Helium the Superfluid*

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In the view of many who cultivate physics, its principal aim is to prove all things subject to a few short general laws. Yet whenever this aim is achieved in some restricted field, there is likely to follow a strange and depressing result. Reduced to law, the field becomes dull; the thrill of discovery and the pleasure of simplification are gone, it may even be hard to sustain a proper delight in the beauty of order. But if when this stage has been reached an exception is found at last, the excitement is all the greater because of the scope of the law; and the proof of the rule has prepared the setting for the exception.

Now of all the fields of physics, two of those most thoroughly subject to rules of long standing are the flow of heat through matter and the flow of fluids through tubes. Liquid helium when exceedingly cold—within 2.2° of the absolute zero—violates these rules in a thoroughgoing way, and therefore is for the present the most exciting fluid in nature. It is far out of the common in respect of magnitudes also; by which I mean, that it is a much better conductor of heat, and also flows much more readily, than anything else which at present is known. To have come on a fluid like this is like finding unexplored land in the midst of an ancient community, or a tract of primeval prairie among the corn fields of the Middle West.

It must at once be said that there are two forms of liquid helium, one of which is unremarkable. The temperature which I gave as 2.2° — it is called the "lambda-point" by a suggestion of Ehrenfest's—is the transition not between liquid and gas, but between the ordinary and the extraordinary liquid. These are known as helium I and helium II. The transition between the two was detected first by a kink in the curve of density against temperature—not a very notable kink, but still the effect was observed as early as 1911 (by Onnes). Most of the measurable qualities (density, dielectric constant, specific heat, heat of vaporization, surface tension, conductivity for heat, viscosity) change appreciably as the liquid passes over from one to the other form: With some the change is trivial, with others tremendous. But there is no latent heat of transformation detectable, though the methods of detection are very delicate;† and this makes people hesitant to speak of the two as "phases."

The only distinction between the two that "meets the eye" was recorded in 1932 at Toronto. To express it in words written later by Wilhelm, Misener and Clark: "Helium I visibly boils as it is being evaporated, in a way similar to ordinary liquids; but immediately the lambda-point is passed, boiling stops, and the liquid appears to be absolutely quiescent." This text suggests how the temperature of liquid helium is varied in experiments where variation is desired: there is a pump incessantly working away at removing the gaseous helium which is steadily boiling off, and if the experimenter wishes to cool or to warm the liquid, he speeds up the pump or slows it down. There is another and a famous method for attaining still lower temperatures, that of "adiabatic demagnetization," which is applied not to the helium directly but to a paramagnetic salt adjoining it; but at the temperatures reached by this it appears (though little is known as yet) that helium II is no longer so remarkable. The methods of measuring temperature, in these far-from-familiar ranges, are very interesting and not always quite reliable; but this subject would absorb the whole article if I were to give it a chance, and so we must for the present accept the temperature-values as given.

Now we turn to the strangest of the phenomena and the simplest of the experiments: the "creeping films" of helium II and the observations which reveal them.

None of the facts in this field is more striking than the easiest to be observed of all. If a cup partly full of helium II is lowered partway into a

* A lecture given before the Colloquia of the Case School of Applied Science and the University of Rochester, and elsewhere.

† Had the latent heat been as much as 0.001 as great as the specific heat of the liquid near the lambda-point, it would have been detected.

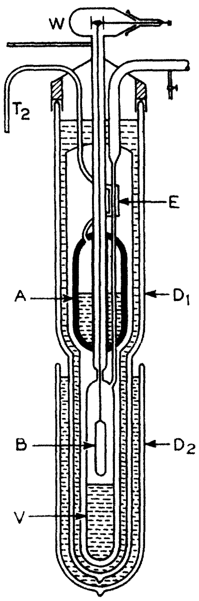

FIG. 1. Cryostat for the study of the creeping film of liquid helium II (Daunt and Mendelssohn: *Proceedings of the Royal Society*).

bath of the same liquid, the levels within and without come gradually and exactly to the identical height, as if there were a perforation in the cup or a siphon over its rim—but there is no siphon and there is no perforation. The cup need not even be partially full to start with—it can be empty initially, and still the liquid will climb invisibly over its rim from the bath. If it is suddenly lifted so that the inner level is now above the outer, it drains back into the bath. Indeed even if lifted completely out of the bath the cup continues to drain, and the liquid is found eventually hanging or dripping in droplets from the outer side of the bottom.

