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Beta-Decay

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1. INTRODUCTION

THE experiments which established the existence of naturally and artificially radioactive emissions of negative and positive electrons by atomic nuclei and the critical calorimetric experiments which showed that part of the energy released during a β-decay process escapes in some yet undetected form were completed before 1933. In that year there was introduced the currently accepted picture of the β-process, in which Pauli's *neutrino hypothesis* occupies a central position. The assumption is that in the process a nucleon is transformed from a proton into a neutron (or vice versa) with the simultaneous creation of a positron (or negatron) and an (*anti*) *neutrino*. The latter particle is hypothesized to be the carrier of the missing energy and the failure to detect it after its emission is ascribed to its having no charge, probably no magnetic moment, and only small non-electromagnetic interactions with other particles.

The experimental basis for the hypothesis thus had originally a rather negative character. Its strength lay in the fact that the only apparent alternative was to discard the law of conservation of energy (also that for angular momentum) for the individual processes. Such an alternative became very implausible when it was shown in numerous specific cases that a maximum β-energy exists which fits into energy balances as the constant energy release in the given process. Evidence of a more positive character is afforded by the successes of Fermi's (F4)* theory of β-decay, which is based on the neutrino hypothesis. The results of this theory have shown, for instance, that the observed distribution of energy among the β-particles is just such as is to be expected

* See references at end of article.

when the energy is shared between an electron and a neutrino. More satisfyingly direct observation of the neutrino was undertaken by Crane and Halpern (C4) and Allen (A5). These investigators attempted to observe the recoils of nuclei from neutrino emissions. Allen's work seems the most nearly conclusive. He made his observations on Be^7, a nucleus suggested by Kan Chang Wang (K5) and many others since it is a light nucleus undergoing K capture and therefore having no electron emitted together with the neutrino. Allen found recoils somewhat too energetic to be due to the known γ-ray and moreover could find no γ-rays coincident with the recoils. Unfortunately, the quantitative aspects of the method are subject to relatively large corrections. Thus the present status of the neutrino can be summed up as follows: Although the detection of an individual neutrino has perhaps not yet been carried out in a completely decisive way, the neutrino hypothesis seems to be the only one which can correlate the known facts, and it has probably done this, through the Fermi theory, completely enough to justify the assumption. A recounting of the successes and of the present limitations of the Fermi theory will occupy most of this review.

Comparatively recent experiments concerned with the nature of processes allied with β-decay are the detection of K capture, the production of artificial isomers, and the discovery of nuclear fission. K capture is a natural extension of positron radioactivity. The latter process involves the absorption by a nucleon of one of the infinitely many electrons in negative energy states, the vacancy so caused being observed as a positron. The absorbed electron might also be one of the bound, atomic electrons, especially out of the K shell, which has the highest density near the nucleus. The vacancy thus created would be easily observable only through the x radiation emitted when it is subsequently filled. The evidence on this point is summarized in Section 8.

One case of nuclear isomerism has been known since 1921 (H1), but the prevalence of the phenomenon has not been appreciated until numerous cases of it have been recently turned up among the artificially produced nuclei. A pair of nuclei are experimentally recognized as isomeric when in spite of being identical in their constituents, they decay with different periods; in fact, one of the pair may even be a naturally occurring stable nucleus (D5). Weizsäcker (W7) has shown that such an occurrence is to be expected theoretically if the two lowest states of a nuclear system differ little in energy and greatly in angular momentum. Such properties make the time required for the radiative transition ordinarily expected from the upper state to the lower comparable in length to β-decay periods. Thus the clarification of the phenomenon is essentially a problem in γ-decay. For this reason, a more complete review of the subject will not be attempted here, especially since still adequate reviews are given elsewhere (W1, B5).

The newest means of producing beta-activities is the nuclear fission process discovered by Hahn and Strassmann (H2). It differs from the conventional nuclear disintegrations induced by bombardment in that the target nucleus splits into two nearly equal parts instead of emitting a single nucleon, α-particle, or γ-ray. It has so far been successful only with the heaviest nuclei as targets. Its chief interest for β-decay lies in the fact that it offers a method of obtaining radioactive nuclei too different in constituency from naturally occurring ones to be produced by the ordinary disintegration processes.

Another group of experiments which may turn out to have an important bearing on β-decay is that concerned with the meson. The possibility of their connection with β-decay arose from theoretical considerations due to Yukawa (Y1). These began with the idea that the β-emitting properties of nucleons might also be made to account for the forces between nucleons in the same way that the electromagnetic field simultaneously accounts for the creation of photons by charged particles and for the forces between these particles. However, the charge with respect to the electron-neutrino field which must be given to nucleons to account for the slow β-decay is found to be too small to account for the magnitude of the strong nuclear forces. Yukawa sought to avoid this difficulty by proposing two steps for the β-process. First, the nucleon emits a single, new kind of particle, which will here be called the meson, with a strength adjusted to account for nuclear forces. Then, the meson, in turn, may emit an electron and transform into a neutrino

with a strength adjusted to account for the rate of β-decay. The meson itself would not be detected in the usual β-process if its rest mass were too large to allow its creation from the energy available. It happens that this mass is related to the range of nuclear forces, which leads to a value for it of approximately 200 times the electron mass. Subsequent experiments seemed to make plausible the essential features of Yukawa's picture. Particles of about the meson mass have been discovered in the cosmic rays (S7, A4). Further, they were proved to decay (R6). However, difficulties have been encountered in fitting together *quantitatively* the experimental facts about mesons, the current deductions concerning nuclear forces, and the rate of β-decay (see, for example, B4).

Besides the more fundamental inquiries into the general nature of β-decay outlined above, there have been performed an enormous number of experiments multiplying and clarifying specific instances of the phenomenon. Several compendia of the results of such work have been successively published (for example, D4, L6, L5, S3). The most general methods used to associate observed decay periods with the proper isotopes and to establish the nature of the particles have been too long practiced to need further exposition here.

One newly-developed technique for clarifying the inter-relation of several types of radiation from a single nucleus perhaps deserves special mention. This is the coincidence counting method by which the association of a β- and a γ-ray or of two γ-rays, following immediately upon each other in a disintegration, can be shown. In the method, the coincident emissions are detected by a pair of Geiger-Müller counters which are connected to an amplifying and recording circuit which responds only to simultaneous discharges of both counters. To identify the energies of the associated radiations, absorbers may be interposed or a magnetic spectrometer used with one of the counters as detector. These types of experiments have been carried out by Norling (N3), Feather and Dunworth (F1), Langer, Mitchell, and McDaniel (L1), and others. The last-named collaborators have also introduced a method of obtaining the energy of a γ-ray coincident with another particle by measuring triple coincidences caused by the second particle and by the passage of a Compton electron through two counters between which absorbers may be interposed.

The measurement of β-γ-coincidence rates as a function of β-energy is a much less ambiguous means of proving the complexity of a β-spectrum than by trying to analyze the superposed continuous spectra themselves. A simple spectrum must give a coincidence rate per β-particle which is independent of the β-energy since all its particles leave the residual nucleus in the same state. A definite deviation from constancy is conclusive proof of an alternative β-process to a different residual state.

On the theoretical side, almost all investigations of β-decay have been interpreted in terms of the Fermi theory of the process mentioned above. Fermi's formulation is not completely unique, and much of the discussion concerns the experimental evidence for and against its various possible modifications. Interest has centered chiefly at two successive points at which the different versions diverge most widely. The discussion at the first point has crystallized to choosing between Fermi's original version, constructed according to a criterion of simplicity, and a suggested modification, known best as the K-U theory, which was constructed according to the apparent demands of existing experimental evidence. At a second point, the theory is subdivided according to the angular momentum changes that are allowed during the β-decay. The two interesting versions of these are Fermi's original selection rules and a modification in them introduced by Gamow and Teller (G1). Here they will be known as Fermi rules and G-T rules. The concern of Sections 2, 3, and 5 will be to recall the formulation which enables precise mathematical statements of the various versions.

The chief result of Section 3 is a formula for the energy distribution of the β-particles emitted in so-called "allowed" transitions. The experimental evidence for these cases is reviewed in Section 4. In Section 5 theoretical and experimental bases for distinguishing between allowed and forbidden transitions are established. The energy distributions to be expected in forbidden transitions are given in Section 6, and also their comparison with the best-measured cases is carried out there. In Section 7, a more thorough inquiry into the absolute probability of β-decay

is made, especially insofar as it is affected by the states of the parent and product nuclei. In Section 8 a brief resumé of the theoretical and experimental facts concerning K capture is given. Finally, in Section 9 conclusions which have developed concerning the questions mentioned in the preceding paragraph are summarized.

2. FERMI'S FORMULATION

Fermi (F4) put the theory of β-decay into mathematical form by adding to the Hamiltonian for the nuclear system a perturbing energy term like the following, for each constituent nucleon:

$$H=G\{(O^L\varphi\psi^*)O^HQ+(O^L\varphi\psi^*)^*O^{H*}Q^*\}. \quad (1)$$

Q is defined as an operator which, when applied to a wave function describing the initial nucleus, substitutes for it one in which a proton replaces a neutron. Q^* causes a nucleon to make the opposite transition. ψ and φ are wave functions of the electron and the antineutrino, respectively, each to be evaluated at the position of the transforming nucleon. In O^L and O^H lies the lack of uniqueness of the formulation. Each may be an operator affecting in some arbitrary way the operands, that is, O^L affects ψ and φ, O^H the wave function of the nucleon. G is a constant, to be empirically evaluated, which measures the strength of the interaction between the nucleon and the electron-neutrino field and consequently determines the scale on which β-decay takes place. Instead of merely advancing the form (1) of H as a hypothesis, Fermi put his original introduction of it on a more elegant basis by fitting it into a formalism analogous to the quantization of the electromagnetic field into photons. Thus the form (1) of H insured the simultaneous appearance of an electron and a neutrino during the transformation of a nucleon. The concrete merits of Fermi's theory will rest, however, on the direct experimental test of the expression for H.

From a given H, it is a straightforward matter to obtain the rate of β-decay. The probability per unit time PdW of the ejection of a β-particle with energy between W and $W+dW$ will be given by:

$$PdW=(G^2/2\pi^3)\langle|H|^2\rangle_{Av}pW(W_0-W)^2dW. \quad (2)$$

Here and throughout this article all quantities are made dimensionless in the following way: All masses are given as multiples of the electron rest mass m, lengths are given in units of $\hbar/mc$, and unit time is $\hbar/mc^2$. Thus W is the total (including rest) energy of the electron in units of mc^2, and $p=(W^2-1)^{\frac{1}{2}}$ is the electron momentum in units of mc. W_0mc^2 is the energy released by the decaying nucleus, so that $(W_0-W)mc^2$ is the part of the energy borne off by the neutrino, its mass being assumed zero in accord with the energy balance for several cases of β-decay (K4, B7, C2). *With GH_k being defined as* (1) applied to the kth nuclear particle:

$$\langle|H|^2\rangle_{Av}=(2p^2)^{-1}\int d\omega\sum_{lmj}\left|\int dv\sum_k V^*H_kU\right|^2. \quad (3)$$

The first integration $\int d\omega\cdots$ is over the directions of the neutrino momentum, a form convenient if φ is represented as a plane wave of unit density. The first summation $\sum_{lmj}$ is over the angular momentum quantum numbers of the electron, appropriate if its wave function ψ is expressed in spherical coordinates and is normalized to one in a sphere of unit radius. The $\int dv\cdots$ stands for integration over all internal nuclear coordinates including the space and spin coordinates of the transforming nucleon and therefore also of the electron and neutrino. U and V are the wave functions of the initial and final nuclei, respectively, properly symmetrized so as not to distinguish between identical neutrons or protons which may make the transition. The mean life τ of the β-decay will be given by:

$$1/\tau=\int_1^{W_0}PdW, \quad (4)$$

and expressed in seconds it is $\tau(\hbar/mc^2)=(1.24\times10^{-21})\tau$ sec.

After introducing H in the form (1), Fermi's next step was to adopt criteria from which the operators O^L and O^H could be determined. He first imposed the requirement of relativistic invariance as is appropriate in a theory involving a neutrino of vanishingly small rest mass. Next, as the simplest assumption possible, he chose $(O^L\varphi\psi^*)$ to consist merely of (relativistically invariant) bilinear combinations of the Dirac

components $\psi_1, \psi_2, \psi_3, \psi_4$ of ψ with $\varphi_1, \varphi_2, \varphi_3, \varphi_4$ of φ as *exemplified* by:

$$(O^L\varphi\psi^*) = (\psi^*\beta\varphi) \equiv \psi_1^*\varphi_1 + \psi_2^*\varphi_2 - \psi_3^*\varphi_3 - \psi_4^*\varphi_4. \quad (5)$$

Here β is one of the Dirac matrix operators, its effect as an operator being defined by (5). The form (5) transforms like a scalar, that is, it remains completely invariant in a Lorentz transformation. The ψ and φ components can be arranged into four other such combinations (B5), having the transformation properties of (*b*) a polar vector, (*c*) a tensor, (*d*) an axial vector, (*e*) a pseudoscalar, which is also a complete invariant except that it changes sign in a mirroring transformation. These transformation properties will be found to have great importance for the selection rules. Of course, the energy H must always remain a scalar and that can be arranged by choosing O^H in such a way that (V^*H_kU) takes on the following forms for the choices of ψ, φ combinations named in the parentheses:

$$(Scalar): \quad S = (V^*\beta Q_k U)(\psi^*\beta\varphi); \quad (6a)$$

$$(Polar\ V.): \quad V = (V^*Q_kU)(\psi^*\varphi) - (V^*\boldsymbol{\alpha}Q_kU)\cdot(\psi^*\boldsymbol{\alpha}\varphi); \quad (6b)$$

$$(Tensor): \quad T = (V^*\beta\boldsymbol{\sigma}Q_kU)\cdot(\psi^*\beta\boldsymbol{\sigma}\varphi) + (V^*\beta\boldsymbol{\alpha}Q_kU)\cdot(\psi^*\beta\boldsymbol{\alpha}\varphi); \quad (6c)$$

$$(Axial\ V.): \quad A = (V^*\boldsymbol{\sigma}Q_kU)\cdot(\psi^*\boldsymbol{\sigma}\varphi) - (V^*\gamma_5Q_kU)(\psi^*\gamma_5\varphi); \quad (6d)$$

$$(Pseudosc.): \quad P = (V^*\beta\gamma_5Q_kU)(\psi^*\beta\gamma_5\varphi). \quad (6e)$$

In these expressions the operators β, $\boldsymbol{\alpha}$, $\boldsymbol{\sigma}$, γ_5 may be regarded as merely shorthand for representing bilinear combinations such as (5). $\boldsymbol{\alpha}$ is a vector whose three components are the Dirac α matrices. $\boldsymbol{\sigma}$ differs from the usual Pauli spin matrices only in being doubled to four rows and four columns. β is the fourth Dirac matrix, and $\gamma_5 = -i\alpha_x\alpha_y\alpha_z$. Once some one of the forms (6) for (V^*H_kU) is settled upon, the problem becomes unique and the calculation of $\langle|H|^2\rangle_{Av}$ in principle straightforward.

It may be that no single one of the interaction forms (6) is the correct one, taken by itself. But any expression linear in each of the wave functions U, V, ψ, and φ must be expressible as a linear combination of the forms (6) if it is to be Lorentz invariant. Accordingly, the properties of such generalized expressions can be judged from the results obtained with the forms (6) separately, except insofar as interference between the forms in the linear combination becomes important.

Taking one of the forms (6) singly is the natural generalization of Fermi's original choice, which was to take the polar vector form (6b). In doing this, he allowed himself to be led by the analogy of the electromagnetic field; the interactions in the latter case are expressed in terms of a polar four-vector potential. But other points of view can be assumed, which make certain linear combinations seem more natural. One can be led to such a different point of view by the meson theory; Wigner and Critchfield are responsible for another.

Critchfield and Wigner (C5) have pointed out that the β-process may be viewed as a simultaneous creation of four particles: for example, a proton, an electron, and a neutrino in positive energy states and an unobservable neutron in a negative state. The fact that each is a particle with spin $\frac{1}{2}\hbar$ and accordingly can be equally well assumed to possess a continuum of negative states lends support to this picture. Now one is led to treat the four particles on an equal basis. The interaction form is expected to have some fundamental symmetry with respect to interchanges of any pair of U, V, ψ, and φ. It turns out that a completely symmetrical expression cannot be formed, but also satisfactory is the antisymmetrical combination

$$S - A - P \quad (7)$$

of the forms (6). Of course, such a treatment of β-decay ignores any connection of the meson with it, whereas Fermi's original choice of a single one of the forms (6) puts the nucleons (U and V) on one level and the light particles (ψ and φ) on another, in consonance with the two-step, meson process of β-decay.

The means by which the forms (6) may enter into the meson theory can be most readily seen in the "classical" treatment of the meson field analogous to the classical relativistic formula-

tions of the gravitational and electromagnetic fields which ignore the existence of quanta and are therefore only a first approximation (B4).

Assuming that inter-nucleon forces are derivable from a potential field requiring, like gravitation, only a scalar Φ for its complete specification, we let

$$\nabla^2\Phi-\frac{1}{c^2}\ddot{\Phi}-\kappa^2\Phi=-4\pi\rho. \tag{8}$$

Here ∇^2 is the Laplace operator, the dots on Φ represent time derivatives, c is the light velocity, κ is a constant, and ρ is defined as a "charge" density which serves as a source of Φ. Equation (8) differs from the usual relativistic generalization of the Poisson equation only in the addition of the term $-\kappa^2\Phi$, which was introduced by Yukawa (Y1) in order to replace the static centro-symmetric, point charge solution of the Poisson equation,

$$\Phi=g/r,$$

with

$$\Phi=ge^{-\kappa r}/r, \tag{9}$$

which is in better consistency with the short-range nuclear forces. Thus the known range of the forces sets the value of $1/\kappa$ at approximately $2\cdot10^{-13}$ cm, and their known strength gives g the approximate value $5\times(4.8\cdot10^{-10})$ erg$^{\frac{1}{2}}$-cm$^{\frac{1}{2}}=5e$, if e is the elementary *electric* charge. The g ($=\int\rho dv$ over the point charge) serves the same role in the meson field as e in electromagnetism, and it is appropriately referred to as the nucleon's "charge with respect to the meson field." From the quantum-mechanical point of view, (8) is a Gordon-Klein equation for the wave function of a quantum with a rest mass $\mu=\kappa\hbar/c\approx185$ electron masses.

Now, if the mesons play the role in β-decay discussed in Section 1, the possibility of their decay into an electron and a neutrino adds another sink to the meson field, so that $\rho=\rho_n+\rho_l$, ρ_n representing the charge on nucleons and ρ_l the contribution to the sources and sinks of the field due to the light particles. From (8), it is readily seen that ρ must be a *scalar* quantity, and therefore it may be assumed that

$$\rho_l=g'(\psi^*\beta\varphi), \tag{10}$$

after considering the properties of (5). Analogously,

$$\rho_n=g(V^*\beta U) \tag{10a}$$

may be used in place of the crude point charge picture. Then the addition to the energy of the system due to the influence on the existence of light particles of a meson field having nucleons as sources is given by:

$$\mathcal{H}=\int dv_l\rho_l\Phi_n\approx\int dv_l\rho_l\int dv_n\rho_n(e^{-\kappa r}/r)$$

$$=gg'\int\int dv_l dv_n(e^{-\kappa r}/r)(V^*\beta U)(\psi^*\beta\varphi). \tag{11}$$

Thus is introduced essentially the form S of (6a). The exponential in the integrand in effect requires that ψ and φ be evaluated at the position of the nucleon, $r=0$, in consistency with Fermi's procedure.

