The Specific Ionization of High Speed Particles

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THE theory of the ionization of an atom by a moving charged particle was first given by J. J. Thomson[1] and N. Bohr.[2] The energy imparted to the atomic electron was calculated from the impulse of the electric field of the passing charged particle. When the distance of approach is small, the impulse will be large and the binding force of the atom for its electrons can be neglected. The duration of the impulse is important in determining the chance of ionization of the atom. As the speed of the ionizing particle is increased the impulse given to the atomic electrons will become smaller. The formula given by Bohr was of the form

$$N=(A/\beta^2)\log k\beta^2, \tag{1}$$

where N is the number of ion pairs per cm of path, β the ratio of the velocity of the ionizing particle to the velocity of light, and A and k are constants. The values of A and k were given explicitly for atomic hydrogen in terms of atomic constants. For other elements the values of the constants could only be estimated.

The application of quantum mechanics to Bohr's relativistic solution of this problem has been made by Bethe,[3] Williams,[4] Møller,[5] Oppenheimer,[6] Bloch[7] and others. There is a general agreement that when a limit is set on the maximum energy lost to a secondary electron, the total number of ion pairs per cm produced by all secondaries with energies below this limit is given by an expression of the form

$$N=\frac{A}{\beta^2}\left(\log k+\log\frac{\beta^2}{1-\beta^2}-\beta^2\right). \tag{2}$$

This is called the probable ionization.[4] In the experiments where the number of ion pairs is obtained by counting the number of drops in the Wilson cloud-chamber track the limiting energy of the secondary is determined by the ability to count dense clusters of drops. The limit can be as high as 10^4 ev.

The continuous curve in Fig. 1 is a plot of Eq. (2) in which the constant k has been chosen as approximately 10^5, a value estimated as appropriate for nitrogen. The general shape and variation of the curve is not very sensitive to the choice of k. The constant A has been chosen to make the magnitude of the theoretical curve agree with the observations of Corson and Brode.[8] It is not easy to make a direct determination of the β for a high speed particle. It is usual to measure the radius of curvature, ρ, in a magnetic field, H. The quantities β and $H\rho$ are related by the equation

$$H\rho=\frac{mc\beta}{e(1-\beta^2)^{\frac{1}{2}}}. \tag{3}$$

For a particle of a given mass there is a definite relation between the value of $H\rho$ and β. The

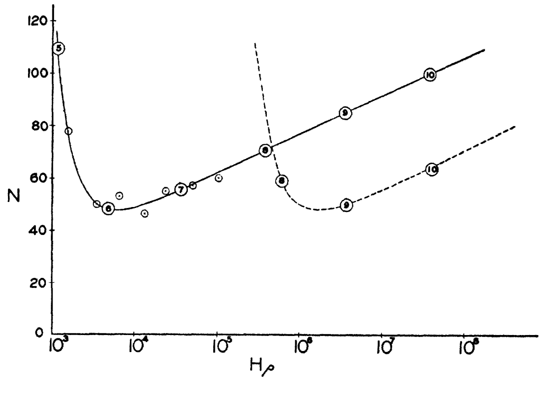

FIG. 1. The theoretical curve for the probable specific ionization for electrons in air in ion pairs per cm of track at 76 cm and 0°C is shown in the continuous curve as a function of Hρ in gauss cm. The dotted curve is the probable specific ionization for a particle of mass 250× the mass of an electron. The numbers in the circles along the curves indicate the power of 10 for the energy in electron volts. The small circles are the experimental observations of Corson and Brode (reference 8).

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[1] J. J. Thomson, Phil. Mag. **23**, 449 (1912).

[2] N. Bohr, Phil. Mag. **30**, 581 (1915).

[3] H. Bethe, *Handbuch der Physik*, Vol. 24; 1 (Springer, Berlin, 1933), p. 523.

[4] E. J. Williams, Proc. Roy. Soc. **135**, 108 (1932).

[5] Ch. Møller, Ann. d. Physik **14**, 531 (1932).

[6] J. R. Oppenheimer, Phys. Rev. **47**, 44 (1935).

[7] F. Bloch, Ann. d. Physik **16**, 285 (1933).

[8] Corson and Brode, Phys. Rev. **53**, 773 (1938).

value of N has therefore been plotted as a function of the observed quantity $H\rho$.

