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Recent Research in Meson Theory

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1. INTRODUCTORY REMARKS*

A MATHEMATICAL theory of mesons in interaction with nuclear particles can today be based only on the quantum theory of fields. Classical theories may occasionally serve as an illustration, but in the actual problems quantum theory is, of course, indispensable: mesons must be described by quantized meson fields. This has the grave consequence that meson theory shares with quantum electrodynamics the well-known defects, and the search for remedies to these defects is indeed a prevailing feature in recent literature on meson theory. There are more modest and more pretentious theories, regarding the requirements of internal consistency and relativistic invariance. The most unsophisticated approach to the problems is to apply the ordinary perturbation methods (expansion with respect to an interaction parameter) and to omit higher order terms although they may be large or even infinite. General correspondence arguments, or the hope of justification by future theories, may be quoted in favor of this procedure, which imitates the usual procedure in quantum electrodynamics where it seems to give correct results. However, the nuclear forces, which are supposed to be caused by the interaction of the nucleons with the meson field, are rather strong forces in comparison with the electromagnetic forces, which means that the expansion parameter of perturbational calculations in meson theory is considerably larger than the corresponding parameter in quantum electrodynamics (Sommerfeld's fine-structure constant). Therefore, it may be thought that the conditions are, in meson theory, more favorable to an expansion into falling rather than rising powers of the coupling parameter. This kind of expansion is characteristic of the *strong coupling theories*.[1] Here the self-energy problems present themselves in a somewhat different form: If the nucleon is assumed to have a small but non-vanishing radius a, the convergence of the strong coupling expansions is improved by choosing smaller values of a; but in the limit of a point source, $a\rightarrow 0$, the inertia effects of the meson field become infinite.[2] No way to avoid this dilemma has been discovered so far, and this has been obstructive to a relativistic description of nucleons in the strong coupling theories. On the other hand, several attempts have been made recently to *subtract the infinities* of the quantized field theories without violating the relativistic requirements. We mention here[3] the λ-limiting procedure, eventually supplemented by Dirac's new method of field quantization, Heitler's and Peng's theory of radiation damping, and Stueckelberg's theory which is based on Heisenberg's S matrix scheme.

* See the appendix for a summary of notation.

[1] See Section 4.

[2] Except in the case of the scalar field; cf. footnote 26.

[3] For quotations see section 5.

All these subtraction theories are essentially weak coupling theories, using expansions according to rising powers of the interaction parameters.

Even from this superficial survey of the present situation it is clear that "meson theory" and "meson theory" may be very different theories. This should always be remembered in confronting the theory with experimental results. Moreover, a particular theory may be apparently adequate in one respect and quite insufficient in another. If, for instance, certain theoretical cross sections are found to be in agreement with experimental data, such a statement will hardly prove very much, whereas it might be much more valuable to state a definite disagreement enabling us to rule out particular theories. Therefore, in the following report on recent theoretical work, such items have been selected which are apt to reveal the deficiencies and weaknesses of the various theories rather than their positive achievements. The balance of such a review is rather embarrassing: none of the theories worked out so far proves to be entirely satisfactory. Does this mean that meson theory should be rejected altogether? One will certainly hesitate to do so today, considering the unsettled condition of the quantum theory of fields. It should also be kept in mind that the *basic ideas* of meson theory are supported, though only qualitatively, by several striking facts: (1) the approximate equality of the range of nuclear forces and the Compton wave-length of cosmic-ray mesons, (2) the β-decay of the meson which has been found more or less in accordance with Yukawa's predictions, and (3) the anomalies of the magnetic moments of the proton and the neutron, which seem to indicate the presence of a meson cloud around the nucleon. All this may be encouraging for further theoretical work. But a definite judgment on meson theory must be postponed until a very marked progress in relativistic quantum theory is accomplished.

2. CONCERNING THE CHOICE OF THE HAMILTONIAN

Cosmic-ray experiments provide some information about the fundamental properties of mesons, but more complete and reliable evidence would be highly desirable. Meson *masses* (rest masses) ranging[4, 5] from about 20 to nearly 1000 electron masses have been reported; but, unfortunately, the accuracy of most measurements seems to be rather poor, and the reality of the mass spectrum may, therefore, be doubted. In meson theory one generally assumes *one*, or at most *two*, mass values. In order to determine the *spin* of cosmic-ray mesons, the shower production by mesons[6] has been compared with the theoretical predictions;[7] the result is in favor of spin 0 or $\frac{1}{2}$ and seems to preclude spin 1; but spin 1 mesons with shorter lifetimes may be present in the higher layers of the atmosphere.[8] Spin $\frac{1}{2}$ mesons are only acceptable in meson pair theories, where the elementary process assumed is the emission by the nucleon of a meson *pair* (instead of a single meson, as in the ordinary or Yukawa theory). However, the pair theories fail to account for the spin dependence of the nuclear forces,[9] and we want to disregard them here. So we are left with mesons of spin 0 (scalar or pseudoscalar fields) and possibly spin 1 (vector or pseudovector fields).

The existence of *neutral mesons* ("neutrettos") will be difficult to prove from cosmic-ray observations alone. It is well known that meson theory has to assume neutral mesons in order to account for the charge independence of nuclear forces which has been inferred from the proton-neutron and proton-proton scattering data (equality of the ^1S-potentials in both cases). There are two kinds of theories which warrant automatically the charge independence of the forces: Kemmer's charge-symmetrical theory which introduces both charged and neutral mesons in a symmetrical manner,[10] and Bethe's "neutral theory" in which only neutral mesons are supposed to have an interaction with nuclear particles.[11] This neutral theory, however, can hardly be said to conform to

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[4] H. Maier-Leibnitz, Zeits. f. Physik **112**, 569 (1939). See also J. A. Wheeler and R. Ladenburg, Phys. Rev. **60**, 754 (1941).
[5] L. Leprince-Ringuet, lecture to the Physical Society, Zurich, 1945.
[6] M. Schein and P. S. Gill, Rev. Mod. Phys. **11**, 267 (1939); R. E. Lapp, Phys. Rev. **64**, 255 (1943).
[7] R. F. Christy and S. Kusaka, Phys. Rev. **59**, 414 (1941); S. B. Batdorf and R. Thomas, Phys. Rev. **59**, 621 (1941).
[8] H. Snyder, Phys. Rev. **59**, 1043 (1941).
[9] W. Pauli and Ning Hu, Rev. Mod. Phys. **17**, 267 (1945); J. M. Blatt, Phys. Rev. **69**, 285 (1946).
[10] N. Kemmer, Proc. Camb. Phil. Soc. **34**, 354 (1938).
[11] H. A. Bethe, Phys. Rev. **55**, 1261 (1939).

the primary ideas of meson theory. In fact, the range of nuclear forces and the β-lifetimes of nuclei are in such a theory determined by the mass and β-lifetime of a neutral meson, which is entirely different from the charged mesons observed in cosmic radiation. So just those features which made Yukawa's theory so attractive, when in 1937 the cosmic-ray meson was discovered, get completely lost in the neutral theory. Moreover, this theory fails to explain the magnetic anomalies of the nucleon, since a neutral meson cloud gives no contribution to the magnetic moment. The neutral theory should, therefore, not be taken too seriously. The following discussion is confined to charge-symmetrical theories, except in cases where general problems are more easily exemplified by other theories.

If only *one* kind of meson field is admitted, the *scalar* field can be discarded since it cannot interact with the nucleon spin; hence, for instance, there would be no explanation for the magnetic anomalies of the proton and neutron, apart from other deficiencies. Little attention has been paid to the *pseudovector* theory, presumably on account of the fact that it gives rise to repulsive forces in both S-states of two-body problems according to the weak coupling approximations;[12] but it seems that this theory has never been investigated by means of the strong coupling approximation. As to the *pseudoscalar* and *vector* theories, the difficulties are less obvious. It will be our main task to discuss the more subtle problems in which these difficulties appear.

For reference purposes we list in the appendix the Lagrangians of the pseudoscalar and vector fields in interaction with a nucleon field.[12]

3. THE TENSOR-FORCE. MØLLER-ROSENFELD AND SCHWINGER MIXTURES

We start with the discussion of the tensor-force problem, because here we can simply make use of the plain perturbation theory (usual second-order approximation). In fact, since only arguments concerning sign and order of magnitude are essential, both the strong coupling and the subtraction theories lead to the same conclusions. For the same reason we need only consider the static forces (which are derived assuming the nucleons to be at rest). Eventual close distance forces (δ-potentials) are always supposed to be eliminated by virtue of suitable additional terms in the Lagrangian.

With these restrictions, the two-body potentials resulting from the pseudoscalar and vector, charge-symmetrical theories can be written

$$W_{ps}=(\boldsymbol{\tau}'\cdot\boldsymbol{\tau}'')\left[\tfrac{1}{3}(g_{ps}\mu_{ps})^2(\boldsymbol{\sigma}'\cdot\boldsymbol{\sigma}'')+g_{ps}{}^2Tr\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}\right]\frac{\exp(-\mu_{ps}r)}{4\pi r},\quad (1)$$

$$W_v=(\boldsymbol{\tau}'\cdot\boldsymbol{\tau}'')\left[(g_v\mu_v)^2+\tfrac{2}{3}(f_v\mu_v)^2(\boldsymbol{\sigma}'\cdot\boldsymbol{\sigma}'')-f_v{}^2Tr\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}\right]\frac{\exp(-\mu_v r)}{4\pi r}.\quad (2)$$

$\boldsymbol{\sigma}'$, $\boldsymbol{\sigma}''$, $\boldsymbol{\tau}'$, $\boldsymbol{\tau}''$ are the spin and isotopic spin vector operators of the two nucleons, and T is the tensor-force operator

$$T=\sigma_r'\sigma_r''-\tfrac{1}{3}(\boldsymbol{\sigma}'\cdot\boldsymbol{\sigma}'')$$

(the suffix r indicates the projection on the relative coordinate vector). Introducing the total spin vector $\mathbf{s}=\frac{1}{2}(\boldsymbol{\sigma}'+\boldsymbol{\sigma}'')$, we have

$$(\boldsymbol{\sigma}'\cdot\boldsymbol{\sigma}'')=2\mathbf{s}^2-3,\quad T=2(s_r{}^2-\tfrac{1}{3}\mathbf{s}^2).\quad (3)$$

The eigenvalues of the operator $(\boldsymbol{\sigma}'\cdot\boldsymbol{\sigma}'')$ are -3 (spin-singlet, $\mathbf{s}^2=0$) and $+1$ (spin-triplet, $\mathbf{s}^2=2$); equally the eigenvalues of $(\boldsymbol{\tau}'\cdot\boldsymbol{\tau}'')$ are -3 (charge-singlet) and $+1$ (charge-triplet).

