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Surface Roughness and Sliding Friction

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I. INTRODUCTION

IT was early recognized that sliding friction was most probably due either to adhesion or to the roughness of solid surfaces. Coulomb (1)[1] decided in favor of the latter alternative, and his view was more or less generally accepted for some 130 years. Then the adhesion theory of friction became, and still is, predominant. It is the purpose of this paper to review the evidence for and against the rival interpretations. Its first part answers the questions on what is meant by surface roughness and by solid contact, and gives cursory information on some physical effects and some methods of measuring roughness. In its second part the meaning of sliding friction is defined, the recent theories and experiments on friction are discussed, and the conclusion is reached that Coulomb's decision is still justified, and the modern view must be abandoned.

II. SURFACE ROUGHNESS

1. General Remarks

The smoothest surfaces known are those of clean liquids. Because of the molecular movement, they are not mathematical planes, but they are definite within the amplitude of vibrations of the molecules constituting the surface.[2] This kind of roughness is not considered here; in the kinetic theory of liquids it was made responsible for internal friction of liquids, but the external friction of solids is attributed to a roughness on a much larger scale.

If the lines in Fig. 1 represent the profile of a metal A and of a cleavage face of a single crystal B, the distance $h_{\max}$ between the lowest and highest point on the surface is many times as large as an atom, an ordinary molecule, or the amplitude of its vibrations. A plane solid surface

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[1] References in ( ) are given in the list at the end of the article.

[2] If the transition between liquid and vapor is gradual, each plane defined by, say, constant density or constant composition, is definite within this amplitude.

is an approximation of the kind of the perfectly rigid body dealt with in theoretical mechanics. Most phenomena involving solid surfaces are affected by their roughness and can be used to estimate the latter, but each is affected in its own way and allows the determination of only one (or one set of) parameter(s) of the roughness. The most extensive discussion of roughness and its measurement can be found in the book by Schmaltz (2); shorter reviews have been given, e.g., by Way (3) and by Schlesinger (4). Here only a few points are considered which have a more immediate bearing upon the theory of friction.

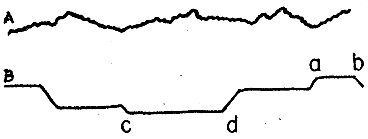

FIG. 1. Profile of a metal A and of a cleavage face of a single crystal.

2. Methods of Measurement

The methods of roughness measurement may be classified as those which do and those which do not involve a contact between two solids.

a. Methods Involving No Solid Contact

1. Optical methods.—Most optical methods belong to this class, and reflection methods form their most important subdivision.[3] If the surfaces were perfectly smooth a light pencil making an angle $-i$ with the normal would be reflected only in the direction $+i$, and the intensity of light reflected under any other angle would be exactly nothing. But the real surfaces are not smooth, and the behavior of the reflected light differs according to kind and degree of roughness.

Suppose first that the surface looks like curve B in Fig. 1. As practically the whole surface exposed to incident light consists of parallel plateaus, the light will be reflected in one direction only but the effect will differ from that produced by a smooth surface insofar as the light reflected from the summits (e.g., *ab*) will show a phase difference against that reflected from the bottoms of valleys (e.g., *cd*). This case was worked out by Fresnel [quoted by Jentzsch (5) (see also Wood (6))] who found that the path difference is $2h \cdot \cos i$ if h is the level difference between summit and bottom, and the phase difference therefore $(2h \cdot \cos i)/\lambda$, λ being the wave-length of the incident light. It is seen that the disturbance produced by the plateaus is less important the greater the wave-length. Jentzsch and Wood observed a similar effect, but the surfaces they used (glazed paper, dark glass) are not likely to have a shape like B in Fig. 1.

In another approximation the solid surface is pictured as an agglomeration of numerous facets inclined to the main plane under various angles, each of these facets reflecting light according to the classical law; the phase differences within the reflected beam are neglected. If the intensity I_j of the beam reflected in the direction j is compared with that I_i specularly reflected, the ratio I_j/I_i is a function of A_j/A_i, A_j and A_i being the total areas of the facets (2) making angles $\frac{1}{2}(j-i)$ and 0, respectively, with the main plane.

A third model assumes solid surfaces to be covered by either balls or hemispherical pits (7). It is easily seen that the ball surface reflects light within a larger solid angle than a pit surface does. Macroscopical surfaces of the ball and pit types conform to the calculation, but apparently not many observations have been made on surfaces possessing only microscopical roughness.

All three methods outlined above assume the reflecting facets to be large enough for formation of a regular wave front; if the facets are of colloidal dimensions, the light is scattered, not reflected, and it is even more difficult to estimate the shape of the surface from its behavior in incident light. A valuable review on light scattering by colloids can be found in Freundlich's *Kapillarchemie* (8) [see also reference (9)], while reflection by opaque bodies is treated by Harrison(10).

An extreme case of the reflection of light would be given by electron diffraction. Unfortunately the information on solid surfaces which so far has been gained by using the electron diffraction technique is rather one-sided. With a few exceptions all the patterns produced by what was

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[3] An interferometric examination of cleavage surfaces has just been described by Tolansky (4a).

intended to be reflection from surfaces are due to refraction in protuberances rising above the main plane; moreover, the protuberances must be very thin lest they absorb the impinging electrons. In order to get reflection patterns at all, nearly grazing incidence must be employed, and this arrangement results in a large area of a more or less smooth surface being covered by shadows of prominent hills. Thus, the electron diffraction patterns are not affected by, and give no information about, a major portion of the solid surface and reveal the crystal structure and the crystal orientation of a relatively small area only (11). However, the electron diffraction technique brought a welcome confirmation of the fact that sharp peaks are present even on well-polished surfaces.

2. *Total surface area.*—A very different function of surface roughness is determined by measurements of the total surface area of a solid. In contrast with electron diffraction which automatically selects favorable spots and neglects everything else, the total area is representative of the whole surface. On the other hand it is not discriminating: Obviously surfaces of various types of roughness, of various height, steepness, and frequency of the hills, etc., can give nearly identical values of the total area.

