A Theory of World-Wide Periodic Variations of the Intensity of Cosmic Radiation

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INTRODUCTION

THE existence of small world-wide variations of the intensity of cosmic radiation is well known. These variations are of two kinds: periodic and nonperiodic. Notable examples of the former are:

(a) The diurnal variation depending on solar time, studied experimentally by Hess and his collaborators,[1] the world-wide character of which was established by Thompson[2] and Forbush.[3]

(b) The diurnal variation depending on sidereal time. Fundamental problems of the theory of cosmic radiation are bound up with this effect, which has been recently discussed elsewhere[4] and with which we shall not be concerned further here.

(c) The world-wide seasonal variation studied by Hess,[5] Compton and Turner,[6] Gill,[7] Forbush[8] and more recently by Millikan and Neher.[9]

(d) The variation of intensity with period of 27 days discussed by Hess and his collaborators,[1] by Graziadei,[10] by Gill,[7] and by Kolhörster.[10]

Of the nonperiodic effects the most important seems to be that associated with magnetic storms, the world-wide nature of which has been strikingly brought out by Forbush.[11]

In this paper we propose to develop certain theoretical considerations bearing on the three periodic effects mentioned above, with particular reference to the problem of the sun's permanent magnetic field.[12] The treatment of the magnetic storm effect will be reserved for another occasion.[13]

THE SUN'S PERMANENT MAGNETIC FIELD AND COSMIC RAYS

Hale's painstaking studies of the Zeeman effect of solar spectral lines[14] led to the probable existence of a permanent magnetic field at the surface of the sun of strength (at the equator) of between 10 and 30 gauss. If the assumption is made that this field has a predominant dipole component, the difficulty arises that apparently the field decreases with distance faster than with the inverse cube of the distance from the dipole. The observations at different levels, however, are so difficult that no final conclusions seem to be justified, and none indeed have been drawn by a great many astrophysicists. If, however, a predominant dipole component of the field exists, then, as already pointed out,[12] a number of observable consequences should arise, in particular periodic variations of the intensity of cosmic rays of the type mentioned in the introduction. Preliminary calculations led to certain disagreements with experiment which have already been pointed out.[15] Thus, while the calculated and observed magnitude and phase of the solar diurnal variation agreed at high latitudes, both disagreed at low latitudes; and for the seasonal variation, while the amplitudes agreed, the phase was just reversed for the Northern Hemisphere. Further, the observed close relation between magnetic storms, on the one hand, and sunspot activity, on the other, remained completely dark. It will be shown in

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1 V. F. Hess, H. T. Graziadei and R. Steinmaurer, Sitz. Ber. Akad. Wiss. Wien **143**, 313 (1934); V. F. Hess, A. Demmelmair and R. Steinmaurer, *ibid.* **147**, 89 (1938).

2 J. L. Thompson, Phys. Rev. **54**, 93 (1938).

3 S. E. Forbush, Terr. Mag. and Atmos. Elec. **42**, 1 (1937).

4 A. H. Compton and I. A. Getting, Phys. Rev. **47**, 817 (1935); W. F. G. Swann, Phys. Rev. **51**, 718 (1937); M. S. Vallarta, C. Graef and S. Kusaka, Phys. Rev. **55**, 1 (1939); A. H. Compton, J. Frank. Inst. **227**, 607 (1939); M. S. Vallarta and R. P. Feynman, Phys. Rev. **55**, 506 (1939).

5 V. F. Hess, Sitz. Ber. Akad. Wiss. Wien **144**, 53 (1935).

6 A. H. Compton and R. N. Turner, Phys. Rev. **52**, 799 (1937).

7 P. S. Gill, Phys. Rev. **55**, 429 (1939).

8 S. E. Forbush, Phys. Rev. **54**, 975 (1938).

9 R. A. Millikan and H. V. Neher, Science **89**, 398 (1939).

10 H. T. Graziadei, Sitz. Ber. Akad. Wiss. Wien **145**, 495 (1936). W. Kolhörster, Physik. Zeits. **40**, 107 (1939).

11 S. E. Forbush, Phys. Rev. **51**, 1108 (1937); also reference 8.

12 M. S. Vallarta, Nature **139**, 839 (1937); L. Jánossy, Zeits. f. Physik **104**, 430 (1937); M. S. Vallarta, J. Frank. Inst. **227**, 28 (1939); P. S. Epstein, Phys. Rev. **53**, 862 (1938).

13 See preliminary note by O. Godart, Phys. Rev. **55**, 875 (1939).

14 G. E. Hale, F. H. Seares, A van Maanen and F. Ellerman, Astrophys. J. **47**, 206 (1918).

15 See in particular, M. S. Vallarta, J. Frank. Inst., reference 12.

the sequel that all these objections disappear in a more complete theory, so that the observed periodic variations of the intensity of cosmic radiation may be taken as an argument for the existence of the sun's permanent magnetic dipole field, and the correlation just mentioned is not an argument against it.

The fact that the intensity of cosmic rays remains constant beyond a certain geomagnetic latitude (about 50°) independently of altitude[16] may be interpreted to mean that the sun's field is capable of keeping all primary rays of energy below the limit 2×10^9 ev (for electrons or protons) away from the earth, and this leads immediately to the value 10^{34} gauss-cm^3 for the sun's dipole moment, which again gives an equatorial magnetic field at the sun's surface of about 30 gauss, in good agreement with Hale's measurements.

