Corrections and Additional Remarks to our Paper: The Influence of the Expansion of Space on the Gravitation Fields Surrounding the Individual Stars

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MR. Max Wyman has drawn our attention to several mistakes in the formulas of the second part of our paper. These mistakes were caused by a change in notation in the final preparation of the manuscript. This explains why they do not affect the validity of the results.

The corrections to be made are:

Page 122, Part II, Line 3 reads $-e^{\mu}$ instead of e^{μ}.
Line 5 reads $\tau=b_0+b_1r^{-\frac{1}{2}}+\cdots$ instead of $\tau=b_1r^{-\frac{1}{2}}+\cdots$.
Equation (2.4.2) reads $+\frac{3}{2}\tau_r$ instead of $-\frac{3}{2}\tau_r$.
Equation (4.3.1) reads $b_0+k_1T^{*-1}P^{-\frac{1}{2}}+\cdots$ instead of $k_1T^{*-1}P^{-\frac{1}{2}}+\cdots$.
Equation (7) add b_0 to the coefficient of dt^2.
Page 123, Line 4 reads -1 instead of 1.

Mr. G. C. McVittie has drawn our attention to the following papers by himself and by G. Järnefelt,

McVittie, M. N. R. A. S. **92**, 499–518 (1932),
McVittie, M. N. R. A. S. **93**, 325–339 (1933),
G. Järnefelt, Ann. Acad. Sc. Fenn. **A1**, 12, 3–38 (1942),
G. Järnefelt, Ann. Acad. Sc. Fenn. **A40**, 3 (1940).
G. Järnefelt, Ark. f. Mat. Astron. o. Phys. **27A**, 15 (1940),

of which we were unfortunately not aware. They did already reach the conclusion that the field in the neighborhood of an individual star is not affected by the expansion and the curvature of space.

We think, however, that our paper has not been superfluous, since it simplifies the treatment of the problem and makes its gist appear clearly. This simplification has been achieved by the following artifice: Instead of introducing an inhomogeneous distribution of matter in the entire space we only replace the dustlike matter in the interior of a part of space G with a spherical boundary, by a mass situated at the center of that sphere, or rather by a singularity representing that mass.

From the proof that the field thus obtained can be transformed into a Schwarzschild field it follows that inside a star and in a certain neighborhood of the star everything behaves as if the star were embedded in a space without curvature and without expansion.

Since the publication of our paper it has become clear to us that altogether every non-static centrally symmetric solution of the field equations can be transformed into the Schwarzschild solution. The following is the sketch of a proof:*

We first show that if μ is independent of time for a particular value of $r(r=\rho)$ then the solution can be transformed into a static solution.

From Eq. (2.3) we see that for $r=\rho$

$$\mu_{rt}=0.$$

Differentiating Eq. (2.3) successively with respect to r we see that for $r=\rho$ and all n

$$\frac{\partial}{\partial t}\frac{\partial^n}{\partial r^n}\mu=0.$$

Hence $\mu_t\equiv 0$ for all r.

Considering this Eq. (2.2) becomes

$$r(\mu_{rr}+\tfrac{1}{2}\mu_r^2+\tfrac{1}{2}\mu_r\nu_r)+(2\mu_r+\tfrac{1}{2}\nu_r)=0.$$

or

$$\nu_r=-(4\mu_r+2r\mu_{rr}+r\mu_r^2)/(1+r\mu_r).$$

Hence ν_r is independent of t, and ν is of the form

$$\nu=F(r)+\varphi(t).$$

Our solution is now of the form

$$ds^2=-e^{\mu(r)}\delta_{ik}dx_idx_k+e^{F(r)}e^{\varphi(t)}dt^2.$$

By choosing a new time coordinate so that $dt'=e^{\varphi/2}dt$ this becomes

$$ds^2=-e^{\mu(r)}\delta_{ik}dx_idx_k+e^{F(r)}dt'^2,$$

which is static. Since it is known that the only centrally symmetric static solution is the Schwarzschild field, we now have the Schwarzschild solution.

* We have noticed subsequently that this theorem was already proven by G. D. Birkhoff in his book, *Relativity and Modern Physics*, pp. 253–256.

Now we have to show that a given solution of the field equations (2) can be transformed so that for a fixed $r(r=\rho)$, μ is independent of t. Considering transformation (9) we see that

$$\exp[\mu^*(r,t)]=\exp[\mu(U^2r,V)]U^2$$

in the new coefficient of $-\delta_{ik}dx_idx_k$. We want to choose U and V so that the Eqs. (10) are satisfied and so that for $r=\rho$

$$\mu_t^*(\rho,t)=0$$

or

$$\mu(U^2\rho,V)+2\log U=c. \tag{A}$$

This is one functional equation in two variables U, V which will in general have infinitely many solutions.

We shall have to show that Eqs. (10) will be compatible with this boundary condition. Equation (10.4) can be written

$$U_r^2\left(-2r+4r^2e^{(\mu-\nu)}\frac{U_t^2}{V_t^2}\right)$$
$$+U_r\left(-2U+4rUe^{(\mu-\nu)}\frac{U_t^2}{V_t^2}\right)+e^{(\mu-\nu)}U\frac{U_t^2}{V_t^2}=0,$$

or

$$\left(U_r+\frac{1}{2r}U\right)^2+\frac{U^2}{4r^2}\left(-1+2re^{(\mu-\nu)}\frac{U_t^2}{V_t^2}\right)^{-1}=0. \tag{B}$$

This equation can be solved for U_r if

$$-1+2re^{(\mu-\nu)}\frac{U_t^2}{V_t^2}<0 \tag{C}$$

But from Eq. (8.2) we have

$$e^{\nu*}=-2re^{\mu}U_t^2+e^{\nu}V_t^2$$
$$=-e^{\nu}V_t^2\left(-1+2re^{(\mu-\nu)}\frac{U_t^2}{V_t^2}\right).$$

Therefore the inequality (C) will be satisfied and Eq. (B) can be solved for U_r. We now know that after we have determined $U(\rho,t)$ and $V(\rho,t)$ so that the boundary condition (A) is satisfied we can determine $U_r(\rho,t)$ from Eq. (B) and $\partial^nU/\partial r^n(\rho,t)$ from the differentiated Eq. (B) (eliminating V_{rt} with the help of (10.1) and (10.2)).

After having determined U we immediately get V from Eq. (10.1) and the boundary condition (A). Thus it is shown that every centrally symmetric solution represents a Schwarzschild field.

We have of course omitted any discussion of the existence of real solutions of the boundary conditions (A) or of the convergence of the Taylor expansion for U which we obtained from Eq. (B).

Concluding remark: In the cosmologic theory it has been assumed so far, that for a consideration of this kind stellar matter could be replaced by a continuously distributed "dust." The considerations of McVittie, Järnefelt, and ourselves give a subsequent justification of that assumption. If one takes this assumption for granted then the theorem of the identity of every dynamic centrally·symmetric solution with the Schwarzschild solution already leads to the conclusion that in the planetary realm everything behaves as if there existed no cosmic expansion or curvature.