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PROBABLE VALUES OF THE GENERAL PHYSICAL CONSTANTS

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By Raymond T. Birge
University of California, Berkeley

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Introduction

Some of the most important results of physical science are embodied, directly or indirectly, in the numerical magnitudes of various universal constants, and the accurate determination of such constants has engaged the time and labor of many of the world's most eminent scientists. Some of these constants can be evaluated by various methods. Each has been investigated by various persons, at various times, and each investigation normally produces a numerical result more or less different from that of any other investigation. Under such conditions there arises a general and continuous need for a searching examination of the *most probable* value of each important constant. The need is general since every physical scientist uses such constants.

The need is continuous since the most probable value of to-day is not that of to-morrow, because of the never-ending progress of scientific research. These remarks appear to the writer so self-evident that the mere statement of them may be deemed superfluous. However, in spite of these facts, an investigation of the values of general constants in current use in the literature reveals a surprising lack of consistency, both in regard to the actually adopted values and to the origin of such values. This is probably due to the fact that it is almost impossible to find a critical study of the best values, sufficiently up-to-date to be really reliable, and sufficiently detailed to explain the inconsistencies found among older tables.

The situation is much better in the case of selected groups of constants. Thus the best value of the atomic weight of each element is determined annually by certain atomic weight committees, and the need of such a list of atomic weights is obvious to every chemist. There is certainly a similar need in the case of the even more important constants such as the velocity of light, the charge of the electron, the Planck constant h, etc. In attempting to respond to this need, the writer has become only too well aware of the intrinsic difficulties involved, but at the same time he has become increasingly convinced of the existence of the need itself. The present investigation was undertaken only at the express request of others, and the results given here should be considered more as a presentation of the situation than as a final solution of the problem. To obtain a satisfactory and thoroughly reliable judgment in such matters, there is required the unbiased cooperation of many persons situated in scientific laboratories throughout the world. In the preparation of this paper, I have endeavored to obtain such cooperation by means of an extensive correspondence, but because of a necessary limitation of time, such correspondence has been confined almost entirely to this country. In addition I have received valuable advice and suggestions from various persons on the campus of this University. To the many persons—too numerous to name individually—who have thus aided by voice or by letter in the preparation of this paper, I desire to express my sincere thanks. At the same time I wish to request a continuance of such cooperation, not only from those who have already aided, but from all readers of the present article. The character of such assistance, and its need, will be clearer after a discussion of the situation.

The decision as to the most probable value, at a particular time, of any given constant, necessarily demands a certain amount of judgment. It however demands, far more, a knowledge of the facts, and such knowledge is not always easy to obtain. Similarly, each investigator uses a certain amount of judgment in the selection of his data and in the final conclusions reached. Here also, apparently, it is the facts which are even more important. For example, there is scarcely an important constant which does not require, in its evaluation, a knowledge of the values of certain other constants. I call these "auxiliary constants." What value of each auxiliary constant did the investigator use? Is this still the best value, and if not, what effect will the required revision have on the final result? The writer has found, to his surprise, that in many cases it is quite impossible to determine, from a printed article, what values of certain auxiliary constants actually were adopted. Often it is not possible to determine even the formula used in calculating the final result. Many approximations are made in actual calculations, and these are ordinarily justified by the precision—or better, lack of precision—of the investigation in question. Occasionally, however, an investigator makes an approximation which is not justified by the circumstances of the case, and which produces an appreciable change in the final result. The precise method of calculation employed also affects the result, and often the adopted method is capable of improvement.

In the preceding paragraph I have listed several sources of possible error in a final published value, quite distinct from the excellence of the experimental work itself. Whether it is worth while for the reviewer to attempt to improve a result by investigating such potential sources of error, depends entirely on the particular case considered. Thus if an investigator has spent many years collecting certain experimental data, and the reduction of these data, to give the final desired result, can be made in several weeks, by the most reliable analytic methods, while a less trustworthy method of calculation can be made in several days, it seems obvious which method of calculation should be employed. Yet the writer has found in the literature cases where an appreciable part of the accuracy obtained by years of the most skillful experimental work has been sacrified, naturally unintentionally, by approximate or ill-advised methods of computation. In such cases it has seemed imperative to make a recalculation of the results, and this has accordingly been done. In many cases no recalculation was possible, since the original data needed for such a calculation had not been published. In other cases it was impossible to determine even what method of calculation had actually been employed.

In the present paper the writer has, then, attempted to carry out the following general procedure

(1) Each general constant has been determined from the available data, beginning with that constant whose value depends least on other constants. The value thus adopted has then been used *consistently* in the calculation of each succeeding constant for which it is an "auxiliary constant." No attempt has been made to compare the results of different investigators until these have been made properly comparable by the use of the same value of each auxiliary constant. Failure to make such revision, in the present paper, in certain cases, is due to lack of available information on the matter, and similar failure in the case of previous articles of this nature may well be due to a similar cause. This point I should like to emphasize above all others.

(2) Each constant has been calculated from the available data by the use, as far as possible, of formulas which involve no approximations. Various specific cases will appear in the later sections of approximations which have thus been eliminated, as well as cases in which, from lack of information, it has not been possible to make an intelligent revision.

(3) Each constant has been recalculated, whenever it seemed necessary, by analytic methods—usually by the method of least squares. The proper reduction of data is a very extensive subject, and is treated in detail in numerous text-books and articles. It seems however desirable to call specific attention to certain points.

