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# The Mechanism of Electrical Discharges in Gases of Low Pressure*

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* Because of the troubled conditions in Holland the authors of this paper have not had an opportunity to read proof.—The Editors.

## I. Introduction

DURING the past ten years the mechanism of a number of electrical discharges in gases has been made partially clear in terms of the fundamental processes between electrons, atoms or molecules, ions, photons, the walls, and the electrodes. In this review we intend to treat mainly the results of this work. It is not our purpose to give a full account of the literature,[1] but we will discuss especially such phenomena as are essential for understanding the mechanism of discharges at low pressure. We deal somewhat more extensively with those subjects which are discussed only briefly in most books on gas discharges,[2] as e g. the present material with respect to the Townsend discharge. Problems that have been treated extensively in these books will be summarized briefly or omitted.

We shall not discuss chemical reactions occurring in discharges, intermittent and alternating-current discharges nor the influence of a magnetic field. Also discharges at a high pressure (of the order of one atmosphere) will be left out of account. The spark discharge has been reviewed recently in this journal by Loeb.[3] For the explanation of the arc discharge at high pressure a point of view is important (especially the thermodynamic equations of Boltzmann and Saha) which is quite different from that which is taken to explain the low pressure discharge. The boundary between low and high pressure is not sharp, but depends on the experimental conditions; often it lies between 20 and 200 mm pressure.

### §1. General survey

For the fundamental processes we refer to the reviews in this journal by Compton and Langmuir, Smyth, and Brode.[4] In the discussion of the discharges we shall try to evaluate the extent to which each of these processes is essential for the mechanism of the discharge.

For the explanation of discharges in terms of the fundamental processes of ionization and excitation, it is often desirable to know the properties of a small electron current through the gas. In several cases it is possible to calculate approximately the velocity distribution of the electrons (Section II) and with the aid of this distribution again other properties (Section III) as e.g. the ionization coefficient $\alpha$ of Townsend (or $\eta$, §10). By the combination of $\alpha$ with other

[1] For the literature on gas discharges from 1930–1937, see G. Mierdel and R. Seeliger, Die Physik **2**, 67 (1934); **6**, 79 (1938).

[2] J. S. Townsend, "Electricity in gases, 1915," *Handbuch der Physik*, Vol. 14, 1927, especially p. 171 (R. Bär), p. 324 (A. Hagenbach); R. Seeliger and G. Mierdel, *Handbuch der Experimentalphysik* Vol. 13, Part 3, 1929; J. J. and G. P. Thomson, *Conduction of Electricity Through Gases*, I, 1928; II, 1933; K. K. Darrow, *Electrical Phenomena in Gases*, 1932; A. v. Engel and M. Steenbeck, *Elektrische Gasentladungen*, I, 1932; II, 1934; R. Seeliger, *Einführung in die Physik der Gasentladungen*, 1934; E. L. E. Wheatcroft, *Gaseous Electrical Conductors*, 1938.

[3] L. B. Loeb, Rev. Mod. Phys. **8**, 267 (1936).

[4] K. T. Compton and I. Langmuir, Rev. Mod. Phys. **2**, 123 (1930), cf. E. W. Pike, Physik. Zeits. **33**, 457 (1932); H. D. Smyth, Rev. Mod. Phys. **3**, 347 (1931); R. B. Brode, Rev. Mod. Phys. **5**, 257 (1933).

coefficients an explanation of the breakdown potential becomes possible.[5]

The discussion of the discharges themselves is divided into two main parts. The first part (Section IV), is devoted to discharges in which the influence of space charge is negligible or small (*Townsend discharge*). The *breakdown potential* is the most important topic of this section.

In the second part (Sections V–IX) we discuss the discharges which are essentially determined by space charges, in this case the equation of Poisson ($\Delta V = -4\pi\rho$) is of primary importance (*glow* and *arc discharges*). The *positive column* (Section VIII) belongs to glow as well as to arc discharges.

For the discussion of the influence of the equations of Laplace and Poisson on the current in some simple cases, we may refer to the review of Langmuir and Compton[6] in this journal.

## §2. Schematic characteristics

In order to illustrate our subdivision of the gas discharges, and to give some definitions, a schematic form of the discharge characteristic $V=f(i)$ is given in Fig. 1, which applies to flat, parallel electrodes. The product of the pressure $p$ and the electrode separation $d$ is taken large enough that the discharge is not "obstructed" (§39), and on the other hand small enough, that no positive column or anode glow occurs (for the rare gases $2<pd<20$ mm·cm). In this case the voltage $V$ on the tube from $E$ to $K$ (Fig. 1) is almost equal to the cathode fall.

For low values of $V$ (e.g. 10 volts) the gas is a very good insulator. Nevertheless, even between cold electrodes without photoelectric and auto-electronic emission, a very small current flows, which is due to ionization by external agents, such as cosmic rays. In air at 1 atmosphere, for example the saturation value is about $10^{-18}$ amp./cm$^2$. This current is subject to statistical fluctuations ($A$ in Fig. 1). When a supplementary source of electrons exists such as photo-emission from the cathode, this part of the characteristic is shifted to the right ($A'$ in Fig. 1, dot-dash).

With increasing voltage the current increases by secondary ionization, at first slowly, later very rapidly; the voltage for which the initial current is increased by a very large factor (e.g. $10^8$) is called the *breakdown potential* $V_B$. For the case in which the primary electrons are due to cosmic rays only, this point is reached at say $B$ in Fig. 1; for the photoelectric current somewhere at $B_1$. In the latter case the breakdown potential may usually be defined as the potential $V_{B1}$, for which the current is maintained, when the radiation is stopped (*self-sustaining current*). When the photoelectric current is not too large, $V_B = V_{B1}$. In the region $BC$ or $B_1C$ the value of $dV/di$ is practically zero.

When the discharge from $B$ or $B_1$ onward is nonintermittent, it is called a *Townsend discharge*. After a longer or shorter almost horizontal part $BC$ or $B_1C$, the characteristic may be falling ($CD$) or rising ($CD'$). With further increasing current the space charge becomes so large that a cathode fall develops. The transition from $D$ or $D'$ to $E$ is often formed by an intermittent discharge. From $E$ to $H$ the discharge is called a *glow discharge*.

The *starting potential of the glow discharge* $V_G$ is equal to the "breakdown potential," when the characteristic of the Townsend discharge is negative ($V_G = V_B$) but larger in the contrary case ($V_{G'} > V_B$).

In the glow discharge the ionization of gas

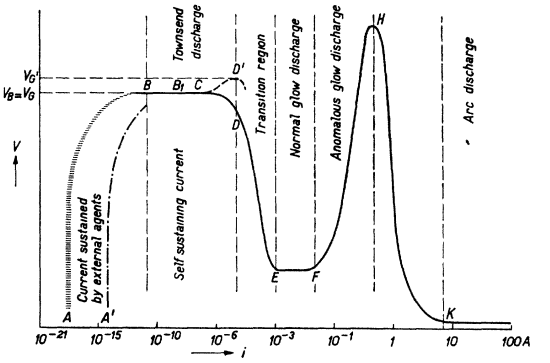


FIG. 1. Schematic characteristic of a gas discharge between flat parallel plates. Scale for the current: logarithmic between 100 and 0.1 amp., still more contracted for lower values of $i$. —·—·— current with photo-emission from the cathode; — — — ($CD'$) positive characteristic of the Townsend discharge; the breakdown potential $V_B$ is lower than the starting potential of the glow discharge $V_G$; ——— ($CD$) negative characteristic of the Townsend discharge; $V_B = V_G$.

[5] For the quantitative explanation we shall use, however, the experimental values of $\alpha$ or $\eta$ which are more reliable.

[6] I. Langmuir and K. T. Compton, Rev. Mod. Phys. **3**, 191 (1931).

molecules[7] is caused by electrons, liberated from the cathode mainly by positive ions ($\gamma$-process).

From $H$ to $K$ a falling characteristic occurs, and the cathode fall can decrease to very low values equal to or below the ionizing potential of the gas. From $H$ onward the electron emission of the cathode is no longer determined by the $\gamma$-mechanism only, but by other emissions, e.g. thermal or autoelectronic emission. The part of the characteristic between $H$ and $K$ is often discontinuous; only with thermionic emission from the cathode a continuous transition can occur. From $H$ onward, or according to other physicists from $K$ onward, the discharge is called an *arc*. Because of the existence of transition regions the distinctions between Townsend discharge, glow discharge, and arc are by no means sharp.

## II. Velocity Distribution of the Electrons

The velocity distribution of the electrons is of fundamental importance as in most discharges the current is mainly an electron current and the excitation and ionization of gas atoms is in large part caused by electrons. The calculation of the velocity distribution is thus one of the important steps for the understanding of gas discharges; we shall discuss here in which cases it can be calculated from the fundamental processes.

If an electron current traverses a gas in which an electric field $E$ exists, the velocity distribution will be almost isotropic if the potential difference per mean free path of the electrons $E\lambda$ is small in respect to the electronic energy $\epsilon$. The number of electrons per cm$^3$ with energy between $\epsilon$ and $\epsilon+d\epsilon$ will in general be a function $\rho(\epsilon, x)$ of the energy $\epsilon$ and the distance to the cathode $x$. If the electron current density is constant and $x$ is large enough, a *steady distribution* $\rho(\epsilon)$ independent of $x$ will exist. In this case the mean gain of energy due to the electric field is equal to the mean loss of energy in collisions with gas atoms.

### §3. Elastic energy losses

Before dealing with the velocity distribution in the case that only elastic collisions occur, we shall first discuss the mean energy loss of an electron (mass $m$) when it collides with a gas atom (mass $M$).[8]

We assume at first that the gas atoms are at rest and that all electrons have the same energy $\epsilon$. The mean energy loss per collision is given by

$$\Delta\epsilon = -\frac{2m}{M}\epsilon\int_0^\pi f(\theta)(1-\cos\theta)d\theta, \qquad (1)$$

where $f(\theta)$ is the probability that the direction of the electron after collision makes an angle $\theta$ with the original direction.

If the collisions take place in the same way as between hard spheres, we get ($\delta = 2m/M$).

$$\Delta\epsilon = -2m\epsilon/M = -\delta\epsilon. \qquad (2)$$

In the case that the atoms are not at rest but have a Maxwell velocity distribution, $\bar{\epsilon}_g = 3kT/2$, we find

$$\Delta\epsilon = -(2m\epsilon/M)(1-4\bar{\epsilon}_g/3\epsilon). \qquad (3)$$

If the electrons also have a Maxwell distribution with mean energy $\bar{\epsilon}$

$$\Delta\epsilon = -(8m\bar{\epsilon}/3M)(1-\bar{\epsilon}_g/\bar{\epsilon}).^9 \qquad (4)$$

If the energy distribution is not Maxwellian, the factor may differ somewhat from 8/3.

### §4. Velocity distribution with elastic collisions only

We shall treat the steady velocity distribution of electrons in a small electron current through a gas in a homogeneous electric field $E$, assuming that the current density is constant (no electron being lost), and that the collisions between electrons and gas atoms occur elastically.

If the energy of the electrons is large with respect to the atomic energy, the mean loss per collision according to Eq. (2) is $\delta\epsilon$; the total loss of all the electrons of energy $\epsilon$ (velocity $v$) per cm$^3$ per sec. is:

$$\rho(\epsilon)\delta\epsilon v/\lambda = \rho(\epsilon)\delta\epsilon(2\epsilon/m)^{\frac{1}{2}}/\lambda,$$

in which $\lambda$ is the mean free path of the electrons.

In a steady distribution this should be equal to the energy taken from the field $j(\epsilon)E = J(\epsilon)eE$

[7] We shall often use the word molecules in its general sense, meaning monatomic as well as polyatomic particles.

[8] K. T. Compton and I. Langmuir, Rev. Mod. Phys. **2**, 210 (1930); W. de Groot and F. M. Penning, *Handbuch der Physik*, Vol. 23, second edition, p. 234 (1933).

[9] A. M. Cravath, Phys. Rev. **36**, 248 (1930).

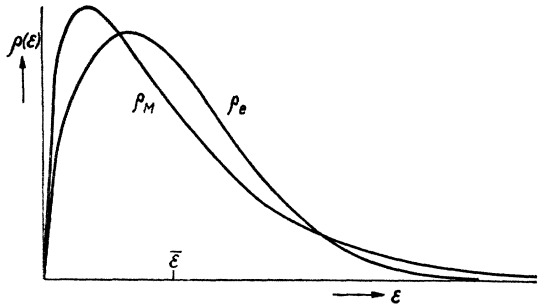


FIG. 2. Distribution-function according to Maxwell (Eq. (11)) and according to Eq. (8) for the same mean energy $\epsilon$ and the same electron concentration.

($j$=electron current density, $J=j/e$ is the number of electrons passing one cm$^2$ normal to the field per sec., $E$=electric field strength).

This gives as a first equation

$$J(\epsilon)eE=\rho(\epsilon)(2\epsilon/m)^{\frac{1}{2}}(\delta\epsilon/\lambda). \tag{5}$$

As a second equation between $J$ and $\rho$ we have the mobility equation,[10] which in its general form is given by

$$J(\epsilon, x)=-\frac{\lambda}{3}\left(\frac{2\epsilon}{m}\right)^{\frac{1}{2}}\rho_x'-\frac{\lambda eE}{3}\left(\frac{2\epsilon}{m}\right)^{\frac{1}{2}}\rho_\epsilon' +\frac{\lambda eE}{3m}\left(\frac{m}{2\epsilon}\right)^{\frac{1}{2}}\rho. \tag{6}$$

In the case of a steady distribution $\rho_x'$ is zero and a combination of Eqs. (5) and (6) gives ($\lambda$ assumed constant)

$$\rho_\epsilon'=\frac{\rho}{2\epsilon}-\frac{3\delta\epsilon}{\lambda^2e^2E^2}\rho, \tag{7}$$

$$\rho(\epsilon)=C\epsilon^{\frac{1}{2}}\exp\left[-\frac{3\delta\epsilon^2}{2\lambda^2e^2E^2}\right] =C\epsilon^{\frac{1}{2}}\exp\left[-0.55\frac{\epsilon^2}{\bar{\epsilon}^2}\right], \tag{8}$$

$$J(\epsilon)=C_1\epsilon^2\exp\left[-\frac{3\delta\epsilon^2}{2\lambda^2e^2E^2}\right], \tag{9}$$

$$C=\frac{\lambda eE}{\delta}\left(\frac{m}{2}\right)^{\frac{1}{2}}C_1=\left(\frac{3\delta}{2\lambda^2e^2E^2}\right)^{\frac{3}{2}}\frac{2n_e}{\Gamma(3/4)}, \tag{10}$$

[10] J. C. Maxwell, Phil. Trans. **157**, 73 (1867); G. Hertz, Zeits. f. Physik **32**, 298 (1925); V. A. Bailey, Zeits. f. Physik **68**, 834 (1931). See also reference 13.

where $n_e$ is the electron concentration and $\Gamma$ is the gamma-function.

It is seen that for given gas $\rho$ is a function only of $E\lambda$ or of $E/p_0$ where $p_0$ is the pressure reduced to 0°C.[11] Fig. 2 shows a Maxwell distribution $\rho_M$ and the distribution (8) $\rho_e$ for the same mean energy $\bar{\epsilon}$ and the same concentration $n_e$.

$$\rho_M(\epsilon)=C\epsilon^{\frac{1}{2}}e^{-\epsilon/kT}. \tag{11}$$

It is seen that the number of fast electrons is much larger for the Maxwell distribution.

Thus far $\lambda$ has been assumed to be a constant; if $\lambda$ is a function of $\epsilon$ we get from (7):[12]

$$\rho(\epsilon)=C\epsilon^{\frac{1}{2}}\exp\left[-\frac{3}{e^2E^2}\int_0^\epsilon\frac{\delta\epsilon}{\lambda^2}d\epsilon\right]. \tag{12}$$

In the derivation of these formulae we supposed that the collisions between electrons and atoms take place as between hard spheres. If this is not the case, the probability for scattering of the electrons under an angle $\theta$ with the primary direction being $f(\theta)$, we may use the same formulae if we put instead of the value $\lambda$ according to the normal definition of the mean free path a somewhat different value $\lambda'$, given by

$$\lambda'=\lambda\Big/\int_0^\pi f(\theta)(1-\cos\theta)d\theta. \tag{13}$$

If the energy of the electrons is not very large in respect to the energy of the atoms (the gas temperature being $T$), the energy loss per collision is no longer $\delta\epsilon$ but we must use Eq. (3). In this case we get with constant $\lambda$:

$$\frac{\rho_\epsilon'}{\rho}=\frac{1}{2\epsilon}-\frac{\epsilon}{kT}+\frac{\lambda^2e^2E^2/3\delta k^2T^2}{\epsilon+\lambda^2e^2E^2/3\delta kT}, \tag{14}$$

$$\rho(\epsilon)=C\epsilon^{\frac{1}{2}}e^{-\epsilon/kT}\left(\epsilon+\frac{\lambda^2e^2E^2}{3\delta kT}\right)^{\lambda^2e^2E^2/3\delta k^2T^2} \tag{15}$$

We have two limiting cases:

(a) $\epsilon\gg\lambda^2e^2E^2/3\delta kT$, (b) $\epsilon\ll\lambda^2e^2E^2/3\delta kT$.

In the case (a) $\rho(\epsilon)$ of Eq. (15) is almost equal

[11] The pressure $p_0$ is proportional to the gas density. It is the gas density and not the gas pressure which is essential in all the processes involved here.

[12] Eq. (12) gives a Maxwell distribution if $\delta\epsilon/\lambda^2$ is a constant.

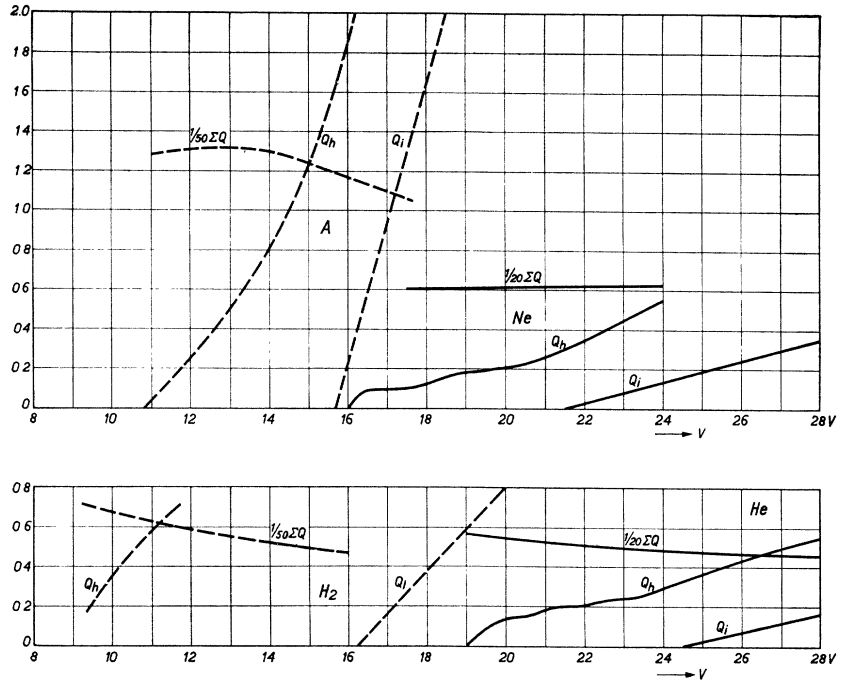


FIG. 3. Excitation, ionization, and total efficiency as a function of the electron energy; A (- -), Ne (—), He (—) (Maier-Leibnitz) and $H_2$ (- -) (Ramien).

to a Maxwell distribution; in case (b) Eq. (15) becomes the same as Eq. (8).[13, 14]

The formulae of this section are deduced by a number of authors in different ways.[15]

## §5. Efficiencies for excitation and ionization

For the calculation of the distribution function in the case that inelastic collisions also occur we must know the probability for such a collision. We will first deal with these probabilities.

In a collision between an electron and a gas molecule several processes $a$, $b$, $c$, $\cdots$ may occur. For these processes we define the efficiencies $Q_a$, $Q_b$, $Q_c$ $\cdots$ so that the number of processes $a$ which occur when the electron passes a distance $ds$ through the gas is equal to:

$$Q_a p_0 ds, \tag{16}$$

where $p_0$ is the gas pressure in mm Hg reduced to 0°C.

It should be emphasized that (16) is to be used only for $ds \ll \lambda$. $Q_a$ is the efficiency for process $a$ per unit path at unit pressure.[16] In this way we define $Q_i$ the efficiency for ionization, $Q_h$ the efficiency for excitation of electronic levels, $Q_v$ the efficiency for excitation of vibrational levels[17] (only for molecular gases), $Q_r$ the efficiency for an elastic collision.

If necessary we can also define efficiencies for other processes. The total collision efficiency is equal to $\Sigma Q$ and the mean free path of the electron is $\lambda = 1/p_0 \Sigma Q$.

[13] The last result can be found by taking the logarithm of Eq. (15) and developing $\log(1+x)$ for small values of $x$ as far as the term in $x^2$. If we go one term further in the development we get the value of $\rho$ in Eq. (8) multiplied by $(1+3\delta^2 kT\epsilon^3/\lambda^4 e^4 E^4)$.

[14] It should be remarked that we assumed in the derivation of Eq. (6) the free path of the electrons between the collisions being only curved by the action of the electric field $E$. The force between an electron and a polarizable atom was neglected, which is probably allowed for the case of formula Eq. (8) but it seems to be dangerous to use this equation if the electron energy is of the same order of magnitude as the atomic energy (see P. Langevin, Ann. chim. phys. **5**, 245 (1905)).

[15] F. B. Pidduck, Proc. Lond. Math. Soc., series 2, **15**, 89 (1915), Phys. Rev. **53**, 197 (1938); M. J. Druyvesteyn, Physica **10**, 61 (1930), **1**, 1003 (1934), **3**, 65 (1936); A. M. Cravath, Phys. Rev. **46**, 332 (1934); B. Davydov, Physik. Zeits. Sowjetunion **8**, 59 (1935), **9**, 433 (1936); P. M. Morse, W. P. Allis and E. S. Lamar, Phys. Rev. **48**, 412 (1935); L. S. Ornstein, H. Brinkman and T. Hamada, Proc. Acad. Sci. Amsterdam **39**, 315 and 484 (1936); J. S. A. Townsend, Phil. Mag. **22**, 145 (1936), formula (29); J. A. Smit, Physica **3**, 543 (1936); R. Lichtenstein, Physik. Zeits. **39**, 646 (1938); L. B. Loeb, *The Distribution of Electron Energies in a Gas in an Electrical Field* (1938).

[16] The effective cross section per $cm^3$ at a pressure of 1 mm is $Q_a$ $cm^2$; the effective cross section for one molecule is $0.283 \times 10^{-16}$ $Q_a$ $cm^2$.

[17] The excitation of rotational levels by an electron is always negligible in the cases to be considered, as a consequence of the large amount of angular momentum which in this process must be transferred from the electron to the molecule, cf. H. Ramien, Zeits. f. Physik **70**, 353 (1931).

The probability per collision for process $a$ is equal to $Q_a/\Sigma Q=K_a$. In this way we shall define: $K_i$ the probability for ionization, $K_h$ the probability for excitation, etc.

It is a matter of choice whether one describes the different processes with $Q_a$ or with $K_a$. In the calculations we will use $K_a$. The efficiencies $Q_a$ are often used to describe the experimental results. For these results we must refer to previous reviews.[2-4] We shall, however, separately mention the recent experimental results for $Q_h$ in He, Ne and A, according to Maier-Leibnitz[18] and, for a small voltage range, also those for $H_2$ of Ramien.[18] Fig. 3 summarizes their results for $Q_h$; together with the values of $Q_i$ and $\Sigma Q$ used by these authors. It would be very useful if these three quantities, which determine practically the whole behavior of a small electron current through a gas, were known also for other gases.

## §6. Velocity distribution with excitation and ionization

In almost every discharge the energy losses due to excitation are very important. To derive the steady distribution function $\rho(\epsilon)$ we must distinguish two cases. If the energy loss in a collision is small with respect to the energy of the electrons (as in the case of a molecular gas if only the vibrational levels of the ground state can be excited) we may use Eq. (12) if we take

$$\delta\epsilon=2m\epsilon/M+\Sigma_n K_{vn}\epsilon_{vn}, \tag{17}$$

in which $K_{vn}$ is the probability of excitation of the $n$th vibrational level (with energy $\epsilon_{vn}$).

If, however, the energy loss in one excitation is large, as is always the case with monatomic gases, then we must distinguish two different equations for $\rho$, for $\epsilon<\epsilon_h$ ($\epsilon_h$ is the lowest excitation energy), the equation does not differ much from Eq. (8) or (12); for $\epsilon>\epsilon_h$ an approximate equation is given by:[19]

$$\frac{d^2\rho}{d\epsilon^2}-\frac{3K_h(\epsilon)}{\lambda^2e^2E^2}\rho=0. \tag{18}$$

If $\lambda$ is constant and $K_h$ can be represented by a linear function of $\epsilon$, the solution is given by a sum of two Bessel functions.

If ionization also occurs, the electron current will increase with the distance to the cathode $x$, so that no steady solution can be obtained.[20] If, however, as is often approximately the case, $\rho$ can be written as the product of a function $\rho_1(x)$ and a function $\rho_2(\epsilon)$ we can still use the solution of Eq. (18) for $\rho_2$, substituting $K_h+K_i$ for $K_h$. In this way the distribution function was obtained for He and Ne.[19] Fig. 4 shows the result for He at $E/p_0=3$ to 10 $V$/cm·mm according to Smit.

In some cases a simpler approximate solution can be used. If we take for $K_h$ a constant, namely the experimental value of the probability for excitation at an energy $\epsilon_2$ between the excitation and ionization energy, e.g. $\epsilon_2=\epsilon_h+2(\epsilon_i-\epsilon_h)/3$ the solution of Eq. (18) is:

$$\rho_2(\epsilon)=C\exp\left\{-\frac{[3k(\epsilon_2)]^{\frac{1}{2}}\epsilon}{\lambda eE}\right\}. \tag{19}$$

The dashed lines in Fig. 4 give the distribution function in He calculated with Eq. (19) and with

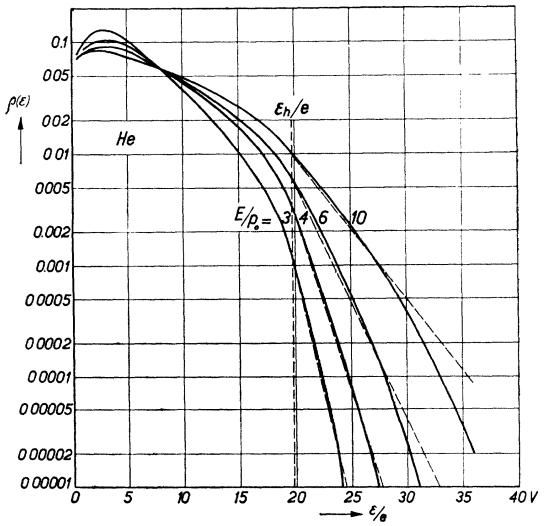


FIG. 4. Distribution function in He for $E/p_0=3$, 4, 6 and 10 (Smit); dashed lines according to Eq. (19).

[18] H. Maier-Leibnitz, Zeits. f. Physik **95**, 499 (1935); H. Ramien, Zeits. f. Physik **70**, 353 (1931). These results were obtained with the electrical method; values of $Q_h$ determined optically give only the excitation efficiency of a few levels and not the total excitation efficiency.

[19] M. J. Druyvesteyn, Physica **3**, 65 (1936) (Ne); J. A. Smit, Physica **3**, 543 (1936) (He).

[20] We have assumed in this section that no electrons are lost. In the positive column, however, a steady electron distribution can occur with ionization present. In this case the number of electrons formed by ionization is equal to the number lost by reaching the wall.

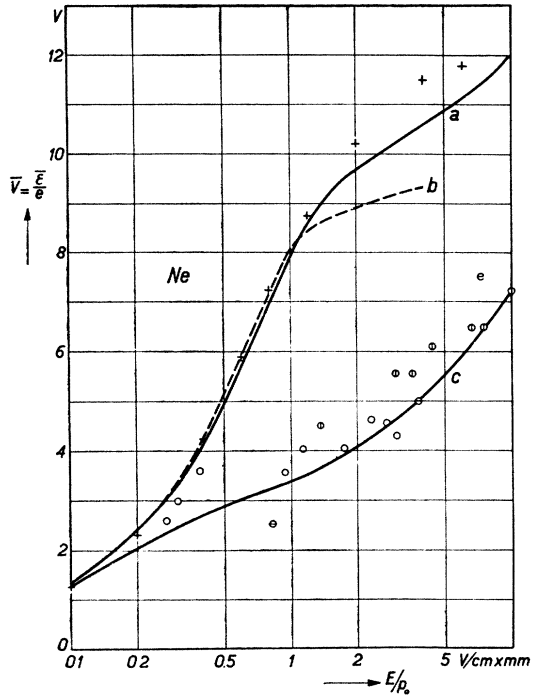


FIG. 5. Mean energy of the electrons as a function of $E/p_0$ in Ne. *a* (Druyvesteyn) and *b* (Allen) calculated for small current, *c* (Mierdel) calculated for large current, + experimental results for small current (Townsend), O for large current (⊖ Seeliger and Hirchert, O Druyvesteyn, ⊙ Heldt, as cited by Mierdel).

the values for $K(\epsilon_2)$ and $\lambda$ for $\epsilon=\epsilon_2$ as given by Smit.[19] We see that Eq. (19) is a rather good approximation. As the use of an exponential function is much simpler than the use of a Bessel function, a number of properties can at least in approximation be derived with Eq. (19).

## §7. Interaction

Thus far the electron concentration has been assumed to be small; if this concentration is large enough, the interaction between the electrons and between electrons and ions governed by their Coulomb forces must be considered. Of course such high concentrations can only occur in a homogeneous field if positive ions are present in the same concentration as the electrons. In an encounter between an electron and a positive ion the energy of the electron changes only little, so that the distribution function is practically not influenced by these encounters.[21] In encounters between different electrons, however, the energy change may be large.[22] As it is difficult to take this interaction into account quantitatively, attempts have been made heretofore only to calculate roughly the steady distribution function of electrons when, besides the electron interaction, elastic collisions alone occur. The influence of the interaction depends on a quantity $B$ which is given by:[23]

$$B=\frac{3\pi ne^2}{\lambda E^2}\ln\left[\frac{\epsilon+\frac{1}{2}\bar{\epsilon}}{e^2(\pi n)^{\frac{1}{3}}}\right]. \tag{20}$$

When $B$ is larger than about 5, the interaction is so important that the distribution is almost Maxwellian. If $B$ is smaller than 0.1 the interaction may be neglected and the distribution function (8) is obtained.

In calculating the interaction it was assumed that high frequency plasma oscillations were absent.

We wish to remark that at present we do not know whether in a plasma with a high value of $B$ these oscillations always exist or whether they have an influence on the distribution function.

## §8. Mean energy of the electrons

*Theory.*—If the energy distribution of the electrons is known, the mean energy $\bar{\epsilon}$ can be found by

$$\bar{\epsilon}=\int_0^\infty \rho(\epsilon)\epsilon d\epsilon \Big/ \int_0^\infty \rho(\epsilon)d\epsilon. \tag{21}$$

In the case of the distribution function (8) we get

$$\bar{\epsilon}=\frac{\sqrt{2}\Gamma(5/4)}{\Gamma(3/4)}\frac{\lambda eE}{(3\delta)^{\frac{1}{2}}}, \tag{22}$$

$$\bar{V}=\frac{\bar{\epsilon}}{e}=0.605\frac{\lambda p_0}{\delta^{\frac{1}{2}}}\frac{E}{p_0}. \tag{23}$$

[21] For the influence of the interaction on the mobility of electrons an opposite situation exists. Here the encounters between different electrons do not change the mean drift velocity, while the encounters with positive ions do alter the mobility. See §9, Eq. (28).

[22] In this case a relaxation length $s$ may be defined, which the electrons should have passed in order that an arbitrary distribution function becomes almost Maxwellian:

$$s=C\bar{\epsilon}^2/n_e e^4.$$

For $C$ somewhat different values are found by different authors, see R. Rompe and M. Steenbeck, Ergebn. d. ex. Naturwiss. **18**, 257 (1939).

[23] B. Davydov, Physik. Zeits. Sowjetunion **12**, 269 (1937); W. R. Haseltine, J. Math. Phys. **18**, 174 (1939); M. J. Druyvesteyn, unpublished.

In the case that also inelastic collisions occur the formulae are more complicated. Line *a* in Fig. 5 and the circles in Fig. 6 for He and Ne are calculated from the distribution functions. Allen[24] calculated the mean energy of the electrons in the rare gases in a way analogous to the one indicated above, but with some other approximations; line *b* in Fig. 5 gives the result for Ne.

So far we have discussed the case of an electron current of very small density. If, however, the density is large, as in a positive column of high current density, the electron interaction is so important that the electrons will have a Maxwell distribution. In this case the mean energy can be calculated by a method other than that indicated above, as the distribution function of the electrons is now known *a priori*. From the energy balance equation (the energy given to the electrons by the field being equal to the energy losses in elastic and inelastic collisions with the gas atoms) Mierdel[25] calculated the mean energy of the electrons in the rare gases as a function of $E/p_0$. Fig. 5, line *c*, shows his results in Ne. It is seen that the results obtained in this way differ much from the results of the calculation without interaction that is given by curve *a* and *b*.

If, by accident, the distribution function for an infinitely small current is almost a Maxwell distribution,[12] the interaction will not change the distribution function and the mean energy will be the same for low and high current density.

*Experiment.*—Townsend[26] has measured the mean energy of the electrons as a function of $E/p_0$ in a number of gases for a very small electron current. The lines of Fig. 6 and the crosses in Fig. 5 show his results. For He and Ne the agreement with this calculation is seen to be good.

In a positive column the mean energy of the electrons can be measured with the collector method of Langmuir.[27] As the electrons have in most cases a Maxwell distribution we can define an electron temperature $T_e$, the relation between mean energy and electron temperature being:

$$\bar{\epsilon} = 3kT_e/2.$$

Mierdel summarizes the results of these measurements in the rare gases.[15] Fig. 5 (circles) shows the experimental results in Ne which are in good agreement with the calculation on the basis of the Maxwell distribution.

The difference between the mean energy in the case with no interaction (crosses) and with large interaction (circles) is seen to be large.[28]

## III. Electron Mobility, Ionization and Excitation in a Small Current Through a Gas

We shall now discuss some properties of an infinitely small electron current in a gas in a homogeneous electric field. On the one hand we will try to explain these properties on the basis of the elementary processes between electrons and gas atoms, together with the velocity distribution of the electrons; on the other hand we shall use the properties of a small electron current to explain some properties of self-sustained gas discharges.

The properties of a small electron current

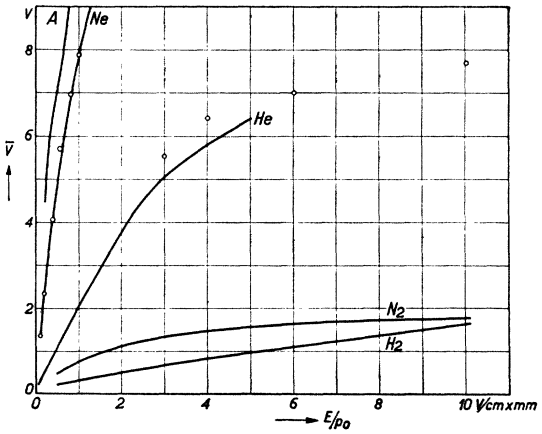


FIG. 6. Mean energy as a function of $E/p_0$ for different gases at small currents. The curves are according to Townsend, the circles are values calculated by Druyvesteyn (Ne) and Smit (He).

[24] W. P. Allis and H. W. Allen, Phys. Rev. **52**, 703 and 707 (1937).
[25] G. Mierdel, Wiss. Ver. Siem. Konz. **17**, 71 (1938).
[26] J. S. Townsend, *Motion of Electrons in Gases* (1925); J. Frank. Inst. **200**, 563 (1925).
[27] I. Langmuir and H. Mott-Smith, Gen. Elec. Rev. **27**, 449, 538, 616, 762 and 810 (1924); R. Seeliger and R. Hirchert, Ann. d. Physik **11**, 817 (1931); M. J. Druyvesteyn, Zeits. f. Physik **81**, 571 (1933).
[28] While in Ne and A the difference between the mean energy for small and large current is found to be large, in He this difference is small.

which will be treated in this section are: the mobility of the electrons, the ionization coefficient of Townsend for electrons and the distribution of the energy losses of the electrons between elastic losses, excitation, ionization and speeding up of the electrons (energy balance). All these quantities are functions of $E/p_0$ as $E/p_0$ is proportional to the potential difference per mean free path of the electrons.

A number of similar quantities, as the mobility of positive and negative ions, the formation of negative ions and the recombination of ions will not be discussed here. For these subjects we must refer to the literature.[29]

## §9. Mobility of the electrons

*Experiment.*—A number of authors have measured the drift velocity of electrons in a gas in a homogeneous electric field. The drift velocity ($w$) is a function of $E/p_0$, the relation between drift velocity and mobility ($\mu$) being:

$$w=\mu E=\mu p_0(E/p_0)=f(E/p_0). \tag{24}$$

In general the results obtained in different measurements show a rather good agreement and therefore we shall discuss only those of Nielsen and Bradbury[30] as being the latest and probably the most accurate. Fig. 7 shows the drift velocity as a function of $E/p_0$ in a number of gases; the dotted line gives a theoretical line to be discussed later on. It is seen that in a large domain of $E/p_0$ values, the drift velocity is given by an almost linear function of $E/p_0$ not passing, however, through the origin.[31]

*Theory.*—The mobility of electrons may be calculated on the basis of the energy distribution as discussed in Section II. If the distribution function of the electrons is known and if the mean free path $\lambda$ is a constant, we find the drift

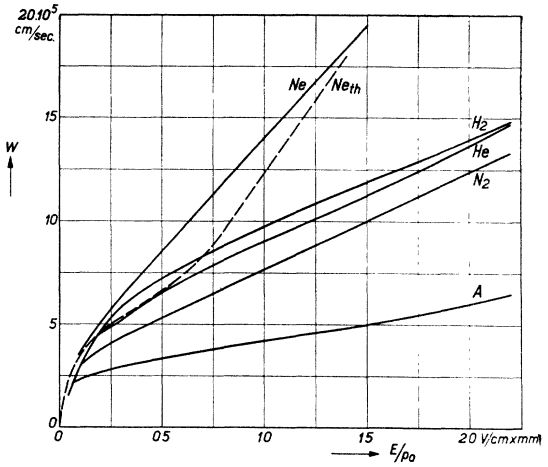


FIG. 7. Drift velocity ($w$) as a function of $E/p_0$ in He, Ne, A, $H_2$, $N_2$. The calculated values are given by $Ne_{th}$ (- - -).

velocity $w$ according to Eq. (6) with:

$$w=-\frac{\int_0^\infty J(\epsilon)d\epsilon}{\int_0^\infty \rho(\epsilon)d\epsilon}=\frac{2\lambda eE}{3(2m)^{\frac{1}{2}}}\frac{\int_0^\infty \frac{\rho(\epsilon)}{\epsilon^{\frac{1}{2}}}d\epsilon}{\int_0^\infty \rho(\epsilon)d\epsilon} \tag{25}$$

We can distinguish different cases.

(*1*) If the energy distribution of the electrons is a Maxwell distribution with mean energy $\bar{\epsilon}$, then $w$ is given by:

$$w=\frac{2\lambda eE}{(3\pi m\bar{\epsilon})^{\frac{1}{2}}}=0.655\frac{\lambda eE}{(m\bar{\epsilon})^{\frac{1}{2}}}, \tag{26}$$

$$w=2.75\times10^7\frac{\lambda p_0}{\bar{V}^{\frac{1}{2}}}\frac{E}{p_0}\ \text{(practical units)}. \tag{27}$$

In the last formula $w$ is in cm/sec., $\bar{V}$ in volts ($\bar{V}=\bar{\epsilon}/e$) and $E/p_0$ in $V/\text{cm}\cdot\text{mm}$.

A more rigorous calculation gives in Eqs. (26) and (27) a somewhat different factor; namely, in Eq. (26) 0.58 and in Eq. (27) $2.43\times10^7$. In this form Eq. (26) becomes the same as the well-known mobility equation of Langevin.[32]

The Maxwell distribution may be caused by different factors. It occurs for example at such low values of $E/p_0$, that the energy of the electrons is equal to the energy of the gas atoms

[29] K. Przibram, *Handbuch der Physik*, XXII[1], second edition (1933), p. 343; A. M. Tyndall, *The Mobility of Positive Ions in Gases* (Cambridge, 1938).

[30] N. E. Bradbury and R. A. Nielsen, Phys. Rev. **49**, 388 (1936); **50**, 950 (1936); **51**, 69 (1937).

[31] While in §8 we discussed the mean electron energy as a function of $E/p_0$ both for a small and for a large current density, we discuss the experiments on the mobility of the electrons only for a small current density. The few measurements of $\mu$ for the case of a large current density will be discussed in connection with the positive column (§54).

[32] See L. B. Loeb, *Kinetic Theory of Gases* (1927).

($\bar{\epsilon}=3kT/2$). In this region of $E/p_0$ values, however, (for Ne, $E/p_0<4\times10^{-3}$) hardly any measurements on $w$ are available. In the positive column often a Maxwell distribution of the electrons will exist. In this case a correction to Eq. (26) must sometimes be applied, as the scattering of the electrons by the positive ions cannot always be neglected in comparison with scattering by the gas atoms. We can, however, still use (26) if we take instead of $\lambda$ the value $\lambda''$:

$$\lambda''=\left[p_0\Sigma Q+\frac{\pi ne^4}{4k^2T^2}\ln\frac{2.7kT}{n_p^{\frac{1}{3}}e^2}\right]^{-1}, \qquad (28)$$

$n_p$ being the concentration of the positive ions;[33] for $Q$ see §5.

(2) If the energy of the electrons is much above the energy of the atoms, but still so low that only elastic collisions occur, Eq. (8) can be used. We find:

$$w=0.59\lambda eE/(m\bar{\epsilon})^{\frac{1}{2}}. \qquad (29)$$

This equation differs but little from (26), the formula for a Maxwell distribution. In the case of distribution (8) the value of (22) for $\bar{\epsilon}$ may be used and we find the drift velocity proportional to $(E/p_0)^{\frac{1}{2}}$:

$$w=\frac{(6\delta)^{\frac{1}{2}}(\pi\lambda eE/m)^{\frac{1}{2}}}{3\Gamma(3/4)}, \qquad (30)$$

$$w=3.2\times10^7\,\delta^{\frac{1}{2}}(\lambda p_0\cdot E/p_0)^{\frac{1}{2}}\ \text{(practical units)}. \qquad (31)$$

In the last formula $E$ is in $V$/cm, $w$ in cm/sec. Eq. (30) is analogous to a formula of Compton.[34]

(3) If inelastic collisions also occur, a more complicated distribution function is obtained and $w$ can be found by numerical integration. The dotted line for Ne in Fig. 7 was obtained in this way.[35] If $\lambda$ varies with the energy, the equations become more complicated; for these cases we refer to Allen,[24] who discussed the mobility in the rare gases.[36]

[33] S. D. Gvosdover, Physik. Zeits. Sowjetunion **12**, 164 (1937); M. J. Druyvesteyn, Physica **5**, 561 (1938); W. Elenbaas, Physica **5**, 568 (1938).

[34] K. T. Compton, Phys. Rev. **22**, 333 (1923); K. T. Compton and I. Langmuir, Rev. Mod. Phys. **2**, 123 (1930).

[35] M. J. Druyvesteyn, Physica **4**, 464 (1937).

[36] For He Smit (unpublished, see reference 19) calculated $\mu p_0$ to be almost constant $6.1\times10^5$ for $4<E/p_0<10$ $V$/cm·mm. Nielsen found for $E/p_0=4$ experimentally $\mu p_0=7.9\times10^5$.

## §10. Coefficients of ionization, excitation, etc.

The *efficiencies* of §5 define the number of processes of a certain kind per cm, measured along the zigzag path traveled by an electron of a given energy. For several calculations it is necessary to know also the mean number of processes per volt potential difference passed by the electron. This number we call the *coefficient* for the process in question. When the energy distribution has become steady [$\rho_2(\epsilon)$, §6] we may define the following coefficients: $\eta$ = ionization coefficient (volt$^{-1}$) $=\alpha/E$, where $\alpha$ (cm$^{-1}$) is the ionization coefficient of Townsend.[37] For in the case of gas mixtures, where the ionization by metastable molecules is important (§12), $\eta$ may be divided into: $\eta_e$ = ionization coefficient (per volt) for the direct ionization by electrons and $\eta_h$ = ionization coefficient (per volt) due to the indirect ionization by metastable molecules. $\xi_{h1}$, $\xi_{h2}$ = excitation coefficient for the 1st, 2nd, etc. excited electron level; in gas mixtures only those excitations are considered which do not give rise to ions; $\xi_{v1}$, $\xi_{v2}$ = excitation coefficient (per volt) for the 1st, 2nd, etc. excited vibrational level; $K$ = coefficient (per volt) for the total number of collisions.[38]

## §11. Determination of the ionization coefficient $\eta$

For the determination of $\eta$ usually an apparatus containing two parallel plane electrodes is used, where photoelectrically an electron current $i_0$ is liberated from the cathode. The electron current $i$ increases in the direction of the anode and, according to the definition of $\eta$, the increase of $i$ per potential difference $dU$ is

$$di=\eta i\,dU. \qquad (32)$$

When the ions, excited atoms, and radiation quanta due to the electron collisions, did not

[37] The reasons we use here the coefficient per volt and not the coefficient per cm are the following: (1) $\eta$ itself is a function of $E/p_0$, while in the case of $\alpha$, $\alpha/p_0$ is a function of $E/p_0$; (2) the numerical values of $\eta$ to be considered lie in a much smaller range than those of $\alpha/p_0$; (3) the relation between $\eta$ and the breakdown potential $V_B$ is simpler than that between $\alpha/p_0$ and $V_B$.

[38] As the probability for the formation of a negative ion in a collision is usually very small, we shall neglect this process; it may become important for the breakdown potential of air and oxygen at high pressures, cf. K. Masch, Arch. f. Elektrotech. **26**, 587 (1932); Zeits. f. Physik **79**, 672 (1932); F. M. Penning, Ned. T. Natuurk. **5**, 33 (1938).

give rise to new electrons, the anode current $i$ would be equal to:

$$i=i_0e^{\eta(V-V_0)}, \tag{33}$$

where $V$ is the anode potential. The constant value $V_0$ in this equation is due to the circumstance that the energy distribution becomes steady only at a certain distance from the cathode. In reality, however, secondary processes are not to be neglected, and therefore the value of $i$ increases more rapidly than given by formula (33). This has been interpreted as due to extra ionization in the gas or as an extra liberation of electrons from the cathode. As will be discussed in §19, in most cases the last-mentioned mechanism is the more important. So, for the present, the secondary processes are taken into account by a coefficient $\gamma$, giving the number of extra electrons liberated from the cathode per ion formed between the electrodes. So (33) changes into

$$i=i_0\frac{e^{\eta(V-V_0)}}{1-\gamma e^{\eta(V-V_0)}}. \tag{34}$$

In the method for the determination of $\eta$, as used originally by Townsend,[2] the anode current $i$ is measured as a function of the anode potential $V$ at constant value of $E=V/d$, so that the

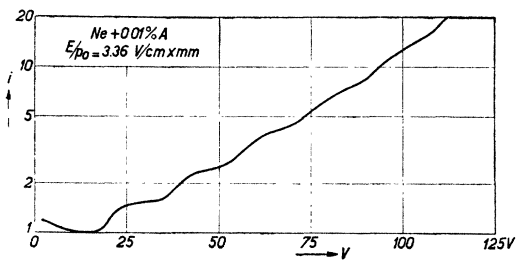


FIG. 8. Steplike increase of $i$ as a function of $V$ at constant $E/p_0$ due to ionization of A atoms by metastable Ne atoms.

anode distance $d$ has to be increased proportionally with $V$. Now for $\eta$, $\gamma$ and $V_0$ those values are chosen which give the best agreement between Eq. (34) and the experimental values.

For very small values as well as for very large values of $E/p_0$ the determination of $\eta$ becomes difficult. At low values the curve for $i$ as a function of $V$ shows, especially for some rare gas mixtures, more or less steplike increases, due to the more or less sudden increase of ionization at definite values of $V$. As long as the electron energy is $<eV_i$, the electrons are not able to ionize gas molecules and Eq. (34) certainly does not hold, but also for $V>V_i$ up to rather large values of $V$ deviations occur. Fig. 8 gives an example[39] where the steplike character of the curve is due to ionization of A atoms by metastable Ne atoms (§12). In this case the value of $\eta$ is calculated from the values of the current $i$ at higher values of $V$ with the aid of a set of preliminary values of $\eta$, $\gamma$, and $V_0$. In Fig. 8 Eq. (34) holds only for $V>100$ volts. For lower values of $V$, $\eta$ becomes not only a function of $E/p_0$ but also of $V$.

For larger values of $E/p_0$ the steplike character of the curve $i=f(V)$ disappears more and more, and this difficulty for the determination of $\eta$ no longer exists. For values of $E/p_0$, however, much larger than the value where the breakdown potential has its minimum (§22), it is impossible to increase the electrode distance sufficiently to get a stationary energy distribution of the electrons, as breakdown occurs at small electrode distances. Therefore also at very high values of $E/p_0$, the experimental value of $\eta$ is not only a function of $E/p_0$ but also of $V$.

## §12. Values of $\eta$ for the rare gases

In the measurements of $\eta$ for the rare gases, the purity of the gas is of much importance. Excitation of rare gas atoms ($A$) often brings them into a metastable state $A_m$ with a high energy $eV_m$ and a long lifetime. When now admixed gas molecules ($A'$) have an ionization potential $V_i'$, satisfying the conditions

$$V_i'<V_m, \tag{35}$$

then the metastable atoms of the main gas are able to ionize the molecules of the admixture according to:

$$A_m+A'\rightarrow A+A'^{+}+\text{electrons}. \tag{35a}$$

The probability for this process is so large that very small admixtures may have considerable influence (see Fig. 9 for the case of Ne-A). Consequently older measurements where for example Hg vapor was present in the rare gas, give values of $\eta$ which may be orders of magnitude in error.