In a qualitative discussion of the two-nucleon problem it is convenient to consider the tensor interactions as small perturbations. Neglecting the tensor-force in a zero-order approximation, the interaction W_{ps} is attractive both in the ^3S state $[(\boldsymbol{\sigma}'\cdot\boldsymbol{\sigma}'')=1,\ (\boldsymbol{\tau}'\cdot\boldsymbol{\tau}'')=-3]$ and in the ^1S state $[(\boldsymbol{\sigma}'\cdot\boldsymbol{\sigma}'')=-3,\ (\boldsymbol{\tau}'\cdot\boldsymbol{\tau}'')=1]$, and the same is true for W_v provided that $(g_v/f_v)^2$ is chosen sufficiently small.[13] In the next approximation, on account of the coupling between spin and orbital

[12] Cf. N. Kemmer, Proc. Roy. Soc. **166**, 127 (1938), where "charged" theories, including the pseudovector theory, are investigated. In the charge-symmetrical theory, only the isotopic spin factor in the potential has to be altered in the well-known way (see reference 10).

[13] The term $\sim(g_v\mu_v)^2$ in W_v arises from the interaction of the longitudinal vector mesons with the nucleons. In the strong coupling vector theory, no such term appears in W_v if $(g_v/f_v)^2<4/3$; cf. G. Wentzel, Helv. Phys. Acta **16**, 551 (1943).

motion, the Schrödinger function of the triplet state ($\mathbf{s}^2=2$) loses its spherical symmetry and becomes a $^3S+{}^3D$ mixture, and the corresponding charge distribution of the deuteron gives rise to an *electrical quadrupole moment*. Now, for this state, the factor of s_r^2 (or T) is negative in W_{ps}, positive in W_v. Thus, the *pseudoscalar* theory favors *larger* values of $(\mathbf{r}\cdot\mathbf{s})^2$, and, therefore, the charge distribution in the deuteron ground state will be *stretched* along the spin axis ("cigar shape"), in accordance with the sign of the quadrupole moment as it was determined experimentally by Rabi and his co-workers.[14] On the other hand, the pure *vector* theory gives the wrong sign and can, therefore, be ruled out.

Unfortunately, the pseudoscalar theory too proves to be unsatisfactory, if a rigorous calculation is attempted. A first difficulty arises from the r-dependence of the tensor-force potential,

$$r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}\frac{\exp(-\mu r)}{r}=\left[\frac{3}{r^3}+\frac{3\mu}{r^2}+\frac{\mu^2}{r}\right]\exp(-\mu r). \quad (4)$$

Such a potential, with its strong singularity at the origin ($r=0$), is inadmissible in wave mechanics (there would be no lowest energy level). It is true that at short distances the non-static forces are no more negligible, but they do not improve the situation. In the λ-limiting theory (cf. Section 5a) the potentials (1), (2) have been derived only for distances r large compared with a certain critical length,[15] and perhaps there may be some hope that the subtraction theories will succeed in eliminating those inadmissible singularities. But for the time being, the only remedy (in the pure pseudoscalar theory) is to cut off the potentials at shorter distances, by adopting an extended source model of the nucleon. This, of course, means falling back on a non-relativistic description of the nucleon.

According to (1) the isotropic potential in W_{ps} is the same for the ^1S state as for the deuteron ground state ($^3S+{}^3D$), since in both cases $(\boldsymbol{\tau}'\cdot\boldsymbol{\tau}'')(\boldsymbol{\sigma}'\cdot\boldsymbol{\sigma}'')=-3$; the difference of the two energy eigenvalues (that is to say, practically the whole binding energy of the ground state) must be caused by the tensor-force. Now, if the potentials are cut off at *small* distances only (cut-off radius $a\ll\mu^{-1}$), the tensor-force is still quite strong at small and intermediate distances, and one will suspect that the energy difference of the two states will turn out to be much too large ($\gg 2$ Mev). In fact, Ferretti[16] has proved that, in order to fit the two energies, the cut-off radius has to be chosen unreasonably large ($a>\mu^{-1}$), no matter what potential is assumed for $r<a$. In this respect, even the subtraction devices, as for instance the λ-limiting device, can be of no help. This seems to justify a final verdict on the pure pseudoscalar (charge-symmetrical) theory.

There remains the possibility of mixing different meson fields (by adding their Lagrangians, cf. the appendix). In particular, mixtures of pseudoscalar and vector fields have been considered by Møller and Rosenfeld,[17] and by Schwinger.[18] The two-body forces are found to be plainly additive ($W_{\text{mix}}=W_{ps}+W_v$), both in weak and strong coupling approximations. From (1) and (2) it is readily seen that in the pseudoscalar-vector mixture theory the inadmissible singularities of the tensor-force can be removed without "cutting-off." Møller and Rosenfeld proposed to assume $\mu_{ps}=\mu_v$, $|g_{ps}|=|f_v|$; in this case the tensor-force terms in the static potential W_{mix} cancel completely, and relativistic corrections must be made responsible for the electrical quadrupole moment. However, Ning Hu,[19] by a more rigorous investigation of the relativistic corrections, has proved that, choosing the constants such that the inadmissible singularities cancel, the quadrupole moment vanishes in the same approximation. On the other hand, Schwinger[18] has observed that for eliminating the r^{-3} and r^{-2} singularities it is sufficient to assume $|g_{ps}|=|f_v|$, but $\mu_{ps}\neq\mu_v$. This is easily verified by expanding the expression (4) into rising powers of μr:

$$r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}\frac{\exp[-\mu r]}{r}=\frac{3}{r^3}-\frac{\mu^2}{2r}+\cdots;$$

[14] J. M. B. Kellogg, I. I. Rabi, N. F. Ramsey, and J. R. Zacharias, Phys. Rev. **55**, 318 (1939) and **57**, 677 (1940).
[15] W. Pauli, Phys. Rev. **64**, 332 (1943).
[16] B. Ferretti, Ricerca Scient. **19**, 993 (1941), appendix. Another failure of the pure pseudoscalar theory, regarding Yukawa's theory of the β-decay, has been pointed out by E. C. Nelson, Phys. Rev. **60**, 830 (1941).
[17] C. Møller and L. Rosenfeld, Kgl. Danske Vid. Sels., Math.-fys. Medd. **17**, No. 8 (1940).
[18] J. Schwinger, Phys. Rev. **61**, 387 (1942).
[19] Ning Hu, Phys. Rev. **67**, 339 (1945).

inserting this expansion in (1) and (2) one finds that in $W_{ps}+W_v$ (with $|g_{ps}|=|f_v|$) only the admissible r^{-1} singularities remain. The sign of the tensor-force is now determined by the sign of $(\mu_{ps}^2-\mu_v^2)$, and it is easily seen from what was said above, that the correct sign of the deuteron quadrupole moment is obtained by assuming $\mu_v>\mu_{ps}$. Schwinger has pointed out that this result is satisfactory in connection with cosmic-ray evidence: Although pseudoscalar (spin 0) and vector (spin 1) mesons will be produced simultaneously in the high atmosphere, a rapid decay of the vector mesons can be expected, and only pseudoscalar mesons will be observed at low altitudes; so there is no difficulty in explaining Schein's and Gill's results concerning the shower production by mesons.[6]

It may be remarked here, that according to Schwinger's theory the vector meson must indeed be highly unstable, even if an eventual β-decay is disregarded. Consider for instance a negative vector meson in the vacuum. Applying Dirac's "theory of holes" to the proton, we have to assume all negative energy levels of the proton to be filled up in the vacuum state. Now the following third-order transition (speaking in terms of perturbation theory) can take place: First the negative vector meson is absorbed by any one of the negative energy protons, whereby this proton is transformed into a neutron of positive energy, leaving a hole in one of the negative energy proton levels. In a second step the neutron, by emitting now a *pseudoscalar* negative meson, becomes a proton again and drops back into the hole. In order to restore the energy balance, a photon will have to be emitted by any one of the charged particles involved. As a result, ignoring the virtual intermediate states, we find that the initial vector meson is decomposed into a pseudoscalar meson and a photon; this process is indeed compatible with momentum and energy conservation, if the vector meson is heavier than the pseudoscalar one, according to Schwinger's assumption. The probability of this process has not been calculated so far, but it is certainly large enough to explain a very rapid decay of the vector mesons in cosmic radiation.[20] Unfortunately, this does not exempt us from the necessity of assuming an additional, independent instability of the vector meson with respect to the β-decay. In fact, it is well known that Yukawa's theory of the nuclear β-decay requires a meson decay probability at least a hundred times larger than that of the observed cosmic-ray mesons.[21] So, if the latter are supposed to be pseudoscalar mesons, we must assign the higher decay probability to the vector mesons. Certainly, such a new assumption made *ad hoc* does not improve the internal consistency of the theory; on the other hand, it may be granted that it fits remarkably well into Schwinger's picture.

