REVIEWS OF

MODERN PHYSICS

VOLUME 10 | OCTOBER, 1938 | NUMBER 4

Cosmic-Ray Intensity *and* Geomagnetic Effects

THOMAS H. JOHNSON

The Bartol Research Foundation of the Franklin Institute, Swarthmore, Pennsylvania,
and
Research Associate of the Carnegie Institution of Washington, Washington, D. C.

TABLE OF CONTENTS

1. Introduction

SEVERAL historical accounts[1–20, 273] of the development of the cosmic-ray investigation have been published from time to time and to these the reader is referred for a complete bibliography. Here we will summarize the salient features of the present state of our knowledge and the methods of obtaining data, mentioning only the results of investigations which seem to the writer to contribute most significantly to the modern point of view.

Corlin[8] distinguishes three periods in the investigation. The discovery period began in 1901 when C. T. R. Wilson[21] and Elster and Geitel[22] found residual ionizations in closed vessels containing gas. Investigations of the cause of this ionization culminated in the demonstrations by Gockel[23] (1910), Hess[24] (1913) and Kolhörster[25] (1914) that the effect increased with elevation above the surface of the earth as could be explained if it were caused by a penetrating radiation coming into the earth's atmosphere from some cosmic source. It was this characteristic of the effect that suggested the hypothesis of the existence of *cosmic rays* or *hohenstrahlen.* The second period of the investigation was characterized by an agreement on the part of all observers as to the existence of the cosmic radiation, and the investigations during this period were directed to the task of finding what energies the cosmic rays had, and to the study of such characteristics as would lead to an indication of where in the universe these radiations were being produced. To these ends measurements were made of the absorption coefficients of the radiations in the atmosphere and in various other materials, and the periodic and secular variations of its intensity with time were studied. Interpretations were made, almost universally, with the expressed or tacit assumption that the radiation was analogous to the gamma-radiation from radioactive materials.

Since the problems of the second period are still largely unsolved, this period might have been considered as extending through the present, were it not for the radical change in the point of view brought about by the experiments of Skobelzyn[26] (1929) and of Bothe and Kolhörster[27] (1929). These experiments revealed the existence of corpuscular, electrically charged rays which were identified as cosmic rays by the fact that their penetrating ability was equal to that deduced from studies of the variation of cosmic-ray ionization with thickness of superposed matter. This discovery led to an entirely new group of experimental investigations belonging to the third period. The investigations of this period have now established the point of view that the primary cosmic rays are largely, if not entirely, corpuscular, and much has been learned regarding the secondary radiations generated by the primaries as they pass through the atmosphere and other matter. Several summaries have been made of the evidence leading to the proof of the corpuscular hypothesis.[4–7, 12]

In its present status the problems of the cosmic-ray investigations divide themselves into two principal categories. On the one hand are those problems bearing upon the nature of the primary rays, their energy distribution, and the places and processes of their origin, and on the other hand there are the problems relating to the interaction of cosmic rays with matter. The latter belong more properly to the more general field of radiation and nuclear physics, since in their consideration one is not concerned, except technically, with the fact that the rays used in the study happen to be of cosmic origin, yet it is practically impossible to discuss the evidence bearing upon the problems of the first class without becoming involved with those of the second. However, the present survey will be made with the more or less restricted point of view of the problems of the first group, and the consideration of questions of interaction of the rays with matter will be limited. The object of the investigation from the present point of view is to determine what types of rays impinge upon the upper surface of the atmosphere, what their energies are, and how the intensity depends upon the various cosmic and terrestrial parameters of which it is a function. These include the depth of the atmosphere; the geomagnetic parameters, latitude, longitude, direction and field strength; and, finally, any other parameters affecting the intensity as shown by the periodic and secular variations of the cosmic radiation with time.

2. Definition of Units of Cosmic-Ray Intensity

For the measurement of cosmic-ray intensity four types of instrument have been developed, the ionization chamber or electroscope, the single point or tube counter, trains of coincidence counters, and the cloud expansion chamber. In this section the interpretation of measurements made by each of these instruments will be considered in their relation to the cosmic-ray intensity.

All measurements of the cosmic radiation depend upon the detection of the ionization produced by the rays. In the ionization chamber the ions are drawn to the electrodes and are measured as an electric current. In the tube counter the ions formed by each ray trigger an electrical discharge which is recorded as a count. The number of counts in a given time is equal to the number of rays which have passed through the counter tube. In the cloud chamber the ions produced by the rays act as centers of condensation for the formation of visible droplets and the track of each ray may be observed.

In experiments with cloud chambers or counter tubes a practical measure of the cosmic-ray intensity is the number of tracks or the number of counts per unit of time. In the cloud chamber or with coincidence counters, which will be described fully at a latter stage, the number of rays within a given range of directions may be determined. This number depends, of course, upon the size of the detecting volume. The fundamental quantity for expressing the *unidirectional cosmic-ray intensity* is the number j of ionizing rays per unit solid angle per second crossing through a unit area placed normally to the direction of incidence. If the cloud chamber is provided with a magnetic field whose lines are parallel to the direction of observation the curvature of the tracks may be measured, and with knowledge of the mass and charge of the ray the energy $\mathcal{E}$ may be deduced. The number of rays of energy between $\mathcal{E}$ and $\mathcal{E}+d\mathcal{E}$ per unit solid angle per cm² per sec. will be designated by $j(\mathcal{E})d\mathcal{E}$, where $j(\mathcal{E})$ is called the *unidirectional spectral intensity*. It follows that

$$=\int_0^\infty j(\mathcal{E})d\mathcal{E}. \quad (1)$$

The number of rays from all directions of energy between $\mathcal{E}$ and $\mathcal{E}+d\mathcal{E}$ per sec. per cm², which cross an area normal to the direction of incidence, or in other words, the number of rays which pass through a sphere of unit cross-sectional area per sec., is

$$J(\mathcal{E})d\mathcal{E}=\int j(\mathcal{E})d\mathcal{E}d\omega$$

$$=\int_0^{2\pi}\int_0^{\pi/2} j(\mathcal{E}) \sin \zeta d\zeta d\varphi d\mathcal{E}. \quad (2)$$

where ζ and φ are zenith and azimuthal angles. $J(\mathcal{E})$ is called the *total spectral intensity. The total intensity* or the number of rays of all energies through a sphere of unit cross-sectional area per sec. is

$$J=\int_0^\infty J(\mathcal{E})d\mathcal{E}.$$

If the average number of ion pairs produced by a ray of energy $\mathcal{E}$ along a unit length of its path is designated by $N(\mathcal{E})$, then in a vessel of arbitrary shape the rays of energy between $\mathcal{E}$ and $\mathcal{E}+d\mathcal{E}$ whose paths lie within the cylinder which intersects the walls of the vessel in the elements of area da_1 and da_2 and whose length in the vessel is L produce

$$N(\mathcal{E})j(\mathcal{E})d\mathcal{E} \cos \theta_1 \cos \theta_2 da_1 da_2/L$$

ion pairs per sec. within the vessel, where θ_1 and θ_2 represent the angles between the normals to the elements of area and the axis of the cylinder. Since the solid angle subtended by the area da_1 from a point in da_2 is $d\omega=da_1 \cos \theta_1/L^2$ and the element of volume within which the ionization takes place is $dV=da_2 \cos \theta_2 L$ the number of ion pairs formed per sec. within the cylinder may also be written

$$N(\mathcal{E})j(\mathcal{E})dVd\omega d\mathcal{E}.$$

The number of ion pairs produced within the vessel per sec. by rays of energy between $\mathcal{E}$ and $\mathcal{E}+d\mathcal{E}$ per unit volume is

$$N(\mathcal{E})j(\mathcal{E})d\mathcal{E}d\omega,$$

a quantity which is independent of the shape or the size of the vessel. The quantity $i(\mathcal{E})=N(\mathcal{E})j(\mathcal{E})$

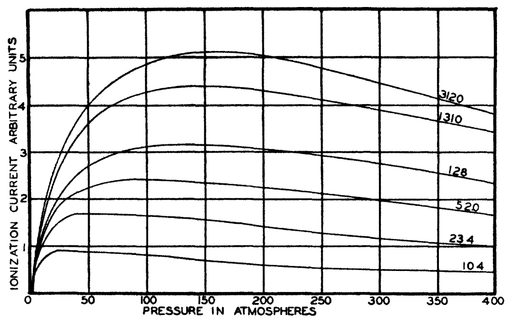


FIG. 1. The variation with pressure of the ionization current in an air-filled chamber under constant irradiation by gamma-rays. Parallel plate electrodes provided uniform collecting fields which are given in volts per cm by the numbers attached to each curve. (Erikson, reference 30, 1908.)

is a convenient measure of the cosmic-ray intensity and will be known as the *unidirectional spectral ionization intensity*, or the *unidirectional spectral ionization*. The *unidirectional ionization intensity* or the number of ion pairs per cc per sec. formed by cosmic rays of unidirectional intensity j is

$$i=\int_0^\infty N(\mathcal{E})j(\mathcal{E})d\mathcal{E}=Nj, \tag{3}$$

where N is the average ionization per cm of path defined by $N=i/j$. The number of ion pairs produced by rays of energy between $\mathcal{E}$ and $\mathcal{E}+d\mathcal{E}$ from all directions is

$$I(\mathcal{E})d\mathcal{E}=N(\mathcal{E})J(\mathcal{E})d\mathcal{E}.$$

$I(\mathcal{E})$ is the *total spectral ionization intensity*. The number of ion pairs per cc per sec. produced by rays of all energies and directions is called the *total ionization intensity* and is given by $I=NJ$. When recombination of ions is neglected, the current in an ionization vessel of volume V in which I ion pairs per cc per sec. are produced is $I\epsilon V$, where ϵ is the electronic charge.

The number of rays of energy between $\mathcal{E}$ and $\mathcal{E}+d\mathcal{E}$ which pass through a horizontally placed unit of area, or the *normal spectral intensity*, is

$$J_\perp(\mathcal{E})d\mathcal{E}=\int_0^{\pi/2}\int_0^{2\pi} j(\mathcal{E})d\mathcal{E}\sin\zeta\cos\zeta d\zeta d\varphi. \tag{4}$$

The *normal intensity* of rays of all energies is

$$J_\perp=\int_0^\infty J_\perp(\mathcal{E})d\mathcal{E}.$$

The energy carried across unit horizontal area by rays of energy between $\mathcal{E}$ and $\mathcal{E}+d\mathcal{E}$ is $\Psi_\perp(\mathcal{E})d\mathcal{E}=\mathcal{E}J_\perp(\mathcal{E})$; $\Psi_\perp(\mathcal{E})$ is the *normal spectral energy flux*. The *normal energy flux* is

$$\Psi_\perp=\int_0^\infty \mathcal{E}J_\perp(\mathcal{E})d\mathcal{E}.$$

In a column of air of infinite depth and of unit cross section the total ionization per sec. is proportional to the normal energy flux, since an equivalent amount of energy is spent within this column in the production of ions. If k is the energy expended in the production of one ion pair and $I(h)$ is the total ionization intensity at depth h then the normal energy flux at depth h_1 is

$$\Psi_\perp(h_1)=k\int_{h_1}^\infty I(h)dh. \tag{5}$$

The unidirectional intensity i, or j, in the vertical direction may be evaluated from the corresponding total intensity I, or J, if the variation of the total intensity with depth h is

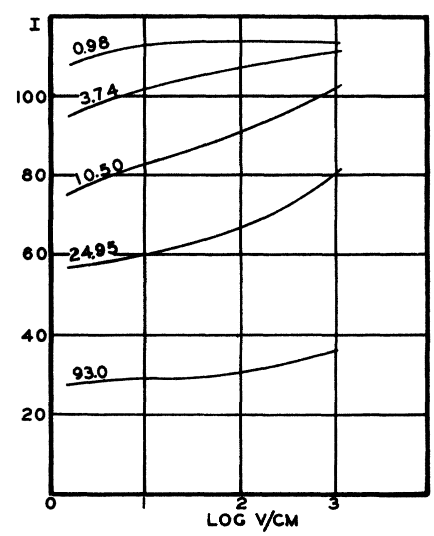


FIG. 2. The variation with the collecting field of the ionization current in an air-filled chamber under constant irradiation by gamma-rays. Parallel plate electrodes provided uniform electric fields from 1.5 volts per cm to 1000 volts per cm. The pressure in atmospheres is indicated by the numbers attached to each curve. Ordinates: ionization current per atmosphere in arbitrary units. Abscissae: $\log_{10}$ of the collecting field in volts per cm. (Bowen, reference 36, 1932.)

also known. A transformation of this kind is known as a Gross[28] transformation and it can be made if the assumption is valid that the unidirectional intensity i is a function of the atmospheric path $x=h \sec \zeta$ alone. Making the substitution $\sin \zeta d\zeta = x^2dx/h^3$ in Eq. (2) and introducing i in place of j by means of (3) we obtain

$$I(h)=2\pi h\int_h^\infty (i(x)/x^2)dx. \qquad (6)$$

Differentiation of (6) with respect to h leads to the required expression for the vertical intensity, $i(h)$, namely

$$i(h)=I(h)\frac{(1-h(dI/Idh))}{2\pi}=G(h)I(h), \qquad (7)$$

where $G(h)$ stands for the Gross reduction factor

$$(1-hdI/Idh)/2\pi. \qquad (8)$$

As Eckart[29] has pointed out, (7) leads to the following graphical method for constructing the curve representing the variation with depth of $2\pi i(h)$ when the $I(h)$ curve is known. At the abscissa h draw the tangent to the I curve, continuing it until it intersects the I axis at $h=0$. This intercept is then equal to the ordinate of the curve representing $2\pi i(h)$ for the abscissa h.

3. Technique of Measurement of the Cosmic-Ray Ionization

Many of the investigations of the intensity of the cosmic radiation and its dependence upon the several parameters of which it is a function have been made by measuring relative values of the ionization currents in a gas-filled chamber suitably shielded to eliminate the gamma-rays from radioactive materials, and without any attempt being made to reduce these currents to some standard condition of shield thickness, gas pressure, etc. In such investigations it is customary to assume that the relative values of the currents indicate the relative intensities I of the cosmic radiation. On the other hand, the results are frequently reduced to absolute values and are expressed in terms of the number of ions per cc per sec. at one atmosphere of air. However, in comparing such reduced values obtained by different observers for a given set of external conditions (barometer, latitude, shield thickness, etc.) the disparity of the values reported seems to indicate that the reduction has not produced a figure entirely independent of the

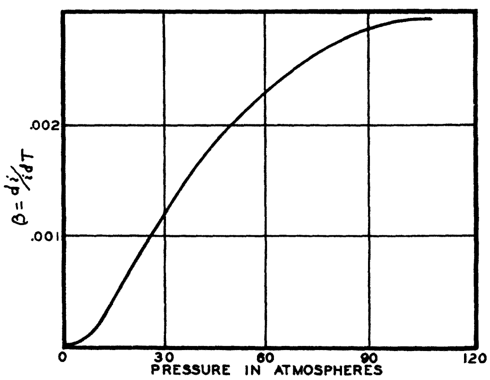


FIG. 3. The variation with pressure of the temperature coefficient of the ionization current, $\beta=dI/IdT$ in air at room temperature. (Gingrich, reference 33, 1932.)

particular instrument. It is hardly necessary to suggest that a reduction to some standard, independent of the instrument, is desirable and is even essential if the absolute measurement of the intensity is to have some definite significance. On the other hand, there appear to be great difficulties in making a reduction to completely standard conditions, and even relative intensities seem to depend in a degree upon the particular instrumental conditions. It is thus necessary to consider in some detail the various factors effecting the ionization in a closed vessel under constant irradiation. These factors will be briefly discussed in the following three sub-sections.

a. Dependence of ionization upon the gas, the gas pressure, the collecting field and the temperature

The variation of the number of ions collected under constant irradiation by cosmic or gamma-rays as a function of the electric field and the temperature has been extensively investigated[30–72, 272] both experimentally and theoretically. If the probability of any molecule of the gas becoming ionized is proportional to the radiation intensity the ionization per atmosphere should be independent of the pressure as long as the absorption of the primary radiation in the gas

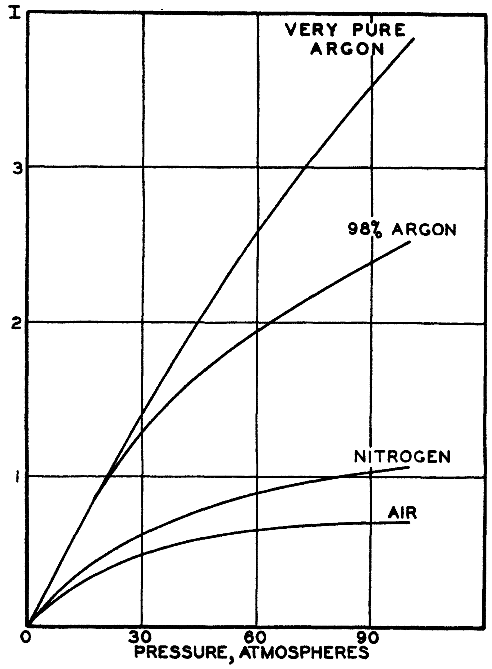


FIG. 4. The variation with pressure of the ionization current in various gases, showing the effect of impurities in the case of argon. Ordinate: ionization current in arbitrary units. Abscissae: pressure in atmospheres. (Compton, Wollan and Bennett, reference 39, 1934.)

itself can be neglected. At low pressures this expectation is fully realized, but as the pressure is increased beyond several atmospheres, the number of ions collected begins to increase less than proportionally with the pressure, it eventually reaches a maximum and then at very high pressures it decreases again. The pressure at which the maximum is reached, and the current at all pressures, depends upon the collecting field (Figs. 1 and 2), the temperature (Fig. 3), the kind of gas (Fig. 4), and possibly the material of the walls and the size of the vessel. Although other proposals have been made, there is now a quite general acceptance of the view that recombination of the ions before collection is the principal cause of the dependence of the current upon these factors, but there is no general agreement as to what type of recombination is the most important. Three different types are recognized:[69] *preferential* recombination, where the electron recombines with the parent ion before diffusing out of its field of attraction; *columnar* recombination, where positive and negative ions produced by the same ray unite before they are drawn from the column of dense ionization; and *volume* recombination, which takes place after the ions become randomly distributed throughout the volume of the vessel. Each type of recombination reacts differently to changes of temperature, pressure and electric field. Possibly all three types are important under the conditions used in some cosmic-ray measuring electroscopes, and it may be for this reason that theoretical treatments which have not considered all types have met with only partial success.

Preferential recombination has been considered by Compton, Bennett and Stearns,[64] and by Bragg and Kleeman.[65] If an electron is to escape the attractive field of its parent ion it is necessary for it to reach a distance of the order

$$r_0 = e^2/kT$$

before it attains thermal equilibrium with the gas; beyond this distance the attractive potential is less than the average thermal energy and the electron escapes by diffusion. At ordinary temperatures and at a pressure of one atmosphere in air this distance is of the order of the mean free path so that it is only at the higher pressures where this type of recombination becomes of importance. In the rare gases where electrons have anomalously long free paths because of the Ramsauer effect the electrons retain their original energy for a longer period and preferential recombination becomes important only at correspondingly higher pressures. On the other hand in these gases the free path is sensitive to the presence of impurities. Fig. 4 shows the ionization in various gases illustrating the difference between nitrogen and argon and between pure and impure argon.

The attractive field of the ion at the distance r_0 is of the order of 40,000 volts per cm. At distances where the field of mutual attraction is of the order of that ordinarily applied for collection of ions in an ionization chamber, i.e., about 100 volts per cm, the thermal energy so far exceeds the attractive potential that the preferential recombination of electrons which have reached this distance is an unimportant consideration. This form of recombination is thus not much affected by the fields ordinarily applied.

The effect of the temperature upon preferential recombination comes into consideration through the dependence upon T of the distance r_0. The theory of the temperature effect based upon this assumption, according to Compton, Bennett and Stearns is in satisfactory agreement with the data.[33, 34, 64] (See Fig. 3.) Recently Onsager[272] has carried through a more rigorous analysis of the diffusion of ions in the combined field of the collector and the parent ion and has found that the observed variations of ion current with collecting field can be largely explained.

Volume recombination[70–72] is important only in the case of extremely high radiation intensities and low collecting fields. It is rarely, if ever, a consideration in cosmic-ray measurements.

Columnar recombination has been considered on the theoretical side by Jaffé,[66] Zanstra[67] and Gross;[68] Clay[50–53, 70] and his associates have invoked this form of recombination as an explanation of the principal trend of the ionization-pressure curve. This point of view, however, has not met with a general agreement among other investigators.[59, 71] According to the theory, columnar recombination should be sensitive to those factors which determine the rate of separation of ions immediately after their formation, i.e., to the pressure, the temperature, the electric field and to the tendency of the electrons to attach themselves to neutral gas molecules. Unlike the other types of recombination this type should be sensitive to the direction of the electric field with respect to the paths of the rays. This is an effect, however, which has not been found.[59]

Recombination bears principally upon the problem of reducing the measured currents to the absolute number of ions, I, originally liberated by the radiation. Fortunately the fraction of the ions lost by recombination does not depend upon the intensity of the radiation to an important degree and it is not necessary to make corrections for recombination in experiments where relative intensities of cosmic radiation are of interest. This important point was partially established by Millikan[45] and Hoffman[44] who found that the same pressure reduction factors could be used for a wide range of gamma-ray intensities or for cosmic rays at sea level; Swann[43] found the same reduction factor could be applied for cosmic rays of mountain-top or of sea-level intensity. Thus the ionization current can be represented as the product of two factors, one depending upon the radiation intensity and the other depending upon the pressure. The pressure factor is the same for cosmic rays of different intensities or for gamma-rays. Since the loss of ions by recombination varies with the pressure these experiments showed that the fraction thus lost was the same for all of these radiations. This is in sharp contrast with the ionization produced by alpha-rays[257] for which the variation with pressure follows a quite different law, presumably because of the greater importance in this case of columnar recombination.* More recent studies[111] based upon cloud chamber evidence have indicated that columnar and preferential recombination probabilities cannot be expected to undergo variations with the changes in the quality of the cosmic radiation experienced, for example, in the experiments made at different elevations, for it has been shown that all types and energies of cosmic rays produce about the same density of ionization along their tracks and that the initial energy imparted to the ionized electron is also practically invariant. According to some results of Hopfield,[35] however, it is possible that relative cosmic-ray intensities would be measured differently by ionization chambers filled with different gases. In his experiments the ratio of the ionization observed in argon to that in air was 1.4 times greater when produced by a gamma-ray source than when the cosmic rays were used, and one might anticipate that the changes in the quality of the cosmic radiation with elevation would have a similar effect. No satisfactory explanation of this effect has been given.

Added to the effects of recombination, the variation of the ionization current with pressure may also contain a saturation effect due to short range particles emitted from the walls of the vessel. As the pressure is increased to the point where such particles fail to traverse completely

* The effect referred to is the difference between the total ionization by alpha-particle contamination and the ionization per atmosphere produced by cosmic or gamma-rays. Since the dimensions of the vessel are generally greater than the alpha-particle range there is no increase with pressure of the ionization from this source.

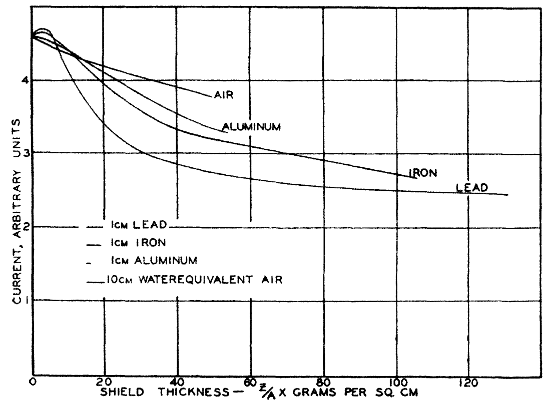


FIG. 5. The variation with shield thickness of the ionization in a thin-walled vessel when various materials are used in the shield. The unit of thickness is chosen as that which contains 6.06×10^{23} electrons per cm^2. (Schindler, reference 76, 1931.)

the vessel, their ionization no longer increases with the pressure. This is an effect which is readily confused with recombination and has, in fact, been offered[48–52, 62] as an explanation of the main trend of the pressure-ionization relation.

It is beyond the scope of the present review to enter into further details of the study of the ionization in gases at high pressures. The effects are not completely understood and it is doubtful if full acceptance can be made of any of the various methods which have been used to deduce from the measurements the number of ions actually produced by the radiation.

b. Dependence of the ionization upon the material and thickness of the walls of the vessel

The wall effect mentioned in the preceding section involves short range particles and it was invoked as an explanation of the saturation of the ion current at high pressures. There is another more generally recognized wall effect which manifests itself through a dependence upon the wall material and wall thickness of the number of secondary rays in equilibrium with the primary beam. This effect, known as the *transition effect*, was first investigated in the case of gamma-rays by Marsden,[73] and later by Workman.[74] Cosmic-ray transition effects have been studied by Steinke[75] and Schindler,[76] by Hoerlin[77] and by Street and Young.[78] Closely akin to this effect are the cosmic-ray showers and their dependence upon the thickness and material of the medium, the investigations of which have recently been reviewed in this journal by Froman and Stearns.[258]

Schindler's measurements of the variation of the cosmic-ray ionization as a function of the material and thickness of the shield are shown in Fig. 5. Here thicknesses are represented on the extranuclear electron scale according to which one unit contains 6.06×10^{23} electrons. The measurements were made at sea level so that zero shield thickness corresponds to one atmosphere of air. The variation with additional air according to an extrapolation is indicated by the curve labeled "air," and the departure of the other curves from this indicates the dependence of the ionization by secondary radiations upon the kind of material through which the radiation is passing.

A simple interpretation of the transition effect has been given by Johnson,[79] and although the assumptions underlying his treatment appear oversimplified from the modern point of view, they contain enough truth to bring out the principal characteristics of the phenomenon, and have the advantage of yielding simple equations representing the variation of the intensity with shield thickness quantitatively consistent with Schindler's measurements. In lead, for example, the radiation incident from above produces secondaries, the probability of production being a characteristic of the material, and these cause the observed ionization. As the lead is increased in thickness, the first secondaries, together with those from the air, are absorbed and new secondaries are formed. It was assumed that the production coefficient was a function of the material of the medium, and the absorption coefficient of secondary rays depended upon the medium in which the ray was produced as well as upon the medium in which it was being absorbed. The total number of secondaries as a function of the distance x in the second medium (m) was then given by

$$S = S_n + S_m = S_{n0} e^{-\mu_m{}^n x} + \beta_m P(e^{-\nu_m x} - e^{-\mu_m{}^m x})/(\mu_m{}^m - \nu_m), \quad (9)$$

in which $\mu_m{}^n$ is the absorption coefficient of secondaries produced in medium n and absorbed in medium m, P is the intensity of the primary radiation and ν_m its absorption coefficient in medium m. β_m is the production coefficient of secondaries in medium m and S_{n0} is the number of secondaries from medium n at the interface. The final equilibrium concentration of secondaries, if we neglect for the moment the absorption of primaries, is proportional to the ratio of the probability of production β_m to the probability of absorption $\mu_m{}^m$. This ratio, according to the analysis, is less in substances of high atomic number than in substances of low atomic number and the lower currents under lead, for example, as indicated in Fig. 5, are to be understood as resulting from a lower density of secondary radiation; not a greater absorption of the primary radiation.