[39] A. A. Kruithof and F. M. Penning, Physica **4**, 430 (1937).

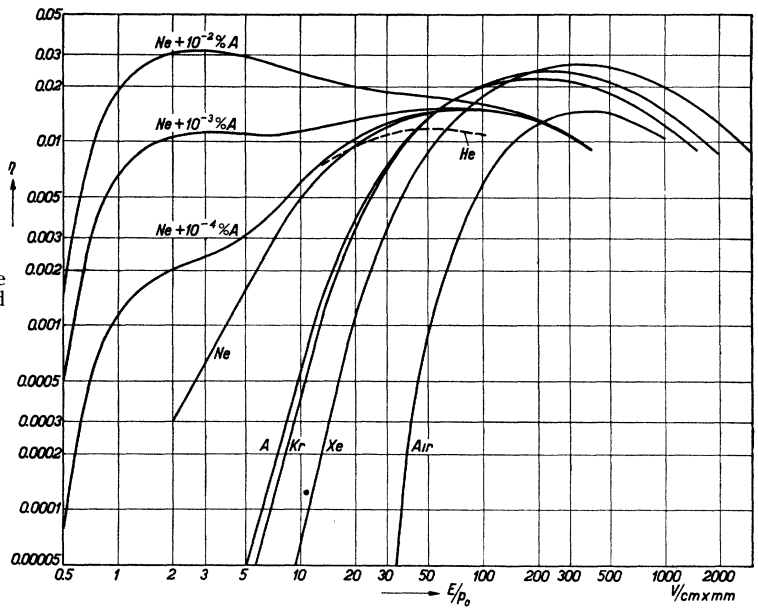


FIG. 9. Values of $\eta$ for the rare gases, some Ne—A mixtures and air.

This phenomenon of ionization by metastable atoms is also of importance in the study of pure gases, as a small admixture, obeying condition (35) may be used as indicator for the concentration of metastable atoms.[40]

Table I and Fig. 9[41] show the results for $\eta$ in the rare gases, carefully purified from admixtures satisfying condition (35), and for some Ne-A mixtures. For comparison also the $\eta$-values of air have been plotted. The values of $V_0$ to be used for the rare gases[42] in Eq. (33) and for the calculation of $V_B$ are shown in Fig. 10. It should be emphasized that, when the complication for small and large values of $E/p_0$ mentioned above, are left out of account, the values of $\eta$ are, also for the Ne-A mixtures, a function of $E/p_0$ only,

TABLE I. *Values of 100 $\eta$ for the rare gases as a function of $E/p_0$ ($v/cm \times mm$).*

| $E/p_0$ | He | Ne | A | Kr | Xe | Ne+ 0.01%A |
|---|---|---|---|---|---|---|
| 0.5 | | | | | | 0.155 |
| 0.6 | | | | | | 0.46 |
| 0.7 | | | | | | 0.88 |
| 0.8 | | | | | | 1.28 |
| 0.9 | | | | | | 1.65 |
| 1.0 | | | | | | 1.94 |
| 1.2 | | | | | | 2.38 |
| 1.5 | | | | | | 2.79 |
| 2.0 | | 0.031 | | | | 3.07 |
| 2.5 | | 0.046 | | | | 3.15 |
| 3.0 | | 0.064 | | | | 3.14 |
| 4 | | 0.11 | | | | 3.05 |
| 5 | | 0.16 | 0.005 | 0.003 | | 2.91 |
| 6 | | 0.22 | 0.009 | 0.006 | 0.001 | 2.78 |
| 8 | | 0.36 | 0.026 | 0.018 | 0.002 | 2.56 |
| 10 | | 0.50 | 0.057 | 0.041 | 0.006 | 2.40 |
| 12 | | 0.62 | 0.11 | 0.079 | 0.014 | 2.27 |
| 15 | 0.81 | 0.77 | 0.21 | 0.16 | 0.036 | 2.13 |
| 20 | 0.95 | 0.96 | 0.38 | 0.34 | 0.11 | 2.01 |
| 25 | 1.03 | 1.10 | 0.56 | 0.54 | 0.21 | 1.93 |
| 30 | 1.10 | 1.20 | 0.75 | 0.71 | 0.34 | 1.88 |
| 40 | 1.16 | 1.35 | 1.08 | 1.04 | 0.62 | 1.81 |
| 50 | 1.18 | 1.42 | 1.35 | 1.32 | 0.88 | 1.75 |
| 70 | 1.16 | 1.48 | 1.71 | 1.71 | 1.32 | 1.66 |
| 100 | 1.10 | 1.49 | 1.98 | 2.04 | 1.80 | 1.53 |
| 150 | | 1.40 | 2.17 | 2.32 | 2.22 | 1.46 |
| 200 | | 1.30 | 2.22 | 2.42 | 2.47 | 1.34 |
| 250 | | 1.18 | 2.19 | 2.41 | 2.60 | 1.22 |
| 300 | | 1.07 | 2.13 | 2.36 | 2.64 | 1.11 |
| 400 | | 0.89 | 1.98 | 2.24 | 2.60 | 0.89 |
| 500 | | | 1.81 | 2.09 | 2.50 | |
| 700 | | | 1.54 | 1.82 | 2.30 | |
| 1000 | | | 1.23 | 1.47 | 1.98 | |
| 1500 | | | 0.90 | 1.08 | 1.57 | |
| 2000 | | | | 0.84 | 1.27 | |
| 2500 | | | | | 1.05 | |

[40] E. W. Pike, Phys. Rev. **49**, 513 (1936).

[41] J. S. Townsend and S. P. McCallum, Phil. Mag. **17**, 678 (1934) (He). According to our definition of $\eta$ Townsend's values of $(\alpha-\beta)/E$ were used. A. A. Kruithof and F. M. Penning, Physica **3**, 515 (1936) (A); **4**, 430 (1937) (Ne, Ne—A mixtures); A. A. Kruithof, Physica, in press, (Kr and Xe); K. Masch, Arch. f. Elektrotech. **6**, 587 (1932), Zeits. f. Physik **79**, 672 (1932) (air); F. H. Sanders, Phys. Rev. **41**, 667 (1932); **44**, 1020 (1932); **45**, 346 (1934) (air). Cf. also J. S. Townsend and S. P. McCallum, Phil. Mag. **6**, 857 (1928) (Ne); I. I. Glotov, Physik. Zeits. Sowjetunion **12**, 256 (1937) (Ne—A mixtures). For the experimental method see also A. Gosseries, Physica **6**, 458 (1939). For Figs. 9, 10 and 19 we are indebted to Dr. A. A. Kruithof.

[42] In the measurements for air no $V_0$ has been taken into account. For Ne with A admixtures $\leqq$0.01 percent the value of $V_0$ for pure Ne has been assumed.

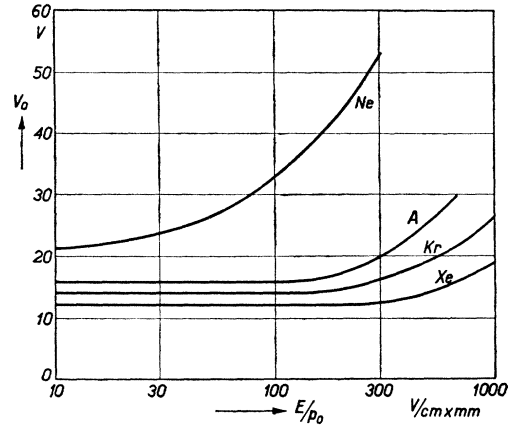


FIG. 10. Values of $V_0$ in Eq. (33) for the rare gases.

This is confirmed by the experimental results.[43]

For pure gases and air the decrease of $\eta$ with decreasing $E/p_0$ is due to the increasing ratio of excitations to ionizations, the decrease at the other side of the maximum value of $\eta$ to the increase of the electron energy, which is delivered ultimately to the anode. (See further §14 and §15.)

In mixtures of Ne with a small amount of A the excitation energy is partly used for the ionization of A according to the reaction (35a). Fig. 9 shows that this effect may increase the value of $\eta$ very considerably. We will discuss it somewhat more in detail.

We denote the admixture in volume parts by $a$ (equivalent to $100a$ percent). For $a<10^{-4}$ the direct collisions between electrons and atoms are not influenced by the admixture, therefore $\eta_e$ in the mixture[44] is, with a good approximation, equal to the value of $\eta$ in pure Ne at the same value of $E/p_0$. Hence, when $\eta$ is known for the admixture, also $\eta_h=\eta-\eta_e$ is known. The results for $\eta_h$ show that the proportion between $\eta(a_1)$ and $\eta(a_2)$ for definite values $a_1$ and $a_2$ of the admixture, is a function of $a_1$ and $a_2$ only and does not depend upon $E/p_0$. Consequently, when for $a_2$ a constant value e.g. $10^{-4}$ is taken, $\eta_h(a)$ is equal to

$$\eta_h(a)=b(a)\eta_h(10^{-4}), \qquad (36)$$

where $b(a)$ is independent of $E/p_0$. This relation can be deduced also theoretically. For $a<10^{-4}$ the values of $\eta_h(a)$ may be calculated from the values of $b(a)$ given in Table II and those of $\eta_h(10^{-4})=\eta(10^{-4})-\eta(0)$ from Table I.

From the values of $\eta$ for Ne-A mixtures with $a<10^{-4}$ estimates may be made concerning some elementary processes,[45] *viz.* the probability $q$ that an excited Ne atom passes into a metastable state and the ratio between the probability of the destruction of a metastable Ne atom by collision with a Ne atom on the one hand and with an A atom on the other (provided that in the last process the A atom is ionized). For the last-mentioned proportion the value $2.4\times10^{-5}$ is found; in connection with the known probability[46] for the destruction by a Ne collision ($1.6\times10^{-5}$) and the known cross section of an A atom, for the probability of destruction by an A collision (in which process the A atom is ionized) a value $\frac{1}{2}$ is calculated. The values of $q$ are given in Table III.

A survey of $\eta$ for the whole mixture-range between pure Ne and pure A is given in Fig. 11, where constant values of $\eta$ are plotted in the form of niveaulines. For low values of $E/p_0$, according to Table III the value of $q$ is nearly 1; moreover, the probability that a metastable Ne atom ionizes an A atom also may become nearly 1. In this way the excitation losses, which cause the low value of $\eta$ in pure gases, may be reduced very considerably in mixtures. The largest value of $\eta$ (0.037) for Ne-A mixtures, being at the same time the largest value of $\eta$ found up to now in any gas or gas mixture, occurs at $a=10^{-3}$, $E/p_0=3$.

TABLE II. *Value of b in Eq. (36).*

| $a$: | $3\times10^{-7}$ | $10^{-6}$ | $3\times10^{-6}$ | $10^{-5}$ | $3\times10^{-5}$ | $10^{-4}$ |
|---|---|---|---|---|---|---|
| $b(a)$: | 0.020 | 0.056 | 0.14 | 0.33 | 0.65 | 1.00 |

TABLE III. *Value of the probability q that a* Ne *atom, when excited, comes into a metastable state.*

| $E/p_0$: | 2 | 5 | 10 | 15 | 20 v/cm·mm |
|---|---|---|---|---|---|
| $q$ : | 0.92 | 0.80 | 0.60 | 0.44 | 0.33 |

[43] The deviations found by R. Hellmann, Zeits. f. Physik **91**, 556 (1934) ($N_2$) for low values of $p_0$ are perhaps due to the small values of $V$ used in this region (see the end of §11).

[44] For the definition of $\eta_e$ and $\eta_h$ see §10.

[45] A. A. Kruithof and M. J. Druyvesteyn, Physica **4**, 450 (1937).

[46] W. de Groot and F. M. Penning, *Handbuch der Physik*, Vol. XXIII[1] (2nd ed., 1933), p. 167.

Figure 9 shows that for some admixtures (e.g. $a=10^{-5}$) the curve for $\eta$ has two maxima. The maximum at $E/p_0=70$ is due to the direct ionization of Ne atoms by electrons, the maximum at $E/p_0=3$ to the ionization of A atoms by metastable Ne atoms. The fall of $\eta$ for $E/p_0<2$ may be ascribed to energy losses in elastic collisions with the atoms. These features may be understood on the basis of the electron energy balance as discussed in §15.

## §13. Values of $\eta$ for diatomic gases

The behavior of $\eta$ for molecular gases has been reviewed recently in this journal by Loeb.[3] It has been found by his co-workers that also in these gases small admixtures may have a large influence on the value of $\eta$. Bowls[47] found for $400<E/p_0<800$ an increase of $\eta$ of about 15 percent when Hg vapor was added to pure $N_2$, and of about 30 percent when sodium was introduced into the discharge tube. Still larger variations of $\eta$ by this kind of admixtures were found by Hale[47] in the case of $H_2$. The influence of Na is, according to Hale, due to some unknown volatile Na compound; in the case of Hg the mechanism has not yet been cleared up.

The values of $\eta$ for air, plotted by way of example in Fig. 9 were determined by Masch[41] and by Sanders.[41] The agreement between their results is good. It is not known, whether in this case also the Hg vapor present has influenced the values of $\eta$. In connection with the circumstance that these values of $\eta$ for dry air are often used in combination with breakdown potentials for moist air, it is of importance to state that Kruithof[48] found no perceptible difference for $35<E/p_0<90$ between the $\eta$-values for dry and those for moist air.

## §14. Calculation of $\eta$

If the energy distribution function of the electrons $\rho_2(\epsilon)$ is known (for simplicity we shall omit the index 2 and write $\rho(\epsilon)$), the number of ionizations per cm³ per sec. is given by

$$J_i=\int_{\epsilon_i}^{\infty}\rho(\epsilon)\left(\frac{2\epsilon}{m}\right)^{\frac{1}{2}}\frac{K_i(\epsilon)}{\lambda}d\epsilon. \tag{37}$$

$K_i(\epsilon)$ is the ionization probability. For a pure gas $\eta$ can be found from (37) and (25) as

$$\eta=\frac{J_i}{E\int_0^{\infty}J(\epsilon)d\epsilon}=\frac{3}{\lambda^2e^2E^2}\frac{\int_{\epsilon_i}^{\infty}\rho\epsilon^{\frac{1}{2}}K_i(\epsilon)d\epsilon}{\int_0^{\infty}\frac{\rho}{\epsilon^{\frac{1}{2}}}d\epsilon}. \tag{38}$$

The value of $\eta$ also can be calculated from the energy balance equation (§15). For the case of a monatomic gas in the region, where the elastic

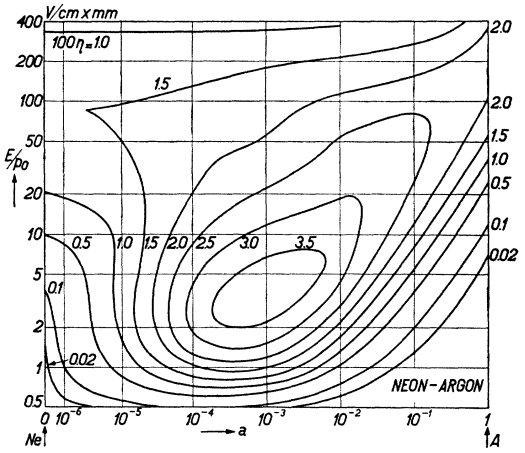


FIG. 11. Curves for constant $\eta$ in Ne−A mixtures ($100\eta$ being given as parameter at each curve) as a function of $E/p_0$ and "$a$" (volume parts A). In order that the points for "$a$"$=0$ could be plotted, a linear scale has been taken for log $(10^{-6}+a)$. (Kruithof and Penning.)

losses can be neglected, Eq. (45) which will be discussed later reduces to[49]

$$\eta=\frac{1}{1+r+(\bar{\epsilon}/\epsilon_i)}\frac{1}{V_i}, \tag{39}$$

where $r$ is the ratio of the energy loss by excitation to that by ionization ($\Sigma_n\xi_{hn}V_{hn}=r\eta V_i$).

For Ne the value of $\eta$ was calculated numerically with (39) for values of $E/p_0$ between 5 and 30 v/cm mm and found in good agreement with the experiments (see Fig. 12, circles).

If we use the approximate distribution function of Fig. 13, namely: $\rho$ constant for $\epsilon<\epsilon_h$ and

[47] W. E. Bowls, Phys. Rev. **53**, 293 (1938); cf. D. Q. Posin, Phys. Rev. **50**, 650 (1936) ($N_2$). D. H. Hale, Phys. Rev. **55**, 815 and 1121 (1939) ($H_2$).

[48] Unpublished.

[49] If the distribution function is known only approximately for $\epsilon<\epsilon_h$ we can better use (39) than (38) as in (39) this part of the distribution function is only needed to evaluate $\bar{\epsilon}$, while in (38) it gives the main term in the denominator.

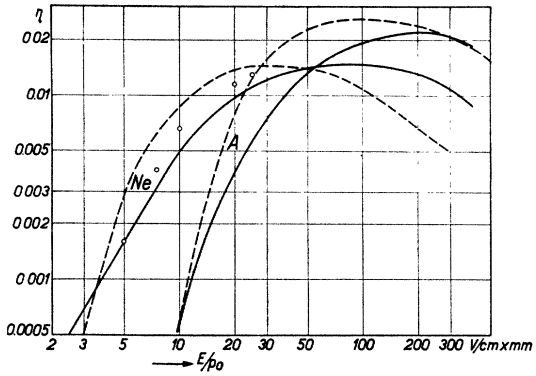


FIG. 12. $\eta$ as a function of $E/p_0$ in Ne and A. Experimental (—), simplified calculation (- - -), and more rigorous calculation for Ne (circles).

for $\epsilon > \epsilon_h$, $\rho$ given by Eq. (19); and if we take almost linear functions for $K_h$ and $K_i$:[50]

$$K_h = a_h \frac{\epsilon - \epsilon_h}{(\epsilon\epsilon_h)^{\frac{1}{2}}}, \quad K_i = a_i \frac{\epsilon - \epsilon_i}{(\epsilon\epsilon_i)^{\frac{1}{2}}}, \tag{40}$$

we get

$$r = \frac{a_h}{a_i}\left(\frac{\epsilon_h}{\epsilon_i}\right)^{\frac{1}{2}} \exp\left[\frac{(3k)^{\frac{1}{2}}}{\lambda eE}(\epsilon_h - \epsilon_i)\right] = \frac{a_h}{a_i}\left(\frac{\epsilon_h}{\epsilon_i}\right)^{\frac{1}{2}} \exp\left[\frac{Cp_0}{E}\right], \tag{41}$$

$$\bar{\epsilon} = \frac{\epsilon_h}{2[1+\lambda eE/(3k)^{\frac{1}{2}}\epsilon_h]}\left[1 + \frac{2\lambda eE}{(3k)^{\frac{1}{2}}\epsilon_h} + \frac{2\lambda^2 e^2 E^2}{3k\epsilon_h^2}\right]. \tag{42}$$

By substituting Eqs. (41) and (42) in Eq. (39) we get a formula for $\eta$ which, for small values of $E/p_0$ ($<20$ v/cm·mm in the case of Ne), reduces to

$$\eta = (a_i/a_h V_i) e^{-Cp_0/E}, \tag{43}$$

where the exponent is the same as in the original formula for $\alpha/p$ derived by Townsend. Fig. 12 shows $\eta$ in Ne and A[51] according to Eqs. (39)–(42) (dashed lines), together with the experimental values. Table IV gives the values of $\epsilon_2$ in Eq. (19), $\lambda p_0$, $K(\epsilon_2)$, $C = (3K)^{\frac{1}{2}}(\epsilon_i - \epsilon_h)/\lambda p_0$ and $a_h/a_i$, as they were used in these calculations (see Fig. 3).

[50] The $\epsilon^{\frac{1}{2}}$ in the denominator of (40) is chosen to facilitate the integrations. The approximations used are of a different kind for (19) (excitation probability a constant) than for (40) (probability of excitation and ionization, almost linear functions of $\epsilon$).

[51] The calculated He curve is close to the Ne curve.

It is seen that the form of the curves, as well as the difference between Ne and A, are well described by the simplified calculation of Eqs. (39)–(42). The decrease of $\eta$ for small $E/p_0$ values is due to the increase of $r$ (increased excitation) while the decrease of $\eta$ for large $E/p_0$ values is due to the increase of $\bar{\epsilon}$. The difference between Ne and A is due to the different $C$ values (for small $E/p_0$) and the different $a_h/a_i$ values (in the $\eta$-maximum). The differences between the experimental and the calculated results is partly due to the simplification of Eq. (19) (see the circles in Fig. 12).

In mixtures of Ne with a small A percentage ($a < 10^{-4}$) an equation for $\eta$ can also be derived from the electronic velocity distribution.[45] Here also the excited Ne atoms give rise to $A^+$ ions, in the way indicated in §12. If the elastic energy losses are taken into account the form of the $\eta$-curve for $E/p_0 < 5$ v/cm·mm resembles the experimental curves of Fig. 9.

Emeléus, Lunt and Meek[52] calculated $\eta$ in another way. They assumed *a priori* the form of the distribution function, e.g. the Maxwell distribution. By using the experimental values of the mean energy $\bar{\epsilon}$ and the drift velocity $w$ of the electrons they found $\eta$ by integration. While the values in A differ very much from the experimental results, in $H_2$ and air the agreement is good, showing that in the latter case the distribution cannot differ much from a Maxwell distribution.

## §15. Electron energy balance

When the energy distribution of the electrons is steady, the energy $idU$, received by the electron current $i$ from the electric field in passing a potential difference $dU$, is equal to the energy lost by the electrons in collisions plus the energy $\bar{V}di = \bar{V}\eta i dU$ which the newly formed electrons must receive in order that the mean energy $e\bar{V}$

TABLE IV. *Values used for the calculation of* $\eta$.

| | $\epsilon_2/e$ | $\lambda p_0$ | $K(\epsilon_2)$ | $C$ | $a_h/a_i$ |
|---|---|---|---|---|---|
| He | 22.9 v | 0.10 cm | 0.024 | 12.6 | 1.33 |
| Ne | 19.8 | 0.082 | 0.016 | 13.4 | 1.07 |
| A | 14.4 | 0.0157 | 0.015 | 59 | 0.46 |

[52] K. G. Emeléus, R. W. Lunt and C. A. Meek, Proc. Roy. Soc. **A156**, 394 (1936).

of the electrons remains the same. In connection with the definitions of §10 this leads to the energy balance equation:

$$i\,dU = i\eta V_i dU + i\Sigma_n \xi_{hn} V_{hn} dU + i\Sigma_n \xi_{vn} V_{vn} dU + kf\bar{V} dU + \eta \bar{V} dU \quad (44)$$

or

$$1 = \eta V_i + \Sigma_n \xi_{hn} V_{hn} + \Sigma_n \xi_{vn} V_{vn} + kf\bar{V} + \eta\bar{V}, \quad (45)$$

where $f$ is the fraction of the mean energy lost per elastic collision. The right-hand side of Eq. (45) gives successively the fraction of the electron energy used for ionization, excitation of electronic levels, excitation of vibrational levels, elastic collisions and increase of the kinetic energy of the electron stream.

In §14 this equation with $\Sigma_n \xi_{vn} V_{vn} = 0$ and $kf\bar{V} = 0$ has been used to calculate the value of $\eta$ from the fundamental processes. Here we start from experimental values of $\eta$ as being more reliable, and use Eq. (45) to calculate more precise values for the other energy losses. $V_i$, $V_{hn}$, $V_{vn}$ are known for many gases; $f$ is taken as $8m/3M$ (§3); further we use the experimental values of $\eta$ (§12 and §13) and $\bar{V}$ (§8). In this way, for the rare gases $\Sigma_n \xi_{hn} V_{hn}$ may be calculated from Eq. (45) and so all the fractional energy losses in Eq. (45) are known. For $N_2$ and $H_2$ also $\Sigma_n \xi_{vn} V_{vn}$ may be found by combining Townsend's energy loss determinations[26] with those for the efficiency $Q_{vi}$ of the vibrational excitation according to Harries[53] and Ramien.[18] The results of these calculations for Ne-A mixtures, $N_2$ and $H_2$ are given by Penning,[54] Fig. 14 gives a few examples.

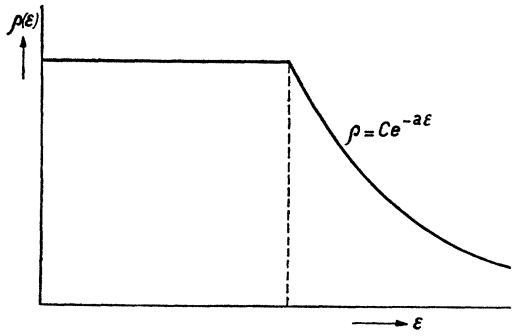


FIG. 13. Simplified distribution function of the electrons.

[53] W. Harries, Zeits. f. Physik **42**, 26 (1927); W. Harries and G. Hertz, Zeits. f. Physik **46**, 177 (1927).
[54] F. M. Penning, Physica **5**, 286 (1938).

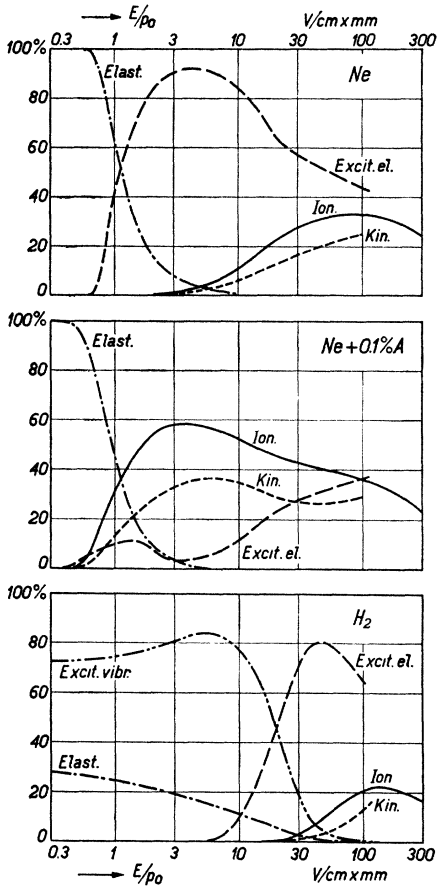


FIG. 14. Fraction of the total energy received by the electrons from the electric field which is spent in elastic collisions (Elast.), excitation of electronic level (Excit. el.), excitation of vibrational levels (Excit. vib.), ionization (Ion.) and increase of kinetic energy of the electrons (Kin.).

As the mean fractional energy loss due to ionization is $\eta V_i$, the maximum for the curves "Ion" in Fig. 14 corresponds to the maximum of $\eta$ in the curves of Fig. 9. The figures show that this maximum is a consequence of the following circumstances. For pure gases the decrease of $\eta$ at low values of $E/p_0$ is due to the excitation of electronic energy levels, as also in the case of $H_2$ and $N_2$. For Ne+0.1 percent A, however, it is due to the elastic energy losses, as here nearly all the excitation energy is transformed into ionization-energy. For the high values of $E/p_0$, the decrease of $\eta$ is, in the case of pure gases, a consequence of the increase of the kinetic energy of the electron stream, this energy being delivered ultimately to the anode of the discharge tube.

For Ne+0.1 percent A the decrease with increasing $E/p_0$ is due to the excitation of higher levels in Ne, which are less efficient in the production of $A^+$ ions.

The maximum of $\eta V_i$ amounts in pure gases to 35 percent. In Ne-A mixtures it increases with increasing A percentage until a maximum of 60 percent is reached in Ne+0.1 percent A. Fig. 14 shows that in this case the excitation energy losses are practically zero, the rest of the energy being used for the acceleration of the newly formed electrons of the stream.

The mean fractional energy loss in the excitation of electronic levels in $H_2$ is roughly in accordance with the results of Lunt and Meek,[55] who calculated them from the elementary processes. The loss due to excitation of vibrational levels may become as high as 80–90 percent in $H_2$ (Fig. 14) as well as in $N_2$.[56] It should be remarked, however, that the course of the vibrational excitation curve for each of these gases depends wholly on the correctness of one, rather indirect, determination of the probability for the vibrational excitations.

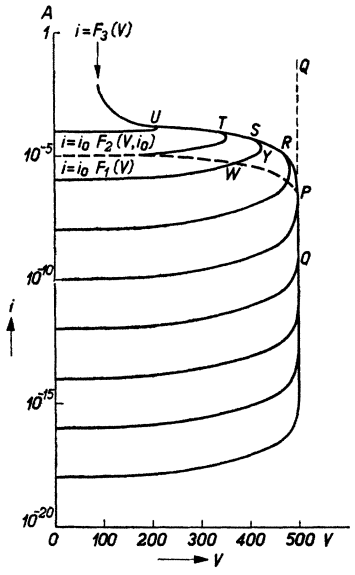


FIG. 15. The three different cases of the dependence of $i$ on the primary current $i_0$ from the cathode (plotted at $V=0$) and $V$. Below the dashed line $PW$, $i=i_0F_1(V)$, above the dashed line $i=i_0F_2(V, i_0)$; along $OPSU$ $i=F_3(V)$, independent of $i_0$. The figure is schematic for $i>10^{-5}$ amp. and $i_0>10^{-8}$ amp. and in the neighborhood of $V=0$ or $i<10^{-8}$ amp. the values were calculated for Ne between parallel plates at values of $p_0d$ of about 40 mm×cm. The unsustained characteristic is for $i<10^{-5}$ amp. in accordance with measurements at $p_0=20$ mm, $d=2$ cm, plate diameter $=8$ cm.

## IV. Breakdown Potential and Townsend Discharge

### §16. Definition of the breakdown potential

In order to arrive at a definition of the breakdown potential $V_B$ we consider the increase of the current $i$ as a function of the tube voltage $V$ for different values of a primary current $i_0$ from the cathode. Fig. 15 gives a schematic[57] survey, the smallest value of $i_0$ being that due to cosmic radiation and natural radioactivity ($\sim 10^{-19}$ amp./cm², a few electrons per sec.). With respect to the connection between $i$, $i_0$ and $V$ three cases are to be distinguished:

$$(A) \qquad i=i_0F_1(V),$$

$$(B) \qquad i=i_0F_2(V, i_0),$$

$$(C) \qquad i=F_3(V).$$

Case $A$ occurs below the dashed line $PW$ in Fig. 15. For sufficiently high values of $i/i_0$ the value of $V$ becomes independent of $i$ ($OP$ for $i_0=10^{-14}$) and the characteristics would remain vertical (dotted line $PQ$) further on, if there were no space-charge deformations of the electric field or cumulative processes (§32) proportional to $i^2$, the function $F_1(V)$ being represented above $P$ by $PQ$. Above the dashed line $PW$, however, the real current due to the complications just mentioned no longer obeys Eq. $(A)$; here $i/i_0$ becomes also a function of $i_0$ (case $B$). This occurs e.g. in the part $WYS$ of the characteristic with $i_0=10^{-6}$. With further increasing current this characteristic debouches into the self-sustained characteristics $i=F_3(V)$ (case $C$), which is independent of $i_0$ (part $STU$ for $i_0=10^{-6}$). It is seen from Fig. 15, that in the characteristics with low values of $i_0$ ($<10^{-10}$)

[55] R. W. Lunt and C. A. Meek, Proc. Roy. Soc. London **A157**, 146 (1936).

[56] In the case of $H_2$ this is contrary to the results of Lunt and Meek, reference 55. These authors, however, assume that the elastic energy loss per collision in $H_2$ is a fraction 15.6 $m/M$, whereas we took 2.67 $m/M$. It seems to us that this value of Lunt and Meek is much too high.

[57] An experimental characteristic for low $i_0$ is given in Fig. 16.

only two parts occur,[58] corresponding to the cases $A$ and $C$, while for higher values of $i_0$ the three parts corresponding to $A$, $B$ and $C$ are present.

The part of the characteristic where $V$ diminishes strongly with increasing $i$ is often, especially at high pressures, intermittent;[59] the intermittencies start at higher values of $i$ when $i_0$ is increased.[60]

In the study of breakdown the following cases are to be distinguished.

*I. $i_0$ very small (e.g. due to cosmic radiation only).*—For the rare gases and other gases at low pressures the characteristic of Fig. 15 becomes vertical at currents of the order of $10^{-12}$ amp ; here $i$ may increase by a factor 10 for an increase of $V$ smaller than 0.001 volt which is below the experimental accuracy in $V$. It is clear that in this case the definition of §2 may be used.[61] the breakdown potential is that potential at which the primary current has increased by a very large factor e.g. $10^8$. According to the equations given above this means:

$$i/i_0 = F_1(V_B) \sim 10^8. \tag{46}$$

In practice, for the cases considered here, this condition is, within the limits of experimental accuracy, identical with[62]

$$F_1(V_B) = \infty, \tag{47}$$

so that for the definition of $V_B$ either (46) or (47) may be used.

The condition $F_1(V_B) = \infty$ may be expressed in a somewhat different way. One electron, starting from the cathode, forms on its way to the anode a certain number of ions ($N_i$), excited atoms, and photons. These secondary products again liberate a number of electrons $M$ from the cathode.[63] This number, being the factor with which the electron from the cathode is multiplied after one transition from cathode to anode, we call the multiplication factor $M$.[64] It is clear that

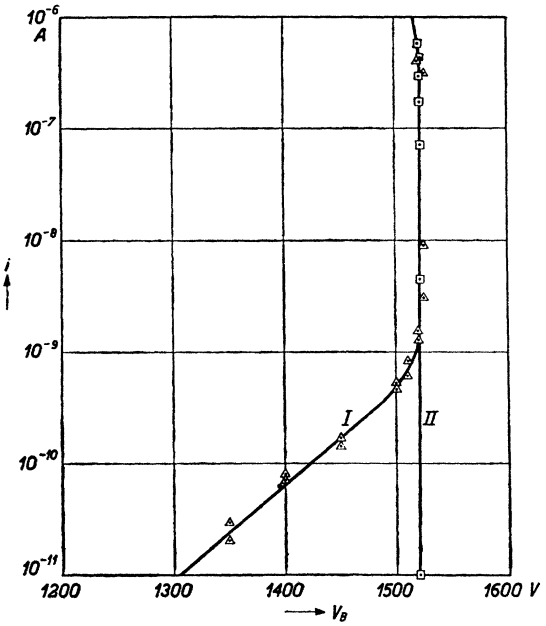


FIG. 16. Townsend discharge between parallel plates with (I) and without (II) a spurious discharge to the glass wall. (Ne, $p_0$ about 150 mm, $d=2$ cm, plate diameter 8 cm).

the current becomes self-sustained, i.e., independent of $i_0$, at the potential for which

$$M = 1. \tag{48}$$

On the other hand, with a primary current $i_0$:

$$i = i_0(N_i+1)(1+M+M^2+\cdots),$$

which, for $M=1$, becomes infinite. This should be the case, as even the slightest increase of $M$ above 1 gives an exponential increase of the current with the time. Thus we see that the condition $M=1$ for the passage of a self-sustained current is identical with (47).

[58] At very low current $i$ and even $i/i_0$ is subject to statistical fluctuations (cf. §25 and §29) which are left out of account here.

[59] G. Valle, Physik. Zeits. **27**, 473 (1926); F. M. Penning, Physik. Zeits. **27**, 187 (1926); A. J. Rothe, Physik. Zeits. **31**, 520 (1930); W. Schulze, Zeits. f. Physik **78**, 92 (1932).

[60] F. M. Penning, Physica **11**, 183 (1931); this circumstance is the reason that radiation of the cathode here may diminish the *mean* value of $i$ (negative photo-effect).

[61] In literature the word breakdown is sometimes used also in another sense, e.g. for the discontinuous transition from glow-to-arc discharge (especially in mercury vapor). In this case we shall use the expression "ignition of the arc discharge."

[62] It is hardly necessary to remark that this condition is not at all identical with $F_3(V) = \infty$ which gives the potential at which the current becomes infinite according to the real characteristic.

[63] Among these ions those formed by metastable atoms are to be included (cf. §12). When secondary electrons are formed by positive ions or photons in the gas, $M$ has to be defined in a somewhat different way.

[64] $M$ is identical with the "Ionisierungsanstieg" in one "Ionisierungsspiel" as introduced by Rogowski (see e.g. Physik. Zeits. **33**, 797 (1932).

We conclude that, for very small values of $i_0$, at low pressures one of the following definitions may be given: $V_B$ is the potential at which: (1) $F_1(V)$ becomes very large (e.g. $10^8$); (2) $F_1(V)=\infty$; (3) a small, self-sustained current passes ($M=1$); (4) the current becomes independent of $i_0$; (5) the potential becomes independent of the current.

In actual measurements (1), (3), (4) and (5) are used; often for the sake of checking it is desirable to see whether (5) is satisfied. A rougher method is to see where: (6a) $V$ has its maximum for a negative characteristic of the Townsend discharge and (6b) $(di/dV)/i$ has its minimum for a positive characteristic (cf. the curve $B$, $CD'$ in Fig. 1).

It should be emphasized that in the cases considered here the influence of space charge and multiple processes on the value of $V_B$ is altogether negligible within the limits of experimental accuracy, as the point of the characteristic where $V$ becomes independent of $i$ occurs at very low currents. The situation changes, however, in the cases II and III which will be discussed now.

*II. Larger values of* $i_0$.—For larger values of $i_0$ ($i_0>10^{-14}$ in Fig. 15) the point where $i/i_0=10^8$ lies outside the region in which $i/i_0=F_1(V)$. Then the definition (1) of $V_B$ is no longer valid; the other definitions given above lead to different values of $V_B$ and even to values which may be dependent on the resistance in the tube circuit, as with definition (3) when the persistence of the current at the suppression of $i_0$ is taken as criterion.

For larger values of $i_0$ the safest way seems to be to use definition (6), although, for a positive characteristic, this may give a rather arbitrary value of the potential.

The larger values of $i_0$ at the cathode may be due to: (1) Photoelectric emission (comp. §24); (2) autoelectronic emission; (3) thermal emission, an extreme example being the case of a heated filament as cathode; of course here very different conditions prevail at breakdown (comp. §41); (4) electrons and ions from an auxiliary discharge;[65] in this case the primary current need not be limited to the cathode. A special example of this type of current ("predischarge" current) was observed in some breakdown experiments[66] as shown in Fig. 18. This figure at the same time may serve as a proof that in Ne even at 150 mm pressure $V$ is practically independent of $i$ from $i=10^{-11}$ to $10^{-7}$ amp.

*III. Breakdown potential as gases such as air at high pressures (e.g., 1 atmos.).*—In this case even at $i/i_0=10^8$ the value of $V$ may not yet become independent of $i$,[67] so that our definition (1) does not hold. Here the ions formed by even one electron passing from cathode to anode seem to be able to modify the electric field sensibly; moreover the large electron and ion density may lead to cumulative processes.[68] Here the breakdown may be influenced by space charge. For this case lying beyond the scope of our review, we refer to the articles of Loeb[3] and Rogowski.[69] It seems not superfluous to remark here that the considerations of these authors on the influence of space charge on breakdown do not apply to the case of low pressures and low $i_0$ to which we limit ourselves mainly further on in this section.

## §17. Breakdown potential between electrodes of various shapes

The breakdown potential depends among other things on the shape and separation of the electrodes and on the gas density. Now Townsend[70] has pointed out that $V_B$ remains the same if all dimensions of an arrangement are increased by the same factor $k$ and at the same time the gas density is reduced in the ratio $1/k$. In this case, at $V_B$ the value of $E/p_0$ at corresponding points of the two arrangements remains the same. As the potential difference $E\lambda_e$ per mean free path also remains equal the electrons per-

[65] C. Deimel, Physik. Zeits. **37**, 610 (1936).

[66] F. M. Penning and C. C. J. Addink, Physica **1**, 1007 (1934).

[67] F. H. Sanders, Phys. Rev. **41**, 667 (1932); **44**, 1020 (1933); **45**, 346 (1934). The difference in behavior between air and the rare gases at high pressures is due to the circumstance that at low values of $E/p_0$ the value of $\gamma$ for air becomes very small ($<10^{-10}$) while in the rare gases $\gamma$ does not decrease below $10^{-4}$ (cf. §20).

[68] G. Holst, Proc. Roy. Acad. Amsterdam **36**, 271 (1933).

[69] W. Rogowski, Arch. f. Elektrotech. **25**, 551 (1931); **26**, 643 (1932); Physik. Zeits. **33**, 797 (1932); Zeits. f. Physik **100**, 1 (1936); W. Rogowski and A. Wallraff, Zeits. f. Physik **106**, 212 (1937); **108**, 1 (1937).

[70] J. S. Townsend, *Electricity in Gases* (1915), p. 365.

form the same number of ionizations and excitations on their way from cathode to anode and also the number of electrons liberated per ion, etc., from the cathode remains constant. As we limit ourselves to the case where space charges and cumulative processes have no influence on the breakdown, the change in current density is irrelevant. Consequently the breakdown potential for arbitrary electrode arrangements of the same shape is a function of $ap_0$, where $a$ is a linear dimension of the arrangement. For infinite parallel plates at a distance $d$ this condition reduces to [71]

$$V_B = F(p_0 d) \text{ (law of Paschen).} \qquad (49)$$

As to the influence of a magnetic field on the breakdown potential, which will be left out of account, we remark that the influence is only large when $r_H < \lambda_e$, $r_H$ being the radius which an electron with energy $eV_e$ in vacuum describes under the influence of a magnetic field $H$ ($r_H = 3.37 V_e^{\frac{1}{2}}/H$, $V_e$ in volts, $H$ in oersted) and $\lambda_e$ the mean free path of the electrons.[72] For similar electrode arrangements $V_B$ remains the same when $ap_0$ and $H/p_0$ are left constant.[73]

In the following we shall at first discuss the breakdown potential and Townsend discharge between infinite parallel plates[74] (uniform electric field), many of the considerations applying also to electrodes of other shapes. A few supplementary remarks will be made on the case of nonuniform fields, especially that between coaxial cylinders.

### *A. Homogeneous Electric Field*

### §18. Breakdown potential $V_B$ and elementary processes ($\eta$ and $\gamma$)

When at increasing voltage the electron current increases only by the ionization due to primary electrons then, according to Eq. (33), $i = i_0 e^{\eta(V-V_0)}$. The value of $\eta$ in this formula is, however, at low gas pressures far too small to give at $V = V_B$ the large value of $i/i_0$ (e.g. $10^8$) needed for the breakdown. Besides this primary process therefore one or more secondary processes are necessary, for which in §11 a coefficient $\gamma$ was introduced, leading to Eq. (34) instead of Eq. (33). The function $F_1(V)$ defined in §16 becomes:

$$\frac{i}{i_0} = F_1(V) = \frac{e^{\eta(V-V_0)}}{1-\gamma(e^{\eta(V-V_0)}-1)} \qquad (50)$$

and according to definition (2) of §16, the value of $V_B$ is given by:

$$\gamma(e^{\eta(V_B-V_0)}-1) = 1. \qquad (51)$$

It was stated that the value $V_B$ following from $F_1(V_B) = \infty$ was practically equal to the value $V_n$ following from $F_1(V_n) = 10^n$, where $10^n$ is a very large number e.g. $10^8$. This is proved for Ne by Fig. 17 where the values of $(V_B - V_n)/V_B$ have been plotted for $n = 10, 8, 6, 4$ as a function of $p_0 d$. Since in the determination of $V_B$ an accuracy of 0.1 percent may be called high, it is clear that in the case of Ne for the "very large factor" any number might be taken between $10^4$ and $\infty$.

Equation (51) shows that $V_B$ may be calculated from $\eta$, $\gamma$ and the correction $V_0$; conversely $V_B$ is also often used for the calculation of $\gamma$ (§§20, 21). For this reason we discuss the nature of the secondary processes $\gamma$ in connection with the breakdown potential.

### §19. Nature of secondary processes ($\gamma$)

There are two possible places where electrons may be liberated by secondary processes:

I—from the cathode.
II—in the gas (by ionization of gas molecules).

There are three possible agents involved: (a) positive ions or neutral molecules originating from positive ions by charge-transfer, (b) radiation quanta, (c) excited and especially metastable molecules.[75]

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[71] The law of Paschen is a special case of the similarity principle, treated in §32.

[72] M. Wehrli, Ann. d. Physik **69**, 285 (1922); F. M. Penning, Physica **3**, 873 (1936). F. M. Penning, Physica **4**, 71 (1937).

[73] E. Brüche and A. Recknagel, Zeits. f. tech. Physik **18**, 139 (1937).

[74] For the breakdown between parallel plates in a cylindrical tube (positive column) see M. Steenbeck, Wiss. Veröffentl. Siemens **15**, 32 (1936); G. Mierdel and M. Steenbeck, Zeits. f. Physik **106**, 311 (1937).

[75] For the sake of convenience we shall refer in the following only to metastable molecules which, as a consequence of their long lifetime, have much more effect than ordinary excited molecules.

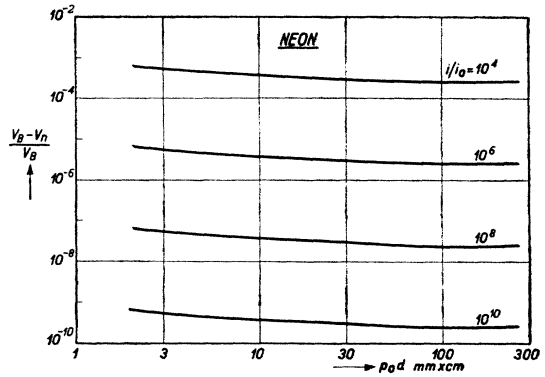


FIG. 17. Values of $(V_B-V_n)/V_B$ as a function of $p_0d$ at different values of $i/i_0$ in neon, calculated from Eq. (50) with $\gamma=0.05$ and $\eta$ according to Table I. $V_n$ = potential difference where $i/i_0=10^n$, $V_B$ = potential difference where $i/i_0=\infty$, $i_0$ = primary current from the cathode.

This gives together six cases (Ia, Ib, $\cdots$IIc), the theoretical possibility of each of them being established unambiguously by experiments on the fundamental processes. The question interesting us, however, is not the *possibility* but the *probability* for a given process to occur at the values of $E/p_0$ and in the gases to be considered.

In the past mainly two theories have been used, namely IIa (Townsend[2]) and Ia (especially for rare gases at low pressures; Holst and Oosterhuis[76]). We first admit both these processes, calling: $\gamma$ the average number of electrons liberated per positive ion from the cathode, $\zeta$ (analogous to $\eta$) the ionization coefficient for the positive ions, $\zeta dU$ being the number of ionizations of an ion in passing through the potential difference $dU$.[77]

When $i_e$, $i_p$ and $i$ are, respectively, the electron current, the positive ion current and the total current we have

$$i=i_e+i_p, \tag{52}$$

$$di_e=(\eta_e i_e+\zeta i_p)dU. \tag{53}$$

The ionization coefficient for the electrons is called $\eta_e$ here instead of $\eta$. For plane parallel plates the boundary conditions may be written as

$$U=V_0, \quad i_e=i_0+\gamma i_p, \tag{54}$$

$$U=V, \quad i_e=i, \tag{55}$$

where $V_0$ has the same meaning as in §11. Taking into account that $i$ is independent of $U$, one finds for the solution of these equations

$$i=i_0\frac{e^{(\eta_e-\zeta)(V-V_0)}}{1-\dfrac{\eta_e\gamma+\zeta}{\eta_e-\zeta}(e^{(\eta_e-\zeta)(V-V_0)}-1)}. \tag{56}$$

The breakdown potential ($V_B$) follows, according to this equation, from

$$\frac{\eta_e\gamma+\zeta}{\eta_e-\zeta}(e^{(\eta_e-\zeta)(V_B-V_0)}-1)=1. \tag{57}$$

For

$$\frac{\eta_e\gamma+\zeta}{\eta_e-\zeta}=\gamma' \quad \text{and} \quad \eta_e-\zeta=\eta. \tag{58}$$

Eq. (57) reduces to (51) and (56) to (34), so that both mechanisms mentioned above or a combination of them give exactly the same formula for the breakdown potential and also for the increase of the current in the experiments for the determination of $\eta$ and $\gamma$ according to Eq. (57). In the case of ionization by positive ions in the gas the coefficient following directly from the experiments is $\eta=\eta_e-\zeta$; as $\zeta$ is always $\ll\eta$ the reduction of $\eta_e$ to $\eta$ has only the character of a correction.

The meaning of (57) may be generalized to allow also the liberation of electrons from the cathode by radiation quanta or metastable molecules (cases Ib and Ic). For the interpretation of the experiments one may then also use Eq. (56) or (57), provided that $\gamma$ is no longer interpreted as the number of electrons liberated *by* one ion, but as the number calculated *per* ion arriving at the cathode. When the number of radiation quanta formed *per* ion equals $N_r$, the number of metastable molecules $N_m$, and the number of electrons liberated in the mean *by* one ion, quantum, or metastable molecule $\gamma_p$, $\gamma_r$ and $\gamma_m$, respectively, then

$$\gamma=\gamma_p+f_r\gamma_r N_r+f_m\gamma_m N_m. \tag{59}$$

Here $f_r$ and $f_m$ are geometrical factors[3] representing the fraction of the photons and metastable

[76] G. Holst and E. Oosterhuis, Phil. Mag. **46**, 1117 (1923).

[77] The ionization coefficient $\beta$ of Townsend is equal to $\zeta E$, analogous to $\alpha=\eta E$.

atoms reaching the cathode. It is now seen that Eqs. (51) and (57) hold also for this case.[78]

From the considerations given above we conclude that in the cases Ia, Ib, Ic and IIa a breakdown occurs at the *finite* potential difference $V$ satisfying Eq. (56). This need not be true for IIb and IIc as in these cases the local distribution of the new electrons is symmetrical with respect to the place where the primary process of excitation occurs. Therefore, when the distance between the place of excitation and the place of secondary ionization is not too large or the probability for the process is small, the secondary process may be interpreted as an increase of $\eta$, as was done for the ionization by metastable atoms in Ne-A mixtures (§10). Between parallel plates with moderate and low pressures the processes IIb and IIc will occur only in gas mixtures for in pure gases the energy of the radiation quanta and excited molecules is practically always lower than the ionization potential.

For pure gases therefore only the possibilities Ia, b, c and IIa need to be discussed. It was shown that in cases Ia and IIa the formulae for the increase of $i/i_0$ as a function of $V$ were exactly the same, so that in this respect no decision as to the mechanism is possible. For the cases Ib and Ic a somewhat different increase should be possible, but up to now no measurements of $i/i_0$ have been accurate enough to settle this point. Therefore the decision must be taken on other grounds.

