Effect of Small Stresses on Magnetic Properties

R. M. BOZORTH AND H. J. WILLIAMS
Bell Telephone Laboratories, Murray Hill, New Jersey

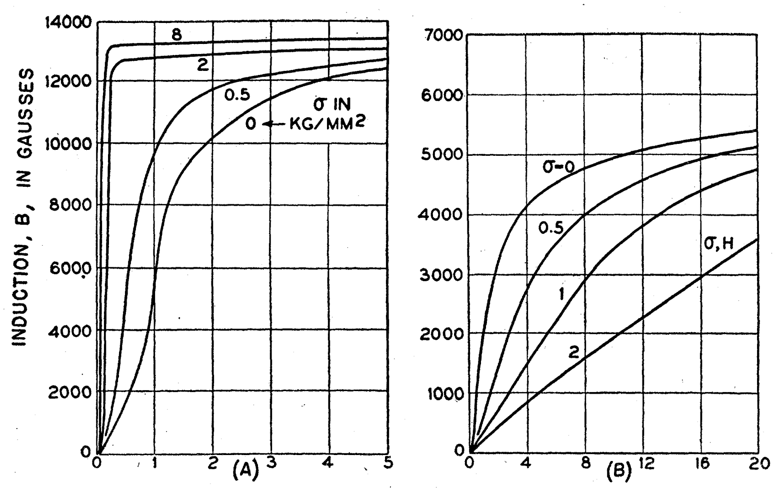
MEASUREMENTS are here reported briefly on the changes of magnetization that occur when very small stresses are applied or removed. This has been background work to help in our understanding of the behavior of magnetic materials in apparatus developed as a part of the war effort.

Before discussing the effect of small stresses, however, it seems appropriate to review some of the more general phenomena regarding the effect of stress on magnetization.

GENERAL

The magnetic properties of most ferromagnetic materials change with the application of stress to such an extent that stress may be ranked with field-strength and temperature as one of the *primary factors* effecting magnetic change. In some materials a tension of 10 kg/mm² (14200 lbs./in.²) will increase the permeability in low fields by a factor of 100; in other materials the permeability is decreased by tension; and in still others (e.g., iron) the permeability in low fields is increased and that in higher fields decreased. In all materials the saturation induction is unaffected by a stress within the elastic limit and is believed to be affected by any stress at all only when it causes change of phase in the material.

In discussing the effect of a unidirectional stress, it is convenient to divide materials into two classes in which the induction is respectively increased or decreased by tension. Materials of the first class are said to have *positive magnetostriction*, and those of the second *negative magnetostriction*, because of the increase or decrease in length that occurs in these materials when they are magnetized. Nickel is a good example of a material having negative magnetostriction, and 68 permalloy is a material having positive magnetostriction and high strain sensitivity. Figure 1 shows magnetization curves for these materials with various values of tensile stress. In 68 permalloy we see the rapid rise of induction with field-strength nearly to the saturation value when tension is present. By a stress of 8 kg/mm² the maximum permeability, μ_m, is increased 20-fold, and μ at $H=0.1$ is increased 50 times. The same figure shows the corresponding decrease in permeability with tension in nickel. In taking these curves, the specimen is demagnetized with



FIELD STRENGTH, H, IN OERSTEDS

FIG. 1. Effect of tension on the magnetization curves for (A) 68 permalloy (positive magnetostriction) and (B) nickel (negative magnetostriction). Tension held constant while field strength is varied.

the tension on, and the tension is kept constant during the measurements of B and H.

Although nickel has the greater magnetostrictive change in length when magnetized to saturation, it is not as sensitive to changes in stress as 68 permalloy. This difference may be attributed to the relatively low magnetic anisotropy of 68 permalloy as compared to nickel and should be borne in mind in considering the following data.

As the stress is increased from zero, the largest changes in induction occur, first, near the steepest part of the magnetization curve and with higher stresses near the knee of the curve for nickel and nearer the origin for 68 permalloy. When the tension is high, the magnitude of the change is large and for materials having positive magnetostriction is limited only by the saturation magnetization of the material and the ability of the material to maintain the stress. As the field-strength is increased from zero to a high value, the change of induction due to any constant stress increases from zero, passes through a maximum and approaches zero again as a limit.