According to the modern point of view[81–87] the transition involves changes with the material of the medium of several different effects. These include the production by the soft component electrons of multiplicative showers, the loss of energy through ionization by the less energetic constituents of these showers, the rate of production of soft component electrons by the penetrating particles, and the final equilibrium between penetrating particles and the soft secondaries produced by them. The complete story is a complicated one and it has not yet been satisfactorily worked out in all of its details.

It is clear from Fig. 5 that the value of the ionization current depends critically upon the material and thickness of the shield, and a factor depending upon the shield must be used in reducing measurements made by different instruments to a common level for comparison. In this connection a troublesome element arises in the fact that the shield factor depends upon the quality of the radiation and is therefore a function of the cosmic-ray intensity, for any change of intensity seems also to involve a change of the relative intensities of hard and soft components.

This dependence of the shield factor upon the quality of the radiation is shown in some experiments of Young and Street[78, 80] who have measured the transition effect at different elevations. We need consider here only their results for atmospheric depths 6.9 meters and 10.3 meters of equivalent water. The ionization intensities which they have measured at these two elevations are shown as a function of shield thickness in the A curves of Fig. 6. The difficulty which the situation faces is illustrated by the fact that the ratio of the intensities measured at the two elevations varies with the shield thickness. As an example the intensity at the higher elevation relative to that at sea level was forty percent greater with a 2-cm lead shield than with a 20-cm lead shield.

In an analysis of these results Street and Young have resolved the radiation into two components, the "hard" component and the "soft" component, each of which is homogeneous in the sense that the intensity of each can be expressed as the product of two factors, one

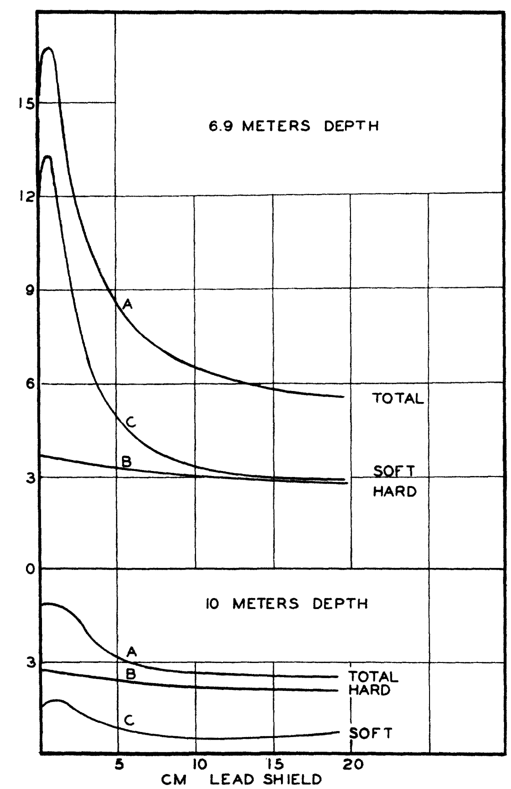


FIG. 6. Analysis of the cosmic radiation according to Young and Street. The curves represent the variation with lead shield thickness of the ionization in a small steel chamber produced by (A) the total cosmic radiation, (B) the hard component and (C) the soft component. The change of the relative proportions of hard and soft components with elevation introduces a dependence of the shield factor upon elevation. Ordinates: ionization current in arbitrary units. Abscissae: thickness of the lead shield in cm. (Young and Street, reference 80, 1937.)

depending upon the atmospheric depth alone and the other depending only upon the shield. Homogeneity in this sense implies that a given shield will reduce the intensity by a factor which is independent of the elevation, or that a given increase in atmospheric depth will reduce the intensity by a factor independent of the shield. A resolution of the radiation into two components has been suggested by a number of other writers and the properties of each component have been extensively investigated.[258] A survey of these studies is beyond the scope of the present writing and it will suffice to remark that the hard component experiences approximately the same mass absorption in all substances whereas the soft component loses energy by radiative collisions for which the probability per atom varies with the square of the atomic number. The effect of the lead shield upon the observed ionization thus depends upon the proportions of the hard and soft components. In order to determine the intensity of each component separately it was necessary for Street and Young to make use of measurements of the relative intensities of each component as a function of the atmospheric depth and thickness of the lead shield. Effects useful for the measurement of relative intensities can be recorded with suitable arrangements of coincidence counters. Thus the frequency of coincidences recorded with counters in vertical line and with 20 or more cm of lead interposed into the paths of the rays is usually regarded as a measure of the intensity of the hard component. With thinner lead screens the coincidences are produced in part by the soft component and their frequency rises abruptly as the lead is reduced in thickness. In the range of shield thicknesses less than 20 cm the hard component can be measured by an extrapolation of the results obtained with greater thicknesses. The intensity of the soft component, on the other hand, is approximately proportional to the frequency of cosmic-ray showers recorded with a disaligned arrangement of counters surmounted by a lead plate.

In their analysis Young and Street have made use of results obtained with the above-described techniques by Woodward[88] who worked at the same elevations as those used for their own measurements of the total ionization intensity. Whereas Woodward actually measured the frequency of coincidences it was assumed for the analysis that these were proportional to the respective ionization intensities I_H and I_S of the hard and soft components. It was then possible to reduce his results to a scale comparable with that in terms of which the total ionization I_T was measured by solution of the following equations which hold for each shield thickness.

$$I_H(h_1)+I_S(h_1)=I_T(h_1), \tag{10a}$$

$$I_H(h_2)+I_S(h_2)=I_T(h_2), \tag{10b}$$

$$I_H(h_1)/I_H(h_2)=F_H, \tag{11a}$$

$$I_S(h_1)/I_S(h_2)=F_S. \tag{11b}$$

F_H and F_S are the ratios of the coincidence rates observed by Woodward. The values of I_H and I_S obtained from the solutions of these equations for each shield thickness have been plotted to construct the B and C curves of Fig. 6. As already indicated the unit of intensity is the same as that used in the plotting of the A curves.

Within the accuracy of the experiments Woodward finds that F_P and F_S are constant with respect to variations of the shield thickness. This fact justifies the hypothesis that the soft and hard components may be individually regarded as homogeneous in the sense in which this property has been defined.

In conclusion, it is not possible to express the total intensity as the product of two attentuation factors, the one depending upon the shield and the other upon the atmospheric path, even though, as has been indicated, such a representation is approximately valid as regards each of the two components. The problem of reducing ionization intensity measurements to a common standard independent of the instrumental conditions must remain, therefore, in an unsatisfactory state.

Although in the above discussion only the measurements of Young and Street have been considered, mention should also be made of similar sets of measurements made over a wide range of elevations with a variety of shield thicknesses by Mott-Smith and Howell,[89] and by Hoerlin.[77]

c. Reduction of cosmic-ray ionization to standard conditions

Although the shield required to protect a cosmic-ray ionization chamber from radioactive radiations while measurements are being made upon land introduces a factor depending upon the elevation which may not be conveniently corrected for, the situation is more favorable for ionization measurements made in airplanes, with free balloons or under pure water. Under each of these conditions the shield may be removed and the wall effect reduced to almost negligible proportions. Using a steel chamber with 3-mm thick walls Millikan[45] has made an absolute determination of the ionization produced by cosmic rays at sea level. In order to eliminate the necessity of correcting for the shield and still to protect the chamber from radioactivity the measurements were carried out at an effective atmospheric depth of 10 meters of water, realized by sinking the chamber in a snow-fed lake at high elevation. The chamber was filled with 30.1 atmospheres of air, but the results were reduced to one atmosphere by applying a factor determined by a direct comparison of the ionization currents produced by a fixed source of gamma-rays when the chamber was filled alternately with 30.1 and one atmospheres of air. After subtracting the "zero current" produced by the radioactive contamination of the walls of the chamber, an effect which was determined by sinking the chamber to such depths that the cosmic-ray ionization was small in comparison with it, Millikan found that the cosmic-ray ionization at sea level was, $I_{\text{sea level}}=2.48$ ions per cc per sec. per atmos. The current was not affected, even at the higher pressures, by changes of voltage and Bowen's[36, 37] results, Fig. 2, indicate that no appreciable increase in current could have been expected at one atmosphere if the field had been increased to 1000 volts per cm. Hence the value 2.48 ions per cc per sec. may be regarded as being independent of the collecting field to within a percent or two. Moreover, Schindler's results on the transition effect in iron (Fig. 5), although not carried back to zero wall thickness, indicate that the wall effect cannot amount to more than a percent or two for the 3-mm thick walls of the Millikan chamber. When a shield of 11 cm of lead was placed around the chamber the sea level cosmic-ray ionization was reduced to 1.75 ions per cc per sec. per atmos.

Using a method by which they sought to correct for the wall effect, Clay and Jongen[52] found 1.10 ions per cc per sec. in air at one atmosphere produced by cosmic rays of sea-level intensity after filtering through 12.5 cm of iron. Clay's method of correcting for the absorption in the shield raised this figure to 1.28 ions per cc per sec. for the ionization in free air, or about half of that found by Millikan.

Compton, Wollan and Bennett[39] have used Hopfield's[35] determination of the ratio of the ionization in 98 percent pure argon to that in air, their own determination of the ratio of the ionization in pure argon to that in 98 percent pure argon, and Millikan's value for the ratio of the ionization in air at 30 atmospheres to that in air at one atmosphere. The combined reduction factor for converting the ionization in 30 atmospheres of pure argon to that in one atmosphere of air is 1/67. With this factor the ionization produced by cosmic rays in Chicago, after filtering through 12 cm lead, converts to 1.22 ions per cc per sec. in one atmosphere of air.[206] This value differs by a puzzlingly large factor from that obtained by Millikan for nearly the same shield thickness as well as from Clay's figure, after applying a reasonable correction for the shield difference.

Discrepancies of the type indicated above, throw considerable doubt upon the accuracy of any of the determinations of the absolute ionization caused by cosmic rays. The discrepancies are of the same order and of a similar character to those encountered in the various determinations of the specific ionization by beta-rays and a suggestion made in the latter connection by E. J. Williams[57] may have some bearing upon the present situation. Williams cites experiments by M. L. E. Oliphant[56] which indicate that the currents may be considerably amplified by electrons emitted from the walls of the vessel and from the collecting electrode as well as from impurities of low ionization potential in the gas under the influence of the positive ions produced by the radiation. This is a factor which could vary considerably from one experiment to another, and it might account for such discrepancies as are observed. In fact Broxon[40] has observed variations in the ionization currents with the lining of the chamber amounting to twenty percent or more.

4. Technique of Measurement of the Number of Cosmic Rays, J

The number, j, of cosmic rays per cm^2 per sec. from unit solid angle in a given direction can be deduced from the counts recorded with a train of coincidence counters or from a count of the tracks appearing in a Wilson cloud chamber. The total number J of rays from all directions can be calculated by integrating j over all angles, or it can be deduced from the counting rate of a single tube or point counter or from an analysis of the short time fluctuations in an electroscope. Whereas the latter two methods are subject to the uncertainty arising from the spurious rays due to radioactivity, the coincidence counter method counts only those ionizing rays which are sufficiently penetrating to pass through the walls of the counters; the penetrating gamma-rays of radioactive origin are unable to discharge a counter except through the conversion of their energy into beta-rays, and the latter are not sufficiently penetrating to pass through the walls from one counter to another. With the usual unshielded counter train, where the counters have thin copper cylinders and standard glass envelopes, cosmic rays of more than about 10 Mev are recorded.

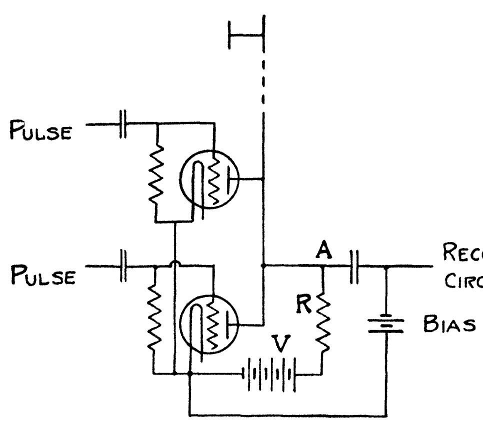


Fig. 7. Schematic diagram of a circuit for selecting coincidences which makes use of the "Rossi" parallel plate connection. Negative pulses from the individual Geiger-Mueller counters are introduced at the left. The characteristic of the circuit is such that the potential of the point A remains sensibly unaltered unless all of the tubes whose plates are connected in parallel receive simultaneous pulses.

a. Coincidence counter determinations of j

The passage of an ionizing ray through each member of a counter train produces simultaneous discharges which may be discriminated by suitable electrical circuits from events in which any number of counters less than the full number of counters in the train are discharged. Various vacuum tube circuits for this purpose have been suggested,[27, 259, 260, 262–264] but the ones now used almost universally are based upon the parallel plate connection suggested by Rossi.[261] (Fig. 7.) The selecting resistance R is adjusted in relation to the plate resistance of a single vacuum tube in its normal state so that when negative pulses are impressed upon the grids of all but one of the tubes, the current carried by the other tube is still sufficient to produce an iR drop across the resistance nearly equal to the full battery potential. If, on the other hand, negative pulses are received on the grids of all of the tubes the iR drop across the resistance disappears and the grid of the output tube receives a positive pulse, which results in the recording of a count. The counting rate C/T measures the intensity of the cosmic radiation.

Because of the random character of the impulses a counting rate determined from any finite time of recording is subject to statistical fluctuations which may be calculated from the theory of probability. If $\bar{C}$ is the average number of counts in a time T supposed to be evaluated from an infinitely long period of observation, the probability that the number of counts actually recorded will be C is given[107] by the Poisson equation

$$P(C)=(\bar{C})^C e^{-\bar{C}}/C!, \tag{12}$$

or by the Gauss equation

$$P(C)=(2\bar{C}\pi)^{-\frac{1}{2}} e^{-(C-\bar{C})^2/2\bar{C}}. \tag{13}$$

The Poisson equation is correct for all values of $\bar{C}$, whereas the Gauss equation is accurate only for large values of $\bar{C}$. From the Gauss equation the "standard deviation" or the root mean square of the deviations observed in n trials of equal weight is

$$\left[(1/n)\sum_{i=1}^{n}(C_i-\bar{C})^2\right]^{\frac{1}{2}}$$

$$=\left[\int_{-\infty}^{\infty}(C-\bar{C})^2P(C)dC\right]^{\frac{1}{2}}=(\bar{C})^{\frac{1}{2}}. \quad (14)$$

Thus the standard deviation, or the mean error, of a counting rate based upon C counts is approximately $C^{\frac{1}{2}}/T$. The "probable error" is equal to the standard deviation multiplied by the factor 0.6745:

In the extreme case where the solid angle subtended by the counter train is small and where other correction factors may be neglected, the counting rate is related to the unidirectional ray intensity j in the direction of the principal axis of the counter train by the equation

$$C/T=(A^2/L^2)j,$$

where A is the area of one of the counters and L is the distance between the two extreme counters which determine the aperture or solid angle of the train. In practice some care is required in determining corrections for the following factors: (1) the difference between the actual and the effective area of each counter; (2) the variation of j over the effective aperture of the counter train; (3) the probability that an ionizing ray can pass through the train without producing discharges in all of its counters; (4) the probability that false counts will occur when nearly simultaneous, but unassociated, rays pass through the counters; (5) the probability that two or more associated rays will pass through the train simultaneously and be recorded as a single count, (6) the probability that a group of associated rays from the side will discharge the counters simultaneously although no single ray has traversed the train, (7) the probability that two or more coincidences will occur within the time of action of the mechanical recording device so that only one will be recorded, and, finally, (8) the absorption of rays within the counter walls.

(1) In determining the effective area of a counter Street and Woodward[91] and Froman and Stearns[92] have used the counter under test as the

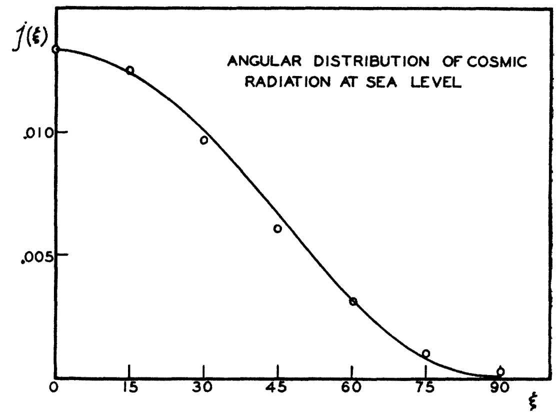


FIG. 8. The zenith angle distribution of cosmic rays at sea level according to Johnson and Stevenson. The experimental points are indicated by the circles and the full curve represents a $\cos^2 \zeta$ distribution.

central member of a triple coincidence train and have observed the change in coincidence rate as this counter was displaced sidewise and endwise by measured distances from the central line of the train. They concluded from their investigations that a cylindrical counter is sensitive over its entire radius but that an end-correction of the order of the radius, depending upon the overvoltage, must be subtracted from the length.

(2) Once the effective dimensions of the extreme counters are known, the effect of the variation of j over the finite aperture can be taken into account by expressing the counting rate in the form of an integral of the contributions of each element of area of the extreme counters. This integral takes the form

$$C/T=\int\int[j(\zeta)\cos^2\psi/r^2]da_1da_2, \quad (15)$$

where da_1 and da_2 are elements of area of the two extreme counters, r is the distance between these two elements and ψ is the angle between r and the principal axis of the train. In practice the integration of (15) must be carried through by numerical or graphical methods and for this purpose it is convenient to express the dependence of j upon the zenith angle ζ by some approximate empirical expression such as

$$j(\zeta)=j(\zeta_0)(1-2\delta\zeta\tan\zeta_0), \quad (16)$$

where ζ_0 is the zenith angle of the central line of the counter train and $\delta\zeta$ is the increment in zenith angle measured from this line for any arbitrary ray direction through the train. This equation represents the variation in j about the mean position as it would be given by the $\cos^2\zeta$ distribution. As indicated in Fig. 8, this is a close approximation to the distribution at sea level. The resulting integral (16) is the coefficient of the intensity along the central line of the counter train in the expression for the counting rate.

(3) The probability that an ionizing ray will pass through a counter without producing a discharge can be expressed as the sum of two other probabilities, (a) that the counter will be insensitive because of not having recovered from a previous discharge and (b) that the ray will fail to produce an ion pair along its path through the counter. The probability (a) depends upon the *recovery time* σ defined as the minimum time interval between the passage of two successive rays for which both rays will produce recordable pulses. σ depends upon the quenching resistance used in series with the counter and upon the capacity of the counter wire and its connections. It may also depend upon the character of the discharge. With an ohmic resistance it is difficult to reduce σ below 10^{-3} sec., but vacuum tube control circuits have been devised[93–95] to shorten this by a considerable factor. The probability that the interval between two successive rays is more than σ so that the second ray will also be recorded is equal to the probability $P(0)$ that no ray will occur in the time σ. According to the Poisson equation (12)

$$P(0)=e^{-\sigma\nu},$$

where ν is the average rate of passage of rays through the counter. This is the chance, as limited by recovery, that any ray will be recorded and is therefore the efficiency E_σ.

The probability (b) that a ray will pass through the counter in the recovered state without exciting a discharge has been shown by Danforth[96] and Cosyns[97] to be equal to the chance that a ray will traverse the path through the counter without producing an ion pair. This probability may be computed from the random distribution of the primary ions and for the purpose of the calculation the specific ionization in hydrogen may be taken as about 6 ions per cm at one atmosphere. Values for other gases are given in Section 5. The calculation of the efficiency again involves the application of the Poisson law and knowledge of the distribution of path lengths through the counter. If N is the specific ionization at unit pressure, the chance that at least one ion will be formed by a ray traversing a path of length L through the counter at pressure p is given by

$$E_p=1-e^{-NLp}. \tag{17}$$

This factor multiplied by the chance of the ray traversing a path of length L may be summed over the distribution of paths through the counter. The geometrical problem of this distribution in the case of a cylinder has been treated at length by Kolhörster and Tuwim,[13] by Cosyns,[97] by Swann,[98] and by MacAdam and Lipman.[99]

If E_σ is the efficiency as limited by recovery and E_p that determined by the pressure in the counter, as in the foregoing discussion, then the combined efficiency is

$$E=E_\sigma E_p.$$

In the case of a train of n coincidence counters the efficiency is E^n.

An experimental determination of the efficiency of a counter can be made by using it as the central member of a triple coincidence train, in which double coincidences between the two extreme counters as well as the triple coincidences are recorded. If C_{13} is the double coincidence rate between the outside counters 1 and 3, C_{123} the triple coincidence rate, A_{13} and A_{123} the rate of double and triple "accidental" coincidences produced by events other than the passage of a single ray through the counters, i.e., coincidences caused by unassociated rays or by sidewise showers of rays, then the efficiency is given by

$$E=(C_{123}-A_{123})/(C_{13}-A_{13}). \tag{18}$$

The accidental rates A_{13} and A_{123} can be determined by subsidiary experiments in which the counters are separated at some distance, for the coincidence rate caused by sidewise showers and unassociated rays is independent of the separation unless concentrated material is near at hand.

Counter efficiencies under normal conditions of operation can be as high as 98 percent, the figure varying with the pressure, size, radiation intensity and the circuit for recording the discharges.

(4) The accidental counting rate[266, 267] due to unassociated rays is a function of the *resolving time* τ. This quantity may be defined as the minimum time separation between an $(n-1)$-fold coincidence in an n-fold coincidence train and the passage of an unrelated ray through the nth counter for which an n-fold coincidence will not be recorded. τ is distinctly a circuit constant falling between 10^{-3} and 10^{-6} sec. in the usual circuits but it is limited by the breakdown time of the counter. If C_1 and C_2 are the counting rates of two single counters connected in coincidence but spatially separated so that the real coincidences caused by single rays and showers may be neglected, the accidental rate of coincidences may be expressed by

$$A_{12}=C_1(1-e^{-C_2\tau})+C_2(1-e^{-C_1\tau})\doteqdot 2C_1C_2\tau. \quad (19)$$

if $C\tau \ll 1$. In practice τ is usually determined by applying Eq. (19) to the accidental rate observed with an arrangement of counters in which the coincidence rate from associated rays is negligible. In a triple coincidence train the accidental rate is given by

$$A_{123}\doteqdot(C_{23}C_1+C_{13}C_2+C_{12}C_3)\tau, \quad (20)$$

where the coincidence rates between pairs of counters are indicated by the subscripts. These, of course, refer to the double coincidences which take place without the third counter being discharged at the same time.

(5) The number of coincidences in a counter train, after correction has been made for the efficiency and accidental counts, is equal to the number of events in which one or more rays traverse the counter train. If j_1 is the number of single rays per sec. per unit solid angle, j_2 the number of associated pairs, etc., then the ray intensity j_0 computed from the observed counting rate of a coincidence train may be expressed by

$$j_0=j_1+j_2+j_3=\sum_\nu j_\nu$$

whereas the actual number of rays per unit solid angle is

$$j=\sum_\nu \nu j_\nu. \quad (22)$$

Writing $r_\nu=j_\nu/j_1$ and $S_1=\sum_\nu \nu r_\nu/\sum_\nu r_\nu$ (23)

we may express the ray intensity by

$$j=S_1 j_0. \quad (24)$$

The shower factor S_1 is a function of the distribution and kind of material over and near the counter train, as well as the area and solid angle subtended. The shower factor also varies with the elevation above sea level, because of the larger proportion of shower-producing radiation at the higher elevations.

(6) Showers of rays which might discharge the counters simultaneously without the passage of a single ray through all counters can be reduced if the counter train is operated in the open, away from massive material above and at the sides of the apparatus. A lead shield around one or more of the counters has also been used for this purpose, its effectiveness depending upon the fact that most of the shower rays are soft. In some experiments conducted in the stratosphere, Swann, Locher and Danforth[125] have used a wall of shielding counters for the same purpose, the circuits having been arranged to prevent the recording of a coincidence in the main train if the shielding counters were also simultaneously discharged. In the usual arrangement of coincidence counters the correction for events of this nature may be determined by observing the count when one of the counters is displaced slightly from the line of the train.

(7) The theory[265–267] of the correction for the finite time of operation of the mechanical recording device is similar to that already given under (3a) in the case of the finite recovery time of a counter. If T is the operation time of the recorder, in practice of the order of 0.01 sec., the efficiency of the recorder circuit can be written

$$E_r=e^{-\rho T},$$

where ρ is the true rate of occurrence of coincidences.

(8) The correction for absorption of rays within the counter train can be determined if a

pair of thin-walled counters is used in a double coincidence train and the counter for which the correction factor is desired is inserted between the two as an absorber. The count with and without the absorber gives the fraction stopped.

For the measurement of relative intensities as a function of direction at a single station, it is only necessary to correct the counting rate for accidentals and for the finite solid angle. On the other hand, when intensities at different elevations are being compared, it is necessary to make a correction for the change in efficiency as well as the accidental rate. These corrections, though troublesome, are always possible.

The absolute intensity of the cosmic radiation for high latitudes in the vertical direction at sea level, reduced to unit solid angle and corrected for all of the effects of significant magnitude, indicated in the preceding paragraphs, has been determined by Street and Woodward,[91] who find $j=0.0133\pm0.0005$ ionizing rays per cm^2 per unit solid angle per sec. In computing the number J of rays per cm^2 from all directions they have used the angular distribution for sea level as determined by Johnson and Stevenson,[100] Fig. 8, from which they find by means of Eq. (1) $J=0.0247\pm0.0009$ ionizing ray per sec. per cm^2. Cosyns[97] has made a similar determination at Brussels and finds $J=0.0266\pm0.0003$ and recently Froman and Stearns[92] have reported similar measurements with a redetermination of the angular distribution and find $j=0.0162\pm0.0005$ and $J=0.0303\pm0.001$. While these determinations are not in agreement with one another to within the probable errors stated, the probable errors have been determined from the measurements themselves and not from the reproducibility of the determination from one time to another. Since variations of the cosmic-ray intensity from one time to another are of the same order as the discrepancy between the value of Street and Woodward and that of Cosyns, a better agreement could hardly have been expected. The value of Froman and Stearns is somewhat higher than could be accounted for in this way, although even with their value included, the spread in the various counter determinations of j is not as great as exists in the various determinations of the ionization I, reviewed in the last section.

The experiments of Street and Woodward, of Cosyns, and of Froman and Stearns have not been corrected for showers according to Eq. (24) but the conditions attending these experiments were unfavorable to the occurrence of many groups of simultaneous rays. A count of the showers recorded in the randomly exposed cloud photographs of Anderson[101] indicate a shower factor $S_1=1.25$. Since his cloud chamber was surrounded by a massive iron magnet the showers were probably more frequent than under the conditions of the counter experiments and it is reasonable to suppose that less than a ten percent correction need be applied to the above values of j on this account. The actual number of rays at sea level is thus very nearly $j=0.015$ ray per sec. per unit solid angle in the vertical.

b. Other determinations of J

No serious attempt has been made to determine J with a cloud chamber. Skobelzyn[26] estimated from the observation of 32 tracks that the rate of occurrence was $J=0.02$ per sec. Though not a precise result, this determination was important in that it indicated an accord between the intensities measured with the cloud chamber and those recorded by counters, and it suggested the use of counters for the control of the expansion chamber.[102]* Probably the greatest uncertainty attendant in a cloud-chamber measurement of intensity is the sensitive time per expansion.

