Average Energy of Beta-Rays Emitted by Radioactive Isotopes

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For those radio elements whose beta-ray spectra have been adequately investigated, the average kinetic energy $\bar{E}_\beta$ is computed by graphical methods. $\bar{E}_\beta$ is calculated also as a function of the atomic number Z and the spectral limit E_o by means of Fermi's theory for allowed transitions, and compared to these experimental results. It is found that only the well-known experimental values of $\bar{E}_\beta$ for the forbidden spectrum of RaE deviates very markedly from the calculated one.

IN many biological and medical studies where radioactive isotopes are administered to animals or human beings it is important to estimate rather closely the energy absorbed by living tissues. It has been shown by Marinelli[1] that in a tissue mass of linear dimensions large compared to the range of the beta-particles, wherein an isotope is totally disintegrated and uniformly distributed during its lifetime, the beta-radiation dose D_β can be expressed by

$$D_\beta = 88\bar{E}_\beta TC \qquad (1)$$

where D_β is given in equivalent roentgens (one equivalent roentgen ~ 83 ergs per gram of tissue); $\bar{E}_\beta$ is the average kinetic energy of the beta-rays in Mev; T is the half-life in days; and C is the isotope concentration in microcuries per gram of tissue.[2]

In order to make use of Eq. (1) C must be determined by the investigator. In the literature there are data for T and the maximum kinetic energy E_0 for each isotope,[3] but to our knowledge $\bar{E}_\beta$ is explicity stated[4,5] only for RaE and As^{76}. This inconvenience was readily overcome by proper computation of $\bar{E}_\beta$ from experimentally determined beta-ray spectra published in the literature;[6] the results are collected in Table I. Those spectra which are complex were decomposed according to the scheme of Kurie, Richardson, and Paxton.[7] This procedure involves always a certain degree of arbitrariness, but it is not essential to the determination of $\bar{E}_\beta$; nevertheless it was carried out in order to extend the comparison with theory; for the same purpose component spectra of Sc^{44}, Ir^{194}, UX_2, and UZ have been also included.

Only reports on magnetic spectrometer investigations from 1939 on have been considered here, in view of the complications arising by back-scattering from the spectrometer source-mounting, not always recognized before that time. Reports based on cloud-chamber measurements also have been discarded because of their well-known limitations.

For the convenience of the reader, T, E_0, and the range in water[8] of the beta-rays of energy E_0 are included in the table. The type of radiation emitted by each isotope is indicated in column 4 by conventional symbols except that zero is used to indicate the absence of gamma-radiation. Column E_β refers to the average kinetic energy of the component spectra, whereas $\bar{E}_\beta$ represents the average kinetic energy of those complex spectra which could be broken down completely.

In the case of Cd^{107} and In^{114}, the internal conversion of the gamma-ray belonging to those

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[1] L. D. Marinelli, Am. J. Roentgenology and Radium Therapy **47**, 210 (1942). In this reference the decimal point in the expression for D_β (Eq. 2) is misplaced.

[2] By microcurie is meant the amount of radioactive isotope which disintegrates at the rate of 3.7×10^4 atoms per second.

[3] G. T. Seaborg, Rev. Mod. Phys. **16**, 1 (1944).

[4] A. Flammersfeld, Zeits. f. Physik **112**, 727 (1939).

[5] G. L. Weil, Phys. Rev. **62**, 229 (1942).

[6] The experimental ordinates $N(H\rho)$ of the beta-ray spectra are converted to $N(W)$ by multiplying them by $W/H\rho$. Curves of $N(W)dW$ and $N(W)WdW$ *vs.* W thus obtained are planimetered, the ratio of the areas yielding $\bar{E}_\beta+mc^2$.

[7] Kurie, Richardson, and Paxton, Phys. Rev. **49**, 368 (1936).

[8] The range of the spectra was calculated by Feather's formula with the constants recently determined by Bleuler and Zünti. (E. Bleuler and W. Zünti, Helv. Phys. Acta **19**, 137 (1946).