Figure 2 shows that the behavior of annealed

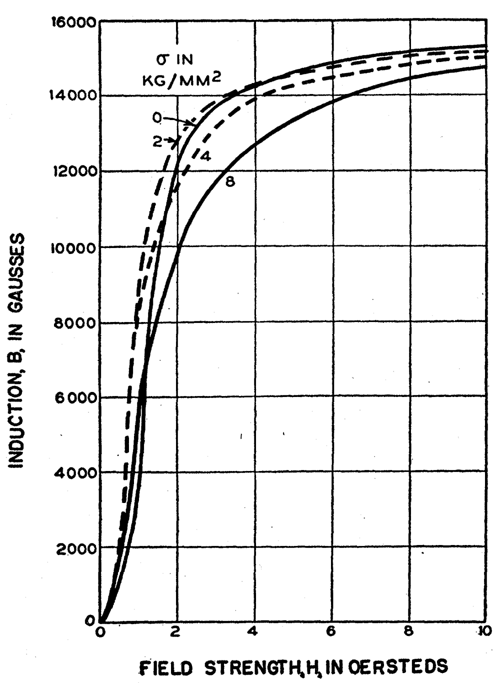


FIG. 2. Effect of tension on the magnetization curve of iron. Maximum stress is below elastic limit.

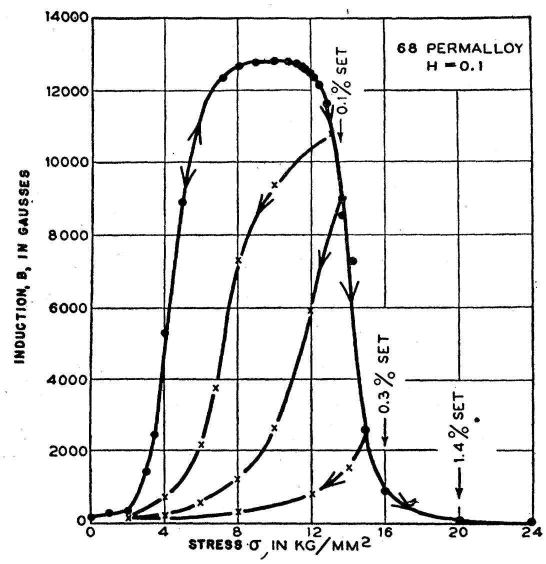


FIG. 3. Effect of stress within and beyond elastic limit. H held constant while σ is varied. B falls below maximum value when permanent set occurs, provided material has positive magnetostriction.

iron is more complicated. Low tensions cause an increase in induction when this lies below the knee of the curve and a decrease for higher inductions; high tensions cause an increase in B for points near the origin of the curve and a decrease for other points (the Villari reversal). Below the knee of the magnetization curve ($B = 13{,}000$), there is a definite value of the tension at which the change of induction with tension, $dB/d\sigma$, changes sign, and this tension varies from very small values near the knee to about $\sigma = 10$ kg/mm² at $B = 1000$ to 2000.

STRESS BEYOND ELASTIC LIMIT

When the field-strength is held constant and increasing tension is applied to a material having positive magnetostriction, the induction increases until the *elastic limit* is reached and then decreases continuously to a low value as the material is hardened by plastic deformation. The data of Fig. 3 refer to the specimen of 68 permalloy of Fig. 1, but here the tension is increased from zero to about twice the elastic limit. The induction in a field of 0.1 oersted increases until it is almost equal to the saturation induction ($B_s = 13{,}300$) and then decreases when plastic flow begins. When the stress is decreased, the curve is retraced provided the maximum

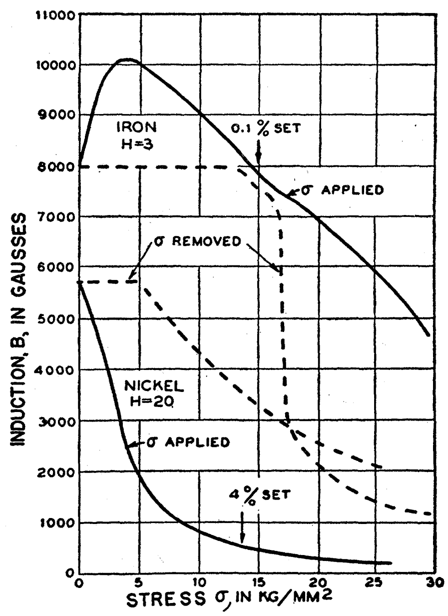


FIG. 4. Effect of low and high stresses on magnetization of iron and of nickel. Elastic limit can be determined from broken lines indicating B present after σ has been applied and removed with H held constant.

stress has not exceeded the elastic limit; if it has, a new curve is followed—the induction continually decreases with the stress.