Unfortunately the measurement itself is still very uncertain. A technique often used consists in taking the adsorption isotherm of a substance on the solid surface to be tested, assuming a point of the isotherm to correspond to a complete unimolecular layer and attributing to the adsorbed molecule some definite cross section [see the recent review by Emmet (12)]. This procedure is open to objection. Since a large majority of solid surfaces are not uniform (either chemically or crystallographically, or both), there is no reason to suppose that a complete monolayer exists on any one of them; it is well possible that one face of a crystal adsorbs a multimolecular layer before another face starts building up its first monolayer [see references (13) and (14)].[4] The decision as to the point on the isotherm and to the cross section to be selected also must be somewhat arbitrary. Thus it is possible to calculate (16) that the real surface area A_r of cold-rolled steel, which probably consists of iron oxide particles, perhaps also of graphite flakes, etc., is identical with its geometrical area A_g. More probable values have been obtained, e.g., by Paneth and Vorwerk (17), for the A_r/A_g of $PbSO_4$ crystals: 6.4/3.34 and 10.7/6.1. Emmett (12) let glass beads, 7×10^{-4} cm in diameter, adsorb nitrogen at about $-180°$. The geometrical area was 0.34 m^2/g, the area given by adsorption on once-cleaned beads was 0.53, and that of twice-cleaned beads 0.75 m^2/g. Frazer, Patrick, and Smith (18) found no measurable adsorption of toluene (their experiments were not sensitive enough to detect a monolayer) on fire-polished glass surface, but they found considerable adsorption on glass washed with acids. This enhancement of adsorption is attributed to the silica gel formed on the surface by acid.

An electrochemical method seems to give more consistent values for the total surface area. The electrostatic capacity of a condenser is proportional to its area and inversely proportional to its thickness. As the equivalent thickness of the electric double layer at solid/electrolyte boundary, as far as we know, is independent of the roughness of the solid, the equation $A_r/A_g = C_1/C_0$ should hold; C_1 is the capacity per cm^2 of the geometrical area, and C_0 the capacity per cm^2 of liquid mercury surface, as the real and the geometrical areas are identical for clean liquids. The area A_r given by this relation would be expected to be smaller than the area accessible to adsorbed gas atoms since the equivalent double layer is thicker than, say, the nitrogen molecule, and many a double layer would not register asperities even of 10 or 20A.[5] But Bowden and

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[4] An instructive example of soiling a surface by polishing is described by Hothersall and Hammond (15).

[5] A striking example of the sensitivity of the effective roughness to the registering device is offered by another electrochemical effect—the anodic polishing (20). When a metal anode dissolves at a low current density its roughness does not change in any definite direction [although changes of roughness caused by a uniform dissolution have been described, see Rayleigh (21)]. At low current densities the layer of solution modified by the anodic reaction—that is, the layer which has a different composition, a different concentration, or is "polarized" in any other way—is so thin that it can uniformly coat the hills and valleys; the dissolution can proceed at a uniform rate along the whole surface, and the shape of the latter can remain unchanged. At high current densities (in a suitable solution) the anode becomes bright, in fact brighter than after ordinary polishing. When the current density is high, the modified layer of the solution, which probably consists

Rideal (19) found for A_r/A_g of a polished silver electrode the value 16; it was 50 for silver etched with nitric acid. Apparently, their mercury was not free from relevant impurities which lowered its capacity, but Veselovskii (22) could confirm their capacity values for silver. If from his results the ratio A_r/A_g is calculated from the modern values for the capacity of Hg electrodes, the values are near 5 for a silver foil and near 15 for etched silver.

b. Methods Involving a Contact Between Solids

1. Tracer methods.—There are several methods of roughness measurement involving a contact between two solids. One group uses tracer needles (stylus) which are drawn over the surface; their displacements normal to the surface are magnified by an optical lever [(23), (24)] or by an electric device (25). The result of a test is a curve representing the profile of the surface along the line chosen, although the representation exaggerates the unevenness. Since, for reasons of mechanical strength, the needle cannot be too thin, it cannot penetrate, and therefore register, narrow cracks and valleys; the diameter of most tracers mentioned in the literature is 10^{-3} cm or more, and a surface consisting of pits (as envisaged by Canac) less than 10^{-3} cm in diameter would appear perfectly smooth to a tracer instrument. The area which can be explored by a tracer needle in a reasonable time is very small. If we assume the diameter of the needle to be 10^{-3} cm and the length of its path 1 cm (it is shorter in many instruments), the area touched by the needle is 10^{-3} cm². A great advantage of the tracer curves is the immediate appeal to our senses which they make; after having seen such curves one is less prone to treat solid surfaces as solidified mathematical planes.

From the profile of a surface, its bearing-area curve can be obtained (26). If y is the ordinate of a point on the surface, reckoned from the bottom of the deepest pit (for which $y=0$), and x is the distance of a point from the point of origin along the main plane (i.e., not along the real surface), the abscissa defined by $y=0$ lies wholly within the solid, whereas only p percent of the line defined by $y=y_1$ (see Fig. 2) is within the solid. Values of p plotted against y form the bearing-area curve. Square root of the bearing area would have been a more correct designation (for more or less isotropic surfaces); when p is, say, 1 percent, the bearing area is only 0.0001 of the geometrical area involved. Three instances of bearing-area curves are reproduced in Fig. 3; the upper curve refers to a lapped, the middle one to a ground metal surface, and the lowest line was taken on a Johansson gauge (26).

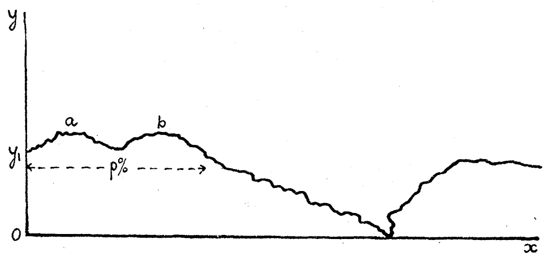

FIG. 2. Profile of a surface. The ordinate is reckoned from the bottom of the lowest pit.

An interesting idea is utilized in the thetameter of Tornebohm (27). The depth of an impression produced by a ball in a plate in the range of elastic deformations is connected with the curvature of the ball and the elastic constants of the materials by the well-known equations of H. Hertz [see, for example, reference (28)]. These equations assume, however, that the surfaces are perfectly smooth or, in other words, that the applied load is distributed over the whole (apparent) area of contact. In reality, the surfaces are rough. When the ball is placed on the plate, the load is concentrated around the summits of the hills (see a and b in Fig. 2), and the hills are considerably depressed by loads which would be too small to indent noticeably a perfectly smooth surface. Therefore, if the ball is loaded and its displacement downwards measured, the latter is larger than the displacement calculated according to Hertz. The difference of these displacements would be a roughness measure, easy to visualize. Tornebohm chose instead the ratio of the apparent to the real elasticity, a ratio which has no immediate geometrical significance. Two other drawbacks are inherent in the thetameter data. One is the insufficient knowledge of the real elasticity (i.e.,

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often of gas bubbles trapped in the pits of the surface, is thick and covers the valleys by an insulating cloud; the summits remain exposed and are rapidly worn away.

of the elasticity of the thin surface layer). The other is due to the friction between the ball and the walls of the pit being produced by the ball; this friction also is a function of the roughness of the surfaces in contact; its effects are discussed in O'Neill's book (29).