Other estimates[103] of the value of J have been made on the basis of single and coincidence counter measurements but the precision is in no other case comparable with that of the three determinations cited above and they will not be discussed in detail.

The determination of J from analysis of the fluctuations of the ionization current in an electroscope is considered in detail in the next section. It depends upon the determination of N, the specific ionization per cm of path of the ionizing particles, and upon the ionization according to Eq. (3).

---

* Mott-Smith and Locher,[274] and Johnson, Fleischer and Street[275] also showed that a close correspondence existed between the coincidences of Geiger-Mueller counters and the appearance of tracks in the cloud chamber.

5. Determinations of the Specific Ionization N by Cosmic Rays

When an ionizing particle passes through matter it produces N_p primary ions per cm by direct interaction with the atoms of the medium. Some of the electrons so liberated have sufficient energy to produce other secondary ions and the total ionization N_t is the sum of the primary and secondary ionizations. In discussing the experimental results it is necessary to distinguish between N_t and N_p because of the different statistical distributions of the two types of ions. Whereas the primary ions are randomly distributed along the track of the primary ray the secondary ions occur in clusters which are often of such density that in some experiments the ions do not individually manifest themselves. The energy lost by the primary ray is equal to the sum of the energies of the primary ions, or, since the energy is ultimately spent in the production of secondary ions with an average expenditure of k per ion pair, the total energy lost by the primary rays may also be expressed by

$$-dT/dx = kN_t;$$

the energy lost by the excitation of atoms without ionization is included in k.

A third quantity more nearly comparable with the ion count observed in the cloud chamber with delayed expansion, and known as the "probable specific ionization," has been introduced by Williams[57] and Oppenheimer[90] to designate the sum of the primary ions and those secondary ions which are produced by electrons of less than some critical energy, the value of which depends somewhat upon the experimental conditions and is of the order of 10^4 volts. In cloud-chamber studies secondary electrons of greater energy, curled up in the magnetic field, produce such dense clusters of ions that the resolution of the individual ions is impossible.

The quantum theory of the energy loss of high speed particles by ionization has been given by Bethe[104] who finds a rate of loss of energy given as a function of $\beta = v/c$ by

$$-dT/dx = (A/\beta^2)[\log K + \log \beta^2/(1-\beta^2) - \beta^2]. \quad (25)$$

The value of K depends inversely upon the effective ionization potential of the gas and upon whether it is the probable or the primary ionization that is being described. For the primary ionization in hydrogen, K is of the order of 10^5, and $A = 4Z^2e^4n/mc^2$ where n is the electron density of the gas and Ze is the charge on the ray. According to (25) the ionization falls off inversely as β^2 in the range $\beta \ll 1$. It passes through a minimum at $\beta = 0.97$ (energy $= 3\ mc^2$) and it increases logarithmically with higher energies. At 10^9 volts the theoretical ionization by an electron is 1.7 times its minimum value. In hydrogen the theoretical value of N_p is 4.5 ions per cm at the minimum and 8 ions per cm at 10^9 volts. The theory of more complex atoms and molecules meets with analytical difficulties in regard to the value of K, but this is not critical in its effect upon the relative values of the ionization at different energies nor upon the absolute values. In the case of N_2 Corson and Brode[113] find that $K = 2\times 10^4$ brings (25) into good agreement with the results (see Fig. 9).

There are several experimental methods for the determination of the specific ionization, but the values obtained diverge so widely that there remains considerable doubt as to what the true value of this important constant is. The methods fall into two groups according to whether the value of N obtained approximates more closely to the total ionization or to the primary ionization. Of those methods which give the total ionization, perhaps the most direct is to combine the measurement of the ray intensity with that of the ionization intensity. By the use of Millikan's[45] figure, $I = 2.48$ ions per sec. in air and Street and Woodward's[91] figure $J = 0.0247$ ray per sec., we obtain from Eq. (3), the value $N_t = 100$ ions per cm. It should be pointed out that the ray intensity in the Millikan electroscope may be a little higher than under the conditions of Street and Woodward's experiment, because of the secondary shower rays generated in the electroscope walls. It is also noted that Street and Woodward's value for J has not been corrected for showers and should, therefore, be increased about ten percent. The value of N_t must, therefore, be reduced and it is probably of the order of 80 to 90 ions per cm. The principal uncertainty in this figure is in the value of I

since this is affected by recombination and electrode effects which are difficult to estimate. Compton, Wollan and Bennett's value for J reduced to zero shield, would make $N_t \doteqdot 60$ ions per cm.

Combining his own determination of J with Clay's[52, 48] value for the ionization in argon,

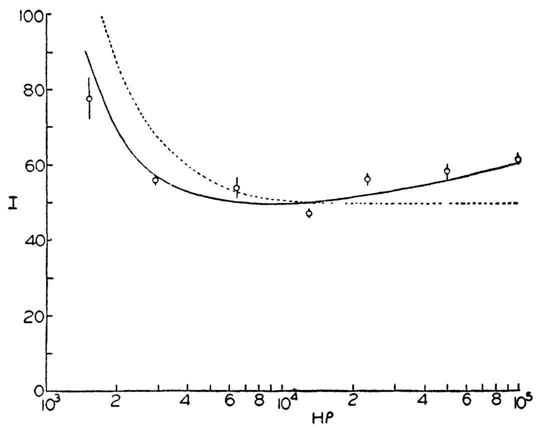


FIG. 9. The number of ions (twice the specific ionization) per cm in N_2. The continuous curve is the theoretical equation (25), the dotted curve represents the classical inverse-square-of-the-velocity law and the points indicate the droplet counts of Corson and Brode.[118] Ordinates: ions per cm which are also proportional to the rate of loss of energy $-dT/dx$. Abscissae: the magnetic rigidity or momentum plotted logarithmically.

Cosyns[97] computes $N_t = 71.4$ ions per cm. Again no correction has been made for simultaneous rays and the correction for this effect in Clay's experiments in which thick iron shields were used must be an important consideration.

A method which avoids the uncertainty of the number of secondaries generated in the walls of the electroscope has been used by Kolhörster and Tuwim.[103] They have placed a single counter inside the electroscope shell and have determined the ray intensity J from its counting rate. The use of a single counter for this purpose involves an elaborate geometrical analysis of the path distribution of rays through the counter in relation to the angular distribution of the radiation and the orientation of the counter. From their analysis of the experimentally determined counting rates and ionization currents they have computed

$N_t = 135$ ion pairs per cm under 10 cm of lead.

They have not considered simultaneous rays through the counter although there must have been many showers originating in the lead shield. Thus it is possible to account qualitatively for their high value.

The method of fluctuational analysis of the ionization in a chamber for the determination of the ray intensity has the advantage of the Kolhörster-Tuwim method in that the ray intensity is measured inside the chamber simultaneously with the measurements of the ionization. In this way an estimate of $N = 110$ ions per cm was given by Messerschmidt.[105] A more rigorous analysis of the fluctuations in a spherical chamber was given by Evans and Neher.[106]

In this case the angular distribution of the radiation does not enter into consideration since the average path through the chamber is the same for all directions. A ray passing at a distance p from the center of the chamber of radius R will have a path length

$$l_p = 2(R^2 - p^2)^{\frac{1}{2}}. \tag{26}$$

The average number of paths between p and $p+dp$ per observation period t when the ray intensity is J, is

$$dx_p = 2\pi J t p dp \tag{27}$$

and the ionization produced by one such path is

$$q_p = l_p N. \tag{28}$$

The average number of pairs of ions produced in the vessel in the time t is

$$Q = \int_0^R q_p dx_p = \bar{q} J \pi R^2 t, \tag{29}$$

where $\bar{q} = (4/3)RN$ is the average ionization per path.

The mean square of the deviations δ in the number of rays dx_p, according to the Gauss equation is, as already stated in (13),

$$\int_{-\infty}^{\infty} \delta^2 P(\delta) d\delta = dx_p \tag{30}$$

and the mean square deviation in the corresponding element of ionization is $q_p^2 dx_p$. The fluctuations in the elements of ionization are mutually independent and their squares are additive. Hence the mean square deviation of the total ionization is

$$\Delta Q^2 = \int_0^R q_p^2 dx_p = 2\pi R^4 N^2 J t, \tag{31}$$

or, dividing through by Eq. (29)

$$\Delta Q^2 / Q = \tfrac{3}{2} NR = \tfrac{9}{8} \bar{q}. \tag{32}$$

The quantity $\Delta Q^2/Q$ may also be determined from the residuals from the mean of a series of measurements in which t is sufficiently short so that the statistical fluctuations are large compared with the variations of cosmic-ray intensity arising from meteorological or cosmic causes. Thus Eq. (32) may be used for the evaluation of N.

Another equation, giving the precision of a measurement made with an ionization chamber, follows immediately from (29) and (31);

$$\Delta Q/Q = \tfrac{3}{4}(2/\pi)^{\frac{1}{2}}/R(Jt)^{\frac{1}{2}}. \quad (33)$$

Thus far no consideration has been given to the occurrence of showers. Since these consist of several nonindependent rays the fluctuations resulting from them will be greater than would arise from the random distribution of independent rays. The average total ionization in the time t caused by showers consisting of ν rays may be written

$$Q_\nu = \nu\bar{q}J_\nu t = Q_1\nu r_\nu, \quad (34)$$

where J_ν is the number of showers per second in which there are ν rays through the chamber, r_ν is defined as in Eq. (23) as the ratio of the frequency of ν-ray showers to the frequency of single rays, and Q_1 is the total ionization in time t due to single rays.

The fluctuations in the part of the ionization produced by the ν-ray showers may be expressed by an equation analogous to Eq. (31)

$$(\Delta Q_\nu)^2 = 2\pi\nu^2N^2R^4J_\nu t = \nu^2 r_\nu(\Delta Q_1)^2. \quad (35)$$

Since the fluctuations produced by each type of shower are independent they must be compounded by taking the sum of their squares, and

$$(\Delta Q)^2 = \sum_\nu(\Delta Q_\nu)^2 = (\Delta Q_1)^2\sum_\nu \nu^2 r_\nu. \quad (36)$$

On the other hand, the average ionizations caused by each type of shower are summed to give the total ionization;

$$Q = Q_1\sum_\nu \nu r_\nu. \quad (37)$$

The final equation for the determination of N, analogous to Eq. (32) is

$$(\Delta Q)^2/Q = \tfrac{3}{2}NRS_2, \quad (38)$$

where the shower factor S_2 is defined by

$$S_2 = \sum_\nu \nu^2 r_\nu / \sum_\nu \nu r_\nu. \quad (39)$$

The number of events per cm² in which one or more rays pass through the chamber may be expressed as in Eq. (21) by

$$J_0 = \sum_\nu J_\nu = J_1\sum_\nu r_\nu. \quad (40)$$

This number corresponds with the number of counts per cm² which would be recorded by a counter of the dimensions of the ionization chamber. Having determined N from Eq. (38), J_0 may be determined from Eq. (37), which in expanded form may be written

$$Q = (4\pi/3)NR^3tJ_0S_1, \quad (41)$$

where S_1 is the first shower factor defined by (23). Dividing (41) through by the time and by the volume it reduces to

$$I = NJ_0S_1. \quad (42)$$

When showers are considered, the root mean square deviation is given, analogously with Eq. (33) by

$$\Delta Q/Q = \tfrac{3}{4}(2/\pi)^{\frac{1}{2}}/R(J_0tS_1/S_2)^{\frac{1}{2}}. \quad (43)$$

The shower factors S_1 and S_2 can be estimated from the multiplicity of rays observed in the cloud chamber. On the basis of randomly exposed photographs of Anderson,[101] Locher[108] and Skobelzyn[26] and by taking into consideration the effect that the shielding of the Millikan and Neher electroscope might have upon these factors, the following estimates may be given for sea level:

$$S_1 = 1.3; \qquad S_2 = 2.0.$$

With these values of the shower factors the fluctuations observed by Evans and Neher for fifteen-minute periods of observation reduce to $N = 65$ ions per cm,

$$J_0 = 0.021 \text{ events}; \quad J = S_1J_0 = 0.027 \text{ rays per cm}^2.$$

This value of J is probably a little higher than would have been found by counters similarly shielded according to Street and Woodward's results, and the value of N is correspondingly lower.

Still another method for the determination of N_t has been used by Swann.[109] A cylindrical ionization chamber of length L was connected through a vacuum tube amplifier to a short period galvanometer which recorded the irregularities of ionization produced by the passage of single cosmic rays through the chamber. By a suitable correlation between the sizes of these irregularities and the distribution of path lengths in the chamber, it was possible to identify a particular size of irregularity with the passage of a ray through the long dimension of the chamber. From the known length of path, the measured irregularity in the ionization, and the pressure in the chamber, Swann found for a pressure of one atmosphere

N_t(argon) = 50 ions per cm,
N_t(oxygen) = 32 ions per cm,
N_t(nitrogen) = 34 ions per cm.*

Droplet counts in cloud-chamber photographs provide still another method for the determina-

* These data represent corrected values based upon later experiments (see reference 276), the results of which have not yet been published in full detail. Higher values found in the first experiments were to be associated with electron pairs which also made their appearance as a subsidiary peak in the later experiments.

tion of the specific ionization. Locher[108] mentions several reasons for mistrusting the results obtained in this way, pointing out that droplets produced by the secondary electrons and Auger electrons occur in clusters and are difficult to count accurately. Another difficulty is that some of the ionization is produced at a distance from the track through the intermediary of x-rays. Finally, there is no electric field for the separation of the ions and preferential and columnar recombination may assume considerable proportions. The confusion of the droplets in clusters may be largely overcome by the use of instantaneous spark photographs taken with a camera of small aperture for good depth of focus. With this improvement in technique Locher[110] obtained counts on tracks of unknown energies (not magnetically resolved) ranging from 30.3 to 79 ion pairs per cm in argon at 68 cm pressure. When converted to air at standard temperature and pressure the values range from 27 to 71 pairs per cm. Anderson and Neddermeyer[111] have reported 31 ion pairs per cm in air by direct count of the droplets, whereas the loss of energy of single rays while passing through a lead plate has a minimum value corresponding to between 120 and 140 ion pairs per cm (if 31 volts is the average energy spent in the production of one ion pair). Since radiation losses occur infrequently and in large amounts it is probable that the lowest energy losses observed in a thin lead plate correspond to the total specific ionization N_t, whereas the direct droplet count with the usual technique may correspond more closely with the primary ionization, N_p.

Kunze[112] has reported droplet counts of 19 ion pairs per cm which he believes represents the primary ionization.

Corson and Brode[113] have made counts on tracks diffused by delayed expansion so that the droplets were well separated from one another. They point out that their count gives the "probable" ionization, i.e., the ions produced by the primary and by secondaries of less than a certain critical energy. The higher energy secondaries which occur only rarely, produce dense clusters of ions and these are not sufficiently resolved for an accurate count. They have studied the variation of the specific ionization with the magnetic deflectability or the energy of the primary particle, and within the probable errors the results agree with the theoretical prediction that the ionization is a minimum at about $\mathcal{E}=3mc^2$, increasing slightly in close agreement with the theory for higher energies up to $30mc^2$. The ionization in air which they determine from the diffuse tracks is about 50 ions per cm at the minimum or 25 ion pairs. Some counts on sharp tracks gave 14 to 18 droplets per cm and these were interpreted as indicating the primary ionization. From the observed rate of change of $H\rho$ with range, assuming 32 volts is lost per ion pair, the total ionization must have been about twice the observed probable ionization, or about 50 ion pairs per cm in air.

A fifth method for the determination of the specific ionization N depends upon measuring the efficiency of a Geiger-Mueller counter. In calculating the probability of the production of one or more ions by the ray along its path through the counter, as in Eq. (17), one assumes a random spatial distribution, and the result must correspond to the primary ionization N_p. Danforth and Ramsey[96] find 21 ion pairs per cm in standard air and 6.2 per cm in H_2. Cosyns[97] has carried through an elaborate analysis of the path distributions through the counter and finds values of N_p by this method which are in fair agreement with those of Danforth and Ramsey. For hydrogen he finds $N_p=5.96$; for helium $N_p=5.96$; and for argon $N_p=29.4$. All of these values pertain to cosmic-ray ionizing particles. Using the beta-rays from UX_1 whose mean energy is 1 Mev, Cosyns finds $N_t=8.2$ in hydrogen.

Table I presents a summary of the various experimental determinations of N.

6. Determination of k, the Energy Expended per Ion Pair

Because of its connection with the determinations of the energy loss of cosmic-ray particles a brief resumé of the evidence as to the value of k, the average energy per ion pair, will also be given. The best measurements of this constant have been made by observation of the total ionization produced in a vessel by a beam of

TABLE I. *Summary of the Experimental Determinations of N.*

Method	Gas	N	Remarks
Ratio of Ion to Ray Intensity			
Street and Woodward—Millikan	Air	100	Not corrected for showers.
Street and Woodward—Millikan	Air	90	With estimated correction for showers.
Froman and Stearns—Millikan	Air	83	Not corrected for showers.
Cosyns—Clay	Argon	71	Ionization corrected for "wall rays."
Kolhörster and Tuwim (single counter and electroscope)	Air	135	Uncorrected for showers.
		104	Corrected for showers with $S_1=1.3$.
Evans and Neher (fluctuations)	Air	65	Corrected for showers.
Messerschmidt (fluctuations)	Air	110	Corrected for showers.
Ionization by single rays			
Swann	Air	50	
	O_2	32	
	H_2	34	
Droplet count in cloud chamber			
Skobelzyn	Air	40	
Locher	Air	36	Number of droplets in clusters estimated or counted.
Locher	Argon	30–79	
Kunze	Air	19	Clusters counted as one.
Anderson	Air	31	
Corson and Brode	Nitrogen	50–65	Probable ionization.
Counter efficiency			
Danforth and Ramsay	Air	21	Corresponds to primary ionization.
Cosyns	H_2	5.9	Corresponds to primary ionization.
	He	5.9	
	Argon	29.4	
	H_2	8.2	1-Mev electrons from UX_1.

electrons of known energy (usually a few thousand volts) when the vessel is filled with gas to a pressure sufficient to absorb completely the electron stream. In this way Lehman and Osgood[114] found a value of k in air of 45 volts per ion pair. This value was corroborated by Schmitz,[115] but Buchmann,[116] by the same method, found a value $k=32$ volts and this value was also confirmed in a very careful piece of work by Eisl.[117] The cause of such discrepancies, as are here indicated, according to E. J. Williams,[57] must lie in the measurement of the total ionization and he mentions a suggestion of Gurney and Oliphant[56] that electron emission from electrodes and impurities may contribute appreciably to the observed ionization. Here, as in the measurement of the ionization by cosmic radiation, there must be a source of uncertainty which has not yet been definitely discovered.

7. The Geomagnetic Cosmic-Ray Effects and the Corpuscular Hypothesis

Introductory remarks

Although practically all interpretations[118–122] of the cosmic radiation prior to 1929 were made on the assumption that the primary rays were photons, the evidence supporting this assumption was, of course, very incomplete and the point of view was justified mainly by the fact that gamma-rays from radioactive substances were known to be more penetrating than electrons of the same energy. The shape of the ionization-depth curve[123–126] at the top of the atmosphere, its approximately exponential form at greater depths, and the discovery by Anderson[127, 128] of photon-produced secondary electrons also suggested that at least a good portion of the primary rays were photons, and it was only in the light of more recent knowledge of the radiative properties of high energy electrons and of the creation of positive and negative pairs by high energy photons that these same effects could be given the alternative interpretation[85–87] based upon the assumption that the primary rays were electrons.

As early as 1924 C. T. R. Wilson[129] pointed out that beta-rays of sufficiently high energy might produce the observed cosmic-ray effects and he sought processes for the origin of such particles in thunderstorms. In 1927 Skobelzyn[26] found high energy beta-rays in the Wilson cloud chamber; their frequency was sufficient to account for the whole of the cosmic-ray ionization and their energies were at least an order of magnitude higher than those of the known radioactive radiations. In 1929, Bothe and Kolhörster[27] showed by coincidence counter experiments that the penetrating power of the ionizing corpuscles was about equal to that deduced from the variation with depth of the total cosmic radiation and this finding prompted them to suggest that the ionizing particles with which they were dealing were the primary cosmic rays themselves. They also pointed out that the rays of lower energy should be excluded from the equatorial zone by the earth's magnetic field and that this would account for the lower intensity near the equator, already indicated by the experiments of Clay.[130]

Although Clay[130, 131] had found consistent evidence for an equatorial dip in the intensity of about eleven percent on three different voyages between Java and Holland, the failure of others to find variations over the earth's surface in polar latitudes,[132–135] and Millikan and Cameron's[136] finding of nearly equal intensities in California and Bolivia* caused many, including Clay himself, to question the reality of the effect, and it was not until the world-wide survey of Compton and his associates[137–141] and of numerous other individuals[77, 142–156] that the true nature of the effect was realized.

In 1931 Rossi[157–159] showed that Stoermer's[160–165] theory of the orbits of electrons in the magnetic field of the earth would lead to the prediction of an east-west asymmetry in the cosmic-ray intensity if the primary rays were preponderantly of one sign of charge. Employing an arrangement of double coincidence counters he looked for an effect of this nature in Florence, Italy,[157, 158] but the result was negative. In the following year Johnson and Street[167, 11] made similar experiments on Mt. Washington, 51°N, 8 meters atmospheric depth, with an arrangement of triple coincidence counters which afforded a higher precision and a better angular resolution than the arrangement used by Rossi. They observed a significantly greater intensity from the west than from the east and although the difference was small, their positive results did much to hasten further work in the equatorial zone. In 1932, the western excess was confirmed by a number of experimenters[168–186] working in both equatorial and high latitudes. Briefly stated, these experiments showed that the excess of the western over eastern intensity increased with zenith angle and attained a maximum at about 45° from the zenith. In the equatorial zone the excess at sea level amounted to approximately fifteen percent; at higher elevations the equatorial asymmetry was somewhat greater. The asymmetry decreased towards higher latitudes and at 50°N it had fallen to a value of the order of two or three percent. The study of the asymmetry confirmed the corpuscular hypothesis of the nature of the primary radiation and revealed that the primaries instrumental in producing effects near sea level were largely positively charged.

8. Theory of the Geomagnetic Effects

a. Introductory remarks

The theory of the orbits of electrons in the dipole magnetic field of the earth had received a considerable development as early as 1904 by Stoermer[160] who was at that time especially interested in the aurora borealis. His results showed that for rays of a given energy there was an inaccessible equatorial zone. At a certain critical latitude rays began to arrive within a narrow cone about the western horizon, in the case of positive charges, and as the latitude was increased this cone broadened out until finally at a second critical latitude it encompassed the

* The points on Millikan and Cameron's ionization-depth curve obtained in Bolivia actually fell five or six percent below those obtained in California, but the precision of the measurements was not sufficient to indicate to the authors that this difference was significant.

entire hemisphere. Although from his analysis the inaccessible regions were well defined, it was not clear[197] just what intensity would be realized just inside of the accessible cone, and it was only after Lemaitre and Vallarta[187] and Swann[188] had pointed out the applicability of Liouville's general dynamical theorem to the problem that a connection was established between the dynamical theory of the orbits and the problem of the intensity of the cosmic radiation as a function of latitude and direction. Liouville's theorem implies that the number of primary cosmic-ray particles of a given momentum incident from unit solid angle per cm^2 per sec. is the same in all accessible directions in the accessible regions of space; and the intensity problem was thereby resolved into the problem of finding for what range of momenta a given direction in a given region is accessible. In Stoermer's theory, and to an approximation, in the more rigorous orbital theory of Lemaitre and Vallarta, this range of momenta extends continuously from a lower limit to infinity and the total intensity, j, defined as in Section 2 as the number of rays per cm^2 per sec. per unit solid angle of all momenta above the atmosphere may be expressed as a definite integral of the primary momentum distribution. If all rays have the same charge and mass we may use the energy instead of the momentum, and express the total intensity in the form

$$j=\int_{\mathcal{E}_c}^{\infty} j(\mathcal{E})d\mathcal{E}, \tag{44}$$

where $j(\mathcal{E})$ is the unidirectional spectral intensity in external space and $\mathcal{E}_c$ is the lower limit of the energies for which the direction in question is accessible. In a given latitude $\mathcal{E}_c$, for positive rays, increases from west to east and for a given direction $\mathcal{E}_c$ decreases with increasing latitude. Thus the variations of the intensity giving rise to the geomagnetic effects make their appearance in the theory through the variations of the lower limit of the integral (44).

b. Stoermer's theory of the excluded regions

To find the lowest energy for which a given region is accessible it is necessary to consider the equations of motion[163–165] of an electrically charged particle in the magnetic field of the earth. This field is assumed to be representable by a dipole of strength $-M$ located at the magnetic center of the earth and oriented from north to south along the geomagnetic axis Z whose positive direction is northward. The equations

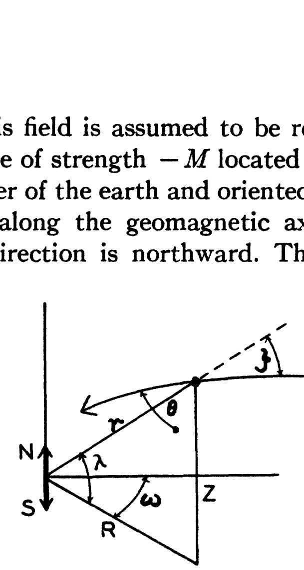


FIG. 10. Diagram showing the significance of the coordinates used for describing the motion of a charged particle in the field of a dipole. ζ is the zenith angle of the direction from which the ray approaches, θ is the angle between the forward direction of the ray and the projection of its orbit upon the meridian plane, ω is the longitude and λ is the latitude of the instantaneous position of the ray.

of motion are handled most readily when expressed in the cylindrical coordinates (see Fig. 10)

$$R=r\cos\lambda,\quad z=r\sin\lambda,\quad \text{and}\quad \omega,$$

where r is the radial distance from the center of the earth to the particle, λ is the geomagnetic latitude and ω is the longitude for which the positive direction is westward. The unit vectors $\mathbf{R}_1$, $\mathbf{Z}_1$, ω_1 then constitute a right-handed set. The magnetic field in these coordinates is given by

$$\mathbf{H}=-(2M/r^3)(1-3R^2/2r^2)\mathbf{Z}_1 -(3MRz/r^5)\mathbf{R}_1. \tag{45}$$

The equation of motion of a particle of charge e e.s.u., mass m and velocity v is

$$d^2\mathbf{r}/dt^2=(e/mc)(\mathbf{v}\times\mathbf{H}), \tag{46}$$

where the cross represents vector multiplication.