When the material, like nickel, is magnetostrictively negative, the point on the curve at which plastic flow begins in tension is not readily determined. Data illustrating this point are given in Fig. 4 which includes also the data for iron. The elastic limit for magnetostrictively negative materials is more evident when one plots the induction for a given field-strength determined after the stress has been applied and removed; here an overstrain of one percent is easily discernible, and under favorable circumstances, considerably less than 0.1 percent can be detected. A curve obtained in this way for nickel is shown in the figure by a broken line, and similar data are given for iron.

It has already been mentioned that 68 permalloy is especially sensitive to stress and that there are marked differences between the magnetization curves when σ and H are applied in different orders. We shall designate by (σ, H) the curve obtained by first applying the stress and then the field, by (H, σ) the curve with those applied in reverse order. Figure 5 shows (H, σ) curves for low fields when $\sigma = 4$ kg/mm². The maximum slope of the (H, σ) curve is about $B/H = 10^6$, and both this high slope and the shape of the curve bring to mind the ideal magnetization curve, which for comparison has been determined on this same specimen. As nearly as one can tell from the data, these two curves coincide.

As pointed out years ago by Ewing, there is often marked hysteresis in the relation between stress and magnetization with constant field-strength. As illustrations of the complicated interrelations between σ, B and H, Fig. 6 shows a B, H loop of 68 permalloy taken with a stress $\sigma = 4$ kg/mm² continually maintained (broken line), and a loop with the same stress released and then applied at various constant values of H (solid line). On the right is shown a similar loop taken with no stress applied except at certain values of H at which it is applied and immediately released.

SMALL STRESSES

When a small stress is repeatedly applied to and removed from a magnetic material subjected to a steady field, the change in induction B_σ so produced is proportional to σ. As may be noted in Fig. 7 the B_σ, σ curve goes through the origin without change in slope and as σ increases, it approaches a limiting value of B_σ. The data of this figure refer to 45 permalloy to which was

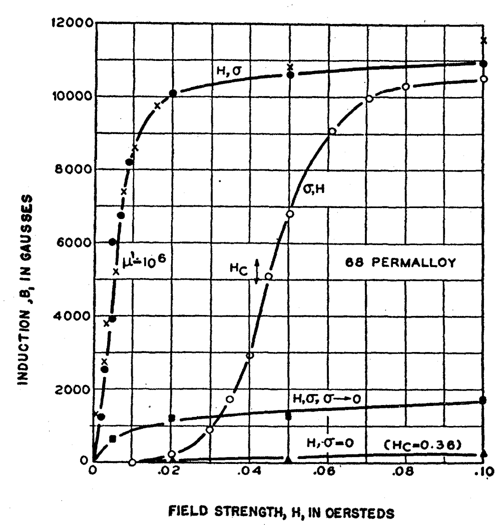


FIG. 5. Magnetization curve depends on order of application of stress and field, with order indicated by symbols. Data refer to 68 permalloy with $\sigma = 4$ kg/mm². Crosses indicate ideal magnetization curve.

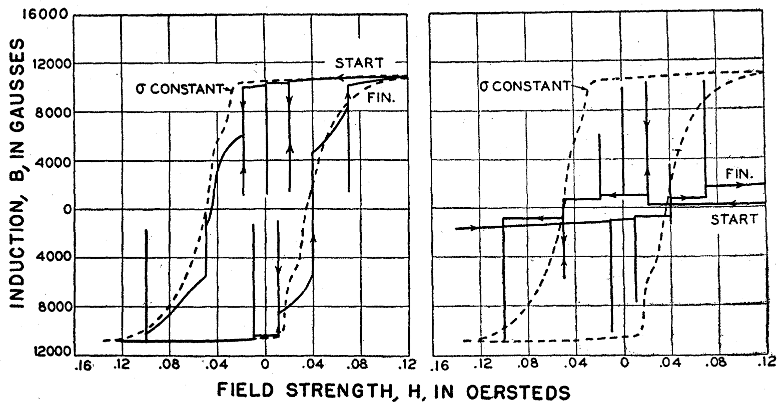


FIG. 6. Hysteresis loops for 68 permalloy. Broken lines show loop with constant tension, $\sigma=4$ kg/mm². In figure at left solid line shows loop taken with $\sigma=4$ at start, and stress removed and reapplied at certain values of H during traversal of loop. At right loop is taken with $\sigma=0$ except stress applied and removed at various values of H.