2. *Contact between flat surfaces.*—Both the tracer and the thetameter techniques involve contacts between surfaces of very different curvatures. Rougness can be estimated also by observing a contact between two flat surfaces. As the surfaces touch at a few points only, a slit is left between them and its dimensions can approximately be determined by several methods.

One of these methods was worked out by Heidebroek (30) although he did not realize the significance of his results (31). A plate under examination is pressed against a standard (stationary) plate covered with a liberal amount of lubricant. The experiment consists in separating the plates by a normal force and noting the time required to any value of the force. If the plates are at the beginning of the experiment separated by a liquid layer h_2 and at its end by a layer h_1 cm thick, the time t required for this movement is

$$t=\frac{3\eta a^2}{4F}\left(\frac{1}{h_2^2}-\frac{1}{h_1^2}\right);$$

F is the tension (force per unit area) applied, a is the radius of the moving plate (the stationary plate is larger), and η is the viscosity of the liquid (32), (33). If the experiment begins when the surfaces are in contact, h_2 is a measure of the clearance remaining between them, which approximately is equal to the height of a few highest hills on the rougher surface. The value h_2 computed in this way increases from finely to coarsely ground surfaces and also with the radius a of the moving plate. In the sole experiment quantitatively reported by Heidebroek h_2 was 1.7×10^{-3} cm (for cast steel).

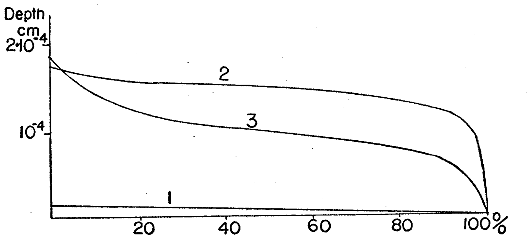

FIG. 3. Bearing-area curves. 1: Johansson block; 2: lapped surface; 3: ground surface.

A similar technique was used already by Nicolau (34), who pressed a metal cup against the surface to be tested and measured, in terms of pressure difference, the amount of air leaking through the slit between the surface and the edge of the cup.[6] The apparatus was calibrated by use of clearances of known dimensions and gave then the thickness of the slit in absolute units. This thickness again is closely related to the height of the highest hill, but it is likely to be smaller than h_2 because the apparent area of contact is smaller. The values found by Nicolau range between 7×10^{-5} and 7×10^{-4} cm for various finishes of three sorts of steel; the finish is important but the hardness of steel is not.

The clearance can be estimated also by measuring the electrostatic capacity of the condenser formed by the parallel surfaces in contact. Intensional roughness measurements of this kind have been published by Perthen (37). The computed distances between two plates were 3.7×10^{-3} cm for scraped, 1.2×10^{-3} cm for turned, and 7×10^{-4} cm for milled metal surfaces. Similar experiments had been performed by Watson and Menon (38) but they did not correlate their results with surface roughness. The thickness of the insulating layer in the system used by Watson and Menon (three flat steel pegs on a silvered plane) was not uniform, the dielectric constant of the insulator was doubtful, but the calculated "maximum height" of hills was in a surprising agreement with Perthen's later values: 9×10^{-4} to 3×10^{-3} cm.

Lewis (39) compared maximum heights obtained from capacity measurements with the average height of protuberances determined by a tracer method. For the average heights of 7.5×10^{-6} and 7.5×10^{-5} cm the maximum heights were 3.1×10^{-3} and 5.5×10^{-3} cm. These values have been found at a pressure of 1.4 kg per cm^2; both the maximum heights themselves and their differences for different surfaces were smaller at higher pressures.

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[6] An apparatus of the same type (Bekk smoothness meter) is widely used in the paper industry. Christiansen (35) and Rayleigh (36) used this method to estimate the distance between polished glass surfaces in contact.

3. Electrical contacts.—Another electric method gave, perhaps, the best information so far available about the nature of contact between solids, or at least between metals. It was worked out and employed by electrical engineers in connection with a study of electric switchgear and coherers. Holm (40) and Windred (41) reviewed the history of this study.

If two surfaces which are really smooth and really metallic are brought into contact, no contact resistance should be detectable; the electric resistance of the joined parts should be equal to the sum of the single resistances. Such joints can be realized by soldering or welding, but an additional resistance R is always observed when the contact is established only by pressing the surfaces against each other. The value of R depends on the total load L and not on the force per unit area. When L is raised, R generally decreases, but the rate of decrease depends on the geometry of contact and on the state of the surfaces.[7]

When two flat surfaces touch, the geometrical area of contact is independent of load. It would be natural to expect the conductance $1/R$ of the interface also to be independent of load. In reality $1/R$ is proportional to L. This has been noticed first apparently by Browning (43). He used brass disks in air and in oils. He found $1/R$ almost proportional to L up to a critical load which was higher for a smooth than for a coarse surface.[8] Other metal combinations show the same effect, e.g., nickel plates in air and in oil (45). Often $1/R$ is independent of the (geometrical) area of contact; that is, the joint between disks, of say, 1 cm^2 area offers the same resistance as the joint between disks of 2 cm^2, if the total load is identical in both systems. The analogy between this rule and Coulomb's law of friction is certainly striking. But the fundamental difference between the behavior of contact resistance and that of frictional forces becomes apparent when the load is made to vary periodically from, say, L_1 to L_2 and back ($L_2 > L_1$). It was found that $1/R$ does not resume its previous value when the load completes its cycle; every cycle raises both $1/R_1$ (at L_1) and $1/R_2$ (at L_2), but $1/R_1$ is raised more than is $1/R_2$, and after some 100 cycles $1/R_1$ becomes equal to $1/R_2$, that is, $1/R$ ceases to vary with load (45). The frictional force can be lowered by an appropriate treatment of the surfaces in contact, but it still remains proportional to load.