Now for the rare gases a strong proof for the liberation of electrons from the cathode (process I) lies in the layers which Holst and Oosterhuis[76] discovered in the Townsend discharge through these gases. They are especially sharp at low values of $E/p_0$. Fig. 18 gives an example.[79] These layers clearly show that the electrons are liberated from the cathode and are not produced by ionization in the bulk of the gas. On the other hand Holst and Oosterhuis[76] showed many years ago that in the rare gases at low values of $E/p_0$ the velocity of the positive ions is much too small to ionize gas molecules. This argument has since been strengthened by the discovery that positive ions have a large chance of transfering their charge in a collision without loss of their kinetic energy. For argon the different cross sections are known at present[80] and the value of

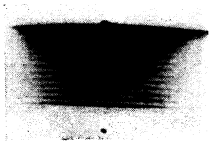

FIG. 18. Layers of Holst and Oosterhuis in neon (cathode below); $p_0=40$ mm; $d=1$ cm; $E/p_0=61$ v/cm$\times$ mm; $i=0.3$ $\mu$a.

$\zeta$ in (58) for positive ions and fast neutral atoms may be calculated roughly. This calculation shows that, with the exception perhaps of very low pressures (much lower than the pressure where $V_B$ has its minimum, §23), the value of $\zeta$ in (58) for ions and fast neutral atoms[81] is very much lower than the value of $\eta_e\gamma'$ found experimentally so that also on this ground case II can be excluded.

The relative importance of the remaining processes Ia, Ib and Ic can be better discussed on the basis of the experimental values of $\gamma$ as a function of $E/p_0$.

## §20. Values of $\gamma$ for the rare gases

The values of $\gamma$ may be determined from the same experiments as were used for the determination of $\eta$ (§11). When $\eta$ and $V_B$ are known, $\gamma$ may be found[82] also from Eq. (51); as, however, $\eta$ figures in the exponent of $e$, the values obtained for $\gamma$ are much less accurate than those for $\eta$.

Figure 19 gives the results for a Cu cathode in several rare gases according to Kruithof,[41] and Fig. 20 the results for several cathode materials in A according to different authors.[41, 66, 83] Even

[78] $\gamma_r$, $\gamma_m$, $N_r$ and $N_m$ may be considered as functions of $E/p_0$, provided the anode potential is equal to $V_B$. The increase of $i$ with $V$ below $V_B$, however, is no longer necessarily described by (56) as $\gamma_r$ and $\gamma_m$ in this case may become also function of $V$ as a consequence of absorption and the finite dimensions of the cathode. This may be taken into account in the way pointed out by Loeb, reference 15.

[79] M. J. Druyvesteyn, Zeits. f. Physik **73**, 33 (1931). These layers also form an optical demonstration of the step-like increase of the current showing itself electrically in Fig. 8.

[80] F. Wolf, Ann. d. Physik **29**, 33 (1937) and previous papers. A. Rostagni, Nuovo Cimento **13**, 391 (1936); **15**, 117 (1938).

[81] The contribution from the neutral atoms originating from charge transfer proves to be still larger than that from the positive ions themselves.

[82] Log ln $(1/\gamma)$ is also approximately equal to the difference between log $(V_B-V_0)$ and log $(1/\eta)$ in Fig. 21 (§22).

[83] R. Schöfer, Zeits. f. Physik **110**, 21 (1938); F. Ehrenkranz, Phys. Rev. **55**, 219 (1939).

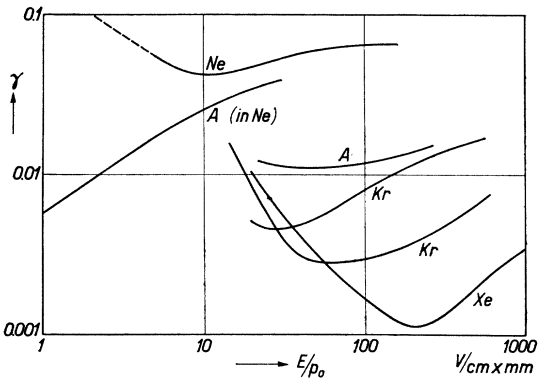


FIG. 19. Values of $\gamma$ for Cu in the rare gases.

with the same cathode material $\gamma$ depends strongly on the surface condition,[84] as is shown by the two curves for Cu[41] in Fig. 20. The lower curve was obtained after a second treatment of the metal by a glow discharge. Another example of the change in $\gamma$ by different treatment of the cathode is given by the Kr curves in Fig. 19. The increase of $\gamma$, at the higher values of $E/p_0$, with decreasing atomic number of the bombarding ions is very probably real; we do not pretend, however, that the ratio of the $\gamma$-values for the different gases is exactly given by Fig. 19 because the surface condition of the cathode may not have been the same.

Although our knowledge of the $\gamma$-values even in the simplest case of the rare gases, is still inexact, it seems possible to draw some conclusions with respect to the $\gamma$-mechanism (Ia, b, c) from the absolute values obtained and from the dependence of $\gamma$ on $E/p_0$. Direct experiments in vacuum[85] proved that rare gas ions of small energy ($<10$ v) are able to liberate electrons from metals, the efficiency roughly increasing with increasing ionizing potential of the gas and with decreasing work function $\varphi$ of the metal. The value of $\gamma_p$ found for Ne and A, in these direct experiments is of the same order of magnitude as the value of $\gamma$ in Fig. 19, so that in this case probably $\gamma=\gamma_p$ and the secondary breakdown mechanism involves the liberation of electrons from the cathode by positive ions (Ia). Also the dependence of $\gamma$ on the kind of gas and electrode surface is in agreement with what should be expected from the direct measurements.

As to the variation of $\gamma$ with $E/p_0$, one should expect that in the case Ia at *high values of* $E/p_0$, $\gamma$ should have the same value as in vacuum, and that it should decrease slowly with decreasing $E/p_0$ as a consequence of back diffusion of liberated electrons to the cathode.[86] The course of $\gamma$ for Ne at $E/p_0>20$ is in accordance with this expectation, so that the hypothesis $\gamma=\gamma_p$ is confirmed in this region.

Also the time required for the development of

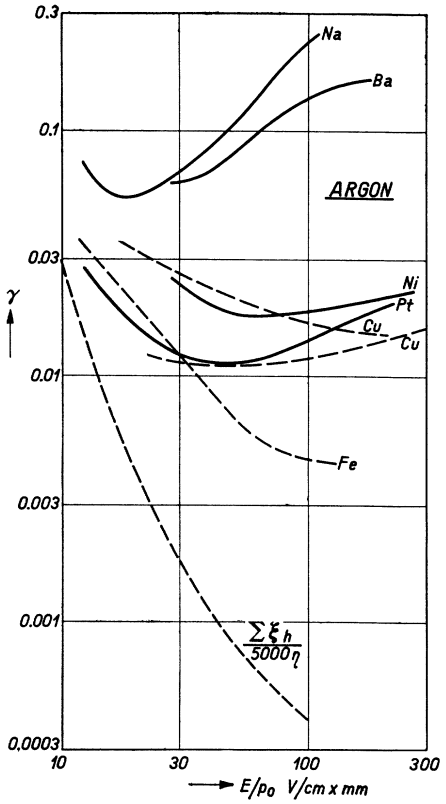


FIG. 20. Values of $\gamma$ for A with different cathode materials, according to Kruithof and Penning (Cu), Penning and Addink (Fe), Schöfer (Ni and Ba), Ehrenkranz (Pt and Na). The proportion of the number of excitations to the number of ionizations is roughly equal to $\Sigma\xi_h/\eta$.

[84] This was also found in the direct measurements of $\gamma$ in vacuum, reference 85. Probably it is also the reason why it is very difficult to find reproducible results for the value of the normal cathode fall even in very pure gases (§35).

[85] F. M. Penning, Proc. Roy. Acad. Amsterdam **31**, 14 (1928); **33**, 841 (1930); Physica (old series) **8**, 13 (1928); M. L. E. Oliphant, Proc. Roy. Soc. **A124**, 228 (1929); **A127**, 373 (1930); M. L. E. Oliphant and P. Moon, *ibid.* **A124**, 388 (1929).

[86] W. Harries and G. Hertz, Zeits. f. Physik **46**, 177 (1927); H. Pose, Zeits. f. Physik **52**, 428 (1928); I. Langmuir, Phys. Rev. **38**, 1656 (1931).

the breakdown, which will be discussed in §25 is of the order of magnitude to be expected if mainly positive ions have to move to the cathode in order to form new electrons.[87]

At *lower values of* $E/p_0$, $\gamma$ increases again and this we ascribe[88] to the liberation of electrons by radiation (case Ib) or metastable atoms (case Ic). In the case of radiation, which according to Kenty[89] is the most favorable, at least for Ne and A, Eq. (59) becomes:

$$\gamma = \gamma_p + f_2 N_r \gamma_r. \tag{60}$$

The following considerations, however, apply as well to the case of metastable atoms; in Eq. (60) $N_r\gamma_r$ has to be replaced then by $N_m\gamma_m$.

The value of $N_r$ is roughly proportional to $\Sigma\xi_h/\eta$, which may be calculated from the energy balance for the electrons (Eq. (45), §15). This number increases rapidly with decreasing $E/p_0$ and so the influence of $N_r\gamma_r$ becomes more and more important, and causes the increase of $\gamma$. Fig. 20 shows that for different cathode materials in the case of A the curve for $\gamma$ goes upward in the same region where $\Sigma\xi_h/\eta$ increases to high values.

The curve for Ne in Fig. 19 may be explained satisfactorily when it is assumed that $\gamma_i$ and $\gamma_r$ decrease with $E/p_0$ in approximately the same way as a current of photoelectrons liberated from the cathode by external radiation (e.g. from a quartz mercury lamp[90]). In this way for a copper cathode at $E/p_0 = 100$ v/cm mm the following figures were obtained:[88]

$$\gamma_i = 0.06; \quad \gamma_r = 0.003 \text{ for Ne},$$
$$\gamma_i = 0.001; \quad \gamma_r = 0.0005 \text{ for Kr}.$$

For neon this value of $\gamma_i$ is in accord with the rather rough results of direct measurements, as mentioned above; the value of $\gamma_r$ is rather high, but still lower than the experimental values of Kenty[91] for the resonance radiation of neon.

Figure 19 gives also the values of $\gamma$ for $A^+$ ions in Ne which can be calculated from the $\gamma$-values of Ne-A mixtures in a somewhat more complicated way. The curve through these points has, contrary to the others, the shape which should be expected for the liberation of electrons by positive ions only. This is in accord with the circumstance that in these mixtures the A ions are formed by the metastable Ne atoms. The value of $N_r$ for A remains very low.

Summarizing, our conclusion is that in rare gases at $100 < E/p_0 < 500$ the secondary breakdown mechanism is mainly liberation of electrons from the cathode by positive ions (Ia), while at lower values of $E/p_0$ the liberation of electrons by radiation (Ib) or metastable atoms (Ic) comes into account. For $E/p_0 > 500$ where the ion energies become large it seems possible that also ionization by positive ions in the gas (IIa) comes into play.

## §21. Values of $\gamma$ for diatomic gases

Recently the values of $\gamma$ have been determined by Bowls[47] for $N_2$ and by Hale[47] for $H_2$ from the increase of $i/i_0$ with increasing electrode separation and constant $E/p_0$. Hale's curves show a considerably more complex nature than those for the rare gases, that for a Na surface having no less than four maxima. According to the author they are due to photoelectric liberation of electrons, conditioned by the excitation of certain lines in the hydrogen spectrum as the average energy passes through peaks in the excitation curves; the absorption of the photons by a hydrogen-sodium compound should explain why the maxima occur at different values of $E/p_0$ for Na and for Pt. A further investigation of these interesting phenomena seems highly desirable.

In air, contrary to other gases, the value of $\gamma$ decreases very sharply at $E/p_0$ about 40 v/cm·mm; at high pressures it becomes[92] even

[87] This does not preclude, however, that at high values of $E/p_0$ and for cathodes with small value of $\varphi$, some electrons may be liberated by metastable atoms. For a photoelectric cathode this was shown by the curves for the frequency response by Kruithof, Philips Tech. Rev. **4**, 57 (1939).

[88] A. A. Kruithof and F. M. Penning, Physica **5**, 203 (1938); cf. L. B. Loeb, reference 2; W. Rogowski and A. Wallraff, Zeits. f. Physik **106**, 212 (1937); A. Wallraff and E. Horst, Arch. f. Elektrotech. **31**, 789 (1937).

[89] C. Kenty, Phys. Rev. **43**, 181 (1933). Recently, however, a preliminary estimate of the efficiency for the liberation of electrons by metastable helium atoms leads to a value of 10 percent (R. Dorrestein and J. A. Smit, Proc. Roy. Acad. Amsterdam **41**, 725 (1938)). For the older literature see Compton and Langmuir, reference 4.

[90] A. A. Kruithof and F. M. Penning, Physica **4**, 430 (1937), Fig. 4.

[91] C. Kenty, Phys. Rev. **44**, 891 (1933).

[92] W. Rogowski, Arch. f. Elektrotech. **25**, 551 (1931).

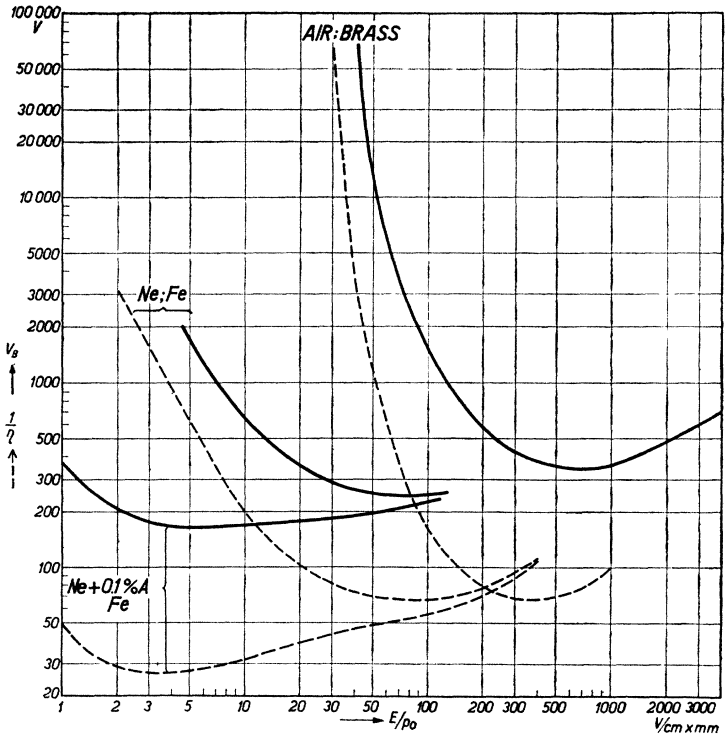


FIG. 21. Values of the breakdown potential $V_B$ (——) between parallel plates and $1/\eta$ (- - - -) as a function of $E/p_0$, the vertical distance between the two curves gives, on the logarithmic scale, the value of ln $(1/\gamma)$.

lower than $10^{-10}$. This may be explained by a kind of absorption, either of the positive ions which become clusters,[93] or of photons.[94]

## §22. Values of $V_B$ as a function of $E/p_0$

The breakdown potential $V_B$ is connected with the elementary processes by Eq. (51). Often $V_0$ is $\ll V_B$ and $e^{\eta(V_B-V_0)}\gg 1$, so that approximately

$$\gamma e^{\eta V_B}=1 \quad \text{or} \quad V_B=(1/\eta)\ln(1/\gamma). \quad (61)$$

According to the law of Paschen $V_B$ is a function of $p_0d$ and therefore also of $V_B/p_0d = E/p_0$. Now in Eq. (61) $\gamma$ appears only as a logarithm and, moreover, in many cases $\gamma$ changes only slowly with $E/p_0$. It is therefore clear that the behavior of $V_B$ will be mainly determined by that of $1/\eta$. In Fig. 21 this is demonstrated for some gases; as the scale is logarithmic in both directions, the vertical distance between the curves for $V_B$ (corrected for $V_0$ when necessary) and $1/\eta$ gives the value of ln $(1/\gamma)$ on the logarithmic scale. It is seen that the minimum in the curve for $V_B$ corresponds to the minimum in the curve for $1/\eta$ (discussed in §12), although it occurs at a somewhat different value of $E/p_0$ in the case of air.[92] Also the large difference in breakdown potential between the different gases at the same value of $E/p_0$ is mainly due to the large difference in the values of $\eta$. For the same gas and different cathode materials the $V_B$ curves are shifted in the direction of the $V_B$ axis; the minimum occurs at roughly the same value of $E/p_0$ (cf. Fig. 25, §23).

The three types of curves shown in Fig. 21 may be characterized roughly as follows. For Ne+0.1 percent A at $E/p_0>2$, the energy of the electrons is mainly used for ionization by intermediate metastable Ne atoms; at $E/p_0<2$ and still more strongly at $E/p_0<1$ the value of $V_B$ rises as a consequence of the elastic energy losses of the electrons. In pure Ne the rise for $E/p_0<30$ is due to excitation losses; in air these are much stronger, therefore the curve in air rises at even higher values of $E/p_0$ and goes up much more steeply. For high values of $E/p_0$ ($\sim$1000) the excitation losses become less and less important as compared with the ionization by the electrons; in this region the differences between the curves are smaller and are determined mainly by the

[93] F. M. Penning, Ned. T. Natuurkunde **5**, 33 (1938).
[94] R. Schade, Zeits. f. Physik **111**, 437 (1939).

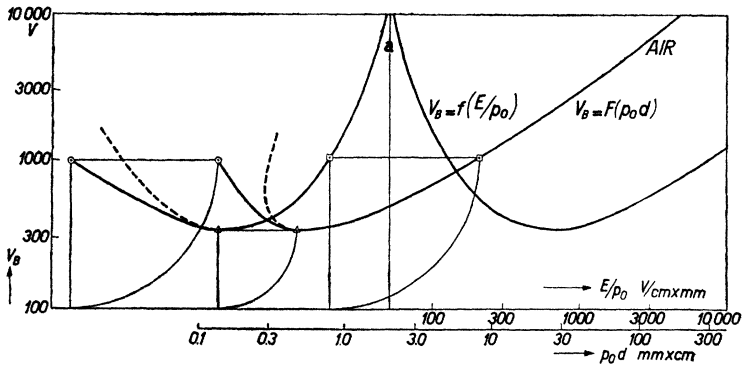


FIG. 22. Transition of $V_B$ as a function of $E/p_0$ to $V_B$ as a function of $p_0d$. The first curve is reflected into the line $a$, and then the points are shifted to the right over a distance (log $V_B-2$). When the curve $V_B=f(E/p_0)$ has the form indicated by - - - -, the curve $V_B=F(p_0d)$ gives for certain values of $p_0d$ more than one value for $V_B$.

ionization probability in the elementary collision process between electrons and molecules. For the different gases these increase in the order He, Ne, $H_2$, $N_2$, A, Hg. Consequently at high values of $E/p_0$ the extrapolated curve for Ne in Fig. 21 intersects that for air. As the order of the curves for $E/p_0 > (E/p_0)_{\min}$ is not changed by the transition from $E/p_0$ to $p_0d$, it appears also in Fig. 23 for $p_0d < (p_0d)_{\min}$ (§23) and in the results given by other authors.[95]

## §23. $V_B$ as a function of $p_0d$

When $V_B$ has to be determined for a given value of $E/p_0$, the field strength should be kept constant and the distance between the plates increased until breakdown occurs. Usually, however, the plate separation $d$ and the pressure $p_0$ (reduced to 0°C) are kept constant and the potential difference is increased. Then according to the law of Paschen (§17) $V_B$ is a function of $p_0d$.

In order to derive the ordinary Paschen curves for $V_B$ as a function of $p_0d$ from the curves given in Fig. 21, we write: $V_B=(E/p_0)(p_0d)$ or:

$$\log p_0d = -\log (E/p_0) + \log V_B.$$

Hence to obtain $V_B=F(p_0d)$ from $V_B=f(E/p_0)$ the curve for $V_B$ as $f(E/p_0)$ must be reflected in the (log $V_B$) axis and each point shifted to the right through a distance log $V_B$. This has been done for the case of air in Fig. 22. The minimum value of $V_B$ appears of course also in the curve $V_B=F(p_0d)$, the corresponding value of $p_0d$ we call $p_0d_{\min}$; obviously the parts right and left of the minimum have interchanged by the transition from $E/p_0$ to $p_0d$.

Figure 23 gives a survey of the $V_B$ values as a function of $p_0d$ for a number of typical cases[96] (logarithmic scale for $V_B$ and $p_0d$).

The ionization by metastable Ne atoms causes the large decrease of $V_B$ by small admixtures of A to Ne and also the double minimum in the $V_B$ curve for Ne+0.002 percent A, corresponding to the double maximum of the $\eta$-curve for the same mixture (§12). This influence of the metastable atoms can be proved by diminishing their concentration whereupon the breakdown potential increases again. The experiment may be performed either by irradiation with light of the main gas as discussed in §24 or by the admixture of a third gas which destroys the metastable atoms of the main gas but is not ionized by them.[97]

[95] R. B. Quinn, Phys. Rev. **55**, 482 (1939) (Ni and stainless steel cathode); H. Klemperer, Zeits. f. tech. Phys. **19**, 270 (1938) (C cathode). The absolute values of $V_B$, however, show very large differences from one author to another, which may be partly due to the difference in $\gamma$. Compare for the case of Hg vapor E. J. Lawton and K. H. Kingdon, Phys. Rev. **50**, 1095 (1936) (abstract). The values given by Klemperer for air and Hg at low $p_0d$ are even of another order of magnitude than those given by other authors.

[96] W. R. Carr, Phil. Trans. Roy. Soc. **201**, 403 (1903) (air, $H_2$; Fe, Zn, Al, brass; $p_0d < (p_0d)_{\min}$; E. Meyer, Ann. d. Physik **58**, 297 (1919) (air; brass); G. Holst and A. N. Koopmans, Vesl. Kon. Akad. Amsterdam **26**, 502 (1917) (air; Ag); H. Ritz, Arch. f. Elektrotech. **26**, 219 (1937) (air; Ni); F. Ehrenkranz, ($H_2$; Pt), Phys. Rev. **55**, 219 (1939); H. Fricke, ($H_2$; Fe), Zeits. f. Physik **86**, 464 (1933); F. M. Penning and C. C. J. Addink, Physica **1**, 1007 (1934) (A, Ne, Ne+A; Fe). For A compare also Fig. 28. The values for $H_2$ and brass according to E. Meyer (cited in B. Frey, Ann. d. Physik **85**, 381 (1928)) in the neighborhood of $(p_0d)_{\min}$ are about 20 percent lower than those of Fig. 23. After the completion of our article F. L. Jones and J. P. Henderson published extensive measurements (Phil. Mag. **28**, 185 and 192 (1939)) for $H_2$ with 6 different cathode materials. Their minimum values of $V_B$ lie between 312 (Staybrite steel) and 256 v (commercial Al); the absolute value of the difference in $V_B$ for two cathode materials remains practically constant up to $V_B=5000$ v.

[97] F. M. Penning, Physica (old series) **10**, 47 (1930); B. Klarfeld, Zeits. f. Physik **78**, 111 (1932).

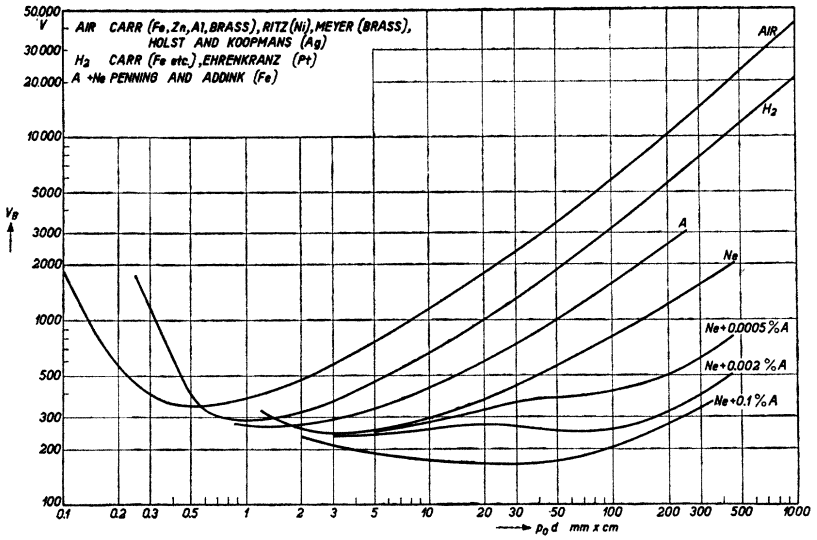


FIG. 23. Typical curves for $V_B$ as $F(p_0d)$ between parallel plates.

Figure 24 shows how in A $V_B$ depends on the cathode material;[96] the differences[98] are a consequence of the different values of $\gamma$ (cf. Fig. 20, which was computed from the curves given in Fig. 24). When $\gamma$ does not change strongly with $E/p_0$, the minimum of $V_B$ should occur for the different cathode materials at the same value of $E/p_0$ (§22) and in Fig. 24 the points of $(p_0d)_{\min}$ should lie on a straight line which makes an angle of 45° with the axes. This indeed proves to be the case.

It may be shown[3] that a certain percentage difference in $\gamma$ has the less percentage influence on $V_B$, the higher the value of $p_0d$ and the smaller the value of $\gamma$ itself. This is in accord with Fig. 24 and also with the circumstance that for high pressures in air the breakdown potential becomes independent of the cathode material. In the rare gases, however, particularly in the neighborhood of $(p_0d)_{\min}$ the influence of the cathode material is so large that, even with the same cathode metal, variations in the surface condition may change $V_B$ considerably. For discharges in vapors (e.g. Hg) or gas-vapor mixtures the situation is still more complicated by the condensation of vapor on the cathode.[99]

The reason for the differences in $V_B$ between the different gases with the same cathode material has been discussed above for $V_B=f(E/p_0)$. Here we mention only the typical difference between Ne+0.1 percent A for $2<p_0d<100$ mm·cm on the one hand and air for $p_0d>100$ on the other (Fig. 23). In the first case the *potential* necessary for starting the discharge is nearly constant; practically the whole of the electron energy is used for ionization and therefore it is mainly the total potential difference traversed by a primary electron from the cathode which is essential for the ion production. For air, at high values of $p_0d$, on the other hand, $V_B$ changes about linearly with $p_0d$ (inclination of the curve in Fig. 23 about 45°). This means that

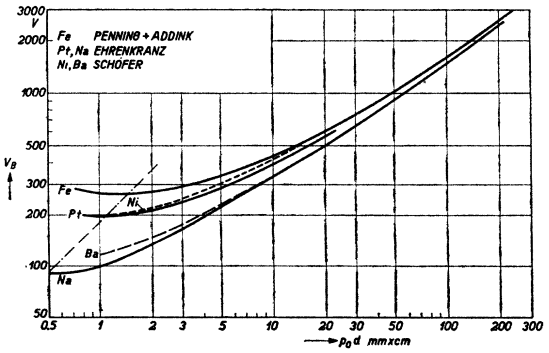


FIG. 24. Breakdown potential $V_B$ of A between parallel plates for different cathode materials (compare Fig. 20). The values of $p_0d$ where $V_B$ has its minimum occur at about the same values of $E/p_0$ (—·—·—).

[98] This difference was one of the first arguments of Holst and Oosterhuis (Physica (old series) **1**, 78 (1921); Phil. Mag. **46**, 1117 (1923)) for the $\gamma_P$-mechanism of the breakdown.

[99] R. Grigorovici, Zeits. f. Physik **111**, 596 (1939); F. Llewellyn Jones and W. R. Galloway, Proc. Phys. Soc. **50**, 207 (1938).

at constant pressure the breakdown *field strength* is nearly constant, whereas the total potential difference traversed by a primary electron changes considerably with $p_0d$. Here $\eta$ changes strongly with $E/p_0$ (Fig. 9); when $E/p_0$ is decreased, $\eta$ decreases so much that a much larger value of $p_0d$ must be taken to form the required number of positive ions. Hence a large change in $p_0d$ corresponds to a small change in $E/p_0$ which means that the starting *field strength*[100] remains nearly constant. In this respect the behavior of air at high values of $p_0d$ resembles that of massive solids.[101]

An abnormal form of the Paschen curve at low values of $p_0d$ was found for He[102] (Fig. 25). Here in a certain region breakdown occurs only when the potential is *lowered* to a definite value. The reason is, that the ionization probability for electrons in He has a maximum at about 100 v energy; for a value of $E/p_0$ where the electron energy in the gas is higher, the decrease of the tube potential will not oppose but favor the starting of the discharge, provided $\gamma$ remains constant.[102]

In this section it has been assumed that the law of Paschen holds. It should be remarked, however, that this is only true when the separation of the plates $d$ is small compared with the diameter $D$. In the contrary case at the same value of $p_0d$ the value of $V_B$ is generally higher with larger plate separations because of sideway losses of electrons and ions.[103] In gas mixtures obeying condition (35) still larger deviations were found than in pure gases; here the loss of metastable atoms which are not directed by the electric field may be larger and also the destruction of metastable atoms at the walls may have greater influence.[66, 104]

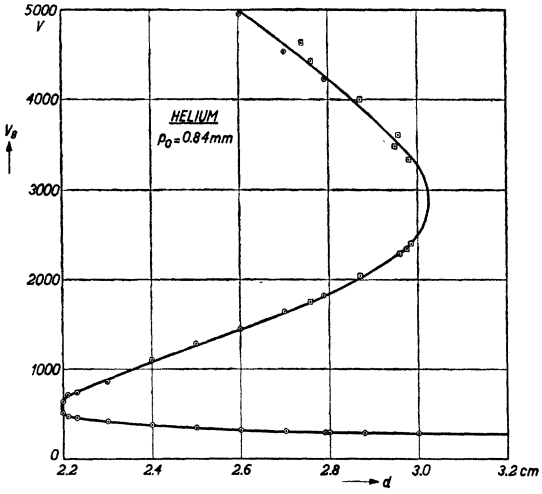


FIG. 25. Breakdown potential $V_B$ as a function of $d$ for He at $p_0=0.84$ mm. For $2.2<d<3$ cm, $V_B$ has three different values, one of them corresponding to breakdown at decreasing anode voltage.

## §24. Influence of irradiation on $V_B$

*Irradiation of the cathode.*—At low pressures irradiation of the cathode diminishes the time lag, but generally has little influence on the value of $V_B$ (§16). Only for large photoelectric cathode emissions $i_0$ of, say, $10^{-8}$ amp./cm$^2$, which may be realized in photoelectric cells or by very strong irradiation, the value of $V_B$ may be diminished considerably. We define, according to §16, the breakdown potential with radiation $V_{Bi}$ for a negative characteristic as the maximum voltage which is reached with increasing current. Then experimentally the difference between $V_{Bi}$ and the value $V_{Bo}$ without radiation proves to be:[105]

$$V_{Bo}-V_{Bi}=C_1 i_0^{\frac{1}{2}} \qquad (62)$$

[100] W. O. Schumann, *Elektrische Durchbruchfeldstärke von Gasen* (1923).

[101] On the other hand it has been shown (W. Ch. van Geel, Ingenieur **50**, E9 (1935)) that for *thin* layers of a solid the breakdown field strength depends on the cathode material.

[102] F. M. Penning, Proc. Roy. Acad. Amsterdam **34**, 1305 (1931). As the plate distance is not small here in respect to the plate diameter (4 cm) the curve of Fig. 25 is not the same as for infinite plates. The abnormal curve for $V_B=F(p_0d)$ and the corresponding one for $V_B=f(E/p_0)$ are indicated in Fig. 22 by dotted lines. The minimum in $p_0d$ for $V_B=f(p_0d)$ occurs when $dV_B/d(E/p_0)=V_B/(E/p_0)$; this corresponds to the maximum of $\alpha=\eta E$ as a function of $E/p_0$, in the case that $\gamma$ remains constant. Generally, however, at high values of $E/p_0$ $\gamma$ increases strongly and this explains why the $V_B$ curve in Fig. 25 bends back again at $d=3$ and $V_B=3000$ v. Apparently in most gases the increase of $\gamma$ is so strong as to mask the influence of the decrease in $\alpha$ (perhaps also $\zeta$ (§19) comes into play); therefore this peculiar form of the Paschen curve is found only in a few cases.

[103] S. P. McCallum and L. Klatzow, Phil. Mag. **17**, 291 (1934). This is perhaps also partly the reason for the deviations from Paschen's law found by Fricke, reference 96, in $H_2$, as for his largest distances $D/d$ is only 2.

[104] F. M. Penning, Proc. Roy. Acad. Amsterdam **34**, 1305 (1931); H. Büttner, Zeits. f. Physik **111**, 750 (1939).

[105] R. Schade, Naturwiss. **24**, 813 (1936); Zeits. f. Physik **105**, 595 (1937). For air at atmospheric pressure this relation had been found and explained earlier by Rogowski and his co-workers, see W. Rogowski and A. Wallraff, Zeits. f. Physik **97**, 758 (1935); **102**, 183 (1936); **108**, 1 (1937); Naturwiss. **25**, 448 (1937); **27**, 302 (1939). Cf. H. J. White, Phys. Rev. **48**, 113 (1935).

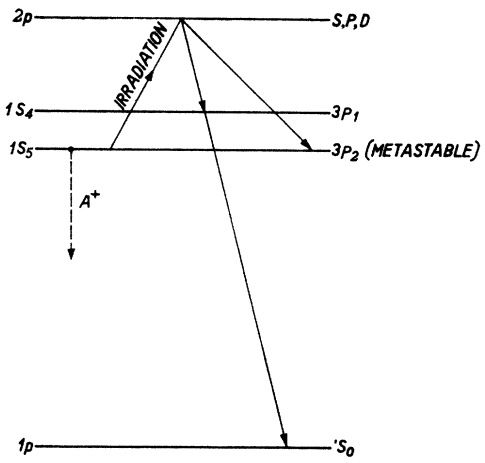


FIG. 26. Decrease of the concentration of metastable Ne atoms ($1s_5$) by irradiation with Ne light ($1s_5-2p$) causes transitions $1s_5\rightarrow 2p\rightarrow 1s_4\rightarrow 1p$.

and the current $i_B$ at the potential $V_{Bi}$:

$$i_B = C_2 i_0^{\frac{1}{2}}, \tag{63}$$

where $i_0$ is the primary current and $C_1$, $C_2$ are constants. The same suppositions which are used to explain the linear negative characteristic of the Townsend discharge (§26), also lead to Eqs. (62) and (63).

*Irradiation of the gas.*—The ionization potential of most gases is so high, that radiation of the corresponding wave-length is unable to penetrate the walls of the discharge tube. Therefore this case, where $V_B$ should be lowered by irradiation, need not be discussed further.

Irradiation of the gas, however, may also increase $V_B$ and decrease the current of the Townsend discharge (negative photo-effect). This occurs in gas mixtures in which the molecules of the admixture are ionized by the metastable states of the main gas (§12) when the discharge space is irradiated by the light of this main gas.[106]

Figure 26 shows the mechanism in the case of Ne with a little A. To increase $V_B$ the concentration of the metastable atoms in the $s_3$ and $s_5$ state[107] should be diminished. This may be done by irradiation of the mixture by the red-yellow neon light (transitions $1s-2p$). This brings a number of the metastable atoms into one of the ten $2p$ states, from which they may return via the $s_4$ level to the normal state and so are lost for ionization of A atoms.

Figure 27 gives an example for the breakdown potential in Ne-A with various intensities of the illuminating neon light. It can be shown that the phenomenon can be described in a simple way by comparing the admixture $a$ in the case of radiation (intensity $L$), with the admixture $a_0$ (without radiation) giving the same value of $V_B$. Then approximately the following formula holds:

$$a/a_0 = 1 + Ll/n, \tag{64}$$

$ndt$ being the probability that a metastable atom is destroyed in the time $dt$ by a collision with the wall or with another Ne atom; $Lldt$ the probability that it is destroyed by the irradiation. For small values of $a_0$ and $L$ the results of Fig. 27 are in accord with this formula.[108]

## §25. Development of the Townsend discharge

As stated in Section 2 the number of primary electrons formed by natural means (cosmic rays, etc.) amounts to only a few per cm³. Hence a certain time elapses between the application of the voltage and the moment that the primary current of a few electrons/sec. has increased to a Townsend discharge of, say, $10^{-7}$ amp./cm². This so-called time lag, consists of two parts: (1) the statistical lag, due to the circumstance that on the average a finite time interval will elapse before an electron will appear which gives rise to sufficient multiplication; and (2) the formative lag, necessary for the development of the already multiplied, but still very small, current into the current of the Townsend discharge; contrary to the statistical lag it has a constant value when it is measured under the same circumstances.

*Statistical lag.*—The statistical time lag was for the first time measured and calculated by Zuber and von Laue.[109] The probability for the starting of the discharge during the time interval

[106] F. M. Penning, Physica (old series) **8**, 137 (1928); **12**, 65 (1932); Proc. Roy. Acad. Amsterdam **32**, 341 (1929); Zeits. f. Physik **57**, 723 (1929); E. W. Pike, Phys. Rev. **49**, 513 and 515 (1936).

[107] Of the two metastable states $s_3$ and $s_5$ for simplicity only one is considered below.

[108] To explain the deviations for larger values of $L$, Pike (reference 106) assumes that the excited $s_4$ level of Ne has also a very long lifetime (about 1/50 that of the $s_5$ level).

[109] K. Zuber, Ann. d. Physik **76**, 231 (1925); M. von Laue, Ann. d. Physik **76**, 261 (1925).

$dt$ proved to be equal to:

$$w(t)dt = WN_0 e^{-WN_0 t}dt, \qquad (65)$$

where $t$ is the time after the application of the anode voltage, $N_0$ the number of primary electrons formed per sec. and $W$ the probability that such a primary electron is sufficiently multiplied. From Eq. (65) follows for the mean statistical lag $t_s$:

$$t_s = 1/WN_0. \qquad (66)$$

The constant $W$ in this equation was calculated by Braunbek[110] on the basis of Townsend's theory and recently in the following more simple way by Hertz.[111] For the sake of simplicity it is supposed that the $N_0$ primary electrons start from the cathode and that they all perform exactly the same number of ionizations $e^{\eta(V-V_0)}-1$ in passing to the anode. Now the mean number of electrons, liberated by the ions due to one

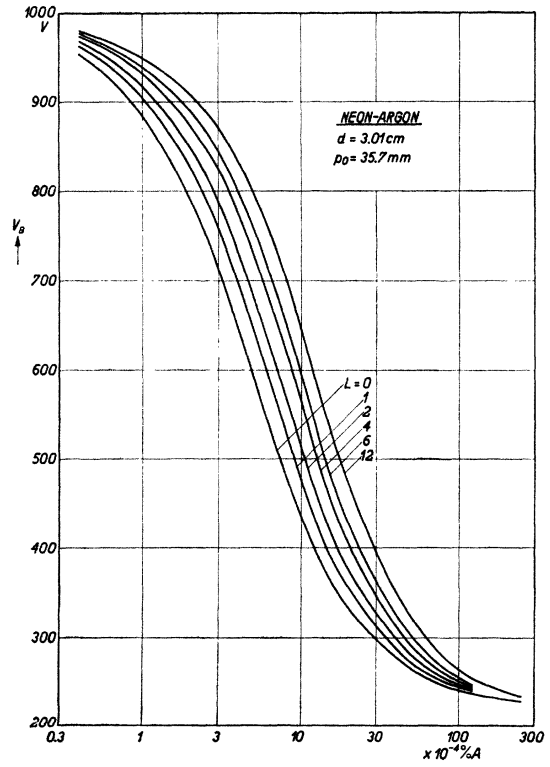


FIG. 27. Breakdown potential $V_B$ of Ne–A mixtures for different irradiation-intensities $L$ (Penning).

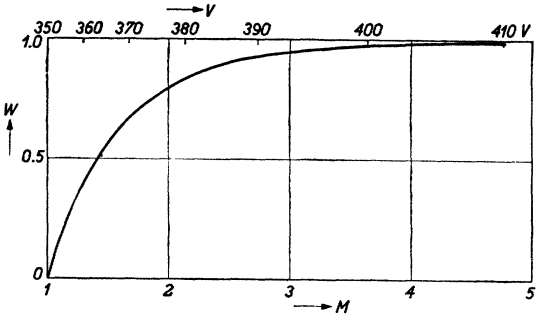


FIG. 28. Values of W, the probability that a primary electron is multiplied into a steady current (Hertz). $M$ = multiplication factor. The upper scale relates to the voltage in a special case (A, $p = 10.6$ mm; $d = 0.8$ cm; $\gamma = 0.02$; $V_B = 350$ v).

primary electron, is equal to the multiplication factor $M = (e^{\eta(V-V_0)}-1)\gamma$ (§16) being 1 for a steady current. This $M$, however, represents only a mean value; for one individual primary electron it is actually equal to 0 or 1 or 2 or another whole number. Each time that it is 0, an electron avalanche stops. The same may also occur for electrons which are liberated by ions of the second, third, etc. ionization cycle. By summing up over all these possibilities the fraction $S$ of the primary electrons which give rise to stopping avalanches may be calculated. Then $W = 1-S$ represents the probability for one electron to start the discharge and it may be shown that:

$$\gamma \ln (1-W)/\ln (1-\gamma W) = M, \qquad (67)$$

which, as $\gamma$ usually is very small, reduces to:

$$-(1/W) \ln (1-W) = M. \qquad (68)$$

Figure 28 gives the value of $W$ as a function of $M$ according to Eq. (67). The upper scale relates to the anode voltages in an example given by Hertz. It is seen that in this case ($N_0 = 10$ electrons/sec.) the time lag decreases from infinity at $V = V_B$ to 1 second for an overvoltage of 2 volts.

The values, found experimentally, are generally still lower, especially shortly after a preceding discharge. This aftereffect has been studied recently by Paetow.[112] One of his results is shown in Fig. 29. Here during 1 sec. a current of 1 $\mu$a was passed between two parallel Ni plates; $t_w$ sec. after extinguishing this current the

[110] W. Braunbek, Zeits. f. Physik **39**, 6 (1926); **107**, 180 (1937).
[111] G. Hertz, Zeits. f. Physik **106**, 102 (1937).
[112] H. Paetow, Zeits. f. Physik **111**, 770 (1939).

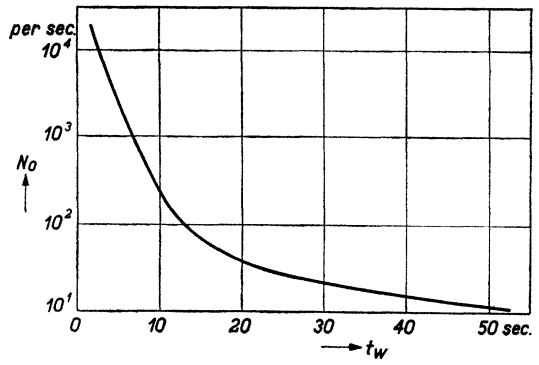


FIG. 29. After-currents ($N_0$ electrons per sec.) according to Paetow; $t_w$=time after the switching off of the main current (1 $\mu$a during 1 sec.).

value of $N_0$ was determined from the statistical time lag $1/N_0W$; the overvoltage being so high that $W=1$ (Fig. 28). Fig. 29 shows that even 50 sec. after the passing of the 1-$\mu$a current the influence had not vanished. The after-current is explained by Paetow as an autoelectronic emission due to surface charges on an impure cathode, analogous to the "spray discharge" (§39). These surface charges could be formed by photons of the preceding main discharge with a wave-length $<2000$A.

*Formative lag.*—While for air at atmospheric pressure and large electrode separations (an extreme case being the lightning discharge) the field distortion by positive ions is considerable (compare §16), we here only consider the case of low pressures, where this field distortion may be neglected and only the electron liberation from the cathode comes into account. In this case the formative lag $t_F$ required for the transition of a small primary current $i_0$ from the cathode into a Townsend discharge may be calculated roughly as follows. One ionization cycle takes a time $t_i$ which is approximately equal to the transit time of an ion from anode to cathode. Therefore a primary current $i_0$ ($N_0$ electrons/sec.) changes in the time $t_i$ after the application of the anode voltage into $i_0(1+M)$, after the time $2t_i$ into: $(1+M+M^2)i_0$ and after the time $t_F=nt_i$ into:

$$i_e=i_0\frac{M^{n+1}-1}{M-1}=i_0\frac{M^{(t_F/t_i)+1}-1}{M-1}. \qquad (69)$$

Generally $i_e/i_0$ will be very large, for example, $10^{10}$ and $t_F/t_i\gg1$ in which case Eq. (69) may be simplified into

$$t_F=t_i\frac{\ln\,(M-1)i_e/i_0}{\ln M}. \qquad (70)$$

Similar formulas were calculated in a more accurate way and tested experimentally by Steenbeck and Schade.[113] The value of $t_i$ derived by these authors from their experiments was of the right order of magnitude and varied in the expected way with the mass of the ions used.[114]

Comparing Eqs. (66) and (68) with (70) we see that $t_S$ and $t_F$ depend in a different way upon $M$ and especially upon $i_0$ (or $N_0$). For the case of the $A$ discharge referred to in Fig. 28, both $t_S$ and $t_F$ were calculated as a function of $N_0$ and $M$ and plotted in Fig. 30. This figure shows that it is possible to determine $t_S$ and $t_F$ independently. For the determination of $t_S$ the value of $N_0$ should be small, for the determination of $t_F$ high. In the experiments of Steenbeck and Schade the high values of $N_0$ were reached by using the after-current from a preceding discharge, referred to above.

## §26. Characteristic of the Townsend discharge

With increasing potential difference the primary electrons due to cosmic radiation, etc., may develop into a small self-sustained continuous current of about $10^{-15}$ to about $10^{-5}$ amp./cm$^2$, which is called the *Townsend discharge* (*BD* or *BD'* in Fig. 1). Space charges have little influence here and a cathode fall has not yet developed.

According to Eq. (50) from the breakdown onward the characteristic should be a straight line parallel to the $V$ axis. As the characteristics of Figs. 1 and 15 show, this is only true in a limited current range. Since the transition to the glow discharge at larger currents is certainly due to space charge, we will here discuss whether this is also the reason for the first small decrease of $V$ which occurs in the Townsend discharge at about $10^{-6}$ amp./cm$^2$.

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[113] M. Steenbeck, Wiss. Ver. Siem. Konz. **9**, 42 (1930); R. Schade, Zeits. f. Physik **104**, 487 (1937). Cf. F. Tank and L. Ackermann, Helv. Phys. Acta **3**, 468 (1930).

[114] Schade's experiments were mainly made with neon which was not absolutely pure. Here a quantitative agreement cannot be expected, as the formative lag depends also on the time which a metastable Ne atom takes to ionize a molecule of the admixture.

In pure gases between parallel plates and values of $p_0d$ which are not too low the experimental characteristic is falling. Now the amount of decrease in $V$ by the field distortion may be calculated with the known values of ion mobilities, etc. from the formula

$$i = i_0 \frac{e^{\int \eta dV} - 1}{1 - \gamma(e^{\int \eta dV} - 1)},$$

which replaces Eq. (50) when the field and therefore also $\eta$ is no longer constant. When $\gamma$, which generally changes only slowly with the field strength, is taken constant it may be shown[115] that the amount of decrease in $V$ below $V_B$, developed with respect to $i$, is equal to

$$V - V_B = -K_1 i^2, \tag{71}$$

where $K_1$ is a positive constant. The first power of $i$ does not appear in Eq. (71) because the decrease of $\eta$ in the region of higher field strength is compensated in first approximation by an increase in the region of lower field strength.

Now it has been stated experimentally[79, 105, 116] that the characteristic for pure rare gases does not obey Eq. (71) but has the form of a straight line:

$$V - V_B = -K_2 i, \tag{72}$$

$K_2$ being a positive constant (Fig. 31).

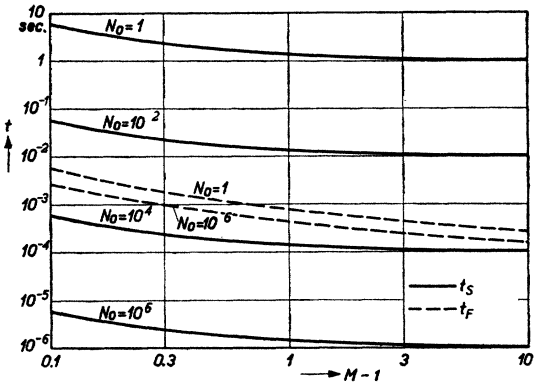


FIG. 30. Values of the statistical lag $t_S$ and the formative lag $t_F$ as a function of the multiplication factor $M$ and the number $N_0$ of primary electrons liberated per sec. from the cathode. The end current $i_e$ has been taken as $10^{-6}$ amp.

[115] A. von Engel and M. Steenbeck, *Elektrische Gasentladungen II* (1934), p. 48.

[116] Compare also J. Taylor, Phil. Mag. **3**, 368 (1927).

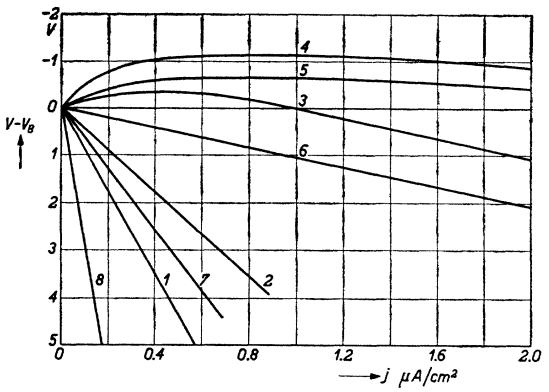


FIG. 31. Characteristics of the Townsend discharge in Ne–A mixtures (Büttner). 1. Ne (pure); 2. Ne+$9\times10^{-4}$ percent A; 3. Ne+$7\times10^{-3}$ percent A; 4. Ne+$3\times10^{-2}$ percent A; 5. Ne+0.13 percent A; 6. Ne+0.7 percent A; 7. Ne+10 percent A; 8. A (pure).

Several mechanisms may be proposed which lead to Eq. (72) and the corresponding[117] Eqs. (62) and (63): (1) a variation of $\gamma$ with $E$ (Rogowski, Schade); (2) an increase of the number $\gamma_r N_2$ of the electrons liberated by radiation quanta, with increasing $E$ at the cathode (Rogowski[118]); (3) an increase of the electron current *between* the plates according to

$$di = \eta i dU + ci^2 dU \tag{73}$$

instead of the formula $di = \eta i dU$ used previously[119] (Schade[120]).

The first explanation may possibly hold for the case of air at atmospheric pressure where the variation of $\gamma$ with $E/p_0$ can be large (§21) but this will not be further discussed here. In the case of the rare gas characteristics, however, it can be left out of account because here the variation of $\gamma$ with $E/p_0$ is much too small[120].