applied a constant "polarizing" field of $H_0=5.6$, corresponding to a biasing induction $B_0=13{,}000$. Data of this kind may also be plotted, as in Fig. 8, with B_σ/σ as ordinate. In this figure the curves refer to 68 permalloy and show that when the polarizing field is small, they rise characteristically to a maximum from a finite ordinate at $\sigma=0$, and then drop asymptotically toward zero. When H_0 corresponds to a point on the magnetization curve well beyond the knee, the

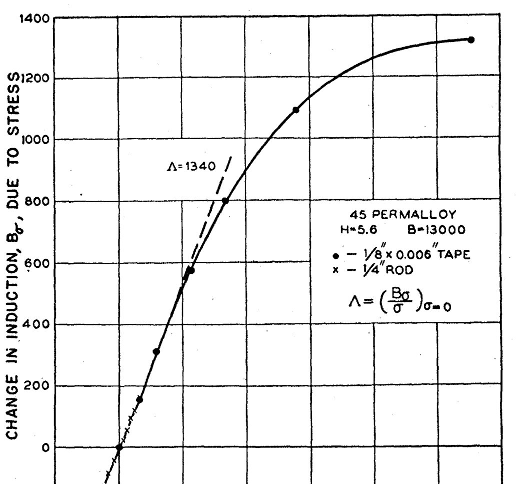


FIG. 7. Change in B, designated B_σ, caused by cyclic stress, σ, in 45 permalloy polarized to induction $B_0=13{,}000$. Λ is defined as B_σ/σ for small stresses. Negative values of σ are for compression.

B_σ/σ curve falls continually from its value for $\sigma=0$. Hysteresis is especially apparent in low fields.

A significant fundamental constant is the quantity

$$\Lambda=(B_\sigma/\sigma)_{\sigma=0}=(dB/d\sigma)_{\sigma=0}.$$

For 45 permalloy the dependence of Λ on the polarizing field-strength is given by the curve of Fig. 9. The curve rises rapidly to a maximum occurring at a field-strength somewhat greater than that for maximum permeability and then falls toward zero. When Λ is plotted against the polarizing induction B_0, it is apparent that its maximum value, Λ_m, occurs when B_0 is somewhat greater than half of the saturation, B_s.

The constant Λ_m, the maximum value of Λ obtained by plotting it against H_0 or B_0, has been determined for ten iron-nickel alloys and the results are apparent in Fig. 10. The cross-over near 81 percent nickel is obviously associated with the vanishing magnetostriction of this alloy. The maximum near 60 percent nickel is apparently associated with the low magnetic anisotropy of alloys near this composition. The minimum near 90 percent nickel is due to a combination of factors—the changes of saturation magnetization, of magnetostriction, and of anisotropy with composition. The quantitative relation between Λ_m and these quantities is discussed later in more detail.

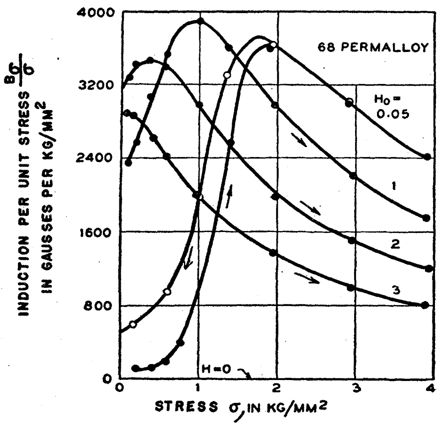


FIG. 8. Change in B per unit stress for various magnitudes of stress. Data used for extrapolation to $\sigma=0$ to determine Λ.

DOMAIN THEORY

The domain theory affords a convenient picture of the changes that occur in a magnetic material as affected by magnetic field and stress, and it also can be used as a basis for quantitative calculations which give interesting relations between the various fundamental constants involved. Domain theory will now be used to calculate both Λ as a function of B_0 for a given material, and Λ_m as dependent on the nickel content of Fe-Ni alloys.

The general picture of a magnetic material is that it is composed of a great many small regions, each one of which is magnetized to saturation in some one direction, e.g., a direction of easy magnetization in the crystal in which the domain lies. When the material is unmagnetized, they are oriented in all directions so that the net magnetization of the body is zero. They are changed by the action of applied magnetic fields and stresses and at saturation are all aligned parallel to the field. When the net magnetization is more than about one-half of saturation, the changes in magnetization produced by stress or by a field occur mainly by rotation of the direction of magnetization in each domain.