After many variations of load the conductance of flat-flat contacts becomes proportional to their geometrical area (which is independent of L). According to Holm (46) this condition can be attained at once for (slightly oxidized metal) contacts between ball and plate or between two crossed cylinders, i.e., when very high pressures deform the original surface. When the pressure is high enough to cause plastic flow of the contacting parts, the geometrical area of contact A_g is proportional to load, and the conductance $1/R$ is also proportional to L. The specific conductance $1/RA_g$ is a constant depending on the nature of the interfacial layer. It is independent of the voltage applied as long as the current density remains low; at high current densities the contact surface is heated so much that irreversible changes occur; an electric welding takes place, and the resistance drops.

When the metal contacts are well cleaned the conductance $1/R$ between ball and plate or between crossed cylinders is not proportional to the geometrical area A_g of contact. The issue is complicated by the spreading resistance which always occurs when the cross section of a conductor changes abruptly as, for example, when a thin wire abruptly joins an infinite plane or when an infinite plane is touched by a sphere. This spreading resistance is inversely proportional to the radius of the constriction, i.e., to $\sqrt{A_g}$. Since for plastic deformation A_g is proportional to L, the conductance $1/R = \text{const.}\ \sqrt{L}$. Conditions at which this equation approximately holds apparently have been realized by Holm (40) [see also (47)], although later experiments by Holm and Meissner (48) do not confirm his findings. The spreading resistance operates also in soldered or welded joints; its magnitude is independent of the nature of the actual contact; it must be eliminated or taken account of before the physics of the interface is deduced from the behavior of $1/R$.

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[7] The effect of load on the electric resistance between nut and bolt has been measured by Wenner *et al.* (42).

[8] A similar effect of load is observed also for the thermal conductance of flat metallic contacts (44).

The spreading resistance may be partly responsible also for the contact resistance of flat contacts. If the current passes chiefly through protuberances, a spreading resistance can occur at the base of each of them. Thus, it depends on roughness and has a bearing on the problem of friction. If, under some given conditions, n protuberances (bridges) connect the surfaces in contact, and the average radius of a protuberance is r, the combined conductance $1/R$ is proportional to nr. As experiment shows that $1/R$ is proportional to load (within limits, see above), nr as well appears to be proportional to L. That would be a very interesting rule, but, as Binder (49) points out, the electric resistance offered by the protuberances themselves cannot be neglected, and the increase of $1/R$ with L must be due partly to the shortening of the bridges caused by higher loads, which is measured in the thetameter technique. (See Section II, 2, b1.)

Both the spreading resistance and the resistance within the protuberances are metallic resistances. If by profound cooling the resistivity of the metal is decreased, say, hundredfold, these resistances are reduced in the same ratio. Hence, the temperature coefficient of the contact resistance can be used to decide whether this resistance is located within the metal or in an insulating layer between metal surfaces. Holm and Meissner (48) measured R between very carefully cleaned crossed gold rods in a vacuum at 20° and at low temperatures (−253° to −271°). At low loads (1–10 g) the resistance R was independent of temperature, and at higher loads (35–400 g) it was reduced by cooling to only about one-half of its value at 20° instead of to about 0.01. Obviously even at room temperature the major part of the resistance at low loads, and a considerable fraction of it at high loads, is due to an insulating layer.

Holm and Meissner calculated the specific resistance of this insulating layer and found it to be about 10^{-9} to 10^{-8} ohm cm, almost independent of temperature and the metal used.[9] [Holm and Meissner (50) also investigated nickel-nickel and copper-copper contacts]. On very clean surfaces this layer consists probably of adsorbed or chemisorbed oxygen. Frenkel (51) had worked out a theory attributing the contact resistance between metals to a gas layer always present on their surfaces. His theory was developed by Holm and Kirschstein (52). If it is substantially correct, the thickness of the adsorbed layer is only about 5×10^{-8} cm.[10]

If by a prolonged degassing at, say, 1100°C the gas layer is to a considerable extent eliminated, $1/R$ ceases to be nearly independent of temperature. The resistance between degassed crossed platinum wires in vacuum is at −253°C only about 0.06 that at 20°C. The contact is, therefore, mainly metallic (48). Since the relation between R and L has not been sufficiently investigated, it is not known whether the contact was along the whole geometrical area of contact or only on a fraction of this area.

The study of electric contacts thus shows that only by a rigorous treatment in high vacuum can a metal-metal boundary be established, that is, some metal atoms of one surface made to approach metal atoms of the other surface within a distance comparable with the interatomic distances inside the metal. It is uncertain whether non-metallic surfaces can attain this degree of cleanness. Metal surfaces which are exceptionally clean in the usual sense but not persistently degassed are covered by an insulating layer several molecules thick. All ordinary solid surfaces are coated with a much thicker layer of moisture, grease, etc.

III. SLIDING FRICTION

1. Coulomb's Theory of the Mechanism of Friction

Coulomb (1) himself gave only a brief indication of the mechanism by virtue of which the roughness of solid surfaces produces a frictional force. Our present knowledge of the roughness effects makes possible an amplified description.

When a slider is placed on a support it rests on relatively few legs. These legs are deformed by the weight of the slider, and the total elastic

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[9] The precision of the experiments does not warrant a more detailed discussion of the actual values.

[10] According to Rayleigh [(53), (54)] the nearest approach of two quartz surfaces is about 13×10^{-8} cm. This value is calculated from the intensity of light reflected from a boundary between polished quartz crystals in air. The medium filling the clearance is assumed to have the refractive index 1.

force caused by the deformation is equal to this weight. The number of legs and the extent of their distortion depend on the kind of roughness; if the maximum height of the protuberances is much larger than their average height, the number of legs will be small and their deformation considerable; when the maximum is only slightly greater than the average height, the legs will be numerous and little deformed. The magnitude of load is only important when the load is very small or very large. This is due to the shape of the bearing-area curves of all the usual surfaces[11] (see Fig. 3). At moderate loads the positive magnitude $-dp/dy$ is very large; that is, a small deformation causes a large increase of the bearing area (which is proportional to p^2); this results in the pressure (load divided by the real area of contact) being almost independent of load, and as the deformation is (approximately) proportional to the pressure, the deformation as well is almost independent of load. At low loads the variation of deformation with load is considerable, and at high loads the over-all pressure (load divided by the geometrical area) is so high that cracks or plastic deformations occur, and the profile of the surface is again a function of load.