If there is a permanent dipole moment of the sun, its influence on the intensity of cosmic radiation observed on the earth will be of two different kinds: directly, insofar as the solar field affects the motion of charged primaries, and indirectly, because of the influence of the solar field on terrestrial magnetism and related phenomena. We consider now the former. In the *first* approximation the earth can be considered as a point in the solar field, surrounded by a "sphere of action" of the earth's magnetic field, of radius approximately 250,000 km. Within this sphere of action, the earth's magnetic field is the controlling factor, outside of it the sun's field determines the motion of primary cosmic particles. With respect to the sun there is then a Störmer cone of allowed directions[17] which is the only one which has to be considered, because there is no shadow effect.[18] This is a right circular cone, with axis perpendicular to the plane containing the sun's dipole. The solid angle of this cone, with respect to the sun, is given by $2\pi(1+\sin\theta_s)$ where $\sin\theta_s$ is given by Störmer's classic formula:

$$\pm\sin\theta_s=\frac{2}{r_s\cos\lambda_s}-\frac{\cos\lambda_s}{r_s^2},$$

where λ_s is the latitude of the earth with respect to the solar magnetic equatorial plane. The plus sign refers to positive particles and the minus to negative. The relation between r_s and the energy in störmers, r_e,[19] is given by the formula

$$r_s=(M_e/M_s)^{\frac{1}{2}}(D/R)r_e,$$

where M_e and M_s are the magnetic dipole moments of the earth and sun, respectively; and D, R are the distance from the earth to the sun, and the radius of the earth, respectively.

In the *second* approximation we consider the earth to be a solid sphere with no magnetic field. The solar Störmer cone is then cut by the horizontal plane at a given point of the earth P and all directions within the earth become forbidden. We take the origin of a polar coordinate system at the center of the earth, with its axis parallel to the neglected earth's dipole, and the longitude counted from a plane containing the sun and a line through the given point on the earth P and parallel to the axis of the coordinate system. Thus the longitude is approximately

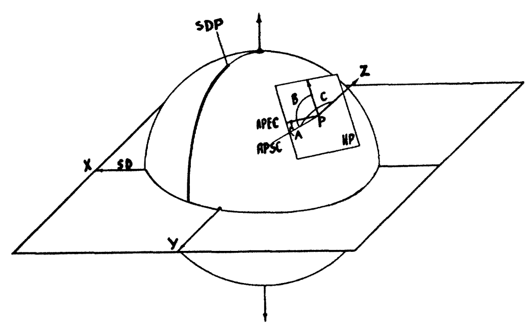

FIG. 1. Coordinate system and the angles A, B, C.

measured by the solar time. Through the center of the earth we draw a line parallel to the sun's dipole, the direction of which is defined by the angles u, v. The latter have in general three periods, corresponding to the earth's rotation around its axis (24 hours), the sun's rotation (27 days) and the rotation of the earth around the sun (365 days). Let $\cos A$, $\cos B$, $\cos C$ (Fig. 1) be the direction cosines of the axis of the solar cone with respect to the east-west line at the point P, the north-south line and the zenith

[16] A. H. Compton, Phys. Rev. **43**, 398 (1933); M. Cosyns, Nature **134**, 616 (1936); H. Carmichael and E. G. Dymond, Nature **141**, 910 (1938); I. Bowen, R. A. Millikan and H. V. Neher, Phys. Rev. **53**, 855 (1938); T. H. Johnson, Phys. Rev. **54**, 151 (1938).

[17] C. Störmer, Pub. Univ. Obs. Oslo, No. 10, 1934.

[18] E. J. Schremp, Phys. Rev. **54**, 154 (1938).

[19] G. Lemaître and M. S. Vallarta, Phys. Rev. **49**, 720 (1936).

direction, respectively. Then we have

$$\cos A = \sin \delta \sin (\varphi - \varphi_0)$$
$$\cos B = -\cos \lambda \cos \delta + \sin \lambda \sin \delta \cos (\varphi - \varphi_0)$$
$$\cos C = -\sin \lambda \cos \delta - \cos \lambda \sin \delta \cos (\varphi - \varphi_0),$$

where λ and φ are the latitude and longitude of P, δ is defined by

$$\cos \delta = \cos \lambda_e \cos w$$

and

$$\tan \varphi_0 = \tan w / \sin \lambda_e.$$

In these formulas λ_e is the latitude of the sun in the polar coordinate system defined above and w is given by

$$\tan w = (\tan u / \sin v) \cos \lambda_e - \cot v \sin \lambda_e.$$

The fraction of a unit hemisphere at P included within the solar Störmer cone at P is then given by

$$\frac{1}{\pi}\left[\cos^{-1}\left(\frac{\cos C}{\cos \theta_s}\right) + \sin \theta_s\left(\cos^{-1}\left(-\frac{\tan \theta_s}{\tan C}\right)\right)\right]$$

with

$$\theta_s + \pi/2 \geq \pi/2 - C \geq -(\theta_s + \pi/2)$$

and for $|C| < |\theta_s|$ but $\cos C < 0$ then the fraction is $1 + \sin \theta_s|$; while for $C| < |\theta_s|$ but $\cos C > 0$ the fraction vanishes.