In the present investigation I have endeavored to obtain not only the most probable value of a given constant, but also its "*probable error*." Some estimate of the probable error is often quite as important as the constant itself, but published lists of constants contain, in general, no precise estimate of error. The term "probable error" is a technical one, applied properly only when the errors are distributed on a Gaussian error curve. It is defined as the numerical magnitude which the *true error is as likely to exceed as not*. This is a definite concept and as such furnishes an *objective* criterion of error. The writer strongly advocates its use in all cases. Expressions such as "the maximum error," "the limit of uncertainty," "the possible uncertainty," etc, are inherently vague and technically undefinable. It may be noted, in passing, that in some cases one cannot tell whether the published error is intended to represent a probable error or a limit of error.[1]

With a Gaussian error curve ($y=e^{-x^2}$), there is one chance in 4.5 that the true error is at least twice the probable error, one in 22 that it is at least three times the probable error, one in 142 that it is at least four times, etc. One can arbitrarily choose, as the limit of error, a quantity which the true error will exceed only once in a thousand times. This quantity, called the "huge error," is 4.9 times the probable error. One can however equally well adopt a ratio of one to the million, or one to the hundred, as defining the limit of error. One of the greatest difficulties in the present work has been the proper weighting of various results, one of which is stated in terms of "limit of error," another in terms of a "probable error," another possibly with the error undefined, and still another with no definite statement as to accuracy. This is quite aside from the reliability or correctness of the method used by each investigator in arriving at his stated error.

Now in order to evaluate the probable error it is necessary to use the method of least squares. One great objection, it appears to me, to certain methods which have been proposed as substitutes for least squares, is that they give no objective criterion for the error. These substitute methods often have the additional objection that they give no unique solution of the data. The investigator is permitted to combine the data in various different ways, each leading to a different result.[2] A third objection is that they are usually less reliable than least squares. Graphical methods combine all of these objections. They are never as accurate as an analytic method, and have no unique solution. Not only will each different person obtain in general, a different result

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[1] German scientists often use the term "mean error." This is a quantity which, when multiplied by 0.6745, gives the probable error.

[2] This is quite aside from the question of *weighting*, which *does* require the judgment of the investigator and should be made in all cases.

from the same set of data, but the *same* person will obtain a different result, each time he makes such a graph of a given set of data. Graphical methods are invaluable for a preliminary survey of the results, especially in order to determine, if possible, the proper *functional* form to be used. They should, however, never be employed in the final accurate evaluation of important constants.[3]

The chief objection to least squares work is the time required. This objection is hardly valid for simple functions such as $y=a_{00}$, and $y=a_{01}+a_{11}x$, and as a matter of fact, the great majority of all calculations involved in this paper concern only these two simple functions. The least squares formula for the value of a_{01} and of a_{11} can be found in numerous texts. The corresponding formula for the *probable error* of these two quantities (r_{01} and r_{11}) is, however, practically never given. One can find the general formula applicable to any function, and the particular formula for r_{00}, but not the explicit formulas for r_{01} and r_{11}. Since the error formulas involve quantities occurring in the original formulas for a_{01} and a_{11}, I quote these also. If p is the weight of any observation,

$$a_{01}=\frac{(\Sigma px)(\Sigma pxy)-(\Sigma py)(\Sigma px^2)}{D}$$

$$a_{11}=\frac{(\Sigma px)(\Sigma py)-(\Sigma p)(\Sigma pxy)}{D}$$

where

$$D=(\Sigma px)^2-(\Sigma p)(\Sigma px^2).$$

Then

$$r_{01}=0.6745\left[\frac{(\Sigma pv^2)(\Sigma px^2)}{(n-2)D}\right]^{1/2}$$

$$r_{11}=0.6745\left[\frac{(\Sigma pv^2)(\Sigma p)}{(n-2)D}\right]^{1/2}$$

where n is the number of observations, and v, the residual, is the difference between the observed y and its value as calculated from the values of a_{01} and a_{11} just given. The well known expression for the probable error of a weighted average is

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[3] Because graphical methods have been used in the literature, in one or two such cases, the writer has made some objective experiments on this point. With a set of data which could be satisfactorily represented by the equation $y=a_0+a_1x$, it was found that the value of a_0 and also of a_1, as determined from a graphical solution, deviated *on the average* from the least squares values by just the probable error of the least squares solution. This result was obtained with the cooperation of a number of persons having considerable experience in the drawing of graphs. For real experts the graphical error can probably be made as small as one-half the least squares probable error. Now one of the most fundamental rules of computing is that the error introduced by the calculation itself shall be negligibly small, compared to the probable experimental error. A graphical solution obviously fails to satisfy this criterion in the case cited, and I believe that the result deduced is fairly typical of graphical solutions of simple functions.

$$r_{00}=0.6745\left[\frac{(\Sigma pv^2)}{(n-1)(\Sigma p)}\right]^{1/2}.$$

Before closing the discussion of methods of calculation it seems necessary to call attention to the fact that one must use some judgment in applying the method of least squares. Otherwise the results may well be absurd. Such a solution applies *only* to observations which are affected merely by accidental errors of observation. If a particular observation deviates too widely from a smooth curve,[4] it should be rejected before attempting to treat the data by least squares. If the data are so few in number that the residuals cannot be expected to fall on any symmetrical error curve, the results of least squares are relatively uncertain. Fortunately one can judge this uncertainty. Thus the "probable error of the probable error," or the "uncertainty of the probable error," is given by

$$ur=0.4769\, r/n^{1/2}$$

where r is the probable error, u its proportional uncertainty, and n the number of observations. This shows that with only two observations, the probable error has an even chance of being in error by more than 33.7 percent of itself,[5] while for nine observations this drops to 15.9 percent, and for 25 observations to 9.5 percent. These results lead naturally to a consideration of the number of figures to be retained in all experimental values. The standard rule of computing is that one should retain two doubtful figures and should express the probable error to two significant figures. Thus 126.944 ±0.046. This rule is based on the preceding formula. If one wrote, for instance, 126.94±0.05, the change in the probable error is nine percent of itself, and in the number also, in this case, nine percent of the probable error. These arbitrary changes are of the same magnitude as the uncertainty of the probable error, for 25 observations, and so are not justified.