Returning to the two-nucleon problem, we recall that Schwinger's assumption $\mu_v>\mu_{ps}$ leads to the correct sign of the tensor-force. However, the theory does not stand the more stringent test of a numerical evaluation. Jauch and Ning Hu[22] have carried out a numerical integration of the wave-mechanical two-body problem with the Schwinger mixture potential ($W_{\text{mix}}=W_{ps}+W_v$, $|g_{ps}|=|f_v|$, $\mu_{ps}<\mu_v$) assuming $g_v=0$ (no coupling of longitudinal vector mesons). The mass of the pseudoscalar meson is taken from cosmic-ray measurements to be 177 electron masses. Then, the mass ratio μ_v/μ_{ps} and the coupling parameter $|g_{ps}|$ can be determined from the binding energy of the deuteron ground state and the energy of the ^1S state;[23] the mass ratio is found to be 1.6. Now the electrical quadrupole moment of the ground state can be computed: the value found is only *one-third* of the experimental value. The discrepancy becomes even larger if a $(g_v\mu_v)^2$ term is admitted in (2), since such a term would by itself account for some fraction of the $^1S-{}^3S$ energy difference; therefore, a weaker tensor-force would be needed, and a smaller quadrupole

[20] J. Hamilton, W. Heitler, and H. W. Peng, in their paper on the genesis of cosmic radiation (Phys. Rev. **64**, 78 (1943); cf. also W. Heitler and P. Walsh, Rev. Mod. Phys. **17**, 252 (1945)) do not mention the above process. It is true that their work is based on Møller's and Rosenfeld's mixture theory ($\mu_v=\mu_{ps}$). But as soon as the two mesons are supposed to have different masses, both β- and γ-decay of the heavier particle may occur in competition. (*Note added in proof:* The γ-decay is now being investigated by M. Verde.)

[21] H. A. Bethe and L. W. Nordheim, Phys. Rev. **57**, 998 (1940); E. C. Nelson, Phys. Rev. **60**, 830 (1941). The result is essentially the same in the strong coupling or subtraction theories.

[22] J. M. Jauch and Ning Hu, Phys. Rev. **65**, 289 (1944).

[23] For the neutron-proton scattering cross section at low energies Jauch and Ning Hu adopt the value 14.8×10^{-24} cm^2 which is probably too small; but this has hardly any effect on the numerical conclusions.

moment would result. Possibly non-static forces and other corrections may improve the situation, but it is hard to believe that they can account for the larger part of the quadrupole moment.

Of course, other mixture ratios g_{ps}/f_v (with cutting-off of the then reappearing r^{-3} singularities) might be tried, or even mixtures involving other types of meson fields. But no convincing theory can be obtained by simply increasing the number of arbitrary constants and adapting them to the experimental requirements.

4. STRONG COUPLING THEORY

a. Spin-Inertia, Isobar States

In Section 1 it has been mentioned that the use of an *extended source model* of the nucleon is unavoidable in the strong coupling theory. Although, therefore, only a non-relativistic theory of nucleons can result, one outstanding advantage of the extended source model theories is that they afford a straightforward approach to the field-inertia problems. In fact, it is well known that Lorentz's electromagnetic theory of the electronic mass is based on the conception of an extended electron, whereas in point source theories the electromagnetic mass turns out to be either infinite (limit $a\rightarrow 0$) or zero (subtraction theories). Of course, it is doubtful how much of the field-inertia concepts will survive in the future, but just because no definite prediction is possible it may be considered worth while to study the theoretical possibilities even within the limited scope of a non-relativistic theory.

In the electron theory it is customary to consider only the inertia with respect to translational accelerations. However, in the case of a nucleon surrounded by a meson field, since the nuclear forces are known to be spin-dependent, also rotational accelerations become of importance. Let us, for the moment, consider a *classical* particle in interaction with a field. If the particle is interpreted as an extended source of the field, its self-energy can be calculated in a non-relativistic approximation. In general, the self-energy will comprise a translational term ($\mathbf{p}^2/2m$) and a rotational term (for instance $\mathbf{P}^2/2C$, where $\mathbf{P}$ = angular momentum). If no rotational forces are active, the second term is constant and can be ignored. This may be true for the electron, but the situation is different in the nucleon case, on account of the spin-dependence of nuclear forces. At least from the standpoint of the extended source model theory—disregarding arguments taken from subtraction theories—there is no reason why the rotational term should be insignificant. In quantum theory, where the angular momentum is quantized, the rotational energy will give rise to excited states with higher spin values. If the field is charged and coupled with the charge coordinate ("isotopic spin") of the nucleon—e.g., in the charge-symmetrical theories—the charge degree of freedom also contributes to the "rotational" self-energy, and the excited states can belong to higher charge values too. Of course, this result may be expressed by saying that mesons can be bound by the "bare" nucleon, thus giving rise to compound nucleon states with higher spin and charge values. These states are usually called "*isobar states.*"

A rigorous quantum-mechanical derivation of the rotational self-energy has been achieved only by means of the *strong coupling approximation.* As to the mathematical method, we must content ourselves with a few hints. If a nucleon of infinite mass is considered, the nucleon field density ($\phi^*\phi$ in the notation of the appendix) can be replaced by a given source function $\delta_a(\mathbf{x})$ (of spherical symmetry, vanishing outside a sphere of radius $\sim a \ll \mu^{-1}$, $\int d^{(3)}x\delta_a = 1$), and the Hamiltonian reduces to $H = \text{const.} + H^0 + H', H^0$ describing the free mesons and H' their interaction with the nucleon,

$$H' = g\int d^{(3)}x\delta_a \sum_n O_n\psi_n$$

(ψ_n = meson field components, O_n = operators involving nucleon spin and isotopic spin operators, and g = coupling parameter, for instance, $g = |g_{ps}| = |f_v|$ in the Rosenfeld-Møller or Schwinger mixture theory). Since g is supposed to be large, a first requirement is to make the large interaction term H' diagonal: a unitary operator S (matrix with respect to the spin and isotopic spin indices) is determined such as to make $SH'S^{-1}$ diagonal. The operator H^0, depending on the fields canonically conjugated to the ψ_n's, does not commute with S, but successive canonical transformations can be performed to diagonalize the total Hamiltonian in the form of an expansion with respect to falling powers of the coupling parameter g.[24] In the pseudoscalar and vector theories (including pseudoscalar-vector mixtures) the condition which warrants the rapid convergence of this expansion is[25]

$$g \gg a \tag{5}$$

("strong coupling condition"). The highest order term ($\sim g^2$) is the static self-energy of the nucleon. The next following terms describe interactions of the free mesons with the (compound) nucleon; for instance, the scattering of mesons by nuclei is determined by the terms $\sim g^0$. The rotational self-energy appears among the terms $\sim g^{-2}$. In the charge-symmetrical pseudoscalar, vector, and mixture theories, it is formally equal to the kinetic energy of a spherical top

$$H_{\text{rot}} = \frac{1}{2C} |\mathbf{P}|^2 \tag{6}$$

(C is the moment of inertia and $\mathbf{P}$ the angular momentum of the spherical top). The "spin-inertia" C depends on the nucleon radius a and the coupling parameter g according to $C \sim g^2/a$. $|\mathbf{P}|$ and the projections of $\mathbf{P}$ on a space fixed and a body fixed axis (speaking in terms of the spherical top picture) have half-integer eigenvalues: j, m, n ($|m|$, $|n| \leqq j$). With respect to the physical nucleon, j and m are ordinary spin quantum numbers, and $n+\frac{1}{2}$ is the charge of the nucleon state. The eigenvalues of H_{rot} are

$$\frac{1}{2C} j(j+1),$$

$j=\frac{1}{2}$ ($m=\pm\frac{1}{2}$, $n=\pm\frac{1}{2}$) corresponding to the ordinary proton and neutron states, and $j \geqq \frac{3}{2}$ to the excited states or "isobars." The excitation energy of the lowest isobars is $\frac{3}{2}C^{-1}$. Choosing appropriate numerical values for a and g, one may expect the excitation energy to be in the order of magnitude of 50 Mev.[26]

It goes without saying that the question, whether such proton isobars really exist or not, is of greatest importance in connection with the self-energy problems, and it is to be hoped that experiments with high energy machines will soon decide this question. A theoretical estimate of the *excitation probability* can be obtained from the strong coupling theory. The excitation by photo-dissociation of the deuteron has been investigated by Jauch;[27] he finds that a few percent of the photo-nucleons should appear in excited states, if the γ-ray energy is somewhat above the excitation threshold. According to a paper by Lopes,[28] a similar yield is to be expected in high energy collisions of protons or neutrons.

b. Magnetic Moments and Mass Difference of Proton and Neutron

So far we have disregarded the interaction of the charged particles with the electromagnetic field. If we now consider a nucleon, for example, in a homogeneous magnetic field $\mathbf{H}$, the self-energy will contain an additional term $-M|\mathbf{H}|$, M being the magnetic moment of the nucleon. The calculation of M provides a very severe test for every meson theory, since it is one of the principal claims of the theory to explain the magnetic anomalies of the proton and neutron as being caused by the presence of the meson cloud.

Turning to this problem, we have to address our attention to the electric currents located in the meson cloud. In the strong coupling theory, we here meet with a difficulty which is inherent to the extended source model: As Pauli and Dancoff[29] have observed, one cannot define the electric density functions in such a way that the continuity equation is satisfied inside the source (where $\delta_a(\mathbf{x}) \neq 0$). Unfortunately, in these theories, the region inside the source gives a very

[24] Cf. G. Wentzel, Helv. Phys. Acta **13**, 269 (1940) and **14**, 633 (1941): charged scalar theory. R. Serber and S. M. Dancoff, Phys. Rev. **63**, 143 (1943): charged scalar and neutral pseudoscalar theories. W. Pauli and S. M. Dancoff, Phys. Rev. **62**, 85 (1942): symmetrical pseudoscalar theory. W. Pauli and S. Kusaka, Phys. Rev. **63**, 400 (1943): pseudoscalar, vector, and mixture theories. G. Wentzel, Helv. Phys. Acta **16**, 551 (1943): symmetrical vector theory with 2 coupling parameters.