In the *third* approximation the earth is considered together with its own magnetic field. Then the east-west line is the axis of the terrestrial allowed cone,[20] and A in the previous formulas is the angle between the axis of the solar and the terrestrial cones. The longitude φ is now no longer measured by the solar time, but account must be taken of the deflection of the particle by the earth's magnetic field within the sphere of action defined above. This deflection is given by

$$\Delta \varphi = \int_{r_0}^{40 r_0} \left(\frac{2\gamma_1}{r^2 \cos^2 \lambda} - \frac{1}{r^3}\right) d\sigma,$$

where for purposes of actual calculation σ and λ are expressed as functions of r.[21] This integral has been actually calculated only for equatorial orbits,[22] in which case φ can be expressed either as a function of γ_1 and r_e, or θ_e and r_e, or θ and γ_1. For orbits not contained in the equatorial plane the above integral cannot be calculated analytically, but must be found by numerical or mechanical integration. We may note, however, that the definition of θ_e and γ_1 does not depend on latitude, so that as a first approximation we may assume the deflection for given γ_1 and θ_e is the same for all latitudes. But the energy belonging to a latitude λ is given by

$$\frac{1}{r_e'} = \frac{1}{\cos^2 \lambda}[\gamma_1 \pm (\gamma_1^2 - \cos^3 \lambda \sin \theta_e)^{\frac{1}{2}}],$$

which may be interpreted in the sense that the deflection at a given latitude of a particle of given energy corresponds to the equatorial deflection of a particle of different energy. Not all values of θ_e are always allowed because of the existence of the solar cone. It remains now to find the relation between the angles θ_s and θ_e. An easy calculation gives

$$\sin \theta_s = \sin \theta_e \sin \delta \sin (\varphi - \varphi_0) - \sin \delta \cos \theta_e \cos (\lambda - \eta) \cos (\varphi - \varphi_0) - \cos \delta \cos \theta_e \sin (\lambda - \eta)$$

where η has the same significance as in previous papers on the theory of the allowed cone.[21] As δ depends slightly on the position of the sun relative to the earth, θ_s depends on the time of the day and on the season of the year and therefore the phase shift $\Delta\varphi$ depends on t in the same way. In our first approximation θ_s does not depend on η, and we may take the two dipoles parallel, that is to say, $\theta_s = \theta_e - \varphi$. The mean phase shift $\langle \Delta\varphi \rangle_{\text{Av}}$ for different times of the day given by φ and for energies $r_e = r_s/2.09$, is expressed by (Fig. 2)

$$\langle \Delta \varphi \rangle_{\text{Av}} = \frac{\int_{\varphi_1}^{\varphi_2} \Delta \varphi \cos (\theta_e - \varphi) d\theta_e}{1 + \sin (\theta_e - \varphi) \Big|_{\varphi_1}^{\varphi_2}},$$

[20] For a review of the theory of the allowed cone, see M. S. Vallarta *An Outline of the Theory of the Allowed Cone of Cosmic Radiation* (Univ. of Toronto Press, 1938) or (with less detail) J. Frank. Inst. **227**, 1 (1939).

[21] For the significance of the symbols used see G. Lemaître and M. S. Vallarta, Phys. Rev. **49**, 719 (1936); or M. S. Vallarta, reference 20.

[22] C. Graef and S. Kusaka, J. Math. Phys. **17**, 43 (1938); M. S. Vallarta, C. Graef and S. Kusaka, Phys. Rev. **55**, 1 (1939).

where $\varphi_1=\varphi-\pi/2$, $\varphi_2=\varphi+\sin^{-1}(2/r_s-1/r_s^2)$. Thus we may calculate the average phase shift for each energy and each time of the day.

Before proceeding with the calculation of the different world-wide periodic changes of intensity mentioned in the introduction, we note that all phenomena directly attributable to changes of position of the solar dipole relative to the earth can be felt only at high latitudes, because the energies for which the solar Störmer cone may vary appreciably are between 0.2 and 0.35 störmer and therefore particles of such energy cannot arrive at the earth at latitudes below roughly 40°. The origin of seasonal and diurnal variations at low latitudes will be investigated in later sections.

The 27-Day Period of Intensity Variation

If the solar magnetic dipole does not coincide with the solar axis of rotation, the solar magnetic latitude of the earth must change with a period equal to the period of rotation of the sun, or 27 days. To calculate this variation of latitude we compare the latitude at the instant when the solar dipole makes the smallest and the largest angle with the ecliptic. Calling the angle of rotation u_1 we have for the variation of $\sin\theta_s$,

$$\Delta\sin\theta_s=\frac{2\sin(\lambda_s+u_1)\sin u_1}{r_s}\times\left(\frac{2}{\cos\lambda_s\cos(\lambda_s+2u_1)}+\frac{1}{r_s}\right)\quad (r_s<1)$$

We now calculate the mean amplitude of this variation, choosing the mean values of δ, φ and φ_0 in such a way that the horizontal plane and the terrestrial cone divide in two proportional parts the total energy and the energy cut off on account of the solar rotation, with $\lambda_s=0$. In other words, on the average the solar dipole is perpendicular to the ecliptic. Then the percentage change of the solid angle of the allowed solar cone is

$$-\Delta\Omega=\frac{\sin^2 u_1}{r_s}\left(\frac{2}{\cos 2u_1}+\frac{1}{r_s}\right).$$