This was observed in 1922 and by Kamerlingh Onnes himself; but most of what is known in detail was found in recent work in Oxford by Daunt and Mendelssohn. I show their sketch of their cryostat as Fig. 1. The central "tube" contains the bath of liquid helium II at V, and B is a symbol of the cup or "beaker" suspended from the winch above. Around and over the bath are protective sheaths of liquids not quite so cold but still very chilly: helium I at A, hydrogen in D_1 and nitrogen in D_2. These make it next to impossible for heat to be conducted to the bath; but ordinary daylight may vaporize the helium II too rapidly for convenience, and these experiments were done in the light of neon tubes. Observation by eye, with or without the aid of a cathetometer, is made upon the rise or fall of the menisci in beaker and bath. In the pictured apparatus, the beaker being about a centimeter in breadth, the rates of fall or rise were of the order of millimeters per minute.

Let the beaker be filled to a level higher than that of the bath. Slowly and steadily it empties itself, till either the two levels are equated or else the experimenter suddenly plunges the cup down so deep that the interior level becomes the lower, whereupon instantly (so far as can be told) it proceeds to fill itself out of the bath at just the same rate as previously it was emptying. This is vividly shown in Fig. 2, where also one sees how nearly constant the rate is—a remarkable fact! There are, it is true, departures from constancy of rate if either the inner or the outer level is within a centimeter or two of the beaker-rim; there is also a slight upward trend of the rate of transfer with the difference of height between the levels, too small to be seen in Fig. 2. The fact that there are such departures is less striking by far than the fact that they are slight enough to be neglected in the first analysis. Indeed the efflux from the beaker and the rate thereof are not contingent on the presence of the liquid bathing the outside of the wall. The outflow continues when the beaker is lifted quite out of the bath, and it continues also—and at the identical rate—when the beaker is girdled by a wire heated to such a degree that the film cannot pass over or even approach it without being vaporized.

Now having twice employed the word and concept "film" I must make haste to justify it. To cite one of the aptest of the Daunt-Mendelssohn experiments: the beaker was designed with a cylindrical hollow of which the radius had a particular value R_0 down to a certain depth below the rim, and a smaller value R_0/a below that

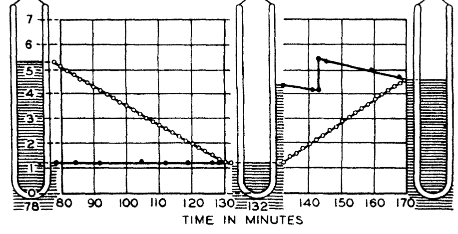

FIG. 2. Rise and fall of meniscus in beaker and bath (Daunt and Mendelssohn).

depth: a was 3.58. As the meniscus in its fall passed by the point of sudden narrowing, its rate of fall changed quickly from one steady value to another and a higher one (Fig. 3). Let us form first a hypothesis destined to be proved wrong by the experiments, and then a happier one. Suppose the rate of efflux to be proportional to the exposed area of liquid (as it should be, if the effect is a simple evaporation). It would then alter in the proportion $1:a^2$ at the point of sudden narrowing; but at the same point, the rate of fall of the meniscus entailed by any constant rate of efflux would alter in the proportion $a^2 : 1$; the actual rate of descent would therefore not change at all. But the rate of descent *does* change, and so the first hypothesis is false. Moreover it changes in sensibly the ratio $a : 1$ (actually, 3.57 to 1 was observed, a remarkable agreement!) and this shows that the rate of efflux varies as the circumference of the beaker, i.e., as the breadth of wall in contact with the outflowing liquid. This is the prime quantitative reason for believing in a film of constant speed (or, not to go one single further step beyond the data, a film of constant product-of-thickness-and-speed) clambering up and over the wall which dips into the helium II.

Taking this as established, we can think about choosing a numerical measure for the effect. It must be the rate of efflux or influx of helium II per unit breadth of overflowed wall. According to Daunt and Mendelssohn this is a sharply varying function of temperature, as Fig. 4 displays: imperceptibly small at the lambda-point, it rises with falling temperature to the value 1.5 cc per cm per second at 1.5°, and thence onward to 1.0° is sensibly constant. The measurements were made with beakers of glass, and accordingly

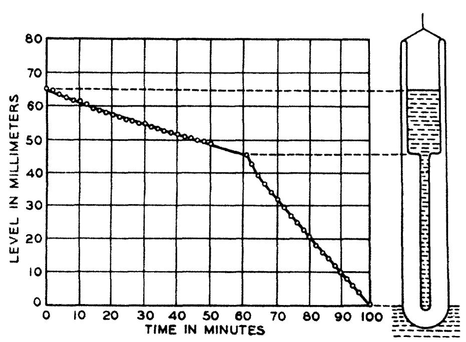

FIG. 3. Dependence of rate of fall of meniscus on diameter of beaker (Daunt and Mendelssohn).