Neglecting terms that are not important in the case where the maximum transfer of energy is limited to about 10^4 volts, Eq. (2) will be valid for a particle of any mass. All particles with the same β would in that case have the same probable ionization. From Eq. (3) a proton with the same β as a particular electron will have the same ionization as that electron but will have a value of $H\rho$ that is 1850 times that of the electron. Since in Fig. 1 the value of $H\rho$ is plotted logarithmically, it is only necessary to displace the curve by the logarithm of 1850. For a mesotron of mass 250 times the mass of an electron the probable ionization N is given by a curve displaced from the solid line curve by the logarithm of 250. The dotted curve represents the ionization for a mesotron of mass 250 times the mass of an electron.

Methods of Observation

A knowledge of the number of particles passing through an ionization chamber in unit time combined with the total ionization in the same time is sufficient to determine a value of the average specific ionization. The number of particles can be estimated in terms of the count of a Geiger Counter but a correction must be made for showers because the Geiger counter gives one count when several simultaneous particles have passed through it. The mean ionization must be corrected for recombination of the ions which will be appreciable if high pressures and low voltages are used.

With a linear amplifier one may measure the number of ions collected when a single particle passes through an ionization chamber. The time of collection must be short enough to resolve each particle. This requires pressures that are not too high and limits the ionization from a single particle. There must be some allowance made for the shape of the chamber as each track will traverse a different path length in the ionization chamber.

The fluctuation of ionization may also be used to estimate the number of particles and hence the specific ionization. In this calculation it is important that corrections be made for the frequency of showers of different magnitudes and for the variation in length of the tracks in the chamber.

The efficiency of Geiger counters when filled with gas at a low pressure will give a measure of the primary ionization provided no ions are released at the walls by the penetration of the particle and provided that every ion in the gas will trip the counter.

For high speed β-rays, the measurement of the current of β-particles together with the ionization produced by them has been used to measure their average specific ionization. The inability to produce intense sources of high energy particles makes this method inapplicable to particles above a few million volts. The ionization need not be directly measured if one can observe the energy loss in penetration through a given thickness of material. Extensive measurements by a number of observers agree on a value of about 32 ev as the average energy expended in the production of an ion pair. This value shows very little variation with speed of the ionizing particle, and one is probably justified in applying it to all cosmic-ray particles.

The C. T. R. Wilson cloud chamber can be used to measure both the primary and the probable ionization along the tracks of ionizing particles. When the particle passes through the chamber just after the expansion, condensation will take place on the ions before they can diffuse appreciably from their points of formation. A primary ionization in which the ejected electron has sufficient energy to produce a dozen secondary ions will appear as only one large blob along the primary track. A count of the number of blobs gives one directly the specific primary ionization. If the measurements are made in a magnetic field the primary ionization may be measured as a function of the energy of the particles. If the ionizing particle passes through the chamber a short time before the expansion, the ions will spread by diffusion so that after expansion each ion will be the nucleus of a drop of liquid which can be resolved and counted in most cases. With a time interval of 0.2 sec., blobs of 250 ions can be counted. This represents a secondary of about 10^4 ev. The measurement of the radius of curvature in a magnetic field is somewhat impaired by the diffusion of the tracks.

THE MEAN VALUE OF THE SPECIFIC IONIZATION

In the measurement of the average value of the specific ionization it is necessary to restrict the velocity of the particle to values that are between about 10^5 and 10^{10} ev. At lower velocities the specific ionization rises very rapidly and would cause considerable changes in the average values if they were included. The scarcity of very high energy particles makes the upper limit unimportant.

The value for the cosmic-ray ionization at sea level in ions per cc per sec. in air at one atmosphere and 0°C has been determined as follows: Clay,[9] 1.63; Compton, Wollan and Bennett,[10] 1.22; and Millikan,[11] 2.48. The number of cosmic-ray particles or showers per sq. cm per sec. has been determined by Cosyns[12] as 0.0266 and by Froman and Stearns[13] as 0.0303, and by Street and Woodward[14] as 0.0247. It is necessary to correct these values for the probable number of shower particles accompanying each cosmic-ray particle. This shower correction factor has been estimated as about 1.10. A probable value of the number of particles passing through one sq. cm per sec. is 0.028. Combined with Millikan's data this gives $N=90$. With Compton, Wollan and Bennett's observations the value of N is about 50. From Clay's data the value of N is about 65. Because of the difference in the shielding of the ionization chambers these results are not strictly comparable without further corrections.