The ensuing energy lost is not uniformly distributed over the earth because low energies cannot reach low latitudes. We therefore have

$$\frac{\Delta E}{E}=\frac{\int_{r_e}^{\infty}\Delta r\omega'(r_e,\lambda)dr_e/r_e^{5.6}}{\int_{r_e}^{\infty}\omega'(r_e,\lambda)dr_e/r_c^{5.6}},$$

where $\omega'(r_e,\lambda)$ is the solid angle of the earth's shadow cone,[23] and the energy spectrum of the primaries has been taken as $K/E^{2.8}$. Taking the angle between the earth's dipole and the solar dipole as 6°, we have for the amplitude of the 27-day period the values given in Table I,

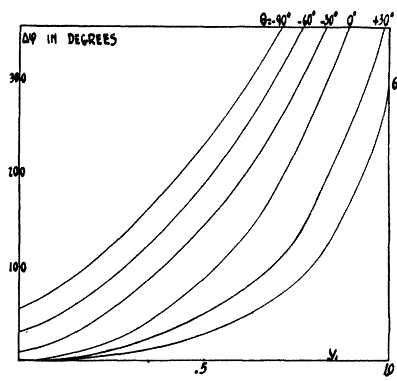

FIG. 2. Magnetic deflection for several values of θ.

which may be compared with Graziadei's value[10] 0.4 on the Hafelekar ($\lambda=50°$ approx.), Kolhörster's value[10] of 0.5 for Berlin ($\lambda=55°$) and Gill's average value of 0.4 over four stations (Cheltenham, Teoloyucan, Huancayo, Christchurch). The agreement is satisfactory. It should be emphasized that the calculated amplitude is quite sensitive to the angle between the two dipoles.

The Seasonal Variation of Intensity

From our present standpoint the seasonal variation arises from three distinct causes, two of which have already been considered by P. S.

TABLE I. *Amplitude of 27-day period.*

λ Geomag.	90°	50°	40°	30°
$\Delta E/E$ (%)	0.7	0.4	0.2	0.05

[23] E. J. Schremp, Phys. Rev. **54**, 158 (1938).

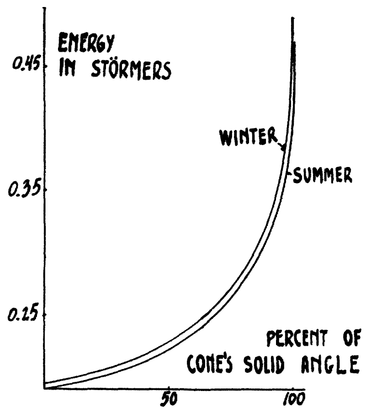

FIG. 3. The distance seasonal effect.

Epstein.[24] The first effect (a) is the periodic change of distance between the earth and the sun (maximum in July, minimum in January) caused by the eccentricity of the earth's orbit, which alone had been considered in our preliminary theory,[12] the second (b) is the yearly change of heliomagnetic latitude of the earth because of the fact that the sun's dipole is not perpendicular to the ecliptic, as already discussed in the previous section, consideration of which had been omitted in our preliminary theory, and the third effect (c) is the change of the geometrical position of the solar cone with respect to a given point on the earth.

Of these effects the first is readily calculated.[25] Bearing in mind (see Fig. 3) that only particles of energy between 0.2 and 0.45 störmer approximately are affected by the sun's field, this effect gives $\Delta E/E = 0.03$ for the polar regions, 0.015 at 50°, 0.003 at 40° and is negligible in the tropical belt. The second effect (b) may be calculated as follows: the axis of rotation of the sun is not exactly perpendicular to the ecliptic[26] but makes with this perpendicular an angle of 7° 10′, with its ascending node in longitude 73° 47′. The mean position of the sun's magnetic dipole over the 27-day period of rotation coincides with the axis of rotation. It is therefore in the earth's meridian plane twice a year in March and September. This means that the mean heliomagnetic latitude of the earth is +7° in September and −7° in March. The corresponding semi-annual change of intensity (Fig. 4) is readily calculated to be 0.004 for the pole, 0.002 at 50° and is negligible elsewhere, in good agreement with Epstein's calculation.

We now proceed to calculate the variation of intensity due to the third effect (c). Since the angle between the zenith direction at a given point of the earth and the solar cone varies with the season, the common region between the earth's shadow cone, and the solar Störmer cone changes with the season, and therefore there must be a seasonal variation of intensity. The mean position of the earth's magnetic axis relative to the ecliptic coincides with the axis of rotation. Hence the angles u, v introduced in a preceding section have the values $u = 90° - 17° \; 42'$, $v = 90° + 7° \; 10' - T$ where T is ecliptic longitude measured from the vernal equinox. The mean value of $\cos C$ is

$$\langle \cos C \rangle_{\mathrm{Av}} = -\sin \lambda \cos \delta,$$

which at the pole reduces to $\langle \cos C \rangle_{\mathrm{Av}} = -\cos \delta = -0.30237$ in spring and $+0.30237$ in autumn. By graphical integration (Fig. 5) we find 0.078 at the pole, 0.040 at 50° and 0.01 at 40°. For positive particles this gives a maximum in spring and a minimum in autumn, and conversely for

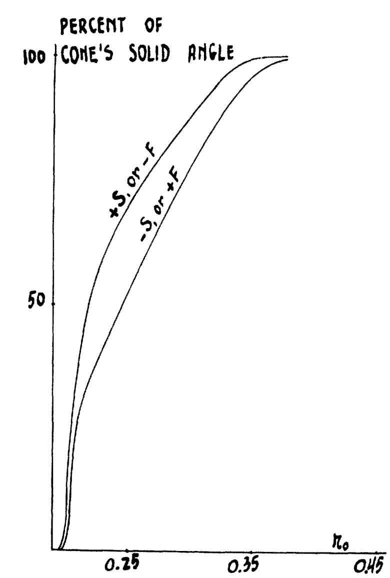

FIG. 4. The semi-annual effect. $+S$, positives in spring; $-F$, negatives in fall, etc.

