{m}-\frac{1}{m^2-1}\right].$$

Values of $N/4\pi$ for various values of $m$ are given in Table V.

### *Yokes and Permeameters*

When the specimen is in the form of a straight rod or tube or tape the demagnetizing field may be partially or completely annulled by connecting the two ends, outside the magnetizing solenoid, with magnetic material of high permeability and large cross-sectional area. Such a yoke may be used for specimens of high dimensional ratio, and measurements made with considerable accuracy.

More often, in the testing of permanent magnets or commercial materials in the form of bars or strips, it is desirable to use a yoke with a relatively short specimen and in this case the demagnetizing field must be taken into account or annulled in some way. One way[9] that has been satisfactorily used to accomplish both of these things is to measure the field with a ballistic galvanometer connected to a coil ("H-coil") of many turns placed in the magnetizing coil near the middle of the specimen but not surrounding it (Fig. 75), and at the same time to magnetize the yoke with the current that passes through the magnetizing coil. The turns through which this current passes are adjusted to overcome the reluctance of the yoke and the air gaps that exist at each end of the specimen no matter how much care is taken to obtain good joints at these points. The search coil used in determining $B$ is wound on the middle of the specimen as usual.

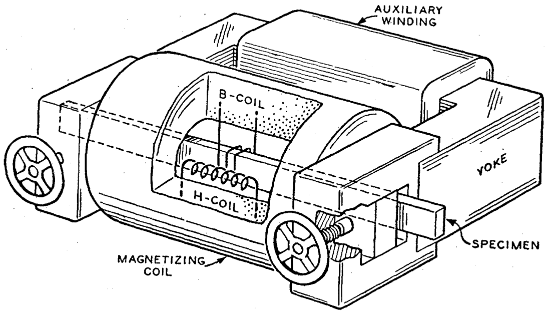

FIG. 75. Simple yoke method of compensating demagnetizing effect of ends of short specimen.

A further step toward increased accuracy was taken in the construction of the Burrows and the Niwa permeameters. These use auxiliary magnetizing and search coils to effect a uniform magnetization of the specimen by compensating for the air gaps that must be present. The induction is determined with a ballistic galvanometer.

Other methods of testing with a yoke have been proposed by F. P. Fahy and by A. Koepsel and these have been used extensively for routine testing of samples of standardized materials of moderate or low permeability. In the Fahy permeameter $B$ and $H$ are determined with the help of coils extending the full length of the specimen and connected to a galvanometer. An auxiliary magnetizing coil is used on the yoke. The Koepsel permeameter has the advantage of being a direct-reading instrument since it requires no galvanometer. To indicate the induction in the specimen a coil of wire is placed on a delicate bearing in an air gap in the yoke, and when a constant current is passed through this coil it deflects in accordance with Eq. (4) by an amount proportional to the field strength in the gap, which in turn depends on the flux in the specimen.

In determining the magnetic properties of materials in high fields, of the order of 300 to 10,000 or more, it is necessary to use some kind of an electromagnet to create the field. A modification[10] of the isthmus method used originally by J. A. Ewing and W. Low is shown in Fig. 76. The heavy square yokes are wound with many turns of heavy wire, and when energized create

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[9] B. J. Babbitt, *J. Opt. Soc. Am.* **17**, 47–48 (1928); C. E. Webb and L. H. Fort, *J. Inst. Elec. Eng.* **67**, 1302–16 (1929).

[10] R. L. Sanford and E. G. Bennett, Bur. Stand. J. Research **10**, 567–573 (1933), **23**, 415–425 (1939); A. Neumann, Zeits. f. tech. Physik **15**, 473–477 (1934).