The same is true for the second explanation which, moreover, does not give the right de-

[117] These mechanisms lead namely, for the case of a photoelectric current, to Eq. (62) and Eq. (63). On the other hand the calculation, giving Eq. (71), leads in the case of a photoelectric current $i_0$ to a decrease of $V_B$ proportional to $i_0^{\frac{1}{2}}$ and a breakdown current $i_B$ proportional to $i_0^{\frac{1}{3}}$.

[118] W. Rogowski, Naturwiss. **27**, 302 (1939).

[119] It should be emphasized that for nonirradiated discharges this change does not lead to other values of $V_B$ according to the definition of §16, as in this case the values of $i$ are always so small that the term with $i^2$ has no influence; compare R. Schade, Zeits. f. Physik **104**, 487 (1937).

[120] R. Schade, Naturwiss. **25**, 568 (1937); Zeits. f. tech. Physik **18**, 595 (1937); Zeits. f. Physik **108**, 353 (1938); R. Schöfer, Zeits. f. Physik **110**, 21 (1938).

pendence of the effect on the pressure $p$ at constant $pd$.

The third possibility was tested extensively by Schade[120] and Büttner.[104] According to these authors the term with $i^2$ in Eq. (73) is due to collisions between two metastable atoms, leading to the ionization of one of them. An estimate of the amount of this effect gives the right order of magnitude and also the right dependence on the pressure $p$ when $pd$ is kept constant. Another argument is the decrease of the slope of the characteristic when small amounts of A are added to Ne (Fig. 31); in this case the metastable atoms are destroyed by the A atoms and the influence of the collisions between two metastable atoms is diminished. For certain Ne-A mixtures even a positive characteristic was found.

One point, however, is not clear. It has been shown definitely by Meissner[121] and others[106] that the concentration of metastable atoms may be decreased by irradiation with the light of the same gas (compare §24). It should therefore be expected that illumination of a Ne discharge with Ne light should decrease the slope of the characteristic. This, however, proved not to be true.[120] Moreover, it is not clear why in the case of Ne, the ionization of A atoms by metastable Ne atoms should be diminished by irradiation (§24) and not the ionization of metastable Ne atoms.[122]

### *B. Nonhomogeneous Electric Field*

### §27. Breakdown condition

For non-uniform fields the value of $\eta$ is no longer constant along the path of an electron and condition (51) for homogeneous fields has to be replaced by:

$$\gamma\left(\exp\left[\int_{V_0}^{V_B}\eta dV\right]-1\right)=1$$

or

$$\int_{V_0}^{V_B}\eta dV=\ln(1/\gamma+1). \qquad (74)$$

When the deviation of the field from uniformity is only small the application of this equation offers no difficulties, but in other cases, for example for a wire in a coaxial cylinder (a "small" and a "large" electrode), the following complications occur.[123]

*Concerning* $\eta$.—(I) The correction $V_0$ in Eq. (74) discussed for parallel plates in Section 11 may be taken as zero in the case of small $E/p_0$ values at the cathode (e.g. cylinder cathode, wire anode) because the electrons are speeded up in a region where they do not ionize. In the case of the "small" electrode as cathode $V_0$ cannot be neglected.

(II) When $d(E/p_0)dx$ is large ($x$=distance to the electrode) the electrons have not the velocity corresponding to their position in the electric field but are faster when moving from high to low fields and vice versa; therefore also the value of $\eta$ is other than that corresponding to the $E/p_0$ at the place of the electron.

(III) When the dimensions $a$ of the "small" electrode, used as anode, are small with respect to the mean free path $\lambda_e$ of the electrons, instead of $\int\eta dV$ in (74) a much larger value has to be taken. After collisions with gas atoms, the electrons generally will have a velocity component perpendicular to the electric field; consequently a collision within a distance $\lambda_e$ from the anode, for $a\ll\lambda_e$ will cause many of the electrons to miss the anode and so describe much longer paths and produce more ionizations than in the case of a large anode.

(IV) In gas mixtures obeying condition (35) the place where a new electron is formed by a metastable atom lies at a certain distance from the point where the metastable atom itself has its origin. In the case of strongly inhomogeneous fields this may have a considerable influence on $V_B$.

*Concerning* $\gamma$.—V. At the same value of $V_B$ the value of $\gamma_p$ with the "small" electrode positive is smaller than that with the "small" electrode negative as a consequence of the difference in $E/p_0$ at the cathode.

VI. Especially at larger pressures the liberation of electrons by radiation or metastable atoms may come into play. In this case $\gamma$ has

[121] K. W. Meissner und W. Graffunder, Ann. d. Physik **84**, 1009 (1927).

[122] This kind of ionization is not a resonance effect; as was shown above it has an efficiency of approximately 1 in the case of Ne—A. The surplus energy can here be taken away by the liberated electron.

[123] Compare L. B. Loeb, J. App. Phys. **8**, 495 (1937).

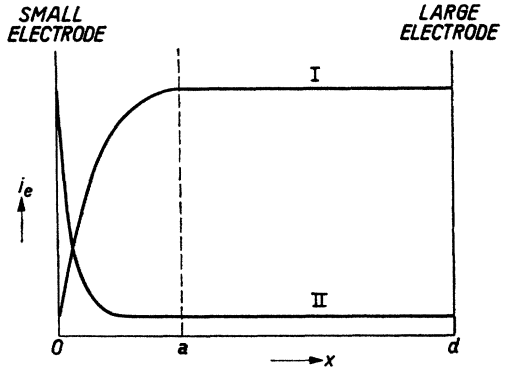


FIG. 32. Electron current $i_e$ in the space between a "small" and a "large" electrode as a function of the distance $x$ to the small electrode. Ionization occurs only for $0<x<a$. The upper curve I is obtained when the small electrode is the cathode, the lower curve II is obtained when the large electrode is the cathode. (Schematic.)

to be written, according to Eq. (59) as:[123]

$$\gamma=\gamma_p+f_rN_r\gamma_r,$$

where $N_r$ is the number of radiation quanta per positive ion arriving at the cathode. The value of $N_r$ may be quite different for the "small" electrode positive and negative, even for the same number of ionizations: $\exp[\int\eta dV]$. This is shown by Fig. 32 where the electron current in the space between the electrodes has been plotted schematically for both cases as a function of the electrode distance $x$, under the assumption that the electron currents at the cathode and the anode and therefore also $\exp[\int\eta dV]$ are the same, and that only for $0<x<a$ is the value of $E/p_0$ large enough to give a measurable value of $\eta$. Now for $a<x<d$, where only excitation occurs, the number of excitations is much larger with the small electrode as cathode, for in this case the region $a$–$d$ is traversed by a strongly multiplied electron current. This effect, however, is again diminished by the influence of the geometrical arrangement which decreases strongly the number of radiation quanta striking the cathode. Nevertheless the combination of both effects e.g. in the case of a wire and a cylinder may give, at the same pressure and potential difference, a larger value for $f_rN_r$ for the wire cathode than for the cylinder cathode.

In the case of low pressures, to be considered in this review, the influence of space charge formation on $V_B$ is probably small when the primary electron emission from the cathode is not artificially increased to high values (compare §16). At high pressure (especially in air) this may be otherwise; the phenomena occurring in this case have been studied recently by Loeb[124] and his co-workers.

## §28. Breakdown between coaxial cylinders

By way of example we now discuss somewhat more in detail the case of coaxial cylinders with radii $R_1$ (inner cylinder) and $R_2$ (outer cylinder). When the correction $V_0$ is neglected, Eq. (74) may be transformed into:

$$\ln\left(\frac{1}{\gamma}+1\right)=\frac{E_1}{p_0}(p_0R_1)\int_{E_2/p_0}^{E_1/p_0}\frac{\eta}{E/p_0}d(E/p_0), \quad (75)$$

where the index 1 refers to the surface of the inner, the index 2 to that of the outer electrode and $E_1R_1=E_2R_2$. When the complications discussed in §27 are left out of account this equation holds both for the inner electrode ("wire") cathode and anode, giving with the same value of $\gamma$ the same value of the breakdown potential $V_B$ in both cases. $V_B$ follows from:

$$V_B=E_1R_1\ln(R_2/R_1)=E_2R_2\ln(R_2/R_1).$$

With $R_2\gg R_1$ and not too low values of $p_0$, $E_2/p_0$ is so small that it may be replaced by 0,[125] as boundary in the integral of Eq. (75). It is clear that in this case the breakdown is determined by the field strength $E_1$ at the wire surface, independent of the radius of the outer cylinder.[126] For the wire cathode this holds apart from the approximations, underlying Eq. (75). For the wire anode, however, $\gamma$ is dependent on $E_2/p_0$ and therefore on $R_2$ so that in this case the value of $R_2$ indeed may have influence.

For all values of $R_1$ and $R_2$, according to the similarity law of §17

$$V_B=f_1(R_1p_0)=f_2(R_2p_0)$$

for $R_2/R_1$ is constant.

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[124] L. B. Loeb and A. F. Kip, J. App. Phys. **10**, 142 (1939); A. F. Kip, Phys. Rev. **54**, 139 (1938); **55**, 549 (1939).

[125] For diatomic gases the replacement of $E_2/p_0$ by 0 gives a better approximation than for the rare gases, as the value of $\eta$ decreases more rapidly with $E/p_0$ (Fig. 9).

[126] L. G. H. Huxley and J. H. Bruce, Phil. Mag. **23**, 1096 (1936).

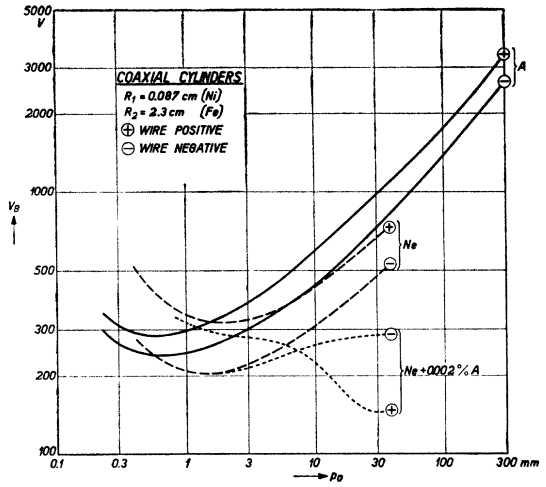


FIG. 33. Breakdown potential $V_B$ between coaxial cylinders with radii $R_1$ and $R_2$ in Ne, A and Ne+0.002 percent A.

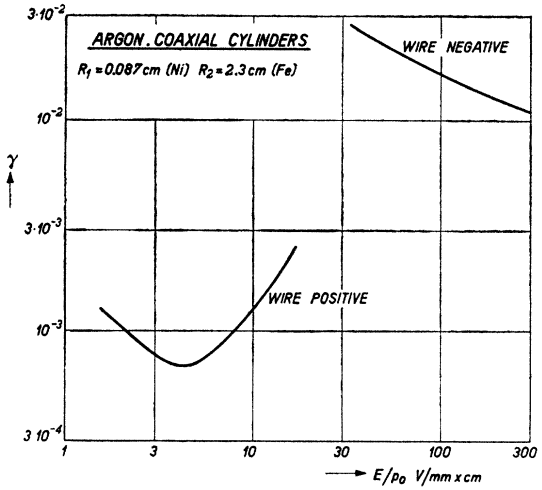


FIG. 34. Values of $\gamma$, calculated with Eq. (14) from the values of $V_B$ in A, given in Fig. 33.

Figure 33 gives a few examples[127] for the values of $V_B$. The curves show the same general features as in the case of parallel plates: a minimum value of $V_B$ at a certain value $(p_0)_{min}$ of $p_0$, an intersection of the curves for Ne and A at low values of $p_0$ and a large influence of a small admixture of A to Ne.

*Pure gases.*—We at first discuss the case of pure gases. The breakdown potential $V_B^-$ for the wire negative is lower than for the wire positive ($V_B^+$). In the neighborhood of $(p_0)_{min}$ this may be due to the difference in the $\gamma$ of the cathode-surface used; in the case of hydrogen Bruce[128] found in this region $V_B^- > V_B^+$. For $p_0 > (p_0)_{min}$, and in pure gases, however, there seems always to be a certain pressure region[129] where $V_B^- < V_B^+$.

When the values of $\gamma$ are calculated from the curves for A in Fig. 33 with Eq. (75) the results, shown in Fig. 34 are obtained. There is a large discrepancy between the values resulting for the wire cathode and the wire anode which cannot be ascribed solely to a difference in the cathode surface. It may be explained on the basis of the effects I, II, V and VI of the preceding section, but the contribution of each of them remains uncertain. For example the value of $\gamma$ is not known with sufficient accuracy.

An example of the case in which the mean free path of the electrons becomes large with respect to $R_1$ (§27, III) is shown in Fig. 35.[127] It is seen that a certain high breakdown potential (wire positive) of say 2000 v, for a wire of 0.005 cm radius is obtained at less than one-tenth of the pressure at which it is obtained for a wire of 0.12 cm radius.

*Gas mixtures, obeying condition (35).*—When a small amount of A is added to Ne, $V_B$ is decreased as a consequence of the ionization of A atoms by metastable Ne atoms. Fig. 33 shows,

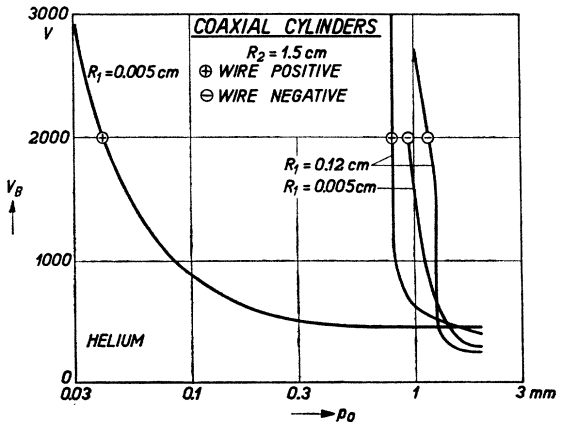


FIG. 35. Breakdown potential $V_B$ between a very thin or a thick wire and a coaxial cylinder.

[127] F. M. Penning, Phil. Mag. **11**, 961 (1931); F. M. Penning, J. Moubis and C. C. J. Addink, Physica (old series) **13**, 209 (1933).

[128] J. H. Bruce, Phil. Mag. **10**, 476 (1930).

[129] In oxygen and air the curves for $V_B^+$ and $V_B^-$ have again an intersection at a value of $p_0$ much higher than $(p_0)_{min}$; H. F. Boulind, Phil. Mag. **18**, 909 (1934); **20**, 68 (1935).

however, that this influence is much larger for $V_B^+$ than for $V_B^-$ so that at higher values of $p_0$ even $V_B^- > V_B^+$. From Fig. 36, where the values of $V_B^-$ and $V_B^+$ are plotted as a function of the argon percentage,[127] it is seen that at the pressure used, the intersection of the $V_B^+$ and $V_B^-$ curves lies at about $10^{-4}$ and 1 percent A.

This different behavior of $V_B^-$ and $V_B^+$ may be explained in the following way. With the wire positive the number of ionizations and excitations due to the avalanche of one electron starting from the cathode increases rapidly with decreasing distance to the wire. Hence the greater part of the metastable Ne atoms lies inside a small cylinder of radius $R_e$ around the wire (Fig. 37). Now the metastable atoms travel a certain distance before ionizing an A atom, this distance being increased further through the diffusion of resonance radiation.[130] When the ionization due to metastable atoms formed at $M$ lie inside a cylinder with radius $R_m$ it is clear from Fig. 37 that a great many of them may lie outside the cylinder $R_e$. The electrons originating in these ionizations have to travel the whole potential difference of the cylinder $R_e$ and so give rise to many more new electrons than when they originated at $M$. This leads to a much larger decrease of $V_B^+$ than of $V_B^-$ when a small amount of A is added to the Ne.

## §29. Corona discharge between coaxial cylinders

The Townsend discharge between a wire and a coaxial cylinder is often called corona discharge because the light is usually concentrated around the wire. Fig. 38 gives a few examples[127] for the characteristic in A with the same electrode configuration as in Figs. 33 and 36 and the wire positive ("positive" corona). The vertical part of the curves shows that here space charge has no influence on the breakdown potential. While at the higher values of $p_0$ the negative corona has a negative characteristic as does the Townsend discharge between parallel plates (§26), Fig. 38 shows that the positive corona has a positive characteristic. This difference may be explained as follows.

*Positive corona.*—The number of ionizations (per primary electron from the cathode) between the wire (radius $R_1$) and a cylinder of radius $R_n$ is given approximately by:

$$\exp\left[E_1R_1\int_{E_n/p_0}^{E_1/p_0}\frac{\eta}{E/p_0}d(E/p_0)\right] \qquad (76)$$

(compare Eq. (75)) and a similar formula with $n=2$ holds for the total ionization between wire ($R_1$) and anode cylinder ($R_2$). With Eq. (76) the percentage $F_i$ of this total number of ionizations occurring between $R_1$ and $R_1+0.2(R_2-R_1)$ at the breakdown potential $V_B^+$ has been calculated and plotted in Fig. 38 (dashed line). The figure shows that at the higher pressures practically all the ionization is limited to the neighborhood of the wire. Now with increasing current the positive space charge diminishes the value of $E$ at the wire (anode) and the number of ionizations in this region. At the cathode side the field is

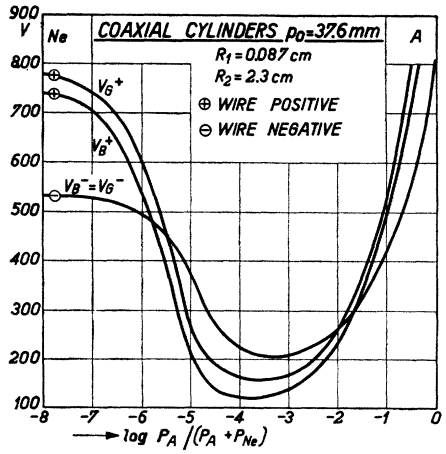


FIG. 36. Breakdown potential $V_B$ and starting potential of the glow discharge $V_G$ for Ne–A-mixtures between coaxial cylinders at $p_0=37.6$ mm.

increased, but as long as the current is small this gives no perceptible contribution to the ionization because in the cathode region $\eta$ is to be neglected. So for small currents the resultant effect of space charge is to diminish the number of ionizations and the voltage has to be increased in order to maintain the current.

The numerical value of this increase $V-V_B$ along the characteristic may be calculated with certain approximations by Townsend's formula[131]

[130] I. Langmuir and C. G. Found, Phys. Rev. **36**, 604 (L) (1930); F. M. Penning, Zeits. f. Physik **78**, 454 (1932); C. Kenty, Phys. Rev. **43**, 181 (1933).

[131] J. S. Townsend, Phil. Mag. **28**, 83 (1914). Compare especially for air at atmospheric pressure W. Rogowski, Arch. f. Elektrotech. **29**, 130 (1935); N. Kapzow, Physik. Zeits. Sowjetunion **11**, 95 (1937).

$$\frac{V-V_B}{V_B}\ln\frac{R_2}{R_1}=(1+\vartheta)^{\frac{1}{2}}-1+\ln\frac{2}{1+(1+\vartheta)^{\frac{1}{2}}} \quad (77)$$

with

$$\vartheta=2.37\times10^{9}\frac{Ip_0}{\mu_p V_B^2}(R_2\ln R_2/R_1)^2 \text{ (volt, amp.)}.$$

$I$ = current per cm length of the wire ($=i/5$ for Fig. 38); $\mu_p$ = mobility of the positive ions (cm/sec. at 1 v/cm and 1 atmos.). If we assume $\mu_p=2.6$ cm/sec.[132] Eq. (77) gives the points indicated by crosses. At the higher pressures the agreement is satisfactory for $10^{-7}<i<10^{-5}$ amp.

When the current increases, Eq. (77) no longer holds because the value of $E/p_0$ at the cathode becomes high enough to give ionization; finally the positive space charge increases the total ionization so that $V$ passes through a maximum ($V_{\max}$), the characteristic becomes negative and the corona changes into a glow discharge. It is clear that for these pressures the starting potential for the negative glow ($V_G^+$) is equal to the maximum voltage reached in the characteristic and is higher than the starting potential for the corona discharge $V_B^+$. In Fig. 36 besides $V_B^+$ $V_G^+$ is also given.

Figure 39 shows the light phenomena for the case of Ne+0.002 percent A, which indicate the field distribution between the cylinder and the wire. At 1 $\mu$a there is a faint light only in the immediate neighborhood of the wire; with increasing current the region of maximum light intensity moves toward the cathode, indicating

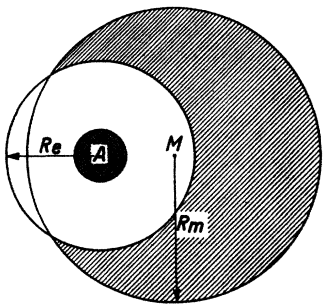


FIG. 37. Ion formation by metastable atoms around a wire A as anode.

that the field strength at the anode is diminishing. At 250 $\mu$a the discharge resembles a glow discharge; the current density, however, is still less than 0.01 times that of the glow discharge, which has developed at 750 $\mu$a. The negative glow is concentrated here in a small spot, the remaining faint light being due to electrons liberated from the cathode probably by positive A ions due to metastable atom collisions and diffusion of resonance radiation.[130].

Up to now we have considered the characteristics for the positive corona only for higher pressures in Fig. 38. At lower pressures the ionization is no longer confined to the neighborhood of the wire and deviations from Eq. (77) are to be expected ($p_0=28.9$ mm). At $p_0=4.61$ mm the value of $E_1/p_0$ at the surface of the wire is already so high, that $\eta$ has passed its maximum; here the positive space charge, which diminishes $E_1/p_0$, increases the number of ionizations and the characteristic becomes negative. At these low pressures $V_B^+=V_G^+$.

*Negative corona.*—Townsend showed that Eq. (77) could be applied also to the negative corona, provided the current is transported through the region of small $E/p_0$ by negative ions. This is the case for electronegative gases as $O_2$ or mixtures with these gases[133] at sufficiently high pressure. So in air at atmospheric pressure both the negative[131] and the positive corona have a positive characteristic.

In the contrary case, when the electrons do not combine to form negative ions, as for example in pure rare gases, the negative corona has, at higher pressures, a negative characteristic, as is easily explained by a reasoning analogous to that for the positive characteristic of the positive corona. In the case of Fig. 38 the characteristic was negative for $p_0<10$ mm, but it could be measured only in a very limited current range because of the transition to an intermittent discharge. For these pressures the starting potential of the glow discharge is the same as that of the corona discharge ($V_B^-=V_G^-$, Fig. 36). At $p_0<1$ mm the characteristic proved to be positive (compare the negative characteristic of the positive corona at the lowest pressure in Fig. 38).

[132] This value is considerably higher than that given usually for $A^+$ ions in the literature (1.4 cm/sec.); possibly this is due to the higher purity of the gas in our case, compare A. M. Tyndall, *The Mobility of Positive Ions in Gases* (1938); M. J. Druyvesteyn, Zeits. f. Physik **73**, 33 (1931)

[133] K. Lange, Arch. f. Elektrotech. **31**, 411 (1937).

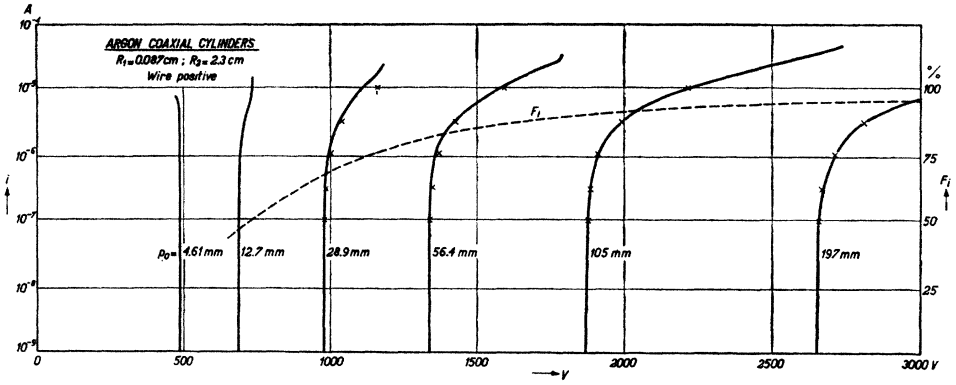


FIG. 38. Characteristics of the positive corona in A between coaxial cylinders. The points indicated by $x$ are calculated with Eq. (77). $F_i$ = percentage of ionizations occurring between $R_1$ and $R_1+0.2(R_2-R_1)$.

We finally remark that a small positive corona current especially in diatomic gases at high pressure, is subject to statistical fluctuations which lead to a statistical extinction of the discharge (Geiger-Müller counter). The higher the current, the higher is the mean time during which the discharge is maintained. For a certain electrode configuration and a gas pressure of 80 mm $H_2$ Werner[134] found that this time changed from $10^{-6}$ sec. at a current of 0.1 $\mu$a to $10^{-2}$ sec. at a current of 1 $\mu$a. According to van Geel and Kerkum and to Schade[135] the extinction of the current is due to statistical fluctuation of the number of electrons liberated from the cathode. When no one of the ions, photons, or metastable atoms, present during a certain time interval in the gas, liberates an electron from the cathode, the current extinguishes. The mean lifetime of the discharge in diatomic gases, calculated in this way, does not agree with the observation, when it is supposed that the electrons are liberated from the cathode by positive ions. For an electron liberation by photons, however, Schade finds a good agreement between theory and experiment.[136] Van Geel and Kerkum try to solve the difficulty by assuming that in the region of low $E/p_0$ the electrons are transformed into negative ions which may recombine with positive ions.

[134] S. Werner, Zeits. f. Physik **90**, 384 (1934); **92**, 705 (1934).
[135] C. van Geel and J. Kerkum, Physica **5**, 609 (1938); R. Schade, Physik. Zeits. **39**, 908 (1938).
[136] The value of $\gamma$ which has to be assumed in this case is much less than in the case of electron liberation by positive ions. Still it seems to us rather low as compared with the values following from Werner's experimental curves.

## V. Discharges Essentially Determined by Space Charge

### §30. General equations. General features of the discharge

In the preceding section a number of properties of the Townsend discharge were roughly derived from the fundamental processes. Our knowledge of the mechanism of discharges with higher current density, however, is much less advanced. This is partly due to the fact that the fundamental processes, essential for the mechanism of the glow discharge and the arc, are only

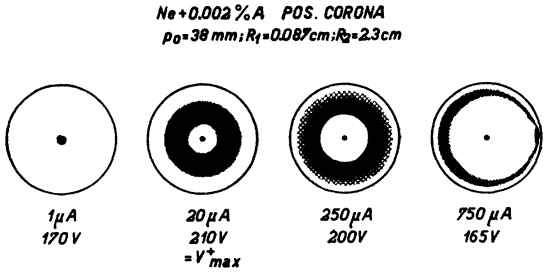


FIG. 39. Light phenomena in the positive corona in Ne—A. At 750 ma the glow discharge has developed.

in small part quantitatively known, and partly to the mathematical difficulties which arise in the description of these discharges.

For discharges at a higher current density the effect of space charge is essential, as it determines largely the electric field, which is calculated from the equation of Poisson:

$$\Delta V = -\operatorname{div} \mathbf{E} = 4\pi e(n_e - n_p). \qquad (78)$$

$V$ is the potential, $\mathbf{E}$ the field strength (a vector) and $n_e$ and $n_p$ the concentrations of the electrons and the positive ions; we suppose the negative atomic ions to be absent. If the numbers of particles crossing 1 cm² per sec. are $\mathbf{J}_e$ and $\mathbf{J}_p$, we obtain for a not too low pressure:

$$\begin{aligned}\mathbf{J}_p &= n_p\mu_p\mathbf{E},\\ \mathbf{J}_e &= -n_e\mu_e\mathbf{E}-D\ \mathbf{grad}\ n_e.\end{aligned} \tag{79}$$

$\mu_p$ and $\mu_e$ are the mobilities, $D$ the diffusion coefficient for the electrons; in most cases the diffusion term for the ions can be neglected.

If the number of ionizations per cm³ per sec. is $z$, we get in the steady state:

$$\text{div}\ \mathbf{J}_p=\text{div}\ \mathbf{J}_e=z. \tag{80}$$

In different cases $z$ can be given by different functions. If only ionization by electrons occurs, if the velocity distribution of the electrons be steady in the sense used in Section II, and if the diffusion term in (79) be neglected, $z$ is determined by the ionization coefficient $\eta$:

$$z=\eta|J_e||E|=\mu_e n_e\eta E^2=n_e f|E|. \tag{81}$$

Since at a constant pressure $\eta$ depends only on $E$, $z/n_e$ will be a function[137] of $E$ only. A combination of Eqs. (78) to (81) and elimination of $J_p$ and $J_e$ give us three equations for $V$, $n_e$, and $n_p$. As yet no attempt at a general solution of these equations in the three-dimensional case has been made.

We shall illustrate the use of these equations for a simple one-dimensional case.[138] We consider the values of $J_p$, $J_e$, etc. in the axis of a long tube with two flat electrodes as a function of the distance $x$ to the cathode, the direction in which the variables are taken positive being indicated in Fig. 40. An equal number of electrons and

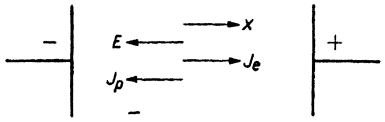


FIG. 40. Directions for positive values of $E$, $J_e$, $J_p$ and $x$.

[137] We may as well assume only that $z/n_e$ is a function of $E$ without taking for $E$ the special form of Eq. (81).

[138] W. O. Schumann, Zeits. f. tech. Physik **11**, 194 (1930); W. Rogowski, Arch. f. Elektrotech. **26**, 643 (1932) and the other literature, cited in Table VIII (§38).

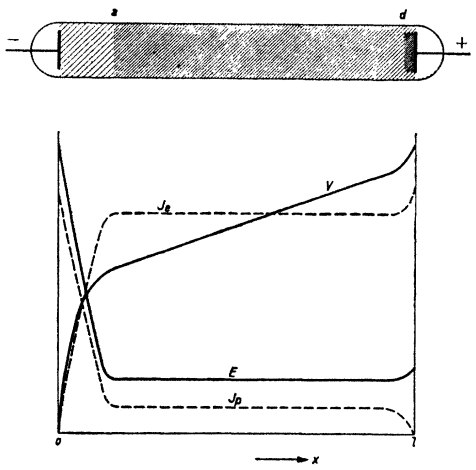


FIG. 41. Light emission, potential ($V$), field strength in the axis ($E$), current density of the electrons ($J_e$) and of the positive ions ($J_p$) that would appear in a long tube if no electron diffusion existed and if the ionization only depended on $E$.

positive ions will go to the wall. Let $J_w$ be this current per cm of the tube. Then we get:

$$\begin{aligned}&dE/dx=4\pi e(n_e-n_p),\\ &\frac{d(\mu_e n_e E)}{dx}-D\frac{d^2n_e}{dx^2}=z-cJ_w,\\ &J=\mu_p n_p E+\mu_e n_e E-Ddn_e/dx.\end{aligned} \tag{82}$$

$c$ is a constant, $J=J_e+J_p$ is the total current-density which is independent of $x$. If we first take $D=0$, set $J_w$ proportional to $n_e$ ($J_w=c_1n_e/c$), take for $z$ Eq. (81) and assume $\mu_e$ and $\mu_p$ independent of $E$,[139] we get for $E$:

$$\frac{d}{dx}\left(E\frac{dE}{dx}\right)=\left(\frac{4\pi eJ}{\mu_e\mu_p}+\frac{E}{\mu_e}\frac{dE}{dx}\right)\left(\mu_e\eta E-\frac{C_1}{E}\right). \tag{83}$$

We will take as boundary condition for the cathode ($x=0$):

$$J_e=\gamma J_p=\gamma(J-J_e) \tag{84}$$

and for the anode ($x=l$):

$$J_p=0,\quad J_e=J. \tag{85}$$

[139] Equation (83) is only slightly altered by accepting other functions for $J_w$, $\mu_e$ and $\mu_p$, e.g. Rogowski, reference 138, assumes $\mu_e$ and $\mu_p$ to be inversely proportional to $E^{\frac{1}{2}}$. He obtains an equation for $E$ which can be obtained from Eq. (83) by changing $EdE/dx$ into $E^{\frac{1}{2}}dE/dx$ and omitting the term due to the wall current. For the theory of the cathode fall Rogowski's assumptions are nearly correct; for the positive column the assumptions made in the text are probably better.

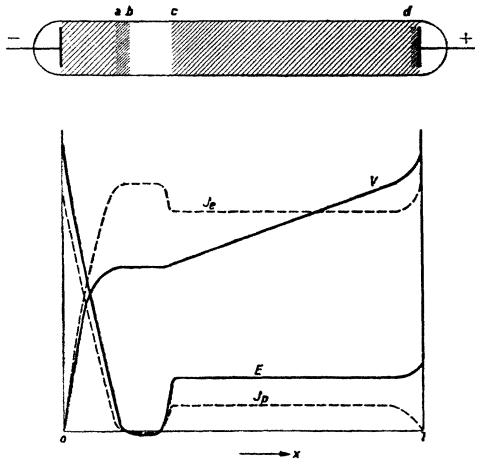


FIG. 42. Light emission, etc. as in Fig. 41 taking into account the influence of the electron diffusion and the dependence of ionization on $E$ and $V$.

The solution of an equation analogous to (83) is discussed by Rogowski.[138] Fig. 41 shows the results for $V$, $E$, $J_e$ and $J_p$ as a function of $x$ the intensity of the light emission in the tube is indicated by different shading. It is seen that three parts may be distinguished; a cathode fall, an anode fall and a part between $a$ and $d$, where $E$ has the constant value $E_1$ (the positive column),[140] the last term in Eq. (83) is zero for this part, $\mu_e \eta E_1^2 = C_1$.

We will now try to estimate qualitatively the influence of the most important omissions: the diffusion of the electrons[141] and the ionization function $z$. As the thickness of the cathode fall is small, a number of fast electrons will pass the plane $a$ in Fig. 41 and the number of ionizations near $a$ will be much larger than is given by Eq. (81). The function $z$ will be not only a function of $n_e$ and $E$, but will depend on the whole field between the cathode and $a$. A high electron concentration will occur between $a$ and $b$ (Fig. 42) and the diffusion term for the electrons will allow an electron current to pass without a field[142] or even in a direction opposite to the field from $b$ to $c$. Since at this moment it is impossible to take these factors into account quantitatively, it can only be guessed that the solution for $V$, $E$, $J_p$ and $J_e$ will be as shown in Fig. 42.

The values of Fig. 42 are qualitatively in agreement with the experimental facts, which show that in a glow discharge the following parts are to be distinguished: The *cathode fall* or cathode dark space (also called Crookes or Hittorf dark space) between the cathode and $a$, with a high field strength; the *negative glow* ($ab$), where ionization and excitation are for the greater part due to the fast electrons arriving from the cathode fall; the *Faraday dark space* ($bc$) where the energy of the electrons is too small to excite gas atoms, the electron current between $a$ and $c$ being largely a diffusion current; the *positive column* ($cd$) with a constant field strength; the *anode glow* near the anode. The three last-mentioned parts may also be absent. Further discussion of these different parts is given in the following sections.

If for any reason the cathode emits a number of electrons much larger than Eq. (84) states, we may obtain an arc with a much lower cathode fall. The part of the discharge between the cathode and $b$ will differ from this part in a glow discharge.

The thickness of the cathode fall in the glow discharge is neither large nor small compared with the mean free path of the ions ($\lambda_p$) but lies between $4\lambda_p$ and $20\lambda_p$. This makes the mathematical treatment difficult. In the arc the thickness of the cathode fall is often smaller than $\lambda_p$, and the mathematical treatment is much simpler.

## §31. Space-charge layers

One of the essential features of discharges can be seen from Figs. 41 and 42: the discharge consists of a part (between $a$ and $d$) with small field strength and small space charge ($|n_p - n_e| \ll n_p$), and layers where the space charge is large ($n_p \gg n_e$ or $n_e \gg n_p$). These layers are generally only near the electrodes and also near the tube wall. For the last layer see Section 53.

The part of a discharge where the space charge is small and $n_p$ almost equal to $n_e$ is usually called a plasma. In a plasma the electrons often have a Maxwell distribution. For a general discussion of

[140] Rigorously $E$ is always larger than $E_1$ and has a minimum; the minimum, however, can be very flat and the difference in the minimum between $E$ and $E_1$ can be made infinitely small.

[141] By taking into account the diffusion term, we get an equation of the third order for $E$ and we need three boundary conditions; as third condition we may take $n_e = 0$ at the anode.

[142] It may be that under certain circumstances a maximum of $E$ will arise near $c$. There are experimental indications for the existence of this maximum.

TABLE V. *Equations for space charge layers.*

| | PARTICLES | VALUE OF $x$ WHERE $E=0$ | (a) $\lambda \gg d$ $v_e=(2eV/m_e)^{\frac{1}{2}}$; $v_p=[2e(V_d-V)/m_p]^{\frac{1}{2}}$ | (b) $\lambda \ll d$ $v_e=\mu_e E$; $v_p=\mu_p E$ |
|---|---|---|---|---|
| I | Electrons or ions | 0 (electrons) or $d$ (ions) | (86) $j=\frac{1}{9\pi}\left(\frac{2e}{m}\right)^{\frac{1}{2}}\frac{V^{\frac{3}{2}}}{d^2}$ | (87) $j=\frac{9\mu V^2}{32\pi d^2}$ |
| II | Electrons and ions Electron current limited by space charge | 0 | more complicated equation | (88) $\left\lvert\frac{j_e}{\mu_e}-\frac{j_p}{\mu_p}\right\rvert=\frac{9V^2}{32\pi d^2}$ |
| III | Electrons and ions Both currents limited by space charge | 0 and $d$ | (89) $\frac{j_p}{j_e}=\left(\frac{m_e}{m_p}\right)^{\frac{1}{2}}$ $j_e=\frac{1.86}{9\pi}\left(\frac{2e}{m}\right)^{\frac{1}{2}}\frac{V^{\frac{3}{2}}}{d^2}$ | (88a) $\frac{j_e}{\mu_e}=\frac{j_p}{\mu_p}$ $j_e=j_p=\infty$ |

the plasma we refer to the review of Rompe and Steenbeck.[143]

The space-charge layers are very important for an understanding of discharges; we shall therefore summarize some equations for space-charge layers in simple cases.[6] We consider two flat parallel plates (the cathode at $x=0$, the anode at $x=d$), the potential difference between them being $V_d$. The cathode emits electrons (mass $m_e$, charge $e$), the anode emits ions (mass $m_p$, charge $e$); the initial velocity is supposed to be zero. We distinguish two cases: (a) no collisions with gas atoms ($\lambda \gg d$), (b) so many collisions with gas atoms ($\lambda \ll d$) that the velocity of electrons and ions is proportional to $E$ ($v=\mu E$; $\mu=$mobility), while no ionization occurs in the layer. The equation obtained when either electrons or ions or both of them are emitted, are summarized in Table V.[144]

In case IIb the electron current is supposed to be limited by space charge; the same formula holds when the ion current is limited by space charge and the electron current arbitrary. It is seen that very large currents can traverse the gas when $j_e/\mu_e$ is almost equal to $j_p/\mu_p$ (case IIIb). This shows that the effect of introducing particles of another sign is much larger in the case of high pressure than in the case of low pressure, the difference between Eq. (86) and Eq. (89) being only a factor 1.86.

[143] R. Rompe and M. Steenbeck, Ergebn. exakt. Naturwiss. **18**, 257 (1939).

[144] For the case (a) we refer to the review of Langmuir and Compton, reference 4; the equations for case (b) are found directly from the Poisson eqnation $dE/dx=4\pi\rho=4\pi j/v=4\pi j/\mu E$.

## §32. Similarity principle

The relations between the variables of a gas discharge may be simplified considerably by using the *similarity principle* which has the following content. When a given discharge $A$ is physically possible, under certain conditions a "similar" discharge $B$ is possible in which the gas density is multiplied by a factor $a$, as compared with case $A$, the linear dimension by a factor $1/a$, the space charge density by a factor $a^2$, etc.[145] The potential differences between corresponding points of $A$ and $B$ are the same, as also are the velocities of the electrons and ions in corresponding points and the current to the electrodes. The most important "conditions" referred to above are that there should be no excitation or ionization of excited atoms by electrons (cumulative ionization or excitation), and no recombination of electrons and ions. For the glow discharge between parallel plates the similarity principle states for example that $V_c$ is a function of $j/p_0^2$ ($j=$current density at the cathode) (§34).

# VI. CATHODIC PART OF THE GLOW DISCHARGE

## §33. Introduction

As for many other discharges a short and at the same time adequate definition of a glow discharge does not exist. Usually it is defined as a discharge with a relatively small current density at the cathode ($<10^{-1}$ amp./cm$^2$), and a cathode

[145] For a general treatment see A. v. Engel and M. Steenbeck, *Elektrische Gasentladungen II* (1934), p. 95–102.

fall of 50 v or more, where the electrons are liberated from the cathode mainly by positive ions ($\gamma_p$-mechanism). According to this definition a "spray" discharge would not be a glow discharge, nevertheless it is usually considered as such and we shall therefore discuss it in this Section 39.

As in the Townsend discharge, in addition to the liberation of electrons by ions ($\gamma_p$), the liberation of electrons by fast atoms, or by radiation or by excited molecules from the Crookes dark space and the negative glow may take place. When the energy of the positive ions becomes large enough they may even produce ionization in the gas, as well as excitation (cathode glow, §34). As we do not know the quantitative influence of these effects, we shall in the following not use the quantity $\gamma_p$ (number of electrons liberated by one ion) but $\gamma$ (number per ion), $\gamma_p$ being usually the greater part of $\gamma$.

One of the most important quantities in the glow discharge is the *cathode fall* $V_c$, the large potential difference between the cathode and the negative glow. It can be measured in different ways which give results differing by a few volts. For most discussions such a difference is immaterial and it is not necessary to state whether the cathode fall is the potential difference between the cathode and the boundary of the negative glow or the middle of the negative glow where a potential maximum exists.

A characteristic feature of the glow discharge is that two cases must be distinguished: a "normal" discharge with "normal" cathode fall and an "abnormal" discharge with "abnormal" cathode fall. In the normal discharge the current goes only to a part of the cathode, the surface of the covered part being proportional to the current; the normal cathode fall is nearly independent of the current and the pressure. If the current is increased above the value at which the whole cathode is covered by the discharge, not only the current density but also the cathode fall will increase and we obtain an abnormal discharge.

For the very extensive literature on glow discharges we refer to the handbooks and reviews.[1,2] In the following we shall discuss only such experiments and theories as seem to us essential for an understanding of the discharge mechanism, summarizing only shortly some of the older results. We shall confine ourselves mainly to discharges with a plane cathode at a sufficient distance from the anode and discuss separately some other cases (§39).

## §34. Optical phenomena

As was shown schematically in Fig. 42, the existence of a cathode fall produces in the gas a number of layers with very different light intensity. We shall not discuss the spectroscopic results but shall by way of example treat the visual phenomena in a Ne discharge.[146] Many of the effects observed appear also in other gases under somewhat different circumstances. In the Ne discharge the light emission is mainly due to the red and yellow $1s-2p$ lines (Fig. 26).

The optical phenomena are different for a cathode fall $V_c$ in the neighborhood of the normal value ($100 < V_c < 200$ v) and for a higher cathode fall ($V_c > 300$ v) (Fig. 43). In both cases a region

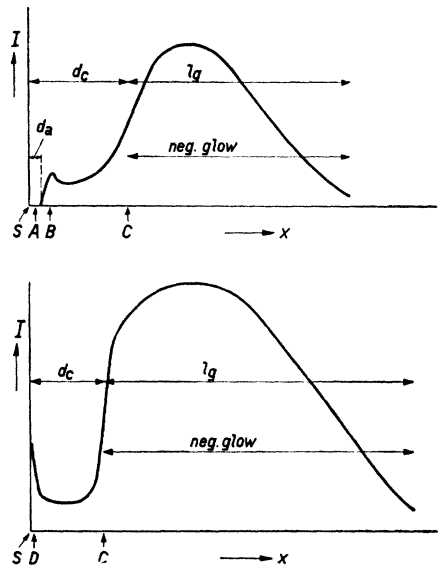


FIG. 43. Light intensity ($I$) as a function of the distance to the cathode ($x$) for a normal (above) and an abnormal discharge ($V_c \approx 400$ v, below) in Ne (schematically). $S$ is the surface of the cathode, $A$ the Aston dark space (length $d_a$), $B$ the first cathode layer, $D$ the cathode glow; the Crookes dark space (length $d_c$) extends from $S$ to $C$.

of high light intensity (*negative glow*; yellow in Ne) is separated from the cathode by a more or less dark space emitting some yellow light

[146] M. J. Druyvesteyn, Physica (old series) **11**, 129 (1931); Zeits. f. Physik **57**, 292 (1929); M. J. Druyvesteyn and N. Warmoltz, Zeits. f. Physik **68**, 378 (1931).

(*Crookes, Hittorf or cathode dark space*). The boundary between the regions, however, is much sharper for the higher than for the lower cathode fall. The distance between this boundary and the cathode is in the following called the thickness of the Crookes dark space $d_c$; it decreases with increasing $V_c$. The light phenomena in the Crookes dark space are different in the two cases: at the lower $V$ we observe, beginning with the cathode, successively a thin absolutely dark layer $A$ (*Aston dark space* or *primary dark space*), a thin yellow layer $B$, which we shall call the *first cathode layer*, and a region of again lower light intensity. After the first cathode layer a second and a third may be faintly observed. At higher $V_c$, the Aston dark space and first cathode layer are no longer visible, but we see a bright red layer on the cathode, which is called the *cathode glow D*.

The light intensity of the negative glow diminishes, after having reached a maximum, in the direction of the anode. If the Ne contains a little impurity, between the negative glow and the positive column a dark space exists (*Faraday dark space*). In pure Ne this Faraday dark space is filled with red light (*aureole*).

The yellow light in the discharge may be ascribed to excitation by electrons. As they leave the cathode with very small energy they cannot excite atoms before having traversed a potential difference approximately equal to the excitation potential for visible light. This explains the Aston dark space. The decrease of light intensity after the passing of the first cathode layer may be ascribed to the circumstance that the excitation function often has a maximum a few volts above the excitation potential. At the low value of $V_c$ the electron current increases markedly in the direction of the anode by ionization, therefore the light intensity too increases, passing gradually into the negative glow. At the higher $V_c$, however, the increase of the electron current in the Crookes dark space is but small (§37). In the negative glow the electron current is large and the field strength small, so the electrons will diffuse here, giving a high concentration and a strong light emission. At still larger distance from the cathode the light intensity decreases again as the electron energy gradually becomes too low for excitation. As at the higher $V_c$ the electrons enter the negative glow with a larger velocity and therefore a longer range than at the lower $V_c$. The length of the negative glow in the first case will be longer.

The red cathode glow $D$, appearing at higher values of $V_c$ and outshining the Aston dark space and first cathode layer is due to excitation by positive ions[147] or fast ions originating from charge transfer. This is shown by the extension of this light behind the cathode when this electrode is perforated (canal rays).

The red light of the aureole is due to the visible resonance radiation $1s-2p$ of metastable atoms,[148] diffusing from the negative glow. It may be due partly to the excitation of these atoms by electrons. Therefore the aureole disappears when the metastable atoms are removed by a small admixture, and reappear again as a *blue* aureole when the admixture is large enough, e.g. in Ne+0.5 percent A. In this case, however, it is probably a recombination spectrum of A.[146]

In different gases the light phenomena differ somewhat; for example, the Aston dark space is not observed in $O_2$ and $N_2$, while in $O_2$ a dark layer exists between the Crookes dark space and the negative glow.[149]

A peculiar light on the cathode for which we suggest the name *cathode light*, is observed when the cathode is covered by an alkali or alkaline earth oxide (MgO, $Na_2O$, $Li_2O$) or by a very thin layer of such a metal.[150] This light consists of the strongest spectral lines of the metal. It seems that excited atoms are sputtered from the cathode by the impinging ions, these atoms having an energy of about 5 ev.

In the literature often some confusion exists between the three totally different forms of light layers near the cathode. The first cathode layer is caused by electron excitation, the cathode glow by ion excitation and the cathode light by sputtering. All three are sometimes called cathode glow.

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[147] A. Güntherschulze and F. Keller, Zeits. f. Physik **72**, 143 (1931).

[148] R. Seeliger and G. Mierdel, Zeits. f. Physik **19**, 230 (1923); W. de Groot, Naturwiss. **14**, 104 (1926); Physica **6**, 53 (1926).

[149] N. M. Carmichael, Phil. Mag. **8**, 362 (1929).

[150] A. Güntherschulze and F. Keller, Zeits. f. Physik **71**, 246 (1931); **75**, 105 (1932); **79**, 563 (1932); H. Sporn, Zeits. f. Physik **112**, 278 (1939); J. Stark and G. Wendt, Ann. d. Physik **38**, 669 (1912); H. Mayer, Phil. Mag. **16**. 594 (1933).

## §35. Results of direct measurements

Quantities which in the glow discharge may be measured directly or nearly directly are (compare Fig. 43): the current density $j$, the cathode fall $V_c$, the thickness of the Crookes dark space $d_c$ and of the Aston dark space $d_a$, the length of the negative glow $l_g$ and the electrode distance $d_{crit}$ at which the anode glow appears. For $d_{crit}$ we may refer to §59. The similarity principle (§32) states that for the same temperature of the gas:

$$V_c = f_1(j/p^2), \quad pd_c = f_2(V_c), \qquad pd_a = f_3(V_c), \quad pl_g = f_4(V_c). \tag{90}$$

Although these relations give a useful means for comparing the results of measurements under different circumstances, large deviations from Eqs. (90) have been stated. These may be due to the heating of the cathode and of the gas, changing the density of the gas[151] or to cumulative effects[152] which disturb the validity of the similarity principle.