The acceleration is always perpendicular to the velocity so that v remains constant throughout the motion and it is convenient to use the orbital distance $s=vt$ instead of the time as the independent variable. The kinetic mass m also remains constant and the equations are not altered by relativity considerations.[166] Eq. (46) may now be normalized so as to free it of the magnetic moment of the earth (M) and of the magnetic

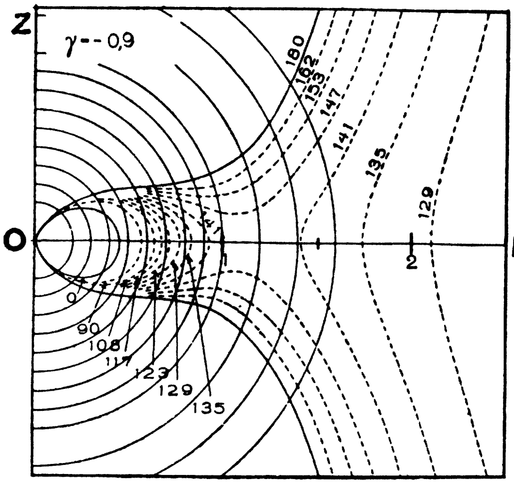


FIG. 11. A typical Q diagram for $\gamma=-0.09$. The contours of constant Q are designated in the figure by values of the angle $180°-\theta$, i.e., the angle between the direction of the ray and eastward drawn normal to the meridian plane. The circles have their common center at the position of the dipole and represent the surface of the earth, radius r_0, corresponding to various values of the rigidity, r_0^2. The units in which r_0 is measured are indicated on the line OR. (Stoermer, references 163–165.)

rigidity, mcv/e, of the ray if a unit of length

$$C=(Me/mcv)^{\frac{1}{2}}\text{ cm} \tag{47}$$

is used. This unit has the physical significance of being the radius of the circular periodic orbit of a ray of rigidity mcv/e in the equatorial plane of the dipole of strength M. The unit of length thus depends upon the rigidity of the ray and any particular assignment of the rigidity fixes the scale to which the radius of the earth must be represented in relation to the regions traversed by the orbits. In every other detail the problem is independent of the rigidity and of the strength of the dipole. So normalized, Eq. (40) reduces to

$$d^2\mathbf{r}/ds^2=\mathbf{v}_1\times\mathbf{H}_1, \tag{48}$$

where $\mathbf{v}_1$ is a unit vector in the direction of the orbit and $\mathbf{H}_1$ is the field due to a dipole of strength -1.

In the system of cylindrical coordinates Eq. (48) has the components

$$d^2R/ds^2-R(d\omega/ds)^2 =(2R/r^3)(1-3R^2/2r^2)(d\omega/ds), \tag{49}$$

$$d^2z/ds^2=-(3R^2z/r^5)(d\omega/ds), \tag{50}$$

$$Rd^2\omega/ds^2+2(dr/ds)(d\omega/ds) =-(2/r^3)(1-3R^2/2r^2)(dR/ds) +(3Rz/r^5)(dz/ds). \tag{51}$$

Equation (51), when multiplied through by R, reduces to

$$(d/ds)(R^2d\omega/ds)=-(d/ds)(R^2/r^3), \tag{52}$$

of which the integral is

$$-R^2(d\omega/ds)=R^2/r^3+2\gamma, \tag{53}$$

where 2γ is the constant of integration. If Eq. (53) is multiplied through by mv, its physical significance becomes evident, for then the left-hand side is the moment of the momentum of the particle about the Z axis. At infinity the term R^2/r^3 vanishes leaving $2\ mv\gamma$ as the initial moment of the momentum. Dividing through by R and realizing that $R(d\omega/ds)$ is the sine of the angle θ between the orbit and its projection upon the meridian plane, we may rewrite Eq. (53)

$$-\sin\theta=R/r^3+2\gamma/R, \tag{54}$$

$$=\cos\lambda/r^2+2\gamma/r\cos\lambda. \tag{55}$$

Eq. (55) shows that the motion of the ray, for a given value of γ, is restricted to those regions of the meridian plane for which

$$-1<\cos\lambda/r^2+2\gamma/r\cos\lambda<1. \tag{56}$$

At any allowed point the component of the momentum perpendicular to the meridian plane is

$$mRd\omega/dt=-mv\cos\lambda/r^2-2\ mv\gamma/r\cos\lambda. \tag{57}$$

The other two components of the motion are obtained by integrating Eqs. (49) and (50). Although this integration cannot be carried out in terms of known functions, the equations may be simplified and shown to represent a type of motion analogous to that of a particle sliding without friction and acted upon by a force derivable from a potential. We define

$$Q\equiv 1-(R/r^3+2\gamma/R)^2=\cos^2\theta. \tag{58}$$

Then, in terms of Q Eqs. (52) and (53) reduce to

$$d^2R/ds^2=\tfrac{1}{2}(dQ/dR);\quad d^2z/ds^2=\tfrac{1}{2}(dQ/dz). \tag{59}$$

In these equations $-Q/2$ plays the role of a potential function for the two-dimensional motion in the R, z plane. It is also noted that if we regard dR/ds, instead of dR/dt, as the R component of the velocity, then the total energy of a particle of unit mass is

$$\tfrac{1}{2}(dR/ds)^2+\tfrac{1}{2}(dz/ds)^2+\tfrac{1}{2}(Rd\omega/ds)^2=\tfrac{1}{2}, \quad (60)$$

whereas the kinetic energy of the two-dimensional motion in the R, z plane is

$$\tfrac{1}{2}(dR/ds)^2+\tfrac{1}{2}(dz/ds)^2=\tfrac{1}{2}(1-\sin^2\theta)=\tfrac{1}{2}Q. \quad (61)$$

Therefore the potential energy of the fictitious particle may be thought of as being equal to the difference between the total energy and the kinetic energy, i.e.,

$$\text{Pot. Energy}=\tfrac{1}{2}(1-Q). \quad (62)$$

With the potential function defined at every point in the R, z plane the orbits defined by Eqs. (59) can be plotted out by numerical or mechanical integration. For this purpose it is convenient to have plotted out the lines $Q=\text{constant}$ and particularly the boundaries $Q=0$ of the accessible regions. A typical example of such a Q diagram is shown in Fig. 11 which has been plotted from Eq. (58) with $\gamma=-0.9$. The boundaries of the accessible regions for various other values of γ are shown in Fig. 12. For values of γ less than -1 the allowed region divides into two parts, a closed inner region and an external region. At $\gamma=-1$ the inner region begins to connect with external space but the excluded region is separated into two parts by narrow horn-shaped areas which run into the center of the dipole at either pole. For $\gamma>0$ the excluded regions merge together and form a spindle-shaped area with an enlarged diameter near the equator.

As already noted, the size of the earth in one of these Q diagrams is determined by the unit C in which its radius is measured, and this, in turn, depends upon the rigidity of the ray according to Eq. (47). The radius of the earth in units of C is given by

$$r_0=a/C, \quad (63)$$

where a is the radius measured in cm. It follows that r_0^2 is the ratio of the rigidity of the ray in question to that of a ray which describes a circular periodic orbit of radius a in the equatorial plane of a dipole of moment M. Thus r_0^2 measures the rigidity if we regard the magnetic moment and radius of the earth as fixed quantities. The

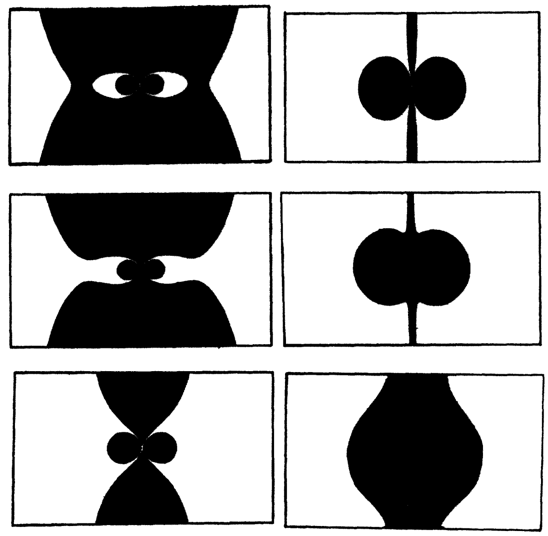


FIG. 12. Cross sections of the regions forbidden to charged particles of various initial angular momenta, $2mv\gamma$, in the magnetic field of a dipole. The position of the dipole in each figure is at the center with its axis pointing vertically. The orbits of particles of the values of γ designated are excluded from the regions traced out by revolving the darkened areas about the vertical axis. Values of γ are, from top to bottom, left-hand side, -1.016, -0.97 and -0.5; right-hand side, -0.05, 0.03 and 0.2. (Stoermer, references 163–165.)

corresponding value of the energy* measured in electron volts is

$$\mathcal{E}=(300m_0c^2/\epsilon)((e^2M^2r_0^4/m_0^2c^4a^4+1)^{\frac{1}{2}}-1); \quad (64)$$

or approximately

$$\mathcal{E}=300Mer_0^2/a^2\epsilon, \quad \text{if} \quad \mathcal{E}\gg 300m_0c^2/\epsilon. \quad (65)$$

Here m_0 and e are the rest mass and charge of the particle, ϵ is the electronic charge in e.s.u. and c is the velocity of light. With $M=8.1\times10^{25}$ gauss cm, $a=6.37\times10^8$ cm, and with $e=\epsilon$ Eq. (65) reduces to

$$\mathcal{E}=59.6r_0^2\ \text{Bev}. \quad (66)$$

* Frequently in the literature r_0 has been called the energy. Here we have avoided this inaccurate though convenient usage by introducing the term rigidity which is accurately measured by r_0^2. Lemaitre and Vallarta have introduced the *stoermer* as a unit in terms of which r_0 is measured. This practice tends to be somewhat confusing since r_0, from the point of view of ordinary space is the ratio of the fixed radius of the earth to the variable length C. However, when we adopted C as a unit of length we thereby made a transformation to a space in which C is fixed and the radius of the earth is variable. It is only in this system that the stoermer unit has a definite significance. The situation is analogous to that of a runner who chooses to measure lengths in terms of his distance from the starting line. He regards this distance as fixed and he sees the meter bar shrink as he runs along.

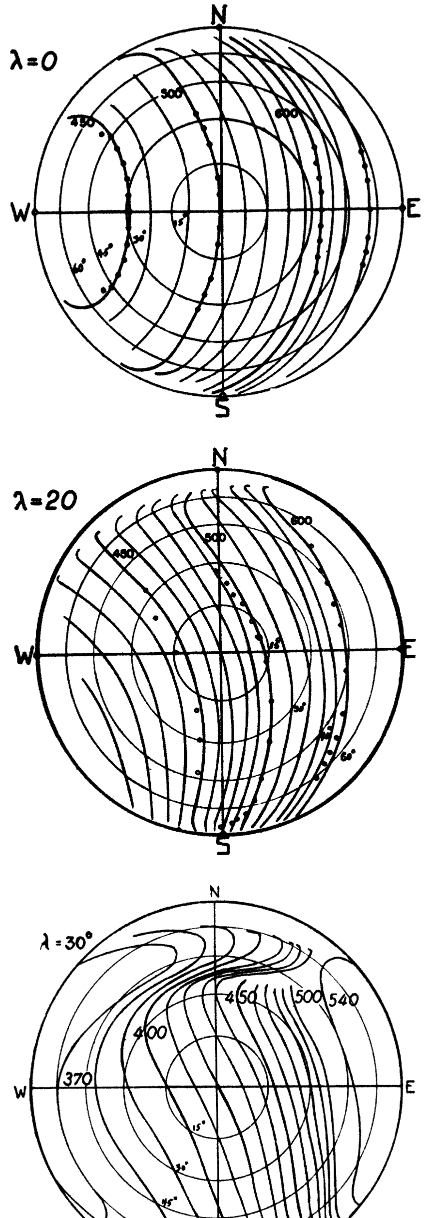


FIG. 13. Orthogonal projections on the plane tangent to the earth of the intersections with a unit sphere of the boundaries of the main cone for positive rays of various rigidities. The allowed directions lie to the left of the boundary. Diagrams are given for three latitudes in the Northern Hemisphere. Corresponding diagrams for negative rays may be constructed by forming the images of these diagrams in a mirror placed along the NS line. Diagrams suitable to the southern hemisphere are the images in a mirror placed along the EW line. Values of 1000 r_c are indicated on the curves. (Lemaitre and Vallarta, reference 195, 1936.)

The energies of various types of rays corresponding to r_0 are given in Table II.[187]

If attention is fixed upon rays of a given value of γ, the Q diagram is fixed. An assignment of the rigidity then determines the radius of the earth r_0 with respect to the Q diagram and the trajectory of the R, z components of the motion may be mapped out for any given initial direction of the motion at infinity. At each accessible latitude on the earth's surface the rays thus defined with respect to rigidity and γ are incident along the surface of a circular cone whose axis is perpendicular to the plane of the meridian and the complement θ of whose angle is defined as a function of the latitude by the following equation derived from Eq. (55) by substituting $r = r_0$;

$$-\sin\theta = \cos\lambda / r_0^2 + 2\gamma / r_0 \cos\lambda. \quad (67)$$

At the latitude of intersection of the earth with the boundary of the inner forbidden region, $\sin\theta = -1$, and positive rays are incident from the western horizon. At the intersection with the boundary of the outer forbidden region, which exists only for values of $\gamma < 0$, $\sin\theta = +1$, and the rays are incident from the eastern horizon. At the intersection with the contour $Q = 0$ the rays are incident within the meridian plane.

Equation (67) is a relation among four variables. In the above discussion attention has been focused upon the variation of the direction of incidence with respect to changes of latitude when the rays considered had fixed values of angular momentum and rigidity. This point of view is convenient in discussing the orbits, but for discussing the intensity we are interested in knowing for what range of the primary energy spectrum a given direction in a given latitude is an allowed direction. In other words, we must consider the variation of r_0 with respect to γ for fixed values of λ and θ, in order to find for what value of r_0 the Q diagram becomes closed and rays are not able to reach the earth from infinity. As already noted this happens when $\gamma = -1$ and the corresponding value of r_0 given by Eq. (67) sets the lower limit of the rigidity of rays which may reach the earth from infinity for the given value of λ and θ. If we designate the lower limit of the variable r_0 for which rays may enter from infinity by r_c the condition of the closing of the Q diagram at $\gamma=-1$ defines r_c according to the equation

$$\sin\theta = 2/r_c \cos\lambda - \cos\lambda/r_c^2. \quad (68)$$

This definition of r_c as a function of λ and θ will be called the Stoermer function, and the cone of allowed directions, given by θ as a function of r_c and λ will be called the Stoermer cone. In accordance with these considerations positive rays of rigidity r_c^2 may reach the earth on only the western side of this cone, and the intensity of positive rays at the boundary consists of rays whose rigidities are equal to or greater than r_c^2. The Stoermer cone for negative rays is given by $-\theta$ and the allowed directions are on the eastern side.

c. The Lemaitre-Vallarta theory of the allowed cone

Whereas no rays of rigidity less than that determined from Eq. (68) can reach the earth, Lemaitre and Vallarta,[187, 190–195] and Stoermer[163, 196] have shown that some orbits of higher rigidities than those specified by this limit fail to connect between the internal and external regions. In these cases γ is just greater than -1 and the Q diagram is open, but nevertheless the orbits in question do not pass through the neck. Lemaitre[192] has shown that such periodic and quasi-periodic orbits entirely disappear for values of γ greater than -0.78856, but for intermediate values of γ, between this limit and -1, it is necessary to investigate the orbits in detail in order to be able to predict from what directions rays of a given rigidity may enter the earth from infinite space. In presenting the results of these investigations it is convenient to divide the sky

TABLE II. *The energies in Bev of various types of rays corresponding to values of the Lemaitre-Vallarta variable,* r_0.

r_0	ELECTRONS	PROTONS	ALPHA-PARTICLES
0.1	0.596	0.1722	0.1842
0.2	2.38	1.618	2.308
0.3	5.36	4.49	7.60
0.4	9.54	8.61	15.64
0.5	14.90	13.97	26.25
0.6	21.45	20.50	39.28
0.7	29.20	28.23	54.6
0.8	38.21	37.19	72.5
0.9	48.30	47.29	92.7
1.0	59.6	58.5	115.2

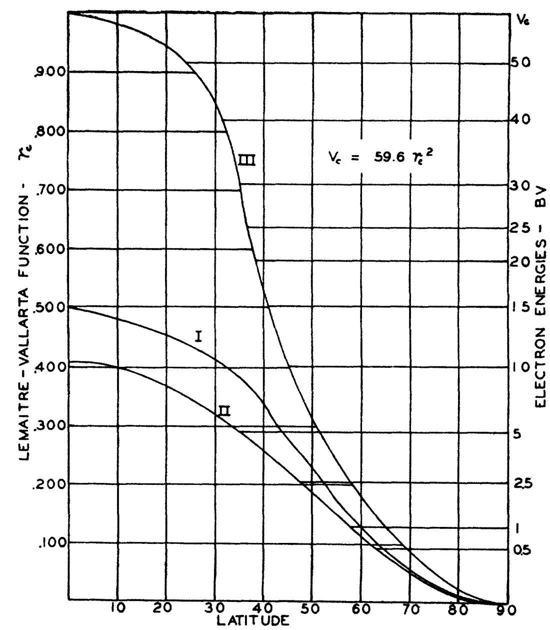


FIG. 14. The Lemaitre-Vallarta function r_c for three directions plotted against latitude. Curve I is for the vertical direction, curve II is for the western horizon and curve III is for the eastern horizon. Electron energies are given along the right margin. (Lemaitre and Vallarta.)

into four regions, (a) the Stoermer cone within which no directions are allowed, (b) the "main cone" within which all directions are allowed, (c) the "penumbra" which lies between the main cone and the Stoermer cone and is crossed by alternating bands of allowed and forbidden directions, giving the effect of partial illumination and (d) the shadow cone which lies close to the horizon adjacent to the nearer pole and within which lie only orbits which have passed one or more times through the earth before their arrival at the point in question.

Since the penumbra is made up largely of excluded directions it is sufficiently precise for most purposes to regard the allowed cone as consisting of that part of the sky lying within the main cone but not included in the shadow cone.[268] The directions of possible entry defined in this way have been computed by Lemaitre and Vallarta[195] for rays of various rigidities and these are represented in the diagrams of Fig. 13. There the curves are the orthogonal projections on the horizontal plane of the intersections of the boundaries of the main cones with a unit sphere whose center is at the position of the observer. The allowed directions for negative rays of the same rigidities are on the eastern side of a corresponding family of cones whose representation may be described as the mirror images of those for the positive rays when the mirror is placed along the NS line. Analogously with Eq. (68) these computations define the minimum value of the rigidity as a function of the latitude and direction for which the rays may reach the earth. The quantity $r_c(\zeta, \varphi, \lambda)$ defined in this way will be called the Lemaitre-Vallarta function, and the cone of irregular shape defined by ζ and φ for constant r_c and λ will be called the Lemaitre-Vallarta cone. The Stoermer cone in a diagram of the type of Fig. 14 is bounded by a straight line parallel to the NS line.

An important property of the family of curves which represent the cone boundaries for rays of various rigidities is that the curves do not cross one another. Thus it follows that any direction accessible to rays of a given rigidity is also accessible to rays of all higher rigidities, and the total intensity in a given direction can be expressed by an integral extending over the spectrum of the primary intensity between the limits r_c and infinity. In Fig. 14 values of the Lemaitre-Vallarta function r_c for three representative directions are plotted against the latitude. Curve I gives the values for the vertical direction and curves II and III the values for the western and eastern horizons. The scale of electron energies for the earth is given at the right. For energies of other types of rays reference may be made to Table II.

For the purpose of calculating the total intensity incident from all directions resulting from a preassigned spectral distribution of primary radiation it is convenient to know the fraction $f(r_0, \zeta)$ of the azimuthal circle at zenith angle ζ which lies within the allowed cone for rays of a given rigidity. These fractions have been scaled from the diagrams of Fig. 13 and are plotted as functions of the zenith angle in Fig. 15. If the spectral distribution of the primary radiation with respect to the parameter r_0 is represented by $j(r_0)$ then the total intensity in a given latitude may be expressed in the form

$$J = 2\pi \int_0^\infty \int_0^{\pi/2} j(r_0) f(r_0, \zeta) \sin \zeta dr_0 d\zeta. \quad (69)$$

d. The geographical distribution of cosmic-ray intensity

Analyses by Schmidt[198] and more recently by Bartels[189] of the earth's magnetic field have shown that it can be approximated by that of an eccentric dipole oriented parallel to the geomagnetic axis, or the axis of uniform magnetization, and situated 342 km from the earth's center in the latitude 6.5° N and longitude 161.8° E. The strength of the dipole is 8.1×10^{25} gauss cm^3. In any given longitude the field of this hypothetical dipole reaches its maximum horizontal strength at the geomagnetic equator, and along the geomagnetic equator the field strength varies between 0.27 gauss at 10° W to 0.371 gauss at 161.8° E. The real field of the earth departs somewhat from the dipole field and especially so in the region of maximum horizontal field strength. The real field of the earth attains its maximum value at 100° E longitude or about 60° west of the position of the eccentric dipole, and the maximum value of the field exceeds that of the eccentric dipole by about eight percent. It is thus reasonable to expect to find some departures of the cosmic-ray intensity distribution from that calculated from the dipole field, but the main features of the distribution are in line with the dipole approximation.

In the case of a centered dipole the latitude effect is the same in all longitudes and the minimum values $\mathcal{E}_c$ of the energy for the vertical direction are given by Eq. (64) in terms of the earth's radius, its magnetic moment, and the Lemaitre-Vallarta function r_c whose variation with latitude is given in Fig. 14. The eccentricity of the dipole affects the energy limits in two ways. In the first place, as pointed out by Lemaitre[199] and Neher,[200] the meridian plane with respect to which the cone of constant r_c is defined is not the vertical plane but one which passes through the dipole axis and the point of observation. The angle η between this plane and the vertical is given by

$$\sin \eta = \alpha \sin \Omega, \quad (70)$$

where α is the eccentricity of the dipole and Ω is the longitude west of the meridian in which the eccentric dipole is situated.

$$\alpha = 342/6370 = 0.0536.$$

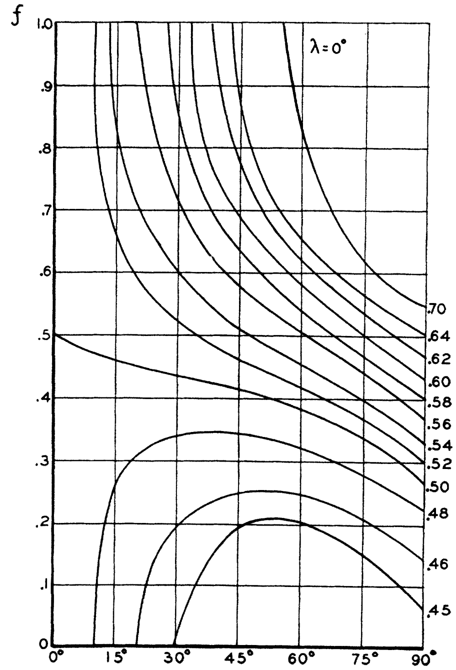
At points along the equator the minimum value of r_c with which cosmic rays may reach the earth along any direction in the east-west plane is given accurately by Stoermer's equation (68). For the vertical direction at the equator this reduces to

$$-\sin \eta = 1/r_c^2 - 2/r_c. \tag{71}$$

Combining (71) with Eq. (70) the value of r_c for the vertical direction is given to a first-order approximation by

$$r_c = 0.5(1 + \tfrac{1}{4}\alpha \sin \Omega). \tag{72}$$

The second and more obvious effect of the eccentricity was pointed out by Vallarta.[201] It involves the variation over the earth's surface of the distance a from the magnetic center. This parameter enters into the determination of the low energy limit $\mathcal{E}_c$ through Eq. (64), (65) or (66). To the approximation contained in (66) the



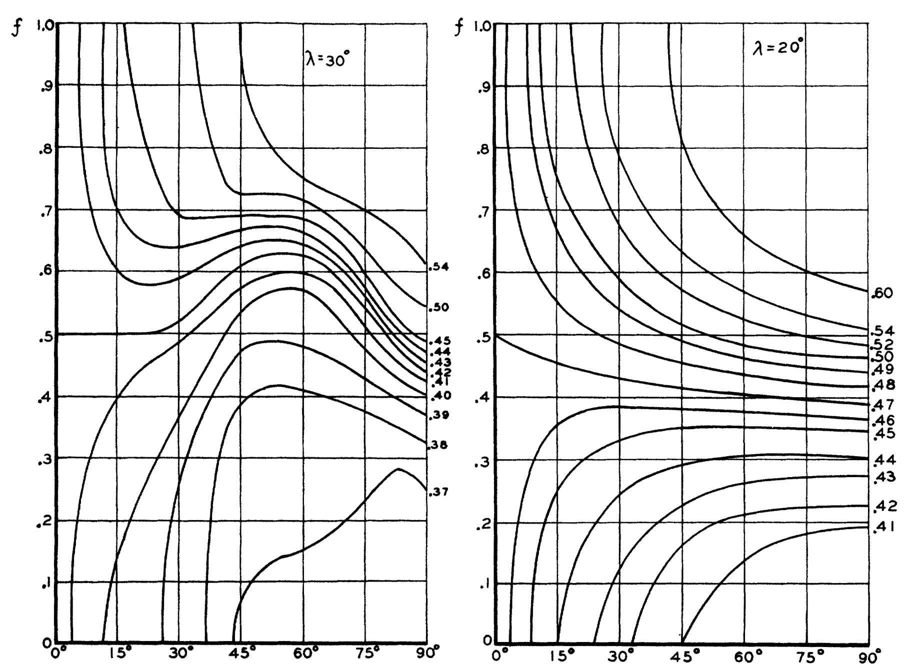


FIG. 15. The ratio f of the azimuthal angle lying within the allowed cone to 2π for various values of r_0 indicated by the numbers attached to the curves. The ratio f is plotted as ordinate against the zenith angle as abscissa. Values are given for three different latitudes.