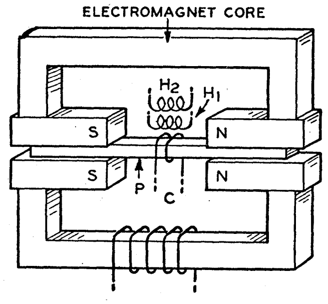

FIG. 76. Permeammeter for measuring $B$ and $H$ in strong fields. Yoke is core of powerful electromagnet having poles NS adjacent to specimen $P$. Field is measured with similar coils $H_1$ and $H_2$, and induction with search coil $C$.

a high field in the space between the pole faces, $N$ and $S$. The specimen $P$ is clamped between the pole pieces as indicated, and is surrounded by a search coil connected to a galvanometer for determining $B$. The field strength, $H$, is determined by measuring the change in the air flux in a coil placed near the specimen ($H_1$), and then in the same coil placed somewhat farther away ($H_2$). The field in this region varies approximately linearly with the distance from the surface of the specimen and its strength at the surface is obtained by extrapolation from the values $H_1$ and $H_2$. By making the core of the electromagnet of laminated material the magnetizing field can be changed quickly enough to make satisfactory ballistic measurements provided the period of the galvanometer is large. In investigations of properties in still higher fields, e.g., 10,000 to 50,000, special methods have been used (see below).

### *Alternating Current Methods*

The principal uses of a.c. testing methods are for determining (1) permeability and energy losses of thin sheet at low inductions, usually not more than a few hundred gausses, using an inductance bridge and (2) energy losses at high inductions, usually $B = 15{,}000$, using the Epstein method.

A diagram of the inductance bridge is shown in Fig. 77. Current of the desired frequency, $f$, is supplied by an oscillator and fed to opposite corners of the bridge through a transformer and a thermal ammeter, $A$. Two equal fixed resistances $R_1$ constitute two arms of the bridge, and in the others are the specimen, $S$, and variable inductance $L$ and resistance $R$. When the bridge is balanced by adjusting $L$ and $R$ until no voltage is detected by the phones $P$ or other instrument connected as shown through a transformer and amplifier, then $L$ and $R$ are equal to the inductance and a.c. resistance of the winding enclosing the specimen. The permeability can be calculated from $L$ and the dimensions of the specimen, the field strength from the current through the core winding, and the energy loss in the specimen from the resistance $R$, correction being made for the losses in the copper wire. In another common modification of the Maxwell bridge the variable inductance $L$ is removed and a variable condenser is connected in parallel with the variable resistor.

To save the application of many turns of wire to the specimen, necessary when low inductions are measured, G. E. Kelsall has used a device indicated schematically at $K$ in Fig. 77. Instead of connecting the windings of a specimen directly to one arm of the bridge, in the usual way, there is substituted a transformer having a many-turn primary and a single-turn secondary. This turn is wound on the specimen also, and the bridge is balanced as before. The core of the transformer is made of high quality laminated material and has low losses which, however, must be taken into account and subtracted from the bridge measurement to obtain the loss in the specimen. This

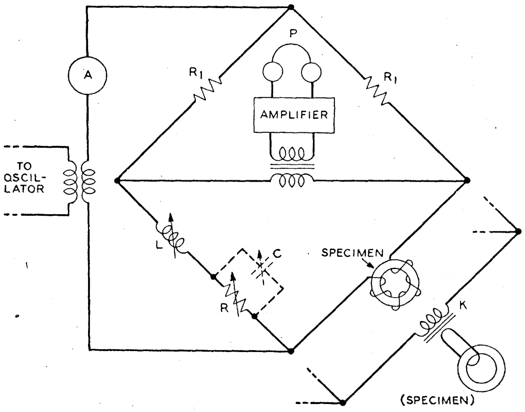

FIG. 77. Inductance bridge for measuring resistance and inductance of winding surrounding specimen. Alternatively, $L$ may be omitted and a variable condenser $C$ inserted in parallel with $R$. Specimen may be connected through transformer $K$ having single turn secondary looped with specimen.

instrument has been adapted for use at high temperatures as well as at ordinary temperatures.

In interpreting the permeability and energy losses measured by a.c. methods one must consider the apparent reduction of permeability due to skin-effect or shielding caused by eddy currents, and the separable losses attributable to eddy currents and hysteresis. This data obtained with the a.c. bridge are especially important in the study of materials used in communication engineering where small magnetic fields are important in the transmission of weak or undistorted signals.