Assume in accordance with Fig. 11 that a single domain is orientated so that its magnetization makes the angle θ with the axis of the specimen along which H and σ are applied. The domain has been rotated into this position by application of a field H_0 which has turned it by the angle $\alpha=\alpha_0-\theta$ from its original orientation which was parallel to the direction of easy magnetization in the single crystal of which the domain is a part. This is calculated from the energy of magnetic anisotropy of the crystal, E_K, and the magnetic energy due to the applied field, as is well known from the work of Akulov. When the vectors H and B lie in a cube plane, the crystal energy reduces to

$$E_K=K(1-\cos 4\alpha)/8,$$

and the energy due to the field is

$$E_H=-HI_s\cos(\alpha_0-\alpha),$$
$$=-(HB_s/4\pi)\cos(\alpha_0-\alpha).$$

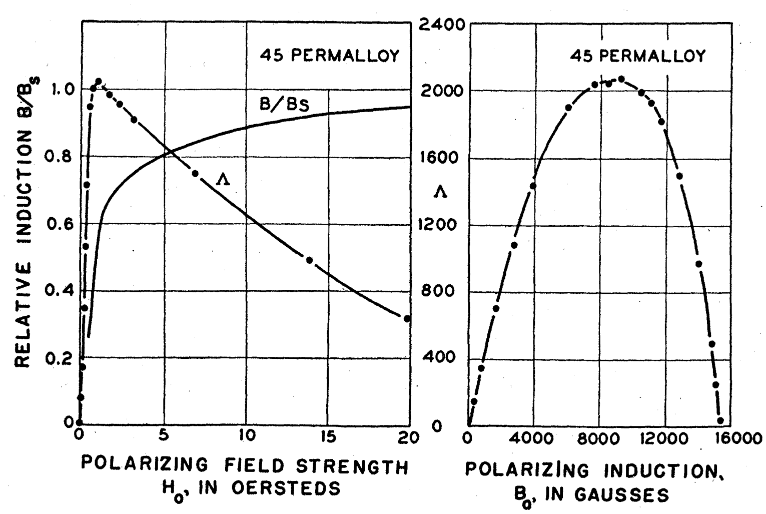


FIG. 9. Typical Λ curves plotted as functions of polarizing field-strength, H_0, and induction, B_0. Highest Λ is Λ_m.

When these energies are added together, they have a minimum value when $H=H_0$ and $B=B_0$ given by

$$H_0=\frac{2\pi K\sin 4\alpha}{B_s\sin(\alpha_0-\alpha)},$$

$$B_0=B_s\cos(\alpha_0-\alpha).$$

Here, K is the anisotropy constant which determines the force necessary to turn the domain from its direction of easy magnetization. B_s is used to designate the saturation intrinsic induction, often written $(B-H)_s=4\pi I_s$.

To find the effect of a small stress on the orientation of the domain, we write the expression for the domain energy which is now composed of three parts E_K, that due to crystal anisotropy determined by K, E_H determined by the magnetic field, and E_σ that determined by the strain. The latter, well known from the work of Becker, is

$$E_\sigma=\tfrac{3}{2}\lambda_s\sigma\cos^2(\alpha_0-\alpha).$$

This shows that the energy is proportional to the stress and to the fractional change in length, λ_s, which occurs during magnetization to saturation, and also, the $\cos^2$ term indicates the twofold symmetry corresponding to stress along a line. Then

$$E=E_K+E_H+E_\sigma,$$

and to determine the orientation at equilibrium,

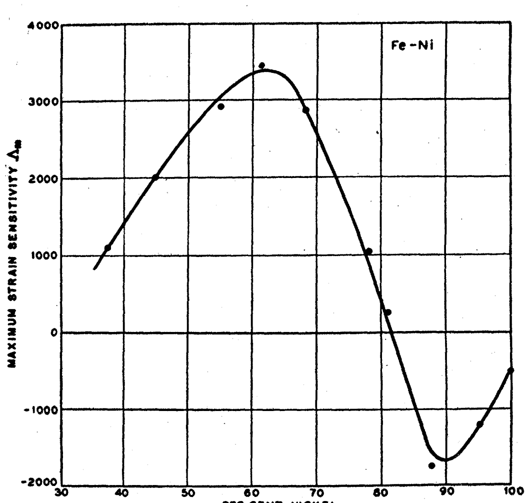


FIG. 10. Λ_m for various iron-nickel alloys.