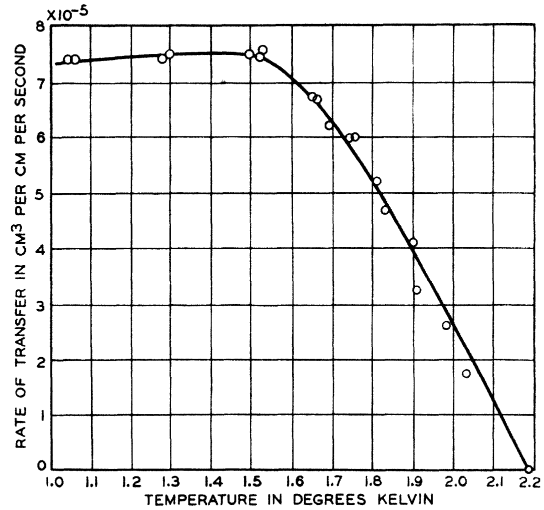

FIG. 4. Dependence of rate of flow of film on temperature (Daunt and Mendelssohn).

it is to the flow of helium II over glass that these figures refer. They may however be correct for the flow of helium II over any smooth surface, since in an experiment with a beaker of copper carefully polished the rate was about the same. A strand of rough copper wires passing from beaker to bath had made the efflux much more rapid in one of the first experiments.

To convert these figures into speeds of flow, one must ascertain the weight of the film per unit area. To do this Daunt and Mendelssohn coiled up a strip of copper having an area of 1000 square centimeters, soldered to it a needle-point, and lowered it until the point dipped into the "bath," which for this purpose was confined to a capillary projecting downwards from the body of the cryostat-tube. The needle was expected to serve as a bridge whereby the helium II would cross to the strip, on which it would presumably spread itself out as a film of the type desired. The level of the liquid in the capillary was measured just as the point was lifted out, and then again when the strip had been raised so high in the tube that it was warm enough to shed its coating, the substance of which presumably found its way back to the capillary. From the difference of these levels Daunt and Mendelssohn computed values of weight-per-unit-area which they translated into values of film-thickness, assuming in so doing that the volume-density of the film is the same as that of liquid helium II in bulk. These values are scattered* between 25 and 50 mμ. The thickness being taken as 50 mμ, the speed of flow is figured as 20 cm/sec. at 1.1°.

Now we are going to consider such topics as the transport of heat by helium II and the passage of the liquid itself through very narrow channels. These are intricate and perplexing, and if I had begun the article with them many a reader might have been discouraged very soon. I deferred them, however, not as a stratagem, but because in studying first the fantastic ease of motion of helium II along a surface we may have become acquainted with the cause of many a curiosity. The creeping films may indeed convert the transport of heat into something very unlike a simple conduction through motionless matter, and the passage of the liquid through a capillary into something very contrasted to a simple viscous flow.

Experiments on both are done with capillaries, usually but not always cylindrical in bore, connecting two broader containers of helium II. In the study of passage, the liquid sinks from the upper container through the tube into the lower, gravity being the impelling force; the temperature is, of course, the same throughout. I shall later state the laws which a classical fluid in such circumstances would obey. In the study of transport of heat, a temperature-difference ΔT is established between the containers, and there is no palpable motion after equilibrium is reached. It is assumed that all the transport is through the column of liquid in the capillary; this at first seems strange, but after all, the glass walls of a capillary are very poor thermal conductors, and the containers themselves are often protected by evacuated spaces. Sometimes (as in the work of Allen and his colleagues at Cambridge and in the later Leyden work) ΔT is steadily maintained by a heater-wire in which electric current is generating heat at a known and steady rate, say Q calories per second. Sometimes (as in the earlier Leyden work) ΔT is raised to an initial value ΔT_0 and then the supply of heat is suddenly cut off; the warmer container thereupon cools down and heat departs from it at the rate $Cd(\Delta T)/dt$ —C standing for the specific heat of the warmer container and its content, which must be known if the method is to be used.