The Epstein or Lloyd-Fisher apparatus is used primarily for measuring magnetic losses in material for power and other transformers operating at high inductions. The material in the form of strips is built into a hollow square. An alternating field is applied, in one winding, to magnetize it to a high induction measured by the voltage induced in a second winding, and the loss is indicated by a wattmeter connected to the circuits of both windings (Fig. 78). Both windings are placed evenly on forms with square sections, and four such forms are placed to form a hollow square. The strips are slipped into the forms and carefully clamped at the corners after being abutted or overlapped there. Standard specifications control the manner of cutting and stacking the specimens, of correcting for the air flux in the coils, and of taking into account the wave form of the exciting current. Measurements are commonly made at $B = 15{,}000$ and at a frequency of 60 cycles/sec. in the United States and 50 cycles/sec. in England.

In principle a simple method of testing a specimen by a.c. is by determining the current-voltage characteristics of a specimen having two windings, the current being a measure of $H$ and the voltage a measure of $B$. Several ways have been used for accomplishing this, involving potentiometric measurements or rectification. From the results one can determine not only $B$ and $H$, but also by taking account of the phase can determine the total energy loss in the material.

## Special Methods

Methods other than those described may be used advantageously for special investigations.

Measurements of very small quantities of materials, of single crystals in definite directions, of paramagnetic or diamagnetic solids, liquids or gases, all require specific arrangements for obtaining results of high accuracy with a minimum of effort.

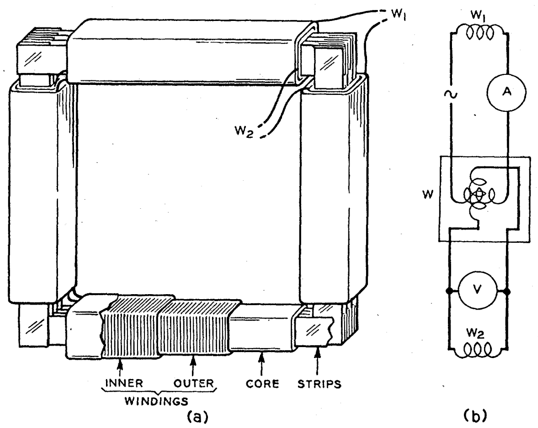

FIG. 78. Epstein, or Lloyd-Fisher, apparatus for measuring power dissipated in a specimen formed from strips. Coils, form wound as shown at (a), are connected as shown at (b) to voltmeter $V$ and wattmeter $W$. Same coils are also used for ballistic measurements.

### *Magnetometers*

Years ago the intensity of magnetization of a specimen was usually measured by the torque it exerted on a small magnetic needle of known moment suspended nearby. The deflection of a light beam by a mirror attached to the delicate suspended system is still a very sensitive indicator of the state of magnetization of a specimen under investigation, but for most purposes the uncertainty in the positions of the magnetic poles of the specimen makes the magnetometer an instrument of low precision.

In principle, the magnetometer measures the field strength at a known distance from the poles of the specimen, in accordance with Eq. (1). It has been used most recently for measuring, with not too great accuracy, the magnetic properties of very small specimens in the form of fine wires or thin films deposited by evaporation or electrodeposition. In the arrangement proposed by H. Tobusch and modified by others (Fig. 79), two oppositely directed needles of equal moment are placed one above the other on the same suspension, and the specimen made with a length about equal to the distance be-