Others of the forms (6) than just (6a) can be incorporated in meson theories of more complicated types than as specified by (8). If the meson possesses internal degrees of freedom (spin), it is not fully describable by a scalar point function such as Φ and may need for its specification also a vector potential $\mathbf{A}$, as does the photon, which has a non-zero spin. Then, *besides* (8), one has

$$\nabla^2\mathbf{A}-\frac{1}{c^2}\ddot{\mathbf{A}}-\kappa^2\mathbf{A}=-\frac{4\pi}{c}\mathbf{j}. \tag{12}$$

Since $(\mathbf{A}, ic\Phi)$ and $(\mathbf{j}, i\rho)$ form polar four vectors, one can put

$$\mathbf{j}=g'(\psi^*\boldsymbol{\alpha}\varphi)\quad\text{and}\quad\rho=g'(\psi^*\varphi) \tag{13}$$

and thus introduce the form V of (6b) into the meson theory.

Still more complicated meson fields may arise if they may also have as sources elementary dipoles such as are introduced into electromagnetic theory by replacing the electric and magnetic fields $\mathbf{E}$ on $\mathbf{H}$ with $\mathbf{D}=\mathbf{E}+4\pi\mathbf{P}$ and $\mathbf{B}=\mathbf{H}+4\pi\mathbf{M}$, $\mathbf{P}$ and $\mathbf{M}$ being electric and magnetic dipole moment densities, respectively. Now (8) and (12) are not sufficient to describe the fields since $\mathbf{E}$ and $\mathbf{H}$ are not merely derivable from Φ and A, but are given by

$$\begin{aligned}\mathbf{E}&=-\text{grad}\,\Phi-\frac{1}{c}\dot{\mathbf{A}}+4\pi\mathbf{P},\\ \mathbf{H}&=\text{curl}\,\mathbf{A}+4\pi\mathbf{M}.\end{aligned} \tag{14}$$

Since the six components of **E** and **H** form a skew-symmetric four tensor of the type used in the interaction form T of (6c), this form can be produced in the meson theory by letting

$$\mathbf{P}=if'(\psi^*\beta\boldsymbol{\alpha}\varphi),\quad \mathbf{M}=f'(\psi^*\beta\boldsymbol{\sigma}\varphi). \tag{15}$$

With these complications, (11) is replaced by

$$\mathcal{H}=\int dv_l\{\rho_l\Phi_n-(\mathbf{j}_l\cdot\mathbf{A}_n)/c+(\mathbf{M}_l\cdot\mathbf{H}_n-\mathbf{P}_l\cdot\mathbf{E}_n)\}$$

$$=\mathcal{H}_V+\mathcal{H}_T+\mathcal{H}', \tag{16}$$

with

$$\mathcal{H}_V=gg'\int\int dv_l dv_n(e^{-\kappa r}/r)\,V, \tag{16a}$$

which would be expected from Fermi's original theory with form V of (6b),

$$\mathcal{H}_T=\tfrac{2}{3}gf'\int\int dv_l dv_n(e^{-\kappa r}/r)\,T, \tag{16b}$$

which is expected from (6c), and

$$\mathcal{H}'=\int\int dv_l dv_n\frac{e^{-\kappa r}}{r}\left\{\frac{1}{\kappa^2r^2}+\frac{1}{\kappa r}+\frac{1}{3}\right\}$$

$$\times\left\{\frac{3(\mathbf{M}_l\cdot\mathbf{r})(\mathbf{M}_n\cdot\mathbf{r})}{r^2}-\mathbf{M}_l\cdot\mathbf{M}_n\right.$$

$$\left.+\frac{3(\mathbf{P}_l\cdot\mathbf{r})(\mathbf{P}_n\cdot\mathbf{r})}{r^2}-\mathbf{P}_l\cdot\mathbf{P}_n\right\}, \tag{16c}$$

in which $\mathbf{M}_n$ and $\mathbf{P}_n$ differ from the $\mathbf{P}_l$, $\mathbf{M}_l$ given in (15) in that V, U replace ψ, φ. $\mathcal{H}'$ is a new contribution to the β-interaction by the meson theory; it vanishes for constant ψ, φ because of the angular dependence but must be considered when the variations of ψ, φ over the nucleus are taken into account. [See Sections 3(b) and 6].

Extension of the original scalar theory, which was completely specified by (8), in a manner similar to that of the last paragraph substitutes for the derived force,

$$\mathbf{F}=-\operatorname{grad}\Phi+f'(\psi^*\boldsymbol{\alpha}\varphi),$$
$$F_l=-\frac{1}{c}\dot{\Phi}+f'(\psi^*\varphi), \tag{17}$$

thus introducing interaction V of (6b) in a new way. Interactions A and P can take the place of V and S, respectively, if a *pseudo*scalar Φ and (therefore) an axial vector F are supposed to describe the meson field. Similarly, if the $(\mathbf{A}, ic\Phi)$ discussed above is assumed to be an axial rather than polar four vector, the four current $(\mathbf{j}, i\rho)$ can be supposed to be the axial vector giving form A of (6d).

To sum up, the simplest generalizations of Yukawa's meson field theory of β-decay produce interaction forms which can be arbitrary (because f' is independent of g') linear combinations of S and V, of V and T, of T and A, or of A and P.

Of course, these attempts to find an *a priori* basis for making a choice among the interactions (6) are at present in a speculative stage. It may turn out, for example, that the meson theory has no connection with the phenomenon of β-decay. We therefore proceed in the succeeding chapters to compare with the experiments the consequences of each of the forms (6) taken separately. This procedure is the simplest one and should perhaps be followed until it becomes evident that the experiments will be capable of distinguishing among different possible linear combinations.

3. THE THEORETICAL ALLOWED SPECTRA

Allowed transitions are defined from the theoretical standpoint as those for which the two following assumptions are valid:

(a) The second terms, if such exist, in the expressions (6) for (V^*H_kU) are neglected. The argument for this rests on the fact that the operators $\boldsymbol{\alpha}$ and γ_5 are known to have eigenvalues of the order of (v/c), v being the velocity of the particle concerned. Nucleons in the nucleus are believed to have velocities of the order of $\frac{1}{10}$ the velocity of light. Accordingly terms containing $(V^*\boldsymbol{\alpha}Q_kU)$ or $(V^*\gamma_5Q_kU)$ will have a magnitude about $\frac{1}{10}$ that of terms with (V^*Q_kU), $(V^*\beta Q_kU)$, or $(V^*\boldsymbol{\sigma}Q_kU)$. The terms neglected here will for convenience be referred to as "velocity terms" hereafter. Further arguments for excluding them at this point will appear when selection rules are discussed in Section 5.

(b) All terms except the first (and largest) one in an expansion of the light particle wave functions ψ and φ into powers of the position vector $\mathbf{r}$ of the transition are neglected. One needs these wave functions in the combination $(\psi^* O^L \varphi)$, in (2), (3), and (6). Then, for example, using plane waves

$$\psi = A \exp[i\mathbf{p}\cdot\mathbf{r}] \quad \text{and} \quad \varphi = B \exp[-i\mathbf{q}\cdot\mathbf{r}],$$

in which $\mathbf{q}$ is the neutrino momentum ($|\mathbf{q}| = W_0 - W$) and which are validly employed for the lightest nuclei, one obtains

$$(\psi^* O^L \varphi) = (A^* O^L B) \times [1 - i(\mathbf{p}+\mathbf{q})\cdot\mathbf{r} - \tfrac{1}{2}(\mathbf{p}+\mathbf{q}\cdot\mathbf{r})^2 + \cdots]. \quad (18)$$

Since the energy release usual in β-decay limits $(\mathbf{p}+\mathbf{q})$ to values of only a few mc units, while $\mathbf{r}$ is of nuclear dimensions and therefore of at most the order of $(1/40)(\hbar/mc)$, the successive terms of (18) grow smaller by factors of at least 10. It will be seen in Section 6 that for heavier nuclei the ratio of the second to the first term is often $(\alpha Z/2)$ instead of $|\mathbf{p}+\mathbf{q}|R$ although further terms resume the value $|\mathbf{p}+\mathbf{q}|R$ for their ratios to their predecessors.

With the approximations (a) and (b), the computation of $\langle|H|^2\rangle_{Av}$ for (2) becomes completely straightforward. Taking plane waves for the neutrino and Coulomb field solutions for the electron wave function, the results for the five cases (6a)$\cdots$(6e) are:

$$\langle|H|^2\rangle_{Av} = F(Z, W)\,|\int \beta|^2, \quad (19a)$$

$$\langle|H|^2\rangle_{Av} = F(Z, W)\,|\int 1|^2, \quad (19b)$$

$$\langle|H|^2\rangle_{Av} = F(Z, W)\,|\int \beta\boldsymbol{\sigma}|^2, \quad (19c)$$

$$\langle|H|^2\rangle_{Av} = F(Z, W)\,|\int \boldsymbol{\sigma}|^2, \quad (19d)$$

$$\langle|H|^2\rangle_{Av} = F(Z, W)\,|\int \gamma_5|^2, \quad (19e)$$

in which $|\int\cdots|^2$ is a symbol for $|\int dv \sum^k V^* \cdots Q_k U|^2$ and

$$F(Z, W) = \frac{4(2pR)^{2s-2} e^{\pi\alpha ZW/p}\,|\Gamma(s+i\alpha ZW/p)|^2}{[\Gamma(2s+1)]^2} \cdot \frac{1+s}{2}. \quad (20)$$

R is the nuclear radius; α the fine structure constant, $1/137$; Z the nuclear charge, to be taken negative for positron emitters; $s = (1-\alpha^2 Z^2)^{\frac{1}{2}}$; and the Γ's are the gamma-functions. $F(Z, W)$ shows the effect of the Coulomb field on the emission of the electrons. It emphasizes slow negatrons and fast positrons, as should be expected. It is exactly unity for $Z=0$ and is not far different from unity for charges as high as $Z=20$. Good approximation formulas for $F(Z, W)$ will be given in Section 5.

The most striking feature of the results (19) is that all the forms (6) give an identical energy dependence for the allowed β-spectrum obtained by using (19) in (2):

$$PdW = (G^2|\int\cdots|^2/2\pi^3) \times F(Z, W)pW(W_0 - W)^2 dW. \quad (21)$$

The nuclear matrix elements $|\int\cdots|$ are expected to be comparatively independent of W and Z and of order unity because of the normalization of the nuclear wave functions, unless some selection rule is violated.

One of the first detailed comparisons of Fermi's allowed spectrum (21) with experiments was published by Uhlenbeck and Konopinski (K2) who showed that many of the existing data indicated that the distribution (21) gave too little weight to slow electrons. They also showed how Fermi's theory could be modified at its most arbitrary point [represented here by the statement leading to (5)] to make it conform to the apparent experimental facts. The K-U modification consisted of the introduction for O^L, in a relativistically invariant manner, of a space-time gradient on the neutrino wave function φ. In the allowed approximation, this led to

$$P_{KU}dW = (G^2|\int\cdots|^2/2\pi^3) \times F(Z, W)pW(W_0 - W)^4 dW, \quad (22)$$

because $\partial\varphi/\partial t = -i(W_0 - W)\varphi$. This differs from Fermi's allowed spectrum (21) only by a factor $(W_0 - W)^2$ which obviously supplies the emphasis on slow electrons sought by Uhlenbeck and Konopinski. A more detailed examination of the Fermi and K-U allowed spectra will be made during the exposition of the experimental data in Section 4.

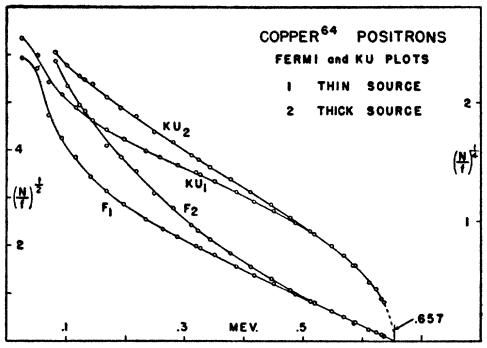


FIG. 1. Kurie plots for the positrons emitted by Cu^{64} according to Tyler (T3). The curves F_1 and F_2 are Kurie plots made in accordance with the Fermi theory [Eq. (21)], by using two sets of data resulting from the use of thin and thick sources, respectively. The same data treated in accordance with the KU modification (Eq. 22) give the curves KU_1 and KU_2. The correct theory should yield a straight line. It is seen that the thin source data support the Fermi theory whereas the thick source distorted the spectrum toward apparent agreement with the KU form over a large central section. The deviations of KU_1 and KU_2 at the upper ends are typical of those mentioned in Section 4(c). In the figure, $(N/f) \equiv N/pWF$.

4. THE EXPERIMENTAL ALLOWED SPECTRA

Great strides toward the elimination of falsifying effects in the measurement of the energy distributions of β-particles have been made since the introduction of Fermi's theory. One step is demonstrated by the work of Richardson (R3) and Alichanian, Alichanow, and Dzelepow (A1). They were among the first to show that previous experimental methods missed many of the slowest electrons in the spectra of heavy elements; they found that RaE, for example, had practically as many electrons of near-zero energy as at the maximum of the distribution. Another, and very difficult, step was the study and elimination of the overemphasis on moderately slow electrons caused by thick sources, work most intensively carried out by Flammersfeld (F7), Lawson (L3), Tyler (T3), and Neary (N1). Tyler's Cu^{64} measurements (Fig. 1) show graphically the effects due to scattering and energy loss in a thick source. Finally, the necessity of especially long runs in the sparse part of the spectra near their upper limits was recognized first in the work of Lyman (L7) and of Langer and Whitaker (L2). The general experience of the various observers seems to point to the conclusion that cloud-chamber data are less reliable than recent β-spectrometer data, a fact which is principally due to scattering in the gas of the cloud chamber and to the statistical fluctuations which are more inevitable in data from that instrument than from the spectrometer. The tendency of the experimental data obtained by more and more improved techniques has been to give less and less support to the criticisms advanced against Fermi's original spectrum (21) by Konopinski and Uhlenbeck.

The comparisons of Fermi's allowed spectrum with experiments are usually carried out by a method introduced by Kurie, Richardson, and Paxton (K4). The quantity $(N/pWF)^{\frac{1}{2}}$, N being the number of observed electrons per unit energy range, is plotted as a function of the energy W. It follows from (21) that this procedure should yield a straight line with an intercept at $W = W_0$ if the Fermi theory is correct. Similarly, $(N/pWF)^{\frac{1}{2}}$ is plotted when the K-U distribution (22) is put to a test. These types of graphs are usually referred to as "Kurie plots." Their chief advantage is, of course, that they make the test of the theory independent of the measurement of W_0. If the theory is accepted, they provide a straight-line-extrapolation method for finding the end point.

Almost all except the most recent experimental energy distributions seemed to give good agreement with the K-U theory over the main body of the spectrum. Most of the agreements should, however, be discounted, for the following reasons:

(a) Many of the agreements noted were provisional in that there was cause for doubting the validity of the allowed approximations [Sections 3(a), (b)] in the specific cases. The distributions actually to be expected for such cases may be very different from the allowed spectrum, as will be seen in Section 6. A criterion for judging whether a given observed spectrum is allowed will be derived in Section 5.

(b) As emphasized most recently by Bethe, Hoyle, and Peierls (B6, see also E2), there are strong reasons for believing that many of the distributions which show deviations from the Fermi spectrum toward the K-U form are in reality superpositions of two or more simple spectra each leading to a different state of the product nucleus. The extent of

this possibility and the evidence for it will emerge in the course of the further discussion below.

(c) A glaring discrepancy in the application of the K-U theory, even to indubitably allowed and simple spectra, appeared when the nuclear masses became accurately enough known to show that the former agreement had been obtained at the cost of assuming energy releases W_0 much larger than the mass differences allowed (K4, B7, C2, H4). Of course, allied with this were systematic deviations of the Kurie plots from linearity near $W = W_0$(L7, L2).

(d) As already indicated, it has developed that errors in the spectrum measurements themselves had been grossly underestimated. How errors due to thick sources tend to distort a Fermi distribution into just the K-U type is demonstrated by Tyler's measurements on Cu^{64} (Fig. 1); the Kurie plots for the Fermi theory give a better straight line with the thin source data, whereas the thick source falsifies the main body of the spectrum toward better agreement with the K-U theory.

Thus, the evidence of the spectra, which has previously comprised the sole support for the K-U theory, now definitely fails to support it.

In^{114}

On the other hand, complete and unambiguous confirmation of Fermi's allowed spectrum is provided in only a small number of cases. The outstanding example is Lawson and Cork's (L4) carefully measured In^{114} spectrum (Fig. 2). It yields by far the most perfect agreement with Fermi's allowed distribution shown by any data. The difficulty characterizing the measurements of most allowed spectra, that either the electron energies are too low for an extensive range of accurate measurement or that the half-life is too short to maintain a good intensity, is for In^{114} fortuitously absent. The 72-second β-decay, appropriately short for the 1.96-Mev energy release, follows upon a 50-day γ-ray, so that measurements can be carried out with leisurely accuracy. Conversion electrons from the γ-ray are easily corrected for because they form only sharp peaks

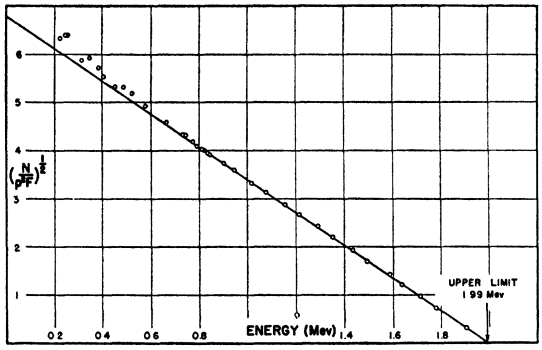


FIG. 2. Kurie plot of Lawson and Cork's (L4) data on In^{114}, made in accordance with the Fermi theory. The near-perfect linearity strongly supports the Fermi spectrum [Eq. (21)]. In this figure, N is the number of electrons per unit *momentum* range.

between 130 and 180 kev. That the In^{114} β-transition is indeed allowed will be shown in Section 5.

Cu^{64}

The Cu^{64} positron spectrum shown in Fig. 1 was measured with the same precautions as In^{114}, but the lower energy release reduced the range in which accurate measurements could be made. No deviation from linearity in the Kurie plot appears down to an energy below 0.3 Mev. The Cu^{64} negatrons, also measured by Tyler, show like results. The deviations below 0.3 Mev may still be due to scattering errors. Because Cu^{64} emits both positrons and negatrons, the evidence that the allowed approximations (a) and (b) of Section 3 are applicable to both transitions is somewhat obscured, but is shown to be largely confirmatory by Table II.

N^{13}

Another allowed spectrum which has apparently been measured with accuracy comparable to the cases above is the positron spectrum of N^{13} (K1, L8, T2). Like Cu^{64}, this shows perfect agreement with the Fermi theory in the upper part of the spectrum from about 0.6–0.7 Mev to the limit of 1.20 Mev. The deviation at low energies, however, seems here to be definitely beyond experimental error. This fact, together with the existence of a 280-kev γ-ray, found by Richardson (R1) and observed to be emitted coincidentally with annihilation radiation by

Lyman (L8), makes plausible the existence of a superposed low energy spectrum. Lyman reports that his results are in agreement with the Fermi theory for a γ- to β-ratio of about 0.25, the limit of the low energy spectrum being taken as 0.92 Mev. Watase (W5) disagrees with this, finding that his spectrum can only be fitted by the Fermi theory if the superposed spectrum has a limit of 0.6 Mev. The importance of this problem as a test of selection rules will be described in Section 7.

C^{11}

This certainly allowed spectrum (see Section 5) was measured by Townsend (T2) and begins deviating slightly from the Fermi form below about 300 kev (end point, 981 kev). The deviation may still be within the accuracy of Townsend's apparatus.