In most experimental work, in addition to the probable error due to purely accidental errors, to which least squares properly applies, there are many other sources of error, constant or systematic. Some of these may be known, others may be quite unsuspected. The investigator endeavors to make some estimate of the magnitude of each such error, and in this case also, it seems greatly preferable to give what appears to be the *probable error*. If each error can be either plus or minus, the final probable error is to be obtained from the square root of the sum of the squares of all the probable errors, including the least squares accidental error.[6] If on the other hand the assumed "limit

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[4] The criterion of "huge error" already mentioned is a convenient one for the rejection of an individual observation, where now the probable error is calculated for any one observation, instead of for the average. Other criteria have been proposed and can be found in texts on methods of calculation.

[5] In the case of only two unweighted observations of a constant quantity, the *mean* error of the average equals merely the common deviation of each observation from the average. It is however more in harmony with the idea of probable error to take this mean error as a measure of the probable error, and this is frequently done.

[6] If the various quantities concerned appear as factors, one uses the proportional error. If they appear as terms, one uses the absolute error.

of error" is given for each source of error, then obviously the final "limit of error" must be taken as the *arithmetic sum* of the component limits of error.[7] Since in many, if not most cases, the uncertain systematic errors are much larger than the least squares probable error, it is sufficient to use only one digit in stating the final probable error and only one doubtful figure in the actual result. This is common scientific practise. Fortunately, in the case of some of the most important general constants, the final error is, on the face of the evidence, due almost entirely to the accidental errors of the individual observations. In such cases one is fully justified in giving the error to two digits. The writer has accordingly stated the errors to one or to two digits according to circumstances Often two digits are used when the first numeral in the error is a 1, 2, or 3, but only one digit for higher numerals. In this connection it is to be noted that more significant figures should be carried in the calculations than it is desired to retain in the final result, in order that this final result may be unaffected by any error arising purely from the calculation. Failure to observe this rule is the cause of several apparent discrepancies in published data.

Before proceeding to the detailed discussion of each constant, attention should be directed to two important sources of information now available.

(1) The International Critical Tables (McGraw-Hill Co., 1926)[8] publish on page 17 of volume I, a list of nine so-called "Accepted Basic Constants," each with its "Uncertainty." On page 20 is given a list of 21 constants derived from these basic constants, and also certain other conventional and experimental constants. One of these conventional constants (the normal atmosphere) is actually an experimental constant, and this with the other experimental constants and the nine basic constants make up essentially the list of constants considered in the thirteen sections (A to M) of the present paper.

Although the ICT list of constants was published in 1926, it had been adopted in 1923, and since then much important work on nearly every constant has appeared. It was prepared with the aid of various scientific societies as well as of individuals. The values given are not claimed to be the best values available at that time, although obviously an attempt was made to obtain the best values. The chief weakness of this list of constants is the lack of any statement as to the origin of each value. By correspondence and in other ways the writer has obtained such information, and specific references to this are made in the various sections to follow.

(2) The Geiger and Scheel "Handbuch der Physik"[9] contains, on pages 487–517 of volume II, an article by F. Henning and W. Jaeger on "The General Physical Constants." On pages 516–517 there is a list of 52 constants, basic and derived, and on the preceding pages, a brief statement as

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[7] All rules for combining errors involve approximations of the type $(1+b)^n = 1+nb$, where b is the error of the quantity 1.

[8] Throughout the present paper, this publication will be abbreviated to ICT.

[9] This publication will henceforth be denoted as HP.

to the theoretical and numerical basis of each value.[10] In their article many approximations and sources of inconsistency in the literature are pointed out, but with one or two exceptions no attempt is made to recalculate data in order to improve the published values. The writer has been greatly aided by this article, and without it and the ICT tables, it is doubtful if the present work would have been attempted.

The Henning and Jaeger article was written in 1926, and so contains more recent information than that in the ICT. Part of this is reflected in the fact that these two tables do not exactly agree on the value of a single basic constant, with the exception of the electronic charge *e*. Even since 1926, however, much new experimental material has appeared, and one result is that practically every constant adopted in the present paper differs more or less in value from that given in either of these two preceding lists. In fact in the case of the great majority of the constants considered in sections A to M, the finally adopted value is based primarily on work which has appeared since 1926. In the case of most of the constants, the situation is now much more satisfactory than it was a few years ago. Specific conclusions on these matters are given in section O.

The values adopted in this paper are based on data available to the writer, from published articles or private communications, on January 1, 1929. Reference to later work has been made in footnotes, and in practically all cases such recent work merely furnishes further confirmation of the adopted values. Because of the great amount of correlation of data, especially in connection with the numerous derived constants, it has not seemed feasible to make any changes in the adopted values of the basic constants, after the date just given. The present section, and the conclusions in section O, are being written in April 1929, and in the latter section especially the general situation is considered as it appeared at the date of writing.

A preliminary report on this work was made to the American Physical Society[11] in December 1928, and at that time a mimeographed table of constants was sent for suggestions to a number of persons. No changes have been made since then in the magnitudes of the constants printed in the abstract, but several of the probable errors have been revised, and this revision affects many of the derived constants.

Section A

THE VELOCITY OF LIGHT IN VACUUM (*c*)

The history of the experimental attempts to measure the velocity of light is a long and brilliant one. An accurate summary of all numerical results to 1927, in which many errors in the literature are corrected, has

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[10] In other volumes of the HP there appear special articles on "*h*" and on "*e*/*m*", as mentioned in the appropriate sections of this paper.