[24] Cf. P. S. Epstein, Phys. Rev. **53**, 865 (1938).

[25] The method of calculation is that devised by M. S. Vallarta, Phys. Rev. **47**, 647 (1935); but with the substitution of the Störmer cone in place of the main cone.

[26] See, for instance, H. N. Russell's, R. S. Dugan's and J. Q. Stewart's *Astronomy* (Ginn and Co., Boston), p. 192.

negative particles, and this holds for the actual season both in the Northern and Southern Hemispheres. If the ratio of positives to negatives were 1/1 the effect vanishes everywhere. For a 3/1 ratio we obtain 0.039 at the pole, 0.02 at 50°, 0.005 at 40°, with a maximum in spring, and this ratio will be assumed here.

The annual variation of intensity with 12-month period, calculated as sketched above, is given in Table II.

TABLE II. *Variation of intensity, 12-month period.*

Station	Geomagnetic Latitude	Calculated Amplitude
Cheltenham	50°N	0.012
Hafelekar	48°N	0.010
Teoloyucan	30°N	0
Huancayo	1°S	0
Christchurch	48°S	0.010

The comparison with experiment (from Gill's data[7]) for the annual variation with 6-month period is given in Table III.

Thus it is clear that the solar magnetic field does not account fully for the seasonal variation of 12-month period, and that, although a seasonal variation with 6-month period is predicted by the present theory, neither the amplitude nor the phase is in good agreement with experiment. It is thus clear that other influences are at work for the seasonal variation in addition to the direct influence of the sun's magnetic field. The 12-month wave, however, indicates already an excess of positives over negatives affected by the sun's field.

The Diurnal Variation

From our present point of view the diurnal variation of intensity would be due to the change of the common solid angle of the solar cone and the earth's cone, caused by the fact that, while the former is fixed with respect to the sun, the latter moves with the earth while it rotates around its axis. We begin by calculating the angle between the axis of the solar cone and the axis of the earth's cone as a function of the solar time of the day, which is readily done by using the formulas already written, and then we determine graphically the common region (Figs. 6 and 7) between the two cones. The result depends strongly of course on the ratio of positive to negative primaries. A summary is given in Table IV. The results in the second column are in good agreement with Epstein's results.[24]

TABLE III. *Variation of intensity, 6-month period.*

Station	λ (Geom.)	Amplitude Calc.	Amplitude Exp.	Maxima Calc.	Maxima Obs.
Cheltenham	50°N	0.001	0.006[7]	Mar. Sept.	June Dec.[7]
Teoloyucan	30°N	0	0.005[7]	—	Apr. Sept.[7]
Huancayo	1°S	0	0.0018[7]	—	Mar. Aug.[7]
Christchurch	48°S	0.0009	0.0036[7]	Mar. Sept.	Jan. July[7]

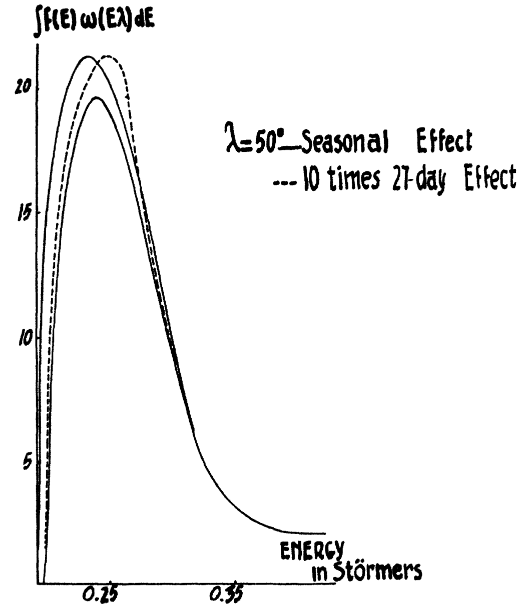

FIG. 5. Illustrating numerical integration of seasonal and 27-day effects.

Our conclusion is that while the direct influence of the sun's magnetic field is amply able to account for both the magnitude and phase of the diurnal variation of intensity at latitudes beyond 40°, additional influences are active in the equatorial belt between 40°N and 40°S. If this

TABLE IV. *Calculated amplitudes of the diurnal variation.*

Geographic Latitude	Ratio of Positive to Negative Primaries: All +	3/1	4/3	Maximum at
90°	0	0	0	—
60°	0.02	0.01	0.0025	15 hr.
50°	0.02	0.01	0.0025	14 hr.
40°	0.015	0.007	0.002	13 hr.
30°	0.00	0	0	—