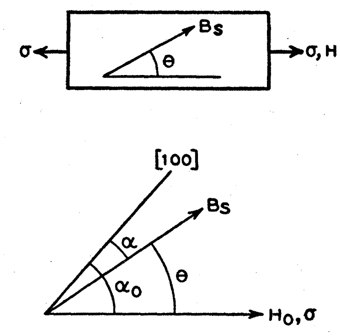


FIG. 11. Diagram for calculating Λ by domain theory. B_s represents induction in one domain oriented as shown with respect to directions of H and σ and of easy magnetization (assumed parallel to cubic crystal axis [100]).

we put

$$dE/d\alpha=0.$$

This gives

$$\Lambda\equiv\left(\frac{dB}{d\sigma}\right)_{\sigma=0}=\frac{3\lambda_sB_s}{2K}(B_0/B_s)(1-B_0^2/B_s^2)\cdot f.$$

Here f is a small factor that varies from 1.37 to 1.60, depending on the original crystal orientation. Putting $f=1.5$, we then have

$$\Lambda=2.2(\lambda_sB_s/K)(B_0/B_s)(1-B_0^2/B_s^2).$$

When K is in ergs/cm³ and B in gausses, Λ is in gausses per dyne/cm². When σ is in kg/mm², Λ is changed by a factor of 10^8.

In a given material, Λ is a maximum when the induction is somewhat over half of saturation, namely $(\sqrt{3}/3)B_s=0.58B_s$, and the value of Λ at its maximum is

$$\Lambda_m=0.77\lambda_sB_s/K.$$

For 45 permalloy the values of the constants $B_s=15{,}500$ and $\lambda_s=25.10^{-6}$ are known to be approximately correct. The value of K is somewhat uncertain but probably lies between 15,000 and 30,000. Using 15,000 and the above values for B_s and λ_s in the corrected expression above, we can calculate the change of induction with stress as a function of the induction. The curve showing this relation is given in Fig. 12 where it is compared with the observations. Fair agreement is obtained for all flux densities. The assumption made in the derivation of the expressions for Λ, that the magnetization is changed by rotation of the domain vector, is not valid at low inductions.

Nevertheless, Λ at the lowest induction can be calculated with some accuracy from the fundamental constants λ_s and K.

The derived relation indicates that for maximum effect of stress on induction the most desirable material is one for which λ_s and B_s are high and K is low. If K is unusually low or if the material is hard worked so that the internal stresses σ_i are high, the resistance of a domain to a force trying to change its magnetization will depend not on the crystal anisotropy constant K but on $\lambda_s\sigma_i$. Under these conditions the above equation does not apply. In general the controlling factor will be K or $\lambda_s\sigma_i$, whichever is larger.

The expression $\Lambda_m = 0.77\ 10^8\,\lambda_s B_s/K$ is plotted on Fig. 13 for the various iron nickel alloys tested, by use of the data shown in the same figure by broken lines. Values of K are taken from unpublished data by H. J. Williams, and the other quantities are well known. Comparison of this calculated curve with the observations as given on the same figure shows a striking similarity in the general features of the curves, and the agreement as to magnitude is not bad. The theoretical curve approaches $\Lambda_m = \infty$ when $K = 0$ and the curve in the figure has been arbitrarily reduced in this region (55 to 70 percent nickel) since the simple assumptions of the theory no longer hold, and Λ is limited by internal strain σ_i, instead of by the crystal forces measured by K, no matter how perfect the anneal. If we assume that the internal strain, σ_i/E, (E is Young's modulus) is equal to the magnetostrictive strain, λ_s, we can calculate the highest expected value of Λ_m to be attained in 65 permalloy provided it has been perfectly annealed. This value is about 6 times that observed.

There is a close relation between the change in magnetization by stress and the change of length by magnetization. When the changes are small and cyclic so that they may be considered reversible, we may apply the thermodynamic relation

$$(\partial\lambda)/(\partial H)_\sigma = (\partial I)/(\partial\sigma)_H,$$

applicable to small reversible changes. This may be put into the form

$$\left(\frac{\partial\lambda}{\partial B}\right)_\sigma = \frac{1}{4\pi\mu_r}\left(\frac{\partial B}{\partial\sigma}\right)_H.$$

Here λ is the fractional change in length and μ_r is the reversible permeability. B is written instead of $B-H$.