Evans and Neher[15] observed the fluctuation in the ionization current and from this estimated N as about 65. Swann's[16] observations on the pulse of ionization in a long ionization chamber give a value of $N=34$. Stuhlinger[17] has observed $N=50$ for shower particles by measurement with a linear amplifier, and $N=35$ for penetrating particles. These measurements indicate a probable value of N between 50 and 60 for the mean of all cosmic-ray particles passing through an ionization chamber.

The early observations on ionization by moving charges indicated that the specific ionization was a function of the velocity of the ionizing particle. For high velocities the variation was shown to follow the Bohr theory, i.e., proportional to $1/\beta^2$. The observations of W. Wilson[18] combined with those of S. Bloch[19] not only confirmed roughly the variation with $1/\beta^2$ from $\beta=0.45$ to $\beta=0.96$ but also gave the limiting value of the specific ionization as about 45 ions per cm of path in air.

C. T. R. Wilson[20] was able to count the number of primary ionizations along sharp tracks. The range of the electrons enabled him to estimate the energy of the particles. The extension of the measurements of primary ionization to cloud chambers with magnetic fields by Williams and Terroux[21] gave the primary ionization as approximately equal to $22\beta^{-1.4}$. Skramstad and Loughridge[22] found for electrons between $\beta=0.89$ and $\beta=0.98$ a value of $19\beta^{-1.15}$. If one multiplies these primary ionization values by the probable number of ions in each cluster, the probable ionization can be found. Multiplying each ionization by factors of 2 and 2.4, respectively, brings these points, except for the highest energy point for each observer, into good agreement with the observed probable ionization by Corson and Brode.[8] From 3600 drops counted along diffuse tracks in argon, Locher[23] has estimated the mean specific ionization in air at 76.0 cm as 68.7 ion pairs per cm at 0°C. The energy of these tracks was unknown.

VARIATION OF SPECIFIC IONIZATION WITH $H\rho$

In none of the work described has the energy of the particles been carried high enough to test conclusively the theory of ionization by high speed electrons which predicts an increase in the ionization above 2 Mev. Kunze[24] found that the

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[9] J. Clay, Rev. Mod. Phys. **11**, 123 (1939).

[10] Compton, Wollan and Bennett, Rev. Sci. Inst. **5**, 415 (1934).

[11] R. A. Millikan, Phys. Rev. **39**, 397 (1931).

[12] M. Cosyns, Bull. Tech. Ass. Ing. Brux. 173–265 (1936).

[13] Froman and Stearns, Can. J. Research **16**, 29 (1938).

[14] Street and Woodward, Phys. Rev. **46**, 1029 (1934).

[15] Evans and Neher, Phys. Rev. **45**, 144 (1934).

[16] W. F. G. Swann, J. Frank. Inst. **217**, 79 (1934). (See also note in T. Johnson, Rev. Mod. Phys. **10**, 211 (1938).)

[17] E. Stuhlinger, Zeits. f. Physik **108**, 444 (1937).

[18] W. Wilson, Prov. Roy. Soc. **85**, 204 (1911).

[19] S. Bloch, Ann. d. Physik **38**, 559 (1912).

[20] C. T. R. Wilson, Proc. Roy. Soc. **104**, 1 and 192 (1923).

[21] Williams and Terroux, Proc. Roy. Soc. **126**, 289 (1930).

[22] Skramstad and Loughridge, Phys. Rev. **50**, 677 (1936).

[23] G. L. Locher, J. Frank. Inst. **224**, 555 (1937).

[24] P. Kunze, Zeits. f. Physik **83**, 1 (1933).