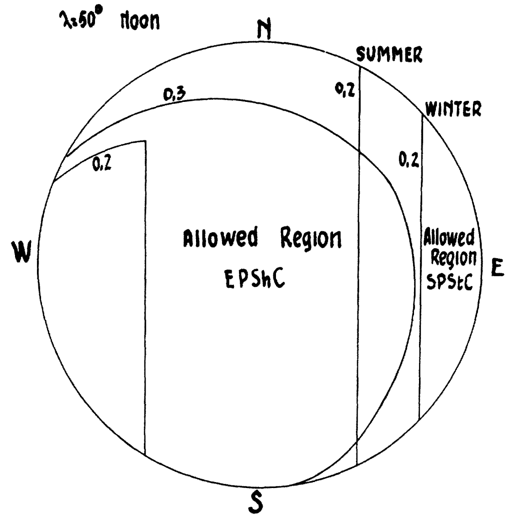

FIG. 6. The solar Störmer cone (*SPStC*) and the earth's shadow cone (*EPShC*). Positive particles.

conclusion is justified, then an additional inference is that the ratio of positives to negatives in the energy interval between 0.2 and 0.35 störmer is more likely 4/3 than 3/1, for from Table IV it is seen that the latter ratio would give at high latitudes a diurnal variation about 10 times greater than is observed.

The Magnetic Field of the Earth and Cosmic Rays

A number of observers have detected a close correlation between the horizontal component of the earth's magnetic field and the intensity of cosmic rays. According to Johnson[27] the proportionality factor $(\Delta I/I)/(\Delta H/H)$ is 15 in the case of magnetic storms, -22 in the case of the seasonal variation, -4 in the case of the diurnal variation. For the secular variation Forbush[8] finds the value 15 for this factor. That there is a causal connection between field and intensity is indicated; the fact that the value and the sign of this ratio is different for the various effects considered points out that the mechanisms involved in each case may be quite different. As far as cosmic rays are concerned, we have to distinguish two kinds of perturbations: Those at large distances are caused by perturbations of the dipole field (changes of dipole moment, position or orientation); at small distances we may have a magnetic field produced by electric currents either outside the atmosphere (Störmer's ring currents, for example) or within the atmosphere (ionosphere). The small distance perturbations, therefore, are not those of the dipole field. A relation between these short range perturbations and the intensity of cosmic rays has been suggested by many investigators,[28] but apparently never studied quantitatively. In this section we propose to examine the relation between the diurnal variation of the earth's magnetic field and of the intensity of cosmic rays.

The diurnal variation of the earth's magnetic field has been systematically studied by Schuster[29] and by Chapman.[30] They distinguish on the earth two components of the diurnal variation of the magnetic potential, the external E and the internal I and find that $E/I \backsimeq 2.5$. At large distances E goes over into I. To be able to carry through this transformation we have made use of the system of ionospheric currents suggested by

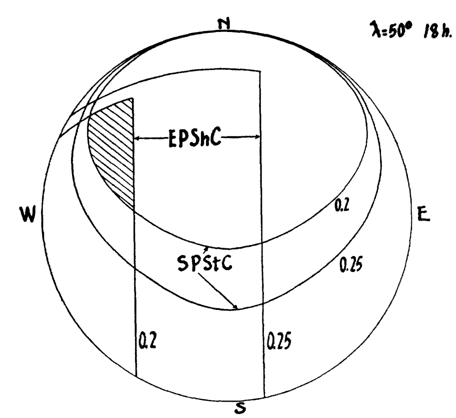

FIG. 7. Common region (dashed) of earth's shadow cone (*EPShC*) and solar Störmer cone (*SPStC*).

Vestine and Chapman,[31] leaving out the currents in the polar caps. Considerations of space oblige us to leave out the details of these calculations. The variation of the total magnetic field at large distances (of the order of a few earth's radii) is negligible (of the order of 10^{-8} gauss), which corroborates Schuster's suggestion that the vari-

[27] T. H. Johnson, Rev. Mod. Phys. **10**, 193 (1938).

[28] R. Gunn, Phys. Rev. **41**, 613 (1932); V. F. Hess and A. Demmelmair, Terr. Mag. and Atmos. Elec. **43**, 7 (1938); S. Chapman, Nature **140**, 423 (1937).

[29] A. Schuster, Phil. Trans. Roy. Soc. **180**, 467 (1889).

[30] S. Chapman, Phil. Trans. Roy. Soc. **218**, 1 (1919).

[31] E. H. Vestine and S. Chapman, Terr. Mag. and Atmos. Elec. **43**, 351 (1938).

ation of the internal magnetic field is mainly induced by the variation of the external field. It seems possible to interpret the small negative residue in the following way: At large distances the earth's magnetism is that of a dipole diminished by the equivalent dipole moment of the external magnetic field. If it is assumed that the sun has a dipole moment roughly parallel to the earth's, the two will repel with a small force, of the order of 10^7 dynes, and a small torque $10^{21} \sin\theta$ dyne-cm. The coupling is therefore small, even negligible. With respect to an observer fixed on the earth, the earth's eccentric equivalent dipole will describe a small circle, while it has a small nutation because of the applied torque, with periods of 24 hours, 27 days and one year. This again introduces small variations in the intensity of cosmic rays, much smaller than those considered thus far. Thus, for instance, the variation of latitude at a given point of the earth is less than 1′, and the change of distance to the equivalent dipole less than 1 km. The conclusion is thus that the change of intensity of cosmic rays due to the variation of the dipole field is negligible.