When a tangential force is applied, the hills on the slider surface have to get over the hills of the support. The hills are deformed again but there is no way yet of calculating the magnitude of these deformations. Several attempts have been made to determine it experimentally; see the recent paper (54a) in which some ingenious experiments are reported and some references given. At any rate, some common slope is established, and sliding consists of alternate lifting of the slider along this slope which acts as an inclined plane, and in its falling between the hills. If the angle formed by the slope and the main plane of the support is α, the component of the pulling force f acting along the slope is $f\cdot\cos\alpha$. The component of the weight W of the slider acting along the slope is $W\cdot\sin\alpha$. When there is no acceleration $f\cdot\cos\alpha = W\cdot\sin\alpha$, and the coefficient of friction

$$\mu = f/W = \tan\alpha. \qquad (1)$$

Equation (1) gives at once the law of friction:[12] The coefficient of friction is independent of load and of the area of contact. Its derivation makes intelligible the limitations of the law. As at very small pressures the deformation is very small, the average $\tan\alpha$ and the coefficient of friction are likely to be larger than at moderate loads. This effect is very familiar; every dust particle clinging to a vertical wall or curtain—thus exhibiting an infinitely large coefficient of friction—can serve as an instance of it. At very large pressures the deformation is so great that the pulling force is employed more for bending and tearing up the hills than for climbing them. But even within the range of moderate pressures, the value of $\tan\alpha$ is not exactly constant for a given combination of slider and support. Since the protuberances on one surface vary in size and shape, it is a matter of chance what value of $\tan\alpha$ is arrived at in a single experiment. However carefully the slider may be placed on the support, at each new contact it will rest, at least partly, on other legs and in the course of sliding interlock with other protuberances. Thus, the coefficient of friction is only statistically determined. That explains the large scatter of its values at even very careful work and the striking fact that the precision of modern friction measurements is in no degree higher than that of the experiments made by Coulomb some 150 years ago.

Equation (1) accounts also for the low temperature coefficient of friction. As a matter of fact, it is uncertain whether μ depends on temperature at all, as long as one is well below the melting points of the substances in contact [(56), (57)]. Other determinations of the temperature coefficient of friction are mentioned below. They do not belong to the dry friction dealt with here.

Coulomb's theory gives a natural explanation also to the observation that the coefficient of friction depends on the material of the sliding bodies scarcely more than on their temperature. In friction tables there is usually only one value (or one range of values) for all metals, and this

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[11] The bearing-area curve of a brush is very different, and Coulomb's law does not hold for the friction between two brushes.

[12] It is usually attributed to Coulomb. Leonardo da Vinci (55) and Amontons (in 1699) already found μ to be independent of load and area but they apparently believed it to be a universal constant ($\frac{1}{3}$ or $\frac{1}{4}$).

value is quite near to those attributed to various woods. The physical properties of metals and woods cover such an enormous range that it would be strange indeed if the coefficient of friction were a function of one or several of these properties and still showed no correlation with the composition of the material.

Equation (1), besides leading to the law of friction, also gives the numerical value, or at least the order of magnitude, of its constant. A quotation from an earlier paper (58) follows: The coefficient μ is a pure number, and *a priori* there is no reason to suppose that it would be somewhere between 1×10^{-1} and 5×10^{-1}; the values, say, 1×10^{-3} to 5×10^{-3} or 1×10 to 5×10 would be as probable. On Coulomb's view, μ is approximately equal to the tangent of the effective angle between asperities and the basic plane. If μ were, say, 1×10^{-3} to 5×10^{-3}, this angle would be 4′ to 18′; if μ were 1×10 to 5×10, this angle would be 84° to 87°. These values would be incompatible with the known roughness of solid surfaces; indeed, they would be sufficient to refute Coulomb's theory. But the observed values require the effective slope of asperities to be 6° to 26°. Their meaning is that it is very difficult to prepare a surface on which no facet is inclined relatively to the basic plane at less than, say, 6°, and on the other hand, that surfaces showing irregularities steeper than 26° are avoided in frictional experiments. Bodies having no smooth surfaces, like earth of friction tables, may have values of over unity.

Qualitative confirmations of Coulomb's theory correlating friction and roughness are too numerous to be mentioned. This correlation is a common knowledge among engineers, the title "Friction and surface finish" of the conference convened by the Massachusetts Institute of Technology in 1940 well illustrates it. A quantitative confirmation will be possible as soon as the effective tan α can be measured independently. Unfortunately so far no method has been discovered for determining just this parameter of roughness. It is possible to compare only the frictional force between two surfaces with their roughness defined and determined by one of the methods listed in Section II. A comparison of this kind has been published by Prévost (59); a similar investigation will be referred to later (60).

Prévost's apparatus consisted of three rings; the top and the bottom rings (made of a nickel steel and polished) rotated in one direction and rubbed against the middle ring kept in position by springs; the extension of the springs was measured and was a measure of the frictional force. The rubbing surfaces were very slightly lubricated. The roughness of the middle ring was determined by the technique of Nicolau (see Section II, 2, b2). When this ring was of steel the coefficient of friction (at a constant pressure of 5 kg/cm^2) rose from 0.12 to 0.14 when the maximum height rose from 3×10^{-4} to 18×10^{-4} cm. A larger variation was observed when a cast iron ring was used at various stages of its grinding; when the maximum height was reduced from 38×10^{-4} to 9×10^{-4} cm, the coefficient of friction decreased from 0.27 to 0.14 (at the constant pressure of 1.3 kg/cm^2).[13]

2. Interface Layers and Their Elimination

The description given above of the mechanism of sliding disregards the phenomena taking place along the small areas of contact like *a* and *b* of Fig. 4. This is generally justified since solid surfaces even when they are in a microscopical contact are still separated by a film of adsorbed gases and vapors (see Section II, 2, b3). The actual shearing occurs between the mobile molecules of a more or less condensed gas or a liquid, not between the (almost fixed) atoms of a solid. The viscosity of the adsorbed film contributes therefore a term to the frictional force at sliding, but this term is negligible since the magnitude of viscous forces is proportional to the area of contact, and this area is only a small

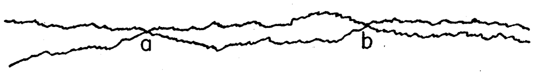

FIG. 4. Surfaces which make contact only on small areas.