*Abnormal cathode fall* $V_c$.—In order to eliminate the heating effect of the cathode, Güntherschulze used in his measurements a very large, massive cathode and low gas pressures.[153] He found that for 1000 v $< V_c <$ 3000 v the current density often could be represented by[154]

$$j/p^2 = a_1 V_c^{a_2},$$

where $a_1$ and $a_2$ are constants for a given pressure. When $a_1$ and $a_2$ are independent of $p$, which is approximately true for $H_2$ and $N_2$, the similarity principle holds. For $O_2$ and the rare gases (especially He) $a_1$ and $a_2$ depend on the pressure and $j/p^2$ is not a function of $V_c$ only.[155] This may be due to cumulative ionization. Fig. 44 shows, by way of example, the increase of $j/p^2$ with $V_c$ for $H_2$ and the deviations from the similarity principle[156] for two cathode materials.[157] As to the influence of the cathode material it may be remarked that $V_c$ generally decreases with decreasing work function of the metal.[158]

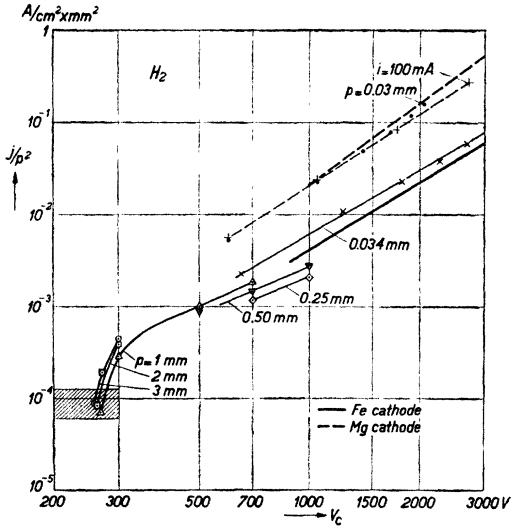


FIG. 44. Values of $j/p^2$ in $H_2$ according to a number of researches of Güntherschulze and associates for Fe and Mg cathode (Mg—Al alloy with oxidized cathode: "electron metal"). The heavy curves give the mean values over an extended range of pressures and currents. The light lines and the points show the results of single series, measured partly at constant current, partly at constant pressure. The hatched rectangle gives the range of the results for the normal cathode fall according to different authors; the points in this rectangle are values of the normal current density.

[151] Although $p_0$ (pressure, reduced to 0°C, proportional to gas density) would be the rational variable, we use in Section V, etc., the pressure $p$, because reduction to 0°C usually is not possible since the temperature is unknown.

[152] In the gases with metastable energy-levels: ionization by collisions of metastable atoms with electrons or other metastable atoms; in diatomic gases: dissociation into atoms followed by ionization of these atoms. Possibly also recombination sometimes may come into account.

[153] A. Güntherschulze, Zeits. f. Physik **59**, 433 (1930) [He, Ne, A, $H_2$, $N_2$, $O_2$; Fe and Mg cathode (electron metal)].

[154] Another empirical formula due to Aston is $j/p_0^2 = a_3(V_c - a_4)^2$ where $a_3$ and $a_4$ are again constants for the diatomic gases but depend on the pressure for the rare gases. F. W. Aston, Proc. Roy. Soc. London **A79**, 80 (1907); F. W. Aston and H. E. Watson, Proc. Roy. Soc. London **A86**, 168 (1912). An equation which may be used for values of $V_c$ down to $V_{cn}$ was given by von Engel and Steenbeck (see §38).

[155] For $V_c <$ 1000 v, conversely, the deviation from the similarity principle for $H_2$ and $N_2$ seems to be larger than for the rare gases (see A. Güntherschulze, Zeits. f. Physik **49**, 358 (1928) (He, Ne, A, $H_2$, $N_2$, $O_2$, Fe cathode)). Possibly this is due to traces of impurities in the rare gases, which at the higher pressures are ionized by metastable atoms of the main gas. This process does not disturb the similarity principle.

[156] The larger deviations for $p = 0.5$ and 0.25 mm in Figs. 44 and 45 are perhaps due to the circumstance that here $d_c$ was no longer small as compared with the diameter of the cathode. The measurements for pressures <0.1 mm were made with a larger cathode in another kind of apparatus.

[157] A. Güntherschulze and W. Bär, Zeits. f. Physik **107**, 730 (1937) (He, Ne, A, Kr, Xe, $H_2$, $N_2O_2$; Mg cathode); A. Güntherschulze and H. Schnitger, Zeits. f. Physik **103**, 491 (1936) ($H_2$ and deuterium, Fe cathode).

[158] A. Güntherschulze, Zeits. f. Physik **24**, 52 (1924); E. W. Pike, Zeits. f. Physik **90**, 319 (1934).

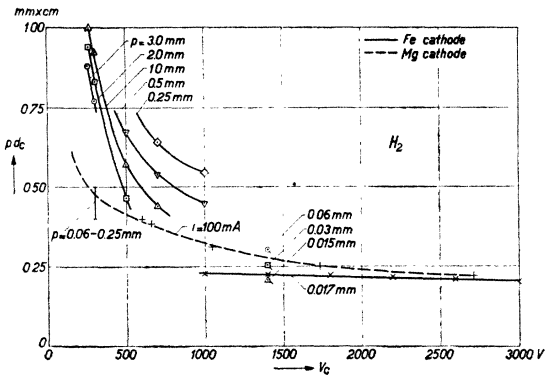


FIG. 45. Values of $pd_c$ in $H_2$ according to Güntherschulze and associates for Fe and Mg cathode (see Fig. 44). The end points of the curves for Fe at $V_c \approx 250$ v correspond to the normal cathode fall.

Besides the deviations from the similarity equations, measurements at the same pressure show large differences in the values of $j$ and $V_c$, especially in the case of the normal cathode fall (see the hatched part in Fig. 44).

*Normal cathode fall* $V_{cn}$.—In the absence of heating and edge effects[159] the value of the normal cathode fall $V_{cn}$ and the corresponding current density are independent of the extent to which the cathode is covered by the discharge.

According to the similarity principle, moreover, $j_n/p^2$ and $V_{cn}$ should be absolute constants for a given combination of gas and (plane) cathode surface. For $H_2$ this proved to be roughly true as long as thickness of the dark space $d_c$ was small as compared with the diameter of the cathode (Fig. 44).

Between different series of measurements, even at the same pressure, large differences were found. The purity of the gas and of the state of the cathode material may have large influence; in Ne, for example, the value of $j_n$ decreased to $\frac{1}{4}$ of its original value by admixing 0.01 percent A.[160] However, even when great care was taken for the purity of the gas and the electrodes as yet no reproducible results for the quantities of the normal cathode fall have been obtained.[161]

*Thickness of Crookes dark space* $d_c$ *and Aston dark space,* $d_a$.—Although Eq. (90) is not exactly fulfilled,[162] Fig. 45 shows, again for $H_2$ as an example,[147,150,157,159] that for values of $d_c$ differing by as much as a factor 100, the difference in $pd_c$ at the same value of $V_c$ is only a factor 2.[156] Here also cumulative processes and heating effects may be responsible for the deviations.

The thickness of the Aston dark space[163] as a function of $V_c$ has only been studied in a few gases; as should be expected $pd_a$ decreases more strongly with increasing $V_c$ than $pd_c$. When in helium $p$ was changed by a factor 100 at a constant value of $V_c$, $pd_a$ remained constant within 20 percent. In our example ($H_2$) only the value of $pd_a$ at $V_c = 300$ volts is known (0.020 mm·cm Mg cathode), being only 1/20 of the corresponding value of $d_c$.

Since at the edge of the Aston dark space the energy of the electrons corresponds to the lowest excitation energy for visible light of the gas, the electric field strength immediately before the cathode may be calculated from the distance, $d_a$.[164]

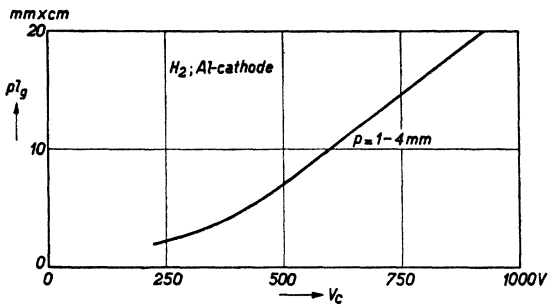


FIG. 46. Values of pressure×length of negative glow in $H_2$ for an Al cathode (Brewer and Westhaver).

[159] A. v. Muralt, Ann. d. Physik **85**, 1117 (1928); measurements with a plane cathode therefore should preferably be made with a guard ring. When the diameter of the cathode glow decreases below a certain value ($\approx d_c$), $V_c$ increases again because of the lateral loss of ions, W. Rogowski, Arch. f. Elektrotech. **25**, 551 (1931); A. Güntherschulze and H. Schnitger, Zeits. f. Physik **77**, 333 (1932); W. Fuchs, Zeits. f. Physik **87**, 139 (1933); A. Güntherschulze, Zeits. f. Physik **49**, 473 (1928) ($H_2$).

[160] F. M. Penning, Physica **11**, 183 (1931); W. de Groot, Naturwiss. **15**, 818 (1927).

[161] E. W. Pike, Zeits. f. Physik **90**, 319 (1934). Compare also J. Taylor, Proc. Roy. Soc. **A114**, 73 (1927); Diss. Utrecht 1927.

[162] An empirical formula for $pd_c$, given by Aston (reference 154) is $pd_c = a_5 + a_6 p\gamma^{\frac{1}{2}}$ or in connection with the formula for $j$, $jd_c = a_5 + a_7(V - a_4)$. For the relation between $p$, $d_c$, and $V$ Aston often noted the relation $jd_c^2 V_c^{-\frac{1}{2}} = \text{constant}$, which has the same form as the space-charge equation for the region of the cathode fall, Eq. (86).

[163] A. Güntherschulze and F. Keller, Zeits. f. Physik **71**, 238 (1931) (He, Ne, A, $H_2$, $N_2$, $O_2$); **71**, 246 (1931) (He with Mg vapor); **72**, 28 (1931) (Kr, Xe); **75**, 105 (1932) (He with Na, Li, Cs, Tl vapor).

[164] F. W. Aston, Proc. Roy. Soc. **A80**, 45 (1907); W. Kossel, Jahrb. d. Radioakt. **18**, 326 (1921); A. Güntherschulze and F. Keller (reference 163). The quantitative results are as yet not satisfying.

*Length of the negative glow $l_g$.*—Recent measurements of Brewer and Westhaver[165] show that $pl_g$ is nearly independent of $p$ for $1<p<4$ mm and $250<V_c<1000$ v and equal to the range of the electrons of energy $V_c$, determined in other experiments. This shows that electrons of approximately this energy enter the negative glow (Fig. 46). As is to be expected $pl_g$ increases with increasing $V_c$.

## §36. Indirect measurements

*Crookes dark space.*—Our knowledge of the electric field, the ion and electron currents, and energies in the Crookes dark space is meager. This is caused by the impossibility of using here the method of the cold probe of Langmuir and Mott-Smith[27] because the field strength is large and the electron distribution deviates widely from an isotropic Maxwell distribution.

The electric field in the Crookes dark space has been measured in a number of ways. While the results of older measurements show large discrepancies, the later measurements agree better.[166] It is very probable that the field strength is largest near the cathode and decreases continuously and monotonously to the negative glow where the field strength is very small. The most recent measurements are those of Ernst with an electron-emitting probe for the case of the normal cathode fall in $H_2$ and A. Fig. 47 shows the potential $V$ and the field strength $E$. $E$ decreases from the cathode to the negative glow along a curve which deviates only near the negative glow from an exponential function.

For the abnormal glow no such reliable measurements exist. In the older experiments sometimes a maximum in the field strength or a potential jump was found, but these effects were very probably due to errors.

Attempts have often been made to measure the positive ion current to the cathode and the energy of the positive ions reaching the cathode by making a small hole in the cathode and measuring the current to a box placed behind this hole. Because erroneous results have been often obtained with this method, we will discuss only the results of two publications which seem to be the most reliable.[167] Campan found for an almost normal cathode fall $i_e/i_p$ at the cathode to be between 6 and 16 percent for $H_2$ and $O_2$. Chaudrhi and Oliphant found for abnormal discharges with a cathode fall of 1000 v or more that ions occurred with all energies up to the value of the cathode fall.

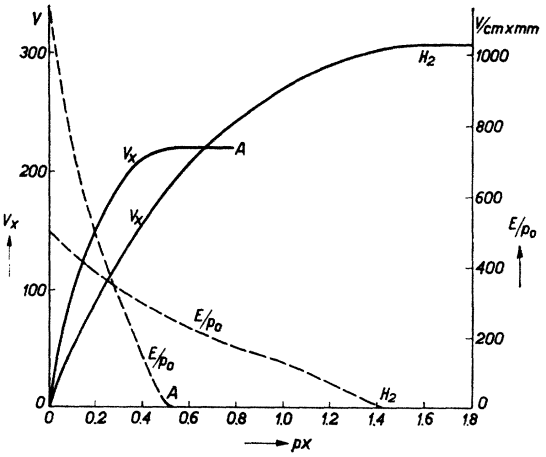


FIG. 47. Potential $V_x$ (—) and $E/p$ (- -) in the normal cathode fall as a function of the product of the pressure and the distance ($x$) to the cathode (according to Ernst).

*Negative glow.*—The field in the negative glow and the Faraday dark space is small. The concentration of the positive ions is almost equal to the electron concentration. The cold probe method can be used here as was done by Emeléus and his collaborators.[168] For the measurements in the neighborhood of the Crookes dark space the results are not very reliable.

It seems probable that a stream of electrons with a high energy enters the negative glow from the Crookes dark space. In the probe measurements this electron group could not be measured very well.[169] The appearance of (Ne) spark lines

[165] A. K. Brewer and J. W. Westhaver, J. App. Phys. **8**, 779 (1937); J. F. Lehmann, Proc. Roy. Soc. **A115**, 624 (1927).

[166] F. W. Aston, Proc. Roy. Soc. **84**, 526 (1910); A. E. M. Geddes, Proc. Roy. Soc. Edinburgh **46**, 136 (1926); W. Steubing, Ann. d. Physik **10**, 296 (1931); W. L. Brown and E. E. Thomson, Phil. Mag. **8**, 918 (1929); K. G. Emeléus, W. L. Brown and H. McN. Cowan, Phil. Mag. **17**, 146 (1934); W. H. Ernst, Helv. Phys. Acta **8**, 381 (1935), where the older literature is cited.

[167] T. I. Campan, Zeits. f. Physik **91**, 111 (1934); R. M. Chaudrhi and M. L. Oliphant, Proc. Roy. Soc. **137**, 662 (1932). The older literature is cited in these publications.

[168] K. G. Emeléus and collaborators, Phil. Mag. **4**, 49 (1927); **7**, 17 (1929); **14**, 355 (1932); **17**, 146 (1934).

[169] In the arc discharge the different groups of electrons could be measured well with the probe method (see §46).

with a high excitation energy proves that, in the (Ne) normal glow discharge, electrons exist with an energy of more than 80 percent of the cathode fall.[170] The concentration of these electrons could not be determined. Another indication of the existence of these electrons is given by the length of the negative glow (§34).

A direct determination of the energy of the electrons entering the negative glow[165] showed that for a cathode fall above 400 v, most electrons had an energy which was almost equal to the cathode fall. Determinations of this kind, however, are not very reliable as a third electrode which disturbs the discharge must be brought into the negative glow.

In the negative glow a potential maximum exists. The potential decreases in the direction of the Faraday dark space by a few volts (1–3 v).

To give an example of the numerical results we cite[168] that in an almost normal discharge in 0.29 mm A (current density $3\times10^{-5}$ amp./cm$^2$) the mean electron energy near the potential maximum was 4 v, the concentration being 1 to $2\times10^8$ per cm$^3$.

The existence of *recombination* in the gas in the negative glow is still a matter of discussion. It seems sure that normally the light emission of the negative glow is largely due to excitation;[146] only in special cases a recombination spectrum is emitted. By putting a third electrode in the negative glow a large ion current can be drawn from the glow (e.g. 1.8 times the discharge current at 960 v in $H_2$) without an increase of the cathode fall.[171] From this experiment it was deduced that a large recombination occurred. We think, however, that this conclusion is not necessary as many ions can diffuse to the anode. A direct calculation of the number of recombinations $R$ per cm$^3$ per sec. from

$$R=\alpha n_e^2,$$

taking for $\alpha$ 2 or $3\times10^{-10}$ as found from experiments in the afterglow,[172] gives generally only a small number of recombinations. More experiments are however necessary to settle this point definitely.

*Energy balance; value of* $\gamma$.—The energy generated per sec. in the cathodic part of the glow discharge is $iV_c$. This power must be dissipated by heating of the cathode, the anode, the walls, the gas and by radiation. The total radiation has never been measured[146] but it will probably be small; the relative heat generated at the anode (§58) and at the walls[173] is small if the discharge is not obstructed by the anode or by the walls. When the positive ion current at the cathode is $i_p$, these ions having the mean energy $e\bar{V}_p$, the energy dissipation at the cathode by these ions is:

$$fiV_c=i_p\bar{V}_p.$$

In a strongly abnormal cathode fall $\bar{V}_p$ may be taken equal to $V_c$ as few collisions of ions with atoms occur in the Crookes dark space, while most of the energy lost by the ions in collisions with the gas atoms is still finally given to the cathode.[174]

By measuring $f$, Güntherschulze and his collaborators[175] could determine $\gamma$ (being the electron current divided by the ion current, both at the cathode) at low pressures as a function of $V_c$ for the abnormal cathode fall:

$$\gamma i_p=i-i_p \quad \gamma=(1-f)/f.$$

They found for $\gamma$

$$\gamma=A(V_c-V_0).$$

$V_0$ lies generally between 100 and 400 v; the value of $A$ depends on the cathode material. This equation may be used only for $V_c\gg V_0$ ($\gamma>0.1$). In the rare gases $\gamma$ attains generally the value 1 for $V_c$ between 2000 and 4000 v.

At higher pressure (1–8 mm) when the cathode fall is in the neighborhood of the normal cathode fall Fischer found that 30 to 40 percent of the input can be used for heating the gas.[176] The

[170] M. J. Druyvesteyn, Zeits. f. Physik **62**, 764 (1930); Physica **1**, 427 (1934).
[171] W. Weizel and H. Fischer, Ann. d. Physik **24**, 209 (1935); H. Fischer, Ann. d. Physik **27**, 81 (1936); K. Geiger, Zeits. f. Physik **106**, 17 (1937).
[172] F. L. Mohler, J. Research Nat. Bur. Stand. **19**, 447 and 559 (1937).
[173] J. W. Beck and K. G. Emeléus, Phil. Mag. **11**, 55 (1931).
[174] I. Runge, Zeits. f. Physik **61**, 174 (1930).
[175] A. Güntherschulze and collaborators, Zeits. f. Physik **37**, 868 (1926); **107**, 642 and 730 (1937); **108**, 780 (1938); **109**, 121 (1938); **111**, 208 (1938).
[176] H. Fischer, Zeits. f. Physik **110**, 197 (1938); **113**, 360 (1939); G. Rudolph, Zeits. f. Physik **111**, 523 (1939).

highest temperature often occurs near the center of the negative glow. At present the measurements are not extensive enough to give a survey of the whole energy balance for the cathodic part of the glow discharge.

## §37. Ionization in the Crookes dark space and in the negative glow

When one electron leaving the cathode gives on the average totally (directly or indirectly) $J_I$ ions in the cathode fall and $J_{II}$ ions in the negative glow, while a fraction $s$ of the ions formed in the negative glow reach the cathode, we have:

$$\gamma(J_I+sJ_{II})=1.$$

As yet no direct experiments have been performed for the determination of $J_I$, $J_{II}$ and $s$. To determine $J_I$ and $J_{II}$, extensive calculations are necessary. As such calculations do not exist and it is not here the place to develop them, we shall only estimate the order of magnitude of $J_I$ and $J_{II}$ in a few examples.[177] We take as two particular cases a cathode fall of 200 v, a value in the neighborhood of the normal cathode fall for most metals, and another case of a cathode fall of 1000 v. Tables VI[178] and VII give the values in these cases for the length of the Crookes dark space $d_c$, the electron mean free path $\lambda$ for an electron energy $eV_c/2$ and the length $l$ in which one ionization occurs for an electron with energy $eV_c$.

TABLE VI. *Values of $J_I$, $J_{II}$ and $J_{pr}$ for $V_c=200$ v.*

| | $pd_c$ | $p\lambda$(100v) | $pl$(200v) | $J_I$ | $J_{II}$ | $J_{pr}$ |
|---|---|---|---|---|---|---|
| He | 1.37 | 0.25 | 0.87 | 2.6 | 6.1 | 0.14 |
| Ne | 0.68 | 0.16 | 0.33 | 3.0 | 4.7 | 0.08 |
| A | 0.30 | 0.05 | 0.095 | 13 | 3.4 | 0.003 |
| $N_2$ | 0.36 | 0.05 | 0.12 | 9.5 | 3.1 | 0.005 |
| $H_2$ | 0.80 | 0.17 | 0.40 | 7.1 | 2.7 | 0.026 |

TABLE VII. *Values of $J_I$, $J_{II}$ and $1/\gamma$ for $V_c=1000$ v.*

| | $pd_c$ | $p\lambda$(500v) | $pl$(1000v) | $J_I$ | $J_{II}$ | $1/\gamma$ |
|---|---|---|---|---|---|---|
| He | 0.50 | 0.80 | 2.23 | 0.25 | 32 | 2.8 |
| Ne | 0.09 | ? | 0.77 | 0.12 | | 3.3 |
| A | 0.055 | 0.15 | 0.27 | 0.22 | 30 | 3.3 |
| $N_2$ | 0.12 | 0.18 | 0.32 | 0.45 | 18.5–27 | 2.6 |
| $H_2$ | 0.30 | 0.56 | 1.3 | 0.26 | 27 | 1.9 |

We see that the length of the Crookes dark space is 4 to 7 times the electron mean free path at 100 v. The number of ionizations in the cathode fall, per electron leaving the cathode, is in He and Ne only about 3, in the other gases about 10. In accordance with this result the fraction of the electrons reaching the negative glow with an energy of 200 v (primary electrons, $J_{pr}$), is in He and Ne about 10 percent, in the other gases often less than 1 percent. The number of ionizations in the negative glow depends on the total ionization $S$ of electrons. This total ionization is the number of ionizations caused by one electron entering a gas with a definite energy and continuing until it has lost all its energy. At a high energy, $S$ is a linear function of the energy. As $\gamma$ is not known in these cases we cannot determine $s$. Assuming $\gamma$ to be 0.15 we would conclude that in He and Ne about half of the ions reaching the cathode came from the cathode fall. In the other gases $\gamma$ would probably be somewhat smaller and it seems likely that a sufficient number of ions are formed in the cathode fall so that none from the negative glow reach the cathode ($s=0$). Under these conditions all the ions formed in the negative glow diffuse to the anode and the walls.

Table VII[179] gives the same results for a cathode fall of 1000 v.

We also give the values of $\gamma$[180] averaged for different cathode metals.[175] We see that now only

[177] See also A. Güntherschulze, Zeits. f. Physik **33**, 810 (1925); E. G. Linder, Phys. Rev. **38**, 678 (1931); K. G. Emeléus and D. Kennedy, Phil. Mag. **18**, 874 (1934).

[178] $J_I$ and $J_{II}$ were found by dividing the Crookes dark space into four parts of equal length, assuming that in each part the potential was constant (80, 140, 170 and 192 v) and calculating the number of ionizations in each part with Eq. (16). Excitations were neglected in the Crookes dark space. The numbers do not change much if we divide $d_c$ into more than four parts. For $J_{II}$ the values of the total ionization $S$ for the different groups of electrons were used. We used the constants from: A. v. Engel and M. Steenbeck, *Elektrische Gasentladungen II*, Table 6; P. T. Smith, Phys. Rev. **36**, 1293 (1930); J. T. Tate and P. T. Smith, Phys. Rev. **39**, 270 (1932); I. Langmuir and H. A. Jones, Phys. Rev. **31**, 357 (1928).

[179] $J_I$ is in this case taken to be $d_c/l$ (1000 v). If $J_I$ is calculated, under the assumption that the field is constant in the Crookes dark space, values are found which are about 1.5 times larger than those given above. The real values will lie between the two. For the literature see reference 178 and A. Güntherschulze and W. Bär, Zeits. f. Physik **107**, 730 (1937); J. F. Lehmann and T. H. Osgood, Proc. Roy. Soc. **115**, 609 and 624 (1927); G. A. Anslow and M. de Blois Watson, Phys. Rev. **50**, 162 (1936).

[180] According to the higher velocities of the positive ions arriving at the cathode, the values of $\gamma$ in this case are much higher than the values occurring at the breakdown, given in Figs. 19 and 20.

few ions are formed in the cathode fall and that most of the ions reaching the cathode must be formed in the negative glow. In A, for example, for each electron leaving the cathode 3.3 ions must reach it. As only 0.2 are formed in the Crookes dark space, the other 3.1 must come from the negative glow, while the other 27 ions formed in the negative glow must either recombine in the gas or go to the anode and the walls. We think that the latter possibility is the more probable.

Summarizing the results of these tables we can state that in the normal discharge often a large part of the ions reaching the cathode are formed in the cathode fall, however in He and Ne some of the ions formed in the negative glow must also go to the cathode unless $\gamma$ is very large in these cases (0.3). In a strongly abnormal discharge ($V_c>1000$ v) the number of ionizations in the cathode fall is much too small and may even in first approximation be neglected. Almost all ions reaching the cathode must in this case be formed in the negative glow.

## §38. Theory

Numerous theories have been proposed to explain the mechanism of the cathode phenomena of the glow discharge. This fact in itself makes it probable that they are only approximations which can be used only in a limited domain.

Before discussing these equations we may make a general remark on the normal cathode fall. Often the cathode fall is given as a function of $j/p^2$ ($j$=current density) by a curve of the type represented in Fig. 48. If this curve shows a minimum, the characteristic features of the normal cathode fall may be explained independently of the special form of the curve $V_c=f(j/p^2)$. For $j>j_n$ ($j_n$=normal current density) the whole cathode is covered by the discharge, $V_c$ rising with increasing $j$. For $j<j_n$ the value of $V_c$ could also increase but since a lower cathode fall results if the discharge only goes to a part of the cathode, $j$ remaining equal to $j_n$, this will occur. The surface of the covered part of the cathode in this region will be proportional to the current.

For the discussion of the theories of the cathode fall we start from the general equations

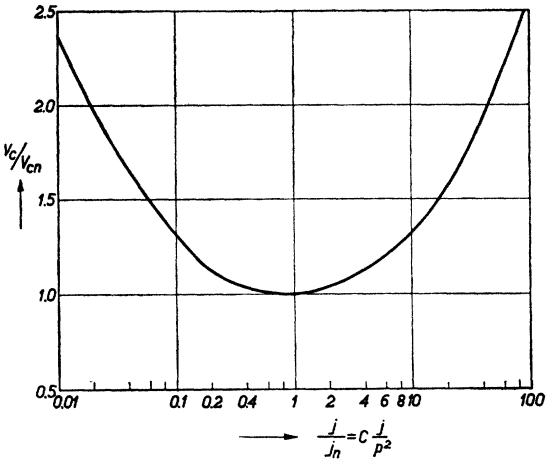


FIG. 48. Calculated values of $V_c/V_{cn}$ as a function of $j/j_n$ (Von Engel and Steenbeck).

of Section V. As here, however, the value of $E/p$ is large, we assume that the velocity of the positive ions is proportional to $(E/p)^{\frac{1}{2}}$ and not to $E/p$ as in Eq. (79). We obtain

$$J_p=n_pv_p=n_pk_pE^{\frac{1}{2}},$$
$$J_e=n_ev_e=n_ek_eE^{\frac{1}{2}}. \tag{91}$$

With $I_w=0$, $\eta E=\alpha$ and $eJ=j$ we now obtain instead of Eq. (83):

$$\frac{dE^{\frac{1}{2}}}{dx}\frac{dE}{dx}=\left(\frac{4\pi j}{k_p}+E^{\frac{1}{2}}\frac{dE}{dx}\right)\alpha. \tag{92}$$

In the case that $\alpha$ is independent of $x$,[181] the solution of (92) is:

$$E^{\frac{1}{2}}=C_1e^{\alpha x}+C_2-6\pi jx/k_p, \tag{93}$$

$C_1$ and $C_2$ being constants which may be found from the boundary conditions at the cathode ($x=0$) and the end of the Crookes dark space ($x=d_c$):

$$x=0, \quad j_p=j/(1+\gamma), \tag{94}$$

$$x=d_c, \quad E=0. \tag{95}$$

A combination of Eq. (78) (Poisson) with (91)

[181] This seems here a sufficient approximation. As the field strength in the Crookes dark space varies strongly with the distance to the cathode (Fig. 47) it is certainly not allowed to use the $\alpha$-values as they were determined for a homogeneous field (see our Table VIII, von Engel and Steenbeck).

and (94) gives, with $k_e \gg k_p$:

$$x=0 \quad E^{\frac{1}{2}}dE/dx = -4\pi j/k_p(1+\gamma),$$
$$C_1 = 6\pi j/\alpha k_p(1+\gamma).$$

$C_2$ follows by substitution of (95) in (93) and gives as solution for $E$:

$$E^{\frac{3}{2}} = \frac{6\pi j\gamma}{\alpha k_p(1+\gamma)}\{e^{\alpha x} - e^{\alpha d_c}\} + \frac{6\pi j}{k_p}(d_c - x). \quad (96)$$

Integration of (96) from 0 to $d_c$ gives the cathode fall $V_c$.

The value of $d_c$ may be obtained by remarking that the positive ion current at the cathode consists of the ions formed in the Crookes dark space plus the ion current entering the Crookes dark space from the negative glow ($j_{pg}$), or:

$$x=0 \quad j_p = j_e(e^{\alpha d_c} - 1) + j_{pg}.$$

Substituting $j_p$ and $j_e$ for $x=0$ this gives:

$$\alpha d_c = \ln\left\{\frac{1+\gamma}{\gamma}\left(1 - \frac{j_{pg}}{j}\right)\right\}. \quad (97)$$

When $\gamma$, $j_{pg}/j$ and $\alpha$ are given, $V_c$ and $pd_c$ may be found from (96) and (97) as functions of $j$. These three quantities $\gamma$, $j_{pg}/j$ and $\alpha$, however, will usually be functions of $V_c$ and $d_c$, so for a complete solution these functions should also be known. A rough estimate of $j_{pg}/j$ may be obtained from Tables VI and VII of §37; for $\alpha/p$ in a first approximation Townsend's equation may be used.

$$\alpha/p = Ae^{-B(pd_c/V_c)}. \quad (98)$$

The method used here is practically the same as used by Weizel, Rompe and Schön,[182] although these authors do not give the solution in the form of Eq. (93). Other authors[182] start from different assumptions, given schematically in Table VIII, which we will now discuss.

TABLE VIII. *List of assumptions used by various authors.*

| | $v_p$ | $\alpha/p$ | $j_{pg}/j$ | $E$ |
|---|---|---|---|---|
| Morse | $k_pE^{\frac{1}{2}}$ | $A'$ | 0 | |
| Rogowski | $k_pE^{\frac{1}{2}}$ | $Ae^{-B(E/p)}$ | 0 | Eq. (92) |
| Thomson and Thomson | $c(V_0-V)^{\frac{1}{2}}$ | $f(E)$ | 0 | $2V_c(d_c-x)d_c^2$ |
| Von Engel and Steenbeck | $k_p'E$ | $Ae^{-B(E/p)}$ | 0 | $2V_c(d_c-x)d_c^2$ |
| Weizel, Rompe and Schön | $k_pE^{\frac{1}{2}}$ | $A'$ | $\neq 0$ | Eq. (92) |
| Scherzer | $k_pE^{\frac{1}{2}}$ | $A'$ | $\neq 0$ | |

[182] P. M. Morse, Phys. Rev. **31**, 1003 (1928); W. Rogowski, Arch. f. Elektrotech. **26**, 643 (1932); J. J. Thomson and G. P. Thomson, *Conduction of Electricity Through Gases II* (1933), Chapter VIII; Phil. Mag. **8**, 393 (1929); A. v. Engel and M. Steenbeck, *Elektrische Gasentladungen II* (1934), §26–31, p. 68; W. Weizel, R. Rompe and M. Schön, Zeits. f. Physik **112**, 339 (1939); **113**, 87 and 730 (1939); O. Scherzer, Arch. f. Elektrotech. **33**, 207 (1939).

*Velocity of the positive ions.*—For this velocity the three possibilities mentioned in the literature, were applied: proportionality to $E$ (slow velocity), to $E^{\frac{1}{2}}$ (higher velocity) and motion without collisions with the energy proportional to the potential difference $V_0 - V$ traversed by the ion ($v_p \propto (V_0-V)^{\frac{1}{2}}$). Experimentally it has been shown[183] that for $E/p < 10$ v/cm mm the velocity is proportional to $E$, $\mu_p p$ (mobility×pressure) being constant, and that for $E/p$ above 10 or 20, the value of $\mu_p p$ begins to rise abruptly. For $E/p > 100$, however, which is of interest in the case of the cathode fall, experiments are almost wholly lacking. In this region $\mu_p p$ is expected to decrease again with $E/p$, $\mu_p$ becoming proportional to $1/E^{\frac{1}{2}}$ and $v_p$ proportional to $E^{\frac{1}{2}}$. For still larger velocities (the limiting case being that without collisions) the velocity does not depend only on the field at the place of the ion but on the whole form of the field traversed by it. In the absence of sufficient information the assumption $v \propto E^{\frac{1}{2}}$ seems to us for the moment the most reasonable one; the result for $E$, given in Eq. (96) is in rather good agreement with the experimental results of Fig. 47.

In the case that no ionization occurs in the Crookes dark space ($\alpha=0$) $E$ can be calculated directly (if the space charge of the electrons be neglected). For the three cases considered above we obtain:

$$v_p = k_p'E, \qquad E = C_1(d_c-x)^{\frac{1}{2}},$$
$$v_p = k_pE^{\frac{1}{2}}, \qquad E = C_2(d_c-x)^{\frac{2}{3}},$$
$$v_p = C(V_c-V)^{\frac{1}{2}} \text{ (free fall)} \qquad E = C_3(d_c-x)^{\frac{1}{2}}.$$

*The origin of the positive ions arriving at the cathode.*—Most authors assume $j_{pg}/j = 0$. This may sometimes be right in the case of the normal cathode fall (Table VI), but it is not justified for higher values of $V_c$ (Table VII). Weizel, Rompe and Schön determine $j_{pg}/j$ by

[183] A. M. Tyndall, *The Mobility of Positive Ions in Gases* (1938), Fig. 31.

invoking the equations of the negative glow, which is a plasma crossed by a beam of fast electrons (§36). The equations for this plasma are analogous to those for the plasma in the positive column. In the negative glow a potential maximum occurs also, but in this case along a direction perpendicular to the cathode. Here also the Boltzmann equation can be used. But we will not discuss these equations since at present it is not possible to calculate $j_{pg}/j$. The different constants and especially those for the beam of electrons crossing the negative glow are not known. A further discussion of the theory of the negative glow probably will throw more light on this point.

*The equation for E.*—Some authors do not use Eq. (92) for $E$ but assume a simple form for it based on the results of Aston and others. Von Engel and Steenbeck, using the values given in Table VIII calculate $V_c$ as a function of $j/p^2$, showing afterwards that in this way the equation for $E$ is approximately fulfilled. It is curious that in their results the constants appear only in the normal cathode fall $V_{cn}$ and the normal current density $j_n$ so that, if we put

$$V_c/V_{cn}=f(j/j_n),$$

one function is found for all gases and all cathode materials. Fig. 48 shows this function. The experiments, however, give different functions for different gases.[184]

Thus far we have only considered the equations for the Crookes dark space. Near the boundary between this space and the negative glow the equations become less satisfactory for here the field strength is small and our assumptions are no longer right. Electrons will also enter this part of the cathode fall from the negative glow and change the field strength near the boundary. A theory with another boundary condition for $x=d_c$ has recently been given by Scherzer.[182]

We will not discuss the older theories of the cathode fall but merely mention an interesting principle, given by Compton and Morse,[185] which states that the potential distribution in the cathode fall takes such a form that the ionization

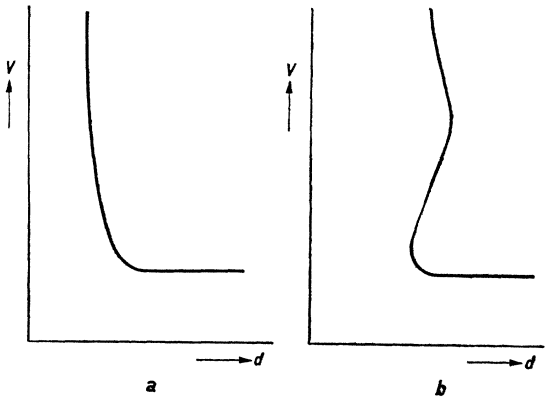


FIG. 49. Increase of the potential difference between the electrodes $V$ as a function of the electrode separation $d$ at constant current (obstructed glow discharge). a, Normal case. b, Abnormal behavior of He (schematically).

of the electrons becomes a maximum, Poisson's equation having to hold. From the foregoing we see, however, that sufficient equations are available and with this principle no free choice can be made.

A comparison of the different theories with experiments is at the moment almost impossible. In each theory some experimental constants are not known and may be chosen almost arbitrarily to give agreement with some experiments. Before a comparison can be made, more data are necessary on the drift velocity of ions, the properties of the negative glow especially $i_{pg}$, the deviations from the similarity equations and the ionization in the different parts of the discharge.

## §39. Special forms of glow discharges

*The "obstructed" glow discharge.*—In the preceding sections we supposed the electrode separation $d$ to be much larger than the thickness of the Crookes dark space $d_c$. When $d$ is decreased, the current being left constant, $V_c$ usually increases; the discharge becomes "obstructed" as the ionization by the electrons diminishes (Fig. 49). This phenomenon is analogous to the increase of the breakdown potential for $pd<(pd)_{\min}$ (§23). In the case of He the curve of $V=f(d)$, at constant $i$, for certain pressures and currents even shows a minimum with respect to $d$, in the same way as the Paschen curve (Fig. 49b).[186]

[184] M. J. Druyvesteyn, Physica **5**, 875 (1938).
[185] K. T. Compton and P. M. Morse, Phys. Rev. **30**, 305 (1927); A. von Hippel, Zeits. f. Physik **76**, 1 (1932); W. Rogowski, Zeits. f. Physik **82**, 473 (1933).

[186] In this region negative characteristics "of the second kind" occur which are stable without series resistance, and

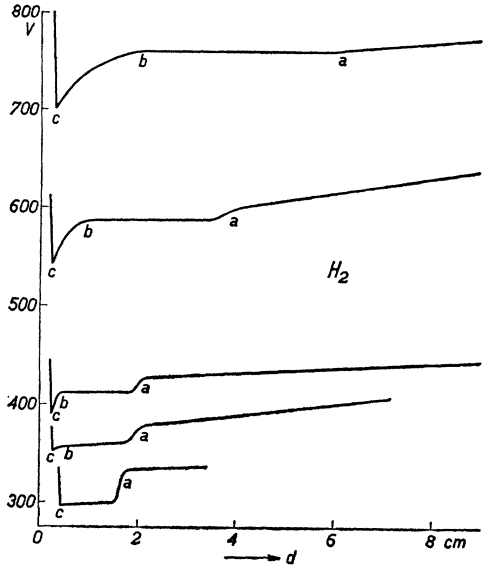


FIG. 50. Tube voltage as a function of the electrode separation $d$ in $H_2$ at different currents. The anode fall vanishes at $a$, the obstructed discharge begins at $c$ (Güntherschulze).

For some gases ($H_2$, He, $O_2$) at higher currents a rather sharp minimum in $V_c$ ($c$ in Fig. 50) occurs with increasing distance before the sharp increase of $V_c$.[187] Near $a$ the anode fall vanishes (§59), between $a$ and $b$ the tube voltage is approximately equal to the cathode fall. At $c$ a sharp decrease of the tube voltage occurs at an electrode separation about 2 $d_c$. Some authors[188] ascribe this minimum to a change of gas density due to the cooling of the gas by the cold anode, but for the moment this explanation is only a hypothesis. The rise of $V_c$ to the left of $c$ is the normal rise already described.

The *hollow cathode* effect occurs when two cathodes are brought so close that the two negative glows flow together. Fig. 51 shows that for the same value of $V_c$ the current density may now be much larger (100 times or more) than for a plane cathode.[189] The ions and perhaps also the metastable atoms and photons formed by the electrons coming from one cathode can now not only reach this cathode but also the other one, so that a much smaller number of ions are lost and the cathode fall will be much smaller than with one plane cathode.

The circumstance that this effect is especially strong in He and Ne is in accord with the result of Table VI, that in these gases the number of ionizations in the negative glow is large compared with those in the cathode fall. Moreover it is clear from Table VII that the influence of the second cathode will be considerable at highly abnormal values of the cathode fall.

Of course the hollow cathode effect will as well occur between two flat cathodes as in a cylindrical one.

For a convex cathode the cathode fall will differ greatly from a plane cathode if the radius of the cathode is small compared to the length of the cathode fall (thin wire).[190]

*The spray discharge.*—Güntherschulze and Fricke[191] discovered an interesting discharge which can be obtained with cathodes of carbon covered by a layer of an insulating powder (e.g. $Al_2O_3$ grains) or of Al covered by anodic oxidation with a coherent layer of $Al_2O_3$. In this case a discharge without Crookes dark space was observed, the negative glow beginning immediately at the cathode surface. The tube potential can be low e.g. $<40$ v, the current density high; $j/p^2$ values were found thousands of times higher than in the glow discharge. For this discharge the name "spray discharge" (*Spritzentladung*) was suggested.

The mechanism seems to be as follows: The electrons are not accelerated in a space-charge layer in the gas, but in a thin solid insulating layer; they enter the gas with an energy approximately corresponding to the potential difference $V$ between cathode and anode when no positive

positive characteristics "of the second kind" which cannot be stabilized with any series resistance. F. M. Penning, Proc. Roy. Acad. Amsterdam **34**, 1305 (1931); Physik. Zeits. **33**, 816 (1932); C. van Geel, Physica **6**, 806 (1939).

[187] A. Güntherschulze, Zeits. f. Physik **36**, 563 (1926); **40**, 414 (1926); **61**, 1 and 581 (1930); F. M. Penning, Zeits. f. Physik **70**, 782 (1931).

[188] H. Fischer, Zeits. f. Physik **110**, 197 (1938). Compare A. v. Engel and M. Steenbeck, *Elektrische Gasentladungen II* (1934), p. 106. For another possible explanation see M. J. Druyvesteyn, Physica **5**, 875 (1938).

[189] A. Güntherschulze, Zeits. f. Physik **19**, 313 (1923); Zeits. f. tech. Phys. **11**, 49 (1930); A. Lompe, R. Seeliger and E. Wolter, Ann. d. Physik **36**, 9 (1939).

[190] R. Seeliger and J. Schmekel, Ann. d. Physik **73**, 249 (1924).

[191] The spray discharge is described in papers of Bär, Fricke, Güntherschulze, Schnitger and K. Wolf, Zeits. f. Physik **86**, 451, 778 and 821 (1933); **92**, 728 (1934); **96**, 551 (1935); **102**, 163 (1936); **106**, 662 (1937); **112**, 148 (1939).

column or anode glow is present. The ionization in this case will be equal to the total ionization $S$ (§37). The current density is given by:

$$j = A\,V^2 e^{-B/V},$$

the well-known equation for cold emission. This emission takes place on the points of the cathode covered by the insulator, the high field strength being due to the ions at the outer surface of the insulator.

In some cases it seems essential that the bottom layer be not a metal but that it have a somewhat higher resistance as is the case for carbon. In other cases this seems not to be necessary. Often the insulating layer has to be chosen surprisingly thick (e.g. $10\mu$), the average value of the field at the metal surface being about $5\times10^5$ v/cm, a rather low value for cold emission. By diminishing the thickness of the insulating layer a continuous transition from the spray discharge to a usual glow discharge can be brought about.

The spray discharge has much resemblance to the vacuum discharge where the positive charge on an insulating layer is brought about by emission of secondary electrons (Malter effect).[192] We wish also to point to the connection between the spray discharge and certain forms of arcs on oxidized metals (§50).

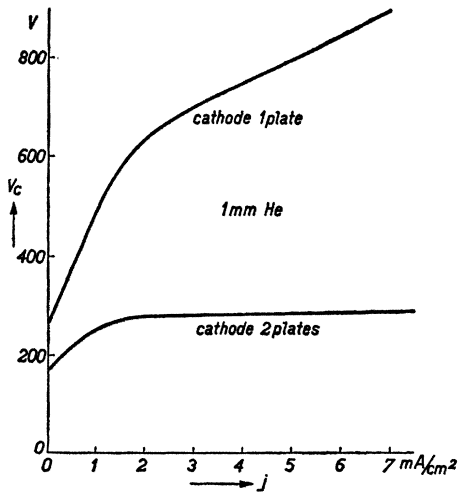


FIG. 51. Cathode fall $V_c$ as a function of the current density $j$ at the cathode, with one plate and with two parallel plates (separation 1 cm) as cathodes. Pressure one mm He (Lompe, Seeliger, and Wolter).

[192] L. Malter, Phys. Rev. **50**, 48 (1936); J. Mühlenpfort, Zeits. f. Physik **108**, 698 (1938).

## VII. Cathodic Part of the Arc[193]

### §40. Definition. Types of arcs

It is difficult to give a rigorous definition of an arc, but roughly speaking an arc may be defined as a discharge with low cathode fall ($V_c$), high current density at the cathode, and a falling or horizontal characteristic of the cathode fall [$V_c=f(i)$]. Further, it is a discharge in which electrons are liberated from the cathode by a supplementary mechanism other than the liberation of electrons by positive ions ($\gamma$ §19) as it occurs in the glow discharge. Which part of the definition is given most weight depends upon circumstances, e.g. in the characteristic of Fig. 1 where a continuous transition from glow to arc occurs; the arc can be assumed to begin at $H$ (as will be done here) or at $K$.

From the large number of experiments at a pressure of about one atmosphere or more we shall mention only those that are essential for the mechanism of the cathodic part of the arc. The positive column and the anodic phenomena for higher pressure are not discussed. The characteristic of the whole arc is discussed in only a few cases because it is usually impossible to deduce the cathode fall from the characteristic. A consideration of the spectrum of the arc and of the reignition potential of a.c. arcs are omitted.

A number of different kinds of arcs occur; of these we shall identify four.

I. In the *thermionic arc* the cathode is heated by the discharge to so high a temperature that the thermionic emission of the cathode is of the same order of magnitude as the arc current. The tungsten and carbon arcs are typical examples.

II. The *arc with externally heated cathode* is also a kind of thermionic arc, but it is not self-sustained. The cathode has the same temperature as in I, but the heating of it is only in small part due to the discharge itself.

III. In the *field current arc* the thermionic emission of the cathode is negligible. The electrons are liberated from the cathode by the high field strength at its surface due to the space

[193] A. Hagenbach, *Handbuch der Physik*, Vol. **14** (1927), p. 324; *Handbuch der Radiologie*, Vol. **4** (1917), p. 211; M. Leblanc(fils), *L'arc électrique* (1922); R. Seeliger, *Handbuch der Experimental Physik XIII*[3] (1929), p. 583; Physik. Zeits. **27**, 22 (1926); K. T. Compton, J. A. I. E. E. **46**, 1192 (1927).

charge of the positive ions. Arcs with liquid metal cathodes and especially Hg arcs are examples of this type (§§48 and 49).

IV. In §§50 and 51 we shall discuss the different *metal arcs* with solid metal cathode (with exception of the W and C arc) for which the mechanism of the electron emission at the cathode is rather hypothetical. Some of the mechanisms proposed will be discussed.

### §41. Ignition of an arc

Self-sustained arcs can be ignited in a variety of ways. The simplest case involves a continuous transition from a glow discharge to an arc as schematically given in Fig. 1. Such transition can take place in the rare gases and $N_2$,[194] and is illustrated in the case of Ne and A in Fig. 52. In this case the cathode was a W sphere on a small W rod; the sphere had an almost constant temperature over its surface and functioned as the sole cathode of the discharge. The descending part of the characteristic begins when the temperature of the cathode surpasses 2000°K, at which point the thermionic emission of W becomes high enough. At a higher pressure the maximum cathode fall is lower (Fig. 52). This is a general feature of the glow-to-arc transition for in the glow discharge the current density is proportional to $p^2$ (§35) and hence at a higher pressure a lower cathode fall is needed for a given cathode heating.

Obviously the transition current can be varied by changing the rate of heat loss from the cathode. Experiments of the above kind are, however, sometimes complicated by the fact that in the transition region another type of arc may occur which is not of thermionic origin.[195] In a.c. arcs Plesse[196] found that the transition from glow to arc was facilitated if some metallic vapor (e.g. from the cathode) came into the gas.

In the cases just discussed, the glow-to-arc transition occurs at a definite current density, in other cases, however, for example, when the

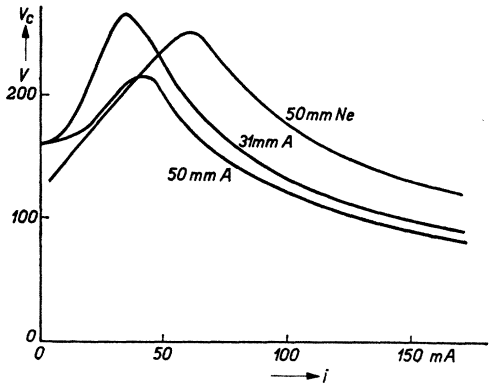


FIG. 52. Cathode fall $V_c$ for the glow-to-arc transition in Ne at 50 mm and in A at 31 and 50 mm between W spheres (diameter 1.8 mm) on W rods (diameter 0.35 mm) (Druyvesteyn).

cathode surface is not uniform, the transition occurs sometimes at a large, sometimes at a small current density; here the transition seems to be a probability phenomenon.[197] With a rod or plate as cathode the arc discharge contracts into a cathode spot which leads in most cases to a discontinuous glow-to-arc transition.

Evidently a discontinuous ignition of an arc will not elucidate the nature of the arc as clearly as a continuous glow-to-arc transition. We will therefore only enumerate a number of other methods by which a self-sustained arc can be ignited: (1) If the cathode of a glow discharge is struck by a small conductor or insulator, an arc can be generated at the point of contact.[197] (2) An easy way of igniting an arc is to separate two metals through which a current passes.[198] (3) An arc can sometimes be ignited by sending a current pulse through a semi-conductor in contact with the metal cathode (ignitron).[199] (4) A capacitative pulse through an insulator can also ignite a cathode spot. As an illustration we

[194] M. Wehrli, Zeits. f. Physik **44**, 301 (1927); Helv. Phys. Acta **1**, 323 (1928); M. J. Druyvesteyn, Zeits. f. Physik **73**, 727 (1932).