Thus in both cases there is transport of a known quantity of heat per unit time (call it Q) down a cylindrical column of liquid with a temperature-difference ΔT or ΔT_0 between the ends thereof. The ratio $Q/\Delta T$ would serve as a measure for the facility of this transport. However in dealing with ordinary conductors of heat, it is the custom to give the value of the ratio

$$\frac{Q}{A(\Delta T/L)}=\frac{QL}{A\Delta T}=\kappa,$$

L and A here standing for the length and the cross-sectional area of the capillary. In an ordinary conductor κ is found to be independent of L and A and ΔT (so long as ΔT is not so great that the properties of the substance vary appreciably along the column). The substance is then behaving like a classical conductor of heat, and it is proper to call κ the "thermal conductivity" thereof. For helium II there are few data as yet bearing upon the dependence of κ on L and A, but the facts about to be quoted will strongly suggest that here we have no classical conductor! Nevertheless the symbol κ and the name of conductivity are still applied to the ratio aforesaid. I will use the former but not the latter.

For helium II, then, the transport of heat along narrow cylindrical columns is in two ways sensational: κ varies with ΔT and Q (which of these last one takes as independent variable is not important); and the values of κ for the lower values of ΔT and of Q are higher by far than the greatest thermal conductivities yet reported for other kinds of matter.

To give examples: the Leyden school (Keesom and Keesom) first reported 190 cal. deg.$^{-1}$ sec.$^{-1}$ cm^{-1} for κ of helium II and 0.00006 for κ of the only-slightly-warmer helium I. Of these two values the latter is like the thermal conductivities of gases at ordinary temperatures, but the former is "about two hundred times that of copper at ordinary temperatures, or about fourteen times that of very pure copper at liquid-hydrogen temperature." Even then, helium II stood out as "by far the best heat-conducting substance we

* The extent of the scattering, i.e., of the discrepancies between different values, is disconcerting; but there is the reassuring feature that the same method, when applied at temperatures above the lambda-point, gave no sign of a film at all, though films of thicknesses as low as one mμ could have been detected.

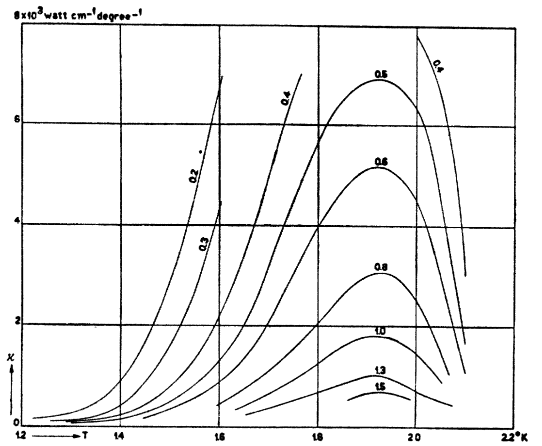

FIG. 5. κ values as function of temperature for various values of heat-flow Q (Keesom and Saris). Divide ordinates by 4.19 to translate into units used in context. Q is expressed in watts/cm².

know"; and how much more so from the later Leyden data which yielded κ-values as high as 1900!

Since κ declines as Q and ΔT increase, the foregoing values correspond to especially low ΔT values, presumably the lowest which could be readily measured; values of 0.001° were lately evaluated "with an accuracy of about 5 percent" at Leyden, the temperature-gradient along the capillary being then about 0.00003° per centimeter. Moreover for given ΔT, κ passes through a maximum in the neighborhood of 1.9° (Fig. 5), and accordingly those values were taken in that temperature-range.* Below 1.0° κ has already dropped down a large part of the way toward "normal" values, as we shall presently see.

I have passed quickly over the problem of measuring ΔT, as though it were so easy as not to be worthy of mention; but the contrary is the case, and there is a remarkable story here.

At Leyden the estimates of the temperatures at the two ends of each capillary were made by methods still deemed reliable (by measuring resistances of wires or susceptibilities of paramagnetic salts). At Cambridge the estimate of ΔT was made by a method then deemed equally reliable, indeed perhaps much more so. Allen and his colleagues arranged the capillary to connect a "bath" similar to that of Fig. 1 with a bulb initially evacuated and shielded well against inflow of heat. The difference in height between the levels in bulb and in bath should then be equal to the difference in pressure between the vapors within the bulb and over the bath, respectively. The vapor-pressure of helium II is a function of temperature previously well determined; thus knowing the temperature of the bath, the observer should be able from that difference in height to determine the temperature of the liquid in the bulb. The bulb was traversed by a heater wire; on sending a current through this wire, Allen and Jones observed that the meniscus in the bulb was forced down, and from its new position they computed ΔT and then κ in the way aforesaid. All seemed well, until when working near their lowest attainable temperature (at 1.08°) they found that when the wire was slightly heated the meniscus in the bulb went up instead of down!