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[13] In a similar investigation by Kehl and Siebel (61) in which, however, the roughness was determined only by the treatment given (lapped or ground) it was found that the seizure occurred at higher loads (300 kg/cm^2) for lapped than for ground surfaces (100 kg/cm^2). Gongwer (62) noticed a difference in the coefficients of friction across and parallel to scratches on ground metal surfaces, but a complete interpretation of his results is difficult.

fraction of the geometrical area of the slider-support boundary.[14]

If by an intensive heating and degassing the adsorbed layers are driven off, the surface atoms of the solids come together without a cushion precluding their interaction. This changes the nature of sliding completely or rather eliminates the sliding altogether (63). When two naked metals are brought in contact, they stick; that is, a force is required to separate them normally to the plane of contact. The force required to start a sliding is high and does not obey Coulomb's law. For example, if this force is determined by measuring the tilt at which sliding begins, it is found that the slider does not move even when the support is vertical, thus simulating an infinitely large coefficient of friction. If into the slider-support system having naked metal surfaces (nickel and platinum have been investigated) argon or nitrogen is admitted, there is still sticking and no sliding; but 0.7 mm Hg of air, moist air, or water vapor is sufficient to make possible a reproducible sliding. Vapors of organic compounds are as effective as air or water.

The experiments by Holm and Kirschstein give a particularly clear picture, as they used for producing naked metal surfaces a technique tested by measurements of the contact resistance. The friction experiments in vacuum performed by Shaw and Leavey (64) and by Bowden and Hughes (65) belong probably to the same class.

3. Adhesion Theory of Friction

Coulomb's theory accounts for: (1) both parts of the law of friction, (2) the numerical value of its constant, (3) the influence of temperature being negligible, (4) the influence of material being small, (5) the influence of roughness being important, (6) the poor reproducibility of frictional measurements, (7) the limitation of the law of friction as regards load, and (8) its breakdown in experiments with naked surfaces. It also helps in understanding the mechanism of lubrication (see Section III, 5 to 7). Nevertheless, objections against it have been raised again and again, and experiments have been reported which seemed to contradict it.

Except for a few objections which have been voiced once and never repeated, the bulk of the theoretical disapprovals of Coulomb's theory was based on the idea that molecular forces between the surfaces in contact could not be neglected [(66)–(71)]. The fallacy [pointed out by Landsberg (72)] of this idea is evident from the experiments reported in Section III, 2: When the contact between two solid surfaces is so intimate that their mutual attraction cannot be neglected, they stick together and cannot be made to slide in conformity with the law of friction. The real friction takes place, under ordinary conditions, during every displacement of solids because their contact is never intimate enough; they are always separated by gas or liquid films which have a negligible shearing strength.

Other reasons, of a more specific nature, can also be advanced against the adhesion theory of friction. (1) The adhesion ought to be discernible as a force operating normally to the interface; if a considerable force is required to overcome the adhesion by displacing the slider parallel to the support, a force of a similar magnitude must be applied to lift the slider clear off support—shearing strength and tensile strength of an adhesive joint usually are of the same order of magnitude (58). In reality no adhesion can be detected when a normal force is applied; Hardy (73), who believed in, and made an extensive search for, a relation between adhesion and friction, failed to find any and had to abandon his belief.

(2) The second objection is closely related to the first. If frictional force measures an adhesion, the magnitude of the latter must be proportional to load, when the load is increased. But when the load is reduced, the adhesion cannot decrease in the same ratio as there is no force available to break all the adhesive joints. Hence, if friction were due to adhesion, it would not have varied in a reproducible way with varying load [(58), (71), (74)]. This behavior is not observed as long as the law of friction is valid (74), but it is known for soft solids and high loads which cause sticking of these solids; Bikerman and Rideal (74) de-

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[14] A convincing estimation of these viscous forces is impossible. Taking probable values for the viscosity and thickness of the adsorbed layer and for the area of microscopical contact, one finds that only in extreme cases can they increase μ by more than 1 percent.

scribed it for paraffin wax, and Ernst and Merchant (71) for lead.

(3) The adhesion theory provides no valid explanation for the law of friction. It is generally agreed that the strength of an adhesive joint is nearly proportional to its area.[15] To account for the law of friction it is then necessary to assume that the real area of contact between slider and support is proportional to load and independent of the geometrical area of contact. The present knowledge of the real area of contact is so uncertain that it is impossible to disprove this hypothesis, but it remains a hypothesis *ad hoc*, and what is known from the behavior of the contact resistance (Section II, 6) does not speak in its favor. It directly contradicts the experiments under decreased load conditions mentioned above.

(4) The adhesion theory does not lead to the observed numerical values of the coefficient μ of friction although Charron (70) and Ernst and Merchant (71) published attempts to calculate μ. According to to Ernst and Merchant, μ would be a function of the heat of fusion. As μ may be identical for wood and metals and there is no heat of fusion of wood, the calculation is not convincing.

(5) The negligible effect of temperature is not in contradiction with the adhesion theory of friction, but the theory can hardly be reconciled with μ being almost independent of the materials in contact. From the point of view of the adhesion theory, if several systems sliding under equal loads exhibit equal frictional forces, then the product $A_r a$ of the real area A_r of contact and attraction a per unit area is identical for all systems. It means either that there exists an unsuspected correlation between A_r and a (but any such correlation is highly improbable since A_r depends on roughness and a does not), or that each A_r and a are identical for materials as varied as plastics, woods, and metals; as far as the magnitude of a is concerned this assumption is certainly wrong.

Only because the absence of a rigid connection between μ and material was disregarded was it possible to attribute friction to fusion (76) or welding (77) between slider and support. The law of friction is valid, and its numerical coefficient can be identical for pairs wood-wood, or platinum-wood, or glass-plastics, etc., in which no welding can possibly occur. It was suggested (77) that fluctuations of the frictional force which take place under certain conditions were a demonstration of formation and breaking of "welded bridges." But these fluctuations are accounted for [(78)–(80)] independently of whatever may be the mechanism of friction; and they are not confined to metal contacts; Kaidanovskii and Khaikin (78) observed them when using brass journals in beechwood bearings, and rubber sliding on glass exhibits periodical changes of friction (81).

(6) The adhesion theory would expect the roughness to be effective, but in the opposite direction to that observed. Rougher surface means a smaller area of actual contact, i.e., a smaller product $A_r a$; i.e., it would give rise to a smaller coefficient of friction.