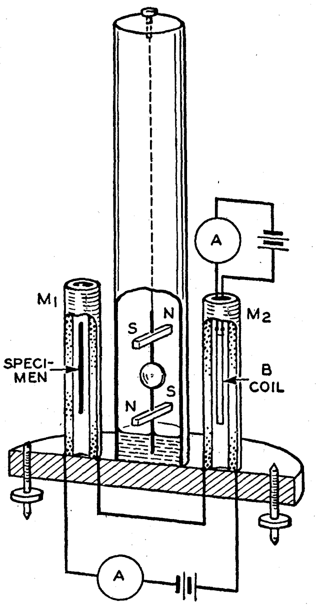

FIG. 79. Magnetometer for small specimen of wire or foil. $M_1$ and $M_2$ are similar magnetizing coils. Moment of specimen is balanced by adjusting current through small coil $B$ placed inside of $M_2$, until suspended needles NS return to original positions as indicated by light reflected from mirror (Tobusch-Bozorth).

tween the needles and placed parallel to the suspension. In this arrangement one pole of the specimen produces a twist on the nearest needle and the other pole a twist in the same sense on the other needle. The resultant twist is neutralized by a small solenoid ($B$-coil) placed opposite the specimen and energized with a current that is proportional to the magnetization of the specimen. In this way films as thin as $10^{-5}$ cm, weighing considerably less than a milligram, have been investigated in fields of various strengths and at different temperatures.

### *Para- and Diamagnetic Materials*

Measurements of weakly magnetic substances usually depend on the translational force exerted on a magnetized body in a non-uniform magnetic field. Faraday suspended the material to be measured between the poles of an electromagnet, displaced from the central position so that the field gradient was large, and used a torsion balance to measure the force pulling the material toward the stronger part of the field or repelling it therefrom. This method was developed by P. Curie and used in his classical researches on the effect of temperature on paramagnetic and diamagnetic materials.

When the field-gradient is $dH/dx$, the force on material of magnetic moment $M$ is $MdH/dx$ dynes, and when expressed in terms of susceptibility and volume (Eq. (3)) the force is

$$f = \kappa v H dH/dx.$$

The force is zero at the central position between the poles of the magnet because here $dH/dx$ is zero, and the force is a maximum at some point as shown by $P_1$ in Fig. 80 and falls off again toward zero at points distant from the magnet.

To avoid the laborious mapping of the field to determine the product $HdH/dx$, and the careful setting of the sample at a given position, C. Chéveneau proposed that the magnet be moved until the sample deviates a maximum amount from its zero position $P$ (Fig. 80), against the restoring force of the torsion balance shown in the upper part of the figure. The deflection is then proportional to the susceptibility and to the maximum value of $HdH/dx$, and the

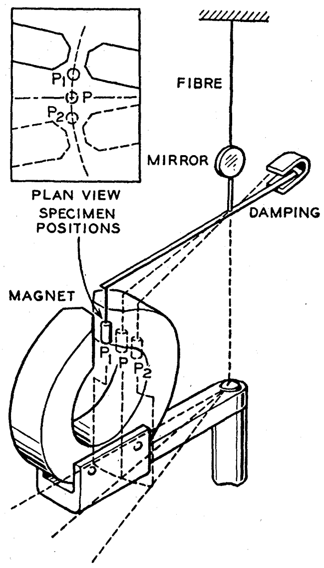

FIG. 80. Curie-Chéveneau balance for dia- and paramagnetic measurements. Specimen is placed on one arm of torsion balance and magnet is rotated in one direction about axis of balance until maximum deflection of balance is recorded, using mirror. Magnet is then rotated in opposite direction and deflection in opposite direction recorded. Sum of two deflections is proportional to susceptibility.

proportionality factor is determined by calibration with a substance of known susceptibility.

Various refinements of these methods have been suggested. In one the torsion is measured electrically by the current in a solenoid that reacts with the field of a permanent magnet to produce a torque on the suspending fibre. P. Théodoridès has used a translational instead of a torsional balance, following the method of P. Weiss and G. Foëx described below. An ingenious quick-reading device has been described by W. Sucksmith.