On making and testing this strange observation, Allen and Jones were willing to discredit their own former conclusions as to the dependence of κ on ΔT. It was not necessary to be so drastic, since at Leyden also κ has been found to decrease as ΔT increases; but it seems likely enough that the numerical values earlier published at Cambridge for the quantity then called κ should be reconsidered in this light. At any rate the interest of the discovery outweighs by far the inconvenience of having to revise the inferences as to the transport of heat!

Taking off the top of the bulb and making it thus again a "beaker" (Fig. 6, left) so as to establish equality of pressure over the liquids within and without, Allen and Jones again observed the rise of level within the beaker when the heat was turned on, "increasing with increasing heat-flow and, for constant heat-flow, increasing with decreasing temperature." It has even been found (by Daunt and Mendelssohn) that the same effect occurs† when there is no hole in the beaker and all the transfer occurs by the film creeping over the rim. One might remember

* Some information about the dependence of κ on L and A (page 260) appears in the paper of Keesom and Saris (Physica 7, 241 (1940)). No dependence on L was found, and at 1.42° and 1.6° no dependence on A. From the plausible notion that heat is carried by a creeping film along the walls, it would follow that κ should increase with diminishing A. At temperatures from 1.84° on upward, a slight increase was indeed found, not however great enough to sustain the notion.

† The level-difference may reach 5 mm!

the effect by thinking that the helium II climbs out of the bath and into the beaker in order to get warm. The experimenters express it however by saying that when heat is flowing one way along a capillary containing helium II, there is a tendency for the liquid to flow bodily the other way. The dependence of this tendency on the radius of the capillary was being studied by Allen and Reekie just at the outbreak of war, and they were finding indications that the tendency is reversed for broader tubes and that near the capillary wall it is not the same as it is in the middle of the lumen.

Related to these facts is the prettiest sight of all in this field, the fountain-like spray of what is called the "fountain-effect." Allen and Jones had taken a tube of glass open at both ends, and submerged it partly in helium II; the upper end, narrowed to a capillary, emerged from the bath; the lower end, broadened and curved in a semicircle, was packed with emery powder (Fig. 6, right). Shining an ordinary 60-watt flashlight on the powder-filled end of the tube, they saw shooting out of the upper end a steady jet of the liquid which broke and fell as a spray! it actually sprang as much as 16 cm above the lip of the tube. Here presumably is the tendency of the liquid to flow against the heat carried to an extreme, because the channels between the powder-grains through which liquid and heat alike must creep are so extremely narrow.

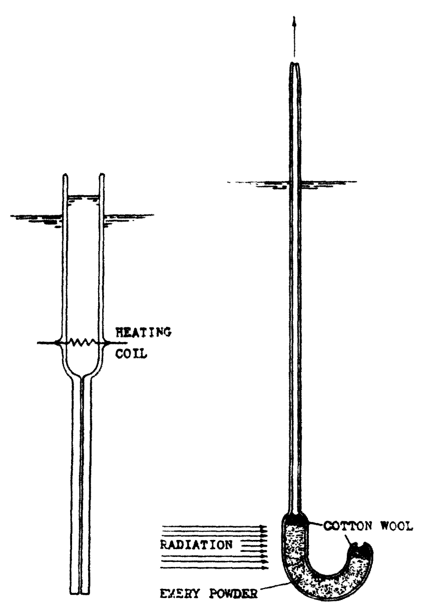

FIG. 6. Apparatus for showing variation of level in beaker when heated (left) and fountain-effect (right) (Allen and Jones; *Proceedings of the Royal Society*).

We are far from having exhausted the transactions of heat with helium II. There is for instance an experiment (of Daunt and Mendelssohn) in which the liquid flows through a porous plug, and the temperature rises (by 0.01° or so) on the side which it is leaving. Perhaps this is a sort of inverse of the phenomenon of the jet. I will, however, leave the subject after describing the trend of specific heat, which is exceptional though not quite so greatly anomalous as the apparent thermal conductivity which we have just considered or the apparent viscosity to which we shall later come.