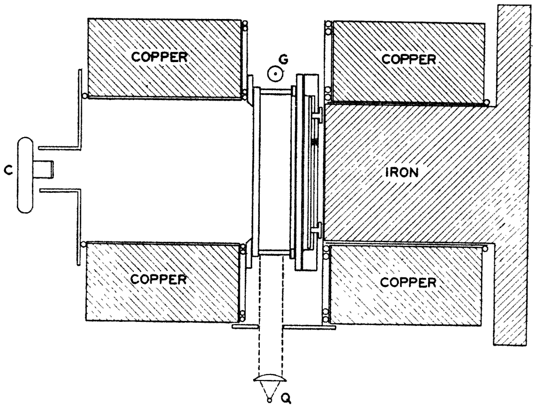

FIG. 2. Schematic diagram of magnet showing the cloud chamber with Geiger counter (G) light source (Q) and camera (C). In the experimental arrangement used there were two Geiger counters one on top and one below the chamber and the lights were at the sides of the chamber.

primary ionization of ten particles with energy about 10^9 ev was about 19 ions per cm. The mean value of the probable ionization observed by Anderson[25] by drop counts on 26 diffuse tracks is 31 ion pairs per cm at S.T.P. The energies of these tracks were all probably above 10^7 ev. The failure of these observers to confirm the rise in ionization for particles of over 2×10^6 ev led to some speculations[6, 26] as to the inadequacy of the theory of ionization by high speed particles.

A study of the specific ionization of particles between 0.1 and 30 Mev was made by Corson and Brode.[8] The energy of the particles was found by measuring the deflection of particles in a magnetic field. The number of ions per cm was found by counting the drops in the Wilson cloud-chamber track. The most accurate curvature measurements can be made on tracks that are very sharp. However, these tracks cannot be used to measure the probable ionization as the drops are not separated in the photograph of the track. In cases where the ionizing particle enters the chamber just after the expansion, the track is sharp enough to count the primary ionization. Such late tracks are rare and seldom lie in the illuminated plane of the chamber so that any length of the track can be counted.

Using a pair of coincidence Geiger counters, one above and one below the cloud chamber, one obtains on nearly every picture a track that passes the whole length of the chamber in the illuminated region. There is a small delay between the arrival of the cosmic ray and the completion of the expansion, about 10^{-2} sec. In this time the ions of the track diffuse and broaden the track so that it is not possible to make primary ionization counts on the track. A delay of 0.2 sec. between the arrival of the cosmic ray and the expansion permits the ions to diffuse so that they are readily countable. This amount of diffusion is, however, not sufficient to disturb greatly the measurement of curvature.

Most of the curvature measurements were made on tracks in an 18-cm chamber in a field of 800 gauss. More recent measurements have been in the 30-cm chamber in a field of 2300 gauss shown in Fig. 2. The curvature of the tracks was measured on a comparator with a micrometer eyepiece. The eyepiece had two parallel hairs which were set so that the center of the track appeared to be midway between them. This measurement was repeated at 15 to 20 positions along the track. A plot of the displacement of the track on a much larger scale than the position along the length of the track should be a section of an ellipse as can be seen in Fig. 3.

As the magnet was inadequately cooled, every other picture was taken without the magnetic field (residual field, 12 gauss). These no field pictures were useful as checks on turbulence. Measurements on these tracks in the 30 cm chamber indicated that curvatures of 40 meters radius and in some cases even more than this could be reliably measured. In the smaller chamber the limit of reliable curvature measurement was about 20 meters radius.

Since the illuminated region was only two cm in depth, it was not necessary to take stereoscopic pictures when measuring tracks extending through the full 30 cm of the chamber. A single Leica camera with a 5.0-cm Elmar lens was used in taking the photographs. The size of image on the film to the track in the chamber was in ratio of 1 to 7.5. The diameter of the circle of confusion for this lens is given by the manufacturer as 0.033 mm or 0.25 mm in the chamber. The apparent diameter of the drops in the chamber was from 0.2 to 0.4 mm depending on

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[25] C. D. Anderson, Phys. Rev. **50**, 263 (1936).
[26] E. J. Williams, Science Progress, No. 121, July, 1936.

the illumination and development. From Stokes law and the rate of fall of these drops in the chamber their diameter is calculated to be about 0.04 mm. Although the tracks are diffuse and about 5 mm in apparent width in the chamber, it was quite possible to estimate the center to 0.1 of a mm. On the film this corresponds to a measurement of 0.013 mm in 0.67.

A curvature of 40 meters in a track 20 cm long corresponds to displacement of the center of about 0.15 mm from the straight line joining the ends of the track. The imperfections of the lens and the fact that the photograph must be taken through a $\frac{3}{8}$-inch plate glass will introduce some fictitious curvatures.[27] Photographs of fine tungsten wires stretched in the plane of focus behind the glass plate were measured for distortion of the image.[26] At a distance of 10 mm from the center of the film a positive curvature of 3.65×10^{-3} cm^{-1} was observed. This corresponds to an apparent curvature of 20 meters radius in the chamber. The curvature correction increased approximately linearly from the center of the film to the edge.