It remains now to estimate the change of intensity of cosmic rays due to the short range forces arising from the ionospheric currents. We consider the atmosphere as a series of concentric spherical shells between which there circulate electric currents of intensity i which follow the parallels. The internal potential is thus $(4\pi/3)(ir/r_0^2)\sin\lambda d\rho$ and the external one is $(-2\pi/3)(i\rho^3/r_0^2r^2)\sin\lambda d\rho$ where ρ is the radius of the shell and r_0 the earth's radius. We assume that within a shell the current is distributed according to the law

$$(4\pi/3)(i/r_0^2) = C\exp\left[-h^2(\rho-r_1)^2\right],$$

where the change of h is chosen so as to agree with the diurnal variation of the earth's field and r_1 is the mean radius of the shell. Now, the ionization of the lowest Heaviside layer diminishes strongly during the night, and the ionization of the F_2 layer is strong only around noon. A permanent ionization in the F_1 layer is assumed by Chapman to account for the permanent part of the external field. Therefore the difference between night and day in the distribution of current in the ionosphere may be characterized

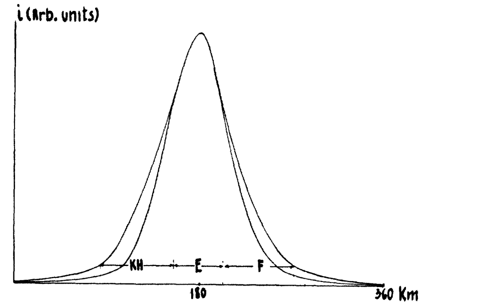

FIG. 8. Assumed representation of ionospheric currents.

by a change of the quantity h in the above formula, to be chosen in such a way that it accounts for the diurnal variation of the earth's magnetic field (Fig. 8). Assuming that the current at ground level is 0.001 of its maximum, and again leaving out the details of our calculations, we obtain for the variation of $\sin\theta$ in Störmer's formula

$$\Delta'\sin\theta = \frac{\kappa}{2}r\cos\lambda\int_r^\infty \exp\left[-h^2(\rho-r_1)^2\right]d\rho + \frac{\kappa}{2}\frac{\cos\lambda}{r^2}\int_0^r \exp\left[-h^2(\rho-r_1)^2\right]\rho^3 d\rho$$

$$(\kappa = 4\pi ie/3mvr_0^2)$$

which, taking into account the diurnal change of h, gives

$$\Delta\sin\theta = -\kappa r\cos\lambda h dh\int_{r_0}^\infty (\rho-r_1)^2 \times\exp\left[-h^2(\rho-r_1)^2\right]d\rho = -\frac{\kappa\sqrt{\pi}}{2h}r_0\cos\lambda\frac{\Delta h}{h},$$

$$= -\cos\lambda/10^3 r_0^2$$

where r_0 is the energy in störmers. Knowing the variation of $\sin\theta$, the change of intensity of cosmic rays may be readily estimated. Thus we obtain $\Delta I/I = 0.004$ at the equator, 0.0037 at 30°. Thus the amplitude of the diurnal variation due to the ionospheric currents should be about 0.002 at 30°, and the maximum should come between 10 A.M. and noon in very good agreement with experiment (see Table V). Thus it appears that the diurnal variation of intensity at low and

TABLE V. *Diurnal variation.*

Station	Geog. Lat.	Calc. Amp. Sun's Field %	Calc. Max. Sun's Field	Ionospheric Calc. Amp. %	Ionospheric Calc. Max.	Observed Amp. %	Observed Max.	Observer
Hafelekar	47°N	0.23	14 hr.	0	—	0.2	13 hr.	Hess,[1] etc.
Pacific Ocean	40°–55°N	0.22	14 hr.	0	—	0.3	14 hr.	Thompson[2]
Cheltenham	39°N	0.2	13 hr.	0	—	0.34	11 hr.	Forbush[3]
Pacific Ocean	25°–40°N	0.1	12 hr.	0.18	11 hr.	0.2	14.5 hr.	Thompson[2]
Pacific Ocean	10°–25°N	0	—	0.19	11 hr.	0.3	14 hr.	Thompson[2]
Pacific Ocean	10°S–10°N	0	—	0.2	11 hr.	0.2	14 hr.	Thompson[2]
Pacific Ocean	10°–25°S	0	—	0.19	11 hr.	0.2	14.5 hr.	Thompson[2]
Pacific Ocean	25°–40°S	0.1	12 hr.	0.18	11 hr.	0.2	13 hr.	Thompson[2]

intermediate latitudes, which is not accounted for directly by the sun's magnetic field, is largely explained by the change of ionization of the ionosphere.

A similar, though somewhat more uncertain, calculation of the 12-month seasonal variation, together with a comparison with experiment, is given in Table VI.

The Relation Between Magnetic Storms, Solar Activity, and the Intensity of Cosmic Rays

The correlation between periods of solar activity, magnetic storms, and changes of intensity of cosmic rays seems to be well established,[11, 32] as well as their world-wide nature. If one accepts the well-known hypothesis of Birkeland and Störmer, according to which particles are projected from the sun, after having acquired their energy by processes possibly connected with the formation of sunspots,[33] and if one accepts a permanent magnetic field of the sun, the difficulty (Vallarta[12]) arises that it is not possible for charged particles of any reasonable energy to leave the sun except in a very narrow band at high latitudes, because most of the sun is within Störmer's forbidden region. A way out of this difficulty is to bear in mind that sunspots produce local magnetic fields of relatively high intensity (of the order of a few thousand gauss) and always occur in pairs of opposite polarity. A particle in the immediate neighborhood of a sunspot, where the local field far outweighs the permanent field, will therefore move under the action of a single pole. As shown by Poincaré the motion is along a geodesic of a narrow right circular cone with vertex at the sunspot, axis and angle depending on the initial conditions. As the particle moves away from the sunspot it comes under the action of the field of the conjugate sunspot of opposite polarity, so that the particle moves in the field of the magnetic dipole equivalent to the sunspot, whose axis is at right angles to a plane containing the permanent dipole.