The trend of specific heat on both sides of the λ-point is shown in Fig. 7 (from the Leyden school). For helium I on the right there is a decline with falling temperature (the upturn near the λ-point being ascribed to the presence of small regions in the fluid where helium I has already been transformed to helium II). At the λ-point there is a sudden upward jump to what proves to be the highest value achieved by helium II, since with further cooling the specific heat drops steadily away. Further down the scale, between 0.8° and 0.2° (so Pickard, Kürti and Simon of Oxford have reported) the specific heat is varying as the cube of the temperature. This is what may be called a normal or conventional behavior for liquids and solids. The fact suggests that helium II may lose its anomalies in becoming exceedingly cold; and indeed, when Kürti and Simon measured the κ-value they found it low—0.022 at 0.5° and 0.002 at 0.2°. Apparently then helium II, instead of being queerer the colder it is, passes through a sort of maximum of queerness not very far below the λ-point, and turns back toward the norm as it approaches the absolute zero.

We are now to study the passage of helium II through capillary tubes; and in so doing we shall be forced along the same mental path as in our study of the transport of heat through this liquid. There we began by assuming conduction of heat to take place in the classical manner; but the quantity which should have been the thermal conductivity turned out to be strangely high and strangely variable, and at the end it was suggested that these results imply that the transport of heat is not altogether by classical conduction. Now we are to begin by assuming helium II to flow in the classical manner of a conventional viscous fluid; but very soon it will be apparent that the quantity which should be the "coefficient of viscosity" is strangely low and strangely variable, and that the best way of dealing with it is to drop the supposition that the liquid is behaving as though it were ordinarily viscous.

But what exactly is this standard, the behavior of a classical viscous fluid flowing conventionally through a tube? It may not be amiss to remind the reader thereof, and in so doing take occasion to define the variables of the experimental problem. The chief dependent variable, which I shall loosely call the rate-of-passage of the liquid, is best defined as the volume-per-second of liquid traversing or emerging from the tube: I denote it by V_s. The velocity of the liquid varies from a maximum in the axis to a minimum at the wall, the latter generally taken to be zero. Dividing V_s by A, the cross section of the tube, one gets a quantity which is a sort of average velocity; usually it is miscalled "velocity" and is denoted by v. The primary independent variables are r and l the radius and length of the tube, and p the pressure-head or pressure-difference between the ends; in the classical fluid it is the pressure-gradient p/l which dominates, not p separately nor l separately. As the viscosity of a fluid usually depends on the temperature T, this last is also an independent variable.

For the ordinary viscous fluid moving through a cylindrical tube, these variables are linked by the formula of Poiseuille

$$V_s = \frac{\pi}{8}\left(\frac{1}{\eta}\right) r^4 (p/l).$$

Thus the rate of passage varies directly as the pressure-head inversely as the tube-length, directly as a high power (the fourth) of the tube-radius. The constant η is the "coefficient of viscosity." Temperature appears in the formula, not explicitly indeed, but implicitly through η. Since usually liquids become more viscous with cold, the expectation is that the rate of passage will fall with falling temperature.

Ordinary liquids conform to all of these rules, but helium II not to any.

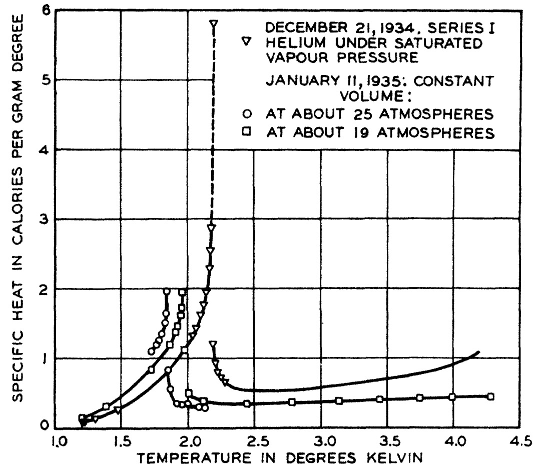

FIG. 7. Specific heats of the two forms of liquid helium, plotted as functions of temperature (W. H. and A. P. Keesom; *Physica*).