[25] We use natural units: $\hbar = c = 1$; then g (g_{ps} or f_v) has the dimension of a length. $a \ll \mu^{-1}$ is always assumed.

[26] In the limit $a \to 0$ the excitation energy would tend to zero ($C \to \infty$). This is why a point source is inadmissible in the strong coupling theories, with the exception of the scalar theory, which makes C independent of a. Cf. R. Serber and S. M. Dancoff, Phys. Rev. **63**, 143 (1943), where the scalar theory is finally rejected on account of the lacking saturation properties of the forces.

[27] J. M. Jauch, Phys. Rev. **69**, 275 (1946).

[28] J. L. Lopes, Phys. Rev. **70**, 5 (1946).

[29] W. Pauli and S. M. Dancoff, Phys. Rev. **62**, 85 (1942)

considerable contribution to the magnetic moments. However, it happens that for the stationary states of the compound nucleon the continuity equation is exactly fulfilled in the limit of infinitely strong coupling (this remark is attributed to Houriet[30]). So, if objection is raised against using Pauli's and Dancoff's density functions, the objection is less serious in the strong coupling theory.

From the charge-symmetrical strong coupling theory, Pauli and Dancoff have derived the following expression for the magnetic moments of the stationary states j, m, n

$$M=\left(\frac{M_0}{2}+\frac{eC}{4}\right)\frac{mn}{j(j+1)} \tag{7}$$

(M_0 = magnetic moment of the bare nucleon, e = unity charge, C = spin-inertia).[31] In particular for the ground states $j=\frac{1}{2}$, $m=+\frac{1}{2}$, $n=\pm\frac{1}{2}$, one has

$$M=\pm\left(\frac{M_0}{6}+\frac{eC}{12}\right), \tag{7a}$$

the $+$ and $-$ signs applying to the proton and neutron states, respectively. According to this formula, the magnetic moments of the proton and the neutron are equal and of opposite sign, in contradiction to experience. The experimental values are

proton: $M_p=+2.79$ nuclear magnetons,[32]
neutron: $M_n=-1.9$ nuclear magnetons.[33]

It is interesting to compare this result of the strong coupling theory with the weak coupling or perturbation theory. In either theory all stationary states are mixtures of substates in which the bare nucleon occurs both in the proton and neutron states:

physical proton = mixture

$$\begin{cases}\text{(a) bare proton} + \text{neutral meson cloud,}\\ \text{(b) bare neutron} + \text{positive meson cloud,}\end{cases}$$

physical neutron = mixture

$$\begin{cases}\text{(a) bare neutron} + \text{neutral meson cloud,}\\ \text{(b) bare proton} + \text{negative meson cloud.}\end{cases}$$

In the weak coupling case, the substates labelled (b) are only small admixtures, and the contribution to the magnetic moment of the bare nucleon amounts to nearly one nuclear magneton in the physical proton state, and is practically zero in the physical neutron state:

$$M_p\cong 1+M_{\text{meson}},\quad M_n\cong -M_{\text{meson}}. \tag{8}$$

This is the theory of Fröhlich, Heitler, and Kemmer,[34] which can be made to agree at least approximately with the experimental data (difficulties arising in the subtraction theories will be discussed later). In the strong coupling theory, however, the situation is entirely changed by the fact that the substates (a) and (b) occur with equal probability amplitudes in either physical state, in the limit of infinitely strong coupling. This is the reason why in (7a) the contribution to M of the bare proton turns out equal and of opposite sign for the physical proton and neutron states.

Of course, the M-value given by Pauli and Dancoff is only the leading term of an expansion with respect to the small parameter a/g. The next correction has been computed by Houriet.[30] The sign of this correction makes $|M_p|>|M_n|$, in agreement with the experimental values, but its order of magnitude ($M_0(a/g)^4$) is much too small. This seems to confirm Pauli's and Kusaka's conclusion that actually the coupling cannot be strong. One must, however, bear in mind that the use of Pauli's and Dancoff's electric density functions may be objectionable in Houriet's calculations, since these are not confined to the limit of infinitely strong coupling. Nevertheless, one has to admit that the strong coupling theory has been unable to account for the experimental facts in this field.

In connection herewith, there is another deficiency of the theory to be mentioned. Since the magnetic moment is determined by the self-energy of the nucleon in a magnetic field, one may with equal right inquire for other electromagnetic terms in the self-energy, even if there

[30] A. Houriet, Helv. Phys. Acta **18**, 473 (1945).
[31] Pauli's and Dancoff's paper deals with the pseudoscalar theory only, but the same formula (apart from an eventual change in the numerical factor $\frac{1}{4}$) results from the vector and mixture theories. Cf. W. Pauli and S. Kusaka, Phys. Rev. **63**, 400 (1943).
[32] S. Millman and P. Kusch, Phys. Rev. **60**, 91 (1941).
[33] L. W. Alvarez and F. Bloch, Phys. Rev. **57**, 111 (1940).
[34] H. Fröhlich, W. Heitler, and N. Kemmer, Proc. Roy. Soc. **166**, 154 (1938).

is no external field assumed. The most interesting term here arises from the Coulomb interaction of the meson cloud with the bare nucleon. From the above mixture scheme it is obvious that the corresponding self-energy term is zero in the physical proton state and negative in the physical neutron state. In the weak coupling version this is of no importance, since the masses of the bare proton and neutron can be chosen such as to cancel this and other terms, and arbitrary values of the masses of the physical proton and neutron can be obtained. But in the strong coupling theory, the bare proton and the bare neutron are present each with 50 percent probability in either of the two compound states, so their contribution to the masses is the same in both cases, and the mass difference of proton and neutron is mainly determined by the above mentioned Coulomb term. Accordingly, the proton should be heavier than the neutron, very much in contradiction with experience.

This argument has been worked out quantitatively by Houriet,[30] and he has also computed the next correction term, developing into powers of a/g. He finds that the wrong result cannot be amended by these corrections, unless very artificial assumptions concerning the masses of the bare nucleons are introduced. This again confirms the conclusion that the coupling cannot be really "strong." But here again, since Pauli's and Dancoff's electric density functions had to be applied, there remains a shade of doubt whether the conclusion is quite unambiguous. In addition, the bare nucleon was assumed to have infinite mass in all these calculations, and this may affect the results to a certain extent. Finally it may be argued that a non-relativistic theory is no reliable basis for self-energy investigations. However, further evidence, pointing in the same direction, can be gathered from the two-body problems.

c. The Deuteron

If the strong coupling approximation method, as outlined in section 4a, is applied to *two* nucleons, the static self-energy ($\sim g^2$) of the two-body system is found to depend on the particle co-ordinates, and by subtracting twice the static self-energy of one particle, the static interaction potential is deduced. In the charge-symmetrical theory this potential can be written as a matrix with respect to the quantum numbers j_1, m_1, n_1, j_2, m_2, n_2 of the two particles. The submatrix corresponding to the ground states ($j_1=j_2=\frac{1}{2}$) is identical with the matrix W resulting from perturbation theory (cf. formulae (1), (2) in section 3), apart from a numerical factor (1/9); but there are additional matrix elements relating to the isobar states.[35] In the limit of infinite isobar excitation energy (spin-inertia $C\rightarrow 0$), the excited nucleon states can be ignored and the strong coupling theory coincides with the weak coupling theory discussed in section 3 (except for a change in notation required by the factor 1/9).

The most elaborate computations on the deuteron problem based on the strong coupling theory have been made by Villars.[36] Imitating Rarita's and Schwinger's well-known deuteron theory[37] which is based on the weak coupling meson theory, Villars replaces the Yukawa potential ($\exp(-\mu r)/r$) and the tensor-force potential (4) by square well potentials of arbitrary depths, both with the same range r_0. Since Villars's theory in the limit $C\rightarrow 0$ becomes identical with the Rarita-Schwinger theory, it may be said to generalize this theory by taking the nuclear spin-inertia into account. The main new feature in the wave-mechanical formalism is that, because of the isobar states, additional components appear in the Schrödinger functions of the stationary states. In the deuteron ground state, for instance, which in the Rarita-Schwinger theory is a ^3S, ^3D mixture, new admixtures of the type ^7D, ^7F, ^{11}F, $\cdots$ appear; and the ^1S state becomes a mixture with ^5D, ^9F, $\cdots$-components. Starting with arbitrary values of the spin-inertia C and the range r_0, Villars first calculates the depths of the two potentials (isotropic and tensor interaction) from the binding energy and the electrical quadrupole moment of the ground state, and with these results he can then determine the energy of the singlet state as a function of C and r_0. The result is, that the singlet state turns out to be stable (owing to a rather large ^5D

[35] For details see M. Fierz, Helv. Phys. Acta **17**, 181 (1944) and **18**, 158 (1945). In the charge-symmetrical theory, the existence of the isobar states does not affect the saturation character of the forces; cf. W. Pauli and S. Kusaka, reference 31, and F. Coester, Helv. Phys. Acta **17**, 35 (1944).

[36] F. Villars, Helv. Phys. Acta **19**, 323 (1946).

[37] W. Rarita and J. Schwinger, Phys. Rev. **59**, 436 (1941).

admixture), unless very small C values are assumed. To raise the energy of this state to the zero level requires an isobar excitation energy $(3/2C)$ of at least 300 Mev.[38] On the other hand, the strong coupling condition (5) puts an upper limit to the excitation energy at about 100 Mev. So we must again conclude that the coupling cannot be really strong, or in other words, that the spin-inertia of the nucleon cannot be very large.

Unfortunately, if the strong coupling hypothesis fails, there is little hope of arriving at a simple and convincing description of spin-inertia effects on the basis of the extended source theory. In fact, this theory proves to be quite impracticable in the intermediate and weak coupling cases. It remains to be seen how the spin-inertia problem presents itself in the point source or subtraction theories.