### *Liquids and Gases*

The force exerted upon a magnetic material in an inhomogeneous magnetic field is also the basis of the methods of L. Gouy and G. Quincke for measuring the susceptibilities of liquids. There are numerous modifications of the original methods and adaptation has been made to the measurement of gases. The force on the liquid in the field is measured by its rise in a capillary tube, against the force of gravity. A simple modification is shown diagrammatically in Fig. 81 which shows the poles of an electromagnet producing a high field at the surface of the liquid, the movement of which is measured by a microscope when the field is applied or removed. In using the method for gases a standard liquid of known susceptibility is employed and the gas fills the remainder of the tube. The force on the gas, caused by the field, adds vectorially to that on the liquid and the displacement of the meniscus is a measure of the resultant.

The force on a column of liquid (or gas) extending from a place where the field is $H_1$ to where it is $H_0$ may be obtained by integration of Eq. (3) to give

$$F = \kappa A(H_1^2 - H_0^2)/2,$$

$A$ being the cross-sectional area of the column. If a column of liquid (susceptibility $\kappa_1$) is opposed by a column of gas (susceptibility $\kappa_2$), the relation becomes

$$F = (\kappa_1 - \kappa_2)A(H_1^2 - H_0^2)/2.$$

Usually $H_1$ is the field strength directly between the poles of the magnet at the boundary between liquid and gas, and $H_0$ at the other end of the column is negligibly small. The susceptibility of gases has also been measured with the aid of a sensitive torsion balance.

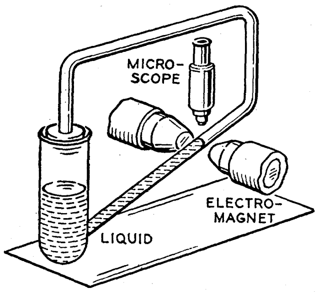

FIG. 81. Susceptibility of liquid is determined by noting (with microscope) the displacement of its meniscus in a capillary tube when the field of an electromagnet is applied in direction at right angles to tube (L. Gouy, G. Quincke).

### *Other Special Measurements*

The measurement of the saturation induction of ferromagnetic materials has been made with great accuracy by P. Weiss and his students in fields up to 20,000 oersteds, at temperatures ranging from about −250 to +800°C. For this purpose they have used some modification of the scheme shown in Fig. 82, in which the force is produced by a field gradient as in the Faraday method, but is measured as a translation instead of a torsion. Such a pendulum magnetometer has a light horizontal beam with the specimen mounted at one end in a region of high field gradient. The force due to the field is balanced by adjusting a current in a solenoid in which is a plunger attached to the other end of the beam, and the point of zero deflection is indicated by a tilting mirror. In a refinement of this method, used by L. W. McKeehan, the specimen (e.g., of a single crystal) is mounted in a strong uniform field, easily measurable, and a readily calculable gradient is applied by means of two thin Helmholtz coils mounted near each other with fields opposing. In this way one avoids the mapping of the field, and also the uncertainty in the direction of the field and the rotational hysteresis in the specimen, difficulties likely to be encountered when a torsion balance is used.

Other methods of measurement in strong fields deserve brief mention. H. DuBois measured the

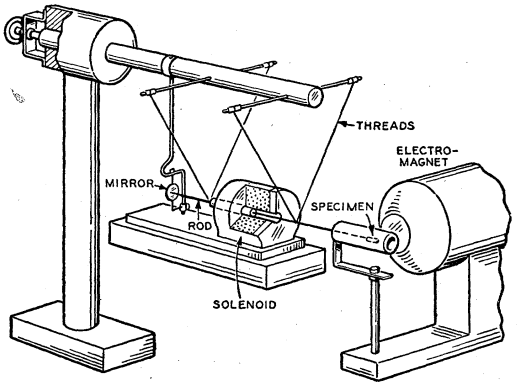

FIG. 82. Pendulum magnetometer has specimen on light frame hung from threads. Inhomogeneous field of electromagnet (half of which is shown) causes horizontal motion of frame, detected by tilting of mirror. Motion is compensated by magnetic plunger in solenoid, current through which is measure of magnetization of specimen (P. Weiss and G. Foëx).

intensity of magnetization by observing the rotation of the polarization of light reflecting from a magnetized surface (Kerr effect). The strength of a field can be measured by noting the change in the electrical resistance of a bismuth wire immersed in the field or by observing the force (opposing gravity) produced on a conductor carrying a known current (Eq. (4)). Special methods have been devised for measureing the magnetic properties of materials in which these vary with direction, as in single crystals.