(7) The adhesion theory accounts satisfactorily for the poor reproducibility of frictional measurements and, perhaps, can account for the limitation of the law of friction as regards load, but it is directly refuted by the breakdown of this law when naked surfaces are examined. Just when adhesion is undoubtedly present and measureable, no sliding is feasible at all.

4. Resistance to Sliding and Sliding Friction

In this section experiments are discussed which aimed at disproving Coulomb's theory and, allegedly, achieved their aim. For a correct estimation of these experiments it is necessary to define friction more rigorously than usual (58). When a knife cuts bread, or a pole is driven into the ground, or a point of a pyramid scratches the specimen in a sclerometric test, only a small part of the resistance is due to friction as conceived by Coulomb, and the bulk originates from cohesion forces of the material which determine its shearing stress, hardness, etc. This resistance, which in its physical nature is so different from Coulomb's friction, can be distinguished from the resistance offered by frictional forces as (1) it generally does not obey the law of friction, or at least its second part stating the independence

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[15] It cannot be exactly proportional to it as large joints are more likely to possess a very weak spot, see reference (75).

of μ from the geometrical area of contact; and (2) it is a function of the material and nearly independent of the conditions of its surface (a knife made of bread does not cut steel loaves, and the force required to slice a cake is not affected by powdering the cake with sugar). It is therefore sufficient, in order to disprove Coulomb's theory, to measure the resistance to scratching or a similar effect and to call it friction. If, however, the term friction is reserved for the effect governed by the law of friction, Coulomb's theory remains its natural explanation.

It must be admitted that the everyday usage of the word friction is wider than the definition given here. The friction between the rubber brakes and the wheels of a bicycle, or between an anchor and the bottom of the sea is different from Coulomb's friction. But if one employs the term friction in the loose sense of the everyday usage, there exists no law of friction and no general mechanism of it.

The resistance to scratching or a related effect has been examined by Hardy and Hardy (66), Tomlinson (67), Bowden and Leben (77), Campbell (82), Gongwer (62), and Ernst and Merchant (71). In their experiments, spheres or the point of a cone were dragged over a flat surface, or two crossed cylinders rubbed together. In all cases the local pressure was very high causing visible scratches and permanent deformations. Since mainly scratch hardness was measured, soft metals showed higher resistances than hard ones [the resistance increases with scratch width which is smaller the harder the specimen, see (83)]. The resistance was also, naturally, independent of roughness [(66), (71)] and of impurities (thin oxide or sulphide films) on the metal surface (82). Ernst and Merchant (71) measured also the variation of the resistance to deformation with temperature and found significant differences for different metals.

Hardy and his followers selected conditions under which resistance to scratching, etc., as distinct from real friction, was measured apparently because they believed that high pressure ensured an approximal equality between geometrical and real area of contact[16] and elimination of the surface films preventing molecular contact between the solids. But these assumptions are not generally valid. A brush pressed against a hanging curtain makes a bulge which does not envelop every bristle; and dust, oxide particles, etc., are not removed in a Brinell hardness test.

5. Monomolecular Layer of Lubricant

Some experiments with lubricated surfaces have been considered to be inconsistent with Coulomb's view and to support the adhesion theory of friction.

Langmuir (85) observed that unimolecular films of greasy substances on solids markedly lowered their coefficient of friction. Similar experiments have been performed later by Bowden and Leben (86), Givens [see (87)], and Hughes and Whittingham (88). Since a unimolecular film cannot appreciably change the profile of a solid surface, Langmuir's observation seems to refute the roughness theory of friction. On the other hand, it easily fits in the adhesion theory as the range of molecular forces is very short, and interposition of one (very long) molecule is sufficient to intercept the forces between two surfaces. But this argument loses its validity if the precise meaning of the term monomolecular is recalled. The cross section of the molecule of, say, stearic acid is about 2.1×10^{-15} cm^2, and a stearic acid film on water in which every molecule occupies about 2.1×10^{-15} cm^2 is a complete monolayer. When it is transferred onto a solid, the amount of acid which has covered 1 cm^2 of the water surface is distributed over 1 cm^2 of the geometrical surface of the solid. The real area being much larger (see Section II, 2, a2), the acid molecules can protect only a fraction of the exposed metal atoms. If, therefore, friction were due to welding, a monomolecular layer of grease would not have been sufficient to prevent it. The high temperature associated with welding would have destroyed the acid anyhow.

The solution of the riddle lies in the additional difference between monomolcular films on solids and on liquids. When the area per molecule of acid on water surface is larger than 2.1×10^{-15} cm^2, it can be shown fairly conclusively that the acid molecules are distributed evenly without

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[16] "The use of a curved surface has the advantage that some measure of definiteness is given to the area of contact" (84).

permanent clusters and voids. There is no evidence for a uniform distribution of film molecules over solids. For built-up multilayers, i.e., for films produced by successive transfer of monolayers from water onto a solid, it has been shown [(89), (90)] that they first form soap films stretched between the hills of the solid surface, which soon burst and give rise to microcrystals or gel particles irregularly dispersed over the solid surface. It is very probable that most monomolecular layers of grease on solids do not deserve the name of a layer at all and, like multilayers, consist of a few relatively thick patches. When a slider moves over a surface mottled with such patches, it may pile them up in front of its leading edge, or the frictional heat may melt several patches and collect them to a drop clinging to the clearance between slider and support because of the capillary forces, etc. Whatever may be the mechanism of lubrication in any special instance, the essential circumstance is that monomolecular layers are not blanketing the surface and cutting through the molecular forces operating between them.[17]

6. Thicker Lubricating Films

Hardy and Doubleday (84) and Hardy and Bircumshaw (33) found that the coefficient of friction between lubricated surfaces—the amount of lubricant being larger than a monolayer—was a linear function of the molecular weight of the lubricant when individual organic compounds were used. They believed this result to be totally inconsistent with Coulomb's theory. However, the linear relationship between μ and molecular weight is observed not for all surfaces and all homologous series. According to Sameshima and Miyake (91) the friction between a polished steel plate and a polished steel drum is independent of the molecular weight of the hydrocarbon (*n*-hexane to *n*-nonane) in which the drum is immersed. According to Sameshima and Tsubuku (92) μ between a glass lens and a microscope slide does not vary with the length of the fatty acid (acetic to nonylic). No effect of the molecular weight for cyclic compounds was noticed by Izemura (93). According to Fogg (94) the lubricating effect of esters on a steel ball-plate system exhibits minima for methyl propionate (formate to valerate having been studied) and ethyl valerate (in the series formate-stearate).