Bethe, Hoyle, and Peierls (B6) have hypothesized composite spectra to explain apparent deviations from the Fermi theory by B^{12}, F^{20}, N^{13}, O^{15}, and F^{17}. However, the spectra of B^{12}, F^{20}, O^{15}, and F^{17} have so far been measured only by the cloud-chamber method. Of these, perhaps the B^{12} measurement should be the most reliable because of the high energy release (12 Mev). Bayley and Crane's (B3) B^{12} data agree well with Fermi's distribution above 8 Mev, but deviate badly at lower energies, by a factor 4 at 2 Mev. Fowler, Delsasso, and Lauritsen's (F5) data agree better with the K-U spectrum, but then with the certainly too large $W_0=27$. A superposed decay to the known (C2) 4.3-Mev level of C^{12} may explain whatever deviation is not ascribable to experimental error, but the γ-ray has not been found (B3). Grönblom (G2 and Section 7) treats the theoretical possibilities for composite spectra for N^{13}, O^{15}, F^{17} and similar nuclei. It will be seen in Section 5 that all the nuclei mentioned in this paragraph are indeed expected to undergo allowed transitions with the exception of F^{20}, which therefore may not be expected to have a spectrum of the allowed form. It is interesting to note that the semi-empirical mass values listed by Barkas (B1) lead to a F^{20}-Ne^{20} mass difference of 7.2 Mev, just the *sum* of the energies of the β-end point (5.0 Mev) and the only known γ-ray (2.2 Mev). This argues for a simple spectrum, its deviation from the Fermi form to be accounted for by the fact that the transition is forbidden.

5. ALLOWED AND FORBIDDEN TRANSITIONS

(a) In Theory

In the allowed approximation (Section 3), there are produced in the formulas for the decay probability certain nuclear matrix elements $|\int \cdots| = |\int dv \sum_k V^* \cdots Q_k U|$, listed in (19). It is obvious, therefore, that the magnitude of the allowed transitions will also depend on how much the operators of the transition, $\cdots Q_k$, operating on the initial state function U, can make it overlap on V, the wave function of the final nucleus. The amount of this overlapping will be determined by the relative symmetry of the initial and final nuclear states and can be expressed in terms of selection rules. Because angular momentum and parity are always conserved, there will exist at least* selection rules with respect to these two quantities. The initial and final nuclei (denoted by subscripts i and f) will each have a specific total angular momentum **J** and either even or odd parity. The eigenfunctions U and V are orthogonal unless $J_i = J_f$ and the parity of the two states is the same. If the states *differ* in these respects then $|\int \cdots|$ vanishes unless the operation . . . alters the symmetry of U to one corresponding to the J and the parity of V. The operation . . . corresponds to a "carrying off" by the light particles of the difference $(J_f - J_i)$ in the angular momentum and of the change in parity.

The polar vector form of interaction** (6b) originally adopted by Fermi led to the nuclear matrix element $|\int 1|$ for allowed transitions. The identity operator 1 certainly does not alter the symmetry properties of U, and hence no difference of total angular momentum or parity between U and V can be allowed. Since the operator β is a scalar just as 1, interaction form (6a) yields the same selection rules for allowed transitions. The pseudoscalar γ_5 changes sign in mirroring transformations so the parity of $\gamma_5 U$ is

* For further possible selection rules cf. Section 7.

** The symbol V used to denote the form (6b) is of course not to be confused with V, the nuclear wave function.

TABLE I. Selection rules.

Interaction form (6):	Fermi (6b)			G-T (6c)		
	Matrix elements	ΔJ	Parity change	Matrix elements	ΔJ	Parity change
Allowed	$\lvert\int 1\rvert$	0	no	$\lvert\int\boldsymbol{\sigma}\rvert$	0, ±1 (No 0→0)	no
First forbidden	$\lvert\int\mathbf{r}\rvert$, $\lvert\int\boldsymbol{\alpha}\rvert$	0, ±1 (No 0→0)	yes	$\lvert\int\boldsymbol{\sigma}\cdot\mathbf{r}\rvert$	0	yes
				$\lvert\int\boldsymbol{\sigma}\times\mathbf{r}\rvert$, $\lvert\int\boldsymbol{\alpha}\rvert$	0, ±1 (No 0→0)	yes
				B_{ij}	0, ±1, ±2 (No 0→0, 1↔0, ½→½)	yes
Second forbidden	R_{ij}, A_{ij}	±1, ±2 (No 1↔0)	no	T_{ij}, A_{ij}	±2	no
				S_{ijk}	±2, ±3 (No 0↔2)	no
	$\lvert\int\boldsymbol{\alpha}\times\mathbf{r}\rvert$	±1	no	$\lvert\int\boldsymbol{\alpha}\cdot\mathbf{r}\rvert$	0→0	no

opposite to the parity of U. Consequently, the interaction (6e) leads to the selection rules $\Delta J=0$, (yes) for allowed transitions, the (yes) signifying that the parity must change.

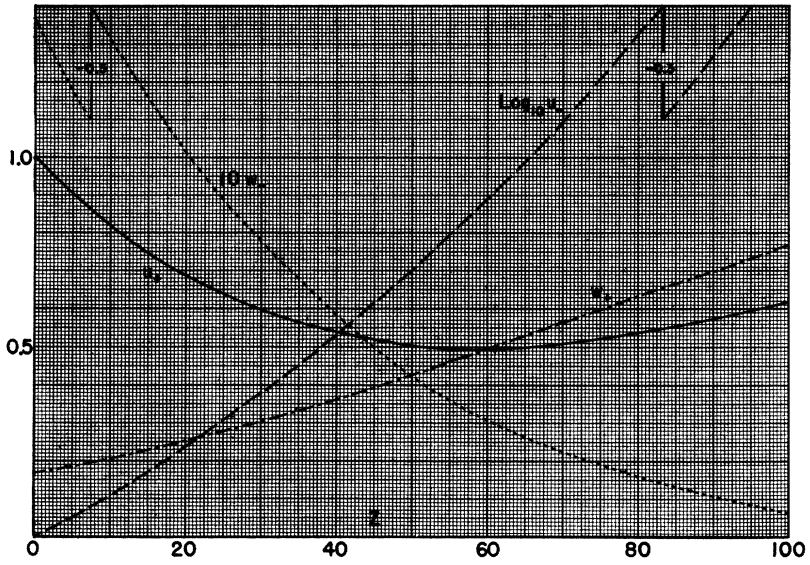
Selection rules very different from the original Fermi ones result from the pseudovector and tensor forms (6c) and (6d). Here, the Pauli spin operator, the axial vector $\boldsymbol{\sigma}$, causes initial states U to overlap on final states V in accordance with the selection rules: $\Delta J=0$, ±1 (no) with $0\rightarrow0$ forbidden. Arguments that these selection rules are to be preferred over Fermi's original ones, or at least that they ought to be superposed on Fermi's on the basis of experimental evidence, were advanced by Gamow and Teller (G1 and below); hence, the name "G-T rules."

Transitions which violate the allowed selection rules discussed above are described as forbidden. In such cases, the nuclear matrix elements obtained with the use of the first term in the expansion (18), giving the allowed transitions [approximation (b) of Section 3], vanish, and so the second term of (18) is to be used. It will, of course, yield transition probabilities smaller than the allowed ones by a factor of about $\lvert\mathbf{p}+\mathbf{q}\rvert^2R^2\approx1/100$. The important fact for the selection rules is that it introduces into the nuclear matrix element the vector $\mathbf{r}$. In Fermi's original interaction form (6b), this will lead to a replacement of $\lvert\int1\rvert$ by $\lvert\int\mathbf{r}\rvert$. The vector character of $\mathbf{r}$, like that of $\boldsymbol{\sigma}$, leads to $\Delta J=0$, ±1 (No $0\rightarrow0$). Since $\mathbf{r}$ is a polar vector, rather than an axial vector like $\boldsymbol{\sigma}$, a change of parity will be required: (yes). The G-T rules for forbidden transitions will be characteristic of quantities formed from $\boldsymbol{\sigma}$ and $\mathbf{r}$. The results will thus contain matrix elements expressible in terms of the irreducible representations: $\lvert\int\boldsymbol{\sigma}\cdot\mathbf{r}\rvert$, a pseudoscalar; $\lvert\int\boldsymbol{\sigma}\times\mathbf{r}\rvert$, a polar vector; and $\lvert\int\sigma_ix_j+\sigma_jx_i-(2/3)\delta_{ij}\boldsymbol{\sigma}\cdot\mathbf{r}\rvert$, a symmetrical tensor with zero spur. The corresponding selection rules are given in Table I.

When forbidden transitions are considered, not only must the smaller terms of the expansion (18) be reconsidered but also the small second terms in (6b), (6c), and (6d) must be readmitted. Only the first term of (18) need be used in evaluating these small velocity terms. This procedure is the proper one because a given case presents the problem of finding a transition probability for a given total spin and parity change; the velocity terms, evaluated with the first term of (18), can be seen to give the same class of selection rules as in the last paragraph. For example, $\boldsymbol{\alpha}$ is a polar vector just as $\mathbf{r}$ and will therefore lead to selection rules for $\lvert\int\boldsymbol{\alpha}\rvert$ the same as for $\lvert\int\mathbf{r}\rvert$.

Of course, in cases for which neither the allowed nor the first forbidden selection rules of the above paragraphs are obeyed, one must continue introducing higher powers of $\mathbf{r}$ from (18), thus obtaining selection rules characteristic of second forbidden, third forbidden, etc., transitions.

Table I contains a summary of the selection rules for allowed, first forbidden, and second forbidden transitions for two of the interaction



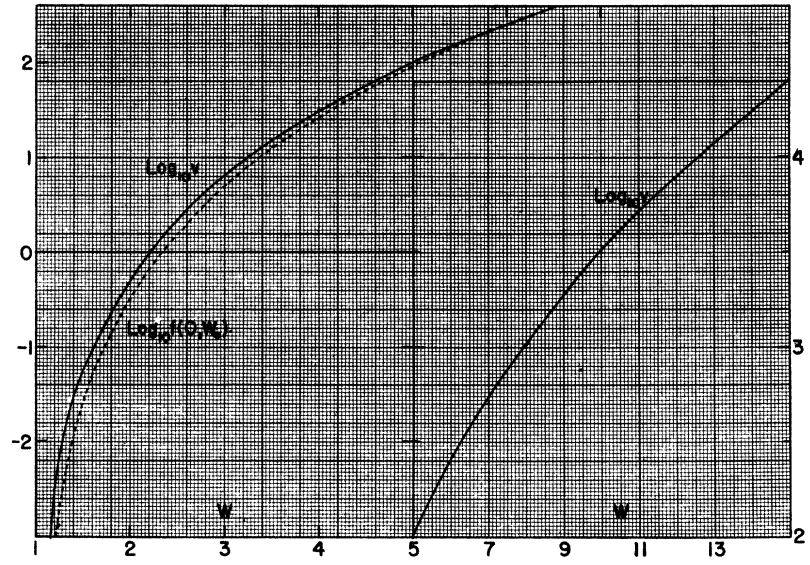


FIG. 3a. The functions $u(Z)$ and $w(Z)$ [Eq. (24a)]. u_+ and w_+ are for positrons; $\log_{10} u_-$ and $10w_-$ for negatrons. For the two curves extending off scale, the affected ordinates are lowered on the graph by 0.3 unit from their proper values, as indicated. FIG. 3b. The function $\log_{10} v(W_0)$, [Eq. (24a)]. The inset is for a contracted scale of energies as indicated. The dotted curve is $\log_{10} f(0, W_0)$ of Eq. (24a).

forms (6). The polar vector form V of (6b) gives Fermi rules and the tensor form T of (6c) gives G-T rules. Selection rules for the other three forms (6) are easily constructed with these as guides (K3). In the columns headed "matrix elements" are given symbols for the matrix elements which will occur in the several cases. In the next columns are given the selection rules for which the corresponding matrix elements will not vanish. The various tensors concerned are, leaving out β's, which are of no practical importance:

$$R_{ij} = |\int x_i x_j - (1/3)\delta_{ij} r^2|, \tag{23a}$$

$$A_{ij} = |\int \alpha_i x_j + \alpha_j x_i - (2/3)\delta_{ij} \boldsymbol{\alpha}\cdot\mathbf{r}|, \tag{23b}$$

$$B_{ij} = |\int \sigma_i x_j + \sigma_j x_i - (2/3)\delta_{ij} \boldsymbol{\alpha}\cdot\mathbf{r}|, \tag{23c}$$

TABLE II. Empirical classification of allowed and forbidden transitions.

Emitter	Radiation	Half-life (sec.)	W_0 (mc^2)	ft	Reference
Group 0A:					
$Be^{7}(1)$	K	5×10^{6}	0.70	2300	H4
$Be^{7}(2)$	K, γ	5×10^{7}	−0.18	5610	H4
C^{11}	β^+	1230	2.86	3460	
$N^{13}(1)$	β^+	750	3.35	5480	L8
$N^{13}(2)$?	β^+, γ	3000	2.8	8100	L8
O^{15}	β^+	125	4.37	4160	
F^{17}	β^+	64	4.78	3350	W8
Ne^{19}	β^+	20.3	5.31	1840	
Na^{21}	β^+	23	6.00	5330	W8
Mg^{23}	β^+	11.6	6.52	3090	
Al^{25}	β^+	7	6.85	2400	W9
Si^{27}	β^+	4.9	7.93	3550	B2
P^{29}	β^+	4.6	8.16	3800	W8
S^{31}	β^+	3.2	8.57	3260	W8
Cl^{33}	β^+	2.4	9.14	3420	W8
A^{35}	β^+	1.9	9.57	3450	W8
Sc^{41}	β^+	0.9	10.68	2480	E1
He^{6}	β^-	0.8	8.25	1160	
C^{10}	β^+	9	7.57	5970	
F^{18}	β^+	6720	2.37	5850	
Al^{26}	β^+	7	6.85	2400	
$Cl^{38}(2)$	β^-, γ	6660	2.3	11000	W4
H^{3}	β^-	10^{9}	1.03	1410	O2
B^{12}	β^-	0.022	24.5	6600	
$(Al^{28}(2)$	β^-, γ	>144	3	>1100	)
Group 1A:					
Li^{8}	β^-	0.88	24.5	2.8×10^{5}	
N^{16}	β^-	8	12.7	1.0×10^{5}	
F^{20}	β^-, γ	12	10.8	0.7×10^{5}	
Na^{24}	β^-, γ	5.3×10^{4}	3.74	12.7×10^{5}	K3
$Al^{28}(1)$	β^-, γ	(>)144	7.45	(>)1.5×10^{5}	
Sc^{44}	β^+	1.4×10^{5}	3.89	1.9×10^{5}	S6
Sc^{48}	β^-	1.6×10^{5}	2.26	2.8×10^{5}	S6
P^{30}	β^+	153	6.87	0.5×10^{5}	
Cl^{34}	β^+	1980	5.90	2.8×10^{5}	
K^{38}	β^+, γ	462	5.50	0.4×10^{5}	
Ne^{23}	β^-	40	9.03	1.0×10^{5}	
Mg^{27}	β^-, γ	612	4.52	0.4×10^{5}	
Al^{29}	β^-	402	5.89	1.0×10^{5}	
Si^{31}	β^-	1.0×10^{4}	4.52	7.6×10^{5}	
A^{41}	β^-, γ	6600	3.94	2.7×10^{5}	
$Ca^{45}(2)$	β^-, γ	$>1.6\times10^{7}$	1.4	$>5.0\times10^{5}$	
Group 2A (and higher A):					
Na^{22}	$\beta^+, \gamma(K)$	$>10^{8}$	2.08	$>1.5\times10^{7}$	
P^{32}	β^-	1.2×10^{6}	4.37	8.6×10^{7}	
$Cl^{38}(1)$	β^-	3330	10.38	2.1×10^{7}	W4
K^{42}	β^-	4.5×10^{4}	7.84	6.8×10^{7}	
Sc^{42}	β^+	1.2×10^{6}	3.74	1.3×10^{7}	
$Ca^{45}(1)$	β^-	$>1.6\times10^{7}$	2.8	$>10.0\times10^{7}$	
Be^{10}	β^-, γ	10^{13}	2.1	0.6×10^{13}	C3
C^{14}	β^-	$>10^{12}$	1.28	$>4\times10^{9}$	R7
Cl^{36}	$\beta^-(\beta^+, K)$	$>10^{8}$	2.4	$>2\times10^{8}$	
K^{40}	β^-	4.5×10^{16}	2.4	9.9×10^{16}	B8
Group 0B:					
S^{35}	β^-	7.6×10^{6}	1.21	1.9×10^{4}	
Sc^{43}	β^+, γ	1.4×10^{4}	3.54	12.0×10^{4}	
Ti^{45}	β^+	1.1×10^{4}	3.35	5.5×10^{4}	A2
V^{47}	β^+	1980	4.72	6.9×10^{4}	

Emitter	Radiation	Half-life (sec.)	W_0 (mc^2)	ft	Reference
Group 0B:					
V^{52}	β^-	234	5.01	4.2×10^{4}	
Mn^{51}	β^+	2760	4.9	11.3×10^{4}	
Mn^{52}	β^+, γ	1260	5.3	6.2×10^{4}	
$Mn^{56}(2)$	β^-, γ	3.1×10^{4}	3.0	9.6×10^{4}	T2
Ni^{59}	β^+	1.3×10^{5}	2.29	6.5×10^{4}	D6
Co^{60}	β^-, γ	640	3.94	3.6×10^{4}	
Cu^{61}	β^+	$>1.3\times10^{4}$	2.76	$>2.4\times10^{4}$	
	K	$>1.3\times10^{4}$	2.76	$>1.0\times10^{4}$	
Cu^{62}	β^+	630	6.1	8.4×10^{4}	
Cu^{64}	β^-	$>4.6\times10^{4}$	2.13	$>6.6\times10^{4}$	
	β^+	$>4.6\times10^{4}$	2.29	$>2.2\times10^{4}$	
	K	$>4.6\times10^{4}$	2.29	$>2.2\times10^{4}$	
Zn^{63}	β^+	2280	5.5	15.5×10^{4}	
Zn^{69}	β^-	3420	2.96	4.3×10^{4}	
Ga^{68}	β^+	4080	4.72	12.0×10^{4}	
Ga^{70}	β^-, γ	1200	4.33	13.0×10^{4}	
Ge^{75}	β^-	4860	2.97	7.6×10^{4}	S1
As^{73}	β^+	1.8×10^{5}	2.17	5.7×10^{4}	S1
$Se^{(81)}$	β^-	1140	3.9	7.7×10^{4}	
Br^{78}	β^+, γ	384	5.5	2.4×10^{4}	
Br^{83}	β^-	8400	3.1	13.5×10^{4}	
$Kr^{(79)}$	β^+	1.2×10^{5}	2.0	1.7×10^{4}	
Y^{88}	β^+	7200	2.34	0.5×10^{4}	
Cb^{94}	β^-, γ	396	3.74	2.8×10^{4}	
Cb^{95}	β^-	4500	3	9.7×10^{4}	
$Mo^{(91)}$	β^+	1020	6.2	12.0×10^{4}	
43^{101}	β^-	540	3.2	1.7×10^{4}	
Rh^{104}	β^-	44	5.5	2.2×10^{4}	
Ag^{106}	$\beta^+(K)$	1470	5.0	4.6×10^{4}	
Ag^{108}	β^-	138	6.5	18.3×10^{4}	
Ag^{110}	β^-, γ	22	6.5	2.9×10^{4}	
In^{110}	β^+	3900	4.13	4.3×10^{4}	
$In^{(111)}$	β^+, γ	1380	4.33	1.9×10^{4}	L4
In^{114}	β^-	72	4.88	2.4×10^{4}	
In^{116}	β^-	13	7.25	3.4×10^{4}	L4
	β^-, γ	3240	2.66	4.8×10^{4}	L4
In^{117}	β^-, γ	7020	4.39	14.1×10^{4}	
Sb^{120}	β^+	1020	4.00	1.0×10^{4}	
$I^{128}(2)$	β^-, γ	>1500	5.2	$>10.0\times10^{4}$	
Ba^{139}	β^-	5160	3.0	18.4×10^{4}	
Group 1B:					
Ca^{49}	β^-, γ	9000	5.5	2.1×10^{6}	
$Sc^{46}(2)$	$\beta^-, \gamma(K)$	$>7.3\times10^{6}$	1.5	$>5.6\times10^{6}$	
Sc^{47}	β^-, γ	2.3×10^{5}	3.15	3.1×10^{6}	
Sc^{49}	β^-	3490	4.53	0.3×10^{6}	
Ti^{51}	β^-, γ	6.2×10^{6}	1.72	1.3×10^{6}	
V^{48}	$\beta^+(K, \gamma)$	$>1.4\times10^{6}$	3.0	$>3.4\times10^{6}$	
$Mn^{56}(1)$	β^-, γ	13,300	6.63	8.8×10^{6}	T2
$Fe^{59}(1)$	β^-, γ	8×10^{6}	1.9	4.2×10^{6}	
$Fe^{59}(2)$	β^-, γ	8×10^{6}	1.5	0.8×10^{6}	
Ni^{63}	β^-, γ	9350	4.72	1.3×10^{6}	
Cu^{66}	β^-	300	6.6	0.2×10^{6}	
Ga^{66}	β^+	3.4×10^{4}	7.1	1.2×10^{6}	
Ga^{74}	β^-	7.9×10^{5}	2.57	4.2×10^{6}	S1
Ge^{71}	β^+	1.1×10^{5}	3.25	0.4×10^{6}	S1
Ge^{77}	β^-	2.9×10^{4}	4.72	4.9×10^{6}	
As^{74}	$\beta^+, \gamma(\beta^-)$	$>1.4\times10^{6}$	2.8	$>1.9\times10^{6}$	
As^{78}	β^-, γ	3780	3.74	0.2×10^{6}	
Br^{80}	β^-	1080	4.92	0.3×10^{6}	
Br^{82}	β^-, γ	1.2×10^{5}	2.37	0.5×10^{6}	
Kr^{88}	β^-	1.1×10^{4}	5.6	5.0×10^{6}	W6
Rb^{89}	β^-, γ	900	8.5	3.8×10^{6}	
Zr^{89}	β^+	2.8×10^{5}	3.0	0.4×10^{6}	
$Mo^{(101)}$	β^-, γ	1140	4.5	0.2×10^{6}	
43	β^-, γ	3300	5.9	2.3×10^{6}	