[11] R. T. Birge, Phys. Rev. **33**, 265 (1929), abstract 6.

been given by de Bray.[1] A good recent account of the experimental methods for measuring c, as well as the numerical results, is that by Ladenburg.[2]

The latest and most accurate direct determination of the velocity of light is that by Michelson,[3] in 1921–26. In fact this investigation so far surpasses in accuracy any of the preceding that it alone need be considered. Michelson[4] lists eight results, obtained from eight sets of observations taken at different times with different mirrors. Each of these results is weighted and the weighted mean is 299796 km/sec. The usual least squares formula for the *probable* error of the weighted mean, I find, gives ±2.6 km/sec. The *mean* error is 3.9 km/sec. Michelson writes 299796±4 km/sec. and thus evidently uses the mean error of the weighted average. No other explicit statement about the error is made. When the various sets of results are collected under the five different mirrors used, the agreement is quite remarkable, all five results varying only from 299797 to 299795 with a mean of 299796 as before. Ladenburg[2] considers somewhat critically the possibility of various constant errors and concludes that all such errors are negligibly small. Aside from the possibility of unsuspected constant errors it is thus evident that the probable error is at most 2.6 km/sec. and may be much less. It seems however more conservative simply to adopt as the probable error the ±4 km/sec given without comment by Michelson, and now being generally used. I accordingly adopt

$$c=(2.99796\pm0.00004)\times10^{10}\,\mathrm{cm\cdot sec^{-1}}. \tag{1}$$

The velocity of short standing electric waves has also been measured and shown to agree approximately with the velocity of light, within the rather large limits of error. Probably the best work on this is by Mercier.[5] He shows that the formation of stationary electric waves on wires is a much more complicated phenomenon than had previously been realized. His measured velocity along a wire is corrected, by the use of various theoretical considerations, to give the velocity in free space. His final result is 299700±30 km/sec for the velocity in free space, but because of uncertainties in the theory, the error may be (and according to Michelson's result actually is) much greater.

The velocity of electromagnetic waves may be obtained *indirectly* from the measured ratio of the electrostatic (es) to the electromagnetic (em) system of electrical units, according to the generally accepted electromagnetic theory of light. The best value of this ratio, which is here denoted by c', is undoubtedly that found by Rosa and Dorsey.[6] They obtained $c'=2.9971\times10^{10}\,\mathrm{cm\cdot sec^{-1}}$. This result is regularly quoted[7] as 2.9971±0.0003.

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[1] E. J. G. de Bray, Nature **120,** 602 (1927).

[2] R. Ladenburg, Hand. d. Exp. Physik **18,** 1–36 (1928).

[3] A. A. Michelson, Astrophys. J. **65,** 1 (1927).

[4] A. A. Michelson, loc. cit. (reference 3) Table VIII.

[5] J. Mercier, J.phys. radium **5,** 168 (1924).

[6] E. B. Rosa and N. E. Dorsey, Bur. Standards, Bull. **3,** 433–604 (1907). See also pp. 605–622 for the various possible methods.

[7] See, for instance, Bur. Standards, Circular No. 60 (1st ed.) 14 (1916) and Ladenburg.[2]

The authors themselves, however, state[8] that the *maximum uncertainty* is 1 in 10000, and Dr. Dorsey has written me that he believes the accuracy is even greater. Their final result is the average of a very large number of individual results, taken at different times, under varying conditions, and showing remarkable consistency. Rosa and Dorsey make in addition an exhaustive study of the sources of systematic error. It therefore seems to the writer that about one part in 30000 is a very conservative estimate for the *probable* error, giving $c' = 2.9971 \pm 0.0001$. This is an interesting conclusion for it indicates that the Rosa and Dorsey value of c' has been more accurate than any direct determination of the velocity of light, up to the time of Michelson's latest work.[9]

This result is however in terms of international electrical units.[10] The relation of these to the absolute electrical units is discussed in section E ahead. Henning and Jaeger[11] show that, to obtain the true ratio between the *es* and the *em* system, in absolute units, the result of Rosa and Dorsey must be multiplied by $p^{1/2}$, where one int. ohm $= p$ abs. ohm. According to section E, $p = 1.00051 \pm 0.00002$. This gives a corrected value of $c' = (2.9979 \pm 0.0001) \times 10^{10}$ cm·sec^{-1}. It is in beautiful agreement with Michelson's recent value of c, and has, on the above assumptions, a probable error only about 2.5 times as large. Hence we conclude that $c' = c$, within limits of error. A strict weighted average of c' and c would give 2.99795, but in view of the uncertainty in the relative probable error of c and c', it does not seem worth while to make this very slight change in Michelson's value.

Section B

THE NEWTONIAN CONSTANT OF GRAVITATION (G) AND THE MEAN DENSITY OF THE EARTH

The HP[1] gives a table of seven determinations of G, ranging from 6.60 to 6.70×10^{-8} dyne·cm^2·g^{-2}. After comparing the most probable values adopted previously by various persons, Henning and Jaeger adopt 6.65. In their list of previous critical reviews, they omit Poynting's value[2] of 6.66 ± 0.01. The ICT[3] adopt as one of their basic constants $G = 6.66 \pm 0.01$.

Since the publication of these reviews, Heyl[4] has made what is undoubtedly the most reliable determination of G. His final result is

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[8] Rosa and Dorsey, loc. cit. (reference 6) page 601.

[9] This of course implies the assumption that c' should *theoretically* be equal to c.

[10] In giving the units of c' as cm·sec^{-1} we have implicitly assumed that the unit of resistance in the *em* system is one cm·sec^{-1}. This is strictly true for the absolute *em* unit, but is not strictly true for the international unit (see section E). Rosa and Dorsey (reference 6, page 601) write more correctly $c' = 2.9971 \times 10^{10}$ cm$^{1/2}$·sec$^{-1/2}$·int $\Omega^{-1/2}$.