TABLE I.

Element	Z	A	Radiation	T (days)	No. of transition	% ***	Allowed or forbid.	E_0 (Mev)	E_β (Kev)	$\bar{E}_\beta$ (Kev)	Maximum range in water (cm)	References and Fig. 1	Element
C	6	11	β^+, 0	0.01415	1		a	0.97 ±0.01	380±40	380±40	0.41	1	C
N	7	13	β^+, 0	0.00703	1		a	1.24 ±0.02	475±45	475±45	0.55	2	N
Na	11	22	β^+, K, γ	1170	1	~100	f	0.575±0.03	225±20	225±20	<0.25	3	Na
		24	β^-, γ	0.61	1		f	1.39 ±0.005	540±20	540±20	0.64	4	
P	15	32	β^-, 0	14.5	1		f	1.712±0.008	695±20	695±20	0.82	5	P
Cl	17	38	β^-, γ	0.0259	3	53	a	4.94 ±0.06	2230±90	} 1390±70	2.70	6.0	Cl
						11		2.79 ±0.06	1190±40			6.1	
						36		1.19 ±0.08	400±35			6.2	
K	19	40	β^-, γ	5·10″	1		f	1.35 ±0.05	490±60	490±60	0.61	7	K
Sc	21	44	β^+, γ	0.167	2*		a	1.47 ±0.02	645±35	——	0.68	8	Sc
V	23	48	β^+, K, γ	16	1	58	a	0.715±0.015	300±25	(300±25)·(0.58)	0.25	9	V
Mn	25	52	β^+, K, γ	6.5	1	35	a	0.58 ±0.03	240±20	(240±20)·(0.35)	<0.25	10	Mn
		56	β^-, γ	0.108	3	50	a	2.81 ±0.05	1240±50	} 890±40	1.48	11.0	
						30		1.04 ±0.03	410±35			11.1	
						20		0.65 ±0.1	280±25			11.2	
Fe	26	59	β^-, γ	47	2	50	f	0.46 ±0.01	150±15	} 120±15	<0.25	12.0	Fe
						50		0.255±0.01	85±10			12.1	
Co	27	55	β^+, K	0.75	1		f	1.5 ±0.05	515±90	(515±90)·(?)	0.70	13	Co
		56	β^+, γ	85	1		f	1.5 ±0.05	655±35	655±35	0.70	14	
		58	β^+, K, γ	65	1	15	a	0.47 ±0.015	195±20	(195±20)·(0.15)	<0.25	15	
Cu	29	61	β^+, K, 0	0.142	1	78	a	1.23 ±0.02	555±40	(555±40)·(0.78)	0.54	16	Cu
		64	β^-, 0	0.53	1		a	0.578±0.003	175±30	} (205±30)·(0.58)	<0.25	17	
			β^+, K, γ		1	30	a	0.659±0.003	265±25			18	
Zn	30	63	β^+, K, γ	0.0271	3	85	a	2.36 ±0.04	1080±50	} (985±40)·(0.98)	1.19	19.0	Zn
						9		1.40 ±0.04	615±30			19.1	
						4		0.46 ±0.03	180±20			19.2	
Br	35	82	β^-, γ	1.5	1		f	0.465±0.01	150±15	150±15	<0.25	20	Br
Cd	48	107	β^+, K, γ	0.28	1	0.31	a	0.32 ±0.01	140±20	(140±20)·(0.003)+92	<0.25	21	Cd
In	49	114	β^-, (γ)	50	1		a	1.98 ±0.03	765±30	940±30	0.98	22	In
Sb	51	124	β^-, γ	60	2	55	f	2.45 ±0.07	980±40	} 660±35	1.24	23.0	Sb
						45		0.74 ±0.03	260±25			23.1	
I	53	130	β^-, γ	0.525	2	45	f	1.03 ±0.02	360±20	} 270±20	0.43	24.0	I
						55		0.61 ±0.03	195±20			24.1	
		131	β^-, γ	8.0	1		f	0.595±0.01	205±20	205±20	<0.25	25	
La	57	140	β^-, γ	1.67	3	12	f	2.12 ±0.08	835±60	} 495±40	1.05	26.0	La
						60		1.40 ±0.04	510±40			26.1	
						28		0.90 ±0.03	320±30			26.2	
Ir	77	194	β^-, γ	0.81	2*		f	2.18 ±0.04	835±50	——	1.09	27	Ir
RaE	83	210	β^-, 0	4.85	1		f	1.17 ±0.005	330±10	330±10	1.51	28	RaE
UX_2	91	234	β^-, γ	0.00079	2*		a	2.32 ±0.005	865±50	——	1.17	29	UX_2
UZ	91	234	β^-, γ	0.28	2**	90	a	0.45 ±0.03	150±20	——	<0.25	30	UZ