(When the flow is between plane-parallel walls instead of through a narrow cylinder, the formula differs from Poiseuille's by only the substitution of the third power of the wall-to-wall distance for r^4 and the substitution of another numerical constant for $(\pi/8)$. This case was realized in a very ingenious way by Giauque, who filled a capillary of glass with hot molten solder which cracked away from the glass when it cooled, owing to the known difference between the thermal coefficients of expansion—the annular passage left between was so slightly curved that its walls could be regarded as plane-parallel. Kapitza also observed the flow between parallel surfaces optically smooth; it was rather a rush than a flow, since even when the surfaces were lying one on top of the other the liquid oozed through so rapidly that no measurements could be made.)

Let us have in mind the magnitudes concerned. In the Allen-Misener experiments, the pressure was varied from 160 down to 5 dynes/cm (the higher figure corresponding to a 15-mm column of liquid helium)—the tube-length, from one mm to 40 cm—the temperature, from 1.15° to 2.18°K, the upper of these temperatures being the lambda-point. As for tube-radius, there were individual tubes of diameters 438 and 153 and 50 and 16.2 microns, but the most striking results were obtained with multiple channels made in an ingenious way, which I must pause to describe.

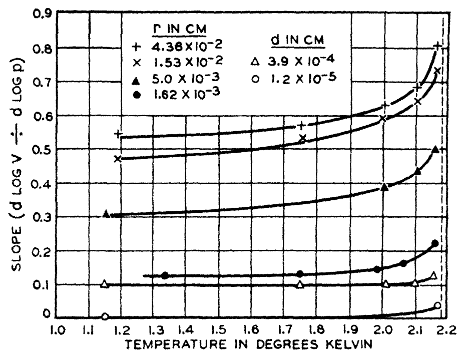

FIG. 8. Variation with temperature of the exponent s in the relation $v = p^s$ obtaining for flow through capillaries (Allen and Misener: *Proceedings of the Royal Society*).

A tightly-bound bundle of wires was slipped into a metal pipe, which then was drawn through a succession of steel dies of decreasing diameter till the wires were mashed together to and even beyond the threshold of deformation. The channels between the wires were taken to be twice as numerous as the wires, and the average cross section estimated by sending through them a fluid known to behave classically and with a known viscosity (ordinary gaseous helium) and using Poiseuille's formula for computing the area. This average cross section corresponded to radius-values of 3.9μ for one and 0.12μ for another bundle of channels, but it is not supposed that these were truly circular in section. Giauque's annular slit between solder and glass had a breadth of about one micron.

We consider first the dependence of rate of passage on pressure, which is to serve as the primary test as to whether a fluid is classical. It is required that v should be proportional to p. Otherwise expressed: *unless the plot of log v against log p is a straight line of slope unity, the fluid is not behaving classically* (*except in one case*), and it is pointless or worse than pointless to try to compute the viscosity.

The cumbrous phrasing of this statement is suggested by the fact that *it is a feature of helium II that* under nearly all conditions *the plot of log v against log p is a straight line but the slope s thereof is not unity*. This fact emerges from the data of Allen and Misener, who found values of slope ranging all the way from 0.8 down to practically zero. Many of these appear in Fig. 8, where s is treated as a dependent variable, T being the independent variable and r the parameter which varies from curve to curve.

It is not surprising that the approach of s to the classical value of unity should be closest for the fattest capillaries and the highest temperatures; it is perhaps surprising that the approach should be so incomplete at best; it *is* surprising, and decidedly so, that in some conditions (the narrowest channels and the lower temperatures) the amount of the pressure-head should make no perceptible difference to the rate of passage! One is reminded of the behavior of the gliding films in which helium II creeps out of beakers; but of this, more later.

Before going on to the dependence of V_s on the other variables, we must be reminded that there is after all another value of s which is not incompatible with a classical fluid. This is the value 0.5, which occurs when the liquid is in "turbulent" motion—that type of motion which occurs when certain limits of speed and other variables are transgressed, and is distinguished by a most irregular and chaotic eddying and whirling throughout the moving mass. Poiseuille's formula with its s value of unity applies to the other extreme case, that of "laminar" flow when the water travels evenly along in straight lines parallel to the axis of the capillary. Allen and Misener observed the s value 0.5 with their shortest and fattest capillaries, and also (which seems strange) for the passage of helium II through tightly-packed powder.