5. SUBTRACTION THEORIES

a. λ-Limiting Process

The primary idea of the λ-limiting theory is most easily explained by referring to the classical problem of the interaction of an electron with its own electromagnetic field. Here we consider the electron as a point source, moving along a given orbit $\mathbf{x}_1(t_1)$. The field, described by the four-potential ψ, comprises the field ψ_1 produced by the electron considered, and the field from other charges: $\psi=\psi_1+\psi_2$. Here ψ_1 is the well-known retarded potential: $\psi_1=\psi^{\text{ret}}$. Now Dirac, Fock, and Podolsky[39] have generalized Maxwell's field equations such as to define $\psi(\mathbf{x}, t)$ for "field times" t different from the "particle time" t_1, with the result that $\psi_1=\psi^{\text{ret}}(\mathbf{x}, t)$ holds everywhere outside the light-cone $(|t-t_1|<|\mathbf{x}-\mathbf{x}_1(t_1)|)$,[40] while ψ_1 vanishes inside the forward light-cone $(t-t_1>|\mathbf{x}-\mathbf{x}_1(t_1)|)$ and becomes equal to the difference of retarded and advanced potentials $(\psi^{\text{ret}}-\psi^{\text{adv}})$ inside the backward light-cone $(t-t_1<-|\mathbf{x}-\mathbf{x}_1(t_1)|)$.[41] The Lorentz force acting on the electron $\mathbf{F}=e(\mathbf{E}+[\dot{\mathbf{x}}_1\times\mathbf{H}])$ may be split into the reaction force $\mathbf{F}_1$ and the external force $\mathbf{F}_2$. The values of $\mathbf{E}$ and $\mathbf{H}$ in $\mathbf{F}$ have to be taken at the point $\mathbf{x}=\mathbf{x}_1(t_1)$, and it is well known that in the ordinary theory, because of the singularities of the retarded field at the point source, $\mathbf{F}_1$ becomes infinite. (The extended source model gives $\mathbf{F}_1=-m\ddot{\mathbf{x}}_1+\frac{2}{3}e^2(\partial^3\mathbf{x}_1/\partial t_1^3)+\cdots$ in the rest system, where m—the electromagnetic mass—tends to ∞ in the limit $a\to 0$.) However, the field generalized for $t\neq t_1$ enables us to define the required limiting values of $\mathbf{E}$ and $\mathbf{H}$ in a different way. Consider the world point $\mathbf{x}=\mathbf{x}_1(t_1)+\boldsymbol{\lambda}$, $t=t_1+\lambda_0$, where $\boldsymbol{\lambda}$, λ_0 is a *time-like* four-vector $(|\lambda_0|>|\boldsymbol{\lambda}|)$, and let the length of this vector tend to zero, while its direction is kept constant $(\lambda/\lambda_0=\text{const.})$. Coming from the forward cone $(\lambda_0>|\boldsymbol{\lambda}|)$, where $\psi_1(\mathbf{x}, t)$ vanishes, we obviously get $\lim \mathbf{E}_1, \mathbf{H}_1=0$, $\mathbf{F}_1=0$; while in the backward cone $\psi_1=\psi^{\text{ret}}-\psi^{\text{adv}}$ is $\neq 0$ but finite everywhere, since the singularities of the retarded and advanced potentials cancel each other, and accordingly a finite limiting value of $\mathbf{F}_1$ results, *viz.* $\frac{4}{3}e^2(\partial^3\mathbf{x}_1/\partial t_1^3)$ in the rest system $((\partial\mathbf{x}_1/\partial t_1)=0)$. Since the numerical coefficient of the radiation damping term $(\sim(\partial^3\mathbf{x}_1/\partial t_1^3))$ is uniquely determined by the requirement of energy conservation, one necessarily has to define $\mathbf{F}_1$ by half the sum of both limiting values (from the forward and backward cones), giving $\mathbf{F}_1=\frac{2}{3}e^2(\partial^3\mathbf{x}_1/\partial t_1^3)$ in the rest system.[41] This device for forming limiting values of field quantities at the world point of an electron can also be applied in *quantum* electrodynamics, e.g., to the four-potential in Dirac's wave equation of the electron.[42] The relativistic invariance of the device is evident in all cases where the limiting values are independent of the directions of the λ-four-vectors chosen; otherwise an appropriate average over these directions restores the invariance. Dirac

[38] With this large value, Villars's theory comes very close to the Rarita-Schwinger or weak coupling theory, the new admixtures being quite insignificant. This remark may explain why in the discussion of the tensor-force in Section 3 it was unnecessary to distinguish between weak and strong coupling theories. For some time Rarita's and Schwinger's "symmetrical" theory seemed to be contradicted by experiments on the angular distribution of neutron-proton scattering at higher energies (E. Amaldi, D. Bocciarelli, B. Ferretti, and G. C. Trabacchi, Naturwiss. **30**, 582 (1942) and Ricerca Scient. **13**, 502 (1942)), and it was hoped that the strong coupling theory might improve the agreement (G. Wentzel, Helv. Phys. Acta **18**, 430 (1945); J. L. Lopes, Phys. Rev. **70**, 5 (1946)). This problem is now obsolete since C. F. Powell has reported that the scattering of 14-Mev neutrons by protons is practically isotropic in the center of gravity system (International Physics Conference at Cambridge, July 1946).

[39] P. A. M. Dirac, V. A. Fock, and B. Podolsky, Physik. Zeits. Sowjetunion **2**, 468 (1932).

[40] We put again $c=1$.

[41] G. Wentzel, Zeits. f. Physik **86**, 479 (1933).

[42] G. Wentzel, Zeits. f. Physik **86**, 635 (1933).

has proposed a somewhat different formulation where the λ-vector appears in the commutation relations of the field ψ;[43] this formalism is easier to handle, but apparently the results are the same. The extension to other quantized field theories is then straightforward.

In the *classical* theory of point electrons the λ-procedure removes *all* infinities; in fact, all classical infinities are directly correlated with the electromagnetic mass which becomes exactly zero in the λ-theory (there is no inertia term in the reaction force $\mathbf{F}_1$). In *quantum* theory, however, several independent infinities appear, usually in the form of divergent momentum-space integrals, with integrands varying asymptotically like $|\mathbf{k}|^n (n \geqq -3)$: $n=-2$ gives the integral which determines the classical electromagnetic mass; the self-energy of a Dirac electron as calculated by Waller[44] involves an additional term with $n=-1$, and Dirac's "theory of holes" combined with quantum electrodynamics gives rise to logarithmic divergencies $(n=-3)$. Now, the λ-limiting process proves to be an efficient tool only with respect to those infinities which belong to *even n* values; all other infinities remain. In particular, the logarithmic divergencies occurring in the hole theory are not at all affected by the λ-device.[45] Because of this failure, Dirac deserted the hole theory and proposed his very intricate new method of field quantization which involves photons of negative energy and "negative probabilities."[46] By this formalism, combined with the λ-limiting process, Dirac actually succeeds in eliminating all infinities; but the problem of physical interpretation is rather puzzling. Recently[47] Dirac has announced an improved form of his theory which seems to avoid this difficulty.

This digression into quantum electrodynamics was necessary to show what services the λ-device can be expected to render and how the further prospects of a theory based on this device may be valued.

Returning now to meson theory, we have to deal with mesons and nucleons, instead of photons and electrons. The "hole theory" formalism must be abolished for nucleons as well as for electrons.

Pauli[48] has applied the λ-formalism to the charge-symmetrical Møller-Rosenfeld mixture theory, using the ordinary weak coupling expansions. Static self-energy and interaction terms of the second and sixth orders in g were computed and proved to be finite (terms $\sim g^4$, which are owing to the zero-point energy of the field oscillators interacting with the nucleons, are mentioned to be finite too in the static approximation). By comparing the second- and sixth-order terms in the two-nucleon interaction, Pauli finds that the weak coupling expansion converges rapidly except for short distances of the nucleons $(r \gtrsim \mu g_{ps}^2)$, where the static approximation (nucleons at rest) is insufficient at any rate.

As to the *spin-inertia*, it is unfortunate that the weak coupling approximation does not lend itself easily to an investigation of this problem. Pauli resorts to studying the *classical* problem of a neutral pseudoscalar field interacting with the spin of a nucleon. The spin vector $\boldsymbol{\sigma}$ is classically interpreted as a unit vector with full freedom of orientation. The equations of motion can have solutions corresponding to periodic motions in which the nucleon spin gyrates freely around a fixed axis:

$$\sigma_3 = \text{const.}, \quad \sigma_1 = (1-\sigma_3^2)^{\frac{1}{2}} \cos\omega t, \quad \sigma_2 = (1-\sigma_3^2)^{\frac{1}{2}} \sin\omega t;$$

at large distances from the nucleon, the meson field ψ behaves either like a spherical wave (if $\omega^2 > \mu^2$), or it decreases exponentially $(\omega^2 < \mu^2)$. While the former case provides a classical description of the scattering of mesons by nuclei, the second alternative pictures the nucleon as being surrounded by a cloud of bound mesons, in close resemblance to the model of the compound nucleon obtained from the strong coupling theory. In fact, if the coupling is assumed to be sufficiently strong and if the nucleon is described as an extended source, energy and angular

[43] P. A. M. Dirac, Ann. de l'Institut H. Poincaré **9**, 13 (1939). Compare also Pauli's report in Rev. Mod. Phys. **15**, 175 (1943), Section 6a.

[44] I. Waller, Zeits. f. Physik **62**, 673 (1930).

[45] Pauli, in his report quoted in footnote 43, stresses the inconsistency of the basic ideas of the hole theory and the λ-device.

[46] P. A. M. Dirac, Proc. Roy. Soc. **180**, 1 (1942). Cf. also Pauli's report, reference 43.

[47] International Physics Conference at Cambridge, July, 1946.