* Only the highest energy spectrum considered.
** Only the lowest energy spectrum considered.
*** % of particles in component spectra of electron-emitters and Zn^{65}, or % of positron emission to K-capture.

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[2] K. Siegbahn and H. Slaetis, Arkiv foer Mat. Astr. o. Fysik **32**, No. 9 (1945).
[3] W. M. Good, D. Peaslee, and M. Deutsch, Phys. Rev. **69**, 313 (1946).
[4, 5] K. Siegbahn, Phys. Rev. **70**, 127 (1946).
[6] N. Hole and K. Siegbahn, Arkiv foer Mat. Astr. o. Fysik **33A**, No. 9 (1946).
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[8] G. P. Smith, Phys. Rev. **61**, 578 (1942).
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[17] A. W. Tyler, Phys. Rev. **56**, 125 (1939).
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[28] A. Flammersfeld, Zeits. f. Physik **112**, 727 (1939); G. J. Neary, Proc. Roy. Soc. London **175**, 71, (1940).
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[30] H. Bradt, and P. Scherrer, Helv, Phys. Acta **18**, 405 (1945).

spectra is nearly 100 percent; therefore the energy of the resulting beta-line is added to the average kinetic energy of the beta-spectrum. Notice must be taken that in the case of positron emission accompanied by K-capture, $\bar{E}_\beta$ equals E_β times the fraction of disintegrations which exhibit positron emission; this is necessarily so on account of the definition of the microcurie adopted above. In order to simplify the task of the reader, in the event that a more accurate value of the fraction be available in the future, $\bar{E}_\beta$ has been expressed as a product in the appropriate column. The average deviations in column 10 are based somewhat empirically upon the accuracy of E_0 and upon the completeness of the spectra as reported by the original investigators.

The calculation of E_β for simple spectra was based on:

$$W_\beta = E_\beta + 1 = \int_1^{W_0} N(W) W dW \Big/ \int_1^{W_0} N(W) dW \quad (2)$$

where $W_0=E_0+1$ in units of mc^2 and $N(W)$ is given by Fermi's formula:

$$N(W)dW=\frac{|M|^2}{\tau_0}F(Z, W)\times(W_0-W)^2(W^2-1)^{\frac{1}{2}}WdW. \tag{3}$$

For $Z=0$ and for allowed transitions, Eq. (2) can be integrated directly, namely:

$$W_\beta=E_\beta+1=\frac{\int_1^{W_0}N(W)WdW}{\int_1^{W_0}N(W)dW}=\frac{W_0(W_0^2-1)^{\frac{1}{2}}(4W_0^4-8W_0^2+49)-(30W_0^2+15)\cosh^{-1}W_0}{(W_0^2-1)^{\frac{1}{2}}(8W_0^4-36W_0^2-32)+60W_0\cosh^{-1}W_0}.$$

For $Z\neq 0$, the expression $F(Z, W)$ was computed in the case of electron emission for $Z=27$, $Z=51$, and $Z=82.2$, and in the case of positron emission for $Z=27$. These calculations were based on the approximations of Nordheim and Yost[9] except in the case of $Z=82.2$ where the formula given by Fermi[10] was used.