[11] HP **2**, 507–508.

[1] Henning and Jaeger, HP **2**, 491.

[2] Poynting, article on "Gravitation," Encyl. Britannica, XI Edition.

[3] ICT. **2**, 17.

[4] P. R. Heyl, Proc. Nat. Acad. Sci. **13**, 601 (1927).

$$G = (6.664 \pm 0.002) \times 10^{-8} \text{ dyne} \cdot \text{cm}^2 \cdot \text{g}^{-2}$$

This result is adopted here. It is based on five separate determinations varying from 6.661 to 6.667.

Assuming $R = 6.371 \times 10^8$ cm as the mean radius of the earth, as given in the HP[1], and[5] $g_{45} = 980.616$ cm·sec^{-2}, one obtains[1] $G \cdot \delta(\text{earth}) = 36.797 \times 10^{-8}$ sec^{-2}, where δ(earth) is the *mean density* of the earth. From the HP result $G = 6.65$, one gets $\delta(\text{earth}) = 5.53$ g·cm^{-3}. With the new result $G = 6.664$, one gets

$$\delta(\text{earth}) = 5.522 \pm 0.002 \text{ g} \cdot \text{cm}^{-3}$$

Section C

RELATION OF THE LITER TO THE CUBIC DECIMETER (1000 cm^3)

The liter is defined as the volume of a kilogram of air-free water at its maximum density. In other words, the maximum density of water is, by definition, one kg·l^{-1}. The kilogram is defined as the mass of the prototype kilogram preserved in Paris. This original prototype kilogram was intended to be the mass of a cubic decimeter (dm^3) of water, at maximum density, but later determinations have shown that there is a slight discrepancy. The various experimental results are discussed by Henning and Jaeger.[1] The mean of the best determinations[2] is 1 liter = 1000.027 cm^3, and this value has been accepted in all recent tables.[3] Henning and Jaeger give no probable error for the result, but one unit in the last place seems a not unreasonable assumption. Hence I adopt

$$1 \text{ liter} = 1000.027 \pm 0.001 \text{ cm}^3 = 1.000027 \pm 0.000001 \text{ dm}^3$$

The maximum density of water $\delta_m(H_2O)$ is accordingly

$$1/1.000027 = 0.999973 \pm 0.000001 \text{ kg} \cdot \text{dm}^{-3} \text{ or g} \cdot \text{cm}^{-3}$$

It should be noted in conclusion, that it is customary to define 1 cc as liter/1000, while 1 cm^3 = liter/1000.027.

Section D

THE NORMAL MOLE VOLUME OF AN IDEAL GAS

(ν_n cm^3 · mole^{-1}, or R_n liter · mole^{-1})

The normal mole volume of an ideal gas is the volume occupied by one gram mole of an ideal gas, at 0°C, under one normal atmosphere pressure.

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[5] This conventionally accepted value of g_{45} is due to F. R. Helmert, Encycl. Math. Wiss. VI, 1B, 96 (1910). See HP **2**, 489.

[1] HP **2**, 491–2.

[2] Three of the best determinations give 1000.029, 1000.026, and 1000.027 respectively.

[3] ICT. **2**, 18, Smithsonian Tables (7th edition), HP, reference 1.

This quantity can theoretically be determined from any real gas, by making the correction necessary to reduce the real gas to an ideal gas. Actually, however, only oxygen is used for accurate determinations. Oxygen is chosen because its atomic weight is 16.000 by definition, and hence there is no error in the resulting value of ν_n, due to error in the atomic weight. This is not the case for any other gas. Moreover, as a result of extensive investigations, the necessary correction to change oxygen to an ideal gas is now known with considerable accuracy.

The ICT[1] gives as one of its basic constants, $\nu_n = 22.4115 \times 10^3$ cm^3. The HP[2] gives $\nu_n = 22.414_5 \times 10^3$ cm^3 or $R_n = 22.413_9$ liters. The discrepancy must be due to the use of different values of $\delta_n(O_2)$, the normal density of oxygen, or of $(1-\alpha)$, the correction factor due to the deviation of oxygen from an ideal gas.[3] Thus

$$\nu_n = \frac{32(1-\alpha)}{\delta_n(O_2)} = \frac{32(1-\alpha)}{L_n(O_2)} 1000.027 = R_n(1000.027) \qquad (1)$$

where ν_n is the normal mole volume in cm^3, R_n the same in liters, $\delta_n(O_2)$ the normal density of O_2, in grams per cm^3, and $L_n(O_2)$ the normal density in grams per liter. All these values correspond to normal gravity ($g_n = 980.665$). It is however customary among chemists to express the experimental results in terms of g_{45} (980.616). Such values will be denoted by ν, δ, L, and R. Thus

$$R = M(1-\alpha)/L \qquad (2)$$

where M is the molecular weight.

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[1] ICT **1,** 17.

[2] HP **2,** 494. See pp. 492–494 for a discussion of the factors involved in the determination of ν_n.

[3] The most general definition of α is $(1/pv)\ d(pv)/d(p)$, (temp = constant), so that it measures the proportional change in pv, per unit change in pressure, and has the dimensions of pressure^{-1}. To make the numerical values more definite, it is customary to write $\alpha = [1/(pv)_1]\ d(pv)/d(p)$, where $(pv)_1$ refers to unit pressure. In investigations on normal density or normal mole volume, it is natural to choose one atmosphere as the unit of pressure. Henning and Heuse (Zeits. f Physik **5,** 285, 1921) use one meter of mercury as the unit of pressure, and denote α by κ_t (see section H). Since the numerical magnitude of α is directly proportional to the size of the unit of pressure, we have $\kappa_t = 100\alpha/76$. On the other hand, Henning (HP **9,** 528) uses the symbol κ_t, but states that p is measured in atmospheres.