[48] W. Pauli, Phys. Rev. **64**, 332 (1943).

momentum of the compound nucleon are again found to be correlated according to formula (6) of Section 4a,[49] the spin-inertia C being always of the order g^2/a; of course, without quantization of the angular momentum, the energy spectrum is continuous, but it has an upper bound because of the condition $\omega^2<\mu^2$. The result is, however, quite different if the point source model in conjunction with the λ-limiting process is applied. In this case, the periodic solutions involving an exponentially decreasing meson field ($\omega^2<\mu^2$) are possible only if the coupling is sufficiently strong ($|g_{ps}|$ must be above a certain critical value which is of the order μ^{-1}). Actually, with the coupling strength estimated numerically from the nuclear forces, this condition is not fulfilled. Translating this result into the quantum language, Pauli concludes that no stationary isobar states exist according to the λ-theory. He also shows that the constant, which according to Bhabha's classical discussion[50] is to be termed the mechanical spin-inertia, is exactly zero in this theory.

Although these results are rather preliminary since they are only based on a classical discussion of a neutral field theory, they seem to confirm that all field-inertia effects are radically cut down by the λ-process. One may wonder whether this tendency is really desirable in view of the requirements of a future theory. Certainly it is an improvement if the electromagnetic mass becomes zero instead of infinite; but should one not rather prefer a finite value which might be identified with the actual electron mass, doing away with the concept of mechanical mass, according to the ideas of Lorentz? Granted that we do not know how to achieve this in a relativistic theory, it still seems that the λ-procedure is a too radical amputation of the field theory, at least with respect to the inertia effects, not to mention its inefficiency in other respects. From the merely theoretical point of view it may, therefore, be doubted whether the predictions of the λ-theory, for instance those regarding the spin-inertia of the nucleon, are very trustworthy, or whether the extended source theories are perhaps a better approximation to reality.

[49] Cf. also J. R. Oppenheimer and J. Schwinger, Phys. Rev. **60**, 150 (1941).

[50] H. J. Bhabha, Proc. Roy. Soc. **178**, 314 (1941).

The only calculations going far enough to allow comparison with experience are those of Jauch[51] concerning the magnetic moments of proton and neutron. Following Pauli, Jauch assumes the coupling to be weak. The perturbational calculations are thus very nearly the same as in the theory of Fröhlich, Heitler and Kemmer[34] and lead to the formulas explained in Section 4b:

$$M_p \cong 1+M_{\text{meson}}, \quad M_n \cong -M_{\text{meson}}. \tag{8}$$

Both in the pseudoscalar and vector theory, M_{meson} involves a momentum-space integral which would be divergent ($+\infty$) in a plain point source theory and which, in the older extended source theory, was made finite by cutting off the high energy part. The λ-limiting process secures the convergence of the integral, at least in the static approximation (infinite nucleon mass, no recoil). However, here again the subtraction proves to be too radical: the finite limiting value of the integral becomes even negative, such as to make the magnetic moment of the neutron positive, in contradiction to experience. The sign can only be altered by introducing the *ad hoc* assumption that the vector meson has an abnormal magnetic moment, even with abnormal sign. Although this assumption, as Jauch remarks, could in principle be checked by independent experiments, it is too artificial to be accepted as satisfactory. We may, therefore, conclude that, although the experimental values of the magnetic moments are in favor of a weak rather than a strong coupling theory (as we saw in Section 4b), it is very unlikely that a consistent and fitting weak coupling theory can be achieved by means of the λ-limiting process.[52]

b. Heitler's and Stueckelberg's Theories

In a second group of subtraction theories, the principal problem is the process of *scattering*, e.g., the scattering of mesons in collisions with

[51] J. M. Jauch, Phys. Rev. **63**, 334 (1943).

[52] One may try to replace the λ-process by other mathematical artifices such as that proposed by Riesz which can probably be incorporated in the quantum theory of fields. Cf. M. Riesz, Comptes rendus du Congrès International de Mathématique Oslo 1936, II, 44; N. E. Fremberg, Lunds Fysiografiska Förh. **15**, No. 27 (1945); T. Gustafson, Lunds Fysiografiska Förh. **15**, No. 28 (1945) and **16**, No. 2 (1946); N. E. Fremberg, Medd. fr. Lunds Univ. Mat. Semin., Vol. 7 (1946).

nucleons. The usual perturbation theory (second-order approximation) neglects the reaction of the scattered meson wave and, therefore, gives a cross section much too large at high energies. Heisenberg[53] first substantiated the significance of the reaction force in the scattering problem by means of a classical extended source theory. According to such classical theories both inertia and radiation damping effects result in a diminution of the scattering cross section. In the λ-limiting and other point source theories where the spin-inertia effects are subtracted, only the diminution due to radiation damping remains which is much less pronounced. In quantum theory, if the extended nucleon model is adopted, the strong coupling approximation method automatically includes all reaction effects; accordingly the cross section becomes very small ($\gtrsim 4\pi a^2$ at all energies, except in the scalar theory). In the subtraction theories to be discussed presently, there is probably a considerable amount of arbitrariness regarding the subtraction of the spin-inertia effects.

In this situation obviously experimental cross section measurements could furnish valuable guidance. Unfortunately, it is difficult to distinguish experimentally the large angle scattering due to nuclear interactions from the multiple Coulomb scattering which is habitually disregarded in meson theory. This is particularly serious in the lower energy region (some 10^8 ev) where the differences of the various theories are most pronounced. From the work of Shutt[54] it appears that the nuclear scattering cross section is somewhat larger than was formerly believed. If this result is correct, it provides another argument against the strong coupling theory, but certainly the accuracy of the measurements is not yet sufficient to distinguish between the competing subtraction theories.

In order to characterize these theories, it is convenient to start from *Heitler's integral equation.*[55] We here reproduce a short derivation of this equation which is attributed to Pauli.[56] In the Schrödinger equation for stationary solutions

$$H\psi \equiv (H^0+H')\psi = E\psi,$$

we treat the interaction H' as a small perturbation and use a representation of ψ which makes H^0 diagonal

$$(q|H^0|q') = E_q(q|1|q')$$

(q = quantum numbers of the vacuum field oscillators). In order to discern the various solutions we write ψ as a matrix $(q|\psi|q_0)$ where q_0 stands for the "initial values" of the variables q, so that in the limit of zero interaction $(q|\psi|q_0) \rightarrow (q|1|q_0)$. The Schrödinger equation may then be written

$$(q|H'\psi|q_0) = (E_0 - E_q)(q|\psi|q_0). \qquad (9)$$

According to Dirac the solutions corresponding to scattering processes are of the form

$$(q|\psi|q_0) = (q|1|q_0) + (q|R|q_0)\,\delta_+(E_q - E_0), \qquad (10)$$

where

$$\delta_+(E) = \tfrac{1}{2}\delta(E) - \frac{1}{2\pi i E} \qquad (11)$$

(in integrals of the form $\int dE f(E)(E-E')^{-1}$, the Cauchy principal value has to be taken at $E = E'$). Inserting this expression of ψ into the Schrödinger Eq. (9), one obtains the following equation for R:

$$(q|H'|q_0) + \sum_{\{q'\}} (q|H'|q') \times (q'|R|q_0)\delta_+(E_{q'} - E_0) = \frac{1}{2\pi i}(q|R|q_0)$$

(on the right-hand side $(E_0 - E_q)\delta(E_0 - E_q)$ and $(E_0 - E_q)(q|1|q_0)$ vanish). Introducing the matrices

$$(q|\bar{R}|q_0) = (q|R|q_0)\delta(E_q - E_0), \qquad (12)$$

$$(q|T|q') = \frac{(q|H'|q')}{E_0 - E_{q'}}, \qquad (13)$$

the last equation can be written

$$H' + \tfrac{1}{2}H'\bar{R} + \frac{1}{2\pi i}TR = \frac{1}{2\pi i}R.$$

Multiplying by T^n and summing with respect

[53] W. Heisenberg, Zeits. f. Physik **113**, 61 (1939).
[54] R. P. Shutt, Phys. Rev. **69**, 261 (1946).
[55] W. Heitler, Proc. Camb. Phil. Soc. **37**, 291 (1941); W. Heitler and H. W. Peng, Proc. Camb. Phil. Soc. **38**, 296 (1942).
[56] W. Pauli, International Physics Conference at Cambridge, July 1946. See also W. Pauli, *Meson Theory of Nuclear Forces* (Interscience Publishers, Inc., New York, 1946).

to n, we get

$$\sum_{n=0}^{\infty} T^n H'(1+\tfrac{1}{2}\bar{R}) = \frac{1}{2\pi i} R,$$

or with the definition

$$\sum_{n=0}^{\infty} T^n H' = K: \tag{14}$$

$$K(1+\tfrac{1}{2}\bar{R}) = \frac{1}{2\pi i} R. \tag{15}$$

Multiplication with $\delta(E_q - E_0)$ gives finally

$$\bar{K}(1+\tfrac{1}{2}\bar{R}) = \frac{1}{2\pi i} \bar{R}, \tag{16}$$

where

$$(q|\bar{K}|q') = (q|K|q')\delta(E_q - E_{q'}). \tag{17}$$

Equations (15) and (16) are integral equations for the "scattered wave" R, since the matrix multiplication (in $K\bar{R}$) involves momentum-space integrations. Equation (16) is Heitler's in a form generalized to include higher order approximations.