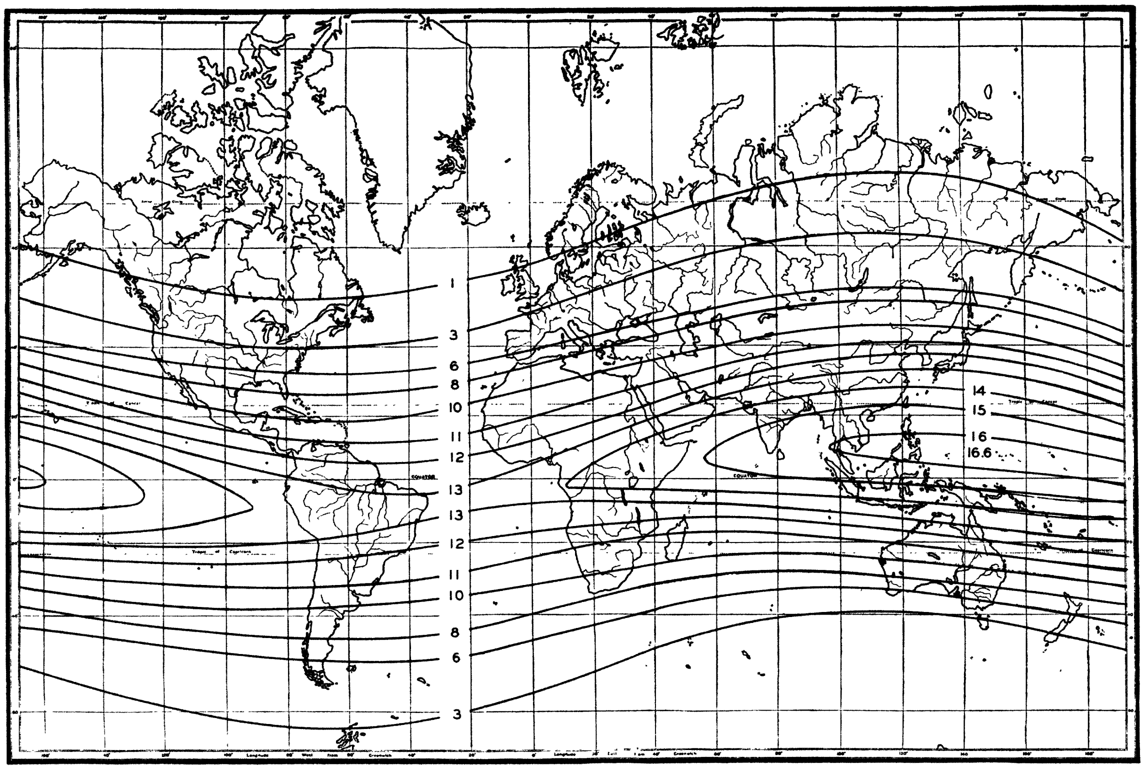


FIG. 16. Lines of equal threshold energies for positive rays entrant in the vertical direction at the earth's surface as calculated from the eccentric dipole whose field approximates that of the earth. The energies are given by the figures attached to the curves in units of billion volts (Bev).

threshold energy is given by

$$\mathcal{E}_c = 59.6 r_c^2 (a_0^2/a^2) \text{ Bev}$$

where a_0 is the equatorial radius of the earth and

$$(a_0/a)^2 = 1 + 2\alpha \cos \Omega. \tag{73}$$

In other latitudes the analogs of Eqs. (72) and (73) are more complex but the bearing of these effects upon the low energy limit is less significant.

Contours of equal threshold energies of vertically entrant positive rays have been computed by taking into account the eccentricity of the dipole, and are plotted in Fig. 16. In view of Eq. (44) these are also lines of equal vertical cosmic-ray intensity in the field of an eccentric dipole. A discussion of the agreement of this theory with the experimental geographical distribution of cosmic-ray intensity is deferred to Section 9.

e. Correlation of geomagnetic cosmic-ray effects[202–204]

If Eq. (44) is extended to take into account both positive and negative primaries and neutral rays of intensity K, and if the low energy limits for positive and negative rays in a given direction and latitude are represented by $\mathcal{E}_c^+$ and $\mathcal{E}_c^-$, respectively, then the total intensity may be written in the form

$$j = \int_{\mathcal{E}_c^+}^{\infty} j^+(\mathcal{E}) d\mathcal{E} + \int_{\mathcal{E}_c^-}^{\infty} j^-(\mathcal{E}) d\mathcal{E} + K. \tag{74}$$

If this equation is to be applied for the interpretation of the observed geomagnetic effects it will be necessary to interpret the spectral intensities $j^{\pm}(\mathcal{E})$ as the distribution functions which represent the intensities at the atmospheric depth of the observations resulting from positive and negative primary rays of energy $\mathcal{E}$, instead of in the sense of the previous definition (Eq. 44) where $j(\mathcal{E})$ stood for the spectral intensity of primary cosmic rays in the interstellar space. Both definitions must be distinguished from that given in Section 2.

Since the low energy limits of the integrals in Eq. (74) are explicit functions of the magnetic moment M and distance a to the magnetic center, and are implicit functions of the latitude and direction through their dependence upon r_c (Eq. 64 or 65)* it follows that the theory can account for five geomagnetic effects, one for each of the five variables upon which $\mathcal{E}_c$ depends. These may be listed as follows: the latitude effect, the zenith angle effect at constant azimuth, the azimuthal effect, the radius or longitude effect, and the magnetic moment or magnetic storm effect. It is convenient to discuss these effects in their differential form. If μ is allowed to represent any one of the five variables, the corresponding effect may be written by differentiation of Eq. (74), as

$$-\partial j/\partial\mu = j^+(\mathcal{E}_c^+)\partial\mathcal{E}_c^+/\partial\mu + j^-(\mathcal{E}_c^-)\partial\mathcal{E}_c^-/\partial\mu, \quad (75)$$

where the coefficients $\partial\mathcal{E}_c^\pm/\partial\mu$ are obtained by differentiating Eq. (64) or (65).

For convenience the following discussion will be limited to the geomagnetic effects for the vertically entrant radiation. In this direction the question of the influence of the atmosphere upon the variations of the intensity with direction are avoided, since

$$(d/d\zeta)(h \sec \zeta) = 0 \quad \text{for} \quad \zeta = 0,$$

and an additional simplification is introduced into the equations because of the relations

$$\left.\begin{array}{l} r_c^+ = r_c^-; \quad (\partial r_c^+/\partial\zeta)_{\mathrm{EW}} = -(\partial r_c^-/\partial\zeta)_{\mathrm{EW}}; \\ \qquad (\partial r_c^+/\partial\lambda) = (\partial r_c^-/\partial\lambda); \\ \qquad (\partial r_c^+/\partial\zeta)_{\mathrm{NS}} = (\partial r_c^-/\partial\zeta)_{\mathrm{NS}} \end{array}\right\} (76)$$

which hold for the vertical. In these relations ζ is considered as varying continuously through the zenith, taking on positive values in the east and north, and negative values in the west and south. The subscripts indicate the azimuth in which the variation is taken.

If we write simply r_c when r_c^+ is intended and use the same symbol with the appropriate adjustment of the algebraic sign when referring to the threshold energy of the negative rays, the following equations are obtained and they express the five* geomagnetic effects in terms of the spectral intensities of positive and negative primaries to the approximation contained in Eq. (65).

I. The latitude effect:—

$$(\partial j/\partial\lambda)/j = -(600M/ja^2) \times[j^+(\mathcal{E}_c)+j^-(\mathcal{E}_c)]r_c\partial r_c/\partial\lambda. \quad (77)$$

II. The east-west asymmetry:—

$$(\partial j/\partial\zeta)_{\mathrm{EW}}/j = -(600M/ja^2) \times[j^+(\mathcal{E}_c)-j^-(\mathcal{E}_c)]r_c(\partial r_c/\partial\zeta)_{\mathrm{EW}}. \quad (78)$$

III. The north-south asymmetry:—

$$(\partial j/\partial\zeta)_{\mathrm{NS}}/j = -(600M/ja^2) \times[j^+(\mathcal{E}_c)+j^-(\mathcal{E}_c)]r_c(\partial r_c/\partial\zeta)_{\mathrm{NS}}. \quad (79)$$

IV. The radius or longitude effect:—

$$(a/j)(\partial j/\partial a) = -(600M/ja^2) \times[j^+(\mathcal{E}_c)+j^-(\mathcal{E}_c)]r_c^2. \quad (80)$$

V. The magnetic moment or storm effect:—

$$(M/j)(\partial j/\partial M) = -(300M/ja^2) \times[j^+(\mathcal{E}_c)+j^-(\mathcal{E}_c)]r_c^2. \quad (81)$$

The unknown quantities in the above set of equations are the two distribution functions $j^+(\mathcal{E}_c)$ and $j^-(\mathcal{E}_c)$. On the basis of these equations the experimental values of the geomagnetic effects may be used to analyze the primary cosmic radiation. From Eqs. (77), (79–81) the spectral distribution of the sum of the positive and negative components may be evaluated whereas Eq. (78) may be used in solving for the spectral distribution of the excess of the

* These equations in which the more general variable r_0 appears are also valid for the particular value $r_0 = r_c$.

* The azimuthal effect for the vertical direction is trivial. Expressions are given, however, for the zenith angle effects in two azimuths.

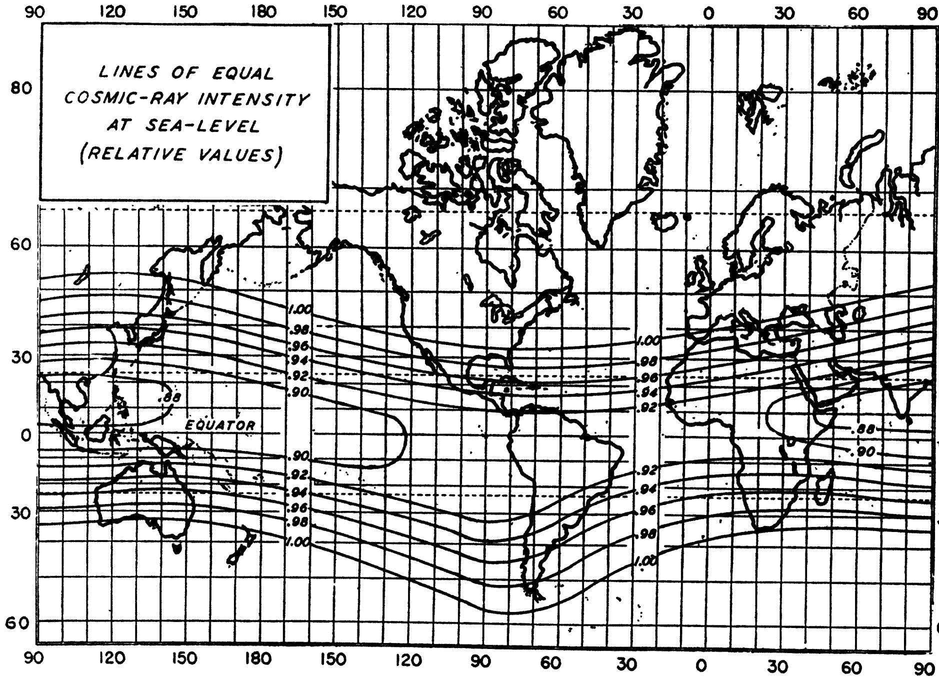


FIG. 17. Lines of equal cosmic-ray intensity according to Millikan and Neher. The latitude effect varies from about 8 percent along the 75th meridian to about 12 percent at 80 E. The longitude effect corresponds to a harmonic term of amplitude 2 percent.

positive over the negative intensity. Since there are three alternative methods for evaluating the spectral distribution of the total intensity, the agreement of the results obtained by the different methods may be regarded as an indication of the validity of the theory as an explanation of the geomagnetic effects. On the other hand, the value of the spectral intensity of the positive excess may be compared with that of the total radiation as an indication of the relative intensities of positive and negative primaries. The actual analysis of the experimental data will be considered in Section 14.

9. Experimental Determinations of the Geographical Distribution of Cosmic-Ray Intensity at Sea Level

In their original world survey of the geographical distribution of cosmic-ray intensities, Compton and his associates[6, 137–141] occupied sixty-nine stations distributed over the range of accessible latitudes, longitudes and elevations. Other expeditions into the equatorial zone were made shortly afterwards by Clay and his associates,[142–145] by Hoerlin,[77, 149, 150] by Prins,[148] and by Hermans and Gueben.[146] A very complete sea-level survey with an automatically recording electroscope placed on board various ships was made by Millikan and Neher[152–154] and, additional data of fragmentary character have been obtained by other observers.[151, 79] What are probably the most reliable data for the latitude effect at sea level have been obtained on a long series of voyages on the Pacific Ocean by Compton and Turner,[206] Fig. 18. It was first noted by Compton[139] that the intensity correlated better with the geomagnetic* than with the geographic latitude, indicating that the variations of intensity were caused by a true geomagnetic effect and were not caused by some other phenomenon, such as, for example, the

* The geometric latitude λ refers to the axis of uniform magnetization which intersects the earth's surface at latitude $\psi = 78° 32'$ N, longitude $\phi = 69° 8'$ W. The geomagnetic latitude is given in terms of the geographic latitude L and the west longitude ω by the formula $\lambda = \sin^{-1}[\cos\psi \cos(\omega - \phi) \cos L + \sin\psi \sin L]$.

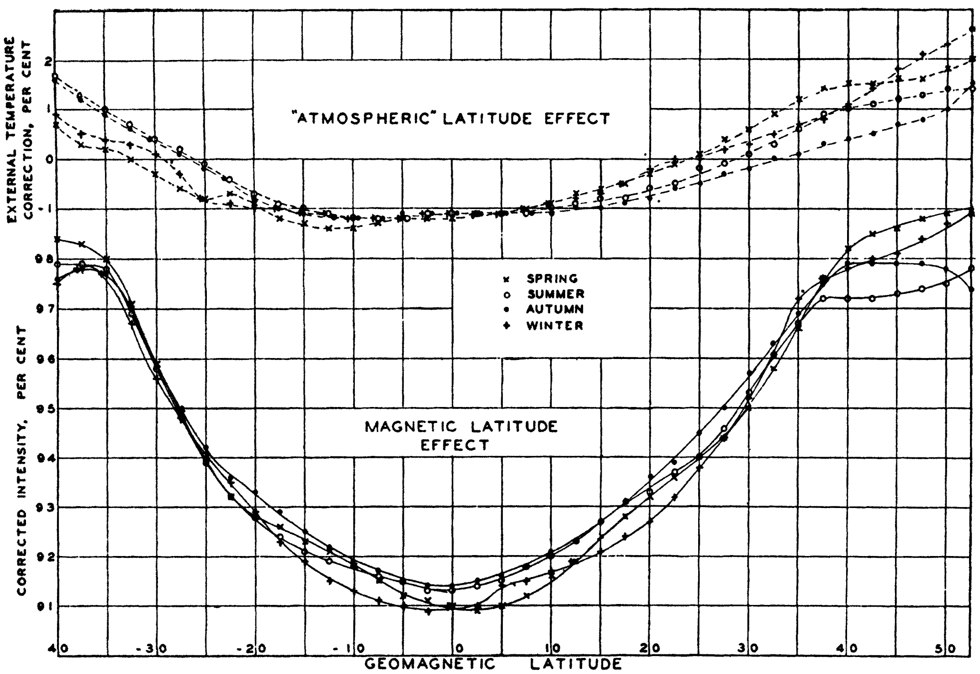


FIG. 18. The sea-level latitude effect showing the part attributed to the external temperature effect and the part attributed to a truly geomagnetic effect. It is noted that the seasonal variations are largely included in the temperature effect. (Compton and Turner, reference 206, 1937.)

temperature which correlates approximately with the geographic latitude. More recent surveys by Clay,[145] Millikan and Neher,[154] and Hoerlin,[77] have shown that whereas along a given meridian the intensity correlates satisfactorily with the geomagnetic latitude, there is an additional variation of the intensity with the longitude. The results of Millikan and Neher[153] which are representative of acceptable values of the intensity are shown in Fig. 18 in which contours of equal cosmic-ray intensity are plotted. A similar chart compiled by Compton[6] from all of the data available at the time showed a somewhat larger variation of intensity in higher latitudes than that indicated in Fig. 17, but the recent results of Compton and Turner[206] are in better accord with the chart of Millikan and Neher, and it is upon this basis that the chart of these authors has been given precedence.

As will be noted from the chart, the magnitude of the latitude effect varies with the longitude from about eight percent in the Atlantic Ocean to about twelve percent in the Indian Ocean. Thus the amplitude of the first harmonic term representing the variation of intensity along the equator is of the order of two percent.

Other data which have been analyzed by Vallarta[201] indicate an amplitude as high as four percent and it is probable that differences in the observed intensities can be expected to occur similar to those appearing among the results of the various measurements of the latitude effect. In the latter instance intensity differences between high and equatorial latitudes ranging from seven to eighteen percent have been reported. Whereas Millikan and Neher[153] have called attention to the consistency of their results from one voyage to another in the same part of the world, others have found variations of a few percent which have been attributed to the radiation itself. Similar fluctuations have also been recorded at fixed stations of a magnitude sufficient to account for most of the discrepancies in the measurements of the latitude effect. The principal effect of this character is one which has been attributed to a correlation of cosmic-ray intensity with the external temperature. The existence of this effect has been confirmed by several observers and an interpretation of the effect has recently been given by Blackett.[271] Hess and Steinmaurer,[208] Schonland, Delatizky and Gaskell,[209] and Compton and Turner[206] have each reported such seasonal variations in which the temperature coefficient was of the order of -0.1 to -0.2 percent per degree C. Johnson and Read[210] have also called attention to fluctuations of similar character causing large variations of cosmic-ray intensity along the Atlantic coast of North America where temperature variations are extreme. Compton and Turner have attributed about three percent of the variation of intensity with latitude to this cause and they find that it also explains the variations in their results from one voyage to the next (Fig. 18). After deducting the temperature effect, about a seven percent variation of the intensity with latitude remains as a true geomagnetic effect.

After allowing for the temperature effect there are certain indications of effects due to local magnetic anomalies. Clay, Bruins and Wiersma,[211] and Johnson and Read[210] have found that the minimum intensity along the 75th meridian is shifted somewhat to the north of the geomagnetic equator, and the latter authors have called attention to a better correlation of the intensity with the horizontal component of the real field of the earth than with the field of the eccentric dipole. A more pronounced example of a similar effect is in the position of the point of minimum cosmic-ray intensity. This occurs about 65° west of the position predicted from the eccentricity of the dipole and very close to the point where the actual field of the earth reaches a maximum. As will be noted in the next section, the magnitude of the longitude effect also indicates the need for invoking the additional strength of the real field of the earth at the position of minimum cosmic-ray intensity. Bruins[212] has attempted to bring these anomalies into the theory by assuming a quadripole component of the earth's field, but Vallarta,[213] on the contrary, finds no indication, from the cosmic-ray observations, of a second harmonic term in the Fourier analysis of the variation of the equatorial intensity with longitude, such as would be expected if the quadripole effect were appreciable.

The latitude effect of the vertical radiation has been measured with the use of coincidence counters by Auger and Leprince-Ringuet,[179] by Clay, Bruins and Wiersma,[211] by Johnson and Read,[210] and by Neher and Pickering.[214] In each case the variations of the intensity of these rays with latitude have agreed fairly well with those found for the total radiation by the electroscope technique although the evidence suggests a two or three percent greater variation in the case of the vertical radiation. Inasmuch as there is a very definite increase in the latitude effect with elevation one must also expect to find a greater latitude effect at sea level when the vertical rays alone are measured.

The latitude effect of rays inclined at a zenith angle of 45° has been studied by Johnson and Read.[210] Such rays in the eastern azimuth were found to undergo about a twelve percent variation, while western rays changed intensity by less than four percent as the apparatus was carried from high latitudes to the equator. The difference in the effects for the two directions correlates satisfactorily with the east-west asymmetries measured in the equatorial zone.

The variation of the intensity of the soft component of the cosmic radiation with respect to the latitude has been inferred from measurements made by Johnson[215] and Read,[210] and by Pickering[216] and Neher[214] with arrangements of apparatus for recording showers of rays generated predominantly by this component in a lead block. At sea level the variation with latitude of the showers was less than that of the total radiation measured with vertical counters, or with ionization chambers, and the plateau of constant shower intensity extended to lower latitudes. The results have been interpreted as indicating that the lower energy field-sensitive primaries of the soft component are unable to penetrate to sea level with sufficient remaining energy to produce recordable showers. A larger latitude effect for the showers has been found by Johnson[215] at high elevations and his results show that the effect at a depth of 6 meters is about equal to that of the total radiation, indicating that the lower energy shower-producing primaries contribute appreciably to the intensity at this depth.

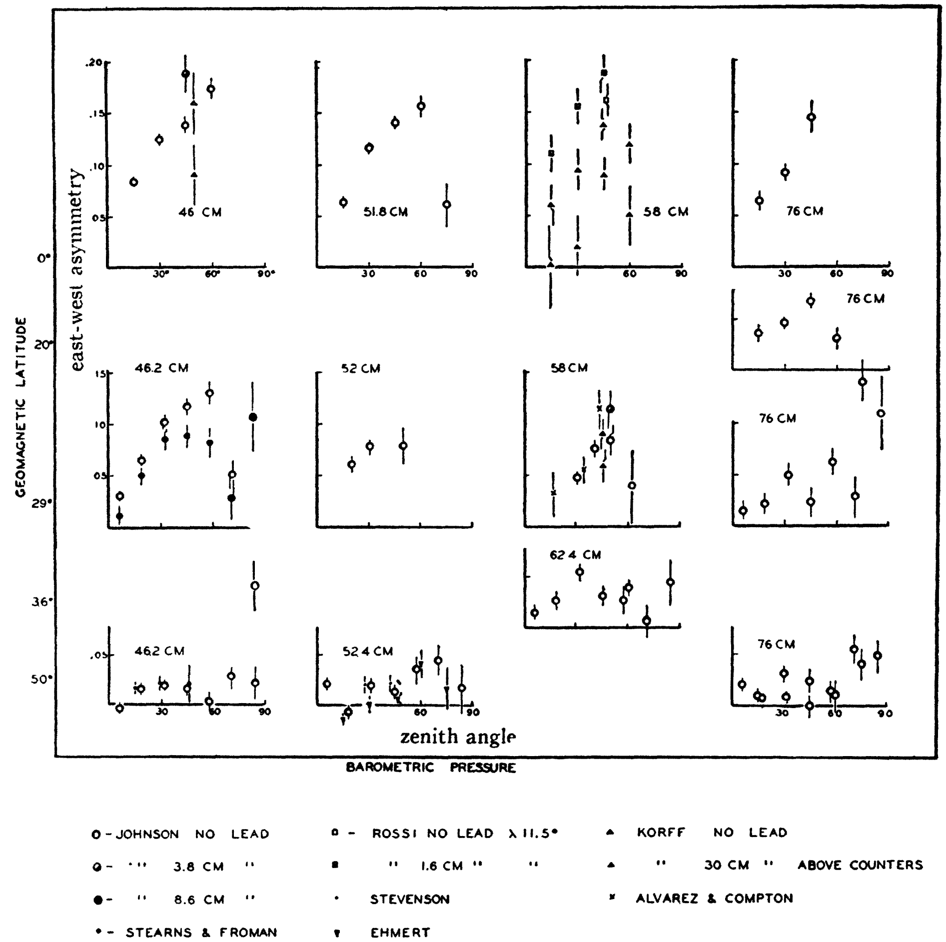


FIG. 19. The east-west asymmetry of the cosmic radiation, defined as the ratio of the excess of western over eastern to the average intensity, plotted against zenith angle. The individual plots are arranged in order of elevation and latitude. (Johnson, reference 186, 1935.)

10. MEASUREMENTS OF THE GEOMAGNETIC DIRECTIONAL EFFECTS

The most important directional effect is the east-west asymmetry, the existence of which is proof that positive primaries contribute more to the intensity in the lower part of the atmosphere, where the asymmetry is known to exist, than do primary negatives. Because of the large variation of the intensity with zenith angle brought about by atmospheric absorption, it is important in measuring the geomagnetic directional effects to compare intensities only in directions for which the zenith angle is constant. In such directions the difference between the intensities in the eastern and western azimuths is caused by the excess of primary positives over negatives in the energy range which extends between the threshold energies of the two directions.[217] This energy range increases with zenith angle and produces an asymmetry which first increases proportionally with the zenith angle, but near the horizon the low energy field-sensitive rays are absorbed to a greater extent than the higher energy background radiation and the asymmetry declines as the zenith angle is increased beyond about 60°. At the equator a greater asymmetry than at higher latitudes results from the greater range of energies between the thresholds for a given pair of directions. (See Fig. 14.) In Fig. 19 the experimental values[186] of the asymmetry, defined

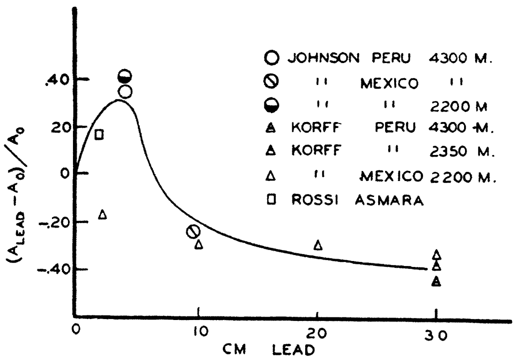


FIG. 20. The effect of lead absorbing screens upon the asymmetry at a zenith angle of 45°. In the experiments of Korff, the lead was inserted above the counter train whereas in those of Rossi and Johnson the lead was placed between the counters. The asymmetry A is defined as the ratio of the excess of western over eastern to the average intensity. A_0 refers to the asymmetry without lead.

as the ratio of the excess of western over eastern to the average intensity, are plotted against zenith angle, and the individual diagrams are arranged in the order of the elevations and latitudes. As noted in the legend some of the points were obtained with lead filtering blocks inserted between the counters while others were obtained without such filters. The theoretical prediction of greater asymmetries at the equator, and the increase of the asymmetry with zenith angle are confirmed. Greater asymmetries at the higher elevations as well as the low values near the horizon indicate that the asymmetrical, like the latitude sensitive, radiation is less penetrating than the total radiation at the same depth.

The absorption of the asymmetric component of the radiation in lead has been studied by Johnson, Rossi and Korff whose results are shown in part in Fig. 20.[186] In Korff's experiments[181] the lead was placed above the counters and his results show a continuous decrease in the asymmetry with absorber thickness. With the lead placed between the counters, as in the experiments of Rossi[178] and Johnson,[185, 186] the asymmetry at first increased and then fell off for absorber thicknesses greater than about 4 cm. Apart from the first rise in the curve which can be accounted for as the elimination by the lead of low energy diffused rays, the decrease in the asymmetry due to the lead is about equal to that produced by an equal mass of air. The experimental points of Korff agree almost exactly with the mass absorption coefficient deduced[186] from the observed absorption of the asymmetric component in air. This characteristic of the asymmetric radiation, contrasted with the Z^2 absorption of the soft component, identifies the asymmetry as a property of the hard component of the radiation.

An asymmetry in the soft component of the cosmic radiation has been sought by Johnson[215] with an arrangement of counters for recording showers. At an atmospheric depth of 6 meters in Mexico his results were conclusive in showing no asymmetry greater than a percent or two, although at sea level in Peru an asymmetry of the shower-producing rays of the same order as that of the total radiation was indicated. Presumably many of the sea-level showers are produced by the asymmetric hard radiation whereas most of the high elevation showers are a result of the soft component. By combining the symmetry of the showers with the fact revealed by the latitude effect (see Section 9) that the primaries involved are field-sensitive it was concluded[215] that the soft component primary rays are equally positive and negative.

A north-south asymmetry in intermediate latitudes was predicted by the more exact orbital theory of Lemaitre and Vallarta.[193, 187] (See Fig. 13.) The cones of allowed directions for rays of a given rigidity in these latitudes show a marked asymmetry with respect to the east-west line and they open up more, with the consequence of higher intensities, on the side away from the pole. Experimental proof of the effect was obtained in Mexico by Johnson[218] whose results were in qualitative agreement with the theory.