TABLE II.—*Continued.*

Emitter	Radiation	Half-life (sec.)	W_0 (mc^2)	ft	Reference
Group 1B:					
$Pd^{(107)}$	β^-	4.7×10^4	3.0	1.2×10^6	
Ag^{112}	β^-, γ	1.2×10^4	5.3	5.4×10^6	
Cd^{115}	β^-, γ	2.2×10^5	3.2	8.2×10^6	
$Sb^{122}(2)$	β^-, γ	$>2.4\times10^5$	2.6	$>3.2\times10^6$	
$I^{128}(1)$	β^-, γ	>1500	5.1	$>0.7\times10^6$	
I^{131}	β^-, γ	6.9×10^5	2.16	3.9×10^6	D3
Cs^{134}	β^-	1.1×10^4	3.0	0.4×10^6	
$Ce^{(143)}$		1.3×10^6	1.2	8.4×10^6	
Dy^{165}	β^-	9000	4.7	5.4×10^6	
Ho^{166}	β^-	1.3×10^5	4.2	4.9×10^6	
W^{187}	β^-, γ	8.3×10^4	3.15	11.0×10^6	
Os^{193}	β^-, γ	3.9×10^6	1.69	3.9×10^6	S4
Au^{196}	β^-, γ	4.0×10^5	1.7	2.0×10^6	
Group 2B (and higher B):					
$Fe^{59}(1)$	β^-, γ	$>4.1\times10^6$	2.76	$>0.3\times10^8$	
Co^{56}	β^+, γ	6.2×10^6	3.66	0.6×10^8	J1
Ga^{72}	β^-, γ	5.0×10^4	6.1	0.3×10^8	
As^{74}	$\beta^-, \gamma(\beta^+)$	$>1.4\times10^6$	3.4	$>0.4\times10^8$	
Rb^{86}	β^-	1.7×10^6	4.05	1.5×10^8	H5
Rb^{88}	β^-	1.1×10^4	5.6	5.0×10^6	W6
Sr^{89}	β^-	4.8×10^6	3.94	3.8×10^8	
Y^{90}	β^-	2.2×10^5	6.1	1.8×10^8	
$Mo^{(99)}$	β^-, γ	2.4×10^5	3.9	0.4×10^8	
$Sb^{122}(1)$	β^-, γ	2.4×10^5	4.45	0.6×10^8	M5
I^{126}	β^-, γ	1.1×10^6	3.2	0.6×10^8	
Eu	β^-, γ	$>3\times10^7$	2.6	$>7.0\times10^8$	
W^{185}	β^-, γ	6.7×10^6	1.9	0.4×10^8	
Group 2B (and higher B):					
Re^{186}	β^-	3.2×10^5	3.1	0.5×10^8	
Re^{188}	β^-	6.5×10^4	5.9	1.5×10^8	
Os^{191}	β^-, γ	1.2×10^5	3.94	0.6×10^8	
Ir^{194}	β^-	6.8×10^4	5.26	1.0×10^8	
Au^{198}	β^-, γ	2.3×10^5	2.6	0.1×10^8	
Rb^{87}	β^-	6×10^{18}	1.26	4.8×10^{16}	
Group 0C:					
Tl^{204}	β^-	253	4.1	2.0×10^5	
AcC″	β^-, γ	291	3.73	1.3×10^5	B5
ThC″	β^-, γ	186	4.51	2.8×10^5	B5
ThB	β^-, γ	3.8×10^4	1.70	1.5×10^5	B5
RaB	β^-, γ	1610	2.27	0.5×10^5	B5
AcB	β^-, γ	2100	2.23	0.6×10^5	L4a
UX_2	β^-, γ	68	5.5	2.7×10^5	B5
U^{237}	β^-, γ	6×10^5	1.5	16.4×10^5	
Group 1C:					
ThC	β^-, γ	3630	5.4	1.3×10^7	B5
RaC	β^-, γ	1180	7.1	1.7×10^7	B5
$MsTh_1$	β^-, γ	2.1×10^8	1.1	1.0×10^7	L4a
$MsTh_2$	β^-, γ	2.2×10^4	4.1	2.5×10^7	L4a
93^{239}	β^-, γ	2×10^5	1.9	0.4×10^7	
RaE	β^-	4.3×10^5	3.3	9.8×10^7	F7
Group 2C:					
Ac	β^-, γ	4.3×10^8	1.4	0.5×10^9	L4a

$$T_{ij}=\left|\int x_i(\boldsymbol{\sigma}\times\mathbf{r})_j+x_j(\boldsymbol{\sigma}\times\mathbf{r})_i\right|, \tag{23d}$$

$$S_{ijk}=\left|\int \sigma_{(i}x_jx_{k)}-(2/5)\delta_{(ij}\sigma_{k)}r^2 -(4/5)\delta_{(ij}x_{k)}\boldsymbol{\sigma}\cdot\mathbf{r}\right|, \tag{23e}$$

in which the enclosure of subscripts in parentheses means that there is a term for each permutation of the subscripts. Some of the matrix elements occurring in the second forbidden transitions repeat the allowed selection rules. Since only the allowed matrix elements will be noteworthy for cases obeying the repeated selection rules, the corresponding second forbidden rules are omitted from the table together with those matrix elements which thus fail to yield new rules.

Transitions with $J=0\rightarrow J=0$ and $J=0\leftrightarrow J=1$ have certain special properties. Thus, $0\rightarrow0$ transitions with parity change are not obtainable through the interactions S and V of (6) in *any* approximation. Similarly, the pseudoscalar and axial vector interactions P and A do not provide for $0\rightarrow0$ (no) transitions. These facts are the results of the impossibility of forming the scalar needed for $0\rightarrow0$ transitions out of an odd number of vectors only one of which ($\boldsymbol{\alpha}$ or $\boldsymbol{\sigma}$) is distinct from the others ($\mathbf{r}$'s). The scalar interaction further prohibits completely $1\leftrightarrow0$ (no) transitions because of the impossibility of forming a vector out of an even number of identical vectors. For similar reasons, the pseudoscalar fails to provide for $1\leftrightarrow0$ (yes) transitions. If it should happen that for a given case the only levels energetically available through the usual β-process require transitions violating rules of the character discussed here, then the nucleus will only decay through some less usual process. In the analogous case of electromagnetic radiation, two-quantum transitions take place. Perhaps, here, for example, a γ-ray would accompany the usual electron-neutrino pair. It is to be noted that the tensor interaction T imposes none of the limitations discussed in this paragraph.

(b) Empirically

Allowed and forbidden transitions are distinguishable *experimentally* through the comparison of the observed half-lives of the β-emitters, after due allowances for differences in

nuclear charge Z and energy release W_0 are made. A specific procedure for this follows. From (4) and (21), one has for the mean life τ in allowed transitions:

$$1/\tau = (G^2|\int \cdots|^2/2\pi^3) f(Z, W_0), \qquad (24)$$

where

$$f(Z, W_0) = \int_1^{W_0} dW W(W^2-1)^{\frac{1}{2}}(W_0-W)^2 F(Z, W).$$

In an approximation (N2) which is valid for $W_0 \gg 1+(\alpha Z)^2/2$:

$$f(Z, W_0) \approx u(Z)(\bar{p})^{2s-2} \times [v(W_0) - w(Z)(W_0-1)^3], \qquad (24a)$$

with

$$u(Z) = \frac{4}{(2s!)^2}(2R)^{2s-2}\frac{\pi|\alpha Z|(1+s)}{|1-e^{-2\pi\alpha Z}|},$$

$$v(W_0) = (W_0^5 - 10W_0^2 + 15W_0 - 6)/30,$$

$$w(Z) = \tfrac{1}{3}\pi|\alpha Z|\,|e^{2\pi\alpha Z}-1|^{-1}.$$

$\bar{p}mc$ is some average of the electron momentum. u, v, and w are plotted in Figs. 3a and 3b. The formula (24a) does not approach the correct $Z=0$ limit although the error is negligible for $W_0 \gg 1$ (in the development, $\frac{1}{2}W^2$ is considered small compared to unity). For $Z \approx 0$, one can use $f(0, W_0)$, plotted in Fig. 3b. This curve is inaccurate for $(W_0-1) \to 0$, in which case one has (W10):

$$f(0, W_0) \approx 0.216(W_0-1)^{7/2} \text{ for } (W_0-1) \ll 1. \qquad (24b)$$

A good approximation for $f(Z, W_0)$ with $Z \neq 0$ and $(W_0-1) \ll 1$ is easy to obtain only for *negatron* emitters ($Z>0$):

$$f(Z, W_0) \approx u_-(Z)(\bar{p})^{2s-2}v(W_0)(1-e^{-2\pi\alpha Z}) \text{ for } (W_0-1) \ll 2\pi^2\alpha^2Z^2. \qquad (24c)$$

The comparable formula for *positron* emitters ($Z<0$) contains so many terms of nearly equal magnitude that it is inconvenient to use. An additional error of less than 10 percent, in most practical cases, is introduced if one uses for positron emitters with $(W_0-1) \ll 2\pi^2\alpha^2Z^2$ just (24c) multiplied with: $\exp(-2\pi|\alpha Z|/[0.6(W_0-1)]^{\frac{1}{2}})$. These last approximations for $f(Z, W_0)$ have a range of validity which overlaps the range of (24a), while both degrees of approximation are still more than adequate for their purpose here. The formula (24a) is adequate for most of the cases to be treated.

Values of the product ft, with t the *half*-life, are given in Table II, for β-emitters whose energy releases and lifetimes are known. The quantity ft should be independent of energy release and charge for allowed transitions, and further, should be distinctly greater for forbidden transitions than for allowed ones. A rough measure of the magnitudes to be expected for the ratio of the ft value of a forbidden transition to the ft values of transitions one degree less forbidden is given by

$$1/|\mathbf{p}+\mathbf{q}|^2R^2 \approx 1/(p^2+q^2)R^2 \approx 1/(W_0R)^2.$$

As mentioned before [Section 3(b)], for heavy enough nuclei this magnitude *may* be replaced by $(2/\alpha Z)^2$ for the ratio of first forbidden to allowed values of ft. A nucleus will be heavy enough if its atomic number Z surpasses a certain critical value Z_c which can be defined by the equation of $\alpha Z/2$ to W_0R. Using for $R(\hbar/mc)$ the value $1.5\cdot10^{-13}\,A^{\frac{1}{3}}$ cm and $A \approx 2Z$, one obtains:

$$Z_c \approx 4\left[\frac{W_0}{\alpha}\left(\frac{R}{A^{\frac{1}{3}}}\right)\right]^{\frac{3}{2}} \approx 1.6W_0^{\frac{3}{2}}. \qquad (25)$$

As will be seen in Section 6, there is at present no way (because of ignorance concerning the precise ΔJ and parity change) of predicting for exactly which nuclei with $Z \gg Z_c$, $(\alpha Z/2)$ replaces (W_0R).

In the first column of Table II are listed the isotopes for which the ft values were calculated. Occasionally, the isotope symbol is followed by the symbols (1) or (2). This signifies that for the isotope in question a complex β-spectrum has been reported. The symbol (1) refers to the high energy β-transition and (2) to the less energetic one. The second column gives the type of radiation emitted: β^- and β^+ signify negatron and positron decay, respectively. K means K capture, and γ signifies that γ-rays are also emitted. When an isotope emits not only the radiation for which the calculation was made, but also other types, these are denoted in parentheses following the symbol of the radiation actually being considered. For example, Cl^{36} emits not only the negatrons to which the data in its row

apply, but also emits positrons and captures K electrons as signified by (β^+, K).

The third column of Table II gives the *corrected* half-lives of the various activities, in seconds. The corrected half-life coincides with the observed half-life only when the isotope emits only one type of radiation and with a single energy release. The time which it is appropriate to use in computing the ft value of complex decays is the inverse of the *partial* decay constant responsible for the transition in question. Accordingly, for a pair of transitions from the same nucleus one uses times t_1 and t_2 determined by the observed half-life $t=(t_1^{-1}+t_2^{-1})^{-1}$ and the intensities I_1, I_2 of the emissions $(I_1/I_2=t_2/t_1)$. In only a few cases are the relative intensities known (Be^7, N^{13}, Cl^{38}, Mn^{56}), this being indicated by the fact that definite values are given t_1 and t_2 in these cases. For all other isotopes the observed half-life is used, even for transitions known to be complex. For these latter cases, the symbol $>$ precedes the half-life value since it is certain that the corrected half-life is greater than the observed one. The error produced by this will rarely exceed a factor 2 or so, because if t_1 and t_2 are too different one of the transitions would have had too low an intensity to be observed.

The fourth column of the table contains the end-point energies of the β-spectra, in units of mc^2, with rest-energy included, consistent with the definition of W_0 in earlier chapters. The quantity ft computed as outlined above is given in the fifth column. The method of computing f for K capture is given in Section 8.

The last column contains the references for the various cases; only references consulted *in addition* to Livingood's (L5) and Seaborg's (S3) summaries are given.

Certain isotopes require special comment.

The complex transitions of Be^7 and N^{13} are treated in more detail in Section 7. There, doubts concerning the existence of complexity for N^{13} will be discussed.

The H^3 decay is the only one in the table for which it was necessary to use the formula (24b) to obtain f.

Following White, Delsasso, Fox, and Creutz (W9), we take the same energy and half-life values for Al^{25} and Al^{26}. These are probably identical within experimental error. Al^{25} has not yet been produced without the accompanying Al^{26} radiation. When Al^{26} is produced by itself $(Na+\alpha-n)$, the lifetime of 7 sec. is observed. This value is about the expected one for Al^{25} from the theoretical standpoint (Section 7), i.e., the energy release is the same as one expects for the difference in Coulomb energy between Al^{25} and Mg^{25}.

Al^{28} decay has not been shown complex by experiment, but probably is complex because the nucleus emits a 2.3-Mev γ-ray, yet its β-energy release is 3.2 Mev exactly as expected from Barkas' *semi-empirical* mass values (B1) for a transition to the ground state of Si^{28}. In a similar way it can be concluded from Barkas' masses that F^{20} and K^{38} undergo *simple* transitions even though they emit γ-rays.

There are a great many cases of γ-radiation which may indicate complex transitions; however, one cannot be sure that the radiation does not *follow* a simple β-transition instead, so the possibility of complexity is ignored. It is ignored even for the natural radio-elements because of conflicting evidence on this point. As pointed out above, such a procedure will not lead to significant errors for the transitions considered.

The elements in Table II are divided into groups A, B, C roughly according to atomic weight, group A consisting of the lightest nuclei, B of heavier ones, and C almost exclusively of the natural radio-elements. OA, OB, and OC are the allowed transitions in each group; $1A$, $1B$, and $1C$ the first forbidden ones, etc., *from the empirical standpoint.* Comparisons of the ft values characteristic of nuclei in OA, OB, and OC give the reason for the grouping according to atomic weight. The ft values in OB and OC are progressively larger than in OA. This phenomenon was first pointed out by Nordheim and Yost (N2). Qualitative reasons for it are discussed in Section 7. There was, of course, some arbitrariness in setting the boundaries between the various groups of Table II. The transition from OA to OB is fairly distinct, in accord with the Wigner picture to be discussed in Section 7; but, for example, the boundary between $1A$ and $1B$ is less definite. Still more arbitrary, however, are many of the distributions of members among the allowed, first forbidden, etc., groups. In

general, an attempt was made to give each group as homogeneous *ft* values as possible. The most uncertainly grouped cases are easily spotted by the wide deviations of their *ft* values from the norm of their groups.

It is necessary to emphasize that the groupings made in Table II are to a great extent provisional, especially because of the lack of distinguishable gaps between the *ft* values of the different classes of nuclei. Many of the assignments of nuclei to the various classes will turn out to be mistaken for several reasons. First, a constant ratio of about 1/100 was assumed to exist between the *ft* values of two classes differing by one degree of forbiddenness. Actually, this ratio is a function of atomic number and energy release, as will be more completely shown in Section 6. The assertion made at several points above, that sometimes the ratio is measured by $(W_0R)^2$ and sometimes by $(\alpha Z/2)^2$, is an indication of this and of the further fact that the ratio also depends on precisely what change of J and parity takes place of the several changes possible at the given degree of forbiddenness. Another factor contributing to the fluctuations of the *ft* values is the operation of further selection rules than those on J and parity. Such rules will be discussed in Section 7. Their violation in a given transition may cause its *ft* value to assume the magnitude characteristic of a more highly forbidden transition (in the theoretical sense according to which the degree of forbiddenness is determined solely by ΔJ and the parity change). A final effect which might be mentioned is one not always expressible in terms of selection rules, that is, the incomplete overlapping of the radial part of the state functions. This may exist even for two states which coincide completely in orientation symmetry and always results in a diminution of the matrix element.

Such tables as II are intended to take the place of the well-known Sargent diagrams (S2). In the diagrams, it is awkward to take into account differences due to atomic number, character of emission, etc. The subdivisions 0, 1, 2 . . . of the table correspond to the first, second, third . . . branches of Sargent's curves. An improved version of Sargent's diagrams has been offered by Evans (E3). Tables of the type given here have been published by Bethe and Bacher (B5), by Nordheim and Yost (N2), and by Itoh (I3).

6. FORBIDDEN β-DECAY

(a) The Spectra

The allowed spectra treated in Sections 3 and 4 cannot be entirely decisive as a test of Fermi's theory. The dependence on energy shown by Fermi's distribution (21) is due almost entirely to a statistical factor (K2, U1):

$$\frac{4\pi p^2 dp}{(2\pi)^3}\cdot\frac{4\pi q^2}{(2\pi)^3}=\frac{W(W^2-1)^{\frac{1}{2}}dW}{2\pi^2}\cdot\frac{(W_0-W)^2}{2\pi^2}, \qquad (26)$$

which can be expected to emerge from almost any conceivable theory involving the sharing of an energy W_0 between a pair of particles. Further, the allowed transitions make no distinctions, at least as far as dependence on energy is concerned, between the various forms (6) which can be given to the theory. There is hope that the forbidden spectra can be made to provide more thoroughgoing tests.