The adequate illumination of a cloud chamber is a problem that has not as yet been satisfactorily solved. For sharp tracks the problem is not so difficult. The drops are concentrated in a much smaller space and cooperate in scattering so that even when a single drop does not scatter enough light, the concentrated column makes a track that can easily be photographed. Increasing the camera aperture to get more light from single drops decreases the depth of focus. At an aperture of *f*: 3.5 the depth of focus was only a little over a centimeter. Larger apertures would limit the region of sharp focus to even a thinner layer. The duration of the illumination is also limited by the action of gravity on the drops. Falling with a velocity of 1 cm per sec. the drops would move the diameter of their apparent image size in the chamber in 0.01 sec. A further increase in time of illumination would not produce brighter images but only longer streaks of light. Incandescent lamps that prove adequate for sharp track photography were not bright enough nor short enough in duration. Quartz capillary arcs operated for one-quarter of a 60 cycle on a system similar to that described by Blackett[28] have been fairly satisfactory.

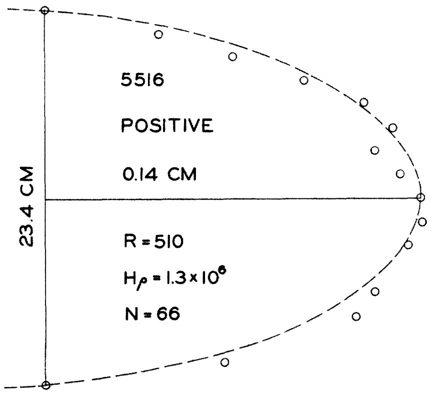

FIG. 3. Plot of the displacement of the center of a diffuse track from a straight line as a function of the distance along the track.

The number of drops per cm in the track may be counted if the ions are allowed to diffuse for some time before the expansion. The time intervals used in photographing most of the diffused tracks were as follows: cosmic ray arrived at $t=0.0$ sec.; clearing field shorted, $t=0.01$ sec.; expansion at 0.20 sec.; lights on at 0.245 sec.; and lights out at 0.250 sec. The delay of 0.20 sec. in the expansion makes it possible to count blobs of ionization up to 250 particles. Larger blobs are not frequently found and any further increase in the time delay would make the measurement of curvature less reliable. Typical tracks are shown in Fig. 4.

Because of the small depth of focus it is not difficult to distinguish ions of the general background that lie in the same region as the track but not in the same plane of focus. If the background density is small compared with the density of drops, one can easily count the number of drops in a region equal in size and adjacent to the track. The density of background drops obtained in this way is subtracted from the density of drops in the region of the track.

The background density can be reduced by reducing the expansion ratio. With nitrogen in the chamber and a vapor in equilibrium with a solution, two parts ethyl alcohol and one part water, the expansion ratio at which appreciable

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[27] Blackett and Brode, Proc. Roy. Soc. **154**, 580 (1936); P. M. S. Blackett, Proc. Roy. Soc. **159**, 1 (1937).

[28] P. M. S. Blackett, Proc. Roy. Soc. **146**, 288 (1934).