11. Magnetic Disturbances and Cosmic-Ray Intensity

As already indicated in Section 8, e, the threshold energy corresponding to a given latitude and direction should undergo a variation with a change in the magnetic moment of the earth. Ever since the discovery of the latitude effect, observers have reported[219–230] correlations between magnetic variations and the fluctuations of cosmic-ray intensity, but from the disparity of the results it is evident that the phenomenon is not one to be simply understood. Typical of such correlations are those recently reported by Hess, Demmelmaier and Steinmaurer[228] as a result of an analysis of records covering a period of one year. Here they have distinguished between four different magnetic effects, some of which correlate positively, i.e., the cosmic-ray intensity increases with the horizontal component of the earth's field, whereas others show the opposite correlation. (1) The *large magnetic storm effect*, discovered with certainty during the April 1937 storm with simultaneous cosmic-ray fluctuations reported[209, 222, 225, 227, 229, 230] from at least seven widely separated observatories, is of a world-wide character and the correlation is positive. (See Fig. 21.) During the April 1937 storm and during several other more recent storms the variations of the cosmic-ray intensity in relation to the magnetic disturbance can be expressed, in order of magnitude, by the ratio

$$\begin{aligned} \Delta I/I\Delta H &= 0.06 \text{ percent per } \gamma;^* \\ H\Delta I/I\Delta H &= 15. \end{aligned} \tag{82}$$

There was surprisingly little variation of this ratio with latitude even in comparing the Innsbruck or Cheltenham observations, where the latitude is normally considered too high for the occurrence of geomagnetic effects, with those at the equator, but there is apparently a wide variation in this ratio from one storm to another. In fact, in one large storm Forbush[230] was unable to notice any change of the cosmic-ray intensity.

(2) A seasonal magnetic effect has been noted by Hess, *et al*[228] and by others[131, 234, 235] in which a negative correlation was found between the monthly mean values of the cosmic-ray intensity and the corresponding means of the horizontal component of field strength. The order of magnitude in this case is given by

$$\begin{aligned} \Delta I/I\Delta H &= -0.1 \text{ percent per } \gamma; \\ H\Delta I/I\Delta H &= -22. \end{aligned} \tag{83}$$

(3) The diurnal effect[231–243] of the cosmic-ray intensity might be linked with the diurnal effect of the horizontal component of field strength as was first suggested by Gunn.[231] The magnitude and the sign of the ratio of these effects depends upon the latitude of the observer, for the magnetic effect in the equatorial zone is opposite in phase to that in polar latitudes, whereas the cosmic-ray variations have their maximum at noon in all latitudes. At Innsbruck where the magnetic fluctuations have the polar phase, the effect is given by

$$\begin{aligned} \Delta I/I\Delta H &= -0.02 \text{ percent per } \gamma; \\ H\Delta I/I\Delta H &= -4. \end{aligned} \tag{84}$$

At the equator the ratio would be of the same order of magnitude but of opposite sign.

(4) Possibly a fourth correlation[228] was found between the daily means of cosmic-ray intensity and the daily means of the horizontal component of the field. During parts of the year this correlation was positive and during other parts negative.

(5) Forbush[230] has recently pointed out the possibility of a fifth correlation between a quasi-persistent 27-day wave in cosmic-ray intensity and the 27-day quasi-persistent wave of magnetic activity associated with the rotation period of the sun.

Although there can be no doubt that correlations do exist between the cosmic-ray intensity

* $1\gamma = 10^{-5}$ gauss.

TABLE III. *Ratios found by various observers of the equatorial to the polar intensity as function of depth below the top of the atmosphere.*

DEPTH (METERS OF WATER)	17	14.5	12.5	10	8	7	6	1
Observer								
Compton—10 cm lead shield				0.88	0.82		0.75	
Young and Street—Average of all shield thicknesses up to 19 cm.				0.86		0.79	0.77	
Hoerlin—Unshielded				0.88	0.81	0.76		
Hoerlin—10 cm iron shield				0.89	0.82	0.78		
Bowen, Millikan and Neher—No shield				0.90		0.84	0.77	0.40
Clay, Bruins and Wiersma—No shield	0.97	0.83	0.82	0.83				

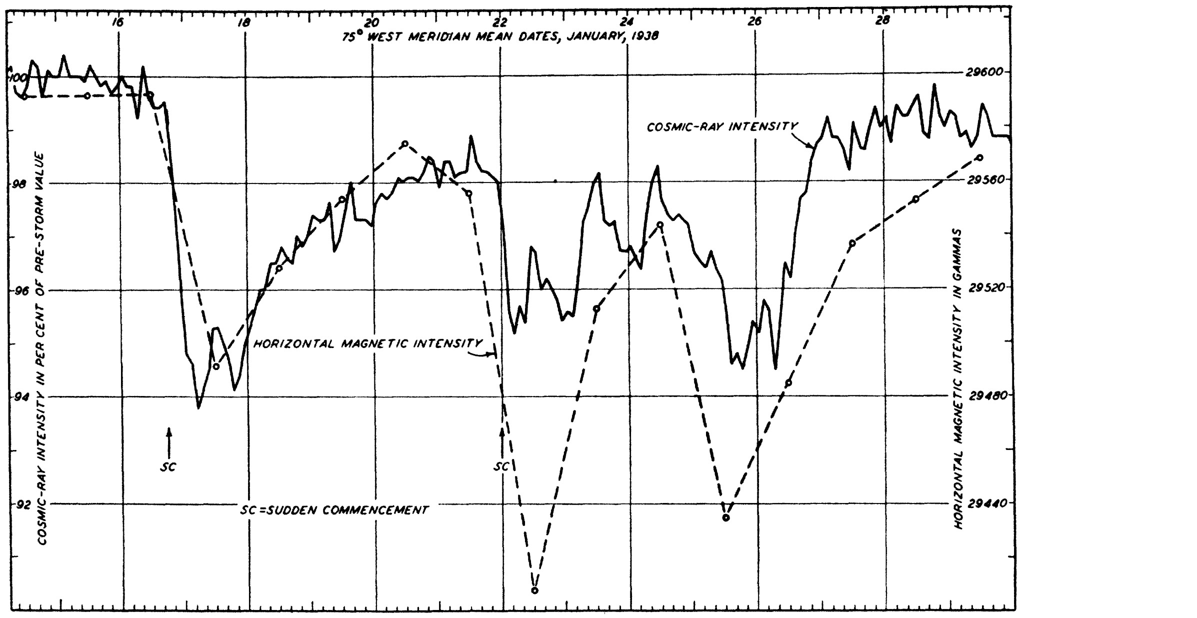


FIG. 21. The correlation between cosmic-ray intensity and horizontal magnetic intensity during a large magnetic storm. The ratio of the change of cosmic-ray intensity to the magnetic disturbance is greatest during the first phase of the storm. (Forbush, reference 230.)

and terrestrial magnetism it does not necessarily follow that the two are causally related and it seems doubtful[232, 202] if the correlations cited above can be interpreted in terms of the Stoermer-Lemaitre-Vallarta theory as a simple change of the magnetic moment of the earth. The most definite magnetic effect is that of the large storms; its magnitude in relation to the theory of the geomagnetic effects, considered in Section 14, is at least 100 times too great.

12. The Absorption in Air and Other Materials of the Field-sensitive Radiation in the Lower Part of the Atmosphere

An increase with elevation of the latitude effect,[77, 80, 122, 139, 155, 156, 211, 244–254] expressed as a percent of the total intensity, has been noted by all who have reported observations. Table III contains some of the values found by several observers for the ratio of the equatorial to the high latitude intensity at the atmospheric depths indicated. The two most extensive sets of observations of the variation with latitude of the absorption of the radiation in the lower part of the atmosphere are those of Compton[139] and of Hoerlin.[77] The ionization intensities I which they have measured at the equator and in high latitudes are plotted logarithmically in Fig. 22 against the atmospheric depth, and it may be noted that the experimental points fall rather closely upon straight lines. Thus the absorption law may be written in the form

$$I = A/h^n, \tag{85}$$

where the exponent n depends slightly upon the latitude. If the exponent appropriate for the representation of the polar intensity I_p is written as n and that for the equatorial intensity I_e is written as $n-\delta$, and if the depth is expressed in units of about 16 meters of water, i.e., the depth at which the polar and equatorial lines intersect, then the ratio of the polar to the equatorial intensities is given by

$$I_p/I_e = \Delta I/I_e + 1 = h^{-\delta}. \tag{86}$$

From the experimental points plotted in Fig. 22 the polar exponent has about the value $n = 2.5$ and the difference between the polar and equatorial exponents is approximately

$$\delta = 0.16.$$

This change in exponent arises from the inclusion in the high latitude measurements of low energy rays in the range $\Delta r_c = 0.5 - 0.35 = 0.15$.

The determination of δ by this method is lacking in precision, but a higher accuracy can be realized from an analysis of the asymmetry measurements. As a matter of fact, the lines in Fig. 22 have been drawn with the results of the asymmetry in mind, and the value $\delta = 0.16$ represents an agreement with all of the evidence bearing upon the absorption of the field sensitive radiation near sea level.

The absorption of the asymmetric radiation can be determined from the variation of the asymmetry with both elevation and zenith angle, and an analysis may be made in a manner analogous to that described above in the case of the latitude effect. With an increase of zenith angle the atmospheric path lengthens but there is also an increase in the energy range within which the asymmetric component lies and to

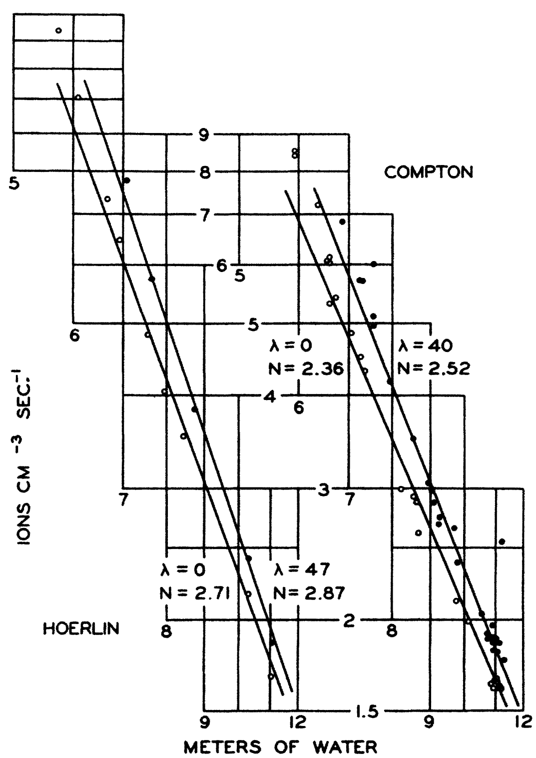


FIG. 22. The variation of the cosmic-ray intensity with elevation at the equator and at high latitudes, (λ) plotted logarithmically. The slopes of the lines are given by values of n. (Compton, reference 139, and Hoerlin, reference 77.)

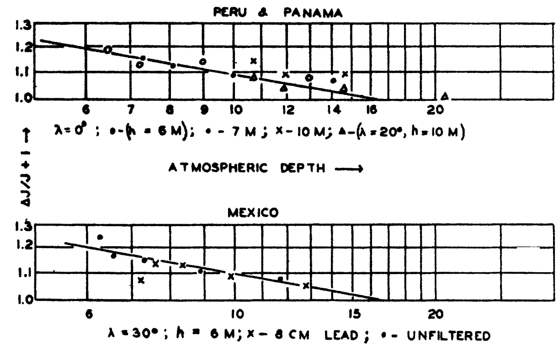


FIG. 23. The asymmetrical component of the cosmic radiation, reduced to a constant energy range, and plotted logarithmically against the atmospheric depth. The lines are drawn with a slope $\delta = 0.16$. Data obtained at different atmospheric depths are plotted with corresponding symbols. Where lead was inserted into the counter train the absorption path has been computed on the equivalent mass scale.

correct for the latter effect each experimental value of the asymmetry, $A = \Delta j/j$, has been divided by the range of r_c encompassed by the threshold values for the two directions involved, and multiplied by 0.15, a value selected as a standard range because it represents approximately the range within which the latitude sensitive radiation is included. These adjusted values of the asymmetry are thus analogous to the latitude effects $\Delta I/I_e$ which occur in Eq. (86). The values of the adjusted asymmetry increased by unity have been plotted logarithmically against the depth in Fig. 23 and it is evident that the line drawn with slope $\delta = 0.16$ well represents the data. It is also noted that this line intersects the axis at $h = 16$ meters and we would expect that at this depth the asymmetry as well as the latitude effect would no longer be found, unless there is some straggling. In this connection it is interesting that Clay's[211] results, given in Table III, also indicate the disappearance of the latitude effect at about this depth.

Inasmuch as δ is to appear in some calculations in the next section, it is important to estimate its probable error. From the scattering of the points in Fig. 23 the range of possible values may be stated as $\delta = 0.16 \pm 0.05$. This is a considerably closer limit of uncertainty than could be set from the analysis of the variations of the latitude effect with elevation, though the fact that there is no indicated disagreement between the rates at which the two effects vary with elevation is some guarantee, as noted by Neher,[184] that the geomagnetic effects have been correctly interpreted. We are thus justified in assuming that the latitude sensitive and the asymmetric components of the radiation have the same properties.

13. Reduction to the Vertical of Geomagnetic Effects Measured with the Electroscope

For purposes of analysis of the energy distribution of the primary cosmic radiation from the observations of the geomagnetic effects, according to the method suggested by Eq. (75), it is important to know what the effects are as they pertain to the vertically entrant radiation. Here the method of Gross, Eq. (7), may be used, for although the uniformity of the radiation in azimuth, assumed in the derivation of Gross' equation, does not exist at the equator, this asymmetry does not seriously affect the analysis. In fact, to the approximation that the asymmetry can be represented by a series of odd harmonic functions of the azimuth, Gross' equation is rigorously valid.

With the ionization-depth relation given by an equation of the form of (85) the Gross factor G is given by

$$2\pi G = 1 - hdI/Idh = 1 + n. \tag{87}$$

The variation of the vertical intensity j with respect to the latitude, λ, or with respect to any of the other geomagnetic variables, is found by differentiating Eq. (7). It is convenient to take the logarithmic derivative

$$dj/jd\lambda = dI/Id\lambda + dG/Gd\lambda \tag{88}$$

$$= dI/Id\lambda(1 + IdG/GdI). \tag{89}$$

If one considers the change in I and G corresponding to the change of latitude between the equator and 40°, for which $\Delta I/I = 0.10$ and

$$\Delta G/G = \delta/(1+n) = (0.16 \pm 0.05)/(3.5) = 0.045 \pm 0.015,$$

the factor in parentheses in Eq. (89) is seen to have the value

$$1 + I\Delta G/G\Delta I = 1.45 \pm 0.15. \tag{90}$$

As already noted in Section 9, there is definite evidence of a greater latitude effect when the vertically entrant radiation alone is measured, and the results are consistent with Eq. (90). The same factor will also apply in reducing the longitude and the magnetic storm effects to the vertical.

14. Analysis of the Primary Rays which Produce the Field-Sensitive Intensity in the Lower Half of the Atmosphere

As already pointed out in Section 8 the geomagnetic effects may be used in analyzing[185, 186, 205] the energy spectrum of the primary radiation. Eq. (75) may be taken as the basis of such an analysis and it is noted that in the case of the latitude effect ($\mu = \lambda$) for the vertical direction

$$\partial\mathcal{E}_c^+/\partial\lambda = \partial\mathcal{E}_c^-/\partial\lambda$$

so that Eq. (75) may be written

$$[j^+(\mathcal{E}_c) + j^-(\mathcal{E}_c)]/j = (\partial j/j\partial\lambda)/(\partial\mathcal{E}_c/\partial\lambda) = (\partial j/j\partial\mathcal{E}_c) \tag{91}$$

or by making use of (90) we have

$$[j^+(\mathcal{E}_c) + j^-(\mathcal{E}_c)]/j = (1.45 \pm 0.15)(\partial I/I\partial\mathcal{E}_c). \tag{92}$$

For the purpose of making just such an analysis as this, Compton and Turner[206] have plotted the intensities which they have measured on the Pacific Ocean (corrected for external temperature) against the vertical threshold energy, as in Fig. 24. The derivative $(\partial I/I\partial\mathcal{E}_c)$ is represented by the dotted curve and, in the spectral region from 7.5 Bev to 15 Bev it is seen to have a constant value of 0.85 percent per Bev range. Accordingly, from Eq. (92), the unidirectional spectral intensity in the vertical direction is

$$(j^+ + j^-)/j = 1.23 \text{ percent per Bev.} \tag{93}$$

with an estimated probable uncertainty of 0.13

percent per Bev. In the energy range below 7 Bev the spectral intensity is considerably less* but there is an indication of an intensity of low energy primary radiation at sea level of the order of 0.2 percent per Bev in the vertical.

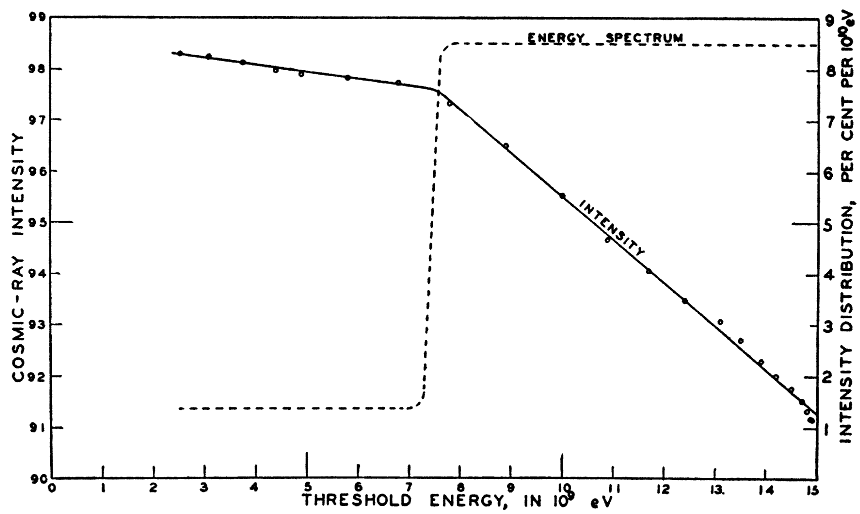


FIG. 24. Spectrum analysis of the cosmic radiation from Compton and Turner's results on the Pacific Ocean. The intensity is plotted against the threshold energy and the derivative of this curve is given by the dotted curve.[206]

By an entirely similar method the east-west asymmetry may be used to determine the spectral density of the excess of the sea-level intensity produced by positive primaries over that produced by negatives. By use of relations (76), Eq. (75), when applied to the east-west effect, may be put in the form

$$[j^+(\mathcal{E}_c)-j^-(\mathcal{E}_c)]/j=(\partial j/j\partial\varphi)/(\partial\mathcal{E}_c/\partial\zeta)=\partial j/j\partial\mathcal{E}_c. \tag{94}$$

The results of the asymmetry measurements have been reduced to a useful form for the purpose of this analysis in Fig. 23. In this figure, it will be recalled, the asymmetric intensity has been adjusted to an energy range of $\Delta r_c=0.15$ or $\Delta\mathcal{E}_c=7.65$ Bev. Since the points for the equator and those for 30° N. fall satisfactorily on the same line the results from the asymmetry studies indicate that the excess of positive intensity, like the total intensity, is uniformly distributed in energy. At a depth of ten meters the asymmetric component, plotted in the figure, has the value

$$\Delta j/j+1=1.10\pm0.02 \text{ per } 7.65 \text{ Bev range.} \tag{95}$$

where the uncertainty has been estimated from the scatter of points. For a range of 1 Bev

$$\Delta j/j=\frac{j^+(\mathcal{E}_c)-j^-(\mathcal{E}_c)}{j}=(0.10\pm0.02)/7.65$$
$$=1.30\pm0.26 \text{ percent per Bev.} \tag{96}$$

The close agreement of this figure with that for the total corpuscular intensity, Eq. (93) indicates that within the accuracy of the analysis all of the field-sensitive intensity at sea level is produced by positive primary rays. The ratio of the total field-sensitive intensity to the positive excess is, from (96) and (93),

$$\beta\equiv(j^++j^-)/(j^+-j^-)=0.95\pm0.22. \tag{97}$$

Thus an upper limit of the negative intensity, allowed by the uncertainty in β, is

$$j^-/(j^++j^-)=(\beta-1)/2\beta=0.09. \tag{98}$$

Values of β less than 1, although allowed by the experimental uncertainty, are in principle impossible.

The fact that δ is the same, whether determined from the latitude effect or from the asymmetry, signifies that there is no indication of a change of β with elevation.

Other estimates of the spectral intensity $j^+(\mathcal{E})+j^-(\mathcal{E})$ may be obtained from the other geomagnetic effects. In the case of the north-

---

* This sudden break in the energy spectrum of the primary rays which produce sea-level effects is of considerable interest, and an explanation[269] for it has been sought in terms of an effect of a solar magnetic field in cutting off the primary rays of lower energy from the space within the earth's orbit. Other explanations[270] have considered the possibility of a process of energy loss in the atmosphere which sets in at this energy and prevents lower energies from reaching sea level.

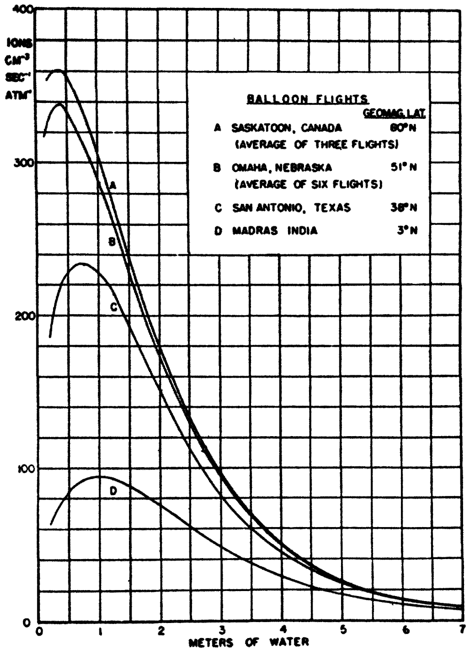


FIG. 25. The cosmic-ray ionization as a function of atmospheric depth in four latitudes. (Bowen, Millikan and Neher.[254])

south effect the measurements[218] are barely sufficient for a reliable numerical estimate. For zenith angles greater than 45° the results are not strictly in qualitative agreement with the theory, but at angles closer to the zenith the data, when adjusted to sea level by the $\delta=0.16$ law, agree with a spectral intensity of 1.2 percent per Bev. Hence, within the rather large uncertainty, the magnitude of the north-south effect is consistent with the spectral intensity of the total radiation as derived from the latitude effect.

In attempting an analysis of the spectral intensity $j^+(\mathcal{E})+j^-(\mathcal{E})$ based upon the longitude effect, it is noted that the Millikan and Neher chart, Fig. 17 shows a variation of 3.5 percent or more in the equatorial intensity, to be accounted for by a ten percent variation of the magnetic radius or a 3.6-Bev variation of the threshold energy. By applying Eq. (90) for reduction to the vertical, the spectral intensity required to explain the longitude effect according to Eq. (92) is

$$[j^+(\mathcal{E})+j^-(\mathcal{E})]/j=1.45\times 0.035/3.6$$
$$=1.4 \text{ percent per Bev.} \qquad (99)$$

This is a slightly greater spectral intensity than that derived from the latitude effect but a slight correction for the effect of the anomalously high magnetic field in the vicinity of the intensity minimum would bring the two into better agreement.

The magnetic storm effect in its relation to the other geomagnetic effects has been discussed by Thompson,[232] Clay and Bruins,[222] and Johnson.[202] A model of the storm field for use in such discussions has been suggested by Chapman,[256] who proposed to consider that the magnetic disturbance is caused by an electric current distributed over a spherical sheet concentric with the earth and flowing along the parallels of latitude from east to west. Such a current system strengthens the magnetic field in the region outside of the current sheet, but weakens it in the internal region. If the current density of the sheet is proportional to the cosine of the latitude, then for all external points its contribution to the magnetic field may be considered as a change in the magnetic moment of the dipole at the center of the earth, designated by δM, whereas for all internal points its contribution to the field is a component parallel to the axis and of uniform intensity. However, when this field is set up, currents are induced in the earth and the total change in the horizontal magnetic intensity on the surface of the earth at the equator can be represented by

$$\delta H=-3\delta M/(2a'^3-a^3), \qquad (100)$$

where a' is the radius of the current sheet, a is the radius of the earth, and the factor 3 takes account of the conductivity of the earth.

In calculating the effect of such a change of moment upon the cosmic-ray intensity it is necessary to make some assumption regarding the magnitude of a'. The simplest of such assumptions is that $a'=a$, or that the current sheet is very close to the surface of the earth. The change in magnetic moment of the earth is then given in terms of the change in the horizontal magnetic intensity at the equator by

$$\delta M=-a^3\delta H/3. \qquad (101)$$

Corresponding to this change in M there is a change in the threshold energy given by Eq. (68), in accordance with which we may write

$$\delta\mathcal{E}_c/\mathcal{E}_c=\delta M/M=-\delta H/3H. \qquad (102)$$

Whence from Eq. (76) the spectral intensity of the cosmic radiation is

$$[j^+(\mathcal{E}_c)+j^-(\mathcal{E}_c)]/j=-(1/\mathcal{E}_c)(Mdj/jdM) = (3/\mathcal{E}_c)(Hdj/jdH). \qquad (103)$$

The large magnetic storm effect described in Section 11 is given, as in Eq. (82), by $HdI/IdH=15$, and converting this to the vertical by means of Eqs. (89) and (90), the spectral intensity derived from the magnetic effect at the equator turns out to be

$$[j^+(\mathcal{E})+j^-(\mathcal{E})]/j=450 \text{ percent per Bev.} \qquad (104)$$

Contrasting this result with that found from the latitude effect, Eq. (93), it appears that the magnetic effect is of the order 300 times too large to be accounted for as an increase in the earth's magnetic moment according to the model suggested. To what extent this interpretation of the storm effect may be remedied by taking a larger radius for the current sheet is a matter concerning which there are no reliable calculations. It is the writer's own view[202] that an explanation of the magnetic effects along these lines is impossible and that some other influence is at work, acting as a common cause for both magnetic and cosmic-ray disturbances.

15. Geomagnetic Effects in the Upper Part of the Atmosphere and the Analysis of the Energy Distribution of the Soft Component of the Primary Radiation

It must be emphasized that the discussion of the preceding sections pertains to the intensity produced at *sea level* by field-sensitive primary rays. A similar analysis of the spectral intensity of the total incoming cosmic radiation is also possible from the data obtained on a series of balloon flights made by Bowen, Millikan and Neher,[126, 252–254] in which a similar technique was used for flights in four different latitudes distributed between the equator and 60° N. The actual results obtained on these flights are contained in Fig. 25 where ionization intensity is plotted against atmospheric depth. Because of the steep descent of these curves near the axis of zero depth it would not be possible to rely upon an extrapolation of the data as a means of determining the primary cosmic-ray intensity, but it is possible to determine the total energy brought in by the cosmic radiation at each latitude by taking the integral under each curve. For this purpose the exact form of the curve in the unexplored layer is relatively unimportant. By taking $k=32$ volts per ion pair, the normal energy flux $\Psi_\perp$ has been computed as in Eq. (5) and plotted against depth in Fig. 26. The ordinates of these curves at $h=0$ are obtained by a short extrapolation and they represent the total incoming energy of the cosmic radiation. This is equal to the energy flux in interstellar space of cosmic rays of energy greater than the threshold energy of the corresponding latitude. From the curves of Fig. 26 the spectral distribution of the normal cosmic-ray intensity may be obtained by the relation

$$\mathcal{E}J_\perp(\mathcal{E})=\Psi_\perp(\mathcal{E})=-\partial\Psi_\perp/\partial\mathcal{E}_c. \qquad (105)$$

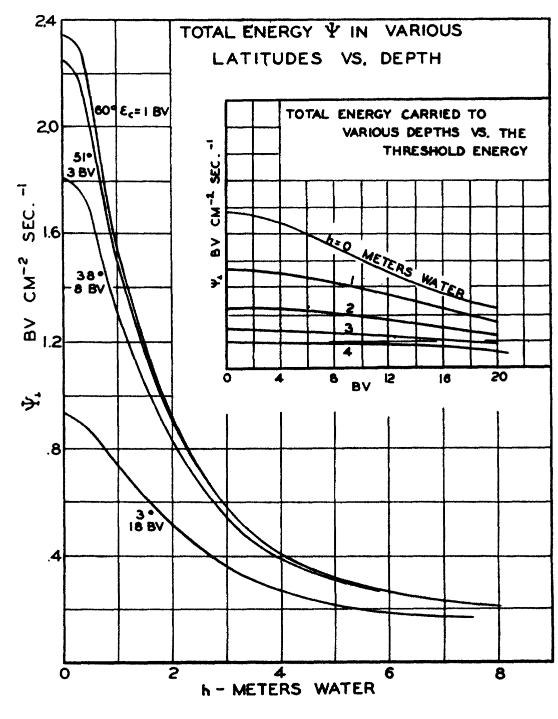


FIG. 26. The normal energy flux (across a horizontal square centimeter) of area as a function of depth and threshold energy, calculated from the data plotted in Fig. 25.