The effect of the molecular weight is even less definite when mixtures are considered. It is well known that small additions of fatty acids considerably improve the lubricating properties of hydrocarbon oils and that, sometimes, larger additions reduce them again [see (95), (96)]. Obviously the connection between lubricating efficiency and the molecular size of lubricant is less rigid than it appeared to Hardy and Doubleday.

From Coulomb's standpoint, since friction is due to roughness, the primary aim of a lubricant is to fill up the valleys on the solid surface and to prevent the hills of one from penetrating into the valleys of the other surface.[18] An ideal lubricant would (*a*) be a liquid so as to produce a truly smooth surface, (*b*) offer a strong resistance to movements normal to the surface (protuberances sinking into hollows), and (*c*) offer a negligible resistance to movements along the surface. There are substances which fulfill all of these conditions. They occur as smectic phases, the molecules of which have the tendency of a spontaneous parallel orientation and, when oriented, have a much higher viscosity in, than across, the direction of orientation [(97), (98)]. Perhaps an analogous phenomenon accounts for the correlation between lubricating effect and orientation tendency observed, e.g., by Finch and Zahoorbux (99) and Andrew (100) for various oils. The effect of molecular weight may also be due to increased orientation. The disturbance of orientation caused by a more intense molecular movement at higher temperatures may be a contributing factor to the temperature coefficient of the lubricating effect, but the experimental material is not definite enough yet to warrant a final conclusion [(33), (56), (101)–(103)].

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[17] In the experiments referred to above [(85)–(88)] at least the average thickness of the soap layer was definite although the local thickness was not controlled, but in several papers in which the expression monomolecular layer is used the average thickness was also unknown and often, probably, equal to several thousand molecular diameters.

[18] The performance of a lubricant in practice depends not only on its lowering the value of μ; a lubricant must also cool, retard corrosion, keep away dust, etc.

However, the connection between the degree of orientation and the efficiency of a lubricant cannot have a general validity since oils are not the only lubricating liquids. Water which possesses no smectic properties is a good lubricant not only for a physicist's system mica-mica (104), but also for industrial systems composed of plastics and metals [(105), (106)], metals and rubber (107), or rubber and rubber (108). On the other hand, graphite, though not liquid, perfectly satisfies the conditions (*b*) and (*c*) above.

Lubricants used for deep drawing of metals [(109)–(111)], in cutting fluids (112), or those termed extreme pressure lubricants, operate undoubtedly by means of another (although related) mechanism, just as the friction met with in cutting, drawing, etc., is different from Coulomb's friction. Here a layer of high crushing and low shearing strength is interposed between two relatively rigid surfaces. In drawing, this layer is produced by filling the oil with a more or less soft powder (graphite, mica, chalk, fireclay, soapstone, borax, flour) or consists of lead or another soft metal [(113), (114)].[19] Cutting fluids and E.P. lubricants produce such layers by chemically reacting with the solids in contact [(82), (112), (116), (117)].

7. Journal Bearings

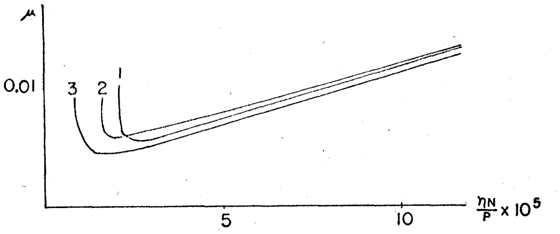

FIG. 5. Relation between coefficient of friction and the ratio $\eta N/P$. Curve *1:* Mean roughness 3.3×10^{-4} cm. Curve *2:* Mean roughness 1.8×10^{-4} cm. Curve *3:* Mean roughness 1.3×10^{-4} cm.

The lubrication of journal bearings has been investigated probably more thoroughly than any other kind of lubrication. If, for a given combination of shaft and bearing, the coefficient μ of friction is plotted against the ratio $\eta N/P$, η being the viscosity of the lubricant, N the number of revolutions per unit time, and P the load per unit projected area of the journal (or the bearing), a curve with a minimum is obtained [(95), (118)]; see Fig. 5 taken (idealized) from (60). It means that at high pressure and/or slow rotation, when the lubricant film is very thin,[20] the viscosity appears to be abnormally high. When the thickness of the fluid film is reduced to the order of magnitude of the asperities on the solid surface, the laminar flow normal to the radius of the shaft becomes impossible, the streaming being broken again and again by hills and peaks.[21] The obstruction thus caused is a probable reason for the abnormally high viscosity of thin films (122). Wittrock (123) observed probably the same phenomenon when he measured the friction between a rotating and an immobile plate at various clearances always filled with lubricant. The apparent viscosity of the lubricant remained constant as long as the clearance was over 5×10^{-4} cm; it was too large for clearances of 1 to 5×10^{-4} cm, and its relative increase was independent of the nature of the lubricant. However, in view of the observations by Bulkley (124) and by Bastow and Bowden (125), a repetition of Wittrock's experiments would be welcome.

A confirmation of the connection between roughness and the minimum on the $\mu-\eta N/P$ curve has recently been obtained by Burwell, Kaye, Van Nymegen, and Morgan [(60), see also (126)]. Steel shafts of various degrees of roughness as determined by a tracer instrument revolved in babbit bearings. The purely hydrodynamical part of the curve (i.e., that at higher $\eta N/P$ values) is scarcely affected by the roughness, but the position of the minimum is: The rougher the surface, the higher is the value of $\eta N/P$ at which the viscosity appears to be abnormally high; see Fig. 5. The effect of roughness is even more striking when instead of μ and

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[19] Atlee, Wilson, and Filmer (115) lubricated with low melting alloys bearings operating in high vacuum x-ray tubes.

[20] Determinations of its thickness from the electrostatic capacity of the condenser formed by shaft and bearing have been made by Schering and Vieweg (119) and Vieweg and Kluge (120).

[21] This effect seems to be of importance for electro-osmosis, streaming potentials, and related phenomena (121).

$\eta N/P$ the numbers $D\mu/C$ and $D^2\eta N/C^2P$ are plotted, D being the diameter of the shaft and C the clearance between shaft and bearing. The value of $D^2\eta N/C^2P$ at which $D\mu/C$ was a minimum was for a turned shaft with an average hill height of 3.25×10^{-4} to 1.25×10^{-4} cm, 475 times as high as for a superfinished shaft having an average hill height of 2.5×10^{-6} cm.

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