At high inductions there is practically no hysteresis in the curve obtained by plotting λ *vs.* B, and also λ is linear when plotted against

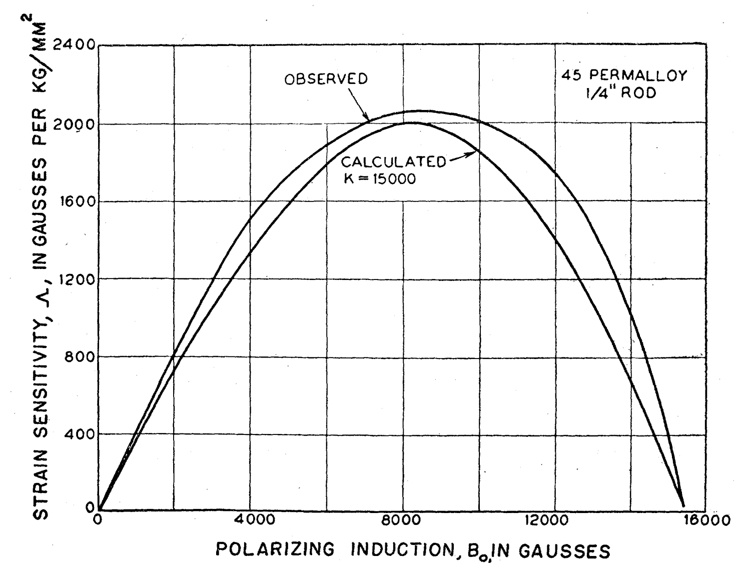


FIG. 12. Comparison of values of Λ as observed and as calculated assuming saturation induction $B_s = 15{,}500$, saturation magnetostriction $\lambda_s = 27\times10^{-6}$ and crystal anisotropy constant $K = 15{,}000$.

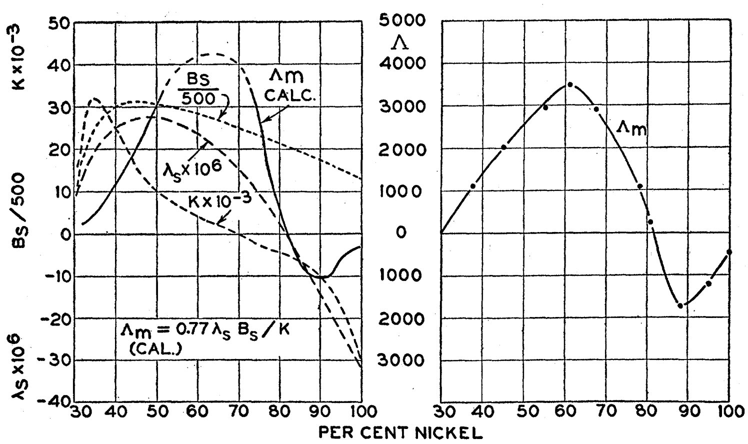


FIG. 13. Λ_m as calculated from data shown by broken lines. Theoretically $\Lambda_m = \infty$ when $K=0$, except as limited by internal strain. Note general similarity of calculated and observed values of Λ_m.

B^2 or, what amounts to the same thing, $d\lambda/dB$ is a linear function of B. That this is so is shown by the broken line in Fig. 14 calculated from Cioffi's data on 45 permalloy. For comparison the uppermost curve shows the values of $(1/4\pi\mu_r)(\partial B/\partial\sigma)$. The thermodynamic relation says that these two curves should be the same at high inductions when hysteresis is absent. Considering the fact that the data were taken on different specimens, annealed in different ways, the agreement is good. The middle curve shows in addition the directly observed values of $(d\lambda/dB)_r$, taken from an unpublished memorandum by L. J. Sivian and S. D. White of the Bell Laboratories, with very small reversible changes in B, using a third specimen of 45 permalloy.

DYNAMIC MEASUREMENTS

Another way to investigate the strain-sensitivity is by the dynamic method. A ring is made of a thin strip of 45 permalloy wound tightly and consolidated with a plastic so that it will vibrate as a unit when it is properly biased with a constant field and excited with an alternating field of small amplitude. It is carefully supported on strings inside of a toroidal box so that it will vibrate freely. Measurements are made of inductance and resistance, using an a.c. bridge, and the results plotted as shown in Fig. 15. Reactance and resistance change rapidly near the resonance frequency, and when impedance is plotted against frequency, one obtains the well-known motional impedance circle.