The matrix K is, by virtue of (13) and (14), defined as a power-series in the coupling parameter g. In the case of the ordinary scattering process (in both states q_0 and q there is just one meson present) the matrix H' has no element corresponding directly to the transition $q_0 \to q$, and the first non-vanishing term in the expansion (14) is $TH'(\sim g^2)$. Therefore, as is easily seen from (15), the weak coupling expansion of $(q|R|q_0)$ equally starts with a second-order term, namely:[57]

$$(q|R|q_0) = 2\pi i (q|TH'|q_0) + \cdots.$$

Dropping higher order terms in R would correspond to a second-order perturbation theory and neglect of the reaction forces. However, the next non-vanishing term in $(q|K|q_0)$, *viz.* $(q|T^3H'|q_0)$, is infinite in the relativistic point-source theory. Here is where *Heitler's subtraction device* sets in: All diverging terms $(q|T^nH'|q_0)$, $n=3, 5, \cdots$, are subtracted, leaving only

$$(q|K|q_0) = (q|TH'|q_0). \tag{14a}$$

More generally, for an arbitrary process $q_0 \to q$, only the first non-vanishing term in the power-series (14), which is always finite, shall be kept (i.e.,

$$(q|K|q_0) = (q|T^{n_0+n-1}H'|q_0), \tag{14b}$$

if n_0 mesons are present in the initial state and n in the final state). With this new definition of K and $\bar{K}$ (17), the integral Eq. (16) is supposed to determine $\bar{R}$ exactly. The term $\frac{1}{2}\bar{K}\bar{R}$ on the left-hand side describes what is left of the reaction force. The solution of (16) can be symbolically written in the form

$$R = \frac{2\pi i \bar{K}}{1 - \pi i \bar{K}}. \tag{18}$$

The relativistic invariance of this subtraction method ensues from the simple fact that all terms in the power-series (14) must have the same transformation properties.[58]

When applied to the scattering of low energy photons by a free electron ($\nu \ll \mu_{el}$), Heitler's theory gives exactly the same scattering intensity as the classical theory with inclusion of the Lorentz damping force $(\frac{2}{3}e^2(\partial^3\mathbf{x}_1/\partial t^3))$. Also for the scattering of mesons by nucleons, the Heitler cross sections calculated according to the various meson theories[59] show a close resemblance to the cross sections derived from classical theories which include radiation damping but omit inertia effects due to the fields. As an example we give Heitler's cross-section formula for longitudinal vector mesons (of momentum k) according to the "charged" vector theory with $f_v = 0$ (see appendix) in the static approximation (no recoil):

$$\sigma = \frac{g_v{}^4 k^4}{4\pi(\mu^2+k^2)} \frac{1}{1+\dfrac{g_v{}^4 k^6}{(4\pi)^2(\mu^2+k^2)}}. \tag{19}$$

The last factor describes the effect of radiation

[57] The first-order matrix elements in $\bar{R}$ vanish on account of momentum and energy conservation.

[58] P. G. Gormley and W. Heitler, Proc. Roy. Irish Acad. **50**, 29A, 39 (1944). See also E. C. G. Stueckelberg and P. Bouvier, Comptes rendus Soc. de Physique Genève **61**, 162 (1944).

[59] For the charged and symmetrical Møller-Rosenfeld mixture theory the various cross sections have been deduced by W. Heitler and H. W. Peng, Proc. Roy. Irish Acad. **49**, 101A (1943).

damping.[60] With increasing momentum this factor becomes important for $k \gtrsim |g_v|^{-1}(4\pi)^{\frac{1}{2}}$ and causes the cross section σ to decrease finally like $4\pi k^{-2}$.

We want to turn now to the more general theory which has been developed by Stueckelberg in the framework of Heisenberg's *S matrix theory*.[61] The S matrix is closely related to the R matrix defined by (10). Putting

$$(q|1|q_0) = (q|\Delta|q_0)\delta(E_q - E_0), \tag{20}$$

where Δ represents a unity matrix with respect to all variables but the total energy, and

$$\delta(E) = \delta_-(E) + \delta_+(E),$$

$$\delta_\mp(E) = \tfrac{1}{2}\delta(E) \pm \frac{1}{2\pi i E}; \tag{21}$$

we can write instead of (10)

$$(q|\psi|q_0) = (q|\Delta|q_0)\delta_-(E_q - E_0) + (q|S|q_0)\delta_+(E_q - E_0), \tag{22}$$

where

$$S = \Delta + R. \tag{23}$$

The two terms on the right-hand side of (22) describe the "incident" and "outgoing" wave, respectively. On account of the energy conservation, only the matrix $\bar{S}$ defined by

$$(q|\bar{S}|q_0) = (q|S|q_0)\delta(E_q - E_0) = (q|1 + \bar{R}|q_0) \tag{24}$$

is actually significant. If we adopt the relation (16) or (18) as a result of wave-mechanical reasoning, we obtain

$$\bar{S} = 1 + \bar{R} = \frac{1 + \pi i \bar{K}}{1 - \pi i \bar{K}}. \tag{25}$$

$\bar{K}$ is a Hermitian matrix and therefore $\bar{S}$ a unitary matrix. Under Lorentz transformations $\bar{S}$ must transform like the unity matrix, since $\bar{S} \to 1$ in the limit of vanishing interaction. The probabilities of all observable processes can be expressed in terms of the matrix elements of $\bar{S}$.

Now Heisenberg, in view of the divergence difficulties of the present theory, wants to free the $\bar{S}$ matrix from all ties which are imposed by the Schrödinger equation, retaining though its general properties, such as unitarity and relativistic behavior. Referring to the wave-mechanical theory outlined above, this means that, although a Hermitian matrix $\bar{K}$ may still be defined by (16) or (25),[62] this matrix can no longer be calculated from a Hamiltonian according to (13), (14), and (17). It goes without saying that such a theory, as long as no other rules are added as a substitute for the Schrödinger equation, is very incomplete; it is like an empty frame for a picture yet to be painted.

In order to fill this frame at least partly, Stueckelberg in the first place postulates *correspondence with a classical theory of radiation damping*.[63] A certain arbitrariness results from the possibility of admitting, in the classical equation of motion, beside the Lorentz damping force, other reaction forces involving fourth and higher time-derivatives of the particle coordinates, such as appear in Lorentz's extended electron theory. This arbitrariness of the classical theory finds its counterpart in the quantum theory. Suppose, for instance, that a finite $\bar{K}$ matrix has been constructed by applying to the expansion (14) a certain subtraction rule, e.g., Heitler's rule, and that the $\bar{R}$ or $\bar{S}$ matrix be defined by (16) or (25), thus maintaining a certain analogy with the wave-mechanical formalism. On the other hand, Heisenberg's theory would suggest to put

$$\bar{S} = \exp(i\eta), \quad tg\frac{\eta}{2} = \pi \bar{K} \tag{26}$$

and to apply the subtraction rule to η rather than to $\bar{K}$. This theory, which would equally be Lorentz-invariant and free of divergencies, would differ from the previous theory in the higher order effects. Even other functions of $\bar{K}$ or η could be distinguished similarly. Once the wave-mechanical theory which leads to Heitler's integral equation is abandoned, one may doubt which of these functions deserves preference. From Stueckelberg's arguments it appears that this

[60] The quantum theory of radiation damping has been independently developed by E. Gora, Acta Phys. Polonica **7**, 159 (1938) and 374 (1939), Zeits. f. Physik **120**, 121 (1943); A. H. Wilson, Proc. Camb. Phil. Soc. **37**, 301 (1941); D. Iwanenko and A. Sokolow, Phys. Rev. **60**, 277 (1941).

[61] W. Heisenberg, Zeits. f. Physik **120**, 513 (1943).

[62] Instead of (25), Heisenberg puts $\bar{S} = \exp(i\eta)$, where η is a Hermitian matrix, and he discusses theories with simple expressions chosen for η (Zeits. f. Physik **120**, 673 (1943)).

[63] E. C. G. Stueckelberg, Helv. Phys. Acta **17**, 3 (1944).

variety of theories corresponds to a variety of classical theories with different reaction forces; in particular, Heitler's theory corresponds to the simplest classical equation of motion which contains third-order derivatives only.

However, the subtraction rule itself is subject to another kind of arbitrariness. Heisenberg as well as Stueckelberg adopt the following subtraction method. It is well known that the infinities always arise from intermediate (virtual) processes in which a certain quantum (photon or meson) is emitted and afterwards re-absorbed; the integral over the resulting intermediate states diverges. The subtraction of these divergent integrals can now be achieved in a relativistically invariant manner by changing the order in succession of the emission and absorption operators[64] (e.g., in T^nH'). If, for instance, all emission operators are written to the left of the absorption operators, a quantum once emitted can never be re-absorbed, and accordingly no divergent integrals appear. But other arrangements of the factors are also possible, differing by finite integrals only.[65]

In order to reduce the number of possibilities, Stueckelberg[66] has introduced another "principle of correspondence" saying that the ordinary *wave-mechanical theory* shall result if radiation damping and other reaction forces can be neglected. This new principle enables him to express $\bar{K}$ as an expansion with respect to the coupling parameter, in which the leading term (chosen for instance according to Heitler's theory) entails an infinite sequence of higher order terms.[67] (Of course, one might equally expand $\eta=2tg^{-1}(\pi\bar{K})$, but $\bar{K}$ is much easier to work with, as is really not surprising on account of Heitler's integral equation.) However, this argument is certainly not sufficient to remove *all* ambiguities in the arrangement of the emission and absorption operators in the higher order terms, not to mention the possibility of introducing there arbitrarily new "leading terms."

Finally there is another condition to be imposed to the expansion of the $\bar{S}$ matrix.[68] It is a characteristic feature of Heisenberg's and Stueckelberg's theories that they want to establish a correlation between initial and final states only, without giving a description in space and time of the actual process which is supposed to be unobservable. Now consider a multiple process involving for instance two nucleons I, II having a macroscopic distance r. According to the general formalism outlined above it can happen that a meson present in the initial state reappears in the final state, but such that the localisation in space and time would lead to the conclusion that the secondary meson has been emitted by the nucleon II *before* the primary meson was absorbed by the nucleon I. For the time interval between the emission and absorption processes, which can be arbitrarily large ($\geqq r$), the theory refuses to describe the state of the system; nevertheless the energy conservation would require the presence of a particle of negative energy during that time interval. Undoubtedly such predictions are wrong. They can, however, be avoided by choosing the higher order terms in the $\bar{K}$ expansion appropriately. It is not quite clear yet to what extent the conclusions drawn from this argument coincide with those obtained from the "second correspondence principle." But, generally speaking, it is to be expected that closest correspondence with the wave-mechanical theory will automatically do away with all "non-causal" predictions, since in a rigorous wave-mechanical theory based on the Schrödinger equation the succession of events in time is necessarily "causal."