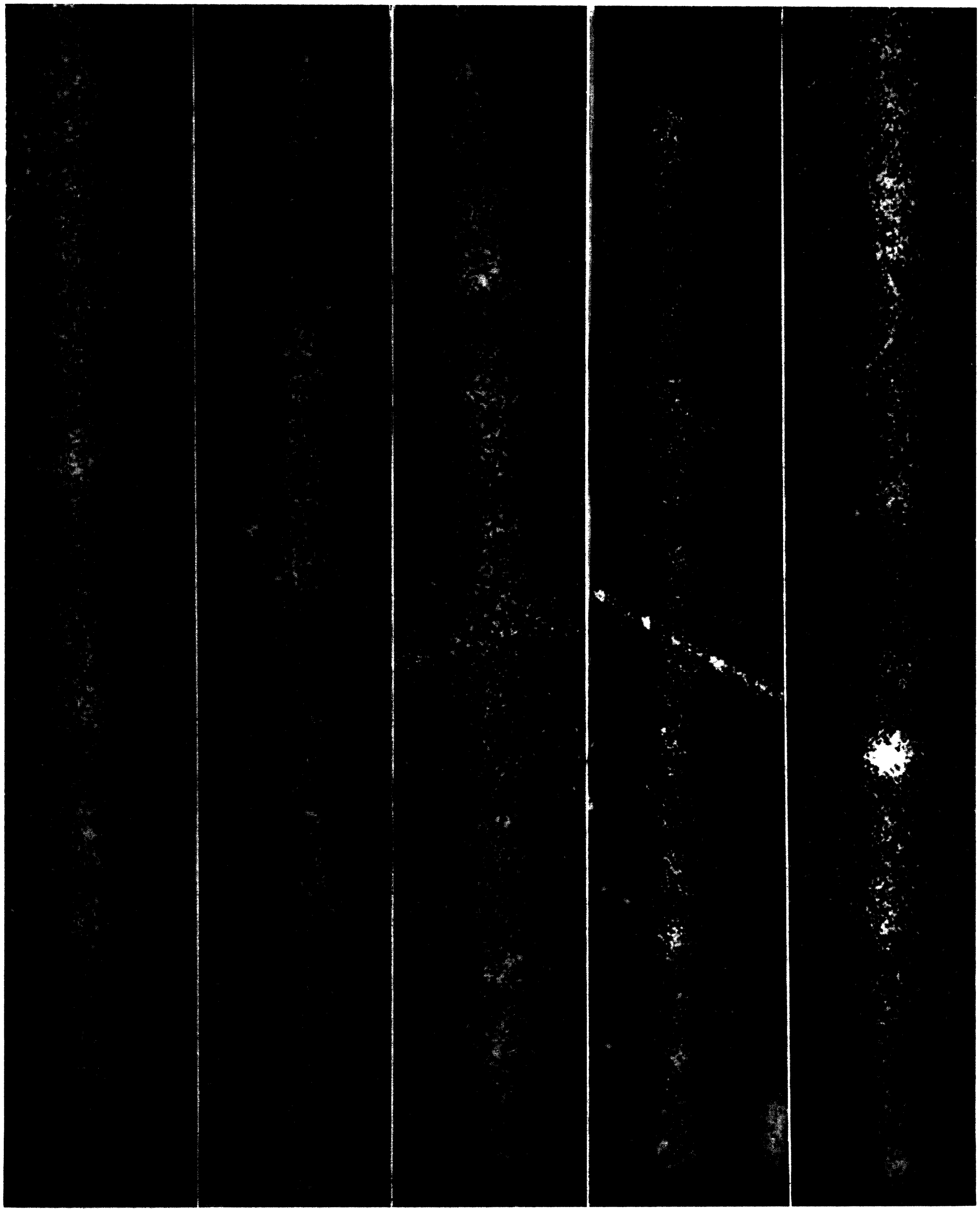

FIG. 4. Typical tracks of cosmic-ray particles diffused by a delay of 0.2 second between the arrival of the cosmic ray and the expansion of the chamber. These pictures are reproduced at double the size as seen in the chamber.

general fog began to form was at 12.5 percent. This value was variable with temperature and with the presence of impurities in the chamber. By connecting the clearing field from one side of the chamber to the other, it was possible to observe the condensation on positive and negative ions. At about 11.3 percent the negative drops are no longer able to form nuclei for condensation although the number of positive nuclei is not appreciably reduced. When the nitrogen is free from oxygen the electrons do not readily attach and the mobility of the negative charges is very high. The electrons move rapidly some distance from the initial track before they attach. In a clearing field between the front glass of the chamber and the rear metal screens, the negatives of high mobility would be drawn out of the illuminated region of the chamber. At 11.5 percent the negative track and the general background have nearly disappeared although the number of drops in the positive track was still between 45 and 60. Changing the clearing field alternately from sidewise to front to back gave the same average count in the positive track of the sidewise field pictures as in the single track seen in the front to back pictures. Under these experimental conditions the total drop count was interpreted as identical with the number of pairs of ions.

With this correction the observations of Corson and Brode[8] are brought into good agreement with the mean values obtained by the other methods. The ratio of probable to primary ionization is therefore about 2.2. The adjustment of the theoretical curve for the probable ionization leads one to expect an approximate increase of about 15 ions per cm in air for each increase of a power of 10 in the energy of the electrons. Electrons of an energy of 10^{10} ev should ionize about twice as many atoms per cm of path as electrons of 10^6 ev.