The results of these calculations are shown by the continuous lines in Fig. 1, where the ratios E_β/E_0 are plotted as a function of E_0 and Z; the line marked $Z=-27$ refers to positron emission. In the same figure, the values of E_β/E_0 which were obtained by graphical computation from the experimentally determined spectra, are represented by points of different types grouped according to the atomic number Z of the emitter. The number associated with each experimental point corresponds to the reference column of Table I. The error of the experimental ratios E_β/E_0 is indicated only in the case of RaE for which both E_0 and E_β are known with the smallest uncertainty. The experimental E_β for the forbidden RaE spectrum, for which Fermi's plot is not straight, is much smaller than the calculated

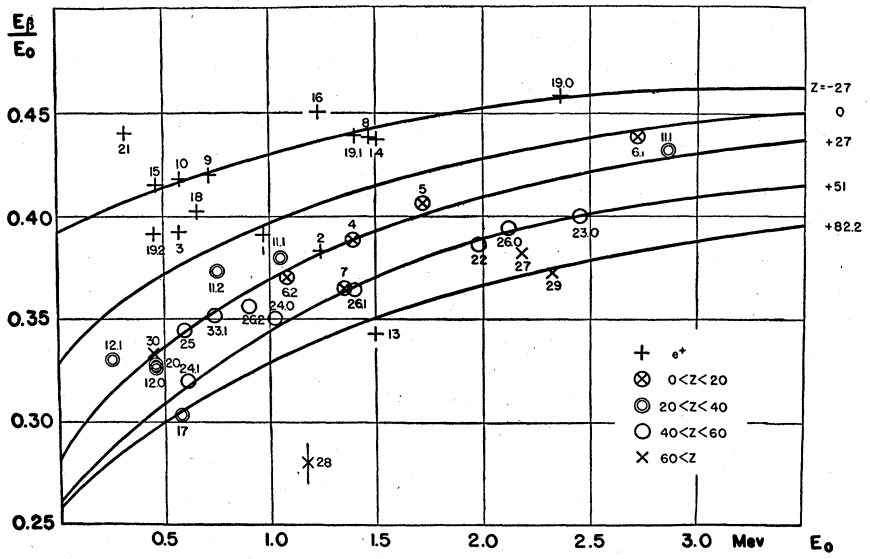

FIG. 1. The ratio of the average energy E_β to the maximum energy E_0 of simple spectra as a function of E_0 and Z.

Continuous lines: Fermi's theory for allowed transitions.
Points: Calculated by graphical methods from experimentally determined β-spectra; numbers associated with points refer to Table I and references printed directly beneath it.

[9] L. W. Nordheim and F. L. Yost, Phys. Rev. **51**, 942 (1937).
[10] E. Fermi, Zeits. f. Physik **88**, 161 (1934).

value for an allowed spectrum of $E_0 = 1.17$ Mev. An analogous situation is presented by Co^{55} for which, however, the experimental data should be considered as preliminary only.

The measured spectrum of several isotopes (N^{13}, K^{40}, Cu^{64}) shows a greater number of low energy particles than predicted by Fermi's theory for allowed spectra; it remains to be seen whether these discrepancies are caused by faulty experimental techniques. In any case, for these elements, the experimental value of E_β is only a few percent smaller than the calculated one.

We may conclude, therefore, that the theoretical curves in Fig. 1 allow an estimation of the average kinetic energy (within the accuracy necessary for biological investigations), for all simple[11] positron- and electron spectra of known E_0 and for those complex spectra in which the disintegration scheme is well established.

It is realized that the contents of this article supply only part of the physical information necessary to biophysicists engaged in the application of radioactive elements to biological investigations. A more detailed analysis of the general problems of radiation dosimetry involved in such studies will be the object of a future publication elsewhere.

[11] In the presence of gamma-radiation simplicity is indicated if the number of beta-gamma coincidences per recorded beta-ray is independent of the beta-ray energy for the whole spectrum.