From the last section (especially Table I), it is seen that a variety of nuclear matrix elements may occur in the expression for the probability of a forbidden transition even with a given spin and parity change and a choice of a particular one of the five interaction forms (6). Prior knowledge of just what matrix elements must be present in the final answer shortens considerably the problem of finding the coefficient of each. The computation was carried out by Uhlenbeck and Konopinski (K3) for each of the forms (6), and an analysis on the basis of the K-U theory was given by Hoyle (H6).

The Uhlenbeck-Konopinski results will be listed here, in terms of a correction factor, $(2/1+s)C/|\int\cdots|^2$, by which the allowed distribution formula (21) must be multiplied to give a forbidden spectrum. The connection of C with $\langle|H|^2\rangle_{Av}$ of Eqs. (3) and (19) is thus defined by:

$$\langle|H|^2\rangle_{Av}=F(Z, W)(2/1+s)C. \qquad (27)$$

In the approximation $\alpha Z\ll 1$, which produces an error of only 10 percent or so even for nuclei as heavy as Ra, the results for the first forbidden

correction factor C_1 for the various forms (6) are (again leaving out the unimportant operators β):

$$C_{1S}=|\int \mathbf{r}|^2 A_-, \tag{28a}$$

$$C_{1V}=|\int \mathbf{r}|^2 A_+ + |\int \boldsymbol{\alpha}|^2 - [i(\int \mathbf{r})\cdot(\int \boldsymbol{\alpha})^* + \text{c.c.}]B, \tag{28b}$$

$$C_{1T}=|\int \boldsymbol{\sigma}\times\mathbf{r}|^2 A_+' + |\int \boldsymbol{\alpha}|^2 - [(\int \boldsymbol{\sigma}\times\mathbf{r})\cdot(\int \boldsymbol{\alpha})^* + \text{c.c.}]B + |\int \boldsymbol{\sigma}\cdot\mathbf{r}|^2 A_-'' + \textstyle\sum_{ij}|B_{ij}|^2\cdot 3a, \tag{28c}$$

$$C_{1A}=|\int \boldsymbol{\sigma}\cdot\mathbf{r}|^2 A_+'' + |\int \gamma_5|^2 - [i(\int \boldsymbol{\sigma}\cdot\mathbf{r})(\int \gamma_5)^* + \text{c.c.}]B + |\int \boldsymbol{\sigma}\times\mathbf{r}|^2 A_-' + \textstyle\sum_{ij}|B_{ij}|^2\cdot 3a, \tag{28d}$$

$$C_{1P}=|\int \gamma_5\mathbf{r}|^2 A_-, \tag{28e}$$

$$\begin{aligned} a&=[(W_0-W)^2+W^2-1]/36,\\ b&=2(W^2-1)(W_0-W)/9W;\\ A_\pm&=12a\pm b+(\alpha Z/2R)^2 + (\alpha Z/3R)[(W-W^{-1})\pm(W_0-W)];\\ B&=\tfrac{1}{3}(W_0-W^{-1})+\alpha Z/2R;\\ A_\pm'&=A_\pm-6a,\quad A_\pm''=A_\pm-8a. \end{aligned} \tag{29}$$

One sees from Eqs. (29) how important a role is played by the nuclear radius R. Its occurrence is due to the evaluation of the electron wave function, which has a singularity at the origin, at the surface of the nucleus. Perhaps some average of the electron wave function in the *interior* of the nucleus ought to be used instead, but no more is known of what such a wave function could be than just that it must connect with the exterior function at the surface.

The correction factors C as given by (28) all contain the term $(\alpha Z/2R)^2$ except for the selection rules $\Delta J=\pm 2$ (yes), which make all the nuclear matrix elements except the tensor B_{ij} of (28d) and (28c) vanish. This verifies the statement made in Section 3(b) and again in deriving (25) that for heavier nuclei $(\alpha Z/2)^2$ may often, but does not always, replace $(W_0R)^2$ as the ratio of the first forbidden to the allowed transition probabilities.

Lack of knowledge concerning the relative magnitudes of the various nuclear matrix elements contained in the expressions (28) makes the investigation of the energy dependence of the factors C difficult except for C_{1S} and C_{1P}. The factors C_{1V}, C_{1T}, and C_{1A} become equally simple only for special selection rules. They will therefore be discussed in terms of arbitrary values for the ratios of the various matrix elements involved. This will of course give unwarranted freedom to the fitting of these factors to experimental data, so conclusions are to be drawn with this in mind.

A few general statements can be made about the dependence on energy of the correction factors (28):

(1a) For $\Delta J=\pm 2$ (yes), possible in the first forbidden approximation only for C_{1T} and C_{1A}, the energy dependence is determined by a of (29). It is representable by a vertical parabola with a minimum at $W=\frac{1}{2}W_0$ having less than half as great a value as at $W=W_0$. The influence of the nuclear charge Z on it is very slight.

(1b) Excepting case (1a), all the C_1's are independent of energy for great enough nuclear charge [$Z\gg Z_c$ with Z_c given by (25)] because the term $(\alpha Z/2R)^2$ becomes dominant. Thus first forbidden spectra of *heavy* elements are nearly identical with allowed spectra, for $\Delta J=0,\ \pm 1$.

(1c) It may happen that first forbidden spectra take the allowed form even for light nuclei according to C_{1V}, C_{1T}, and C_{1A}. If it should happen that $|\int \boldsymbol{\alpha}|^2$ in C_{1V} or C_{1T}, or $|\int \gamma_5|^2$ in C_{1A} has a dominant magnitude, then the C becomes practically a constant.

(1d) C_{1S} and C_{1P}, for which the dependence on energy is definite, have substantially the behavior described in (1a) for $Z\ll Z_c$. For $Z\approx Z_c$, they are monotonically increasing functions of W.

(1e) A definite result is also possible for C_{1T} in $0\to 0$ transitions and C_{1A} in $1\leftrightarrow 0$ transitions. In each case only a single matrix element does not vanish (see Table I). The energy dependence in these cases is very similar to that of C_{1S} and C_{1P} as described in (1d).

Only a small number of forbidden spectra seem to have been measured with sufficient accuracy for the ratio of the observed to the

theoretical allowed spectrum (essentially the factor C) to be adequately reliable. These seem to be limited to Lawson's (L3) Na^{24} and P^{32}, Flammersfeld's (F7) and Neary's (N1) RaE, for all of which source thicknesses less than 4 mg/cm^2 were used. Moreover, the last two nuclei do not emit γ-rays, which could obscure the results.

Na^{24}

Of these only Na^{24} probably undergoes a first forbidden transition (Table II). Its spectrum agrees closely with the allowed distribution (21) from its upper limit of 1.39 Mev down to 0.4–0.5 Mev. Its ratio to the allowed spectrum then begins to rise fairly rapidly, up to 1.5 at 0.15 Mev. Coming at such a low energy, the rise might be spurious on three counts: It may be due to scattering, although that seems unlikely; it may be due to a superposed (allowed) low energy spectrum; or it may represent conversion electrons. The existence of a 1-Mev γ-ray, together with other more energetic ones (R2), would support either of the latter hypotheses. On the other hand, Langer, Mitchell, and McDaniel, (L1) and Feather and Dunworth (F1) find no more β-γ-coincidences per β-particle with low energy β's as partners than with high energy β's, which points to a *simple* spectrum. This seems to leave only internal conversion of the γ-rays to account for extra slow electrons. The intensity of these seems high for conversion in as light a nucleus as Mg^{24}. On the other hand, the γ-rays may very well have a high multipole character to account for their high conversion rate, as will presently be indicated.

For Na^{24}, therefore, it seems likely that the correction factor C must be substantially independent of energy. This rules out the selection rule $\Delta J=\pm 2$ immediately [see (1a)]. According to (25), $Z_c\approx 12$ here so that C_{1S} and C_{1P} become increasing functions of W [see (1d)] and indeed prove to increase so much that they can be definitely ruled out. C_{1V}, C_{1T}, and C_{1A} can all be made to yield approximate independence of energy [see (1c)] by assuming $|\int\boldsymbol{\alpha}|^2$ or $|\int\boldsymbol{\gamma}_5|^2$ to be much larger than any of the other matrix elements. Such assumptions would perhaps not be too implausible. It was seen in Section 3(a) that the ratios $|\int\boldsymbol{\alpha}|^2/|\int\boldsymbol{\sigma}|^2$ and $|\int\boldsymbol{\gamma}_5|^2/|\int\boldsymbol{\sigma}|^2$ are expected to have values of about 1/100, whereas $(W_0R)^2$ and $(\alpha Z/2)^2$, either of which may determine the size of the terms without the $|\int\boldsymbol{\gamma}_5|^2$ or $|\int\boldsymbol{\sigma}|^2$, each have about the magnitude 2/1000. On the other hand, Table II gives Na^{24} an ft value about 400 times as great as the allowed value. This may not all be caused by the violation of the allowed spin and parity rules, however (Section 7).

Thus, if C_{1A} can be said to be correct, Na^{24} must obey the selection rules: $\Delta J=0$ (yes). If C_{1V} or C_{1T} is the right form to use, the selection rules are $\Delta J=0,\ \pm 1$ (yes) and not $0\rightarrow 0$. The information is available that Na^{24} never decays to the ground state of Mg^{24}, which is expected to have $J=0$. The absence of the transition between ground states, in spite of a substantially higher energy release that way, must be due to a high J value in the ground state of Na^{24}. The excited Mg^{24} state to which the transition goes must therefore also have a fairly high J value since it is only first forbidden. A high multipole character and a consequently high conversion rate for the γ-rays in the transition of Mg^{24} down to its ground state are then not too implausible.

To discuss the remaining cases of those quoted above, P^{32} and RaE, it is apparently necessary to consider the correction factors for second forbidden transitions:

$$C_{2S}=\sum_{ij}|R_{ij}|^2\cdot D_-; \tag{30a}$$

$$C_{2V}=\sum_{ij}|R_{ij}|^2\cdot D_+ +\sum_{ij}|A_{ij}|^2\cdot 3a - [i\sum_{ij}R_{ij}A_{ij}^*+\text{c.c.}]\cdot E + |\int\boldsymbol{\alpha}\times\mathbf{r}|^2\cdot A_-'; \tag{30b}$$

$$C_{2T}=\sum_{ijk}|S_{ijk}|^2\cdot(c/36) + |\int\boldsymbol{\alpha}\cdot\mathbf{r}|^2\cdot A_-'' +\sum_{ij}|A_{ij}|^2\cdot 3a +\sum_{ij}|T_{ij}|^2\cdot(1/12)[3D_+-c] - [\sum_{ij}T_{ij}A_{ij}^*+\text{c.c.}]\cdot\tfrac{1}{2}E; \tag{30c}$$

$$C_{2A}=\sum_{ij}|T_{ij}|^2\cdot(1/12)[3D_--c] +\sum_{ijk}|S_{ijk}|^2\cdot(c/36); \tag{30d}$$

$$c=[3(W^2-1)^2+3(W_0-W)^4 +10(W^2-1)(W_0-W)^2]/90,$$

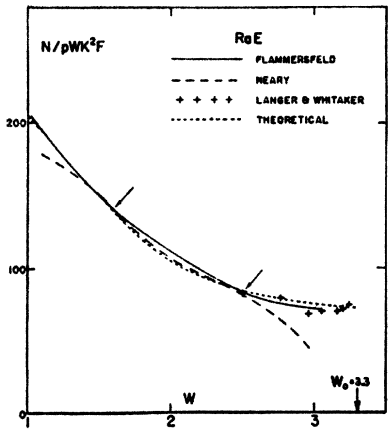


FIG. 4. The ratio of the observed numbers of RaE electrons at each energy W to the number expected according to the allowed spectrum (21). $K=W_0-W$. The dotted curve (K3) is the theoretical result for the ratio as given by C_{2T} (30c) when fitted at the points indicated by arrows by appropriate evaluation of the ratio of two nuclear matrix elements. S_{ijk} and $|\int \boldsymbol{\alpha}\cdot\mathbf{r}|$ were set equal to zero ($\therefore \Delta J\neq 0, \pm 3$). The value $A_{ij}/T_{ij}\approx -5.8$ was found to give a fit as shown. The ordinates are on an arbitrary scale.

$$D_{\pm}=c\pm\frac{36}{5}ab+\frac{\alpha^2Z^2}{48R^2}[W^2-1+4(W_0-W)^2]$$
$$+\frac{\alpha Z}{90R}\{(W-1/W)[10(W_0-W)^2 \qquad (31)$$
$$+3(W^2-1)]\pm(W_0-W)$$
$$\times[6(W_0-W)^2+5(W^2-1)]\},$$
$$E=\frac{1}{30}\left[(W_0-W)^3+\frac{(W^2-1)^2}{W}\right]+\frac{1}{4}bW_0$$
$$+\frac{\alpha Z}{24R}[2(W_0-W)^2+(W^2-1)].$$

C_{2P} depends on the energy exactly as does C_{2S}. The following remarks about the energy dependence of these quantities may be made:

(2a) For $\Delta J=\pm 3$ (no), possible in the second forbidden approximation only for C_{2T} and C_{2A}, the energy dependence is determined by c of (31). It increases very rapidly (by a factor 3 or so between the minimum and a limit of $W_0\approx 4$) for energies beyond the middle of the spectrum. It is uninfluenced by the nuclear charge Z to any appreciable degree.

(2b) Excepting case (2a), all the C_2's decrease nearly as $(W_0-W)^2$ for $Z\gg Z_c$.

(2c) C_{2S} and C_{2P} have a definite dependence on energy which is much like that described in (2a) for $Z\ll Z_c$ [see (25)]. Even for $Z>Z_c$ they exhibit pronounced minima followed by a rapid increase as the energy approaches the limit. The effect mentioned in (2b) is able to set in *completely* only for end-point energies much less than 1 Mev even for the heaviest elements.

(2d) For 0⟶0 transitions, C_{2T} assumes a definite energy dependence much like that of C_{1S}. For 2⟷0 transitions, C_{2A} is definite and very nearly like C_{2S}. Finally, C_{2V} becomes definite for 1⟷0 transitions and gives a distribution much like C_{1S} again. Of course parity changes are not allowed in any of these cases.

P^{32} and RaE

These are both likely to be second forbidden. It is true that the straightforward empirical method employed in constructing Table II puts RaE in the first forbidden class *1C*. However, the actual computation of the magnitude of the RaE decay as reported at the end of this section justifies its classification as second forbidden. The discrepancy is mostly to be attributed to the important part played by the often-mentioned replacement of the factor $(W_0R)^2$ in the order of magnitude of the transition probability with $(\alpha Z/2)^2$, which for RaE is much larger ($\sim\frac{1}{10}$) than the usual value ($\sim 1/100$) assumed in making up the table. It is true also that the RaE ft value is large for class *1C*, though not so large as immediately to suggest classification into group *2C*. It will now be shown that also the second forbidden theory can account for the RaE spectrum whereas the first forbidden theory falls far short of being able to do so.

Both the P^{32} and RaE β-spectra are unaccompanied by γ-rays, and so can be safely treated as simple. Lawson's measurements of the P^{32} spectrum yields a C_2 factor which decreases monotonically with energy (at least to within 0.5 Mev of the end point, see K3, Fig. 5): by a total of about 25 percent between $W=1.4$ and $W=4(W_0=4.37)$. The measurements for RaE are shown in Fig. 4,

from which it can be seen that the experimental C_2 factor decreases quite rapidly with energy. According to (2c), fitting these spectra with the theoretical factors C_{2S} or C_{2P} seems to be out of the question. C_{2A} can also be excluded. This follows from the fact that S^{32} and RaF are even-even nuclei and can be taken to have $J=0$ in their normal states. According to the axial vector theory, then, the transitions must be $2\rightarrow 0$ or $3\rightarrow 0$ in the second forbidden approximation. Referring to (2a) and (2d), one can see that C_{2A} for these selection rules is too much like C_{2S} to be able to represent the data. There remain now only the polar vector and tensor interactions. For the first, the $1\rightarrow 0$ possibility and for the second the $0\rightarrow 0$ and $3\rightarrow 0$ possibilities for the selection rules can be excluded because, according to (2d) and (2a), they give C_2 factors too nearly like C_{1S} or C_{2A}. Thus for every possibility in which a definite energy dependence of the theoretical C_2 value is predicted, that factor fails to represent the data. On the other hand, the factors C_{2V} and C_{2T} (for $J=2\rightarrow J=0$) can both be made to fit the data reasonably well by adjustment of the unknown ratio of the nuclear matrix elements. Moreover, the ratios found to be required for both P^{32} and RaE have seemingly reasonable values. The fitting of C_{2T} to RaE is shown in Fig. 4.

Perhaps the only encouraging feature of this application of Fermi's theory to forbidden spectra is that it is at all able to reproduce the striking deviation of the RaE spectrum from the allowed form, whereas a similarly rapid decrease with energy of the correction factor could not be achieved for a first forbidden transition regardless of the adjustments made in the relative magnitudes of the unknown nuclear matrix elements. According to (2b), the correction factor is approximately proportional to $(W_0-W)^2$ for an element like RaE. This just represents the difference in the Fermi and K-U forms (21) and (22) and accounts for the agreement with the allowed K-U spectrum found by various experimenters.

Finally, it is important to emphasize that the Fermi theory of forbidden β-spectra will require that there be found, probably among the lighter elements, some spectrum with a correction factor which rises rapidly with energy near the end point. This is simply a result of the fact that there should be cases in which the factor $|\mathbf{p}+\mathbf{q}|^2R^2(\approx(W^2-1)R^2$ as $W-W_0=q\rightarrow 0)$ determines the distribution.

(b) The Lifetimes

Integrating the forbidden spectra to obtain the mean life (4) is made difficult by the indefiniteness of the correction factors C for any particular case. Even when spin and parity changes are known or values for them assumed, there remains the task of evaluating the nuclear matrix elements involved. The simplest procedure is one analogous to that usually adopted in dealing with allowed transitions, for which the matrix elements $|\int\cdots|^2$ are treated as having the order of magnitude unity. The analogous procedure for the forbidden transitions is especially complicated by the fact that the matrix elements made up of tensor components (see Table I) were constructed without regard for normalization and might have been defined with arbitrary multiplicative constants. This indefiniteness can have serious effects for tensors of high order when the invariant forms consist of many terms. A simple way to resolve the difficulty is to assume the tensor matrix element to be constructed of unit vectors multiplied with constants giving the orders of magnitude of the actually occurring quantities. Then averages are made over the relative directions of the various independent unit vectors, and an estimate emerges. In this way [see (23) and Table I], one obtains:

$$\sum|R_{ij}|^2\approx(2/3)R^4,\quad \sum|A_{ij}|^2\approx(20/9)(v/c)^2R^2,$$

$$\sum|B_{ij}|^2\approx(20/9)R^2,\quad \sum|T_{ij}|^2\approx(4/3)R^4,$$

$$\sum|S_{ijk}|^2\approx(56/5)R^4.$$

The fourth-rank tensor analogous to (23e), which occurs for third forbidden transitions, yields $\sum|S_{ijkl}|^2\approx(3456/35)R^6$. This shows how important the factor here discussed can become for tensors of high order. One sees from the expressions (28) and (30) that not only the positive definite expressions so far discussed occur, but also cross terms consisting of the sums of many terms, each of which may be positive or negative. About the best one can do at this stage

is to consider all the phases random enough so that the positive and negative terms cancel approximately; thus, the cross terms will be assumed to vanish. An exception will be made for RaE, below, in which case an estimate of the ratio A_{ij}/T_{ij} is available from the spectrum fitting above.

Calculations along the above lines were carried out by Greuling (G3) for several special cases. Marshak (M1a) has also investigated the theoretical periods for cases of highly forbidden decay. His method deviated from that described here chiefly in that he classified the nuclear matrix elements according to the order of the spherical harmonics making up the wave functions rather than according to the transformation properties of the integrands as done here. He also used approximations for the cases sensitive to Z which are limited in their validity to the heaviest nuclei [cf. statement (2b) in this chapter].