We must notice briefly the dependence of V_s on the other variables. As for l: with a constant pressure-head, the rate of passage should diminish towards zero as l is increased—with helium II it diminishes indeed, but so slowly as to suggest a limit greater than zero, as though with a finite pressure the liquid could creep even through a tube of indefinite length. As for the dependence on r, let us think of v the mean velocity in preference to V_s: it ought to increase as r^2 (since V_s ought to increase as r^4) but instead it passes through a minimum and thereafter rises gradually. Let us proceed in imagination from the wider tubes to the narrower: it appears that below about 15μ, the narrowness promotes the flow of the fluid, as though the proximity of the walls encouraged it. As for temperature: the rate of passage falls off with increasing warmth for every capillary except the fattest, and the narrower the passage the more sharply v and V_s fall off as the temperature climbs toward the lambda-point; we see this behavior in Fig. 9. The liquid in the tube of broadest bore is acting like an ordinary fluid growing less viscous as it warms up; but for the others we should have to say the contrary, if the evidence were not convincing that the notion of viscosity no longer has a place.

Returning to the dependence on radius: suppose that for V_s we write a hypothetical formula in powers of r:

$$V_s = Ar + Br^2 + Cr^3 + \cdots$$

and apply it to the curves of Fig. 10, which stand for V_s/r plotted as function of r for various combinations of T and p. For the lower temperature there is indeed an intercept on the axis of ordinates, and therefore a coefficient A which does not vanish. The trend of the curve is linear at first, and therefore B does not vanish; later it is concave-upward, and so the further terms are not to be disregarded. Were the liquid classical, only the term in r^4 would appear, in accordance with Poiseuille's law; how far this is from the truth!

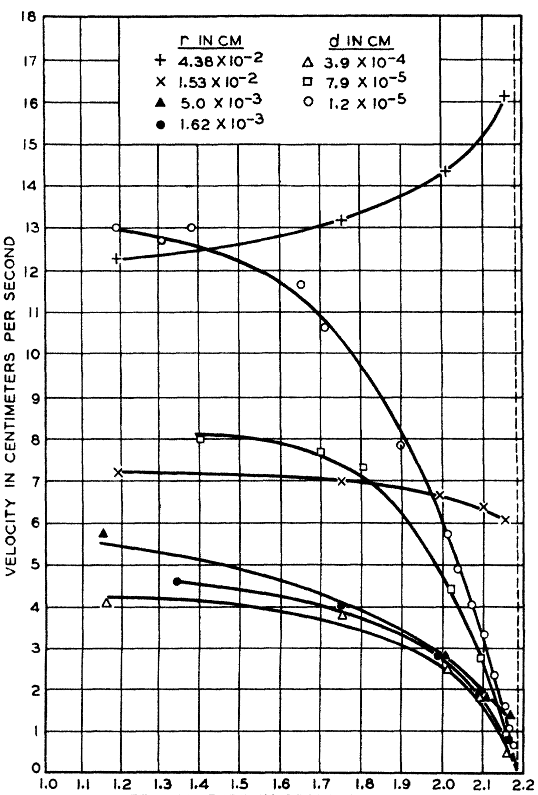

FIG. 9. Variation of flow with temperature for capillaries of different diameters (Allen and Misener).

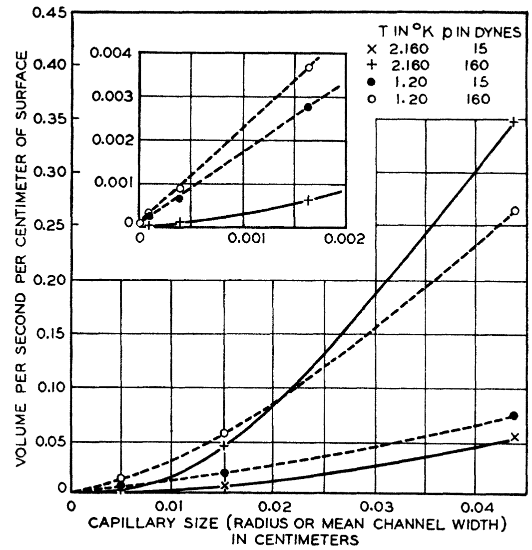

FIG. 10. Variation of V_s/r (see context) with temperature, for capillaries of various diameters (Allen and Misener).

Altogether then we find helium II a very remarkable thing, one of those few to which the word "unique" is properly applied. Theorists have been busy for several years in attempts to explain its qualities. It is to be hoped that the data already in print are sufficient for their purposes; for owing to the location of all but one of the laboratories of which the work has here been cited, it seems likely that we shall not soon get further information.

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