At the top of the atmosphere, where the cosmic rays are uniformly incident from all directions in the hemisphere

$$j(\mathcal{E}_c)=J_\perp(\mathcal{E}_c)/\pi \tag{106}$$

and

$$j(\mathcal{E}_c)=-(1/\pi\mathcal{E}_c)\partial\Psi_\perp/\partial\mathcal{E}_c. \tag{107}$$

Since the experiments completed at present determine only four points on the curve representing the total energy $\Psi_\perp$ as a function of the threshold energy $\mathcal{E}_c$, instead of attempting to carry through the differentiation indicated in Eq. (107) we may try to find an empirical expression of convenient analytic form whose parameters can be determined from the four data. At the time when only three of these points were available an analysis of this kind was made by Johnson[255] in which an inverse power function of the energy was used. With the additional observational datum added by Bowen, Millikan and Neher's[254] *Saskatoon* flights, a somewhat better agreement with the data seems to be realized with an exponential function of the form

$$j(\mathcal{E})=A\exp(-\alpha\mathcal{E}). \tag{108}$$

The equation for comparison with the data is thus:

$$\Psi_\perp=\pi\int_{\mathcal{E}_c}^{\infty}\mathcal{E}j(\mathcal{E})d\mathcal{E}$$

$$=\pi A(\alpha\mathcal{E}_c+1)\exp(-\alpha\mathcal{E}_c)/\alpha^2, \tag{109}$$

where A and α must be assigned so that the values of $\Psi_\perp$ calculated from Eq. (109) agree with those measured. In making this adjustment the values of $\mathcal{E}_c$, corresponding to the vertical direction, may be used for the higher latitudes but this approximation at the equator, where the range in $\mathcal{E}_c$ from east to west is large, would lead to an appreciable error. It is necessary, therefore, to make an assignment of the constants such that an integral of the resulting distribution function which takes into account the allowed cone according to Eq. (69) agrees with the observed value of $\Psi_\perp$. The effect of this more exact treatment is to place the effective threshold energy for the equator about 2 Bev above the value corresponding to the vertical direction, and the values of the constants which yield the observed values of $\Psi_\perp$ according to this method are

$A=0.0075$ cm^{-2} sec.$^{-1}$ per unit solid angle,
$\alpha=0.1$ Bev^{-1}.

The corresponding distribution curves are plotted in Fig. 27; the upper half of the figure shows the distribution of the number of primary particles while the lower half shows the distribution of the energy. One of the latter curves represents the energy $\Psi_\perp$ carried by all rays of energy greater than $\mathcal{E}$ while the other represents the coefficient of $d\mathcal{E}$ in the expression for the energy carried by rays of energy between $\mathcal{E}$ and $\mathcal{E}+d\mathcal{E}$, i.e., $-\partial\Psi_\perp/\partial\mathcal{E}=\Psi_\perp(\mathcal{E})$. The four experimental points are also plotted in the lower half of the figure indicating the agreement of the empirical function with the observations.

The representation of the primary distribution in the form of an analytic function is useful in a number of ways, although, of course, its validity cannot be relied upon in the energy ranges outside of that which is covered by the experimental points. The first application is in a calculation of the total cosmic-ray current. The total number of

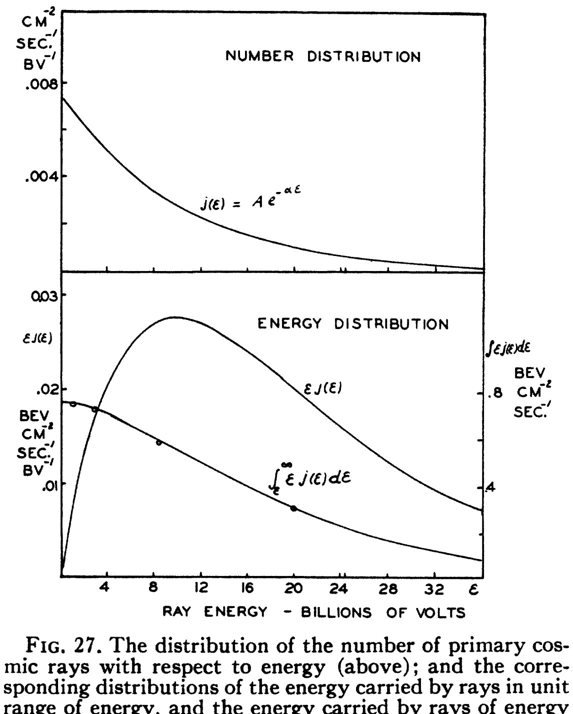


FIG. 27. The distribution of the number of primary cosmic rays with respect to energy (above); and the corresponding distributions of the energy carried by rays in unit range of energy, and the energy carried by rays of energy greater than the values indicated by the abscissae (below). The experimental data of Bowen, Millikan and Neher are indicated by circles.

rays incident upon the surface of the earth can be expressed in the form of the integral

$$4\pi R^2\int_0^{\pi/2} J_\perp(\lambda)\cos\lambda d\lambda$$
$$=[4\pi^2R^2A/\alpha]\int_0^{\pi/2}\exp(-\alpha\mathcal{E}_c)\cos\lambda d\lambda. \quad (110)$$

$\mathcal{E}_c$ as a function of λ is given by the theory of Lemaitre and Vallarta (Fig. 14) from which the total number of cosmic rays incident upon the earth per second is found to be 5.7×10^{17} rays, or a current of 0.09 ampere if all rays were of one sign of charge. The influx of energy is

$$4\pi^2R^2A\int_0^{\pi/2}\int_{\mathcal{E}_c}^{\infty}\mathcal{E}\exp(-\alpha\mathcal{E})d\mathcal{E}\cos\lambda d\lambda$$
$$=91\times10^{17}\text{ Bev per sec. or }1.4\times10^{9}\text{ watts.} \quad (111)$$

Another application of the distribution function is in the calculation of the ionization per cm produced at any depth by a single primary of given energy. The total ionization I due to all rays of energy greater than the threshold $\mathcal{E}_c$ is the experimentally measured quantity but the ionization per cm produced by one ray of energy $\mathcal{E}$ can be calculated according to the sequence

$$N=dI/dJ_\perp=(dI/d\mathcal{E})/\pi(dj/d\mathcal{E})$$
$$=(dI/d\mathcal{E})/(\pi A\exp(-\alpha\mathcal{E})), \quad (112)$$

where $dI/d\mathcal{E}$ can be calculated from the differences in the experimental curves of Fig. 25. The values of N obtained from (112) are plotted in Fig. 28 where the ordinates refer to the average number of ions per vertical centimeter produced by a single primary ray of the energy designated, together with all of its secondaries, when the incidence is randomly directed.

16. Conclusion

In concluding a summary of this character one is impressed by the many contributions to the general body of knowledge that have not been touched upon. For example, sections of this report should have been devoted to (a) the ionization-depth relation and its interpretation, (b) the energy spectrum of the cosmic radiation at sea level and the variation of the spectrum with elevation, (c) the variation of cosmic-ray intensity with time and the many implications of these studies bearing upon the places of origin of the cosmic rays and other cosmological phenomena, (d) the cosmic radiation at great depths below the ground, and (e) theories of the origin of the cosmic radiation. Unfortunately the time at the author's disposal did not permit him to complete these sections, and their writing has been deferred until some future time.

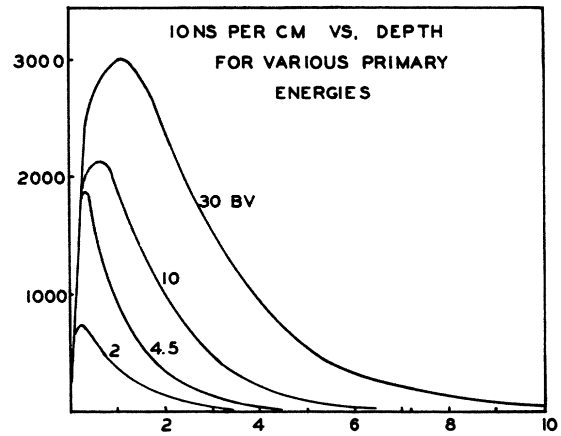


Fig. 28. The specific ionization (ion pairs per vertical centimeter) as a function of atmospheric depth of one randomly incident primary ray of the energy designated and its secondaries. Ordinates are ion pairs per cm of air at NTP. Abscissae are depths in the atmosphere in meters of equivalent water. The results have been reduced from the observations of Bowen, Millikan and Neher.

Finally, the writer wishes to acknowledge the cooperation of his colleagues at the Bartol Research Foundation in making important suggestions and criticisms of the manuscript.

Bibliography

1. P. M. S. Blackett, *The Cosmic Radiation* (Hermann & Cie., Paris, 1935).
2. P. M. S. Blackett, "The Cosmic Radiation," Roy. Soc. Arts. **85**, 893 (1937).
3. K. Buettner, "The penetrating radiation," (Wien-Harms) *Handbuch der exper. Physik* **XXV**$_1$, pp. 479–523 (1928).
4. J. Clay, "The Nature of Cosmic Rays," Proc. Roy. Soc. **A151**, 202–210 (1935).
5. A. H. Compton, "An Attempt to Analyze the Cosmic Rays," Proc. Phys. Soc. **47**, 747–773 (1935); Proc. Am. Phil Soc. **75**, 251 (1935).
6. A. H. Compton, "Recent Developments in Cosmic Rays," Rev. Sci. Inst. **7**, 71–81 (1936).
7. A. H. Compton, "Cosmic Rays as Electrical Particles," Phys. Rev. **50**, 1119–1130 (1936).
8. A. Corlin, "Introductory Chapter to Cosmic Rays in Northern Sweden," Ann. Obs. Lund. No. 4 (1934).
9. K. K. Darrow, "Cosmic Rays—What Physicists Have Learned About Them," Elect. Eng. 221–228, April (1933).
10. G. Hoffmann, "Problems of the Cosmic Radiation," Physik. Zeits. **33**, 633–662 (1932).
11. T. H. Johnson, "Cosmic Rays—Theory and Experimentation," J. Frank. Inst. **214**, 665–689 (1932).
12. T. H. Johnson, "The Nature of the Cosmic Radiation," J. Frank. Inst. **220**, 41–67 (1935).
13. W. Kolhörster and L. Tuwim, *Physical Problems of the Cosmic Radiation* (Leipzig, 1934), 302 pages.
14. L. Leprince-Ringuet, *The Cosmic Rays* (Hermann & Cie., Paris, 1934).
15. E. Miehlnickel, *Hoehenstrahlung* (cosmic rays), (Steinkopff, Dresden and Leipzig, 1938), 316 pages.
16. R. A. Millikan, "On the Cosmic Rays," Ann. de L'Inst. H. Poincare 447–464, (1932).
17. F. Rassetti, *Elements of Nuclear Physics* (Prentice-Hall, New York, 1936), pp. 270–298.
18. E. Regener, "The Cosmic Radiation," Naturwiss. **25**, 1–11 (1937).
19. B. Rossi, *Cosmic Rays* (Hermann & Cie., Paris, 1935).
20. E. G. Steinke, "The Cosmic Radiation," Ergeb. d. Exakt. Naturwiss. **13**, 89–134 (1934); *Handbuch der Physik* **XXIII**$_2$, p. 521 (1933).
21. C. T. R. Wilson, Proc. Camb. Phil. Soc. **11**, 52 (1900).
22. J. Elster, Physik. Zeits. **2**, 116 (1900); J. Elster and H. Geitel, Physik. Zeits. **2**, 560 (1901).
23. A. Gockel, Physik. Zeits. **11**, 280 (1910); **12**, 595 (1911).
24. V. F. Hess, Wien Ber. **120**, 1575 (1911); **122**, 1053, 1481 (1913); **121**, 2001 (1912); Physik. Zeits. **12**, 998 (1911).
25. W. Kolhörster, Verh. d. Phys. Ges. **15**, 1111 (1913); **16**, 719 (1914).
26. D. Skobelzyn, "On a New Kind of Very Fast Beta-Ray," Zeits. f. Physik **54**, 686–702 (1929).
27. W. Bothe and W. Kolhörster, "The Nature of the Cosmic Rays," Zeits. f. Physik **56**, 751–777 (1929).
28. B. Gross, "The Absorption of the Cosmic Radiation," Zeits. f. Physik **83**, 214–221 (1933).
29. C. Eckart, "The Analysis of the Cosmic-Ray Absorption Curve," Phys. Rev. **45**, 851–859 (1934).
30. H. A. Erikson, "Gamma-Ray Ionization in Air and CO_2 as a Function of the Pressure, Electric Field and Temperature," Phys. Rev. **27**, 473–491 (1908).
31. W. Wilson, "Ionization in Gases at High Pressures," Phil. Mag. **17**, 216 (1909).
32. H. F. Fruth, "Ionization by Gamma-Rays in Various Gases," Phys. Rev. **22**, 109 (1923).
33. N. S. Gingrich, "Ionization in Air and CO_2 by Gamma-Rays as Function of Pressure and Temperature," Phys. Rev. **41**, 679 (1932).
34. W. Mächler, "Ionization as a Function of Temperature in Gases at High Pressures," Physik. Zeits. **37**, 211 (1936).
35. J. J. Hopfield, "Gamma-Ray and Cosmic-Ray Ionization in Argon as Function of Pressure and Collecting Field," Phys. Rev. **43**, 675 (1933).
36. I. S. Bowen, "Gamma-Ray Ionization in Air as Function of Pressure and Collecting Field," Phys. Rev. **41**, 24 (1932).
37. E. F. Cox, "Extension of work of Bowen (reference 36) to Other Gases," Phys. Rev. **45**, 503 (1934).
38. S. Chilinski, "Cosmic and Gamma-Ray Ionization in N_2," Phys. Rev. **45**, 309 (1934).
39. A. H. Compton, E. O. Wollan and R. D. Bennett, "Cosmic-Ray Ionization in Very Pure Argon, and Improved Cosmic Ray Meter," Rev. Sci. Inst. **5**, 415 (1934).
40. J. W. Broxon, "Cosmic- and Gamma-Ray Ionization in Various Gases as Function of Pressure and Size of Vessel," Phys. Rev. **27**, 542 (1926).
41. J. W. Broxon, "Cosmic-Ray Ionization in Various Gases as a Function of Pressure," Phys. Rev. **37**, 1320 (1931).
42. J. W. Broxon, "Cosmic-Ray Ionization in N_2 at High Pressures," Phys. Rev. **38**, 1704 (1931).
43. W. F. G. Swann, "Cosmic-Ray Ionization in Air as Function of Pressure at Three Elevations," J. Frank. Inst. **209**, 151–200 (1930).
44. G. Hoffmann, "Cosmic- and Gamma-Ray Ionization in Air, and a Cosmic-Ray Meter," Zeits. f. Physik **69**, 704 (1931).
45. R. A. Millikan, "Cosmic- and Gamma-Ray Ionization in Air and Electroscope Constants," Phys. Rev. **39**, 397 (1931).
46. J. A. Priebsch, "Cosmic-Ray Ionization in Air and Argon as Function of Pressure," Zeits. f. Physik **85**, 448 (1933).
47. P. Pfundt, "Cosmic-Ray Ionization in CO_2," Zeits. f. Physik **85**, 439 (1933).
48. J. Clay, "Cosmic- and Gamma-Ray Ionization in Argon as Function of Pressure and the Effect of the Wall Material," Physica **2**, 111 (1935).

49. J. Clay, Similar studies (reference 48) in other gases, Physica **2**, 811 (1935).
50. J. Clay and M. A. Van Tijn, "Cosmic-Ray Ionization as Function of Pressure and Collecting Field; Interpretation in Terms of Jaffe's Theory of Columnar Recombination," Physica **2**, 825 (1935); **4**, 648 (1937).
51. J. Clay, H. J. Stammer and M. A. Van Tijn, Similar studies (reference 50), Physica **4**, 216 (1937).
52. J. Clay and H. F. Jongen, Similar studies (reference 50), Physica **4**, 245 (1937).
53. J. Clay and G. van Kleef, Physica **4**, 651 (1937).
54. R. M. Sievert, "Gamma and X-Ray Ionization in Air at High Pressures," Nature **129**, 792 (1932).
55. E. G. Steinke and H. Schindler, "Cosmic-Ray Ionization at High Pressures," Naturwiss. **20**, 15 (1932).
56. M. L. E. Oliphant, "Electron Emission from Cold Cathodes Under Positive Ion Bombardment," Proc. Roy. Soc. **A132**, 631 (1931).
57. E. J. Williams, "Discussion of Factors Affecting Ionization Measurements," Proc. Roy. Soc. **A135**, 120 (1932). See also Proc. Roy. Soc. **A139**, 163–186 (1933).
58. J. W. Broxon and G. T. Merideth, "Measurements of Gamma-Ray Ionization in Air at High Pressures and High Gradients," Phys. Rev. **54**, 1–8 (1938).
59. J. W. Broxon and G. T. Merideth, "Analysis of Ionization Currents by Zanstra's Adaptation of Jaffé's Columnar Recombination Theory," Phys. Rev. **54**, 9–17 (1938); Phys. Rev. **54**, 605–608 (1938).
60. J. Clay, "An Analysis of the Cosmic-Radiation Complex," Physica **5**, 94–99 (1938).
61. J. Clay and M. Kwieser, "Ionization in Gases at High Pressures by Gamma-Radiation," Physica **5**, 725–736 (1938).
62. J. W. Broxon, "Theory of the Pressure Variation of Ionization by Range-Particles Emitted from the Walls of a Vessel, Phys. Rev. **37**, 1320 (1931).
63. R. A. Millikan and I. S. Bowen, "Recombination as an Explanation of the Pressure Ionization Relation," Nature **128**, 582 (1931).
64. A. H. Compton, R. A. Bennett and J. C. Stearns, "Theory of Ionization Limited by Preferential Recombination," Phys. Rev. **38**, 1565 (1932); **39**, 873 (1932).
65. W. H. Bragg and R. D. Kleeman, "Theory of Preferential Recombination at High Pressures," Phil. Mag. **11**, 466 (1906); R. D. Kleeman, Phil. Mag. **12**, 273 (1006).
66. G. Jaffé, "Theory of Columnar Recombination," Physik. Zeits. **30**, 849 (1929); Ann. d. Physik **1**, 977 (1929).
67. H. Zanstra, "Adaptation of Jaffé's Theory," Physica **2**, 817 (1935).
68. B. Gross, "Theory of Columnar Recombination," Zeits. f. Physik **78**, 271 (1932); **80**, 125 (1933).
69. W. R. Harper, "Survey of Facts and Theories of the Ionization in Gases at High Pressures," Proc. Camb. Phil. Soc. **28**, 219 (1932); **29**, 149 (1933).
70. J. Clay, "Defense of Columnar Recombination," Phys. Rev. **52**, 143 (1937).
71. I. S. Bowen and E. F. Cox, "Attack on Clay's Interpretation of Columnar Recombination," Phys. Rev. **51**, 232 (1937).
72. L. B. Loeb, *Kinetic Theory of Gases* (McGraw-Hill, 1927), p. 478.
73. K. W. F. Kohlrausch, *Radioaktivität*, p. 139.
74. E. J. Workman, "Secondary Effects in Ionization by Hard Gamma-Rays," Phys. Rev. **43**, 859 (1933).
75. E. G. Steinke, "Shield Effect on Cosmic-Ray Ionization," Zeits. f. Physik **48**, 647 (1928); **31**, 1019 (1930); **42**, 570 (1927).
76. H. Schindler, "Cosmic-Ray Transition Effect," Zeits. f. Physik **72**, 625 (1931).
77. H. Hoerlin, "Latitude Effect of Cosmic Rays at Sea Level and its Change With Elevation up to 6000 Meters," Zeits. f. Physik **102**, 652–677 (1936).
78. J. C. Street and R. T. Young, Jr., "Transition Effects in the Cosmic Radiation," Phys. Rev. **46**, 823–824 (1934).
79. T. H. Johnson, "Interpretation of the Transition Effect," Phys. Rev. **41**, 545–552 (1932).
80. J. C. Street and R. T. Young, Jr., "Cosmic-Ray Measurements in a Small Ionization Chamber; Variation with Altitude and Latitude of the Total Ionization for Various Shields," Phys. Rev. **52**, 552–559 (1937).
81. L. Janossy, "Investigation of the Relation of Shower Frequency to the General Cosmic-Ray Intensity," Proc. Roy. Soc. **A167**, 499–508 (1938).
82. H. J. Bhabha, "On the Penetrating Component of the Cosmic-Radiation," Proc. Roy. Soc. **A164**, 257–293 (1938).
83. H. J. Bhabha and W. Heitler, "On the Passage of Fast Electrons and the Theory of Cosmic-Ray Showers," Proc. Roy. Soc. **A159**, 432–458 (1937).
84. L. Landau and G. Rumer, "The Cascade Theory of Electronic Showers," Proc. Roy. Soc. **A166**, 213–227 (1938).
85. H. Snyder, "Transition Effects of the Cosmic Radiation in the Atmosphere," Phys. Rev. **53**, 960–965 (1938).
86. R. Serber, "Transition Effects of Cosmic Rays in the Atmosphere," Phys. Rev. **54**, 317–320 (1938).
87. J. F. Carlson and J. R. Oppenheimer, "On Multiplicative Showers," Phys. Rev. **51**, 220–231 (1937).
88. R. H. Woodward, "Coincidence Counter Studies of Cosmic-Ray Showers," Phys. Rev. **49**, 711–718 (1936); **49**, 198(A) (1936).
89. L. M. Mott-Smith and L. G. Howell, "Airplane Measurements of Cosmic-Ray Intensity," Phys. Rev. **44**, 4–11 (1933).
90. J. R. Oppenheimer, "Are the Formulae for the Absorption of High Energy Radiations Valid?" Phys. Rev. **47**, 44–52 (1935).
91. J. C. Street and R. H. Woodward, "Counter Calibration and Cosmic-Ray Intensity," Phys. Rev. **46**, 1029–1034 (1934).

92. D. K. Froman and J. C. Stearns, "Altitude Effect on Specific Ionization and Zenith Angle Distribution of Cosmic Rays," Canada J. Res. **16**, 29–40 (1938).
93. H. V. Neher and W. W. Harper, "A High Speed Geiger-Counter Circuit," Phys. Rev. **49**, 940–943 (1936).
94. H. V. Neher and W. H. Pickering, "Modified High Speed Geiger-Counter Circuit," Phys. Rev. **53**, 316–317 (1938).
95. T. H. Johnson, "Circuits for the Control of Geiger-Mueller Counters and for Scaling and Recording their Impulses," Rev. Sci. Inst. **9**, 218–222 (1938).
96. W. E. Danforth and W. E. Ramsey, "Specific Ionization of Cosmic-Ray Particles as Determined by Geiger-Mueller Counter Efficiency," Phys. Rev. **49**, 854 (1936).
97. M. Cosyns, "Study of Geiger-Mueller Counters and Their Application to the Measurement of the Specific Ionization of High Energy Charged Particles," Bull. Tech. Ass. Ing. Brux., 173–265 (1936).
98. W. F. G. Swann, "Distribution of Cosmic-Ray Paths in a Vertical Cylinder," J. Frank. Inst. **216**, 559–581 (1933).
99. D. L. MacAdam and M. R. Lipman, "The Distribution of Cosmic-Ray Paths in a Vertical Cylinder," J. Frank. Inst. **216**, 583–589 (1933).
100. T. H. Johnson and E. C. Stevenson, "Angular Distribution of Low Energy Cosmic Radiation and Interpretation of Angular Distribution Curves," Phys. Rev. **43**, 583 (1933).
101. C. D. Anderson, "Cosmic-Ray Positive and Negative Electrons," Phys. Rev. **44**, 406–416 (1933).
102. P. M. S. Blackett and G. P. S. Occhialini, "Some Photographs of the Tracks of Penetrating Radiation," Proc. Roy. Soc. **A139**, 699–726 (1933).
103. W. Kolhörster and L. Tuwim, "Interpretation of the Experimental Values for the Average Specific Ionization of Cosmic Radiation from Measurements with Ionization Chamber and Tube Counter," Zeits. f. Physik **81**, 435–439 (1933).
104. H. A. Bethe, *Handbuch der Physik*, **XXIV**$_1$, p. 521.
105. W. Messerschmidt, "Fluctuations of the Cosmic Radiation," Zeits. f. Physik **78**, 668 (1932); **85**, 332 (1933).
106. R. D. Evans and H. V. Neher, "Nature of Statistical Fluctuations with Applications to Cosmic Rays," Phys. Rev. **45**, 144 (1934).
107. T. C. Fry, *Probability and its Engineering Uses*, (McGraw-Hill).
108. G. L. Locher, "Cosmic-Ray Particles," Phys. Rev. **39**, 883–888 (1932).
109. W. F. G. Swann, "Measurement of the Ionization per Centimeter of Path by Individual Secondary Cosmic Rays," Phys. Rev. **44**, 961 (1933); J. Frank. Inst. **217**, 79 (1934).
110. G. L. Locher, "Cloud-Chamber Investigations of Some Cosmic-Ray Interactions with Matter," J. Frank. Inst. **224**, 555–582 (1937).
111. C. D. Anderson and S. H. Neddermeyer, "Fundamental Processes in the Absorption of Cosmic-Ray Electrons and Protons," Int. Conf. on Phys., London, p. 179 (1934); Phys. Rev. **50**, 263 (1936).
112. P. Kunze, "Investigation of Cosmic Rays in the Wilson Chamber," Zeits. f. Physik **83**, 1–18 (1933).
113. D. R. Corson and R. B. Brode, "Specific Ionization and Mass of Cosmic-Ray Particles," Phys. Rev. **53**, 773–777 (1938).
114. J. F. Lehman and T. H. Osgood, "Total Ionization Due to Absorption in Air of Slow Cathode Rays," Proc. Roy. Soc. **A115**, 609 (1927).
115. W. Schmitz, "Measurement of Ionization Caused by Electron Rays in Air," Phys. Zeits. **29**, 846 (1928).
116. E. Buchmann, "On the Production of Secondary Electrons by Electrons of 1–30 Kv," Ann. d. Physik **87**, 509 (1928).
117. A. Eisl, "On the Ionization of Air by Cathode Rays of 10–60 Kv," Ann. d. Physik **3**, 277 (1929).
118. H. Kulenkampf, "Remarks on the Absorption Law of the Penetrating Cosmic Rays," Physik. Zeits. **30**, 561 (1929).
119. E. Regener, "The Absorption Curve of the Cosmic Radiation and its Significance," Physik. Zeits. **34**, 306–323 (1933).
120. W. Kramer, "On the Absorption of Cosmic Rays in Water and Analysis of the Absorption Function," Zeits. f. Physik **85**, 411–424 (1933).
121. E. Lenz, "Analysis of the Cosmic-Ray Absorption Curve," Zeits. f. Physik **83**, 194–213 (1933).
122. I. S. Bowen, R. A. Millikan and H. V. Neher, "A Very High Altitude Survey of the Effect of Latitude upon Cosmic-Ray Intensities and an Attempt at a General Interpretation of Cosmic-Ray Phenomena," Int. Conf. on Phys., London (1934); see also earlier papers of Millikan and collaborators.
123. G. Pfotzer, "Coincidence Counter Measurements of the Intensity Depth Curve," Zeits. f. Physik **102**, 23–58 (1936).
124. E. Regener, and G. Pfotzer, "Vertical Intensity of Cosmic Rays by Threefold Coincidences in the Stratosphere," Nature **136**, 718 (1935).
125. W. F. G. Swann, G. L. Locher and W. E. Danforth, "Geiger-Mueller Counter Measurements of Cosmic-Ray Intensities in the Stratosphere," Phys. Rev. **51**, 389–390 (1937); J. Frank. Inst. **221**, 275 (1936).
126. I. S. Bowen, R. A. Millikan and S. K. Haynes, "Precision Cosmic-Ray Measurements up to Within a Percent or Two of the Top of the Atmosphere," Phys. Rev. **50**, 992–998 (1936).
127. R. A. Millikan and C. D. Anderson, "Cosmic-Ray Energies and Their Bearing on the Photon-Neutron Hypothesis," Phys. Rev. **40**, 325–328 (1932).
128. C. D. Anderson, "Energies of Cosmic-Ray Particles," Phys. Rev. **41**, 405–421 (1932).
129. C. T. R. Wilson, "The Acceleration of Beta-Particles in Strong Electric Fields such as Those of Thunder Clouds," Proc. Camb. Phil. Soc. **22**, 534 (1924).
130. J. Clay, "Penetrating Radiation II," Proc. Roy. Acad. Amst. **30**, 1115–1127 (1927); "Penetrating Radiation II," ibid. **31**, 1091–1097 (1928).