Particles with an $H\rho$ above 10^6 gauss cm are quite frequent in the cosmic-ray spectrum, but the measurements of Kunze on primary ionization and of Anderson on diffuse tracks indicate very low values of the probable ionization. Anderson's value of 31 should probably be changed to 62, as the result of the correction mentioned in the paragraph above. The mean ionization must include some particles with $H\rho$ between 10^6 and 10^8, but the mean value of about 60 indicates that there are not many particles with the predicted electron ionization in this region.

The hypothesis that most of the particles with an $H\rho$ above 10^7 are not electrons but are mesotrons brings most of the cosmic-ray ionization data into good agreement. Neddermeyer and Anderson's[29] observations on energy loss have indicated that a large proportion of the particles above $H\rho=10^6$ are mesotrons. From the study of the scattering and energy losses of high speed particles, Blackett and Wilson[30] have concluded that most of the particles with $H\rho$ greater than 10^6 are not electronic in character. If the failure of a particle to produce any secondary electrons in passing through 3 cm of lead be taken as an indication of a mesotron, the observations of Starr[31] would indicate that 90 percent of the particles that tripped the pair of Geiger counters and penetrated the lead block are mesotrons. The fairly well-established theory for the growth of showers predicts that, if these particles were electrons they would in nearly every case emerge from the three-centimeter lead block accompanied by secondaries.

High energy electrons are characterized by their rapid loss of energy in the form of radiation. The calculations of Heitler[32] indicate that electrons of energies above $10^6 H\rho$ will rapidly lose their energy by radiation. The loss of energy by radiation for particles with rest masses greater than the rest mass of an electron will be less than the rest mass of an electron with the same velocity. The ratio of the energy losses is approximately inversely as the square of the ratios of the rest masses. The short range of high $H\rho$ electrons due to the loss of energy by radiation makes it very improbable that any appreciable number of such primary particles would reach the earth's surface with energies in excess of 10^8 ev. The rapid degradation of the primary electrons of 10^{10} ev in the first meter of water equivalent of the atmosphere has been clearly

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[29] Neddermeyer and Anderson, Phys. Rev. **51**, 885 (1937).

[30] Blackett and Wilson, Proc. Roy. Soc. **165**, 209 (1938).

[31] M. A. Starr, Phys. Rev. **53**, 6 (1938).

[32] W. Heitler, *The Quantum Theory of Radiation* (Oxford Univ. Press), Chapter 5.

shown by the measurements of Bowen, Millikan and Neher.[33]

Below an $H\rho$ of 10^5 (50×10^6 ev for electrons) there is no ambiguity as to the nature of a particle whose $H\rho$ and ionization are measurable. In this region the specific ionization of any singly charged particle with a rest mass between 100 and 700 times that of an electron will be much greater than the specific ionization of an electron. In the region above an $H\rho$ of 5×10^6 the measurement of ionization and $H\rho$ is not sufficient to determine the character of the ionizing particle. In this region the ionization of the electron rises and that of the mesotron decreases as the $H\rho$ of the particle is increased. In the ambiguous region between 5×10^6 and $10^6H\rho$ it appears possible to determine some of the properties of the particles by combined electric and magnetic deflection. Experiments of this character are now in progress at the University of California in Berkeley. Above an $H\rho$ of 10^6, low values of the ionization may be considered as evidence favoring a rest mass larger than that of the electron. At $H\rho=10^6$, a value of about 80 ions per cm in air might be expected for electrons and about 50 to 60 for mesotrons. A survey of the ionization in this region is in progress. Counts on a number of tracks in this region have varied from 58 to 63. This value is much lower than the predicted electron value.

A consistent picture of the sea-level cosmic radiation is obtained if one assumes that all of the particles in the $H\rho$ distribution below about 10^5 are electrons, between 10^5 and 10^6 both electrons and mesotrons are found, while above 10^6 most of the particles are mesotrons. The observed $H\rho$ spectrum would in this case be the sum of the electron and mesotron distributions. The failure of previous observers to find a larger specific ionization for particles of about $H\rho=10^7$ compared with the specific ionization of particles of about $H\rho=10^4$ is thus explained. Mesotrons of $10^7H\rho$ do not ionize appreciably more than electrons of $10^4H\rho$. The small value for the specific ionization above $H\rho=10^6$ is evidence in favor of the predominance of mesotrons among the cosmic-ray particles with high $H\rho$.