In computing the theoretical half-lives of P^{32} and RaE, Greuling assumed the tensor interaction and $J=2\rightarrow 0$ (no) for each in accordance with the evidence of the spectra discussed above. He used the value $G=3\times10^{-12}$ for the Fermi constant with G-T rules and $G=5\times10^{-12}$ with Fermi rules (see Section 7).

P^{32}

He obtains for P^{32} a half-life of 5×10^{7} sec. when he uses $(v/c)=\frac{1}{10}$ in evaluating $\sum|A_{ij}|^2$. To bring it down to the observed value, 1.1×10^{6} sec., a value $(v/c)\approx\frac{2}{3}$ must be used, and that does not seem very satisfactory. The disagreement is greatly aggravated with the use of the ratio $A_{ij}/T_{ij}=-2.2$, obtained from the fitting of the theoretical with the observed spectrum as discussed above. It is not improved by discarding the tensor interaction for one of the others. This casts a doubt on the assignment of P^{32} to the second forbidden class. The first forbidden theory yields a life too small only by a factor 2. On the other hand, attempts to fit any of the first forbidden correction factors to the P^{32} *spectrum* have been somewhat less successful than the second forbidden results.

RaE

Here, the agreement is much better. Greuling's procedure yields 10^{6} sec. as compared to the observed 4.3×10^{5} sec. When $\sum|A_{ij}|^2$ is evaluated from the ratio $A_{ij}/T_{ij}=-5.8$, obtained by fitting the theoretical and observed spectra (see Fig. 4), the calculated half-life becomes 4.2×10^{5}, in perfect agreement with the experimental value. Also remarkable is the fact that the ratio $A_{ij}/T_{ij}=-5.8$ is consistent with the evaluations of A_{ij} and T_{ij} as described above, with $(v/c)\approx0.16$.

Rb^{87}

The measured spin of this long-lived (6×10^{18} sec.) nucleus is 3/2 and for the product nucleus, Sr^{87}, it is 9/2. This was the first case in which the spin change in a forbidden β-transition was known. Since $\Delta J=3$ only and so G-T rules would make the decay at most *third* forbidden, it became of critical interest to see whether the extremely long life could be accounted for by G-T rules (K3a). First, it can be verified that the transition is not second forbidden, according to the G-T rule $\Delta J=3$ (no); Greuling's procedure yields a half-life 2000 times too short. The value of $W_0=1.26$ was employed. For the third forbidden G-T rule $\Delta J=3$ (yes), a half-life of 0.6×10^{18} sec. is obtained, using $(v/c)\sim0.1$. This is in adequate agreement with the experimental value. Somewhat small values, such as this, are easily understandable since the nuclear matrix elements can plausibly be smaller than expected. Moreover, such high powers of the nuclear radius R and the end-point energy W_0 enter that the results are very sensitive to small uncertainties in these quantities.

The spin change $\Delta J=3$ could be either third forbidden or fourth forbidden according to Fermi rules. The third forbidden approximation yields again $0.3\approx10^{18}$ sec. for the half-life. Thus the Fermi rules account for Rb^{87} almost as well as the G-T rules. The fourth forbidden approximation gives a life roughly 10^{4} times too long.

It may be noted that according to both the Fermi and the G-T rules, the parities of Rb^{87} and Sr^{87} must differ.

K^{40}

The spin of this nucleus has been measured (Z1) as 4 and the residual Ca^{40} nucleus has an even number of neutrons and an even number of protons so that it has probably the spin 0.

For $\Delta J=4$, G-T rules yield either a third forbidden transition with change of parity or a fourth forbidden decay with no parity change. Fermi rules will require either a fourth or a fifth forbidden transition.

There is a discrepancy between two measurements of the maximum energy. Bramley and Brewer (B8) give $W_0=2.4$ while Henderson (H5a) reports the much higher value $W_0=3.6$.

Marshak and Greuling are in substantial agreement in their calculations of the K^{40} period on the basis of the third forbidden approximation of the tensor interaction. The values are 2.3×10^{17} sec. for $W_0=2.4$ and 1.0×10^{15} sec. for $W_0=3.6$. The second one of these would be in satisfactory enough agreement with the experimental value if one remembers again that too small theoretical values are easily accounted for.

With $W_0=2.4$, the fourth forbidden approximations yield lifetimes about a thousand times too long. If the low value of the energy is accepted, this would signify that the polar vector interaction is to be dismissed since it could give no shorter life. This potentially important argument for the G-T rules and the tensor interaction is, however, quite spoiled by the possibility that actually $W_0=3.6$. Greuling finds that with this large value for the β-energy, both the tensor and polar vector interactions are able to account for the experimental value of the period, in the fourth forbidden approximation. This is in contradiction to Marshak's results, which present somewhat too long lives in these cases. The disagreement is partially due to Marshak's approximations, which are actually valid only for much heavier and less exothermic nuclei than K^{40}, and partially due to differences in estimating the nuclear matrix elements. A similar disagreement did not occur for the third forbidden approximation because it happens that in such a case (the spin change as large as possible for the given degree of forbiddenness) the results are practically independent of the nuclear charge Z [see statements (1a) and (2a) above].

The results of this section show that some of the most definite conclusions concerning the question of which of the interactions (6) is the correct one may be provided by the forbidden transitions. The existing evidence of the forbidden spectra, particularly that of RaE, helps to narrow down the possibilities to the tensor and polar vector interactions, obeying G-T and Fermi rules, respectively. The evidence of the forbidden lifetimes is at present less clarifying. A decision concerning the β-energy of K^{40} would be a most helpful remedy for the situation.

7. ADDITIONAL SELECTION RULES

It was seen in Section 5 how the explicit dependence of β-transitions on the character of the nuclear state functions U and V could be expressed in terms of selection rules. This dependence was there discussed only insofar as it was expressible in terms of selection rules for the total angular momentum $\mathbf{J}$ and the parity. It could be done to a great extent without inquiry into the specific forms of U and V because the conservation laws for $\mathbf{J}$ and parity are exactly valid and because the dependence on them is chiefly determined by the *light particle* wave functions.

The exact validity of the conservation laws guarantees the existence of mutually orthogonal nuclear states, each labelled by a distinct value (quantum number) of J and a definite parity. Then conservation of $\mathbf{J}$ and parity during β-transitions is taken care of through the vanishing of the nuclear matrix elements, $|\int\cdots|=|\int dv\sum^k V^*\cdots Q_kU|$, as caused by the orthogonality of any two states V and $(\cdots Q_kU)$ when the J and parity of these differ. By $(\cdots Q_kU)$ is meant the state resulting from the operation on U by the operator $(\cdots Q_k)$, which may be the operator in any of the nuclear matrix elements $|\int\cdots|$ mentioned in Sections 3 and 5.

The light particle wave functions, rather than the U, V, determine the comparative magnitudes of two transitions differing in their J and/or parity changes. They supply the operators $(\cdots)$ which match $(\cdots Q_kU)$ to V in J and parity. These operators are supplied with certain coefficients, as, for example, $|\int\mathbf{r}|$ in Section 5 is supplied with the coefficient $|\mathbf{p}+\mathbf{q}|$. Because of the largeness of the light particle wave-lengths compared to nuclear dimensions, these coefficients are the most variable and therefore the most decisive factors in comparing transitions

occurring with different J and parity changes. On the other hand, they are alike in magnitude for two transitions which are both allowed or both equally forbidden. In the latter cases the more specific characteristics of U and V gain in importance.

The further dependence of the transitions on U and V may still be expressible in terms of *additional* selection rules. For this, further dynamical variables than **J** and parity must be conserved, at least to some approximation. Such further good quantum numbers might be supplied, for example, by the total orbital angular momentum $\mathbf{L}=\sum \mathbf{l}_k$ and/or the total spin $\mathbf{S}=\sum \mathbf{s}_k$, the $\mathbf{l}_k$ and $\mathbf{s}_k$ denoting individual (not necessarily individually constant) angular momenta of the nucleons. To discover precisely what additional quantum numbers can be used requires a knowledge of the character of the stationary states of nuclei.

A complete exposition of the methods used to determine the characteristics of the state functions U, V, $\cdots$ is not appropriate here. Only the results which are important for the problem of β-decay will be briefly outlined.

There are two not altogether independent procedures which have been followed most successfully:

A—Use of the Hartree model with Russell-Saunders coupling.

B—The Wigner group-theoretical method.

A. There are two classes of results following from the Hartree model which are of importance here.

A(1) The first class of results is the order in energy of the Russell-Saunders terms for nuclei. The R-S classification of states according to the eigenvalues of **L** and **S** is familiar from atomic spectroscopy. The energy of each R-S term of a nucleus is computed from the spin-independent parts of the known inter-nucleon forces. The spin-dependent parts subsequently serve to split terms of equal **L**, **S** but different **J**. Such computations have been carried out by Feenberg and Wigner (F3) and Rose and Bethe (R5) for the nuclei between He^4 and O^{16}. The L, S, J values they obtained for the lowest terms are given here by the usual spectroscopic symbols:

He^6, Be^8, Be^{10}, C^{10}, C^{12}, C^{14}, O^{16}:	$^1S_0$
Li^6, B^{10}, N^{14}	: $^3S_1$
Li^7, Be^7, Be^9	: $^2P_{3/2}$ ($^2P_{1/2}$ near)
Li^8	: $^3P_2$
B^{12}	: $^3P_0$
B^{11}, C^{11}, C^{13}, N^{13}, N^{15}, O^{15}	: $^2P_{1/2}$ ($^2P_{3/2}$ near).

The experimental evidence of magnetic moments indicates a partial failure of these results to give the true character of the terms. The B^{11} ground state is very likely to be $^2P_{3/2}$ (M4). Li^6, B^{10}, and N^{14} probably have a $^3D_1$ character superposed on the $^3S_1$, indicating a partial breakdown in the conservation of L (F2, M4).

A(2) The second class of results of interest here consists of the wave function U, $V\cdots$ to be used in the nuclear matrix elements. Such wave functions were of course also necessary for the computation of energies in *A*(1). According to the Hartree model, each nucleon in a first approximation moves independently, except as limited by the Pauli principle, and its energy is determined solely by its own distance from a central point. This leads to the conservation of the individual angular momenta $\mathbf{l}_k$ and $\mathbf{s}_k$ and consequently to a total zero-order wave function separable into individual particle space and spin functions with spatial and spin-space symmetries as determined by $\mathbf{l}_k$ and $\mathbf{s}_k$. A great many sets of the $\mathbf{l}_k$, $\mathbf{s}_k$ eigenvalues, of course, will lead to the same energy in the Hartree approximation. According to the Russell-Saunders coupling, the chief perturbations will destroy the constancy of the individual $\mathbf{l}_k$ and $\mathbf{s}_k$, leaving **L** and **S** only conserved and causing differences in the energy of states with different pairs of L, S values. These Russell-Saunders states will have wave functions approximately representable by linear combinations of the zero-order Hartree functions; the destruction of the individual l_k and s_k as quantum numbers will thus be represented by the presence in the single R-S state of all the sets of l_k, s_k values

consistent with the given L, S of the state. For the purposes they have fulfilled in the theory of β-decay so far, no more need be assumed about the individual particle functions than just their spatial (l_k value) and spin (s_k valüe) symmetry, their normalizability, and their equality for neutrons and protons. By the methods familiar from atomic spectroscopy, the proper combinations of the individual wave functions for a given R-S term and consistent with the Pauli principle, are straightforwardly found.

The Hartree wave functions described in $A(2)$ were used by Grönblom (G2) to evaluate the matrix element $|\int\boldsymbol{\sigma}|$ for He^6 and for several nuclei with $T_z=\frac{1}{2}(N-Z)=-\frac{1}{2}$, where N is the number of neutrons in the decaying nucleus. Wigner (W10), however, showed that the detailed assumptions characteristic of the Hartree model are not necessary either for Grönblom's or for many other results. Essential are only the symmetry properties such as are given to the total state function in combining the individual particle functions so as to conform to the R-S term character (a type of symmetry with respect to rotational and mirroring transformations) and to the Pauli principle (a type of symmetry with respect to interpermutations of nucleons). These symmetry properties are compactly classifiable by the methods of group theory as applied by Wigner. His results will now be stated as briefly as possible. It will be seen that they do not replace the results listed in $A(1)$, but they do make a further exposition of the results obtained from $A(2)$ unnecessary.

B. Wigner manages to classify nuclear states according to their membership in a system of multiplets analogous to the singlets, triplets, and higher spin multiplets familiar from atomic spectroscopy. Within one of the *electronic* multiplets of the atom, $S=|\mathbf{S}|$ is a constant because the atomic Hamiltonian is at least approximately independent of S. An interesting property of these multiplets is that radiative transitions are forbidden between members of different multiplets (having different S values).

Nuclear spectroscopy is more complicated because of the existence of neutron and proton states of the nucleon in addition to its parallel and anti-parallel spin states. Because the number of the character (C1) eigenstates of the nucleon is thus two, the same as the number of eigenvalues possessed by the spin operator $\boldsymbol{\sigma}$, dependence on the neutron-proton character of the nucleon may be put in terms of an operator $\boldsymbol{\tau}$ precisely like $\boldsymbol{\sigma}$ except that by definition it affects only character coördinates and not spin coördinates. $\boldsymbol{\tau}$ is commonly called the isotopic spin.

To find supermultiplets for nuclei, analogous to the multiplets for atomic electrons, Wigner assumes that the nuclear Hamiltonian is in first approximation independent of both ordinary and isotopic spin. In this approximation, only the operation of the Pauli principle keeps all nuclei with a given number of particles A from complete equivalence regardless of their neutron-proton ratios. The assumptions have some justification in the empirical findings that neutron-proton and proton-proton forces are nearly equal and that the main part of the forces has a Majorana (spin-independent) character. Dependence on isotopic spin is introduced by the Coulomb forces which become important for heavier nuclei.

Because of the added variable $\boldsymbol{\tau}$ by which states can be labelled, more quantum numbers are needed to label a supermultiplet than merely S. The situation is more nearly analogous to the atomic case in which not merely $\mathbf{S}$ but $\mathbf{S}$ and $\mathbf{L}$ are conserved; then the eigenvalues of $\mathbf{S}$, $\mathbf{L}$, and $(\mathbf{L}\cdot\mathbf{S})$ can serve to label states. Wigner employs the eigenvalues of

$$T_Z=\tfrac{1}{2}\sum_{k=1}^{A}\tau_{zk}=\tfrac{1}{2}(N-Z),\quad S_Z=\tfrac{1}{2}\sum\sigma_{zk}$$

and

$$Y_Z=\tfrac{1}{2}\sum\sigma_{zk}\tau_{zk}$$

to distinguish the members of a supermultiplet. Obviously the simple designations singlet, doublet, etc., are unable to characterize the multiplet as a whole; Wigner uses the symbol $(P\,P'\,P'')$. P represents the maximum value in the supermultiplet of any one of the three quantum numbers T, S, Y. P' is the highest value of a second one of these which is consistent with the value P of the first chosen. P'' is the largest value of the third quantum number which can be found among the supermultiplet's members having P

and P' for the values of the first two. Altogether, in a given supermultiplet ($P\,P'\,P''$) there is a member for each combination of the following S_Z, T_Z, Y_Z values:

$$S_Z = -P, -(P-1), \cdots, +P$$

with

$$T_Z = -P', \cdots, +P'$$

with

$$Y_Z = -P'', \cdots, +P''.$$

Besides for each member ($S_Z\,T_Z\,Y_Z$) there is a member having any permutation of the three values. For example, the supermultiplet ($\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2}$) has the 8 members with ($S_Z\,T_Z\,Y_Z$) values: ($\frac{1}{2}\,\frac{1}{2}\,\pm\frac{1}{2}$), ($\frac{1}{2}\,-\frac{1}{2}\,\pm\frac{1}{2}$), ($-\frac{1}{2}\,\frac{1}{2}\,\pm\frac{1}{2}$), ($-\frac{1}{2}\,-\frac{1}{2}\,\pm\frac{1}{2}$). It will be most important to fasten one's attention on the fact that $T_Z=\pm\frac{1}{2}$ nuclei have such supermultiplets and that in these states the total spin is $S=\frac{1}{2}$ with components $S_Z=\pm\frac{1}{2}$.

A number of facts brought out by Wigner which are important here follow:

$B(1)$ Nuclei differing in mass number A by an integral multiple of 4 possess like systems of supermultiplets. This is consistent with the fact that the Pauli principle allows just four combinations of values σ_z, τ_z to exist for a given space state.

$B(2)$ When the energy in a first approximation (spin forces and Coulomb forces neglected; neutrons and protons not distinguished) is estimated as a function of the character of a supermultiplet, the lowest supermultiplets turn out to be:

($\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2}$) for $A=4n\pm1$,
(0 0 0) for $A=4n$,
(1 0 0) for $A=4n+2$.

The supermultiplet ($\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2}$) contains terms with $T_Z=S=\frac{1}{2}$ and $T_Z=-S=-\frac{1}{2}$. (0 0 0) has only the single term $T_Z=S=Y_Z=0$. (1 0 0) has $T_Z=0$, $S=1$ and $T_Z=0,\ \pm1$ with $S=0$. The two different spins S present in this supermultiplet show that it will be split by spin forces. This does happen, depressing the $S=1$ state below the others. Any one of the terms mentioned here may still, of course, have quite different characteristics for different nuclei in the ground state, according to the value of J each nucleus may have.

$B(3)$ The perturbation caused by Coulomb repulsion makes the term of a supermultiplet with the highest T_Z (fewest protons for the given A) most stable. Supermultiplets with larger P appear higher in the order of first approximation energies as estimated by Wigner. These two facts make it always most probable that a nucleus with a given T_Z will have for its ground state a supermultiplet for which $P=T_Z$, P being the highest possible value of T_Z for that supermultiplet. For heavy enough nuclei, the Coulomb effect may become sufficiently strong actually to pull the term with $T_Z=P$ belonging to a higher supermultiplet with a larger P down below the supermultiplets of $B(2)$, thus keeping the high T_Z nucleus from radioactivity. An examination of the stable nuclei reveals just when the Coulomb forces become strong enough to do this. Among the $A=4n+1$ nuclei, P^{33} is unstable and Cl^{37} stable as the $T_Z=\frac{3}{2}$ term of the ($\frac{3}{2}\,\frac{1}{2}\,\frac{1}{2}$) supermultiplet falls below the ($\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2}$) supermultiplet of $B(2)$; the $T_Z=\frac{5}{2}$ term of the ($\frac{5}{2}\,\frac{1}{2}\,\frac{1}{2}$) supermultiplet falls below both between radioactive Ca^{45} and stable Ti^{49}. Among the $A=4n+3$ nuclei the same phenomena occur between A^{39}, Ca^{43}, and between Sc^{47}, V^{51}. Among the $A=4n$ nuclei, the $T_Z=2$ term of the (200) supermultiplet falls below the (000) supermultiplet between $A=32$ and 44; the $T_Z=1$ term of the intermediate (110) multiplet is never stable, the (200) falling lower than it while it is still unstable relative to the (000) multiplet. Finally, among the $A=4n+2$ nuclei there is first the fall of the singlet $T_Z=1$ term of the (100) supermultiplet below its triplet term [see $B(2)$] between radioactive C^{14} and stable O^{18}. Then $T_Z=3$ of (300) becomes more stable than (100) between $A=42$ and 62; an intermediate (210) multiplet is never stable. All these facts are useful in judging the supermultiplet character of any given light radioactive nucleus.

$B(4)$ The matrix elements $|\int 1|$ and $|\int \boldsymbol{\sigma}|$, characteristic of Fermi and G-T rules, respectively, vanish for inter-multiplet transitions; only the transitions between fine

structure components of the same supermultiplet are allowed. This allowedness in general requires more than just adherence to the J and parity selection rules; a distinction may be maintained by referring to transitions consistent with the inter-combination rule of this paragraph as not only allowed in the sense heretofore used, but also favored. For heavier nuclei, the supermultiplet picture is expected to break down to some degree (Coulomb forces no longer mere perturbations), and unfavored but allowed transitions will have almost as great probability as the favored ones.