Within limits of error, the isothermal pv has been shown to be a linear function of p, for the so-called permanent gases O_2, N_2, H_2, etc., so that for such substances α is independent of p but is a function of temperature, and is more properly written α_t. The linear extrapolation of pv to $p = 0$ gives then $(pv)_0 = (1-\alpha)\ (pv)_1$. Now in the limit $p = 0$, any gas becomes, by definition, an ideal gas. Hence $(pv)_0$ is the constant pv of an ideal gas, and $(1-\alpha)$ is the factor which converts the real $(pv)_1$ corresponding to unit pressure, into the ideal $(pv)_0$, both at some definite temperature. $(1-\alpha)$ is often denoted by $(1+\lambda)$, and $(1-\alpha)$ or $(1+\lambda)$ may be defined as the ratio $(pv)_0/(pv)_1$ (See reference 7, ahead). Frequently v is so chosen (in magnitude or unit) that $(pv)_1$ is unity. α(or κ_t) is then numerically (but *not* dimensionally) the *slope* of the pv isothermal (see HP **9,** 528 and 538).

Henning and Jaeger[4] give a full table of experimental values of L, taken from a critical summary by Moles.[5] The mean value is $L = 1.42892$ $g \cdot l^{-1}$. They give also a table of values of $(1-\alpha)$, leading to the mean value $(1-\alpha) = 1.00092$. This table does not contain a more recent result by Baxter and Starkweather[6] ($L = 1.42901$), but this is included in the discussion by Henning and Jaeger, and raises the mean L to 1.42893. From this, and the value of $(1-\alpha)$ just quoted, the HP gets its value of ν_n. The more recent values of $(1-\alpha)$, however, average 1.00086, and this, taken with the Baxter and Starkweather value of L, gives $\nu_n = 22.4119 \times 10^3$ cm^3, in close agreement with the ICT value.

Since the HP article was written, Moles[7] has critically discussed the most probable value of ν_n. This article, as well as all recent experimental work is in turn discussed in the latest (8th) report of the German committee on atomic weights.[8] Moles obtains an average $L = 1.42892 \pm 0.00002$, but in getting it, he quotes[9] $L = 1.42892$ as a more recent value of Baxter and Starkweather.[10] Their true value is 1.42897, and the correct mean L is accordingly 1.42893 ± 0.00002, as pointed out by the committee.[8] Moles'[7] mean value of $(1-\alpha)$ is 1.00091 ± 0.00003, and these last two values give $R = 22.4148 \pm 0.0007$ liters. The discussion of the German committee, however, indicates that the recent results of Baxter and Starkweather[10] ($L = 1.42897$, and $1-\alpha = 1.00092$) are the most trustworthy. They give $R = 22.4144$ liters.

Very recently Baxter and Starkweather[11] have recalculated their 1926 data in a more logical manner and obtain $L = 1.428965$ grams per liter, $(1-\alpha) = 1.000927$. These values are adopted here. As just noted, Moles assumes 2×10^{-5} as the absolute probable error in L, and 3×10^{-5} as the absolute probable error in $(1-\alpha)$. The latter error seems to be a conservative estimate, but in the case of L, it seems better to take 3×10^{-5} as the error, in view of the difference between the adopted value and the older values. Hence we write

$$L = (1.428965 \pm 0.000030) \text{ gram} \cdot \text{liter}^{-1} \; (g = 980.616)$$
$$1 - \alpha = 1.000927 \pm 0.000030.$$

This gives

$$R = 22.4146 \pm 0.0008 \text{ liter} \cdot \text{mole}^{-1} \; (g_{45} = 980.616)$$
$$R_n = 22.4135 \pm 0.0008 \text{ liter} \cdot \text{mole}^{-1} \; (g_n = 980.665)$$
$$\nu_n = (22.4141 \pm 0.0008) \times 10^3 \text{ cm}^3 \cdot \text{mole}^{-1} \; (g_n = 980.665).$$

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[4] HP **2,** 493.

[5] E. Moles, J. chim. phys. **19,** 100 (1921).

[6] G. P. Baxter and H. W. Starkweather, Proc. Nat. Acad. Sci. **10,** 476 (1924).

[7] E. Moles. Zeits. f. anorg. allgem. Chem. **167,** 40 (1927).

[8] Berichte, **61B,** 1 (1928).

[9] E. Moles, reference 7, page 46.

[10] G. P. Baxter and H. W. Starkweather, Proc. Nat. Acad. Sci. **12,** 699 (1926).

[11] G. P. Baxter and H. W. Starkweather, Proc. Nat. Acad. Sci. **14,** 57 (1928).

Section E

RATIO OF INTERNATIONAL (INT) TO ABSOLUTE (ABS) ELECTRICAL UNITS

For practical convenience, the ohm, ampere, and volt have been defined, by international agreement,[1] in terms of definite physical apparatus.[2]

The international ohm (int. ohm) is defined as the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grams in mass, of a constant cross-sectional area and of a length of 106.300 centimeters.

The international ampere (int. amp.) is the unvarying electric current which, when passed through a solution of nitrate of silver, in accordance with certain specifications, deposits silver at the rate of 0.00111800 of a gram per second.

The international volt (int. volt) is the electrical pressure which, when steadily applied to a conductor the resistance of which is one int. ohm will produce a current of one int. amp.