131. J. Clay, "The Earth Magnetic Field and the Corpuscular Nature of the Cosmic Radiation," Proc. Roy. Acad. Amst. **35**, 3 (1932); **33**, 711 (1930).
132. W. Bothe and W. Kolhörster, "Comparative Measurements of the Cosmic Radiation in the North Sea," Berl. Bericht. **26**, 450–456 (1930).
133. R. A. Millikan, "On the Question of the Constancy of the Cosmic Radiation and the Relation of these Rays to Meteorology," Phys. Rev. **36**, 1595–1603 (1930).
134. K. Grant, "Observations on the Penetrating Radiation in Antarctica," Nature **127**, 924 (1931).
135. A. Corlin, Lund Obs. **II**, 58 (1930); Arch. f. Mat. o. Fys. (1930).
136. R. H. Millikan and G. H. Cameron, "High Altitude Tests of the Geographical, Directional and Spectral Distribution of Cosmic Rays," Phys. Rev. **31**, 163–173 (1928).
137. A. H. Compton, "Variation of Cosmic Rays with Latitude," Phys. Rev. **41**, 111–113 (1932).
138. A. H. Compton, "Progress of Cosmic-Ray Survey," Phys. Rev. **41**, 681 (1932).
139. A. H. Compton, "A Geographic Study of Cosmic Rays," Phys. Rev. **43**, 387–403 (1933); Trans. Am. Geophys. Un., p. 154 (1933); Scient. Month. **XXXVI**, 75 (1933).
140. A. H. Compton, J. M. Benade and P. G. Ledig, "Further Geographic Studies of Cosmic Rays," Phys. Rev. **45**, 294(A) (1934).
141. R. D. Bennett, J. L. Dunham, E. H. Bramhall and P. K. Allen, "Intensity of Cosmic-Ray Ionization in Western North America," Phys. Rev. **42**, 446–447 (1932).
142. J. Clay and H. P. Berlage, "Variation of Cosmic Radiation with the Geographic Latitude and the Earth's Magnetism," Naturwiss. **20**, 687 (1932).
143. J. Clay, "The Corpuscular Nature of the Cosmic Radiation and the Geomagnetic Effect," Naturwiss. **21**, 43 (1933).
144. J. Clay, "Earth-Magnetic Effect and the Corpuscular Nature of the Cosmic Radiation," Proc. Roy. Acad. Amst. **35**, 1282–1290 (1932).
145. J. Clay, P. M. van Alphen and G. G. t'Hooft, "Magnetic Latitude and Longitude Effects of Cosmic Rays," Physica **1**, 363–382, 829–838 (1934).
146. L. Hermans and G. Gueben, "Variation of Cosmic Radiation with Latitude," Bull. Roy. Soc. Liège No. 5 (1934).
147. M. A. Dauvillier, "Researches on Cosmic Rays; Effects at Scoresby Sound During the Polar Year," J. de phys. et rad. **5**, 640–648 (1934).
148. J. A. Prins, "Latitude Effect of Cosmic Rays," Nature **132**, 781 (1933).
149. H. Hoerlin, "Latitude Effect of Cosmic Radiation," Nature **132**, 61 (1933).
150. H. Hoerlin, "Variation of Cosmic-Ray Intensity with Height at Different Latitudes," Physik. Zeits. **35**, 793–795 (1934).
151. B. Rossi, I. Ranzi and S. de Benedetti, "Scientific Expedition to Eritrea for the Study of Cosmic Radiation," Ricerca Scient. **5**, 561–605 (1934).
152. R. A. Millikan, "New Techniques in the Cosmic-Ray Field and Some Results Obtained with Them," Phys. Rev. **43**, 661–669 (1933).
153. R. A. Millikan and H. V. Neher, "A Precision World Survey of Sea-Level Cosmic-Ray Intensities," Science **82**, 574 (1935); Phys. Rev. **50**, 15–24 (1936).
154. R. A. Millikan and H. V. Neher, "The Equatorial Longitude Effect in Cosmic Rays," Phys. Rev. **47**, 205–208 (1935); Proc. Nat. Acad. Sci. **21**, 313–315 (1935).
155. I. S. Bowen, R. A. Millikan and H. V. Neher, "New High Altitude Study of Cosmic-Ray Bands and Determination of Their Total Energy Content," Phys. Rev. **44**, 246–252 (1933).
156. I. S. Bowen, R. A. Millikan and H. V. Neher, "A Very High Altitude Survey of the Effect of Latitude upon Cosmic-Ray Intensities and an Attempt at a General Interpretation of Cosmic-Ray Phenomena," Phys. Rev. **46**, 641–652 (1934).
157. B. Rossi, "On the Magnetic Deflection of Cosmic Rays," Phys. Rev. **36**, 606 (1930).
158. B. Rossi, "Researches on the Action of the Earth's Magnetic Field on the Cosmic Radiation," Nuovo Cimento **VIII**, 3–15 (1931); Rend. Lencei **13**, 47–52 (1931).
159. E. Fermi and B. Rossi, "Action of the Earth's Magnetic Field on Cosmic Rays," Rend. Lencei **17**, 346–350 (1933).
160. C. Stoermer, Vid. Selsk, Skr. Christiana (1904); (1913).
161. C. Stoermer, Zeits. f. Astrophys. **1**, 237 (1930).
162. C. Stoermer, Terr. Mag. **37**, 375 (1932).
163. C. Stoermer, "On the Trajectories of Electric Particles in the Field of a Magnetic Dipole with Applications to the Theory of Cosmic Radiation," Univ. Obser. Oslo. (I) Pub. No. 10, p. 5–19 (1934).
164. L. Vegard, "Northern lights," *Handbuch der exper. Physik* (Wien-Harms), **XXV**$_1$, pp. 418–422.
165. C. Stoermer, "On the Problems of Polar Light," Eegeb. der Kosmische, Phys. (Liepzig) **1**, 1–86 (1931).
166. P. Epstein, "Note on the Nature of Cosmic Rays," Proc. Nat. Acad. Sci. **16**, 658–663 (1930); **17**, 160–161 (1931).
167. T. H. Johnson and J. C. Street, "The Variation of Cosmic-Ray Intensities with Azimuth on Mt. Washington, N. H.," Phys. Rev. **43**, 381 (1933).
168. T. H. Johnson, "The Azimuthal Asymmetry of the Cosmic Radiation, Phys. Rev. **43**, 834–835 (1933).
169. L. Alvarez and A. H. Compton, "A Positively Charged Component of Cosmic Rays," Phys. Rev. **43**, 835 (1933).
170. T. H. Johnson, "The Azimuthal Asymmetry of the Cosmic Radiation in Mexico City," J. Frank. Inst. **215**, 749–753 (1933).

171. T. H. Johnson, "Preliminary Report of Angular Distribution Measurements of Cosmic Radiation in Equatorial Latitudes," Phys. Rev. **44**, 856 (1933).
172. T. H. Johnson "Directional Measurements of the Cosmic Radiation and their Significance," Phys. Rev. **45**, 294(A) (1934).
173. T. H. Johnson and E. C. Stevenson, "Asymmetry of the Cosmic Radiation at Swarthmore," Phys. Rev. **44**, 125–126 (1933).
174. E. C. Stevenson, "Azimuthal Asymmetry of the Cosmic Radiation in Colorado," Phys. Rev. **44**, 855 (1933).
175. J. C. Stearns and R. D. Bennett, "Angular Distribution of Cosmic-Ray Particles," Phys. Rev. **43**, 1038–1039 (1933).
176. S. A. Korff, "Azimuthal Asymmetry of Cosmic Radiation," Phys. Rev. **44**, 515 (1933).
177. A. Ehmert, "The East-West Effect of Cosmic Rays on the Zugspitze," Physik. Zeits. **35**, 20–25 (1934).
178. B. Rossi, "Directional Measurements of the Cosmic Radiation Near the Geomagnetic Equator," Phys. Rev. **45**, 212–214 (1934); Ricerca Scient. **4**, 3 (1933).
179. P. Auger and L. Leprince-Ringuet, "Study of the Variation of Cosmic Radiation Between Latitudes 45°N and 38°S," Comptes rendus **197**, 1242–1244 (1933); Nature **133**, 138 (1934).
180. J. C. Stearns and D. K. Froman, "Azimuthal Asymmetry on Mt. Evans," Phys. Rev. **46**, 535–536 (1934).
181. S. A. Korff, "Penetrating Power of the Asymmetric Component of the Cosmic Radiation," Phys. Rev. **46**, 74 (1934).
182. J. Clay, "Positive and Negative Primary Cosmic Radiation," Physica **2**, 861–869 (1935).
183. B. J. F. Schonland and P. T. Viljoen, "Distribution of Ionizing Particles with Respect to the Magnetic Meridian," Phil. Mag. **16**, 449 (1933).
184. H. V. Neher, "The East-West Effect in Southern India," Phys. Rev. **53**, 328(A) (1938).
185. T. H. Johnson, "Coincidence Counter Studies of the Corpuscular Cosmic Radiation," Phys. Rev. **45**, 569–585 (1934).
186. T. H. Johnson, "Progress of Directional Survey ot Cosmic-Ray Intensities and its Application to the Analysis of the Primary Cosmic Radiation," Phys. Rev. **48**, 287–299 (1935).
187. G. Lemaitre and M. S. Vallarta, "On Compton's Latitude Effect of Cosmic Radiation," Phys. Rev. **43**, 87–91 (1933).
188. W. F. G. Swann, "Application of Liouville's Theorem to Electron Orbits in the Earth's Magnetic Field," Phys. Rev. **44**, 224–227 (1933).
189. J. Bartels, "The Eccentric Dipole Approximating the Earth's Magnetic Field," Terr. Mag. **41**, 225–250 (1936).
190. G. Lemaitre, Ann. Soc. Sci. Brux. **A54**, 162 (1934).
191. L. Bouckaert, Ann. Soc. Sci. Brux. **A54**, 174 (1934).
192. G. Lemaitre, Ann. Soc. Sci. Brux. **A54**, 194 (1934).
193. G. Lemaitre, M. S. Vallarta and L. Bouckaert, "North-South Asymmetry of Cosmic Radiation," Phys. Rev. **47**, 434–436 (1935).
194. G. Lemaitre and M. S. Vallarta, "Geomagnetic Analysis of Cosmic Radiation," Phys. Rev. **49**, 719–726 (1936).
195. G. Lemaitre and M. S. Vallarta, "Allowed Cone of Cosmic Radiation," Phys. Rev. **50**, 493–504 (1936).
196. C. Stoermer, "On the Trajectories of Electric Particles in the Field of a Dipole (II)" Univ. Observ. Oslo. **12**, 5–47 (1934); (III), Astrophys. Norveg. **1**, 1–9 (1934); (IV) ibid. **1**, 115–167 (1935); (V) ibid. **2**, 1–121 (1936); (VI) ibid. **2**, 193–248 (1937). "Envelope of Trajectories from Infinity with Asymptote Parallel to *X* Axis," Norske Videns. Akad. Oslo. **5**, 5–95 (1936); "Envelope of Trajectories with Asymptotes Parallel and Perpendicular to Magnetic Axis," ibid. **6**, 5–91 (1936). "Program for Quantitative Discussion of Electron Orbits," Comptes Rendus Cong. Internat. Math. Oslo, 61–74 (1937).
197. C. Stoermer, "Critical Remarks on Paper by Lemaitre and Vallarta," Phys. Rev. **45**, 835–838 (1934).
198. A. Schmidt, Beitrag z. Geophys. **41**, 346–358 (1934).
199. G. Lemaitre, "Longitude Effect and the Symmetry of Cosmic Radiation," Nature **140**, 23–24 (1937).
200. H. V. Neher, "The East-West and Longitude Effects," Phys. Rev. **47**, 417–418 (1935).
201. M. S. Vallarta, "On the Longitude Effect of Cosmic Radiation," Phys. Rev. **47**, 647–651 (1935).
202. T. H. Johnson, "On Variations of Cosmic Radiation During Magnetic Storms," Terr. Mag. **43**, 1–6 (1938).
203. T. H. Johnson, "Earth-Magnetic Analysis of Cosmic Radiation," Trans. Am. Geophys. Union. 171 (1935).
204. T. H. Johnson, "Correlation of Cosmic-Ray Geomagnetic Effects," Trans. Am. Geophys. Union, (1938); J. Frank. Inst. **226** (1938).
205. H. Zanstra, "Principle of the Cosmic-Ray Spectroscope," Naturwiss. **22**, 171 (1934).
206. A. H. Compton and R. N. Turner, "Variation of Cosmic Rays with Latitude on the Pacific Ocean," Phys. Rev. **51**, 1005 (1937); **52**, 799–814 (1937).
207. M. S. Vallarta, "Longitude Effect of Cosmic Radiation," Phys. Rev. **47**, 647–651 (1935).
208. V. F. Hess and R. Steinmaurer, "Results of Registration of Cosmic Rays on the Hafelekar from Sept., 1931 to Nov., 1932," Berl. Ber. **XV**, 5–24 (1933); **XXII**, 3–16 (1933).
209. B. J. Schonland, B. Delatisky and J. Gaskell, "Intensity Measurements of Cosmic Radiation in Capetown, 1933–1935," Terr. Mag. **42**, 137–152 (1937).
210. T. H. Johnson and D. N. Read, "Unidirectional Measurements of the Cosmic-Ray Latitude Effect," Phys. Rev. **51**, 557–564 (1937).
211. J. Clay, E. M. Bruins and J. T. Wiersma, "The Dutch Cosmic-Ray Expedition Amsterdam to Chile," Physica **3**, 746–756 (1936).
212. E. M. Bruins, "Cosmic Rays and the Earth's Magnetic Field," Physica **4**, 659–666 (1937); Amsterdam thesis (1938).

213. M. S. Vallarta, "Longitude Effect of Cosmic Radiation and the Position of the Earth's Magnetic Center," Nature **139**, 24–25 (1937).
214. H. V. Neher and W. H. Pickering, "Latitude Effect for Cosmic-Ray Showers," Phys. Rev. **53**, 111–116 (1938).
215. T. H. Johnson, "Evidence for Positron-Negatron Component of the Primary Cosmic Radiation," Phys. Rev. **47**, 318–319 (1935).
216. W. H. Pickering, "Geographical Variation of Cosmic-Ray Showers," Phys. Rev. **49**, 945 (1936).
217. M. S. Vallarta, "The Interpretation of the Azimuthal Effect of Cosmic Radiation," Phys. Rev. **44**, 1–3 (1933).
218. T. H. Johnson, "North-South Asymmetry of the Cosmic Radiation," Phys. Rev. **47**, 91–92 (1935).
219. J. Barnothy and M. Forro, "Meteorological-Magnetic Effects in the Cosmic Radiation," Zeits. f. Physik **100**, 742–753 (1936); **104**, 534–539 (1934).
220. J. W. Broxon, G. T. Merideth and L. Strait, "Relations Among the Fluctuations of Cosmic Radiation, Terrestrial Magnetism, Potential Gradient, and Absolute Humidity," Phys. Rev. **44**, 253–257 (1933).
221. J. W. Broxon, "Correlation of Cosmic Radiation with Geomagnetism," Terr. Mag. **39**, 121–125 (1934).
222. J. Clay and E. M. Bruins, "Magnetic Storms and Variations of Cosmic Rays," Physica **5**, 111–114 (1938).
223. A. Corlin, "An Indication of a Correlation Between Cosmic Rays and Geomagnetism," Lund Obs. Circ. Mar. 31, (1931).
224. A. Corlin, "Results of Cosmic-Ray Measurements in Northern Sweden," Physik. Zeits. **31**, 1056 (1930).
225. S. E. Forbush, "On the Effects in Cosmic-Ray Intensity Observed During the Recent Magnetic Storm," Phys. Rev. **51**, 1108 (1937).
226. V. F. Hess and W. Illing, "Terrestrial Magnetism and Cosmic Rays," Nature **135**, 97–98 (1935).
227. V. F. Hess and A. Demmelmaier, "World-Wide Effect in Cosmic-Ray Intensity as Observed in the April 1937 Storm," Nature **140**, 316–317 (1937).
228. V. F. Hess, A. Demmelmaier and R. Steinmaurer, "Relations Between Terrestrial Magnetism and Cosmic-Ray Intensity," Terr. mag. **43**, 7–14 (1938).
229. W. Kolhörster, "Cosmic Rays; Barometer Effect, Variation of Second Kind, and Disturbances Produced by the Earth's Field," Phys. Rev. **53**, 768 (1938).
230. S. E. Forbush, "On Cosmic-Ray Effects Associated with Magnetic Storms," Terr. Mag. **43**, 203–218 (1938).
231. R. Gunn, "Terrestrial Magnetism and the Cosmic-Ray Diurnal Effect," Phys. Rev. **41**, 683 (1932).
232. J. L. Thompson, "Note on Diurnal Variation of Cosmic Radiation," Phys. Rev. **50**, 869 (1936).
233. R. L. Doan, "Fluctuations in Cosmic-Ray Ionization Recorded on Several Meters," Phys. Rev. **49**, 107–122 (1936).
234. J. A. Priebsch and R. Steinmaurer, "Cosmic-Ray Fluctuations," Gerland. Beitrag. z. Geophys. **37**, 296 (1932).
235. E. G. Steinke, "Cosmic-Ray Fluctuations," Zeits. f. Physik **64**, 48 (1930).
236. J. E. Thompson, "Diurnal Effect in Cosmic Rays, Phys. Rev. **52**, 140 (1937).
237. V. F. Hess and W. S. Pforte, "On the Solar Component of Cosmic Rays," Zeits. f. Physik **71**, 171–178 (1931).
238. V. F. Hess, R. Steinmaurer and H. Graziadei, "Analysis of Records on the Hafelekar for 1931–1932," Berl. Ber. (1933).
239. A. Demmelmaier, "The Diurnal Cosmic-Ray Effect," Wien Sitz. Ber. 146 (1937).
240. S. E. Forbush, "On Diurnal Variations in Cosmic-Ray Intensity," Terr. Mag. **42**, 1–16 (1937).
241. V. F. Hess, H. Graziadei and R. Steinmaurer, Wien Ber. **143**, 313–334 (1934).
242. V. F. Hess and H. Graziadei, Terr. Mag. **41**, 9–14 (1936).
243. A. H. Compton, R. D. Bennett and J. C. Stearns, "Diurnal Variation of Cosmic Rays," Phys. Rev. **41**, 119–126 (1932).
244. H. Carmichael and E. G. Dymond, "High Altitude Cosmic-Ray Measurements Near the Magnetic Pole," Nature **141**, 910–911 (1938).
245. A. H. Compton and R. J. Stephenson, "Cosmic-Ray Ionization at High Altitudes," Phys. Rev. **45**, 441–450 (1934).
246. S. N. Vernoff, "On the Study of Cosmic Rays at Great Altitudes," Phys. Rev. **46**, 822 (1934); Comptes Rendus, USSR **14**, 263–266 (1937).
247. E. Regener, "New Results in Cosmic-Ray Measurements," Nature **132**, 696 (1933).
248. E. Regener, "Further Measurements of the Cosmic Rays in the Stratosphere," Physik. Zeits. **34**, 820 (1933).
249. S. A. Korff, L. F. Curtiss and A. V. Astin, "Latitude Effect in Cosmic Radiation at High Altitudes," Phys. Rev. **53**, 14–22 (1938).
250. L. F. Curtis, A. V. Astin, L. L. Stockmann, B. W. Brown and S. A. Korff, "Cosmic-Ray Observations in the Stratosphere," Phys. Rev. **53**, 23–29 (1938).
251. T. H. Johnson, "The Cosmic-Ray Intensity at High Elevations in Northern Latitudes," Phys. Rev. **54**, 151–152 (1938).
252. I. S. Bowen, R. A. Millikan and H. V. Neher, "New Evidence as to the Nature of the Incoming Cosmic Rays etc.," Phys. Rev. **53**, 217–223 (1938).
253. I. S. Bowen, R. A. Millikan and H. V. Neher, "Influence of the Earth's Magnetic Field on Cosmic Rays up to the Top of the Atmosphere," Phys. Rev. **52**, 80–88 (1937).
254. I. S. Bowen, R. A. Millikan and H. V. Neher, "New Light on the Nature of the Incoming Cosmic Rays," Phys. Rev. **53**, 855–861 (1938).
255. T. H. Johnson, "The Intensity of the Primary Cosmic Radiation and its Energy Distribution," Phys. Rev. **53**, 499–501 (1938).

256. S. Chapman, "Cosmic Rays and Magnetic Storms," Nature **140**, 423 (1937).
257. E. Korner, "The Pressure Dependence of the Ionization from Wall Contamination," Zeits. f. Physik **85**, 442–447 (1933).
258 D. K. Froman and J. C. Stearns, "Cosmic-Ray Showers and Bursts," Rev. Mod. Phys. **10**, 133–192 (1938).
259. L. M. Mott-Smith, "An Attempt to Deflect Magnetically the Cosmic-Ray Corpuscles," Phys. Rev. **39**, 403–414 (1932).
260. M. A. Tuve, "A Coincidence Counter Circuit," Phys. Rev. **35**, 651 (1930).
261. B. Rossi, "A Coincidence Counter Circuit," Nature **125**, 636 (1930).
262. T. H. Johnson and J. C. Street, "A Circuit for Recording Coincident Discharges of G.M. Counters," J. Frank. Inst. **215**, 239–246 (1933).
263. L. Fussell and T. H. Johnson, "Vacuum Tube Characteristics in Relation to the Selection of Coincidences of G.M. Counters." J. Frank. Inst. **217**, 517–524 (1934).
264. H. P. Barasch, "An Improved Counting Circuit," Proc. Phys. Soc. Lond. **47**, 824–834 (1935).
265. G. L. Locher, "The Amplification and Recording of Rapid G.M. Counter Impulses," J. Frank. Inst. **216**, 553–558 (1933).
266. C. Eckart and F. R. Shonka, "Accidental Coincidences in Counter Circuits," Phys. Rev. **53**, 752–756 (1938).
267. L. I. Shiff, "Counter Statistics," Phys. Rev. **50**, 88, (1936).
268. E. J. Shremp, "(I) General Theory of the Earth's Shadow Effect of Cosmic Radiation and (II) the Simple Shadow Cone," Phys. Rev. **54**, 153–162 (1938).
269. M. S. Vallarta, "Cosmic Rays and the Magnetic Moment of the Sun," Nature **139**, 839 (1937).
270. P. M. S. Blackett, "Further Measurements of the Cosmic Ray Energy Spectrum," Proc. Roy. Soc. **A159**, 1–31 (1937).
271. P. M. S. Blackett, "Interpretation of the Temperature Effect, Phys. Rev. **54**, 973 (1938).
272. L. Onsager, "Initial Recombination of Ions," Phys. Rev. **54**, 554–557 (1938).
273. R. Steinmaurer, *Ergebnisse der Kosmischen Physik*, Vol. III (1938), pp. 38–112.
274. L. M. Mott-Smith and G. L. Locher, Phys. Rev. **38**, 1399–1408 (1931).
275. T. H. Johnson, W. Fleischer and J. C. Street, Phys. Rev. **40**, 1048 (1932).
276. W. J. G. Swann, J. Frank. Inst. **222**, 269 (1936).

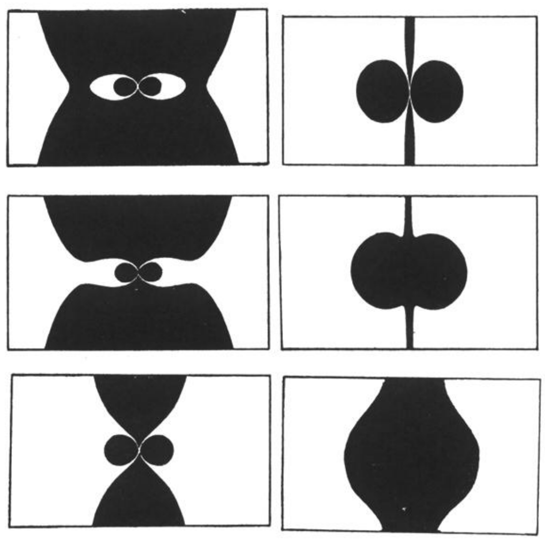


FIG. 12. Cross sections of the regions forbidden to charged particles of various initial angular momenta, $2mv\gamma$, in the magnetic field of a dipole. The position of the dipole in each figure is at the center with its axis pointing vertically. The orbits of particles of the values of γ designated are excluded from the regions traced out by revolving the darkened areas about the vertical axis. Values of γ are, from top to bottom, left-hand side, -1.016, -0.97 and -0.5; right-hand side, -0.05, 0.03 and 0.2. (Stoermer, references 163–165.)