Until now, Stueckelberg and his co-workers have studied mainly simplified theories allowing no comparison with experimental data. In particular, it remains to be seen how the theory can be generalized to include spin-inertia effects. There must be some freedom in this respect according to the freedom in the choice of the corresponding classical and wave-mechanical theories.

Although Heitler's and Stueckelberg's theories

[64] This "change in order" device was first introduced by P. Jordan and O. Klein, Zeits. f. Physik **45**, 751 (1927).

[65] I am indebted to Dr. F. Coester for a clarifying discussion on this and other questions relating to Stueckelberg's theory. A paper by F. Coester and A. Houriet dealing with these questions is being prepared for publication in Helv. Phys. Acta.

[66] E. C. G. Stueckelberg, Comptes rendus Soc. de Physique Genève **61**, 159 (1944), Helv. Phys. Acta **18**, 195 (1945).

[67] E. C. G. Stueckelberg, Helv. Phys. Acta **18**, 211–213 (1945).

[68] E. C. G. Stueckelberg, Helv. Phys. Acta **19**, 242 (1946).

apply primarily to scattering and related phenomena only, i.e., to stationary states belonging to the continuous energy spectrum, there is a possibility indicated by Kramers and Heisenberg[69] to determine the *discrete energy spectrum* too. To exemplify, let us consider the scattering of a particle (meson) by a central field of force (nucleon at rest). Assuming a pure S-wave (i.e., angular momentum $L=0$), the asymptotic behavior of the wave function in ordinary space will be given by

$$\psi_k(r\rightarrow\infty)=\frac{1}{r}(\exp[-ikr] - S(k)\exp[+ikr]). \quad (27)$$

$S(k)=\exp[i\eta(k)]$ determines the shift in phase of the scattered wave caused by the interaction of the two particles. Now the ψ-function can also be defined for complex values of k, by means of an analytic continuation of the function $S(k)$. Along the negative imaginary axis of the k plane we get, putting $k=-i|k|$:

$$\psi_k(r\rightarrow\infty)=\frac{1}{r}(\exp[-|k|r] - S(-i|k|)\exp[+|k|r]) \quad (28)$$

(while $\psi_k(r\rightarrow 0)$ remains finite). If (28) is to be the asymptotic expression of the eigenfunction of a closed state ("meson bound to the nucleon"), we must obviously have $S(k)=S(-i|k|)=0$. The discrete energy eigenvalues (belonging to S states) will, therefore, be

$$E_n=(\mu^2+k_n{}^2)^{\frac{1}{2}}=(\mu^2-|k_n|^2)^{\frac{1}{2}},$$

the k_n being zeros of the analytic function $S(k)$ on the negative imaginary axis.[70] Instead, one can ask for the poles on the positive imaginary axis; these are found at the points $k=+i|k_n|$, according to the general relation $S(k)S(-k)=1$ or $\eta(k)+\eta(-k)=0$.

The connection with the S matrix theory is established by the remark that $S(k)$ is the eigenvalue of the S matrix belonging to the S state ($L=0$) with momentum k. Similarly, in more complicated problems involving more particles or other interactions, the eigenvalues of the S matrix as energy functions can by analytic continuation serve to determine the energy of stationary states in which several particles are bound to each other.[71] Within the framework of wave mechanics this method is but an equivalent of the more customary methods to define the discrete energy spectrum. In the S matrix theory which abolishes the Schrödinger equation, it can be introduced *ad hoc* to correlate the energies of closed states with the scattering of the free particles involved.

In this way, the S matrix describing the scattering of mesons by nucleons according to Heitler's or Stueckelberg's theories can be used to determine "*isobar states*" in which mesons are bound to a nucleon.[72] The zeros of $S(k)$ are found among the zeros of the denominator in the scattering cross-section formula. For example, assuming Heitler's formula (19) (charged vector theory with $f_v=0$, static approximation), the equation

$$(4\pi)^2(\mu^2+k^2)+g_v{}^4k^6=0$$

has one solution of the form

$$k=-i|k_1|\,(0<|k_1|<\mu),$$

saying that a meson may be bound to the nucleon[73] with a binding energy $\mu-(\mu^2-|k_1|^2)^{\frac{1}{2}}$. The excitation energy of such a state, counted from the ground state (mere nucleon), would be $(\mu^2-|k_1|^2)^{\frac{1}{2}}$. It is easily seen that this excitation energy is little less than μ in the strong coupling case ($|g_v|\mu\gg 1$), while it tends to zero in the limit of weak coupling ($|g_v|\mu\rightarrow 0$). This dependence on the coupling strength is quite at variance with that obtained from the extended source theories (cf. section 4a). The obvious explanation of the discrepancy is that the spin-inertia effects, which are decisive for the formation of the isobar states in the extended source theories, are subtracted in

[69] W. Heisenberg, conference at the Zurich Seminar for theoretical physics, 1944, and two unpublished manuscripts available in Switzerland. See also C. Møller, lectures at the University of Bristol, spring term 1946, and Kgl. Danske Vid. Sels. Math.-fys. Medd. **23**, No. 1 (1945).

[70] However, not all of these zeros do necessarily give rise to eigenfunctions; see S. T. Ma, Phys. Rev. **70**, 668 (1946).

[71] Heisenberg also shows how the probability of transitions into these states can be computed from the S matrix elements of related multiple scattering processes.

[72] W. Heitler and Ning Hu, Proc. Roy. Irish Acad., in press; J. Pirenne, conference at the Zurich Seminar for theoretical physics, summer, 1946.

[73] Dr. Heitler has kindly informed me that the charge of an isobar state is either $+2$ or -1 in this theory.

Heitler's theory. Numerically, with the coupling parameters taken from the weak coupling theory of nuclear forces, one should expect the excitation energy to be in the order of 20 Mev only.

From the S matrix of proton-neutron collisions one may in the same way determine the binding energy of the *deuteron*. Here, however, no appreciable influence of the reaction forces is to be expected, on account of the slow variation in space of the deuteron wave-function.

As to the crucial problem of the *magnetic anomalies of the nucleon*, we meet with the difficulty that the electric charge and current distribution of the "meson cloud" is not directly defined in the S matrix theory, although it might be determined indirectly for instance by studying the scattering of a light wave by a nucleon. However, the quantity that should be computed is the energy of a nucleon in a magnetic field, the nucleon being in one of the ground states. These states cannot be associated with a continuous spectrum of meson waves by analytic continuation. Instead one might try to study the behavior of nucleons in oscillating or inhomogeneous magnetic fields such as have actually served to measure the magnetic moments. The Heitler-Stueckelberg theories will have to solve this problem of the magnetic anomalies, before a fair comparison with other theories can be made.

APPENDIX

Notations: Greek indices refer to space-time coordinates ($x_4 = it$).

γ_ν = Dirac's matrices operating on the spin coordinates of the nucleon field. $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$.

i = isotopic spin index (running from 1 to 3 in the charge-symmetrical theory).

τ_i = isotopic spin matrices = Pauli's matrices operating on the charge coordinate of the nucleon field.

ϕ = wave function of the nucleons (8 components, *viz.* 4 each for the proton and neutron states); $\phi^+ = i\phi^*\gamma_4$.

ψ_i, $\psi_{i\nu}$ = (real) wave functions of the pseudoscalar and vector meson fields, respectively.

μ_N, μ_{ps}, μ_v = rest masses of the nucleon, pseudoscalar, and vector meson.

g_{ps}, f_{ps}, g_v, f_v = (real) coupling parameters, all having the dimension of a length.

Natural units are chosen such that $\hbar = c = 1$.

The Lagrangian of a system of nucleons and mesons can be represented by the space integral $\int d^{(3)}xL$, where

$L = L_N + L_{ps} + L_{ps}'$ in the pseudoscalar theory,
$L = L_N + L_v + L_v'$ in the vector theory,
$L = L_N + L_{ps} + L_v + L_{ps}' + L_v'$ in the mixture theory,

$$L_N = i\phi^+\left(\Sigma_\nu\, \gamma_\nu \frac{\partial}{\partial x_\nu} + \mu_N\right)\phi,$$

$$L_{ps} = -\tfrac{1}{2}\mu_{ps}^2\, \Sigma_i\, \psi_i^2 - \tfrac{1}{2}\sum_{i\nu}\left(\frac{\partial \psi_i}{\partial x_\nu}\right)^2,$$

$$L_v = -\tfrac{1}{2}\mu_v^2 \sum_{i\nu} \psi_{i\nu}^2 - \tfrac{1}{4}\sum_{i\mu\nu}\left(\frac{\partial \psi_{i\mu}}{\partial x_\nu} - \frac{\partial \psi_{i\nu}}{\partial x_\mu}\right)^2,$$

$$L_{ps}' = f_{ps}\mu_{ps}\, \Sigma_i\, \phi^+\tau_i\gamma_5\phi\psi_i + g_{ps}\sum_{i\nu} \phi^+\tau_i\gamma_\nu\gamma_5\phi\frac{\partial \psi_i}{\partial x_\nu},$$

$$L_v' = g_v\mu_v \sum_{i\nu} \phi^+\tau_i\gamma_\nu\phi\psi_{i\nu} + \tfrac{1}{2}f_v \sum_{i\mu\nu} \phi^+\tau_i\gamma_\mu\gamma_\nu\phi\left(\frac{\partial \psi_{i\mu}}{\partial x_\nu} - \frac{\partial \psi_{i\nu}}{\partial x_\mu}\right).$$

Erratum

THROUGH error, proper credit for permission to publish the article on "Nuclei Formed in Fission: Decay Characteristics, Fission Yields, and Chain Relationships," by the Plutonium Project, was not given. This article was published in the November issue of the *Journal of the American Chemical Society*, and through the courtesy of the Editors of that Journal we were permitted to use the type in its publication in the October issue of the *Reviews of Modern Physics*.

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