$B(5)$ The conclusion to be drawn from items $B(3)$ and $B(4)$ is that all β-transitions are unfavored. Positron emission increases T_Z; but according to $B(3)$, the ground state of the decaying nucleus belongs to a supermultiplet for which T_Z has its maximum value (P) within the supermultiplet already. Negatron emission within the same supermultiplet, again, tends to be energetically impossible because a lower value of T_Z is given a greater energy by the Coulomb repulsion.

$B(6)$ Two exceptions to the conclusions of $B(5)$ exist. For light enough nuclei $T_Z=-\frac{1}{2}$ exists (the Coulomb repulsions apparently being sufficiently unimportant for the nucleus to hold together until β-decay dissipates the surplus energy). Such a nucleus can only belong to a supermultiplet containing $T_Z=+\frac{1}{2}$ also. Consequently, positron emission, in which a transition from the $T_Z=-\frac{1}{2}$ to the $T_Z=+\frac{1}{2}$ nucleus takes place, is here favored. The second exception consists of nuclei having $A=4n+2$, and $T_Z=0$ for either the initial or final state. The triplet $T_Z=0$ term of (1000) is below its singlet $T_Z=1$ term up to O^{18} according to the evidence in $B(3)$. Thus negatron emission is favored through C^{14} and positron emission from F^{18} up. In addition the singlet $T_Z=-1$ term occurs (C^{10}), underoing a favored positron emission, of course.

The membership of group OA in Table II, which would be assigned to allowed *and* favored transitions from an empirical standpoint, bears out Wigner's conclusions quite well with one exception. All the nuclei in this group except B^{12} are *expected* to be favored according to the preceding paragraph.

The $T_Z=-\frac{1}{2}$ Nuclei

Wigner's (also partly Breit and Knipp's and Grönblom's) results for the squares of the nuclear matrix elements, $|\int 1|^2$ and $|\int \boldsymbol{\sigma}|^2$, are the following:

Transi ion	$\lvert\int 1\rvert^2$	$\lvert\int \boldsymbol{\sigma}\rvert^2$
${}^2S_{1/2}\rightarrow{}^2S_{1/2}$	1	3
${}^2P_{1/2}\rightarrow{}^2P_{1/2}$	1	1/3
${}^2P_{3/2}\rightarrow{}^2P_{3/2}$	1	5/3
${}^2P_{1/2}\rightarrow{}^2P_{3/2}$	0	8/3
${}^2P_{3/2}\rightarrow{}^2P_{1/2}$	0	4/3
${}^2D_{3/2}\rightarrow{}^2D_{3/2}$	1	3/5
${}^2D_{5/2}\rightarrow{}^2D_{5/2}$	1	7/5
${}^2D_{3/2}\rightarrow{}^2D_{5/2}$	0	12/5
${}^2D_{5/2}\rightarrow{}^2D_{3/2}$	0	8/5.

Missing and vanishing combinations are easy to account for on the basis of the total spin and parity rules. The matrix element $|\int \gamma_5|$, because of its requirement of a change in parity during an allowed transition, will lead to new inter-multiplet selection rules; because of the conclusions of Section 6, these are not of sufficient interest to warrant further discussion here.

The quantity ft in Table II is inversely proportional to the square of a nuclear matrix element and accordingly the product $|\int \boldsymbol{\sigma}|^2 ft$ should show more constancy than ft itself if Gamow-Teller rather than Fermi selection rules prevail. The straightforward application of this criterion is not possible, however. The chief reason is that the character of most of the nuclear states is either unknown or very uncertain, as was seen in paragraph $A(1)$ of this section. Another important source of irregularity is always the failure of the present considerations to take into account differences in the radial parts of the various wave functions involved. In addition, the t used in Table II may be in

many cases compounded from decays to an excited as well as to the ground state of the product nucleus. The latter fact is supposed to be true of N^{13} and Be^{7}, of the $T_Z=-\frac{1}{2}$ group, and the ft value given for these in Table II has been appropriately corrected as described in Section 5 and below. Similar occurrences are suspected in several other cases (see closing paragraph of Section 4), but knowledge of the energy of the excited state is lacking. Usually, however, the difference to ft caused by such a complex transition is not so large as to prohibit some general discussion.

Outstanding in the $T_Z=-\frac{1}{2}$ group is the small ft value (1840) of Ne^{19}. Since there is reason to suppose that the transition is $^2S_{1/2}\rightarrow{}^2S_{1/2}$ (spin of F^{19} known to be $\frac{1}{2}$), the extra large value (3) of $|\int\boldsymbol{\sigma}|^2$ for such a transition seems to favor G-T selection rules. As a result, $|\int\boldsymbol{\sigma}|^2ft=5520$. This shall be the value against which will be compared the corresponding quantity for the other nuclei. The small value $f_Kt=2300$ for Be^{7} (1) can also be quite well compensated; $|\int\boldsymbol{\sigma}|^2f_Kt=3800$ for the $^2P_{3/2}\rightarrow{}^2P_{3/2}$ transition, in accord with the Hartree model. If Be^{7} had a $^2P_{1/2}$ ground state, in accord with the findings of Inglis (I1) for the $He_2^4+He_2^3$ model for Be^{7}, then $|\int\boldsymbol{\sigma}|^2f_Kt=6100$, in better agreement with the Ne^{19} value. N^{13} (1) and O^{15} are expected from $A(1)$ to undergo $^2P_{1/2}\rightarrow{}^2P_{1/2}$ transitions. These make $|\int\boldsymbol{\sigma}|^2ft=1830$ and 1390, respectively, definitely too low. Since the probable spins of C^{13} and N^{15} are each $\frac{1}{2}$ (H3, I2, M3), better agreement with the Ne^{19} value can only be obtained by assuming $^2P_{3/2}\rightarrow{}^2P_{1/2}$ giving $|\int\boldsymbol{\sigma}|^2ft=7300$, 5600. In this way, the N^{13} value becomes definitely high and perhaps an admixture of non-contributing $^2D_{3/2}$ should be assumed. A $^2P_{3/2}$ B^{11} would give $|\int\boldsymbol{\sigma}|^2ft\approx5800$ for $^2P_{3/2}$ C^{11}, as the value nearest that of Ne^{19}. With A^{35} undergoing a $^2D_{5/2}\rightarrow{}^2D_{5/2}$ transition (Cl^{35} is known to have $J=5/2$), it yields $|\int\boldsymbol{\sigma}|^2ft=5500$. All the assertions in this paragraph are highly speculative and only the Ne^{19} case can perhaps be regarded as providing some support for the G-T rules in preference to the Fermi rules.

The $T_Z=+\frac{1}{2}$ nucleus H^3 probably undergoes mostly the favored transition $^2S_{1/2}\rightarrow{}^2S_{1/2}$. This leads to $|\int\boldsymbol{\sigma}|^2ft=3\times1410=4230$, in as satisfactory agreement with the Ne^{19} value as perhaps should be expected.

Nuclei with A=4n+2

First it is necessary to point out that all the transitions in this group are probably unfavored by the Fermi rules. [It will be seen below that they are in fact second forbidden with $\Delta J=1$ (no), with Fermi rules.] According to $B(3)$, all the $T_Z=0$ nuclei are expected to have $S=1$ in the ground state, while their parent or daughter nuclei will have $T_Z=\pm1$, $S=0$. But the Fermi matrix element $|\int1|$, containing only scalar operators, vanishes unless $\Delta S=0$ in the transition. *Therefore, the mere presence of He^{6}, C^{10}, and F^{18} in the allowed and favored group OA is an argument for G-T rules.*

All the $T_Z=\pm1$ nuclei involved in the transitions of this group have an even number of neutrons and an even number of protons. From the empirical rule that such nuclei always have $J=0$ and since, according to the above paragraph, they will also have $S=0$, their ground states must be $^1S_0$. The only state with which $^1S_0$ combines in the G-T matrix element $|\int\boldsymbol{\sigma}|^2$ is $^3S_1$. For $^1S_0\rightarrow{}^3S_1$, $|\int\boldsymbol{\sigma}|^2=6$, and for $^3S_1\rightarrow{}^1S_0$, $|\int\boldsymbol{\sigma}|^2=2$, according to Wigner (W10). The difference between these values is in the right direction to explain the very different ft values of He^{6} and F^{18}, say, as given in Table II. For these two cases $|\int\boldsymbol{\sigma}|^2ft=6960$ and 11,700, respectively. If the F^{18} $^3S_1$ state is assumed to have an equal admixture of $^3D_1$, in accordance with the experimental evidence [see $A(1)$] for such admixture in Li^{6}, B^{10}, and N^{14}, then the F^{18} value is reduced by half because $^3D_1$ does not combine with $^1S_0$. On the other hand, no such admixture would be needed by Al^{26} ($|\int\boldsymbol{\sigma}|^2ft=4800$). If the above explanation of the low ft value of He^{6} is correct, the corresponding quantity for C^{10} should be equally small. Instead it leads to $|\int\boldsymbol{\sigma}|^2ft=35{,}800$ which seems too large to be explained away by admixtures.

More serious than the C^{10} difficulty is that presented by Be^{10}, C^{14}, Na^{22}, P^{30} and further members of the family which have such high ft values that they must be put among the forbidden transitions. The acuteness of the difficulty is fully represented by the case of $Be^{10}\rightarrow B^{10}\leftarrow C^{10}$. On the basis of the Hartree central field model, which accounts very well for the known $C^{10}-B^{10}$ energy difference (F2), the three nuclei differ only in that Be^{10} has two neutrons in the

last orbit, B^{10} has a proton and neutron, and C^{10} has two protons in apparently the same position. Yet the Be^{10} state must be very different from both B^{10} and C^{10} because the Be^{10} decay seems to be at least third forbidden compared to the allowed C^{10} transition. The difference needed can hardly be accounted for by Coulomb forces which have always been successfully assumed to represent the entire difference between proton-proton and neutron-neutron forces. Exactly how much the Coulomb forces can account for has been computed by Cooper and Nelson (C3). The level spacing resulting from the Hartree model by neglecting the Coulomb repulsion is about 2 mc^2 from ground (^1S) to ^1D, about 6 mc^2 to ^1F, and 12 mc^2 to ^1G. Marshak (M1a) finds that to obtain a sufficiently highly forbidden $Be^{10}\rightarrow B^{10}$ transition, the ^1F at least must be the ground state of Be^{10}. Yet Cooper and Nelson find that the Coulomb perturbation is able to reduce the gap from ^1S to ^1F only by an energy of the order of 0.1 mc^2.

The inconsistencies in the $A=4n+2$ group may not be the fault of the β-theory. They may mean that the character of the nuclear forces is badly misunderstood or that the Hartree method varies widely in its applicability, even to nuclear structures which are superficially very similar. On the other hand, the difficulty seems to extend to a case too simple to be so misunderstood. This is the reaction important for its astrophysical consequences:

$$H^1+H^1\rightarrow H^2+\beta^+,$$

in which G-T rules should *allow* the $^3S_1$ ground state of H^2 to be formed from a $^1S_0$ collision of the protons. Marshak and Bethe (M1) have shown that if this reaction is not *forbidden*, a discrepancy arises between the calculated and observed values of the radius of the white dwarf star, Sirius B.

Oppenheimer (O3) has suggested a direction in which a modification of the β-theory able to resolve the inconsistencies in the $4n+2$ family might be sought. He points out that the surely allowed transitions (of He^6, C^{10}) differ systematically from the apparently forbidden ones ($He^2\rightarrow H^2$, $Be^{10}\rightarrow B^{10}$, $C^{14}\rightarrow N^{14}$) in that the energy released in the former (3.6 Mev, 3.3 Mev) is much larger than in the latter cases (420 kev, 550 kev, 150 kev). This may mean that there is a threshold energy below which G-T rules do not operate. Such a condition might be achieved if G-T rules apply only to processes in which a new type of neutrino, *with* rest mass, is emitted; the slow processes are to continue to emit massless neutrinos under Fermi rules. The new-type neutrino would be required to be about as massive as an electron to make its emission by Be^{10}, which releases 550 kev, slow enough. A mass of such a magnitude should have an observable effect in energy balances, but these are not known for He^6 and C^{10} with sufficient accuracy to be conclusive. In the cases, such as those in the $T_Z=-\frac{1}{2}$ family, for which more accurately known energy balances have provided no margin for a neutrino rest mass, the Fermi rules must be largely observed. This creates some difficulty for complex β-transitions (below) which are most easily understood with G-T rules. It also means that the processes in which Fermi rules are obeyed must be strong enough to account for the short lives of the $T_Z=-\frac{1}{2}$ family. Thus, the value of the Fermi constant $G=5\times10^{-12}$, derived below, is retained for the Fermi-rule process.

One can now compute the half-life of Be^{10} under the assumption that the transition is $^1S_0\rightarrow{}^3S_1$, and so $\Delta J=1$ (no). This is second forbidden according to the polar vector interaction V and *completely forbidden* in every approximation by the scalar interaction S (see Section 5), which also follows Fermi rules. The procedure of Section 6 then yields a life of the order of 8×10^9 sec. according to the polar vector interaction, to be compared to $\gg10^{10}$ sec. for the experimental value. On the basis of the scalar theory, the long life of Be^{10} would have been explained by the impossibility of the usual emission of an electron and a neutrino; the life would not be infinite only if some less usual process such as mentioned in Section 5 would take place. To compare with these results of Oppenheimer's hypothesis, one has (M1a) from a straightforward application of the tensor interaction (G-T rules) a lifetime greater than 10^{10} sec. only for $\Delta J=2$, in the second forbidden approximation (no parity change).

Value of G

The results of the discussion of the $T_Z=-\frac{1}{2}$ and $A=4n+2$ families, which have transitions

both allowed and favored, enable the evaluation of the Fermi constant G. It is now known that for the $T_Z=-\frac{1}{2}$ nuclei, the matrix elements $|\int 1|^2\approx 1$, so that according to Fermi rules: $|\int 1|^2 ft=0.693(2\pi^3/G^2)=ft$, the ft values of the $T_Z=-\frac{1}{2}$ family being used. These values seem to center at about $ft\approx 3000$, giving $G\approx 5\times 10^{-12}$, for *Fermi* rules. With the G-T rules, $|\int \sigma|^2 ft=0.693(2\pi^3/G^2)$. The $T_Z=-\frac{1}{2}$ family (chiefly Ne^{19}) gives $|\int \sigma|^2 ft\approx 5500$, while He^6 leads to the value 6960, and H^3 to 4230. A compromise between these yields $G\approx 3\times 10^{-12}$, for G-T rules.

Nuclei with A = 4n

All these should undergo unfavored transitions according to Wigner's theory. This seems to be confirmed by the presence of Li^8, N^{16}, F^{20}, Na^{24}, Al^{28} in group $1A$ of Table II.

Li^8

According to A (1), the ground states of Li^8 and Be^8 are $^3P_2$ and $^1S_0$, respectively. The fact that transitions between these states are few would not be a positive confirmation of Wigner's rules because the Fermi and G-T rules would be violated at the same time. But even the G-T rules are violated to a *second* forbidden degree since both ground states are expected to be even and thus the parity unchanged. This is in conformity with the experimental finding (R8, D1) that the β-transition goes rather to an excited state of Be^8; there is a discrepancy of 3.6 Mev between the mass difference Li^8-Be^8 (16 Mev) and the upper limit of the β-spectrum (12 Mev). The excited Be^8 state is probably $^1D_2$ (F3, B10) and also even. A $^3P_2$ (even)$\rightarrow{}^1D_2$ (even) transition is allowed by both the Fermi and the G-T rules. Only the Wigner rules are then left to put the process in group $1A$, by making it unfavored.

In agreement with the conclusion of the last paragraph, that the Li^8 decay is allowed but unfavored is Bayley and Crane's (B3) finding that the β-spectrum has probably the allowed form. The problem is complicated by the great width (0.5 Mev at half-maximum, F6) of the product Be^8 level, which is shown by the spread in energy of the α-particles into which the Be^8 immediately disintegrates. Thus the end point for a given β-process will depend on the energy of the α's, being determined by the $Li^8-2\,He^4$ mass difference *minus* the kinetic energy of the α's. The observed β-spectrum, then, will consist of a superposition of simple components with end points and relative intensities as indicated by the energies of the α-particles and their intensities at each energy. Bayley and Crane reconstructed the β-distribution to be expected from components having Fermi's allowed form and from the experimentally measured α-distribution. They obtained good agreement with the β-spectrum observed directly when they used the α-distribution found by Fowler and Lauritsen (F6). Rumbaugh, Roberts, and Hafstad (R9) observed many more slow α-particles in their work and accordingly the β-spectrum reconstructed from it overemphasized fast electrons. If the reconstruction were done with forbidden shapes for the components, the emphasis on fast electrons would probably be still stronger in view of the high β-energy end point involved.

Rumbaugh, Roberts, and Hafstad discussed the variation of the α-intensity as a function of the β-end-point energy. This intensity should be proportional to the relative intensities of the corresponding β-spectra components and therefore to $f(Z, W_0)\sim W_0^5$ according to Section 5. The authors found just this variation in the higher energy portion of their α-distribution. The same sort of agreement can be found with the Fowler-Lauritsen distribution, since the two groups of experimenters are in approximate agreement as far as the more energetic α-intensities are concerned. The less energetic α's cannot in any case be considered as seriously because important corrections due to their need to penetrate each other's Coulomb barriers ($\sim$1.5 Mev high) must be made.

B^{12}

This is an anomalous member of the $4n$ group in that it seems to belong in group OA of Table II rather than in $1A$. Wigner's rule seems therefore not to operate in this case. It may be that B^{12} undergoes a favored, because first forbidden, transition. In that case, its transition probability should be smaller than the allowed value by a factor $\sim(W_0R)^2$. This factor is much larger than usual in this case because of the great energy

release ($W_0=24$), making $(W_0R)^2\approx 0.1$ for a nuclear radius of 5×10^{-13} cm (probably too large). Under the most favorable circumstances this might lead to an *ft* value of about 10,000 [taking $(ft)_{all'd}=1000$], compared to the experimental 6600 in Table II.

The Hartree model gives the B^{12} ground state a $^3P_0$ (even) character [see $A(1)$] which would make its transition to $^1S_0$ (even) C^{12} *second* forbidden according to G-T rules. Since such a high degree of forbiddenness seems to be out of the question for B^{12}, this can be regarded either as putting in doubt the theoretical result for the state's character or as an argument for Fermi rather than G-T rules. Of course, in neither case, then, would the failure of the Wigner rules be explained.

Heavier Nuclei

Of course, for heavier nuclei, when the Coulomb forces can no longer be treated as mere perturbations, a breakdown of the multiplet system should be expected. The distinction between favored and unfavored transitions should become less. Probably members of the group *B* of Table II are nuclei in which the breakdown has occurred. A residue of Wigner's conclusion $B(5)$ that all β-transitions are unfavored [with the exceptions $B(6)$] seems still to apply to the nuclei of the groups *B*. The *ft* values among the most allowed transitions among these heavier nuclei (group *OB*) are distinctly greater than for group *OA*. The persistence of such a selection rule in such an approximate manner is understandable through an extension of the Wigner model. The breakdown of the multiplet system is approximately representable as a mixing of the characters of various of the multiplet states into the given state. The corresponding wave function becomes a linear combination of the individual multiplet state functions thus acquiring fractions of the distinctive symmetry characteristics of the individual multiplet terms. The combination of a pair of such mixed states will neither be completely orthogonal nor well overlapped.

The tendency toward the growth of the *ft* value with complexity of the nucleus, as demonstrated in Table II by a comparison of the *ft* values for groups *OA*, *OB*, and *OC*, was first pointed out by Nordheim and Yost (N2). Their qualitative explanation was much like that of the last paragraph. They argued that more complex nuclei, capable of more varied configurations, would tend to make state functions more mixed in character, i.e., containing greater numbers of distinct components. A pair of such state functions would have smaller and smaller fractions of such components in common as the complexity increased.

Complex Decay

Applications of the results of $A(1)$ and of the calculated values of $|\int\sigma|^2$ to complex transitions have been carried out by Grönblom (G2), Watase (W5), and Breit and Knipp (B9).