These international units are to be compared with the corresponding absolute units, with which they were of course identical, within limits of experimental error, at the time of adoption in 1908. One abs. ohm $=10^9$ *em* units of resistance, the *em* unit, under the assumption that permeability is dimensionless, being one $\text{cm}\cdot\text{sec}^{-1}$. Measurements of the abs. ohm have been made in a variety of ways, but all methods necessarily involve the measurement of length and time. The abs. ampere is 10^{-1} *em* units, the *em* unit being one $\text{dyne}^{1/2}$, again with the assumption of dimensionless permeability. The older measurements of the abs. ampere were made usually with the tangent galvanometer, but the best measurements have more recently been made with the current balance, the force between a moving and a fixed coil carrying a current being compensated by a known weight (in dynes).

The definition of the int. amp. just given is the primary definition, and I shall follow the ICT in designating the int. amp. so defined, and all quantities involving it, by the symbol "(a)." Now let

$$1 \text{ int. ohm} = p \text{ abs. ohm} \tag{1}$$

$$1 \text{ int. amp. (a)} = q \text{ abs. amp.} \tag{2}$$

then

$$1 \text{ int. coul. (a)} = q \text{ abs. coul.} \tag{3}$$

$$1 \text{ int. volt (a)} = pq \text{ abs. volt} \tag{4}$$

$$1 \text{ int. joule (a)} = pq^2 \text{ abs. joule} \tag{5}$$

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[1] Intern. Conference on elect. Units and Standards (London, Oct. 1908).

[2] For a complete account of electric units and standards, see Circular, Bureau of Standards No. 60 (First edition 1916, second edition 1920). For a short account see Smithsonian Tables (7th edition) pp. XXXVI to XLIV. For a very clear popular description of the principles involved in measuring electrical quantities in terms of absolute units, see F. E. Smith, Phys. Soc. London, Proc. **37**, 101 (1925).

$$1 \text{ int. henry} = p \text{ abs. henry} \tag{6}$$

$$1 \text{ int. gauss} = q \text{ abs. gauss.} \tag{7}$$

The int. ohm can be constructed as a definite laboratory standard. This is not true of the int. amp. (a). Hence the 1908 London conference[1] appointed a committee to determine the e.m.f. of the Weston normal cell, in terms of the int. ohm and int. amp. The final value adopted by the committee (Jan. 1, 1911) was 1.0183 int. volts, at 20°C, which, to avoid ambiguity, is written 1.01830. This is effectively a new definition of the int. volt and to distinguish it, if necessary, from the primary definition, I again follow the ICT in writing int. volt (v). Similarly all units involving the Weston normal cell will be designated by "(v)." Let

$$1 \text{ int. volt (v)} = r \text{ abs. volt} \tag{8}$$

as contrasted with Eq. (4). It is now possible to use the int. volt (v) and the int. ohm to obtain a new (subsidiary) definition of the int. amp. Thus

$$1 \text{ int. amp. (v)} = r/p \text{ abs. amp.} \tag{9}$$

as compared to Eq. (2). Finally, in many investigations, a so-called "semiabsolute" volt has been used. This is defined as the e.m.f. required to force one *abs.* amp. of current through one *int.* ohm resistance. Hence from Eq. (1)

$$1 \text{ semiabs. volt} = p \text{ abs. volt.} \tag{10}$$

From Eqs. (8) and (10) one obtains

$$1 \text{ int. volt (v)} = r/p \text{ semiabs. volt.} \tag{11}$$

We have now to consider the most probable value of p and of q, and the difference, if any, between r and pq (or between r/p and q). These questions are discussed by Henning and Jaeger in the HP[3], and they conclude,

$$q = 1, \qquad p = 1.0005_0 \qquad r = pq = 1.0005_0.$$

On the other hand, the ICT[4] gives

$$q = 0.99993, \qquad p = 1.00052, \qquad r = 1.00042, \qquad \text{while} \qquad pq = 1.00045.$$

Hence $r/p = 0.99990 \neq q$. The correct determination of the best values of p and q is a very technical and extremely involved matter. Unfortunately, as just seen, there is no exact agreement on the subject, even in the most authoritative compilations, and it is therefore necessary to consider the original sources. Part of the present disagreement in the values of p and q is due to the fact that there is no standard international unit of resistance or of voltage. Each national laboratory has its own standards which differ more or less among themselves, and also may change with time. The values of p and q finally adopted here represent, as well as possible, *mean* values

[3] HP **2,** 498–502.

[4] ICT **1,** 18 and 27.

both in respect to place and to time. Fortunately the accuracy of these quantities is so great that any possible error in the finally adopted values is entirely immaterial in its effect on the many constants derived later in this paper.

The numerical relation of the int. and abs. ohm (value of p) rests at the present time chiefly on two extensive investigations, one by Smith,[5] at the National Physical Laboratory (NPL) of England, and the other by by Grüneisen and Giebe,[6] at the German Reichsanstalt. Smith obtained $p=1.00052\pm0.00004$, while Grüneisen and Giebe found $p=1.00051$. The latter investigators estimate their own probable error, as well as that of Smith, as about 3 parts in 10^5. In 1925 a special committee was appointed at the NPL to make a general investigation of the relation of the int. and abs. electrical units. This work is not yet complete, although statements as to progress are given in the Reports of the NPL for 1925, 1926, and 1927. It was stated in 1925[7] that a comparison of various manganin resistances with mercury resistances indicates that the former have all increased in resistance by about 2.5 parts per 10^5, since 1912, or that the mercury standards (defining the int. ohm) are really smaller by this amount. The latter assumption would give $p=1.000495$, in place of Smith's value of 1.00052. In a very recent investigation at the Reichsanstalt, Steinwehr and Schulze[8] evidently assume that the NPL 1925 standards are 2 parts in 10^5 less than the older 1912 standards, giving a mean value of p in exact agreement with the 1920 Reichsanstalt value. Their own experiments in 1928 agree with this same mean value to ± 1 in 10^5. Furthermore, various intercomparisons of standard resistances at the NPL[9] show that the German and American standards lie between the 1912 and 1925 NPL values. Hence it seems quite certain that the best value of p, at the present time, is 1.00051, while the probable error seems to be not more than 2 parts in 10^5.