[195] O. Becker, R. Seeliger and R. Sommermeyer, Ann. d. Physik **24**, 609 (1935); Zeits. f. Physik **102**, 551 (1936); A. von Engel, R. Seeliger and M. Steenbeck, Zeits. f. Physik **85**, 144 (1933).

[196] H. Plesse, Ann. d. Physik **22**, 473 (1935).

[197] J. v. Issendorf, M. Schenkel and R. Seeliger, Wiss. Ver. Siem. Konz. **9**, 73 (1930); J. Slepian and L. R. Ludwig, Quarterly Trans. A. J. E. E. **51**, 92 (1932); F. A. Maxfield and G. L. Fredenhall, Physics **9**, 600 (1938); T. Jurriaanse and M. J. Druyvesteyn, Physica **3**, 825 (1936).

[198] See for these contact arcs: J. Slepian, J. A. I. E. E. **45**, 930 (1926); E. Hutchisson, T. H. Osgood and R. E. Fearon, Proc. Nat. Acad. Sci. **21**, 542 (1935); R. Holm and F. Güldenpfennig, Wiss. Ver. Siem. Konz. **16**, 81 (1937); H. P. Fink, Wiss. Ver. Siem. Konz. **17**, 45 (1938).

[199] J. Slepian and L. R. Ludwig, J. A. I. E. E. **52**, 693 (1933); see also Elec. Eng. **53**, 75 and 1384 (1934); **54**, 942 (1935); G. Mierdel, Wiss. Ver. Siem. Konz. **15**, 35 (1936).

mention that a cathode spot may be generated by separating a metal from a conductor by a thin glass and then applying a voltage pulse.[200]

For an arc with externally heated cathode the ignition problem differs from the cases just discussed, because in this case the cathode emits electrons before ignition (Fig. 60). For the ignition of such an arc the number of ionizations per electron need be only $\leqslant 1$ percent (approximately $(m_e/m_p)^{\frac{1}{2}}$ or $(\mu_p/\mu_e)^{\frac{1}{2}}$, see Table V).[201]

## A. THERMIONIC ARCS

### §42. Tungsten arc. Experiment

The simplest case of a self-sustained thermionic arc is the tungsten arc with as cathode a little W sphere on a thin rod. The temperature is almost constant over the whole sphere and the cathode has a definite area. The thermionic constants of W are well known. Since the electrodes do not evaporate readily and are inert to many gases ($N_2$, $H_2$, rare gases) the experimental conditions are ideal. This arc has been studied extensively, especially by de Groot and by

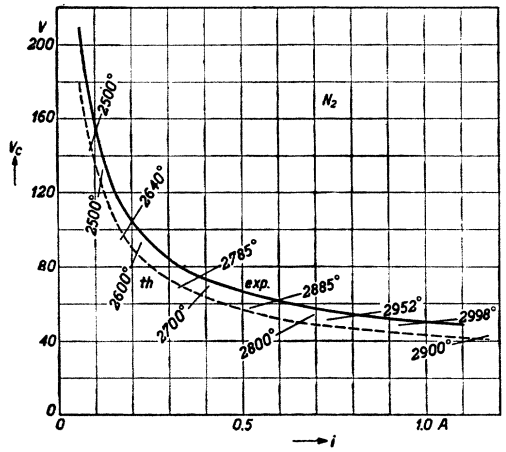


FIG. 53. Characteristic of the cathode fall $V_c$ in $N_2$ at 420 mm (Bächtiger, Helv. Phys. Acta **3**, 335 (1930), Fig. 8). The slope of the isothermic characteristics at the temperature mentioned is also given. The dashed line (*th*) is calculated from Eqs. (132)–(134).

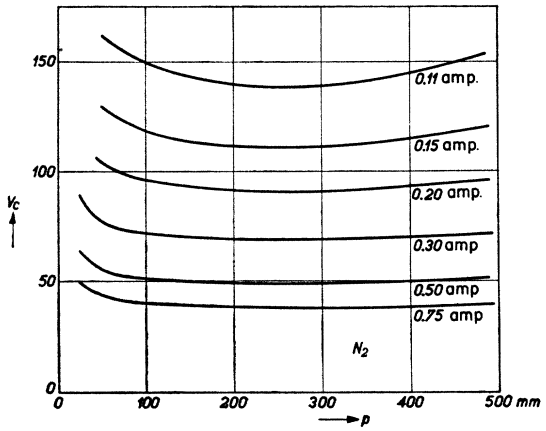


FIG. 54. Dependence of the cathode fall $V_c$ on the pressure $p$ in $N_2$ at different currents (indicated in amp.) (Bächtiger and Wehrli, Helv. Phys. Acta **4**, 31 (1931), Fig. 7).

Wehrli and his collaborators,[202] and we shall deal with it in somewhat more detail than with other forms of arc.

Immediately in front of the cathode lies a space-charge layer of positive ions. The potential difference $V_c$ across this layer is the so-called cathode fall and is the most important feature of the arc. The dependence of $V_c$ on the current is given qualitatively in Fig. 1. Usually the cathode fall is determined by varying the electrode distance at constant current, the minimum voltage on the tube being almost equal to the cathode fall. Fig. 53 shows $V_c$ in $N_2$ as a function of the current $i$. It is seen in Fig. 54 than in a certain range the cathode fall is independent of the pressure.

The temperature of the cathode increases with increasing current and is so high that the thermionic emission of the cathode is of the same order of magnitude as the arc current. At a high current the cathode fall can be almost equal to the ionization potential of the gas, or even, especially in Ne and A, somewhat below it.

[200] Y. Watanabe, H. Kasahara and Y. Nakamura, Elektrotech. J. **2**, 180 (1938); K. J. Germeshausen, Phys. Rev. **55**, 228 (1939).

[201] B. Kirchstein, Arch. f. Elektrotech. **27**, 785 (1933); H. Klemperer and M. Steenbeck, Zeits. f. tech. Physik **14**, 341 (1933); A. Glaser, Jahrbuch A. E. G. **3**, 47 (1932); **4**, 170 (1936).

[202] W. de Groot, Physica (old series) **5**, 121 and 234 (1925); **11**, 307 (1931): (Ne, A, Hg, $N_2$, CO). Most papers by Wehrli and his collaborators have been published in *Helvetica Physica Acta*. The most important seem to be: P. Bächtiger, Helv. Phys. Acta **3**, 335 (1930): dynamic characteristic. M. Wehrli and P. Bächtiger, Helv. Phys. Acta **5**, 106 and 161 (1932): group of primary electrons, and Helv. Phys. Acta **5**, 161 (1932): theory. O. Stübing, Helv. Phys. Acta **8**, 165 (1935): He arc. E. Sieboth, Helv. Phys. Acta **4**, 153 (1931). In these articles other literature is cited.

In Fig. 53 are also shown the slopes of the isothermic characteristics which are obtained by superposing on the direct voltage a small alternating voltage of so high a frequency (above 200 per sec.), that the temperature of the cathode remains constant during a period. If the amplitude of the alternating voltage is large, the whole isothermic characteristic can be determined. Fig. 55 gives an example for a pressure of 49 mm $N_2$ (curve *a*) and for 451 mm $N_2$ (curve *b*). The form at low pressure is analogous to that of Fig. 59 and will be discussed in §46.[203] It is seen that the isothermic characteristic differs greatly from the normal characteristic for which the increase of temperature with increasing current is an essential feature.

At not too high a pressure and not too high values of $V_c$ the length $d_c$ of the cathode fall is smaller than or about equal to the mean free path of the ions $\lambda_p$ and will have a value between those given by Eq. (86) and Eq. (89); $d_c$ is so small that it has only rarely been observed visually ($<0.1$ mm).

### §43. Tungsten arc. Theory

Because frequently in the arc $d_c<\lambda_p$, the theory of the cathode fall of an arc is much simpler than for the glow discharge where $d_c>\lambda_p$. In the arc the electrons from the cathode make hardly any collisions in the cathode fall and hence a homogeneous beam of electrons with an energy corresponding to the cathode fall enters the gas and ionizes it until their energy is exhausted. This beam of electrons from the cathode was found experimentally by Wehrli at a distance from the cathode corresponding to the free path of these electrons. The ionization of this beam is in this case the *total ionization* $S$, as discussed in §37. In many cases $S$ can be represented by a linear function of the energy of the electrons $eV$, namely $P(V-V_i)$.

If a fraction $k$ of the ions formed goes to the cathode, we have:

$$i_p/i_e=kP(V_c-V_i). \tag{99}$$

$V_c$ is the cathode fall, $i_p$ and $i_e$ are the currents of ions and electrons at the cathode. The current $i_e$ is equal to the sum of the thermionic emission $i_{th}$ and the electrons liberated by positive ions:

$$i_e=i_{th}+\gamma i_p, \quad i=i_e+i_p. \tag{100}$$

A combination of these equations gives:

$$\frac{i-i_{th}}{\gamma i+i_{th}}=kP(V_c-V_i). \tag{101}$$

The next equation is given by the energy balance at the cathode which was studied experimentally by de Groot.[202] The energy lost by the cathode is the sum of radiation ($BT^4$) and heat conduction of the W rod and the gas ($CT$). We have, if the accommodation coefficient is $f$,

$$i_p(fV_c+V_i)-i\varphi=\frac{i-i_{th}}{1+\gamma}(fV_c+V_i)-i\varphi$$

$$=BT^4+CT, \tag{102}$$

$$i_{th}=AT^2e^{-b/T}. \tag{103}$$

A combination of Eqs. (101) and (103) gives us the characteristic, which is shown (for $k=1$ and $f=1$) in Fig. 53 by a dashed line.[204] Table IX shows that for a large current the thermionic emission is almost $\frac{2}{3}$ of $i$.

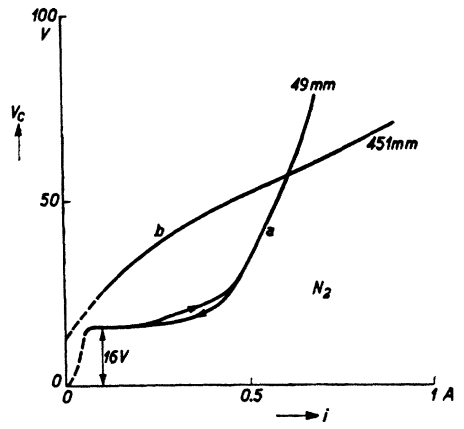


FIG. 55. Isothermic characteristics in $N_2$ at 49 mm and 2763°K (a) and in $N_2$ at 451 mm and 2880°K (b) (Bächtiger, Helv. Phys. Acta 3, 335 (1930), Figs. 37 and 40a).

[203] We will only remark that the form of the isothermic characteristic in $N_2$ at 451 mm differs much from the form at low pressure without giving a suggestion how this difference is to be explained.

[204] In the numerical calculation the electron emission (103) and the total radiation were taken according to C. Zwikker (Physica 5, 249, 319 (1925)); the heat conduction by the rod was calculated with Bächtiger's Eq. (11) in Helv. Phys. Acta 3, 335 (1930). We assumed $V_i=16.7$; $\varphi=4.5$; $\gamma=0.05$ and $P=0.0226$. See K. T. Compton and I. Langmuir, Rev. Mod. Phys. 2, 123 (1930), Table II.

The slope of the isothermic characteristic can be found with Eq. (101), $i_{th}$ being now a constant. It is seen from Fig. 53 that the calculated characteristics do not differ much from the experimental results. However, because several important matters have been neglected the application of our formulae is, in a number of cases, limited.

(1) We assumed $k=1$ and $f=1$. At low pressures $k$ will be smaller than 1 because a number of ions diffuse to the tube walls and to the anode.[205] Especially in He $f$ will be much smaller than 1.[206]

(2) At high current density the number of ionizations can be much larger than is given by (99) because excited atoms will also be ionized (cumulative ionization). This effect will be very large in gases with metastable states (as Ne and A). In Ne de Groot[202] found at a cathode fall about equal to the ionization potential $i_p/i$ to be about $\frac{1}{3}$, practically all ions being formed from metastable atoms. The very low cathode fall in Ne and A is due to this effect.

(3) The current $i_{th}$ will be enlarged by the Schottky effect; this influence, however, can be calculated.

(4) At high pressures or at high cathode fall we will no longer have $d_c<\lambda_p$. The theory will be much more complicated if both ionizations in and outside the cathode fall have to be taken into account.[207]

(5) When the cathode fall is low and the current high, it seems probable that $i_{th}$ becomes larger than $i$. In this case our theory is incorrect (see §47).

If a tungsten rod is used as a cathode the theory is further complicated by the existence of a cathode spot, the area of which depends on the current. This kind of arc would seem to be the next simplest case, but we shall not discuss it in detail but pass immediately to a consideration of the carbon arc.

## §44. Carbon arc

Our insight into the mechanism of the cathodic part of a carbon arc is much less advanced than for a W arc. This is due to a number of circumstances. In the C arc a C rod or plate is always used so that a cathode spot exists; most experiments are performed at a pressure of about one atmosphere (see below); in most cases the composition of the gas is not known, e.g. with a nitrogen filling the gases $N_2$, N, CN, C and $C_2$ are, according to the spectrum, present; moreover, the carbon used often contains unknown impurities. We will therefore discuss only briefly the most important properties of the *C* arc.

*Cathode fall.*—In an arc at one atmosphere the cathode fall is probably about 13 v; often lower values have been found.[208]

The measurement of the cathode fall at this pressure, however, is very difficult for a number of reasons.[208] A sounding electrode to avoid overheating must be swept swiftly through the discharge. However, the arc is then much disturbed by the cooling of the gas by the cold probe (the arc voltage can for example rise 5 v at the moment the probe passes through the arc). Around the probe not only will a thin dark space-charge layer appear, but a large dark

TABLE IX. *Values of $i_{th}/i$ and $i_p/i$ for the case presented in Fig. 53.*

| $T$ °K | $i$ AMP. | $V_c$ VOLTS | $i_{th}/i$ | $i_p/i$ |
|---|---|---|---|---|
| 2400 | 0.063 | 176 | 0.18 | 0.78 |
| 2600 | 0.21 | 87 | 0.35 | 0.62 |
| 2800 | 1.16 | 41 | 0.63 | 0.35 |

[205] The increase of $V_c$ with decreasing $p$ in Fig. 54 for $p<100$ mm may be attributed to this decrease of $k$. Remark also the difference between the two curves of Fig. 55.

[206] C. C. van Voorhis and K. T. Compton, Phys. Rev. **37**, 1596 (1931). K. T. Compton and E. S. Lamar, Phys. Rev. **44**, 338 (1933).

[207] A. von Engel and M. Steenbeck (*Elektrische Gasentladungen II* (1934), p. 119) calculate the characteristic of the W arc supposing that all ions are formed in the cathode fall and that this ionization can be described by an $\alpha$-formula of Townsend. Although this supposition seems to be incorrect under most circumstances, their result is in good agreement with the experiment.

[208] W. B. Nottingham, J. Frank. Inst. **206**, 43 (1928); **207**, 299 (1929); J. L. Myer, Zeits. f. Physik **87**, 1 (1933); R. C. Mason, Phys. Rev. **51**, 28 (1937); C. v. Fragstein and M. Arndt, Ann. d. Physik **33**, 352 (1938). In the last two papers the probe measurements at 1 atmos. are criticized. For the theory of the probe method at higher pressures, see: B. Davydov and L. Zmanovskaja, Tech. Phys. U.S.S.R. **3**, 715 (1936). F. Borgnis, Ann. d. Physik **31**, 497 (1938).

The results of the older methods of determining the cathode fall of an arc, as from the characteristic [$V=f(d)$] or from static probe measurements will not be discussed, as they can give erroneous results.

space is seen in which many collisions between the electrons and gas molecules can occur.[209]

In most cases the potential is measured in the positive column, where a constant field occurs, the extrapolation of this field to the cathode to obtain the cathode fall, may be incorrect.[210]

As the different parts of the discharge are very thin at one atmosphere it is not easy to determine the electric field in the cathode fall by measuring the Stark effect of the spectral lines emitted by atoms in the cathode fall. It is true that a Stark broadening can be observed, but this must be attributed to the high electric fields of ions and electrons in the plasma in front of the cathode fall, the concentration being here very high. The pole effect of the spectral lines of arcs is probably due to this cause.[211] The Stark effect due to the field strength in the cathode fall of an arc has not yet been observed.

*Cathode spot.*—In thermionic arcs as in many other arcs, a sharp cathode spot usually exists. The W arc with a W sphere on a thin rod is the only self-sustained arc without a cathode spot.

The current density at the cathode spot is between 200 and 2000 amp./cm$^2$; when the electrode distance is large enough the current density is according to Güntherschulze 470 amp./cm$^2$; it decreases with decreasing pressure to 100 amp./cm$^2$ at 100 mm.[212] The temperature of the cathode spot is about 3600°K.[213] It seems probable that the thermionic emission of carbon at this temperature is of the same order of magnitude as the observed arc current. If the carbon is impure (e.g. with alkali or alkaline earth impurities) a much larger electron emission or a lower temperature of the cathode spot can occur.

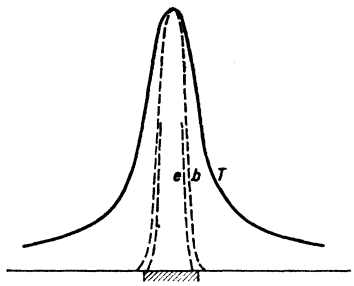


FIG. 56. Temperature $T$, electron emission $e$ and brightness $b$ at the cathode spot, when heat is generated at the shaded part (Seeliger, schematically).

The sharpness of the cathode spot was explained by Seeliger[214] by remarking that the electron emission and the brightness vary strongly with temperature (see the schematic, Fig. 56). It is not yet clear in what way the area of the cathode spot can be determined theoretically. It may be that a minimum value for the cathode fall arises for a definite area of the cathode spot determined by the heat conduction of the cathode, or that the collateral ion diffusion in the gas is essential for this area.

Although adequate measurements to test the thermionic theory for the cathode part of a carbon arc do not exist, there seems no doubt that the C arc is in normal cases of thermionic origin.

## *B. ARCS WITH EXTERNALLY HEATED CATHODE*

### §45. General description

In these arcs the cathode, is heated by an auxiliary current. The cathode is usually a W, WTh, or MoTh spiral, or is oxide-coated. Although these discharges are not self-sustained they are called arcs because the thermionic emission of the cathode is of the same magnitude relative to the arc current as in the arcs discussed in *A*. Because the arc voltage is usually lower than in self-sustained arcs, the name low voltage arc is also used. In some cases after starting the arc the heating current can be switched off. The discharge will remain essentially the same but now since the arc is self-

[209] This phenomenon will not disturb the determination of the space potential much, but it will make a determination of the electron concentration and temperature hardly possible.

[210] In some cases a constant field is found from a position 1 mm distant from the cathode onward but between 0.5 and 1 mm a much larger field strength is sometimes found while the field between 0.5 mm from the cathode and the cathode fall will probably be small (see e.g. the literature on the Cu arc, reference 257).

[211] H. Nagaoka and Y. Sugiura, Jap. J. Phys. **3**, 45 (1924); see also reference 202 and L. B. Loeb, W. A. Hillebrand, H. E. White, R. N. Varney and F. C. Miller, Phys. Rev. **49**, 703 (1936).

[212] G. Granquist, Ges. Wiss. Upsala **20**, 1902; M. Reich, Physik. Zeits. **7**, 73 (1906); A. Güntherschulze, Zeits. f. Physik **11**, 71 (1922); R. Seeliger and H. Schmick, Physik. Zeits. **28**, 605 (1927).

[213] H. Kohn and M. Guckel, Zeits. f. Physik **27**, 305 (1924); E. C. G. Stückelberg, Helv. Phys. Acta **1**, 75 (1928).

[214] R. Seeliger, Physik. Zeits. **27**, 22 (1926).

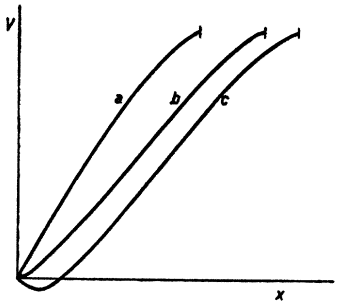


FIG. 57. Potential distribution in the cathode fall of an arc, (a) with Schottky effect, (b) and (c) without Schottky effect.

sustaining the cathode fall will be somewhat higher. Whereas the dependence of the cathode temperature on the current determines in the W arc the form of the characteristic (Fig. 53), the characteristics of the arcs, discussed here, will have quite another form and resemble more the isothermic characteristic of Fig. 55.

The simplest picture we can give of the discharge mechanism of an arc with externally heated cathode is as follows.[215] The cathode emits a thermionic current $i_{th}$. In front of the cathode is a cathode fall of thickness $d_c$, which is much smaller than the electronic free path. Further on an almost field-free space exists where the electrons, leaving the region of the cathode fall, will ionize the gas and form a plasma; the cathode fall will have a value about that of the ionization potential of the gas. If $i_{th}<i$ a Schottky effect at the cathode will enlarge $i_{th}$, the field in the cathode fall will be given by curve $a$ in Fig. 57 and, as no collisions occur in the cathode fall, $d_c$ can be calculated approximately by Eq. (86).[216] If $i_{th}>i$, and the initial energy of the electrons is zero, the field at the cathode will be zero and $j_p/j_e$ will be equal to $(m_e/m_p)^{\frac{1}{2}}$ (curve $b$, Fig. 57, Eq. (89)). If the initial energy of the electrons cannot be neglected a potential minimum will occur in the cathode fall (curve $c$ in Fig. 57).[217] The difference in the thickness of the cathode fall, $d_c$, in these cases is so small (less than a factor 2) that it is unimportant for the following discussion. If still other effects are taken into account, such as the energy of the plasma electrons and ions, the change of $d$ by these causes can also be neglected here.[218]

On the basis of the experimental results we shall extend this picture. We shall also describe arcs with externally heated cathodes for which these assumptions are not valid. Arcs at pressures less than one mm will be considered separately from those at higher pressures although one mm is not a limiting pressure.[219]

## §46. Arcs at a pressure below one mm

In discussing the properties of arcs at low pressure we shall take the arc in A as an example. The low voltage arc is one of the few cases where the thickness ($d_c$) of the dark space-charge layer at the cathode of an arc can be observed, as the current density is relatively small. Table X[220] shows that $d_c$ differs little from the value calculated from Eq. (89) ($d_c$ (theory)).

*Primary, secondary and ultimate electrons.*— From the region of the cathode fall a beam of electrons (primaries) start with a homogeneous

TABLE X. *Thickness $d_c$ and calculated thickness $d_c$ (theory) of the dark-space-charge layer in A. $D$ = thickness of the scattering layer.*

| $p$ MM A | $i$ AMP. | $V$ VOLT | $d_c$ MM | $d_c$ (THEORY) MM | $D$ MM |
|---|---|---|---|---|---|
| 0.01 | 0.1 | 17.5 | 1.5 | 1.0 | — |
| 0.022 | 2.0 | 16.0 | 0.35 | 0.20 | 2.2 |
| 0.050 | 0.25 | 17.0 | 0.86 | 0.60 | — |
| " | 0.5 | 14.8 | 0.44 | 0.38 | 4.5 |
| " | 1.0 | 13.2 | 0.31 | 0.25 | 3.5 |
| " | 2.0 | 13.0 | 0.20 | 0.17 | 2.3 |
| " | 4.0 | 13.0 | 0.12? | 0.12 | 1.6 |
| " | 2.0 | 11.3 | — | 0.15 | 3.1 |
| " | 2.0 | 16.3 | — | 0.20 | 2.4 |

[215] A. Gehrts, Zeits. f. tech. Physik **13**, 303, 350 (1932); Physik. Zeits. **34**, 870 (1933). S. Dushman, Elec. Eng. **53**, 1204, 1283 (1934).

[216] In this discussion the field at $x=d_c$ is assumed to be zero as the field in the plasma is very small with respect to the field in the cathode fall.

[217] I. Langmuir and K. T. Compton, Rev. Mod. Phys. **3**, 191 (1931); C. G. Found, Phys. Rev. **45**, 519 (1934).

[218] I. Langmuir, Phys. Rev. **33**, 954 (1929).

[219] We distinguish these two pressure domains only for practical reasons. Many properties of arcs can be observed in both domains. Under favorable conditions we can, for example, observe the three forms of Fig. 61 at 0.1 mm. The part of $CD$ of Fig. 59 becomes so flat at pressures above one mm (of A) that it is now difficult to determine the position of the point $C$.

[220] Unpublished measurements of N. Warmoltz with an indirectly heated oxide-coated cathode (surface 2.9 cm²) at a constant temperature, only in the last two measurements the cathode temperature was varied.

energy corresponding to the cathode fall.[221] These primaries are scattered and retarded by collisions against the gas atoms and by electron interaction until finally slow electrons, called ultimate electrons, having an isotropic Maxwell distribution, are formed. Between these two groups is found another almost isotropic group of electrons, having about a Maxwell distribution with a mean energy between the primaries and the ultimate electrons. These electrons are called secondary electrons.[222]

At a low current density the primaries degenerate into ultimates by collisions against the gas atoms; at a high current density the interaction between the electrons is important. In §7 we discussed this interaction for an isotropic electron distribution. It has not yet been discussed for a beam of electrons in a plasma but the interaction will of course increase with

FIG. 58. Photograph of a low voltage arc in 0.05 mm A. The cylindrical cathode is viewed end-on and seen as a black circle with a thin bright edge; the figure shows the thin dark space-charge sheath and the relatively dark scattering sheath. The edge of the bulb may be seen in the left-hand bottom corner.

increasing beam intensity. It seems plausible that this interaction is connected to the high frequency oscillations that are found in an electron beam. Because of this interaction, electrons having an energy in excess of the energy corresponding to the cathode fall[223] are also found in the beam.

Beyond the dark space-charge layer another dark layer (between 0.2 and 0.02 mm A) or a luminous layer (between 0.02 and 0.007 mm A) can be seen;[224] we shall call this layer the scattering layer (thickness $D$). Although this layer is

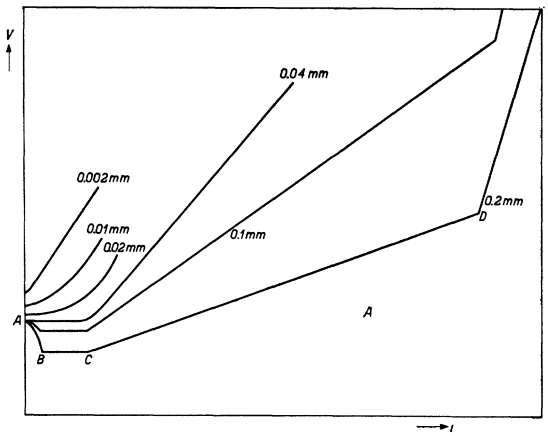


FIG. 59. Schematic characteristic of a low voltage arc with oxide-coated cathode in A at different pressures and a constant thermionic emission. The current scale is indicated thus: point $B$ is at 0.3–0.5 amp. and $C$ is at several amperes.

called dark, it is dark only relative to the light emission of the plasma that is found beyond this layer. Table X shows that the scattering layer is about ten times larger than the space-charge layer; the thickness of both layers is inversely proportional to $i^{\frac{1}{2}}$ ($i$ arc current). Fig. 58 shows a photograph of these layers. We think that the scattering layer is caused by the electron interaction as described above; inside this layer we have an electron beam, outside the layer we have a more or less isotropic electron distribution with only a small drift velocity superposed upon it. As the current density is continuous at the sheath edge (as is also the space potential) the electron concentration must vary discontinuously, and thus cause the sheath. The sharpness of the sheath remains unexplained.

*Characteristics.*—Fig. 59 shows the dependence of the tube voltage, which varies in almost the same way as the cathode fall, on the current at constant cathode temperature.[225]

At a pressure of 0.2 mm A we see first a decrease ($AB$) of the tube voltage from the ionization potential of the gas to a lower value; this decrease is caused by cumulative ionization, the number of ionizations per sec. increasing at a rate greater than $i$. At a low pressure the curve at low current can have another form, see Fig. 60,

[221] In the low voltage arc we have in most cases $d<\lambda_e$ ($\lambda_e$ electronic mean free path).

[222] I. Langmuir, Phys. Rev. **26**, 585 (1925); Zeits. f. Physik **46**, 271 (1927).

[223] F. M. Penning, Nature **118**, 301 (1926); Physica **6**, 241 (1926); A. F. Dittmer, Phys. Rev. **28**, 507 (1926); L. Tonks and I. Langmuir, Phys. Rev. **33**, 195, 990 (1929); L. Tonks, Phys. Rev. **37**, 1458 (1931); **38**, 1219 (1931); H. J. Merrill and H. W. Webb, Phys. Rev. **55**, 1191 (1939). As the high frequency discharges do not fall within the scope of this paper, we shall not discuss the high frequency oscillations occurring in a beam or in a plasma.

[224] N. Warmoltz, Nature **138**, 36 (1936); M. J. Druyvesteyn and N. Warmoltz, Physica **4**, 51 (1937).

[225] M. J. Druyvesteyn and N. Warmoltz, Physica **4**, 41 (1937). The characteristics were measured so fast ($<10^{-4}$ sec.), that no change of the cathode occurred.

where the gradual compensation of the electronic space charge around the cathode by the positive ions can be followed.[226] A cathode fall exists only at the higher current (e.g. at $10^{-3}$ mm above 20 ma). From $B$ to $C$ (Fig. 59) the tube voltage is almost constant. At $C$, $i=i_{th}$ ($i_{th}$ = thermal emission of the cathode). For $i>i_{th}$ a Schottky effect exists at the surface of the oxide-coated cathode. The current of positive ions will increase in respect to the electron current and the tube voltage will increase. At $D$ a much sharper increase sets in. In the domain $BC$ the number of ionizations will be almost proportional to the current, while $i_p/i_e$ will be independent of the current,[227] the consequence being a constant tube voltage. For a pressure below 0.02 mm A it is seen that the tube voltage increases with the current below $C$. This increase for $i<i_{th}$ is probably due to the electron interaction, as at high current density and low gas pressure the primary electrons will have given part of their energy to the ultimates before ionization of a gas atom occurs.

## §47. Arcs at a pressure above one mm

*Visual and electrical phenomena.*—If in an arc at higher pressure and an electrode distance of some cm, the current $i$ and the thermionic emission of the cathode $i_{th}$ are varied, a number of different luminous phenomena can be seen between the electrodes. As these phenomena are especially pronounced in Ne, we take as an example a Ne discharge at a pressure of 10 mm. For $i_{th}<i$ the most intense part of the discharge is concentrated near the cathode (Fig. 61$^{I}$), and a thin space-charge layer can be seen. The tube voltage lies between the excitation and the ionization potential of Ne. If the heating current is increased, so that $i_{th}$ and $i$ become of the same order, or $i_{th}$ is somewhat larger than $i$, we see a red sphere, free from the electrodes, in an aureole of less intense light; the place of the red

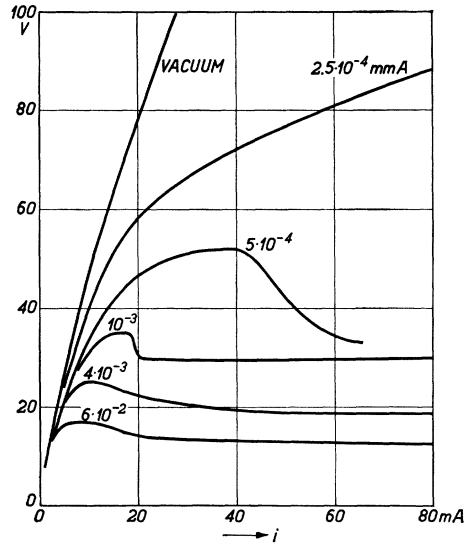


FIG. 60. Characteristic of a low voltage arc at a low A pressure and up to 80 ma (distance of the electrodes 2 cm). Emission of the cathode was several amperes.

sphere is somewhat arbitrary (Fig. 61$^{II}$). The tube voltage is now small, it may be only half of the excitation potential. If the heating current is further increased ($i_{th}>i$), the red sphere is pushed in the direction of the anode, and a red sphere or yellow layer is seen on the anode (Fig. 61$^{III}$). The tube voltage is in the latter case equal to the ionization potential, in the former case it is somewhat lower. The only luminous part of the discharge is now an anode layer. Gabor[228] also described a "dark discharge"

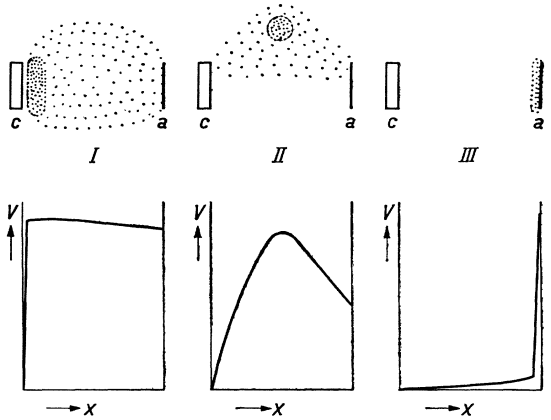


FIG. 61. Light phenomena and potential distribution in a low voltage arc in 10 mm Ne (schematically) for $i>i_{th}$ (I), $i\approx i_{th}$ (II) and $i<i_{th}$ (III). $c$ = cathode, $a$ = anode, $x$ is the distance to the cathode.

[226] The space-charge limited current in vacuum was 20 ma at 78 v. The current, flowing before the maximum voltage (ignition potential of the arc), is very large in this case.

[227] The positive ion current to the cathode is about equal to $(m_e/m_p)^{\frac{1}{2}}$ times the current; however, the correction, which must be applied to this value for the energy of the plasma electrons and negative ions can diminish this value by a factor of 2 or 3. See reference 218, and S. Gvosdover, Physik. Zeits. Sowjetunion **7**, 74 (1935); **8**, 579, 582 (1935).

[228] British Patent 455,967 (1935).

with a very large dark space around an oxide-coated cathode, the gas atoms being first excited at a great distance from the cathode, where in this case a gauze was mounted.

In form II measurements with sounding electrodes[229] have proved the existence of a potential maximum in the red sphere, the potential being about 18.5 v above the cathode potential (in Ne). Between the sphere and the anode a retarding field for the electrons exists (Fig. 62). Only in

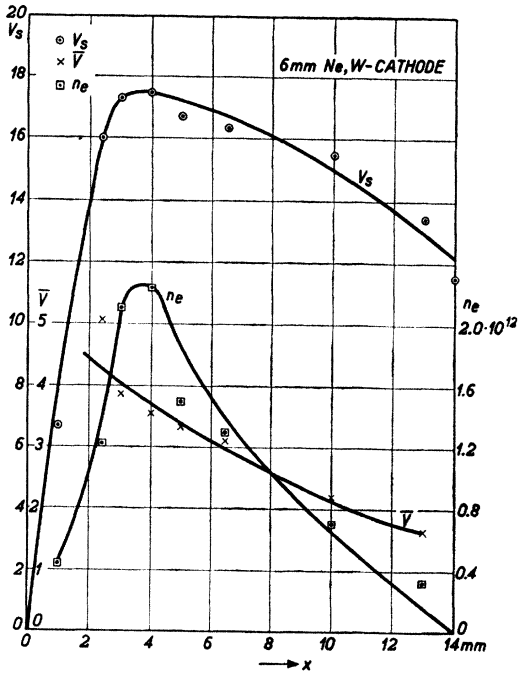


FIG. 62. Space potential ($V_s$), electron concentration ($n_e$) and mean electron energy ($e\bar{V}$) in a low voltage arc in 6-mm Ne with W cathode.

the sphere is the potential high enough for the excitation of atoms. Here metastable atoms are formed that are ionized by a collision with a second electron or with another metastable atom. This cumulative ionization occurs also in many other high current discharges.[230] The electron concentration is so large in the maximum ($10^{12}$ per cm$^3$) that a large electron-interaction occurs and the electrons will arrive at the anode by diffusion against a retarding field, which is roughly described by:[231]

$$V_r = \tfrac{2}{3}\bar{V} \ln (n_{\max}/n_a).$$

Here $n_{\max}$ and $n_a$ are the concentrations in the maximum and near the anode. The potential difference between these places is $V_r$, while $e\bar{V}$ is the mean electron energy, which is supposed to be constant. This constancy is, however, not found experimentally.

*Discussion.*—We will now return to the simple picture of arcs with externally heated cathode which was given in §45. The case of Fig. 61[1] is an extension of this picture to a higher pressure, at which the electrons lose their energy in excitation and ionization at a distance of a few mm from the cathode; and the thickness of the cathode fall is approximately given by Eq. (86).

What is the reason for the other forms of this arc (II and III)? In Table V §31 we have seen that the space charge of the electrons can be compensated by the positive ions in two ways, either the space-charge layer given by Eq. (89) will exist, no collisions occurring in this layer, or the field will be very small and $j_e/j_p$ almost equal to $\mu_e/\mu_p$, many collisions now being essential to give a current with constant mobilities. If $i_{\text{th}} < i$ a field at the cathode will exist and we will have a thin space-charge layer, where no collisions occur, and our simple picture can be used. But for $i_{\text{th}} > i$ the field at the cathode will be zero or negative and we think that here the second case (with constant mobilities) can be approximated if the pressure is high enough. The ions can either be formed in the anode layer or at an arbitrary place in the gas. In the first case the field will be very small between the anode layer and the cathode. Fig. 61 shows also the potential distribution for the three cases. A quantitative discussion of these suppositions has not been given.

It seems plausible that the tube voltage will be especially low for the case II, where a large

[229] K. T. Compton and C. Eckart, Phys. Rev. **25**, 139 (1925); M. J. Druyvesteyn, Zeits. f. Physik **64**, 781 (1930).

[230] In a number of arcs spark lines have been observed. If the pressure is not too low and the cathode fall not too high, it seems probable that the excitation has occurred in more steps, the last step being an excitation of the ion by an electron, the high ion concentration making this process probable. By way of example we may refer to $A^+$-lines in an A low voltage arc (reference 229) and $Cu^+$ lines in a Cu arc in air (W. B. Nottingham, J. Frank. Inst. **207**, 299 (1929)).

[231] The problem is analogous to the sideway electron diffusion in a positive column (see §53). The existence of a large electron current in a low voltage arc makes however an essential difference in the case of the positive column.

diffusion space exists, while the value of $j_p/j_e$ will be smaller in cases II and III than in case I.[232] The current density must of course be higher than a certain value, as an electron interaction is essential for a low tube voltage.

The appearance of a potential maximum and an electron diffusion against a retarding field occurs in many arcs and also in the negative glow and the Faraday dark space of a glow discharge. Usually, however, the retarding field is smaller than in the case just described.

In arcs with an externally heated cathode the properties are often disguised by the appearance of oscillations that occur especially for $i_{th} > i$ but also in other cases.[233] Sometimes these oscillations are connected with a changing arc path as can be observed visually, the frequency being below $10^4$ per sec. However, high frequency oscillations are also found sometimes.

### C. *Arcs with Field Current Emission* (Hg *arc*)

### §48. Experiment

For the arcs with a cathode of liquid metal, e.g. of Hg, Cd, Zn, Sn, Bi or Na it seems probable that the temperature of the cathode spot is so low, that the thermionic emission of the cathode is absolutely insignificant. As the arc on liquid Hg is the most typical example of these arcs, we shall discuss here only this case.

Langmuir[234] supposed that in the Hg arc the electrons were drawn out of the cathode by the high electric field given by the space-charge layer of the positive ions; a high current density being essential for this "field-current arc."

In an arc with a cathode of liquid Hg a small cathode spot[235] is seen (or at high current a number of spots), moving rapidly over the Hg surface. If, however, a metal strip dips into the Hg and the metal is wetted by the Hg, then an *anchored spot* can exist. A number of cathode spots move swiftly along the boundary between the mercury and metal, where the metal is covered by a very thin Hg film,[236] so that a *cathode line* is seen instead of a cathode spot. The velocity of the spots is almost the same as for a free spot (50–150 cm/sec.).[237] As a number of properties are probably about the same for the free and for the anchored spot, we shall discuss both forms at the same time. Fig. 63

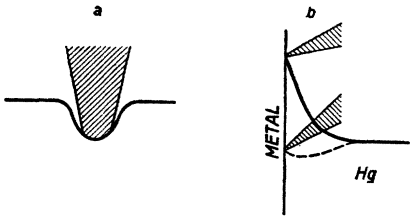


FIG. 63. Form of the Hg meniscus near a free cathode spot (a) or an anchored cathode line (b) for a small and a large current; the probable form of the discharge path is shaded.

shows schematically the form of the meniscus at the cathode spot in these two cases.

The cathode fall of the Hg arc[238] with a free spot was found to be 10.0 v at room temperature, while for an anchored spot it varied from 9.0 to 11.6 v with decreasing gas pressure (from $2\times10^{-2}$ to $4\times10^{-5}$ mm, Fig. 64).

The cathode spot is very small and its area is difficult to measure. The current density is high, about $10^4$ amp. per cm$^2$ (Güntherschulze[239] gives 4000, Kobel[239] 1700–2100, Tonks[236] 3000–18,000 amp. per cm$^2$).

As in thermionic arcs, often a Faraday dark space exists between the cathode spot and the anode glow, or, when the electrode-distance is large, between the cathode spot and the head of

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[232] A. Gehrts and H. Vatter, Zeits. f. Physik **79**, 421 (1932).

[233] H. Kniepkamp, Zeits. f. tech. Physik **17**, 398 (1936); M. J. Druyvesteyn, Physica **4**, 669 (1937); W. Funk and R. Seeliger, Zeits. f. Physik **113**, 203 (1939). In Ne the red sphere can sometimes be seen jumping between two places or in other cases alternatively a red sphere and a yellow anode layer can be observed (II and III in Fig. 61).

[234] I. Langmuir, Gen. Elec. Rev. **26**, 731 (1923).

[235] In the discussions of the Hg arc the word cathode spot is used for the area at the cathode, where the current flows, and also for the thin luminous glow in the gas near the cathode, which is a kind of negative glow. When there is no cause for misinterpretation, we shall use the word cathode spot in both meanings.

[236] L. Tonks, Physics **6**, 294 (1935); N. Warmoltz, unpublished.

[237] When the cathode spot is placed in a magnetic field, the spot velocity can be increased considerably (100 times or more), see: A. Dufour, J. de phys. et rad. [5] **1**, 109 (1911).

[238] E. S. Lamar and K. T. Compton, Phys. Rev. **37**, 1069 (1931); J. Kömmick, Ann. d. Physik **15**, 273 (1932); E. Lübcke, Zeits. f. tech. Physik **14**, 297 (1933).

[239] A. Güntherschulze, Zeits. f. Physik **11**, 74 (1922); E. Kobel, Phys. Rev. **36**, 1636 (1930).

the positive column. A reversed field can occur in the Faraday dark space (see the probe measurements of Fig. 64) and the tube voltage can sometimes be less than the cathode fall and may be as low as 7.3 v for an anchored spot.[240]

The pressure in the discharge may have a wide range of values. Depending on the circumstances, it may be several atmospheres or as low as $10^{-4}$ mm. The low values can be attained by cooling the Hg vapor. The pressure near the cathode spot does not attain so low a value as a consequence of the evaporation of Hg from the spot. For an anchored spot $2.5\times10^{-4}$ gram Hg evaporates per coulomb.[241] For a free spot also a number of fine Hg drops are swept into the discharge. Fig. 63[a] suggests that these drops are thrown from the sharp bending of the liquid Hg near the spot, especially as it moves rapidly and erratically.

The minimum current at which an arc extinguishes at room temperature is about 5 amp. for a free spot and about 0.5 amp. for an anchored spot; this last current is of the same order as the current to one of the spots that move along the cathode line (between 0.2 and 0.4 amp.). The large current necessary for the stable operation of an arc with a free spot seems to be connected with the splashing away of the Hg drops; when a drop falls into the hole in the mercury surface on the cathode spot the arc may extinguish easily. With an anchored spot, the arc lowers somewhat the boundary of the Hg by the force exerted by the discharge upon the mercury[242] (Fig. 63[b]) but the Hg is now much more quiet and no drops are swept away, so that the arc will extinguish at a much lower current. An interruption of the current to the cathode during only $10^{-9}$ sec. is sufficient to prevent reignition,[243]

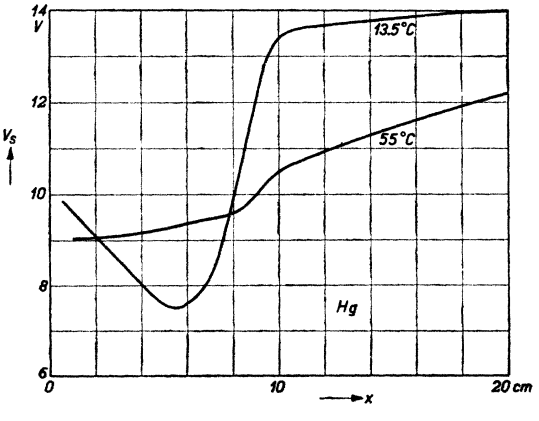


FIG. 64. Space potential ($V_s$) near an anchored cathode line with 10 amp. at 13.5° and 55°C (Kömmnick).

The temperature of the surface of the liquid mercury at the cathode spot is difficult to measure or to calculate directly.[244] The cathode spot emits, beside Hg lines, a continuous spectrum, which has sometimes been attributed to the high temperature of the Hg surface. The temperature would in that case be about 2000°C.[245] It seems, however, that this spectrum is emitted by the highly ionized Hg vapor in the cathode spot (perhaps by recombination) and should not be attributed to radiation of the liquid Hg.

From the evaporation of the spot and the current density the temperature of the cathode spot is found to be between 220° and 300°C,[246] which is far too low for any thermionic emission.

Near the cathode $iV_c\approx10i$ watt is produced, of which $3i$ watt[247] is conducted away by the cathode; only $0.1i$ watt is used for vaporization. The remaining 70 percent of the energy generated near the cathode is ultimately given to the tube walls and the anode, a small part being transformed into radiation. From the energy balance of the cathode we can, according to Eq. (102),

[240] Unpublished experiment of T. Jurriaanse with a C anode. See also A. Gaudenzi, Brown Boveri Mitt. **16**, 303 (1929).

[241] L. Tonks, Phys. Rev. **54**, 634 (1938).

[242] L. Tonks, Phys. Rev. **50**, 226 (1936).

[243] G. Mierdel, Zeits. f. tech. Physik **17**, 452 (1936).

[244] The temperature is determined not only by the heat conduction, but the convection of the liquid Hg is also important (see reference 185 and J. Slepian and A. Toepfer, Physics **9**, 483 (1938)).

[245] E. Lübcke, Zeits. f. tech. Physik **10**, 598 (1929); Brown Boveri Mitt. **16**, 61 (1929). See also reference 236.

[246] If the current density is $10^4$ amp. per cm², 2.5 g Hg evaporates per cm² per sec. To this evaporization corresponds a saturation pressure of 37 mm and a temperature of 223°C. If 20 percent of the current is a positive ion current, the vaporization would be larger, as a number of evaporated atoms go back to the cathode as ions; in this case the temperature of the cathode would be 259°C, while if the whole current was carried by positive ions, it would be 311°C. Although the temperature may be somewhat higher from a number of effects, it seems improbable that it surpasses 350°C.

[247] A. Güntherschulze, Zeits. f. Physik **11**, 74 (1922) gives 2.7 $i$ watt; J. Kawa, C. R. Sol. Pol. Ph. **4**, 27 (1929); N. Warmoltz found for an anchored spot 3.0 $i$ watt (unpublished).

with $\varphi=0$ (see further on) determine roughly the value of $i_p/i_e$ for which we find about 0.2 amp. More detailed discussion would possibly give a rather different value.[248]

### §49. Theory

If in the cathode fall ($V_c$) no collisions occur and the current density for ths ions is $j_p$, for the electrons $j_e$, then the field strength at the (plane) cathode ($E_c$) is given by:[249]

$$E_c^2=16\pi\left[j_p\left(\frac{MV_c}{2e}\right)^{\frac{1}{2}}-j_e\left(\frac{mV_c}{2e}\right)^{\frac{1}{2}}\right]$$

$$\approx 16\pi j_p\left(\frac{MV_c}{2e}\right)^{\frac{1}{2}}. \quad (104)$$

In this $M$ and $m$ are the mass of an ion and an electron, respectively. By taking plausible values (e.g. $j_p/j_e=0.2$, $j_p+j_e=10^4$ amp./cm$^2$), we find for $E_c$ $1.7\times10^7$ v per cm, a value so high that a field current of a few thousand amp./cm$^2$ might exist. As no more definite measurements or theoretical formulae are known, it is impossible to test the theory more precisely.

The cathode fall is about 10 v and it might be thought that it is almost equal to the ionization potential of Hg (10.4 v), this correspondence seems, however, to be fortuitous as at the high current density, occurring in the Hg arc, cumulative ionization will occur and the energy of the electrons leaving the cathode fall needs only be somewhat above the excitation potential of Hg (4.9 v).

For a field current the energy of the electrons leaving the region of the cathode fall will be less than the cathode fall and almost be equal to[250] $e(V_c-\varphi)$, which in our case ($\varphi=4.5$ v) corresponds to 5.5 v, so these electrons have enough energy to excite Hg atoms. Another difference between field current and thermionic emission is that in the last case an electron leaving the cathode cools the cathode by an amount of $e\varphi$, while in the first case almost no cooling will occur.

[248] K. T. Compton, Phys. Rev. **37**, 1077 (1931).
[249] S. S. Mackeown, Phys. Rev. **34**, 611 (1929); O. Becken and R. Seeliger, Ann. d. Physik **24**, 609 (1935).
[250] See reference 248 and a somewhat different discussion in R. C. Mason, Phys. Rev. **38**, 427 (1931).

Theory and experiments of the Hg arc are not yet so far advanced as to derive other properties of this arc from the field current picture; neither can the anchoring of the spot be understood.

## *D. METAL ARCS (ON SOLID METAL)*

### §50. Classification

For the arcs we discussed in the former sections an interpretation of the electron emission of the cathode could be offered; for a carbon and tungsten arc a thermionic emission exists, while for the arcs on liquid metal a field emission is at least a probable hypothesis. For the other metal arcs we can in most cases only guess at a possible mechanism; in some cases it is one of the two mechanisms just mentioned; in other cases none of these is probable and another suggestion must be made for the electron emission of the cathode. This unpleasant state of affairs is, among other reasons, caused by the fact that a metal arc (on one definite metal) can often occur in different forms for which sometimes different mechanisms seem possible. To make this extensive subject suitable for a short discussion we shall at first give a classification of different metal arcs and, although this classification is very dubious, it may help to clear up the different points of view on this subject.