N^{13}

According to the theoretical results of $A(1)$, the lowest states in each of the nuclei N^{13} and C^{13} consist of the doublet $^2P_{1/2,3/2}$, with the ground state having the $^2P_{1/2}$ character. This conclusion about the ground state of C^{13} is supported by the fact that a spin $J=\frac{1}{2}$ for C^{13} is empirically preferable (H3, I2). If the Fermi rules are correct, N^{13} would be expected to undergo β-decay only to the state of the C^{13} doublet having the same character as the N^{13} ground state. On the other hand, G-T rules make possible transitions to both C^{13} states, if the upper one is not too high. The experimental data on these points contain many contradictions. Richardson (R1), Lyman (L8), and Watase and Itoh (W5) find a 280-kev γ-ray to be emitted in a fraction of the disintegrations. The existence of γ-radiation would be consistent with the apparent complexity of the N^{13} spectrum (see Section 4). On the other hand, Valley (V1) fails to find the reported γ-ray. Supporting him are the findings of Schultz, Davidson, and Ott (S2a) that only monoenergetic protons are emitted in the C^{12} (d, p) C^{13} process.

If the existence of the γ-ray is accepted, an explanation of it on the basis of G-T rules can be attempted (G2). A pertinent experimental datum is the number of γ-rays observed per disintegration. Richardson (R1) reports the approximate value 0.4 for this number, while Lyman (L8) obtains from the measurement of gamma-anni-

hilation radiation coincidences approximately 0.2. In attempting to compose a spectrum made up of components following the Fermi shape (see Section 4), and differing by 280 kev in energy, Lyman finds it necessary to assume 0.25 γ-ray per disintegration. [Another measurement of the spectrum (K1) is reported by Watase (W5) to require components differing by 0.6 Mev in energy and 0.4 γ-ray per disintegration, assuming two 300-kev γ-rays to follow the less energetic component.] In computing the theoretical prediction for the ratio of the disintegrations to the excited and ground states of C^{13}, an energy difference between these of 280 kev will be first assumed. If N^{13} is $^2P_{1/2}$ in accordance with $A(1)$, the ratio of the decay constants to the two C^{13} states (the excited one to be denoted by an asterisk) is from (24):

$$f^*|\int\sigma^*|^2/f|\int\sigma|^2=2.7(8/3)/7.3(1/3)=2.96.$$

This disagrees badly with all the data. If a $^2P_{3/2}$ N^{13} state is tried, the ratio comes out to be 0.44 which may be within the uncertainty of the experimental value: 0.2–0.4.

Be^7

Breit and Knipp treated Be^7, which decays by K capture. According to Haxby, Shoupp, Stephens, and Wells (H4), 1.70 mc^2 is released. In roughly one-tenth the disintegrations Li^7 is left in its 450-kev state, as shown by the number of γ-rays of that energy per disintegration (R7), 0.82 mc^2 being left to the neutrino. The theoretical ratio of the decay constants to excited and normal Li^7 can be computed from formulas given in Section 8. The Li^7 states are expected to constitute the $^2P_{1/2,\,3/2}$ doublet with $^2P_{3/2}$ lower in accordance with nuclear spin measurements. Taking $^2P_{3/2}$ for Be^7 in accordance with $A(1)$,

$$f_K^*|\int\sigma^*|^2/f_K|\int\sigma|^2=0.24(4/5)=0.19,$$

to be compared with the very uncertain experimental value $\sim$0.1. With $^2P_{1/2}$ for Be^7 the theoretical ratio is 0.03.

Of course, with Fermi instead of G-T rules, the complex transitions in the above cases could only be understood if the two states of each product nucleus were alike, both having $J=\frac{1}{2}$ in the case of C^{13} and $J=\frac{3}{2}$ in Li^7. Only then would transitions to both be allowed. The existence of two low lying levels so alike in character would be in contradiction to most current ideas about nuclear states. These facts speak strongly for G-T rules and help exclude the polar vector interaction.

8. K CAPTURE

The calculation of the decay constant for the process of K capture seems to have been done first by Yukawa and Sakata (Y2) and independently by others (M6, B5). The absorbed electron, instead of having initially one of a continuum of negative energies* $W<-1$ as in positron emission, has the discrete positive energy $s=(1-\alpha^2Z^2)^{\frac{1}{2}}$ which is its rest energy minus the binding energy in the K shell. The emitted neutrino is monoenergetic, its energy q being given by:

$$\begin{aligned} q&=M_z-M_{z-1}-(1-s)=W_0+s \\ &\approx M_z-M_{z-1} \qquad\quad =W_0+1 \end{aligned} \tag{32}$$

where M_z and M_{z-1} are the energies equivalent to the *atomic* masses of the parent and product nuclei, and W_0 is therefore the energy available for positron emission. For $-s<W_0<1$ (i.e., $1-s<M_z-M_{z-1}<2$) only K capture is energetically possible; for $W_0>1$, positron emission may also take place.

Because of the discreteness in energy the statistical factor (26) for K capture becomes simply

$$4\pi q^2/(2\pi)^3=(W_0+s)^2/2\pi^2.$$

Otherwise the computation of the decay constant differs from that leading to (24) only in the employment of K-electron eigenfunctions for ψ and the introduction of a factor 2 to account for the two K electrons. For allowed transitions one thus obtains

$$1/\tau_K=(G^2|\int\cdots|^2/2\pi^3)f_K, \tag{33}$$

with

$$f_K=2\pi(\alpha Z)^{2s+1}(2R)^{2s-2}(1+s/2s!)(W_0+s)^2. \tag{33a}$$

This expression has already been used in Table II and in the discussion of Be^7 in Section 7. Its dependence on charge and energy release is seen more easily in the approximate formula:

$$f_K\approx 2\pi(\alpha Z)^3(W_0+1)^2. \tag{33b}$$

* Continuing the practice of earlier sections, all the energies are given here in units of mc^2. W and W_0 also retain the earlier meaning: total energy, including rest energy. M_z shall denote, in units mc^2, the energy equivalent to the *atomic* mass, i.e., the mass of the nucleus plus the mass of the Z orbital electrons.

An estimate of forbidden K capture according to a method employed by Bethe (B5) gives for an Lth forbidden transition:

$$f_K{}^L \approx \frac{2\pi}{9}(\alpha Z)^5(W_0R)^{2L-2}\frac{(W_0+L)^2}{1^2\cdot 3^2\cdots(2L-1)^2L}. \quad (34)$$

This is consistent with the findings in Section 4 that first forbidden decay will have about the ratio $(\alpha Z/2)^2$ to allowed decay and that the higher degrees of forbiddenness are successively smaller by the ratio $(W_0R)^2$.

Some important experimental developments with regard to K capture follow:

(1) Alvarez (A3) seems to have been the first to establish the existence of K capture. He detected it by the most unambiguous means for the purpose, the measurement of the K x-ray to be expected from the product element. The decaying nucleus employed was Ga^{67} (84 hr.); no positrons were detectable, yet x-rays characteristic of Zn were found, thus proving the existence of the process. Walke, Williams, and Evans (W3) carried out a similar experimental proof for the 600-day V^{49} isomer, which also emits no detectable positrons yet yields strong Ti K radiation. Apparently, the energy released by both Ga^{67} and V^{49} is insufficient to create positrons.

(2) When positrons compete with the K capture, the detection of product element x-rays will not unambiguously prove the existence of capture unless γ-rays are absent. Internal conversion of the latter likewise excites x-rays. This point was emphasized by Alvarez and applies to V^{48} (16 days) on which he first worked. However, a low intensity of internal conversion electrons as in V^{48} makes it fairly sure that the strong x-rays indicate K capture.

(3) The complete absence of any radiation save positrons and strong product element x radiation as in Cu^{61} (3.4 hr.) proves the occurrence of K capture. A further advantage in such a case is that with the product nucleus always left unexcited (no γ-rays), the energy release in the positron emission measures the release in the K capture. Thus for Cu^{61}, $W_0+2.8$ and

$$f_K \approx 2\pi(29\alpha)^3(3.8)^2 = 0.85.$$

TABLE III. Ratio of positron emission to K capture.

Emitter	t	W_0	f_+/f_K
C^{11}	21 min.	2.86	3600
Sc^{41}	0.9 sec.	10.68	1100
Sc^{43}	4 hr.	3.5	7.6
Y^{88}	2 hr.	2.3	2.4
Sb^{120}	17 min.	4.0	3.0

Since from Section 5, $f \approx 1.5$, the ratio of positron emission to K capture is ~ 1.8 theoretically. Unfortunately, no measurement of this ratio for such an unambiguous case seems to exist.

It is appropriate to remark here that according to the theory above every process of positron emission is accompanied by some K capture, and especially for small energy release and large nuclear charge. A few values of the theoretical f_+/f_K ratio are listed in Table III.

(4) The transitions listed in Table III are all allowed. K capture will very often actually exceed fairly energetic but forbidden positron emission in the following way: Positron decay is often forbidden simply because no level to which it could be allowed is energetically available. For K capture the range of energy available is extended by $2\,mc^2$, which is a considerable extension compared to level spacings especially among heavier nuclei. With some frequency, therefore, forbidden positron decay will be accompanied by allowed K capture plus subsequent γ-radiation which is often well internally converted (because high multipole radiation is easily converted, if its energy is not too high). Cases of this seem to be, for example: V^{48}, Mn^{52}, Zn^{65}, As^{76} (L5).

The positron decay may actually become negligible so that only x-rays, γ-rays, and internal conversion electrons are observed. Such seems to be the case for Cr^{51}, which emits no detectable positrons yet emits strong 1-Mev and perhaps 0.5-Mev γ-radiation (W5).

The general importance of allowed K capture to levels unavailable to positrons as discussed above, may be borne out by the great frequency with which γ-radiation and internal conversion electrons accompany K

capture [Co^{56}, Ga^{65}, Ga^{67}, Y^{87}, Cd (6.7 hr.), Sn^{113}, Te^{121}, Ta^{180}, Hg^{197}]. On the other hand, K-capture processes accompanied by γ-radiation are much more easily detected than when accompanied by x radiation alone.

(5) Sizoo (S5) has made the observation that when two isobars with mass number A and atomic numbers $(Z\pm1)$ are stable, it is likely that the intermediate isobar Z^A decays both by negatron emission and K capture. Because $(Z+1)^A$ is stable against K capture, its atomic mass satisfies the relation

$$M_{z+1}+(s-1)<M_z$$

so that if M_z is larger only by the binding energy of a K electron, $(1-s)$, than it is thus required to be, negatron decay is energetically possible. Likewise, the stability of $(Z-1)^A$ against negatron decay leads to

$$M_{z-1}<M_z,$$

making M_z within the binding energy of a K electron unstable against K capture, for which it is necessary that

$$M_z>M_{z-1}+(1-s).$$

It was pointed out by Bethe (private communication) that Sizoo's arguments can be somewhat sharpened by considering the possibility of the capturing of electrons from the outermost atomic orbits. Since such electrons have negligible binding, the stability condition for $(Z+1)^A$ becomes $M_{z+1}<M_z$, thus making Z^A an electron emitter immediately. [With the stability of $(Z-1)^A$ requiring $M_{z-1}<M_z$, Z^A becomes immediately capable of capturing at least outer electrons.]

Sc^{46} and probably Ta^{180} are examples of β^--K branching. The negatrons emitted by Ta^{180} were originally assumed to be internal conversion electrons following the K capture (O1) because a stable W^{180} was not then known. The possibility that the negatrons were due to β-decay to a stable W^{180} stimulated the discovery of that isotope through spectrographic analysis (A3, D2). Undoubtedly, more examples of β^--K branching would be found if a search for x-rays were a regular part of the procedure of investigating an activity. When

$$M_z>M_{z+1}+2,$$

positron decay as well as K capture can occur. Cl^{36}, Cu^{64}, As^{76}, and perhaps Ag^{106} are examples of this (L5).

(6) Sometimes the presence of a strong K capture accompanying positron decay shows itself in an intensity of γ-radiation too great compared to the positron intensity for all of it to be following upon the positron emission only. Such situations have arisen in Zn^{65}, Mn^{52}, and others (L5). [See paragraph (4).]

9. SUMMARY OF ARGUMENTS

For β-decay theory, next in importance to the confirmation of the general structure of the theory itself, has been the making of a choice between the Fermi and K-U ansätze and between the Fermi and G-T selection rules [more particularly, among the various forms (6) which can be given the theory]. The most important arguments on these points were developed in the course of the discussions in the previous sections. More arguments have been advanced which it is sufficient to mention only in summary.

The K-U criticism and modification of Fermi's theory seems now to be definitely disproved by the following developments:

(1) As shown in Section 4, the forms of the allowed spectra, on the basis of which the criticism and modification were made, now definitely favor Fermi's ansätz.

(2) Also mentioned in Section 4 were the great discrepancies between the mass difference of parent and product nucleus and the K-U values of the energy release as obtained from the spectrum by extrapolation in the Kurie plot.

(3) The relatively good constancy of the ft value for any one of the groups in Table II was obtained from the Fermi theory. The K-U modification would require that $(W_0-1)^2ft$ be more constant. Actually, in the very-well-understood group OA, for instance, $(W_0-1)^2ft$ is a rapidly increasing function with the value 12,000 for C^{11} and 230,000 for Sc^{41}. This argument is frequently quoted in terms of the approximate energy dependence of f. In the Fermi theory, $f\sim W_0^5$; in the K-U modification, a seventh-power law is expected, i.e., $f\sim W_0^7$, approximately.

(4) The K-U distribution for forbidden transi-

tions in light elements turns out to have a sharp maximum at low energy. No such distribution is known experimentally.

(5) The K-U theory of K capture gives

$$f_k \approx 2\pi(\alpha Z)^3(W_0+1)^4.$$

The strong energy dependence of this expression makes itself felt most in the comparison of two K-capture processes with different energy release as in the formation of the two levels of Li^7 from Be^7. One sees from Section 7 that the K-U theory would give at least $5(1.70)^4/4(0.82)^4 \approx 23$ transitions to the normal state for each γ-ray, whereas experiments give ~ 9 for the number. (The experimental value is very inaccurate, however.)

The weight of the evidence has also been to favor G-T selection rules over the Fermi rules, and specifically to favor the tensor form (6c) of the theory.

(1) The fact that He^6 and other light nuclei with $A=4n+2$ undergo allowed transitions can only be understood with G-T rules because the total angular momentum almost certainly changes in these transitions (see Section 7).

(2) The fluctuations in ft shown by the nuclei with $T_z=-\frac{1}{2}$ seem to favor G-T rules somewhat, as was seen in Section 7.

(3) The allowed complex transitions $N^{13} \rightarrow C^{13}$ and $Be^7 \rightarrow Li^7$ plausibly take place to a doublet of levels $^2P_{1/2,3/2}$, one of which can only be reached with a change of total angular momentum (see Section 7).

(4) Gamow and Teller's (G1) original reason for revising the rules was based on the transitions: ThB$\rightarrow$ThC$\rightarrow$ThC′. From the empirical rule that nuclei with even Z and even A have $J=0$ in the ground state, ThB and ThC′ are each expected to have $J=0$. The β-decay from ThB is allowed (see Table II) and is followed by a strong γ-ray; this must be interpreted as showing that most of the transitions go to an excited state of ThC. The latter must have $J=0$ according to Fermi rules, and because the γ-radiation is internally converted at a rate which indicates quadrupole (certainly not dipole) character, the ground state of ThC is expected to have $J=2$. Thus the ground-to-ground transition between ThC and ThC′ should be second forbidden according to Fermi rules. Yet, according to Table II, it is only first forbidden (no strong γ-rays are present so the chief transition is to ground). On the other hand, G-T rules permit $\Delta J=2$ in first forbidden transitions and a greater latitude in the J values generally.

(5) The energy distribution from P^{32} and RaE seems representable only by the polar vector and tensor forms (6) of the theory as was seen in Section 6; the tensor form gives G-T rules, showing that the forms of the forbidden spectra are not inconsistent with these rules.

(6) The half-life of K^{40} has a magnitude corresponding to a third forbidden transition although for it $\Delta J=4$. Such a spin change is possible in the third forbidden approximation only with G-T rules. However, when the highest reported value is taken for the energy release, Fermi rules in the fourth forbidden approximation are still able to give a short enough lifetime, as discussed at the close of Section 6.

A final theoretical conclusion which may be mentioned is that a narrowing down of the β-decay interaction to the tensor form T of (6c) would require a meson of spin 1 according to the simplest versions of the meson theory as outlined in Section 2.

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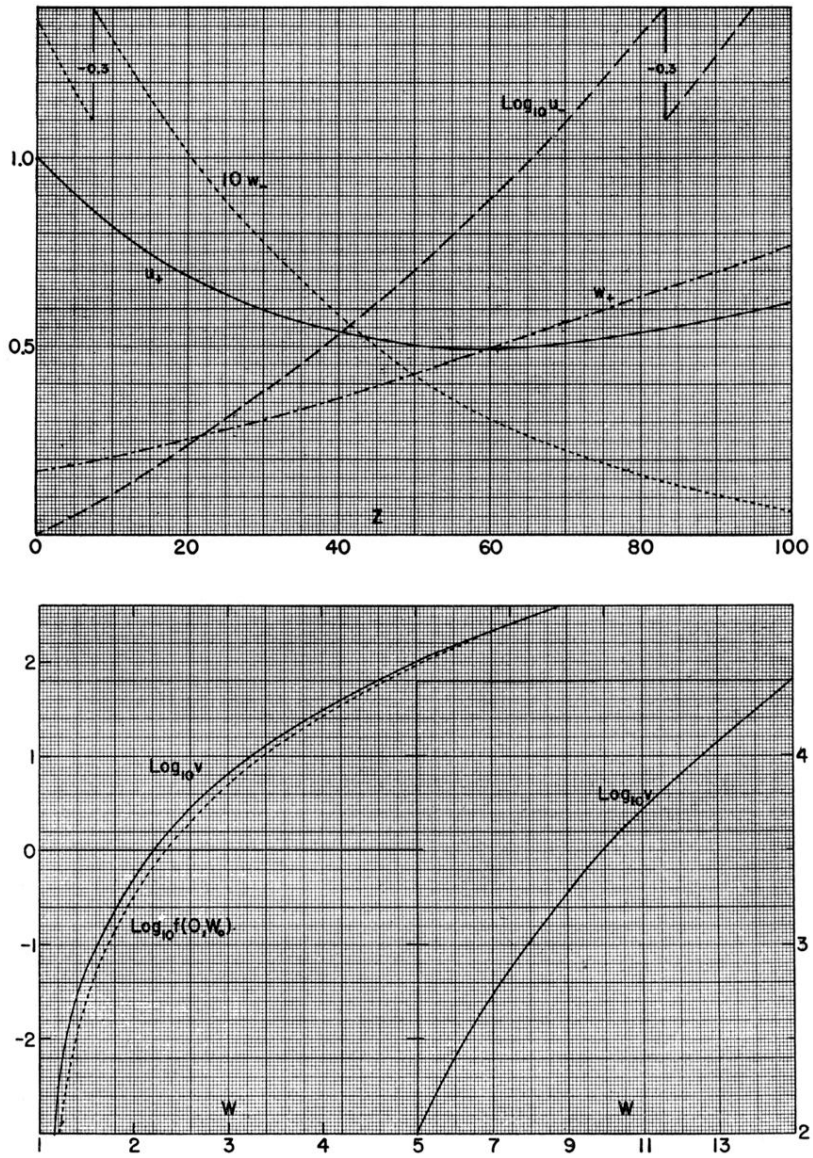

FIG. 3a. The functions $u(Z)$ and $w(Z)$ [Eq. (24a)]. u_+ and w_+ are for positrons; $\log_{10} u_-$ and $10w_-$ for negatrons. For the two curves extending off scale, the affected ordinates are lowered on the graph by 0.3 unit from their proper values, as indicated. FIG. 3b. The function $\log_{10} v(W_0)$, [Eq. (24a)]. The inset is for a contracted scale of energies as indicated. The dotted curve is $\log_{10} f(0, W_0)$ of Eq. (24a).