In weak fields, such as that of the earth, the methods of geomagnetism can be used. In 1936 an apparatus was constructed[11] for measuring and recording rapid changes in fields as small as one gamma ($10^{-5}$ oersted), and during World War II this method was developed to even higher sensitivity. A piece of high permeability material is subjected to an alternating field of moderate amplitude, and if an additional steady field (that to be measured) is also present an alternating voltage of twice the exciting frequency will be produced and can be amplified and recorded. Its amplitude is proportional to the field strength to be determined.

A number of curve tracers have been made for taking magnetization curves and hysteresis loops so that a permanent accurate record may be obtained in a short time. Among the more accurate and convenient of these are the devices of F. E. Haworth, W. Thal and P. P. Cioffi. For the investigation of magnetic properties at high frequencies special techniques of some complexity must be used.

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[11] H. Aschenbrenner and G. Goubau, Hoch: Tech. u. Elek: akus. **47**, 177–181 (1936).

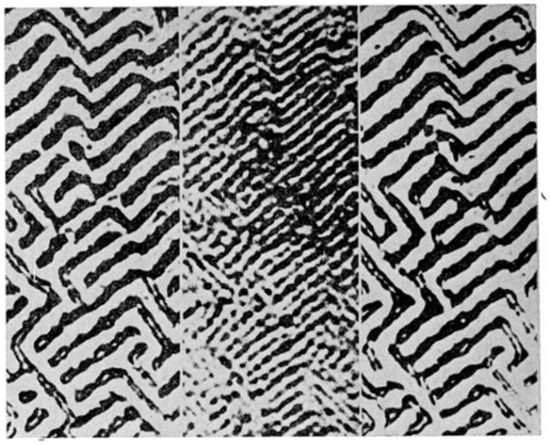
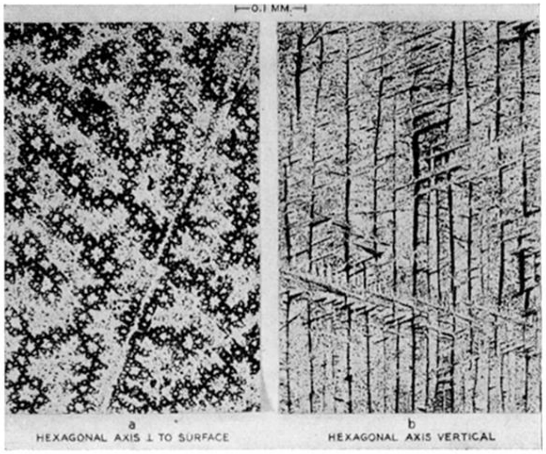


FIG. 35. Pattern of fine powder on surface of iron crystal is caused by domain structure beneath. In middle section magnetization of specimen is zero (McKeehan and Elmore). Magnification about 200 times.



a
HEXAGONAL AXIS ⊥ TO SURFACE

b
HEXAGONAL AXIS VERTICAL

Fig. 36. Powder patterns of cobalt on surfaces perpendicular (a) and parallel (b) to hexagonal axis of crystal (H. J. Williams).

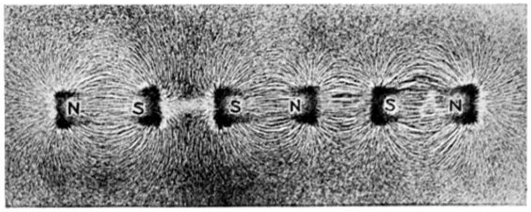


FIG. 4. Iron filings indicate the direction of the magnetic field near a group of bar magnets, show the lines of force emanating from S poles and converging on N poles.