From the constants of this circle one can obtain the so-called magnetostriction constant of the material, and also many other characteristics such as the reversible permeability and the energy loss that occurs during oscillation. From the magnetostriction constant and the reversible permeability and Young's modulus, derived from

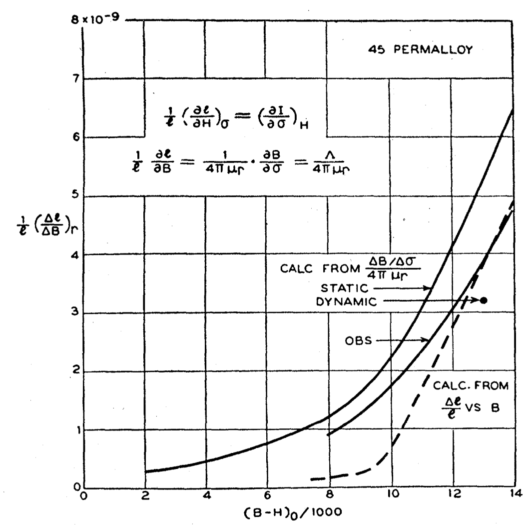


FIG. 14. Comparison of reversible change in length for unit change in induction as determined in various ways on different specimens: (1) directly observed (L. J. Sivian and S. D. White, unpublished), (2) calculated from Λ and reversible permeability determined by static and by (3) dynamic measurements (see Fig. 15), and (4) calculated from normal magnetostriction curve (λ *vs.* B) at high inductions.

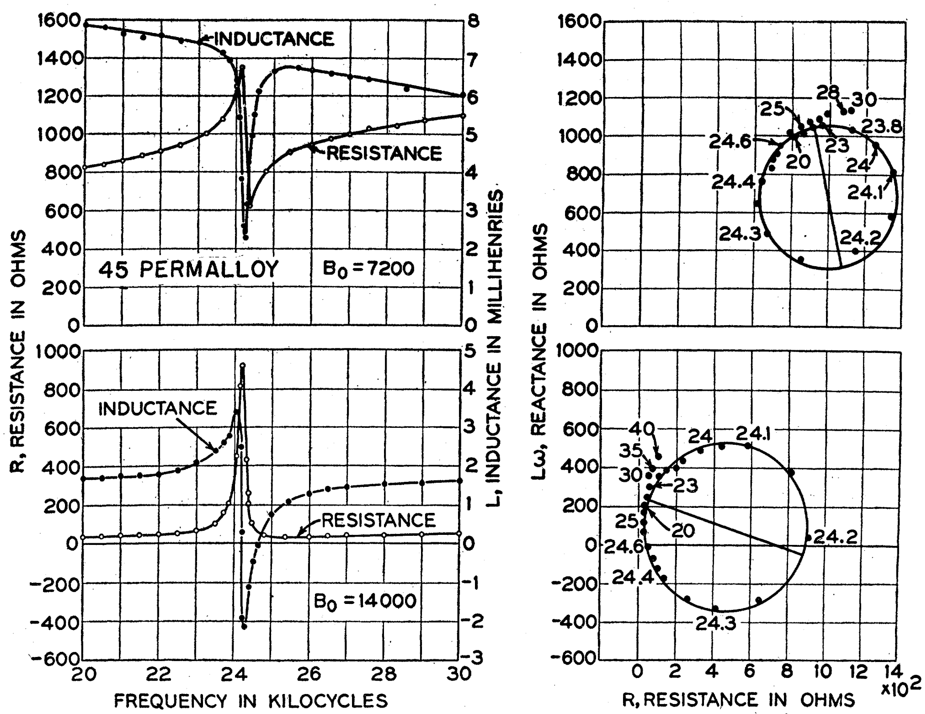


FIG. 15. Representative data on ring specimen of 45 permalloy made of 0.006 in. tape, taken with inductance bridge. Frequency varied through resonance. A variety of data can be obtained from motional impedance circles shown at right.

these a.c. data, we can calculate $d\lambda/dB$ and compare it with this quantity determined in the other ways already mentioned. Referring again to Fig. 14 we see that the agreement is reasonably good, considering the differences in the specimens used.

SUMMARY

Summarizing, we may say that measurements made with small cyclic stresses have established the general form of the Λ *vs.* B curve, showing the cyclic change of induction with stress as dependent on the polarizing induction. This value of Λ is shown to depend on the more fundamental constants λ_s (saturation magnetostriction), B_s (saturation magnetization), and K (crystal anisotropy constant). This dependence is derived from domain theory, and is shown to be valid for 45 permalloy (Λ *vs.* B) and for the iron-nickel series of alloys (Λ_m *vs.* nickel content).