For the classification of these arcs (Table XI) we shall examine especially the temperature of the cathode, the reaction of the arc to a magnetic field and the influence of impurities on the cathode surface. If external means are applied to bring about a movement of the cathode spot, the arc may behave in different ways. If the arc is placed in a magnetic field parallel to the cathode surface the cathode spot may either move swiftly perpendicular to the field over the cathode surface or it may remain at rest or move slowly. We can also try to let the spot move over the cathode

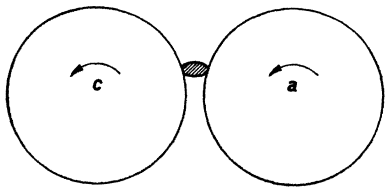


FIG. 65. Arc (shaded) between two polished metal disks which can be rotated rapidly.

TABLE XI. *Classification of arcs at about atmospheric pressure.*

| | SPOT AT REST | | | | SPOT MOVES SLOWLY | SPOT MOVES RAPIDLY | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | WITHOUT EXTERNAL MEANS | | WITH EXTERNAL MEANS | |
| | I | II | III | IV | V | VI | VII | VIII | IX |
| | pure W<br>" C | pure solid Hg<br>" " Cd<br>" " Zn<br>" " Pb<br>" " Mg | normal Cu<br>" Fe<br>" Ni<br>" Ag<br>" Au | Al<br>Sn | impure W<br>" Fe | very pure Cu<br>" " Fe<br>" " Ag | liquid Hg<br>" Zn<br>" Cd<br>" Na | normal Cu<br>" Fe<br>" Ni<br>" Ag<br>" Au | (Al)<br>(Sn) |
| Impurities present | no | no | yes | yes | yes | no | no | | |
| Temp. of cathode | high (3000°K) | low | high (2000°K) | | low | low | low | low | |
| Classification | thermionic | thermionic | | | | field current | field current | field current | |

surface by giving the cathode a high velocity in respect to the anode, e.g. by the method of Fig. 65, which speaks for itself. In this case either the spot will move swiftly over the cathode surface or the arc will extinguish. By these experiments the arcs on solid metals can be classified into three groups.[251] The spot will not move on pure W and C, solid Hg, Cd, Zn, Pb, Mg (column I and II). The spot will move swiftly on liquid metals and on Cu, Fe, Ni, Ag, Au (column VII, VIII). The metals Al and Sn (column IV) occupy a position between the two extremes as the spot moves at a low current and remains at rest at a high current (column IX).

The arcs on a liquid metal may move in the absence of a magnetic field but in general the arcs on solid metals do not. The arcs on solid metals may therefore be expected to have different properties in and out of a magnetic field. The solid metals are classified in two columns, III and VIII, Table XI.

Also in other dynamic experiments, such as the determination of the minimum frequency at which an a.c. arc can remain steady, definite groups are found. Although in various experiments a somewhat different classification is made (e.g. in terms of a chemical reaction between the cathode and the gas or of the different conditions of the electrode surface) the different groups are always clearly distinguished.[252]

We see that in the second column the easily vaporizable metals are found, while the arcs that can be moved easily by external means occur on metals that do not evaporate so easily (column VIII).[253] It is curious that this last group behaves like the arcs on liquid metal in that the spot will move rapidly.

In an experiment, without external means to move the spot, first performed by Doan and his collaborators[254] the difference between columns III and VI was found. When a pure gas (A, Ne, $H_2$) and a very pure cathode (Fe, Cu, Ag) were used the spot moved rapidly over the metal surface and the arc extinguished easily. When the cathode was somewhat impure, e.g. a little oxidized, a normal stable arc was obtained with a steady cathode spot.

[251] This classification is based principally on the literature cited in references 252, 254, and 255 (arcs at about atmospheric pressure). Some of the properties mentioned (e.g. the movement of the cathode spot) will probably depend much on the circumstances (e.g. on the pressure).

[252] H. Stolt, diss. Upsala 1925; Ann. d. Physik **74**, 80 (1924); Zeits. f. Physik **26**, 95 (1924); **31**, 240 (1925); F. C. Todd and T. E. Browne, Phys. Rev. **36**, 732 (1930); W. Ramberg, Ann. d. Physik **12**, 319 (1932); J. J. Sommer, Physik. Zeits. **34**, 324 and 464 (1933); H. Plesse, Ann. d. Physik **22**, 473 (1935); A. Kotecki, Acta Physica Polonica **3**, 105 (1934).

[253] The movement of the cathode spot is probably due to the irregular form of the cathode surface and hence an irregular generation of electrode vapor. At a liquid cathode a deviation from the symmetrical form indicated in Fig. 63a can occur; at a solid cathode sharp points can occur on the surface.

[254] G. E. Doan and E. L. Myer, Phys. Rev. **40**, 36 (1932); Elec. Eng. **51**, 624 (1932); G. E. Doan and A. M. Thorne, Phys. Rev. **46**, 49 (1934); C. G. Suits and J. P. Hocker, Phys. Rev. **53**, 670 (1938); J. D. Cobine, Phys. Rev. **53**, 911 (1938).

We discussed the W and C arcs as thermionic arcs. If, however, some $O_2$ is present or the cathode is oxidized or the gas pressure is low, arcs on these cathodes can be obtained in another form.[255] When a cathode spot moves on the cathode, these arcs are generally not thermionic as the cathode temperature is usually low. Neither are they, in all cases, field current arcs as the current density is sometimes relatively small ($<10$ amp. per cm$^2$). Also on other metals, probably somewhat oxidized, arcs are sometimes found which behave the same as the W and C arcs.[256]

Table XII gives the cathode fall $V_c$, the current density at the cathode $j_c$ and the temperature of the cathode $T_c$ for some metal arcs of normal purity, the cathode spot being at rest or moving slowly (column III of Table XI). Since we criticized these measurements in the discussion of the cathode fall of a C arc we shall not repeat this criticism here. An additional difficulty is, that for metal arcs it is often not stated in the literature whether a pure or an oxidized cathode was used. The data of Table XII[257] therefore must be understood as being provisional.

TABLE XII. *Values of $V_c$, $j_c$ and $T_c$ for some metal arcs.*

| CATHODE | GAS | $V_c$ VOLTS | $j_c$ AMP./CM$^2$ | $T_c$ °K |
|---|---|---|---|---|
| Cu | $N_2$ | 10.5–14.5 (20.5) | $3\times10^3$ | <1900, 2200 |
| Cu | vacuum | | $14\times10^3$ | |
| Fe | $N_2$ | (16.3) | $7\times10^3$ | 2430 |
| Ni | $N_2$ | (11.7) | | 2370 |
| Ne | vacuum | (5) | | low |

[255] R. Seeliger and H. Schmick, Physik. Zeits. **38**, 605 (1927); R. Seeliger and H. Wulfhekel, Physik. Zeits. **31**, 691 (1930); O. Becken and R. Seeliger, Ann. d. Physik **24**, 609 (1935); O. Becken and K. Sommermeyer, Zeits. f. Physik **102**, 551 (1936); F. H. Newman, Phil. Mag. **22**, 463 (1936).

[256] J. Slepian and E. J. Haverstick, Phys. Rev. **33**, 52 (1929).

[257] W. B. Nottingham, J. Frank. Inst. **206**, 43 (1928); **207**, 299 (1929) ($V_c$); E. H. Bramhall, Phil. Mag. **13**, 682 (1932) ($V_c$); J. L. Myer, Zeits. f. Physik **87**, 1 (1933) ($V_c$); P. L. Betz and S. Karrer, Phys. Rev. **49**, 860 (1936) ($V_c$); A. S. Fry, Phys. Rev. **51**, 63 (1937) ($V_c$); A. Güntherschulze, Zeits. f. Physik **11**, 74 (1922) ($j_c$); R. Tanberg and W. E. Berkey, Phys. Rev. **38**, 296 (1931) ($j_c$); K. Langbein, diss. Basel, 1918. Arch. Gen. **46**, 329 (1918) ($T_c$); A. v. Engel and M. Steenbeck, Elektr. Gasentladungen, II, p. 135–136. Most metal arcs (on solid as well as on liquid metal) can either occur in a gas atmosphere or as a vacuum arc, where no gas, except the vapor from the electrodes is present.

## §51. Theoretical considerations

We will now discuss to what extent some metal arcs can be explained and which phenomena seem to be essential. In the C or W arc the electron emission of the cathode is thermionic whether the arcs of column III (Table XI) are of thermionic origin is in our opinion an open question.

In all cases where a high current density of at least $10^3$ amp. per cm$^2$ exists at the cathode, a field emission seems possible. This situation seems to occur especially when the cathode spot moves rapidly over the cathode for then thermionic emission probably can be excluded as the temperature of the cathode under the moving spot cannot rise high enough. For the cases of columns VI, VII and VIII (Table XI) an arc with field emission seems at the moment the most probable explanation.

The great influence of some impurity of the cathode suggests the following hypothesis for an arc mechanism. In §39 we discussed a discharge called "spray discharge," which existed when the cathode was covered with a thin insulating layer. Positive ions on the insulator then give a field strength at the metal surface so high that electrons are emitted through the insulator. Almost no cathode fall exists in the gas. It seems that the same mechanism can occur in arcs with some impurity on the cathode surface as in the arcs of columns III and V.[258,254] Quite another influence of the impurity can occur in arcs with a hot cathode spot, as the thermionic emission, by lowering the value of $\varphi$ can be much increased by impurities. An impurity *under* the cathode spot can increase the emission by raising the temperature of the cathode spot when the impurity has a smaller coefficient of heat conduction than the metal.[259] For the metals of column II in Table XI as yet no satisfying explanation has been proposed.

The influence of the electrode vapor on the properties of an arc is still obscure. As the ionization potential and other properties of metal vapor will be different from these properties of the gas in the tube, it seems clear that an arc with metal vapor will differ from an arc in which

[258] M. J. Druyvesteyn, Nature **137**, 580 (1936).

[259] A. v. Engel, Naturwiss. **23**, 305 (1935).

this vapor is absent. Some authors,[196] however, suppose that the influence of the metal vapor for the cathodic phenomena of an arc in a gas is still more essential than indicated above and that most metal arcs in a gas atmosphere cannot exist without metal vapor. In vacuum arcs, this is a matter of course since no other gas but the electrode vapor is present.

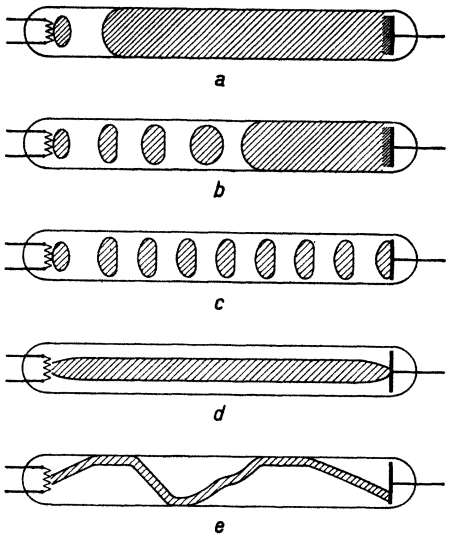


FIG. 66. Schematic drawing of a tube with heated cathode with a homogeneous positive column (a), two forms of striated columns (b and c) and of contracted columns (d and e).

The cathode can give off metal vapor by a number of processes such as evaporation, chemical reaction of the cathode with the gas and sputtering of the cathode by the impinging positive ions. For the origin of the spur of the discharge on the cathode the same processes may be involved. In order to settle the question whether the metal vapor is essential for the arc mechanism, it seems therefore important to know whether or not an arc (e.g. on Cu or Fe) with a cooled highly polished metal cathode in an inert gas would under all circumstances produce an electrode vapor in the discharge. The decrease of the intensity of the spectral lines of the metal by cooling a smooth cathode has been described,[252,260] but the absence of these lines has as far as we know, never been found.

[260] C. G. Suits, Physics **5**, 380 (1934).

Slepian[261] doubted whether an electron emission of the cathode is necessary in all arcs. If the temperature of the gas just before the cathode is so high that a plasma exists even in the first mean free path from the cathode, then the current to the cathode could be an ion current. It seems unlikely, however, that such a high temperature is possible in the gas at so small a distance from the relatively cold cathode.

Still other mechanisms could be suggested *a priori* which could be important for the phenomena at an arc cathode.[262] We shall only mention as an example the suggestion that the probability, of a positive ion liberating an electron from the cathode ($\gamma$) may increase with increasing current density.

Many more experiments, especially under simple circumstances, are needed before a definite theory of the different metal arcs can be given.

## VIII. The Positive Column

The positive column is that part of a glow or arc discharge which exists between the cathodic and anodic parts; normally it extends from the Faraday dark space to the anode fall (Fig. 42). Positive columns are often studied in long cylindrical tubes; usually the properties of the column do not depend on its length. The d.c. positive column can have different aspects (see Fig. 66) which we can classify as follows.

I. *Homogeneous column* (Fig. 66a). The properties of the column (with the exception of the parts near the beginning and near the end) do not depend on the distance to the cathode but depend on the distance to the axis of the tube.

II. *Striated column.* Two cases can be distinguished: (*a*) In rare gases in a limited range of current and pressure a number of striations can occur at the head of the column; the striations, however, soon become vague; at most eight striations have been observed (Fig. 66b). (*b*) In molecular gases or in mixtures containing some molecular gas an unlimited number of striations can sometimes be observed, every striation being equally sharp (Fig. 66c).

III. *Column with moving striations.* This may occur in the rare gases, the striations moving

[261] J. Slepian, Phys. Rev. **27**, 407 (1926).
[262] F. Lüdi, Zeits. f. Physik **82**, 815 (1933).

with a velocity of some hundred meters per sec. to the cathode. In some researches these striations were overlooked and the column erroneously assumed to be homogeneous.

IV. *Contracted column*, which radiates only over a part of its diameter. Here also we must distinguish two cases: (*a*) The column can be

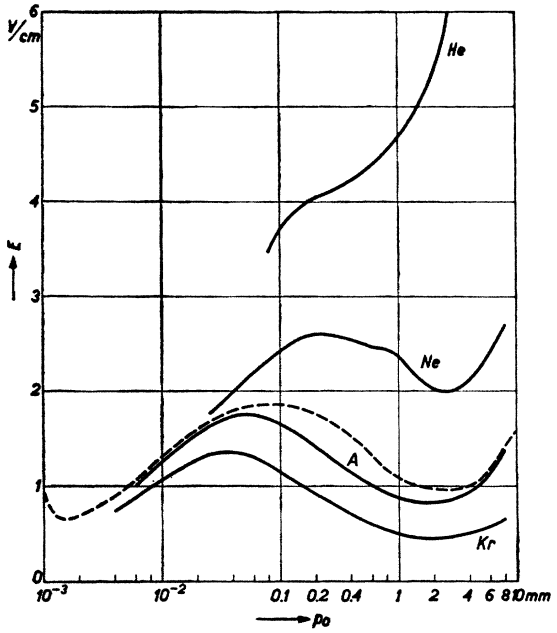


FIG. 67. Gradient $E$ as a function of the gas density $p_0$ in He, Ne, A and Kr at 0.3 amp. ($2R=2$ cm) after Klarfeld (——) and A at 0.15 amp. ($2R=1.95$ cm) after Groos (- - -).

contracted regularly (Fig. 66d). The light emission decreases continuously to zero before the tube wall is reached. This occurs at higher pressures (e.g. above a few cm). (*b*) In some cases the column contracts in a very irregular way (Fig. 66e). A discontinuous decrease of the light intensity exists at the boundary of the discharge path. This occurs for example in a mixture of an electropositive gas with some paraffin vapor.

As the homogeneous column is the most simple and normal form we will consider this first. The striated forms can be treated as a homogeneous column, on which a small oscillating part is superposed, however the average value of most properties (as e.g. the gradient) being, almost the same in both cases.

### *A. Homogeneous Positive Column*

A homogeneous column is a plasma with only a small electric field, having a constant component $E$, called the gradient, parallel to the axis of the tube and a component normal to it which is zero at the axis and increases towards the walls. The difference between the electron and ion concentration is small ($|n_p-n_e| \ll n_p$); we assume usually negative ions to be absent. The electron distribution is often Maxwellian; this can be brought about by the electron interaction, or by other mechanisms (see §7). Near the (insulating) wall a thin space-charge layer will exist, which is caused by the equality of the ion and electron current to the wall.

As the theory of the positive column is rather complicated and treated by combining a large number of equations, while a number of experiments can best be considered in connection with this theory, we will first discuss the two most important experimental properties of a positive column in a pure gas, namely the electrical gradient $E$ and the electron temperature $T_e$. These properties will be discussed both for a homogeneous column and for a column with moving striations.

## §52. Experiment

*The gradient E.*—The electric gradient $E$ in the homogeneous column will in each gas depend on the gas density (the pressure reduced to 0°C being $p_0$), the current $i$ and the radius $R$ of the tube:

$$E=f(p_0, i, R).$$

In *atomic gases* the gradient depends in a peculiar way on the density (Figs. 67 and 68). Above a pressure of a few mm the value of $E$ increases with the density and above a few cm the column begins to contract.

Between pressures of a few mm and $10^{-3}$ mm one or two minima occur; below $10^{-3}$ mm the value of $E$ seems to increase again. The differences between various gases are large. Generally the results of different authors agree well.[263]

[263] A. Güntherschulze, Zeits. f. Physik **41**, 718 (1927); **42**, 763 (1927) (He, Ne, A, Kr, Xe, Hg); P. Johnson, Phil. Mag. **10**, 921 (1930) (Ne); F. Llewellyn Jones, Phil. Mag. **11**, 163 (1931) (He); S. P. McCallum and F. L. Jones, Phil. Mag. **12**, 384 (1931) (A); W. Elenbaas, Zeits. f. Physik **78**, 603 (1932) (Hg); A. Lompe and R. Seeliger,

Usually $E$ increases with decreasing current; the increase is rapid below a current density of about $10^{-6}$ amp./cm$^2$.[264] For a pressure below 0.01 mm (Fig. 69) the dependence of $E$ on the current (above 25 ma) becomes small. In He sometimes a positive characteristic can occur (Fig. 69, below 40 ma).

The gradient $E$ decreases with increasing tube diameter, until at large values of the diameter $E$ becomes independent of it.[265] At a pressure above one mm, the dependence of $R$ can often be described by the principle of similarity (§32), which states that at a constant current $E/p_0$ is a function only of $p_0R$ and $i$:

$$E/p_0 = f(p_0R, i). \qquad (105)$$

It is not yet known how the deviations from this formula, which occur at a lower pressure, depend on the different variables.

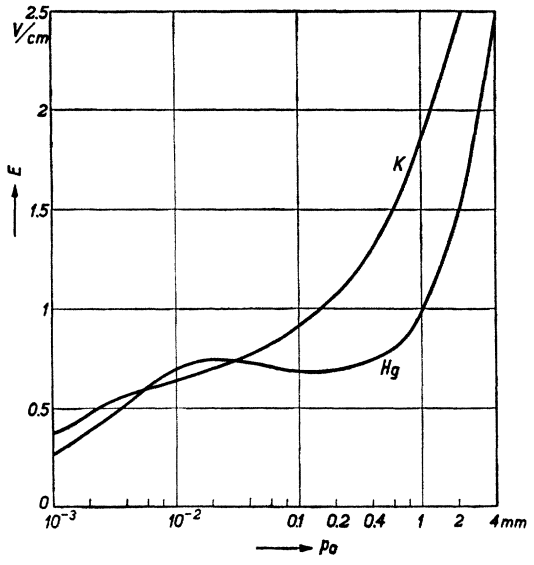


FIG. 68. Gradient $E$ in Hg and K at 0.3 amp. ($2R=2$ cm) after Klarfeld.

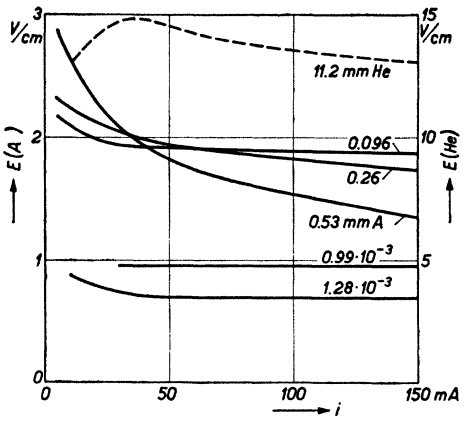


FIG. 69. Gradient $E$ as a function of the current $i$ between 10 and 150 ma at different A pressures after Groos ($2R=1.95$ cm) (——) and in He at 11.2 mm after Lompe and Seeliger ($2R=3$ cm) (- - -) ($E$ scale for He at the right).

In *molecular gases* ($H_2$, $N_2$, $O_2$, $H_2O$) the gradient depends only slightly on the current (between 5 and 100 ma) and the similarity principle seems to apply rather well. Eq. (105) becomes in this case:

$$E/p_0 = C(p_0R)^{m-1}, \qquad (106)$$

in which $m$ lies between 0.56 and 0.67.[266] Sometimes different results are obtained by various authors[267] which is attributed to the differences in the purity of the gas.

*The electron temperature* $T_e$.—While the gradient can be measured rather exactly in different ways, which are not much open to criticism, the electron temperature $T_e$ and the electron concentration are measured by the probe method as developed by Langmuir and Mott-Smith.[27] We shall not discuss this theory here, but shall only point out two difficulties of this method. At higher pressure the collisions in the space charge layer around the probe cannot be neglected,[208] so that at a pressure above a few mm errors may arise. Another difficulty arises in rare gases through the liberation of electrons from the probe by positive ions, excited atoms or radiation. In Hg this last difficulty is almost absent. The electron temperature $T_e$, or the mean electron energy $e\bar{V}$ is measured in the positive

Ann. d. Physik **15**, 300 (1932) (He, Ne, A); O. Groos, Zeits. f. Physik **88**, 741 (1934) (A); W. Ende, Zeits. f. tech. Phys. **15**, 601 (1934) (Hg); B. Klarfeld, Tech. Phys. U.S.S.R. **4**, 44 (1937); **5**, 725 (1938) (Hg, He, Ne, A, Kr, K). See also references 15 and 19. In some cases the column in the rare gases was probably not always homogeneous, as moving striations occurred.

[264] G. Mierdel and W. Schmalenberg, Wiss. Ver. Siemens Konz. **15**, 60 (1936).

[265] The gradient at a value of $R$, so large that $E$ is independent of $R$, is called the normal gradient, see A. Güntherschulze, Zeits. f. Physik **22**, 70 (1924); **77**, 703 (1932). In measuring the normal gradient, however, the great length of the Faraday dark space in a wide tube must be taken into consideration (see §60).

[266] A. Güntherschulze, Zeits. f. Physik **41**, 718 (1927); **42**, 763 (1927). R. Holm, Wiss. Ver. Siem. Konz. **3**, 159 (1923); Zeits. f. Physik **75**, 171 (1932).

[267] G. Gehlhoff, Verh. d. phys. Ges. **21**, 349 (1919).

column by a number of authors,[268] the relation between these quantities being:

$$e\bar{V}=3kT_e/2, \quad T_e=7730\bar{V}_{\text{in volts}}.$$

The temperature $T_e$ usually decreases somewhat with increasing current (just as does $E$). The dependence of the tube diameter is not known. $T_e$ decreases strongly with increasing pressure. Fig. 70 shows this dependence in Hg and A.

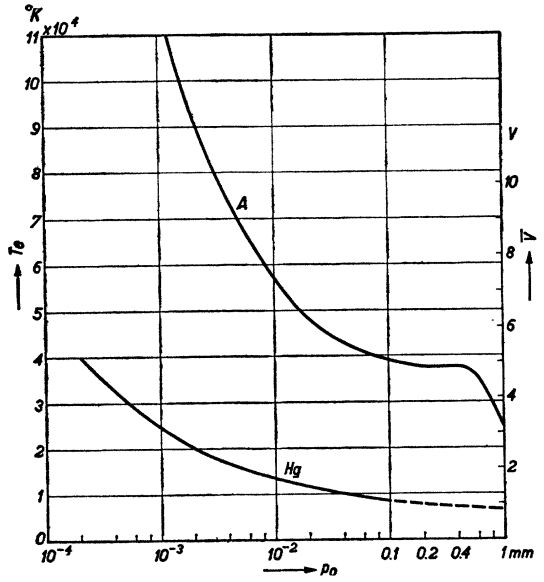


FIG. 70. Electron temperature $T_e$ as a function of the density $p_0$ in Hg at 3 amp. ($2R=3.2$ cm) after Klarfeld and in A at 0.1 amp. ($2R=1.95$ cm) after Groos.

## §53. Theory

Normally the homogeneous column is nearly a plasma, the concentrations of positive ions and electrons being almost equal: $n_e=n_p=n$. We shall suppose that the electrons have a Maxwellian distribution. The number of electrons passing one cm² per sec. in one direction (random current[269]) is almost equal to the number passing in the opposite direction, the (relatively) small difference between these currents being the drift current. The drift current of the electrons can be separated into a current in the direction of the axis and a current normal to the axis, which goes ultimately to the wall. Since the ion current along the axis is small, the electron current along the axis is practically the tube current. As the (cylindrical) wall[270] is generally an insulator the electron current normal to the axis must be equal to the positive ion current in that direction (a bipolar current). Since the diffusion coefficient and the mobility are much smaller for the ions than for the electrons, this bipolar current is only possible if the potential near the walls is negative with respect to the axis of the tube. As volume recombination will be neglected for the present, the current per unit length going to the wall must be equal to the number of ionizations in one cm of the column.

The starting point of the theory of the positive column is the dependence of the concentration $n$ on the distance $r$ to the axis, which can be derived from the reasoning given above.

We call the electronic mean free path $\lambda_e$, the ionic mean free path $\lambda_p$ and the radius of the tube $R$, and recognize two cases: case $a$ where $\lambda_p$ and $\lambda_e \ll R$; case $b$ where $\lambda_p > R$.

*Case a* ($\lambda_p$ and $\lambda_e \ll R$).—In this case, first treated by Schottky[271] the current can be described by a mobility equation.

If $j_e$ is the electric current density of the electrons or the ions perpendicular to the axis ($j$ refers to the number of particles[272]), we have (see §30):

$$\begin{aligned} &j_e=j_p=j, \\ &j=\mu_p n E_r=-D(dn/dr)-\mu_e n E_r. \end{aligned} \tag{107}$$

Elimination of $E_2$ gives:

$$j=-\frac{\mu_p D}{\mu_e+\mu_p}\frac{dn}{dr}, \tag{108}$$

where $\mu_e$ and $\mu_p$ are the mobilities ($\mu_p \ll \mu_e$) and $D$ is the diffusion coefficient for the electrons.

[268] R. Seeliger and R. Hirchert, Ann. d. Physik **11**, 817 (1931) (Ne, A); K. Sommermeyer, Zeits. f. Physik **90**, 232 (1934) (Ne); M. J. Druyvesteyn, Zeits. f. Physik **81**, 571 (1933); O. Groos, Zeits. f. Physik **88**, 741 (1934) (A); B. Klarfeld, Tech. Phys. U.S.S.R. **4**, 44 (1937); **5**, 725 and 913 (1938) (Hg, Ne, A); G. Mierdel, Wiss. Ver. Siem. Konz. (1900).

[269] The random current density (reference 272) $j_r$ is connected to the electron concentration by $j_r=n_e(kT_e/2\pi m)^{\frac{1}{2}}$.

[270] For noncylindrical tubes see: E. Spenke and M. Steenbeck, Wiss. Ver. Siem. Konz. **15**, 18 (1936).

[271] W. Schottky, Physik. Zeits. **25**, 342 and 635 (1924); J. S. Townsend, C. R. Acad. Sci. Paris **186**, 55 (1928); L. Tonks and I. Langmuir, Phys. Rev. **34**, 876 (1929); R. Seeliger, Physik. Zeits. **33**, 273 and 313 (1932); F. Llewelyn Jones, Phil. Mag. **15**, 958 (1933).

[272] In order to avoid confusion with the symbol for the Bessel function we use in this section $j$ and not $J$ for the particle current density.

The increase of $j_r$ with $r$ is caused by ionization. If we assume that the number of ionizations per sec. per cm³ is proportional to $n$ (equal to $\nu n$), we get:

$$d(2\pi r j)/dr = \nu n 2\pi r. \qquad (109)$$

Combination of Eqs. (108) and (109) gives us the Bessel differential equation which, with the Townsend relation:

$$D_e/\mu_e = kT_e/e \qquad (110)$$

leads to the solution:

$$n = n_0 J_0[(\nu e/\mu_p k T_e)^{\frac{1}{2}} r] = n_0 J_0(\omega r), \qquad (111)$$

where

$$\omega = (\nu_e/\mu_p k T_e)^{\frac{1}{2}}$$

and $J_0$ is the Bessel function of the first kind of order zero.

As the concentration near the wall ($n_w$) will be very small in comparison with the concentration ($n_0$) at the axis, we obtain[273]

$$J_0(\omega R) = 0; \quad \omega^2 = \nu_e/\mu_p k T_e = (2.4/R)^2. \qquad (112)$$

The wall current $i_w$ per cm of the tube is, according to Eqs. (108), (110), and (111):

$$i_w = -2\pi R e \mu_p \frac{kT_e}{e}\left(\frac{dn}{dr}\right)\Big|_{r=R} = 7.83 \mu_p k T_e n_0. \qquad (113)$$

The potential distribution across a cross section, $V(r)$, can be found from Eq. (107), as the drift current to the wall is small in respect to the random current:

$$D dn/dr = -\mu_e E n = \mu_e n d V/dr,$$

$$n(r) = n_0 \exp\left[\frac{e\{V(r) - V(0)\}}{kT_e}\right]. \qquad (114)$$

Eq. (114) is the Boltzmann equation, which does not contain any special mobility or diffusion constant and which is also valid in other cases ($\lambda_p > R$).

[273] The concentration $n_w$ can be determined by considering the transition from the plasma to the space-charge sheath near the wall. This determination is rather complicated (see Tonks and Langmuir) but a rough estimation of $n_w$ can be made by determining $n_p - n_e$ with aid of the Poisson equation and Eqs. (111) and (114). We obtain $n_p - n_e = n_e(\omega^2 k T_e/4\pi e^2)\{1 + (J_1/J_0)^2\}$. If we assume $(n_p - n_e)/n_e$ to be 0.3 at the edge of the sheath we obtain $n_w$. With $n_0 = 10^9$, $\bar{V} = 4$ v, $R = 1$ cm we find $\omega R = 2.23$, $J_0(2.33) = 0.094$ and $n_{ew}$ is about 0.1 $n_0$.

*Case b* ($\lambda_p > R$).—In this case, of low pressure, first discussed by Tonks and Langmuir,[271] no mobility exists, the ions falling freely from the moment they are formed ($r_1$) till they reach the wall ($R$). Eq. (109) gives (see Fig. 71):

$$2\pi r n(r) = \int_0^r \frac{2\pi r_1 \nu n(r_1)}{v(r, r_1)} dr_1,$$
$$\tfrac{1}{2} m_p \vartheta^2(r, r_1) = e\{V(r_1) - V(r)\}. \qquad (115)$$

Using the Boltzmann equation, we obtain an integral equation for $V$. We shall not discuss the method for solving this equation but give the results obtained by Tonks and Langmuir:

$$\nu R = 0.772(2kT_e/m_p)^{\frac{1}{2}}, \qquad (I^b)$$

$$i_w = 2.40 e R n_0 (kT_e/m_p)^{\frac{1}{2}}. \qquad (116)$$

These equations are equivalent to the equations ($I^a$) and (113) for the case $\lambda_p \ll R$.

We shall now discuss the other equations for the homogeneous positive column. The mobility equation of Langevin[14,274] gives (see §9):

$$i = \frac{3}{8} \frac{e^2 \lambda_e E}{(2mkT_e/\pi)^{\frac{1}{2}}} \int n(r) 2\pi r dr.$$

We get for the two cases, with $i$ in amp., $\bar{V}$ in volts, $E$ in $v$/cm:

$$i = 5.24 \times 10^{-12} \frac{\lambda_e R^2 E n_0}{\bar{V}^{\frac{1}{2}}} \qquad \lambda \ll R, \quad (II^a)$$

$$i = 8.5 \times 10^{-12} \frac{\lambda_e R^2 E n_0}{\bar{V}^{\frac{1}{2}}} \qquad \lambda > R. \quad (II^b)$$

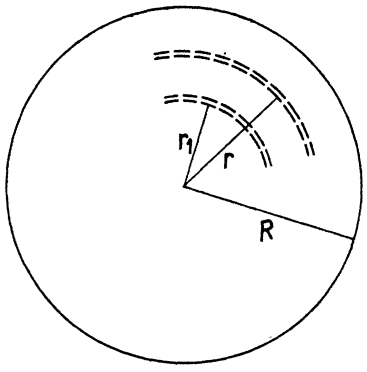


FIG. 71.

[274] We get for $\int_0^R n(r) 2\pi r dr$ in the two cases: 1.36 $n_0 R^2$ (for $\lambda \ll R$) and 2.20 $n_0 R^2$ (for $\lambda > R$).

We have still two other equations to discuss. These, however, cannot always be written down in a simple way. In the case that no cumulative processes occur and excitation and ionization take place only by impact of an electron with a normal atom and all excited atoms radiate, we get simple formulae which of course have only a limited range of application.

The energy balance[275] requires that the input per cm be equal to the sum of the generation of heat at the walls, $P_w$, and in the volume, $P_v$, and also the power radiated, $P_r$, all per cm of tube length.

$$iE=P_w+P_v+P_r. \tag{III}$$

The wall losses are given by:

$$P_w=i_w(V_i+V_s-V_w+(4/3)\bar{V}). \tag{117}$$

The term $V_s-V_w$ is due to the ions, $V_s$ is the space potential near the wall, $V_w$ the wall potential and $V_s-V_w$ the potential of the space-charge layer near the wall;[276] $(4/3)\bar{V}$ is due to the electrons and $V_i$ due to the recombination of ions and electrons at the wall.

At low pressure $P_w$ can be somewhat larger as the ions have some energy when they reach the space-charge layer. Excited atoms can also give some energy to the walls. The heat generated in the volume is often primarily due to the elastic losses of electrons against gas atoms (for $\lambda\ll R$). We get for an atomic gas:

$$P_v=\int_0^R\int_0^\infty n(\epsilon,r)2\pi r dr\left(\frac{2\epsilon}{m}\right)^{\frac{1}{2}}\frac{8\epsilon}{\lambda}d\epsilon$$
$$=2.36(m^{\frac{1}{2}}/M)(e\bar{V})^{\frac{1}{2}}(n_0R^2/\lambda_e). \tag{118}$$

Calling the total excitation probability $K_h$, we may calculate $P_r$ with the simple supposition that the radiation is given by the number of atoms excited per sec. If the excitation energy $\epsilon_h=eV_h$, we obtain

$$P_r=\int_0^R\int_{\epsilon_h}^\infty n(\epsilon,r)2\pi r dr\left(\frac{2\epsilon}{m}\right)^{\frac{1}{2}}\frac{K_h\epsilon_h}{\lambda_e}d\epsilon. \tag{119}$$

If the excitation probability can be approximated by a constant, we get for $\lambda\ll R$:

$$P_r=\frac{4n_0K_h}{\lambda_e}1.36R^2\left(\frac{e\bar{V}}{3\pi m}\right)^{\frac{1}{2}}\left(1+\frac{3V_h}{2\bar{V}}\right)$$
$$\times e^{-3V_h/2\bar{V}}eV_h. \tag{120}$$

If the ionization occurs only from the normal state, and if the ionization probability is given by $p_0C(V-V_i)/V_i$ we get as the fourth equation

$$\nu=4p_0\left(\frac{e\bar{V}}{3\pi m}\right)^{\frac{1}{2}}C\left(1+\frac{4\bar{V}}{3V_i}\right)e^{-2V_i/3\bar{V}}. \tag{IV}$$

If we substitute in Eq. (III) the values obtained for $i_w$, $P_w$, $P_v$ and $P_r$ we get four equations for $E$, $\bar{V}$, $n_0$ and $\nu$ which for the cases $a$ and $b$ considered, can be summarized as follows

$$\nu=a_1\bar{V}/R^2p_0, \tag{I$^a$}$$
$$\nu=a_2\bar{V}^{\frac{1}{2}}/R, \tag{I$^b$}$$
$$i=a_3R^2n_0E/p_0\bar{V}^{\frac{1}{2}}, \tag{II}$$
$$iE=P_w+R^2p_0n_0\bar{V}^{\frac{1}{2}}f(\bar{V}/V_h)+a_4R^2p_0\bar{V}^{\frac{1}{2}}n_0, \tag{III}$$
$$P_w=a_5\frac{n_0\bar{V}}{p_0}\left(V_i+\tfrac{2}{3}\bar{V}\ln a_6\frac{RE}{\bar{V}^{\frac{1}{2}}}+\frac{4}{3}\bar{V}\right), \tag{III$^a$}$$
$$P_w=a_7Rn_0\bar{V}^{\frac{1}{2}}\left(V_i+\tfrac{2}{3}\bar{V}\ln a_8\frac{E\bar{V}}{p_0}+\frac{4}{3}\bar{V}\right), \tag{III$^b$}$$
$$\nu=a_9p_0\bar{V}^{\frac{1}{2}}g(\bar{V}/V_i). \tag{IV}$$

The terms $a_1$ to $a_9$ are constants. Substituting I in IV gives us $\bar{V}$ while II and III give us, if we use $\bar{V}$ and the value of $E$:

$$\bar{V}=f_1(p_0R),\quad E/p_0=f_2(p_0R).$$

Both are independent of the current and satisfy of course the similarity principle (§32). In discussing the validity of these equations we shall see that they can be used only in the simple form given above in a limited range, while for the variation of $\bar{V}$ and $E$ with the current, other less simple equations are necessary.

[275] The energy balance equation may be written in two forms: either the input may be equated to the energy losses of the electrons in collisions against the gas atoms or the method followed in the text may be used. The first method is difficult when cumulative processes occur.

[276] The potential of the sheath is given by $V_s-V_w=(kT_e/e)\ln(2\pi Rj_{rw}/j_w)$. Here $j_{rw}$ is the random electron current density near the sheath and is obtained from $n_w$ (see reference 273). The potential difference between the wall and the axis is found from this same formula by substituting the random current density along the axis in place of $j_{rw}$.

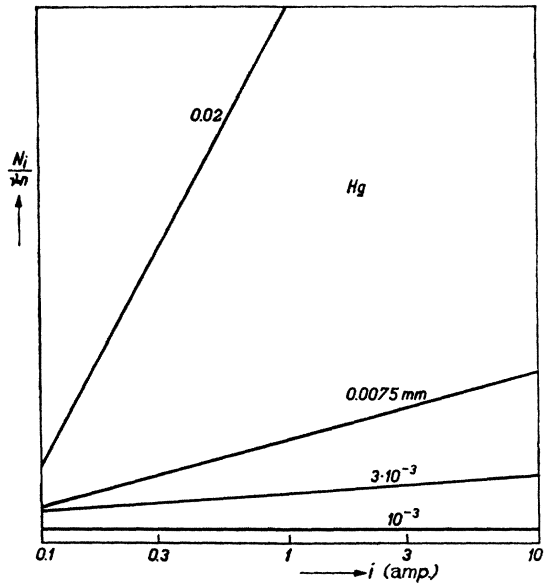


FIG. 72. Number of ionizations per cm³ per sec. (Ni) divided by $\nu n$, $\nu$ being given by (IV), at different pressures (in mm) as a function of the current $i$ (Klarfeld).

## §54. Testing of the theory by experiments

We shall now compare the different equations with the experimental results.

The dependence of the electron concentration on the distance to the axis $r$ was measured in Hg at low pressure by Killian (case *b*) and in Ne at one mm by Druyvesteyn (case *a*). The theoretical dependence was confirmed in both cases. The fact that the electron temperature did not depend on $r$ and the validity of the Boltzmann equation were also confirmed.[277]

Klarfeld also tested the equation for $i_w$ (116). In Ne and A in a limited range of pressure the calculated values of $\bar{V}$ were compared with the experimental results and a rather good agreement was found.[278] The form of the whole of curve of Fig. 70 has not yet been discussed from this point of view.

The Langevin formula (II) was for the greater part confirmed by Killian, Klarfeld and Boeckner and Mohler.[279] A few refinements must, however, be mentioned. Generally the electronic mean free path is not constant but the variation with the electron energy must be taken into account by calculating a mean free path for every value of $\bar{V}$. The mean free path can be reduced by the presence of positive ions (§9) and at low pressure by the wall.[280]

In He and A Klarfeld could calculate $E$ from (II) by measuring $n_0$ and $\bar{V}$. He obtained a curve of the same general form as Fig. 67, and also found $E$ to be of the right order of magnitude.

We now come to Eqs. (III) and (IV), which are often complicated by the appearance of cumulative processes. In Hg the number of ionizations could be described only by (IV) at a pressure of $10^{-3}$ mm. Fig. 72 shows that at higher pressure the number of ionizations per electron increases with the current, probably by ionization of excited atoms.

The energy balance in the rare gases, Na, Cs and Hg vapor was investigated by a number of authors.[281] We can summarize the results as follows: At a low pressure ($2\times10^{-4}$ mm), $P_w$ is the main term and can be as large as 85 percent of the input[282] (see Fig. 73). At moderate pressures (from about $2\times10^{-3}$ to 3 mm) the radiation is often more than half the input and can be as large as 90 percent. At still higher pressure, $P_v$ increases and the column begins to contract. A difficulty in testing the energy balance is that often a part of the radiation lies in a spectral range where measurements are difficult or impossible. The radiation can only rarely be given by the simple formula (119). With III, (118) and (119) and by using the experimental values for $K_h$ and $K_i$ of Fig. 3 Mierdel[25] calculated rather satisfactorily $\bar{V}$ as a function of $E/p_0$ in the rare gases as was shown in Fig. 5 for Ne. Von Engel and Steenbeck[283] found a rather good agreement

[277] T. J. Killian, Phys. Rev. **35**, 1238 (1930); M. J. Druyvesteyn, Zeits. f. Physik **81**, 571 (1933); B. Klarfeld, Tech. Phys. U.S.S.R. **5**, 913 (1938). Klarfeld found a small variation of $\bar{V}$ with $r$.

[278] W. Uyterhoeven, *Elektrische Gasentladungslampen*, Fig. 70; A. v. Engel and M. Steenbeck, *Elektrische Gasentladungen II*, Fig. 53 (in this figure, however, an error occurs in the $T_e$ scale).

[279] B. Klarfeld, Tech. Phys. U.S.S.R. **5**, 725 (1938); C. Boeckner and F. L. Mohler, Bur. Stand. J. Research **10**, 357 (1933).

[280] S. Gvosdover, Physik. Zeits. Sowjetunion **12**, 164 (1937); **13**, 133 (1938).

[281] K. Sommermeyer, Ann. d. Physik **13**, 315 (1932); **28**, 240 (1937); F. L. Mohler, Bur. Stand. J. Research **9**, 25 (1932); M. J. Druyvesteyn and N. Warmoltz, Phil. Mag. **17**, 1 (1934); B. Klarfeld, Tech. Phys. U.S.S.R. **4**, 44 (1937).

[282] In the case that $iE$ can be put equal to $P_w$, Eqs. (I) to (IV) could be solved by Klarfeld, reference 281.

[283] A. von Engel and M. Steenbeck, reference 257, Figs. 54 and 55. See also M. Steenbeck, Zeits. f. tech. Physik **17**, 397 (1936).

with the experimental results for $E$ in $N_2$ and $H_2$.

For the dependence of $E$ and $\bar{V}$ on the current (Fig. 69) it is necessary to take into account the secondary processes.[284] The most important of these processes seem to be: excitation and ionization by impact of an electron against an excited atom or of two excited atoms against each other, collisions of the second kind of an excited atom with a slow electron giving a normal atom and a fast electron, and further, the absorption of radiation in the gas.

To give an example of the influence of these factors on the energy balance we shall assume that only one excited state exists and that the only secondary process is a collision of the second kind of an excited atom with a slow electron. We may write in this case for the energy balance:

$$\begin{aligned} iE &= a_1R^2n_aA + a_2n_0 + a_3R^2n_0 \\ &= P_r + P_w + P_v. \end{aligned} \tag{121}$$

The three terms may be summarized by using the constants $a$. $A$ is the Einstein emission constant, $n_a$ the concentration of excited atoms, $N$ that of normal atoms and $n_0$ the electron concentration in the axis. In the simple case that we assumed, $n_a$ is determined by:

$$\begin{aligned} Nn_0S_{01} &= n_aA + n_an_0S_{10}, \\ n_a &= n_0NS_{01}/(A + n_0S_{10}). \end{aligned} \tag{122}$$

$S_{01}$ and $S_{10}$ are integrals over the distribution function of the electrons multiplied by a probability for excitation or for a collision of the second kind. Using the Langevin equation ($i = a_4R^2n_0E$) we find for the gradient:

$$a_4R^2E^2 = a_1R^2A\frac{NS_{01}}{A + n_0S_{10}} + a_2 + a_3R^2. \tag{123}$$

For $A \gg n_0S_{10}$ the gradient is constant, while if this is not the case, $E$ decreases with increasing $n_0$ or increasing current. A somewhat more extensive equation on the same basis was given by Rompe and Schön.[285]

We think that it will be necessary to correct both Eqs. (III) and (IV) for the presence of different secondary processes to explain the dependence of $\bar{V}$ and $E$ on the current. At present we do not know enough about these processes to undertake this task, but the study of the radiation, ionization, and $n_a$ as a function of $i$, $p_0$ and $R$ will in future make this method of attack possible.

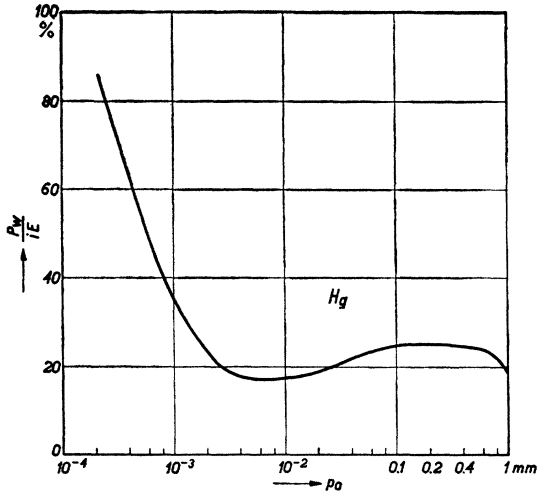


FIG. 73. Wall losses divided by the input as a function of the pressure in Hg, current 3 amp. ($2R = 3.2$ cm) (Klarfeld).

We will now discuss a number of points that were overlooked in the foregoing, but that can sometimes become very important.

At decreasing current the gradient will not only increase because of the absence of secondary processes but also because of the following factors: (1) The limit of the plasma near the wall is not found at $r = R$ as the thickness of the space charge layer must be subtracted from $R$; when the pressure is low enough this thickness can be calculated with Eq. (86); (2) At a low current density, $n_p$ will no longer be almost equal to $n_e$; the influence of this factor can be calculated by the equation of Poisson;[264,271] (3) Thus far we have assumed that the electrons had a Maxwell distribution. At a low current density, when this will no longer be the case, this distribution can be found in the way indicated in Section II. In this case the ionization is not given by Eq. (IV) but can be described[286] with the ionization

[284] When secondary processes occur the number of ionizations is no longer equal to $\nu n$; still Eqs. (111), (113), (114) and (I) remain rather good approximations.

[285] R. Rompe and M. Schön, Zeits. f. Physik **108**, 265 (1938); **111**, 345 (1938).

[286] In §8 we remarked that, if the distribution for an infinitely small current was already Maxwellian, this distribution did not change by interaction. As this state of affairs seems to occur approximately in $N_2$ and $H_2$ (§14),

coefficient $\eta$ (§10):

$$\nu = \eta \mu_e E^2. \tag{124}$$

The Maxwell distribution can as far as we know be caused principally by electron interaction[287] and by collisions of the second kind between excited atoms and electrons. To what extent these two factors can explain the experimental results is still an open question but often the view is expressed that both factors are too small and that high frequency oscillations in the plasma are essential for the establishment of a Maxwell distribution. As all these factors decrease with decreasing current it is certain that at a low current density deviations from the Maxwell distribution will occur.

At very low current the tube voltage will become equal to the breakdown potential.[288]

At high current density and a low pressure we can approach the state of complete ionization of the gas, in this case entirely different conditions occur from those given above.[289]

Until now recombinations of positive ions and electrons in the gas have been neglected. This is allowed in most cases but sometimes they can occur, giving a spectral emission deviating strongly from the normal radiation that is due to excitation.[290]

When $E/p$ is large the drift current will no longer be small in respect to the random current and the electron distribution will not be isotropic.[291]

The heating of the gas by the volume losses can be so high that the gas density in the axis will be much smaller than near the wall.

The electron temperature was assumed to be independent of $r$; in mixtures, however, deviations can occur (§56).

## §55. Radiation

Generally the radiation of the positive column consists principally of the line spectrum of the gas atoms; at not too low a pressure the intensity of the spectral lines in each series falls rapidly with increasing term number. This is in accordance with the absence of recombination, the spectrum being generally an excitation spectrum. Only at high electron and ion concentrations has a recombination spectrum been observed (for example, in alkali vapors) with a series limit continuum (see Mohler[290]).

Other examples of a continuous spectrum in the positive column are: the continuous $H_2$ molecule spectrum (§57) and the continuous spectrum, found in the rare gases at high pressure and low current density,[292] the nature of which has not yet been cleared up sufficiently.

At low pressure and low current density the radiation can be calculated with equations of the form (119) as only direct excitation by electron impact occurs. At a higher pressure an excited atom can in a collision with another atom come in another excited state; the intensity distribution can be altered by this process, but the total radiation will not be much changed as a non-radiating transition to the ground state will seldom occur. At a higher current density also collisions between excited atoms and electrons will occur, giving an ion or an atom in a higher excited state (cumulative excitation or ionization) or (a normal atom through a collision of the second kind). In all these cases the radiation of a special line is still determined by $n_a A$, $n_a$ being the concentration of the particularly excited atoms and $A$, Einstein's radiation probability. This is no longer the case when self-

in this case Eq. (124) may be used as was done by Holm (Zeits. f. Physik **75**, 171 (1932)), who obtained a good agreement with experiment. Also the small dependence of the gradient upon the current in these gases may be connected with this question.