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[33] Bowen, Millikan and Neher, Phys. Rev. **53**, 217 (1938).

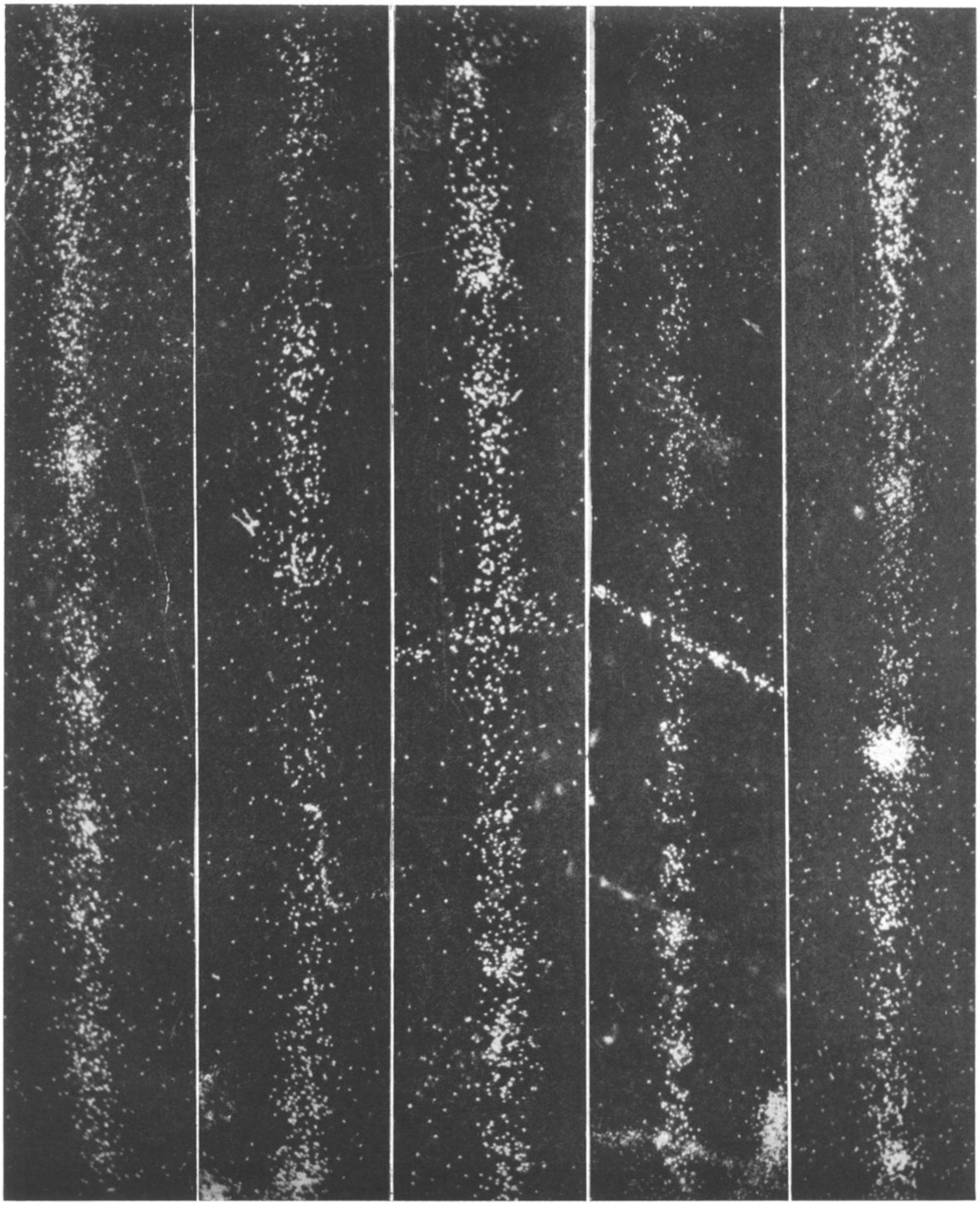

FIG. 4. Typical tracks of cosmic-ray particles diffused by a delay of 0.2 second between the arrival of the cosmic ray and the expansion of the chamber. These pictures are reproduced at double the size as seen in the chamber.