[32] W. Kolhörster, Physik. Zeits. **40**, 107 (1939).
[33] W. F. G. Swann, Phys. Rev. **43**, 217 (1933).

TABLE VI. *12-month seasonal variation.*

Station	Geomag. Lat.	Calc. Amp. Sun's Field %	Calc. Max. Sun's Field	Ionospheric Calc. Amp. %	Ionospheric Calc. Max.	Resultant Calc. Amp. %	Resultant Calc. Max.	Observed Amp. %	Observed Max.	Observer
Cheltenham	50°N	1.2	Apr.	1.5	Jan.	1.2	Feb.	1.4 2.1	Jan. Jan.	Forbush[8] Gill[7]
Hafelekar	48°N	1.0	Apr.	1.5	Jan.	1.1	Feb.	1.8	Jan.	Forbush[8]
Teoloyucan	30°N	0	—	0.8	Jan.	0.8	Jan.	1.0 0.8	Jan. Jan.	Forbush[8] Gill[7]
Huancayo	1°S	0	—	0.6	Jan.	0.6	Jan.	0.1	Oct.	Gill[7]
Christchurch	48°S	1.0	Aug.	1.5	Jan.	1.1	Oct.	1.0 0.3	Aug. Aug.	Forbush[8] Gill[7]

Calculations, which must be left out for lack of space, prove that in this way the temporary field of a sunspot provides a tunnel through the forbidden region of the permanent field through which charged particles can escape the sun and reach the earth. If they have the proper initial conditions they may approach principal periodic orbits in the earth's magnetic field by following along outer nearly asymptotic orbits. In this way they may form a ring of roughly parabolic cross section extending some few earth's radii away from the earth and concentric with it. All other primary particles are now in the combined field of the earth and the ring. Preliminary calculations show that the opening of the allowed cone, and consequently the intensity of cosmic rays, may change by quite an appreciable amount,[13, 34] while the magnetic field changes relatively little. This accounts for the very large variation of intensity during certain magnetic storms. Since periodic orbits are unstable, particles cannot stay on them, but must leave by following asymptotic orbits, either returning to infinity or falling on the earth. This explains why a magnetic storm, and the consequent change of cosmic-ray intensity, starts suddenly and disappears slowly.[13]

[34] S. Kusaka, S. M. Thesis, M.I.T., June, 1938.

Concluding Remarks

It is thus shown that the hypothesis of a permanent field of the sun, together with the known behavior of ionospheric ionization, largely accounts for the observed time variations of the intensity of cosmic rays. The most noteworthy discrepancy still remaining is a phase difference of about three months between the observed and the calculated seasonal variation which possibly may be accounted for by ionospheric phenomena and by the external temperature effect of cosmic rays,[35] the latter according to Blackett's interpretation[36] in terms of the lifetime of the meson and the temperature expansion of the earth's atmosphere. Ionospheric phenomena[37] and the temperature effect should also account for the observed seasonal variation in the tropical belt.

[35] A. H. Compton and L. A. Turner, Phys. Rev. **52**, 799 (1937).
[36] P. M. S. Blackett, Phys. Rev. **54**, 973 (1938).
[37] L. V. Berkner, Terr. Mag. and Atmos. Elec. **41**, 173 (1936).

Discussion

Piara S. Gill, *University of Chicago:* The analysis of data furnished by cosmic-ray meters at four widely separated stations[1] showed that the existence of a 28-day period variation of cosmic-ray intensity was observable at all stations, showing an average amplitude of 0.18 percent. Working on the hypothesis of an interaction of the Störmer cone of the sun with the earth cones, Vallarta and Godart deduce a 27-day period corresponding to the period of the rotation of the sun. According to their calculations the amplitude of this period should vanish for latitudes lower than 30°. We found that the amplitude does not vanish but has a smaller value for latitudes lower than 30°. The values of the amplitudes at Teoloyucan and Huancayo are the same (0.14 percent), which is outside the experimental error. On the other hand, a much higher value (0.33 percent) of the amplitude is found at Cheltenham.

The writer[1] has suggested that these 28-day variations in cosmic-ray intensity may be associated with some surface activity on the sun, such as sunspots. It would then mean that the above variation is superimposed on the 27-day variation in cosmic-ray intensity, discussed by Vallarta and Godart. If the value of the amplitude at Teoloyucan and Huancayo is subtracted from that of Cheltenham, the amplitude at Cheltenham has a value of 0.19 percent. This is in good agreement with the value calculated by Vallarta and Godart near the latitude of Cheltenham.[2]

[1] P. S. Gill, Phys. Rev. **55**, 429 (1939).

[2] Vallarta and Godart have replied to the comments by Gill in their paper as here published, by calling attention to the effect of the ring currents, which give rise to an effect near the equator comparable with, but smaller than, that to be found at high latitudes.