[287] For a positive column, diameter 2 cm, in Ne at 1 mm the interaction will cause a Maxwell distribution ($B>5$, §7) if the current is above 1 amp. At a lower pressure a much higher current is, however, needed.

[288] M. Steenbeck, Wiss. Ver. Siem. Konz. **15**, 32 (1936); Zeits. f. Physik **106**, 311 (1935).

[289] A. W. Hull and H. D. Brown, Trans. A.I.E.E. **50**, 744 (1931); Elec. Eng. **53**, 1435 (1934); L. Tonks, Trans. Electrochem. Soc. **72**, preprint 13 (1937); F. L. Mohler, J. Research Nat. Bur. Stand. **21**, 697 and 873 (1938).

[290] L. J. Hayner, Zeits. f. Physik **35**, 365 (1925); H. W. Webb and D. Sinclair, Phys. Rev. **37**, 182 (1931); H. Krefft, Zeits. f. Physik **77**, 752 (1932); F. L. Mohler, Bur. Stand. J. Research **10**, 771 (1933); **19**, 447 and 559 (1937).

[291] K. G. Emeléus, R. W. Lunt and C. A. Meek, Proc. Roy. Soc. **156**, 394 (1936).

[292] H. B. Dorgelo and T. P. K. Washington, Proc. Roy. Acad. Amsterdam **35**, 1009 (1926); F. M. Penning, Phil. Mag. **11**, 961 (1931); P. Johnson, Phil. Mag. **13**, 487 (1932); J. S. Townsend, Phil. Mag. **14**, 418 (1932); S. P. McCallum, L. Klatzow and J. E. Keyston, Nature **130**, 810 (1932); Phil. Mag. **16**, 193 (1933); Ny Tsi-Ze and Voo Shueg-Ling, Comptes rendus **198**, 356 (1934); S. P. McCallum, Nature **142**, 614 (1938); W. Finkelnburg, *Kontinuierliche Spektren* (1938).

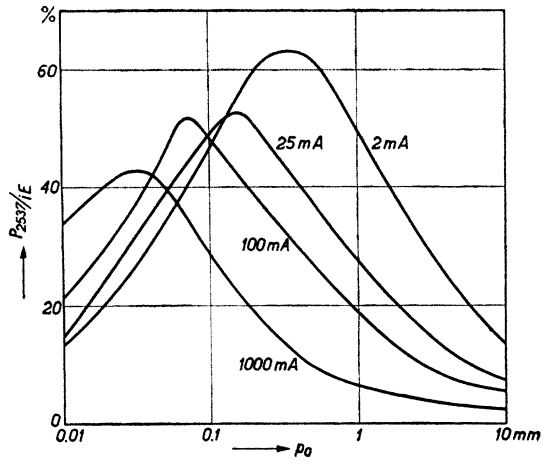


FIG. 74. Percentage of the input that is emitted in the resonance line 2537A in Hg as a function of the pressure at different currents after Rössler and Schönherr ($2R=1.8$ cm).

absorption occurs; in this case the form of the spectral line is important, and this can be changed by the random electric fields caused by the ions and electrons. At first $n_a$ increases with increasing current until later $n_a$ is almost independent of the current. In some cases $n_a$ is determined by a Boltzmann equation with a temperature that is almost equal to the electron temperature. For the results concerning $n_a$ we refer to Ladenburg's article[293] in this journal.

As many processes must be discussed for the radiation and many atomic and spectral properties occur, while the radiation does not yet give so clear a picture of the mechanism of the positive column as the electrical properties, we shall refer to the literature[294] for the details about the radiation and give only a few examples.

Figure 74 shows the intensity of the Hg resonance line at 2537A in percents of the input as a function of the pressure at different currents; it is seen that as much as 60 percent of the input can be radiated in this spectral line. At lower pressure the efficiency decreases because of the increasing wall losses and at high pressure because of volume losses.[295] The decrease with increasing current may be attributed to cumulative excitation, as this causes the higher lines to increase in respect to the resonance line. Fabrikant and Zirg estimated[294] that in one definite case the cumulative excitation to the $6^3P$ level of Hg was about 45 percent of the direct excitation from the ground state. Although this result seems still open to criticism, we think that an important progress along these lines is possible.

Figure 75 shows the dependence of the red Ne lines upon the pressure with different currents. The maximum may be explained in the same way as for the case of Hg. Also a marked dependence of the current caused by cumulative excitation and collisions of the second kind is present. The result, that as much as 15 percent of the input can be radiated in these higher lines, points to

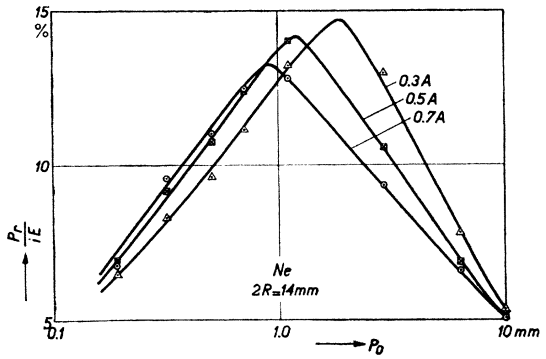


FIG. 75. Percentage of the input emitted in the red and yellow Ne lines as a function of the pressure at different rcurents (Uyterhoeven).

[293] R. Ladenburg, Rev. Mod. Phys. **5**, 243 (1933); H. Kohn, Physik. Zeits. **33**, 957 (1932); K. Krebs, Zeits. f. Physik **101**, 604 (1934); W. Dirbach and K. Krebs, Zeits. f. Physik **110**, 134 (1938).

[294] W. Uyterhoeven, *Elektrische Gasentladungslampen* (1938); E. Lax and M. Pirani, *Handbuch der Physik*, Vol. **19** (1928), p. 379; W. de Groot, Physica (old series) **12**, 289 (1932); **13**, 41 (1933); (new series) **1**, 28 (1934) (Na); M. J. Druyvesteyn, Physica **1**, 14 (1934) (Na); M. J. Druyvesteyn and N. Warmoltz, Phil. Mag. **17**, 1 (1934); Physik. Zeits. **33**, 822 (1932) (Na); B. Klarfeld and I. Taraskow, Tech. Phys. U.S.S.R. **3**, 881 (1936) (Na); F. L. Mohler, Bur. Stand. J. Research **9**, 25 and 493 (1932); **16**, 227 (1936); **17**, 45 and 849 (1936) (Cs); H. Krefft, Tech. Wiss. Abh. Osram **4**, 33 (1936); Zeits. f. tech. Physik **15**, 554 (1934); Licht **4**, 1, 23, 86 and 105 (1934) (Hg); W. Fabrikant, F. Butajewa and I. Zirg, Physik. Zeits. Sowjetunion **11**, 576 (1937); **12**, 324 (1937); **13**, 23 (1938) (Hg); F. Rössler and F. Schönherr, Zeits. f. tech. Physik **19**, 588 (1938) (Hg); J. E. Keyston, Phil. Mag. **16**, 625 (1933) (He); F. Llewellyn Jones, Proc. Phys. Soc. **48**, 513 (1936) (He); H. B. Dorgelo, Physica **5**, 90 (1925) (Ne); W. Elenbaas, Zeits. f. Physik **72**, 715 (1931) (Ne); H. Krefft and E. O. Seitz, Zeits. f. tech. Physik **15**, 556 (1934); Physik. Zeits. **35**, 980 (1934) (Ne); T. Barends, Physica **11**, 275 (1931) (A).

[295] The variation of the radiation with the different variables can be explained principally in two ways, either by the energy balance equation or by the number of direct and cumulative excitations taking also into account collisions of the second kind and absorption.

the large effect of cumulative excitation from the metastable states in this case.

## §56. Mixtures of atomic gases

In a mixture of two gases the values for $E$ and $T_e$ will generally lie between the values for the pure gases. After Eq. (IV) the number of ionizations per sec. depends strongly on $V_i/\bar{V}$. Therefore, in a gas with a low ionization potential the electron temperature will usually be low. For the case that $T_e$ is constant over a cross section, the theory of Schottky was extended to the case of mixtures by Dorgelo, Alting and Boers.[296] Fig. 76 shows that the experimental values of $\bar{V}$ agree

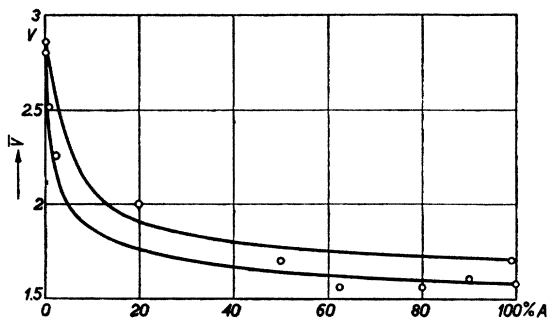


FIG. 76. Mean electron energy $\bar{V}$ in Ne−A mixtures at 5 mm and 2 amp. tube diameter 28 mm. The lines are calculated, by assuming the experimental value $\bar{V}$ for Ne and A (with two different values for $\bar{V}$ in A), the circles were found experimentally (Dorgelo, Alting and Boers).

well with the calculated values in Ne-A mixtures. When the A percentage exceeds a few tenths of a percent practically all ions are $A^+$ ions. Alterthum and Lompe[297] measured the gradient in Ne-A mixtures. While for larger A percentages (above a few percent) the behavior is normal, at a small A percentage other phenomena occur, e.g. the characteristic can become positive.

We shall also describe some of the curious phenomena that occur in mixtures with a fraction of a percent of the gas with the lower ionization potential (especially at a high current density) for a direct-current discharge in a mixture of Ne and Na, when the temperature (Na percentage) is varied.[298]

From 100 to 200°C the Na lines appear faintly near the wall. At increasing temperature the intensity of the Na lines increases. At 250°C both Na and Ne lines are strong, the Na lines, however, only appear near the wall, the Ne lines only near the axis of the tube. Above 280°C Ne light is no longer emitted; only the Na lines are to be observed. We see in Fig. 77 that below 200°C the value of $E$ is almost the same as in pure Ne, while near 240°C a maximum occurs; above 280°C probably all ions will be $Na^+$ ions. In the neighborhood of the maximum in $E$, the value of $T_e$ varies much over a cross section.

The most probable explanation for these phenomena is to be sought in the separation of the mixture along a cross section. As the ionization potential of Na (5.1 v) is much smaller than that of Ne (21.5 v) the relative ionization of Na will be much larger in comparison with that of Ne. As the ions go to the wall the Na concentration in the axis will be much smaller than near the walls; this explains why the Na lines are especially emitted near the wall.[299] The $Na^+$ ions will soon recombine at the wall, as almost all of them are formed in the neighborhood of it; con-

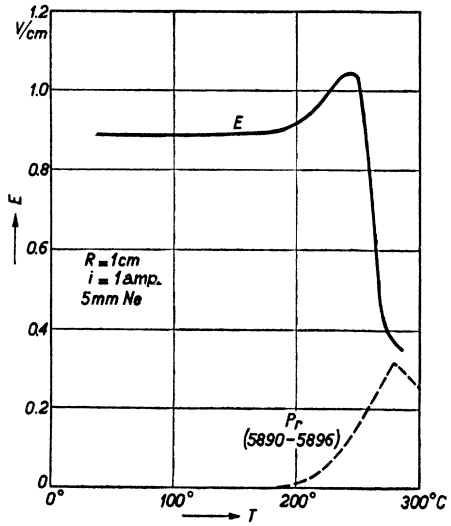


FIG. 77. Gradient $E$ (——) as a function of the temperature in Ne with Na and intensity of $D$ lines of Na (- - -) (Uyterhoeven).

[296] H. B. Dorgelo, H. Alting and C. J. Boers, Physica **2**, 959 (1935).

[297] H. Alterthum and A. Lompe, Ann. d. Physik **31**, 1 (1938). See also L. B. Headrick and O. S. Duffendack, Phys. Rev. **37**, 736 (1931).

[298] W. Uyterhoeven and C. Verburg, Comptes rendus Acad. Sci. Paris **200**, 536 (1935); **201**, 647 (1935); **202**, 1498 (1936); **208**, 503 (1939). See also Uyterhoeven's book, reference 294.

[299] This radial separation is analogous to the separation in the direction of the axis, the gas of lower ionization potential going to the cathode. This phenomenon is called cataphoresis (see M. J. Druyvesteyn, Physica **2**, 255 (1935)).

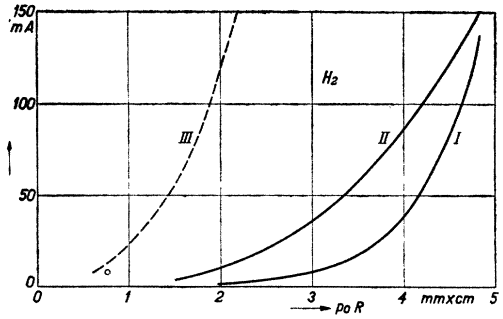


FIG. 78. Different domains for striations in $H_2$ as a function of the current $i$ and the product of pressure and tube radius $p_0R$ drawn from measurements of Günther-schulze and Meinhardt; the circle gives Paul's measurement (Fig. 79).

sequently more $Na^+$ ions must be formed than in the case of a constant Na concentration over a cross section. This effect can cause a maximum in $T_e$ and $E$. In many mixtures these phenomena are observed (e.g. in Ne, Hg[300]) at a small percentage of the gas of lowest ionization potential. The influence of a number of other processes in the behavior of a mixture are not yet known; probably their influence is often large at a low current density. We shall mention only collisions of the second kind as described in §12 and a deviation of the Maxwell distribution for the electrons caused by the large difference of the excitation potentials of the two gases.

### B. *Other Forms of Columns*

### §57. Striated columns

*Experiment.*—As was stated earlier, in rare gases at low current in a small pressure range a limited number of striations can be found at the head of the positive columns. The striations soon become vague. The potential difference per striation was found in A to be 11.9 v in Ne, 18.5 v and in He, 20 v and was found to be independent of the current.[301] The ions are probably formed by ionization of metastable atoms. The observation that the striations soon become vague is analogous to the case of the negative layers at a much higher value of $E/p_0$ (§19).

In *molecular* gases or in a mixture of an atomic gas with some molecular gas a large number of striations can sometimes be observed, all of which are equally sharp. Usually they are sharp at the cathodic side of a striation and vague at the anodic side. The striations are obtained much easier in a gas mixture than in a pure gas. It seems, however, certain that also in pure $H_2$ striations occur. The behavior of these striations in $H_2$ has lately been elucidated considerably by a publication of Güntherschulze and Meinhardt,[302] which we shall discuss first.

Since according to the similarity principle the properties of striations, for example their distance divided by $R(d_{st}/R)$ and the potential difference per striation ($V_{st}=\bar{E}d_{st}$), must be a function only of the current $i$ and the product of the gas density $p_0$ and the tube radius $R$, we can describe the phenomena by distinguishing a number of domains in an $i$, $p_0R$ diagram (Fig. 78). To the right of curve I the column is homogeneous. To the left of curve I red striations appear suddenly over the whole length of the column, $d_{st}$ is about equal to the tube radius and $V_{st}$ about 50 v. curve I is given by:

$$\log i = C_1 p_0 R - C_2. \tag{125}$$

In this $C_1$ and $C_2$ are constants.[303] When we go from curve I to curve II $V_{st}$ and $d_{st}$[304] decrease gradually to about 20 v and $R/2$. Curve II is given by:

$$i^{\frac{1}{2}} = C_3 p_0 R. \tag{126}$$

In passing, the conditions for curve II double striations occur suddenly. The distances between

[300] W. Gerlach and K. Siebertz, Zeits. f. Physik **91**, 37 (1934); K. Siebertz, Zeits. f. Physik **101**, 255 (1936); Ann. d. Physik **27**, 421 (1936); F. M. Penning, Physica **1**, 763 (1934).

[301] F. M. Penning, Zeits. f. Physik **41**, 769 (1926). W. H. McCurdy and P. Dalton, Phys. Rev. **27**, 163 (1926); these authors assume that the Ne percentage in their He causes the striae.

[302] A. Güntherschulze and H. Meinhardt, Zeits. f. Physik **110**, 95 (1938); F. Wehner, Ann. d. Physik **32**, 49 (1910); P. Neubert, Ann. d. Physik **42**, 1454 (1913); Physik. Zeits. **15**, 432 (1914); R. Holm, Wiss. Ver. Siem. Konz. **3**, 159 (1923); Physik. Zeits. **25**, 497 (1924); **26**, 412 (1925); J. Zeleny, J. Frank. Inst. **209**, 625 (1930).

[303] Many differences exist between the results of the authors mentioned under reference 302. Fig. 78 and Eqs. (125) to (128) give the results that seem to us the most probable. An error is, however, quite possible, e.g. Fig. 78 is obtained principally from the measurements of Güntherschulze and Meinhardt. Holm finds other values. The equations are in accordance with the similarity principle as was found by Holm while Güntherschulze and Meinhardt found large deviations from this principle.

[304] As $V_{st}=d_{st}\bar{E}$ and $\bar{E}$ does not vary much with the current, $V_{st}$ and $d_{st}$ are almost proportional at a constant pressure.

the striations are given by:

$$d_{st\ 1-2}=d_{st\ 3-4}=d_{st\ 5-6}=\tfrac{1}{2}d_{st\ 2-3}$$
$$=\tfrac{1}{2}d_{st\ 4-5}=\cdots \quad (127)$$

The uneven striations are sharp, the even are vague. When we pass from curve II the distances $d_{st}$ tend to become equal and near curve III blue striations at equal distance occur for which Goldstein's law applies:

$$d_{st}/R=C/(p_0R)^m. \quad (128)$$

$m$ is of the order of 0.5. $V_{st}$ is about 12 v. In this region probe measurements were done by Paul[305] who found the potential distribution of Fig. 79. Only at the head of a striation is a strong electric field present.

The striations emit mainly the $H_2$ lines, whereas the Balmer lines are weak or absent. Lau and Reichenheim[306] conclude that the recombination of H atoms to $H_2$ molecules which occurs at the walls is important for the appearance of striations. If this recombination does not occur at the wall, the striations disappear. The blue striations, near curve III Fig. 78 emit the continuous $H_2$ spectrum, having an excitation potential of 12.6 v; this is in accordance with Fig. 79. As the ionization potential of $H_2$ is 15.8 v, it is not clear how ionization occurs in this case. It may be due to the ionization of impurities, whose presence is, according to some authors, essential for these striations, which view

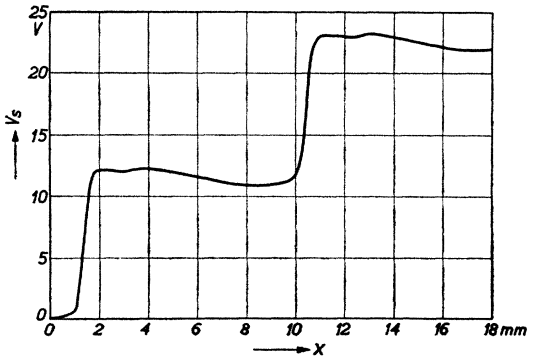


FIG. 79. Space potential $V_s$ as a function of the distance along the tube $x$ for the blue $H_2$ striations after Paul.

[305] H. H. Paul, Zeits. f. Physik **97**, 330 (1935); A. Bramley, Phys. Rev. **26**, 794 (1925).

[306] E. Lau and O. Reichenheim, Ann. d. Physik **3**, 840 (1929); **5**, 296 (1930).

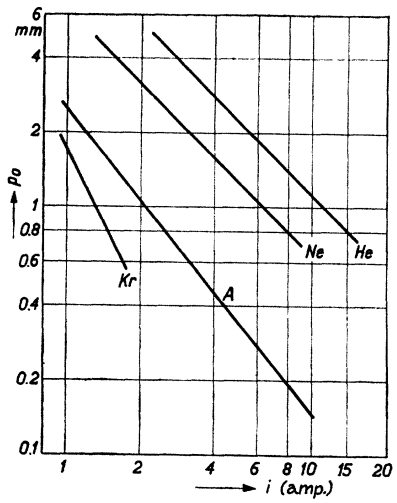


FIG. 80. Maximum current for moving striations as a function of the pressure after Pupp. Small deviations from these curves occur at different diameters.

is, however, not generally accepted. For the red striations impurities are not necessary.

It seems probable that in the positive column in other gases ($N_2$, $O_2$) phenomena occur which are, at least partly, analogous to those in $H_2$. Since for these gases no great progress has been made in the last ten years we shall for the other gases refer to the older literature.[307]

In pure rare gases *moving striations* are often observed, which have been studied especially by Pupp.[308] They can occur only if the current and the density are not too high. The limit where the striations disappear is almost independent of the tube diameter and is roughly given by:

$$i=C/p_0. \quad (129)$$

$C$ is a constant which is, however, different for different gases.

While standing striations disappear above a current density of about $10^{-2}$ amp. per cm$^2$, for moving striations this current is some 100 times larger (see Figs. 78 and 80). At low pressure the

[307] R. Holm, Wiss. Ver. Siem. Konz. **3**, 159 (1923). See also the books under reference 2.

[308] W. Pupp, Physik. Zeits. **33**, 844 (1932); **35**, 705 (1934); **36**, 61 (1935); Zeits. f. tech. Physik **15**, 257 (1934); F. W. Aston and T. Kikuchi, Proc. Roy. Soc. **98**, 50 (1921); T. Kikuchi, Proc. Roy. Soc. **99**, 257 (1921); C. Samson, Zeits. f. tech. Physik **6**, 281 (1925); R. Whiddington, Nature **116**, 506 (1925); **126**, 467 (1929); W. Elenbaas, Ned. T. v. Natuurk. **1**, 16 (1934); B. v. Manen, Physica **1**, 968 (1934); A. H. v. Gorcum, Physica **2**, 535 (1935).

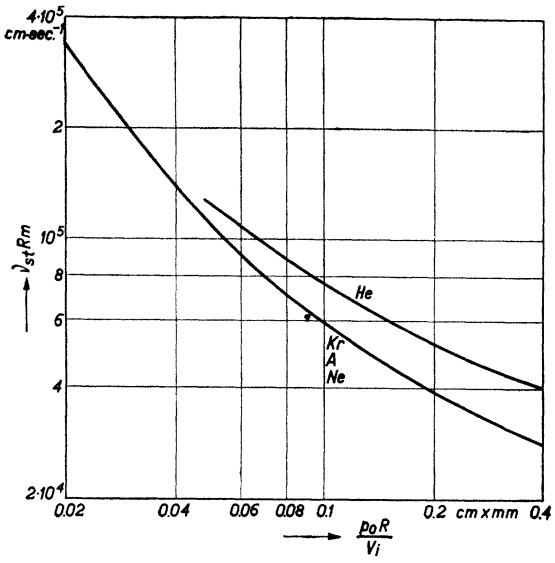


FIG. 81. Values of $\nu_{st}Rm$ for moving striations (Pupp).

striations become vague. The striations are characterized by their distance $d_{st}$ and their velocity ($v_{st}$), often however the frequency $\nu_{st}$ is given by:

$$\nu_{st}=v_{st}/d_{st}.$$

For these properties the following dependencies are found:[309]

$$d_{st}/R=f_1(p_0R, i/R), \quad \nu_{st}R=f_2(p_0R). \quad (130)$$

The frequency varies only slightly with the current and for the different gases almost one curve is found (Fig. 81) if $\nu_{st}Rm$ is considered as a function of $p_0R/V_i$ $m$ being the atomic mass, $V_i$ the ionization potential. $f_1$ lies between about 9 and 3, $\nu_{st}R$ between $2\times10^2$ and $4\times10^4$ giving for the velocity with which the striations move in the direction of the cathode, a value of $10^3$ to $10^5$ cm/sec.

Probe measurements have shown that the gradient, the electron temperature, and the electron concentration are periodic functions of the distance along the tube (see Fig. 82).

*Theory.*—We shall now discuss to what extent the many phenomena observed in striated columns can be roughly explained. The explanations given by different authors often differ much and will not all be summarized here.[310] We shall discuss especially the following points: How is an unlimited number of striations possible? What is the origin of the striations and why are they sometimes moving? Furthermore we shall point to the essential difference between standing striations (molecular gases) and moving striations (rare gases).

In the case of the negative layers (§19) and the standing striations in rare gases the striations are sharp near the cathode but they soon become vague as not all electrons excite atoms at the same place so that the electron distribution broadens out when we go to the anode and atoms are no longer excited only in a layer.

In molecular gases all striations are equally sharp, the form of the head of a striation is probably given by the form of the equipotential surface in the tube. The head of each striation can be equally sharp if the electron energy is very low (e.g. $<0.1-0.5$ v) at the cathode side of a striation. This will especially occur if the field has the form of Fig. 79, where a practically field-free space exists between the striae; but it can also occur if the field between the striae is so small that the mean electron energy will decrease to a very small value before the head of the next striation. The sharpness of the cathodic side of a striation and a fine structure due to different excitation energy which is sometimes observed at the head of a striation points also to the low value of the electron energy just before a striation.[311]

Since for standing striae the similarity principle seems to apply rather well, it is improbable that cumulative excitation or ionization are essential for them.[312] In some cases negative ion formation seems to be an essential feature[313]

[309] Only in long tubes $f_1$ and $f_2$ are almost continuous functions, in short tubes discontinuities arise as generally a whole number of striations are present in the tube.

[310] In reference 307 a number of explanations are discussed, see also: J. J. Thomson, Phil. Mag. **42**, 981 (1921); K. T. Compton, L. A. Turner and W. H. McCurdy, Phys. Rev. **24**, 597 (1924).

[311] In the double striae for which (127) applies the mean energy is probably only small before the head of the uneven striations as only these are sharp. See for the spectral fine structure of striations, reference 302, and W. Finkelnburg, E. Lau and O. Reichenheim, Zeits. f. Physik **61**, 782 (1930).

[312] The number of ionizations in a certain potential interval may be somewhat larger in the field of Fig. 79 than in a homogeneous field.

[313] H. S. W. Massey, *Negative Ions* (Cambridge Physics Tracts, 1938).

since the electric field of Fig. 79 with a sharp double layer seems almost impossible to explain when only positive ions and electrons are present. Direct evidence for negative ions, however, seems not to exist and also the rate of negative ion formation and recombination in the striated column is not known. The appearance of striations in the presence of negative ions may be made plausible in the following way. A small number of negative ions introduced in the axis of a homogeneous positive column cannot reach the walls if for the column the Schottky theory applies and the walls are negative with respect to the axis. In the absence of recombination they can go only to the anode, so that the concentration of negative ions should increase permanently in the direction of the anode. If, however, striations occur it seems at least possible that the field has such a form that the negative ions can reach the wall. Although some attempts of a theory on this basis exist, we do not think that the properties of the column with stationary striations can as yet be explained sufficiently on the basis of this picture. We shall not assume that in every positive column with stationary striations negative ions occur. It seems possible that in certain cases (pure $H_2$) the appearance of electrons with low energy between the striae is sufficient for the occurrence of the striated column.

In mixtures of Hg with some $H_2$[310] the first process is excitation of a Hg atom. The excited atoms react in some way with $H_2$.

In the column with *moving striations* the current density is so large that a Maxwell distribution for the electrons is established between the striations. If this occurs, an unlimited number of striations is possible as these electrons, which have lost only a relatively small energy in collisions with gas atoms in a striation, will be slowed down so much by the establishment of the Maxwell distribution that the next striation is as sharp as the former one. For this process it is immaterial by which mechanism the Maxwell distribution is established.

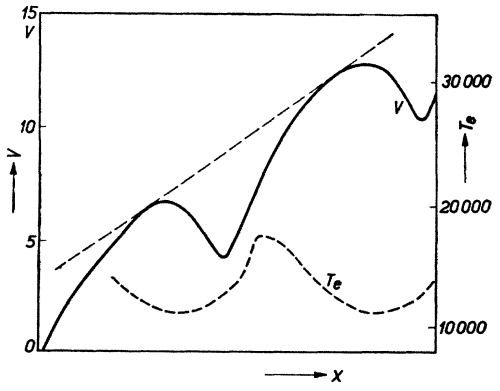


FIG. 82. Potential $V$ and electron temperature $T_e$ as a function of the distance to the cathode $x$ after Pupp.

For rare gases at high current the occurrence of $i/R$ in Eq. (130), which is in contradiction to the similarity principle, as well as the decrease of the gradient with increasing current, point to the importance of cumulative ionization. For this reason the following picture of the moving striations seems probable. When most ions are formed from metastable atoms, a concentration of the column in a number of small parts with higher field strength can make the ionization easier than in a homogeneous column, which is also the case in the red sphere in a low voltage arc or in an anode fall in Ne. At a large current this increase of ionization by concentration in small parts will be smaller and above a certain current, the homogeneous column will have a smaller gradient and a homogeneous column will then appear.

On superposing on the quantities $n_e$, $E$ and $T_e$ of a homogeneous positive column an oscillation as a function of the distance to the cathode, Druyvesteyn[314] found that a solution was possible only if the striations moved to the cathode with a velocity of the same order of magnitude as the drift velocity of the positive ions ($\mu_p \bar{E}$). This is in rough accordance with experiment.

Summarizing, we think that for standing striations a low field strength between the striae is essential. The space charge can be caused by positive ions and electrons only or also by negative ions. This view is in accordance with the validity of the similarity principle. For moving striations in rare gases cumulative ionization is essential. We do not believe however, that it is possible at this moment to give a quantitative explanation of the properties of striated columns on these principles.

[314] M. J. Druyvesteyn, Physica **1**, 273 and 1003 (1934).

### §58. Contracted columns

With increasing pressure the column begins to contract. This *contraction* is probably caused mainly by the heating of the gas, the density near the axis becoming smaller than near the wall, so that $E/p_0$ will be higher in the axis. As this contraction is typical for the positive column at high pressure, we shall not treat it here.

Another form of contraction, the so-called "*pipe*" *discharge* can be caused by mixing with an atomic gas some electronegative gas as $I_2$ or paraffin vapor.[315] The column seems to be surrounded by a mantle of this vapor which forms the wall of the discharge. The lateral edge of the discharge can be sharp.

## IX. Anodic Phenomena

In a gas discharge the cathode surface has to take up positive ions as well as to emit electrons. The anode surface, however, has only to take up electrons and therefore plays a much less essential role in the mechanism of the discharge. This is also the reason why, for example, in a glow discharge with varying electrode separation, the cathodic part generally remains the same whereas the anodic phenomena may show large differences. In discussing these phenomena we distinguish two main cases: (1) a discharge with a positive column, and (2) a discharge without positive column.

### §59. No positive column. Theory

In order to neutralize the space charge of the electrons on their way to the anode, a sufficient number of positive ions should be present. In the negative glow direct ionization by electrons is possible; in the "plasma" of the Faraday dark space, however, the positive ions are (in the absence of a positive column) probably drawn to the anode by a small reversed electric field[316] (§30), the diffusion of electrons against the retarding field being large enough to give a resultant electron current in the direction of the anode.[317]

For large, flat electrodes, filling the cross section of a glass tube the following rough picture may be given.[318] Assuming that the electron concentration $n_e$ depends only on the distance $x$ to the cathode and that no ionization occurs, we can write the electron current density $eJ_e$ the ion current density $eJ_p$ (see §30) to the anode as:

$$\begin{aligned} J_e &= -D_e(dn_e/dx)+n_e\mu_e E, \\ J_p &= n_p\mu_p E, \\ n_e &= n_p. \end{aligned} \tag{131}$$

($D_e$=diffusion constant, $\mu_e$=mobility of electrons.)

When the losses at the wall are neglected and $D_e$, $\mu_e$ and $\mu_p$ are taken constant, from Eq. (131) there follows:

$$\frac{dn_e}{dx} = -\frac{J_e-(\mu_e/\mu_p)J_p}{D_e} = -a, \tag{132}$$

$$n_e = n_0 - ax, \tag{133}$$

which equation is of course only to be applied for values of $x \gg$ length of Crookes dark space. At a certain distance $d_{\text{crit}}$ from the cathode, the electron and ion density become nearly zero; according to (133)

$$d_{\text{crit}} = \frac{n_0}{a} = \frac{n_0 D_e}{J_e-(\mu_e/\mu_p)J_p} \approx \frac{n_0 D_e}{2J_e} = \frac{n_0\lambda_e v_e}{6J_e} = \frac{n_0(\lambda_e p)v_e}{6J_e}\cdot\frac{1}{p}, \tag{134}$$

in which $D_e$ is taken $=\lambda_e v_e/3$, $\lambda_e$=mean free path, $v_e$=mean velocity of the electrons.

For $d<d_{\text{crit}}$ ions and electrons are diffusing to the anode in such numbers that no appreciable space charge is set up at the anode and the anode fall will be approximately zero. For $d>d_{\text{crit}}$, however, the number of positive ions becomes very small and an "anode fall" develops. With increasing $d$ the anode potential $V(d)$ increases rapidly until the electrons have obtained enough

[315] R. Seeliger, Physik. Zeits. **33**, 316 (1932); H. Wulfhekel and R. Seeliger, Physik. Zeits. **34**, 57 (1933); A. Güntherschulze, Zeits. f. Physik **91**, 724 (1934); F. Keller, Zeits. f. Physik **97**, 8 (1935); R. Seeliger and K. Sommermeyer, Zeits. f. Physik **98**, 733 (1936); G. Zimmermann, Zeits. f. Physik **104**, 309 (1937).

[316] K. T. Compton, L. A. Turner and W. H. McCurdy, Phys. Rev. **24**, 597 (1924). For other mechanisms see the end of §60.

[317] An example of a large reversed field of this kind was already treated in §47 (low voltage arc).

[318] M. J. Druyvesteyn, Physica **4**, 669 (1937) (He, Ne and A).

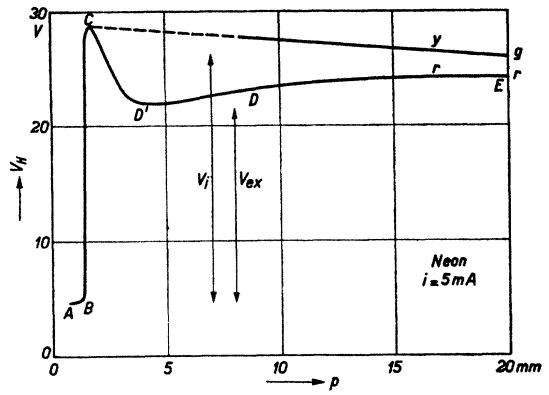


FIG. 83. Anode heating $iV_H$ in a Ne glow discharge as a function of the pressure. $AB$=no anode light; $C$=thin yellow anode glow; $DE$=red sphere on the anode. In the pressure region $DE$ also a discharge with a yellow anode layer was possible (upper curve). $V_i$=ionization potential, $V_{ex}$=lowest excitation potential of Ne.

energy before reaching the anode to form a sufficient number of positive ions. As the ionization takes place at a higher energy than excitation, in this case a luminous layer is present on the anode (anode glow). At a further increase of $d$, $V(d)$ increases only slowly, the anode glow remains on the anode and thus it moves farther from the cathode. At a still higher value of $d$ ($d_{col}$) a positive column develops (often discontinuously, compare below); from this value of $d$ onward the head of the positive column remains at the same distance from the cathode.

From the foregoing it is to be expected that for $d_{crit}<d<d_{col}$ the anode fall should be of the order of the ionization potential $V_i$ of the gas. In electronegative gases, however, where the electrons may combine to negative ions with small velocity and small ionizing power, much larger anode falls may occur.

## §60. No positive column. Experiments

In order to determine the value of $d_{crit}$ and of the anode fall, three kinds of measurements have been performed: (1) determinations of the space potential from collector characteristics, (2) measurements of the heat delivered to the anode by the electrons, and (3) measurements of the variation of the anode potential $V(d)$ as a function of the electrode distance.[319]

[319] Instead of $V(d)$ as a function of $d$ at constant $i$ and $p$, also $V(i)$ with $p$ and $d$ constant or $V(p)$ with $i$ and $d$ constant may be measured.

(1) *Space potential measurements.*—Because of the difficulty of interpretation and the influence of the collector on the discharge, at present no accurate results have been obtained. In a glow discharge collector measurements were performed among others by Güntherschulze and Keller[320] and by Emeléus and associates.[321] The result was that in the presence of an anode glow the space potential in the neighborhood of the anode was much lower (difference of the order of the ionization potential) than the anode potential, while both potentials were about the same when there was no anode glow and the electrode distance was sufficiently small.

Collector measurements in a nitrogen arc discharge between W spheres were performed by Wehrli and Bächtiger[322] with similar results.

(2) *Measurements of anode heating.*—For an electron current $i$, starting from a plasma where the electron mean energy is $e\bar{V}$ and passing an anode fall $V_a$, the heating $iV_H$ of the anode is given by:[323]

$$V_H=\varphi+(4/3)\bar{V}+V_a,$$

where $\varphi$=work function of the anode metal. Fig. 83 gives the results for $V_H$ in a glow discharge in Ne as a function of the pressure.[324] It is seen that, without anode glow $V_H\approx\varphi$ so that both $\bar{V}$ and $V_a\approx 0$; when the anode glow appears, $V_H$ increases by an amount roughly equal to the ionization potential $V_i$, giving $V_a\approx V_i$. At the higher pressures two different forms of the anode light are possible: a thin yellow layer or a red sphere. The latter corresponds to a lower anode fall, probably because here cumulative ionization from the metastable state takes place as discussed already in the case of the low voltage arc (§47) and of the positive column (§57).

Measurements of the anode heating in an arc discharge, performed by de Groot,[202] Wehrli and

[320] A. Güntherschulze and F. Keller, Zeits. f. Physik **81**, 799 (1933) ($H_2$, $N_2$, $O_2$).

[321] K. G. Emeléus, F. D. Greeves and E. Montgomery, Proc. Roy. Irish Acad. **43**, 35 (1936) (Helium); K. G. Emeléus, W. L. Brown and H. McN. Cowan, Phil. Mag. **17**, 146 (1934).

[322] M. Wehrli, Helv. Phys. Acta **3**, 180 (1930); M. Wehrli and P. Bächtiger, Helv. Phys. Acta **4**, 290 (1931).

[323] Compare Eq. (117); in $V_H$ the heating due to positive ions is neglected.

[324] F. M. Penning, Physica (old series) **4**, 380 (1924); **5**, 217 (1925).

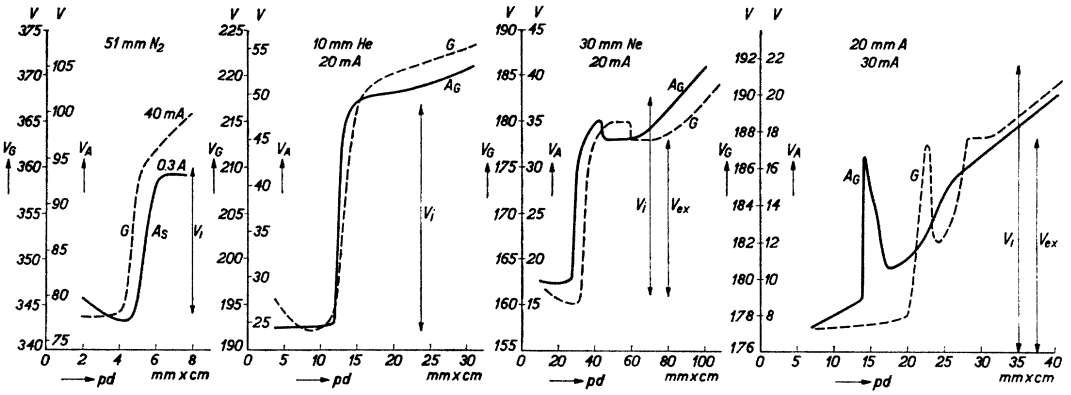


FIG. 84. Values of tube potential as a function of $pd$ ($d$=electrode separation) for a glow discharge $G$, an arc with externally heated cathode $A_G$ and a self-sustained arc $A_S$. Electrodes: for $N_2$ small W spheres in a large glass bulb, for the rare gases a flat anode of 4 cm diameter, nearly filling the cross section of the tube and a similar cathode ($G$) or an oxide-coated filament as cathode ($A_G$). $V_i$=ionization potential, $V_{ex}$=excitation potential.

Bächtiger[325] and Stübing[202] also show a much larger value of $V_p$ for $d>d_{crit}$ than for $d<d_{crit}$.

(3) *Measurements of* $V(d)=f(d)$.—In accordance with §59 $V(d)$ shows a rather sudden increase $\Delta V$ when $d$ is increased above a certain value which we identify with $d_{crit}$[326] ($i$ constant). This increase is accompanied by the appearance of the anode glow. Although $\Delta V$ is not necessarily equal to $V_a$ it gives a rough measure for this quantity. Fig. 84 shows some results for a glow discharge ($G$), an arc with externally heated cathode ($A_G$) and a self-sustained arc with a small W sphere as cathode ($A_S$) in $N_2$,[324] He,[318,202] Ne [318] and A.[318] For $N_2$, He and Ne the value $\Delta V$ is roughly equal to the ionization potential $V_i$. For Ne $V$ decreases suddenly a few volts with a further increase of $d$. This change is accompanied by a transition from the yellow anode layer to a red sphere and may be explained by the occurrence of cumulative ionization in the same way as described under (2). In the case of A Fig. 84 shows that the increase $\Delta V$ is considerably lower than $V_i$ and that $V$ goes through a rather sharp maximum, which suggests an anode fall even much lower than the excitation potential $V_{ex}$ needed for cumulative ionization. Possibly both phenomena are due to oscillations;[327] it is, however, not certain that this is the explanation of all the values of $\Delta V<V_i$ and $<V_{ex}$ occurring in the literature.

In glow discharges in $H_2$, $N_2$, CO and a number of gas mixtures at higher current densities still other phenomena occur (not shown in Fig. 84). In $H_2$ the anode glow may break up into a number of bright spots,[328] also into rings and other more complicated figures[329] or into spots which rapidly move over starlike patterns.[330] These phenomena are very sensitive to small amounts of impurities in the gas and on the electrodes. According to Güntherschulze, Bär and Betz the concentration of the anode fall into smaller spots may be explained in the same way as the appearance of certain striations in the positive column (§57), by assuming that the discharge takes that form in which the total energy loss of an electron along its path to the anode has the smallest value.

The difference between the curves $G$ and $A$ in Fig. 84 may be explained partly by the difference in gas density along the path of the electrons ($p$ being plotted in Fig. 84, and not $p_0$), partly by the difference in the cathodic part of the discharge, $d_{crit}$ increasing with increasing anomalous cathode fall. With variation of $p$ and constant cathode fall the value of $pd_{crit}$ varies only slowly which is in accordance with Eq. (134) when

[325] P. Bächtiger and M. Wehrli, Helv. Phys. Acta **4**, 31 (1931) ($N_2$); P. Bächtiger, Helv. Phys. Acta **4**, 409 (1931) ($N_2$ and $H_2$).
[326] A. Güntherschulze, Zeits. f. Physik **30**, 175 (1924).
[327] W. Pupp, Physik. Zeits. **34**, 756 (1933); W. Funk and R. Seeliger, Zeits. f. Physik **113**, 203 (1939).
[328] C. H. Thomas and O. S. Duffendack, Phys. Rev. **35**, 72 (1930); A. Güntherschulze and F. Keller, Zeits. f. Physik **81**, 799 (1933).
[329] A. Güntherschulze, W. Bär and H. Betz, Zeits. f. Physik **109**, 293 (1938).
[330] E. Kiessling, Zeits. f. Physik **96**, 365 (1935).

$n_0 v_e/J$ remains nearly constant; at low values of $p$, however, where $d>2R$ ($R$=radius of the tube) the disappearance of ions at the walls is the reason that $pd_{crit}$ diminishes,[318] $d_{crit}$ remaining of the order of magnitude of $2R$.

The large influence of small admixtures obeying condition (35), especially at high pressures, is shown[331] by Fig. 85. Here in pure Ne the anode fall occurs at $d<1$ cm, and in Ne+0.0004 percent A at $d>5$ cm; at large distances ($d$) the length of the Faraday dark space in pure Ne is usually only a few mm, in the mixture it is about 20 cm. Probably in the mixture[332] the positive ions in the Faraday dark space are not due only to direct diffusion from the negative glow, but also to the ionization of A atoms in the Faraday dark space by metastable Ne atoms,[333] formed in their turn by resonance radiation from the negative glow.[334, 335] The large difference between the curves $B$ and $F$ for the mixture clearly shows the influence of the negative glow on the anodic phenomena, although it does not prove unambiguously that this ionization is due to resonance radiation.[335]

Although many details need further investigation, the experiments are in agreement with the general picture of the anodic phenomena, given in §59 according to which the disappearance of the anode fall with decreasing $d$, is due to the diffusion of ions from the negative glow. In the rare gases and especially in gas mixtures obeying condition (35) the presence of ions at larger distances may also be due partly to ionization by metastable atoms, the quantitative amount as yet being unknown. According to other authors[336] the disappearance of the anode fall is due to the ionization by fast electrons from the cathode in front of the anode. In the rare gases, however, the values of $d_{crit}$ are so large in comparison with the range $d_e$ of these fast electrons,[165] and the light excitation in front of the anode is so small that this explanation seems very improbable. In the diatomic gases the difference between $d_{crit}$ and $d_e$ is much smaller.

## §61. Anode in the positive column

In the plasma of the positive column the ionization is such that the electron and positive ion densities are equal. An anode in this plasma will alter the state of ionization not appreciably, except for small distances from it, when its dimensions are small with respect to the tube diameter and the mean free path of the electrons. In this case we may apply the theory of Langmuir and Mott-Smith,[337] according to which a positive anode fall develops when:

$$j_r F < i.$$

Here $j_r$=density of the random electron current, $i$=anode current, $F$=surface of the anode. In this case a negative space charge belonging to the positive anode fall builds up around the anode, the outer surface $F'$ of which now collects the electrons. For spherical and cylindrical anodes $F'$ may increase to such a value that $j_r F' \approx i$ and enough electrons arrive at the anode

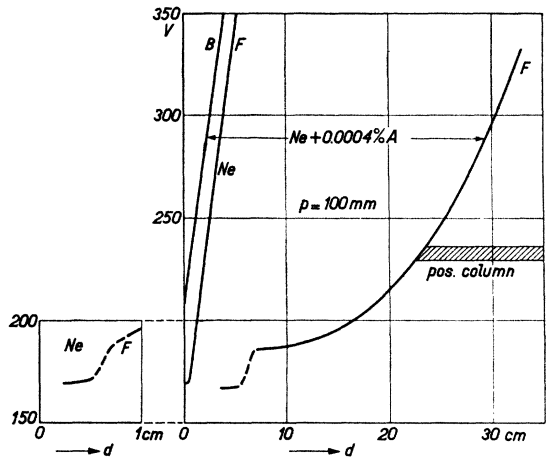


FIG. 85. Development of anode fall and positive column in Ne and Ne+0.0004 percent A at 100 mm pressure. Flat electrodes of 8 cm diameter in a tube of 10 cm diameter. $B$=negative glow on the back side; $F$=negative glow on the front side of the cathode. The curve for Ne is shown to the left on enlarged scale.

[331] F. M. Penning, not previously published.

[332] Also in pure rare gases ionization from a metastable state is possible, either by an electron collision or by a collision with another metastable atom.

[333] M. J. Druyvesteyn, Zeits. f. Physik **57**, 292 (1929).

[334] F. M. Penning, Zeits. f. Physik **78**, 454 (1932).

[335] K. G. Emeléus and O. S. Duffendack, Phys. Rev. **47**, 460 (1935); K. G. Emeléus, Proc. Irish. Acad. **42A**, 31 (1936).

[336] A. Güntherschulze, Zeits. f. Physik **30**, 175 (1924). In the case of a small cathode and a large spherical concentric anode, however, a diffusion of positive ions was assumed by G. Zimmermann, Zeits. f. Physik **91**, 767 (1934); F. Keller, Zeits. f. Physik **97**, 8 (1935).

[337] I. Langmuir and H. Mott-Smith, Gen. Elec. Rev. **27**, 762 (1924); I. Langmuir, Zeits. f. Physik **46**, 271 (1927).

to give the current needed. Not always, however, is this state reached, particularly not in the case of a plane anode. When the anode fall exceeds the ionization potential and the pressure is high enough, a sufficient number of ions may be formed which may lead to a breakdown of the negative space charge before $j_r F' \approx i$ and then a further increase of the anode fall is prevented. The positive anode fall could be established experimentally for a small anode in Hg vapor of room temperature.[337]

In the case $j_r F > i$ one would expect from the same reasoning a considerable negative anode fall; this, however, generally does not occur as the anode now often disturbs the plasma.[338]

In the case of a large anode at the end of a positive column the electron and the positive ion concentration near the anode are smaller than in the plasma, moreover the ion supply from the anode to the column will be too small to neutralize the electron space charge in the column. Therefore here generally also a negative space charge and a positive anode fall builds up into such a strength that a sufficient number of ions is formed adjacent to the anode. This view is supported by experiments of Pupp,[339] which show also that the anode fall can be changed from a positive into a negative value by an artificial ion supply (auxiliary arc) near the anode. In the positive column the mean velocity is much larger than at the end of the Faraday dark space, therefore in the case considered in this section, a lower anode fall may be expected than in that of §59. Experimentally, values of a few volts to a few tens of volts were found. It should be remarked, however, that also in this case the formation of positive ions may take place in anodic oscillations.[339]

When the anode is in the Faraday dark space (§59) we can compare $j_r F$ with $i$, as the Faraday dark space is a plasma as well. Also in this case, according to the circumstances a positive or a negative anode fall can be expected. Experimentally, however, little is known about this equation as here all experiments center about the variations occurring near $d_{\text{crit}}$.

[338] J. v. Issendorf, Zeits. f. Physik **67**, 556 (1931).

[339] W. Pupp, Physik. Zeits. **34**, 756 (1933). A similar potential distribution for a positive anode fall was found by L. P. Smith and P. L. Hartmann, Phys. Rev. **55**, 1145A (1939).

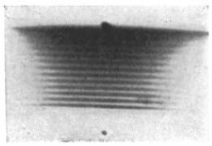

FIG. 18. Layers of Holst and Oosterhuis in neon (cathode below); $p_0=40$ mm; $d=1$ cm; $E/p_0=61$ v/cm$\times$ mm; $i=0.3$ $\mu$a.

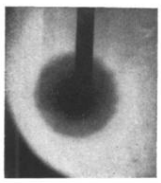

FIG. 58. Photograph of a low voltage arc in 0.05 mm A. The cylindrical cathode is viewed end-on and seen as a black circle with a thin bright edge; the figure shows the thin dark space-charge sheath and the relatively dark scattering sheath. The edge of the bulb may be seen in the left-hand bottom corner.