The most probable value of q is more uncertain, in spite of extensive investigations on the subject. In the older work, the abs. amp., determined with either a current balance or a tangent galvanometer, was compared directly with the int. amp., as measured by a silver voltameter, thus evaluating q of Eq. (2). In other words, there was measured by means of a silver voltameter, operated under certain specifications, the amount of silver, in grams, deposited per sec. by a current of one abs. amp. This mass of silver was then compared with 0.00111800 grams, the defined amount deposited, *under the same conditions*, by one int. amp. per sec.

Such a procedure determines q unambiguously, but does not necessarily evaluate the electrochemical equivalent of silver (E_{Ag}) per abs. coul. As discussed in detail in section J, ahead, the electrochemical equivalent of a substance is the mass actually associated with unit charge, and is indepen-

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[5] F. E. Smith, Phil. Trans. **214,** 27 (1914).

[6] E. Grüneisen and E. Giebe, Ann. Physik **63,** 179 (1920).

[7] National Phys. Lab. Reports p. 94 (1925).

[8] H. v. Steinwehr and A. Schulze, Ann. Physik **87,** 769 (1928).

[9] Nat. Phys. Lab. Reports p. 8 (1927).

dent of experimental imperfections, while the mass deposited in an electrolytic cell per unit charge,—the only quantity we can actually measure,—*is* subject to experimental imperfections. This distinction has no bearing on the value of q, so long as one accepts the official definition of the int. ampere. It concerns only the value of electrochemical equivalents and the resulting value of the Faraday, considered in section J. The matter is mentioned here because it is common practise among investigators to ignore the distinction, and instead of stating their real experimental values of q, to give the so-called corresponding E_{Ag} per abs. coul. ($=0.00111800/q$). The various experimental values of q, determined as explained above, are listed by Henning and Jaeger.[10] They are stated in terms of the assumed corresponding E_{Ag}, the actual values of which vary from 0.0011179 to 0.0011192 grams (i.e., q varying from 1.00009 to 0.99892).

In the later work (1906 to date) the current is measured in abs. amp., usually with a current balance, and this current is sent through an int. ohm resistance, using a Weston normal cell. From the known current in abs. amp. and the known resistance in int. ohms, one obtains the e.m.f. of the Weston cell in semi-absolute volts. By Eq. (11) the ratio of this result to the e.m.f. in int. volt (v), (1.01830 by definition), is r/p. Hence this later work evaluates only r/p, and not q.

The value of the e.m.f. of the Weston cell, in semi-abs. volts, the assumed corresponding electrochemical equivalent of silver per abs. coul., and the true resulting value of r/p, are listed by Henning and Jaeger.[11] Omitting a probably less accurate value by Guthe,[12] the remaining four values of r/p range from 1.00006 to 0.99989. Henning and Jaeger[11] give correctly as 1.01822 semi-abs. volts the Rosa, Dorsey and Miller[13] value of the e.m.f. of the Weston cell, but misquote and use in their averages the resulting E_{Ag} and r/p, giving 0.99995 for r/p in place of the true 0.99992 ($=1.01822/1.01830$). Using 0.99992, the unweighted average of the four investigations[14] is $r/p = 0.99995$. The Bureau of Standards[15] considers only (a) (c) and (d) of reference 14 and gives 0.99991 as the best average value of r/p. The ICT value (0.99990) is based[16] on (a) and (d) only, of reference 14. Henning and Jaeger[11] take the unweighted average of all four values of reference 14, and the writer has done the same, since there seems to be considerable difference of opinion among experts as to the relative weighting of these four values.

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[10] HP **2,** 499.

[11] HP **2,** 500, Table 6.

[12] K. Guthe, Bur. Standards, Bull. **2,** 69 (1906).

[13] E. B. Rosa, N. E. Dorsey, and J. M. Miller, Bur. Standards, Bull. **8,** 269–393 (1912). See page 362.

[14] (a) Ayrton, Mather and Smith (NPL), 1908, $r/p=0.99989$, (b) Janet, Laporte and Jouaust (Lab. Central d'Electricité, Paris), 1908, $r/p=1.00006$, (c) Haga and Boerema (Univ. of Groningen, Holland), 1913, $r/p=0.99994$, (d) Rosa, Dorsey, and Miller (Bur. Standards, Washington), 1912, $r/p=0.99992$.

[15] Bur. Standards, Circular No. 60, page 38 (1916).

[16] Private communication from Dr. N. E. Dorsey.

In particular, it is very probable that (c) should be given a relatively lower weight, but the final average is fortunately not thereby changed.

The next question concerns the equality of r/p and q. Rosa, Vinal and McDaniel[17] determined the e.m.f. of the Weston cell as 1.01827 int. volt (a), by using a silver voltameter and an int. ohm resistance. Hence by Eqs.(4) and (8), knowing 1.01827 int. volt (a) = 1.01830 int. volt (v), $pq/r = 1.01830/1.01827 = 1.00003$. Hence $q = 1.00003\ r/p$. These investigators naturally assumed $r/p = 0.99992$, from reference 14(d). Hence $q = 0.99995$. This is the figure misquoted as r/p, by Henning and Jaeger.[11]

The above result indicates that q differs from r/p by 3 parts in 10,^{5} and that, to agree with the primary int. units, the Weston cell should have been taken as 1.01827 int. volts. But at the Reichsanstalt,[18] the corresponding quantity was found, in 1908, to be 1.01834 int. volts, and in 1922, 1.01831. The average of these three results indicates that the accepted value of 1.01830 int. volts is correct, within limits of error. In other words, $q = r/p$, and one int. volt (a) = one int. volt (v). This agrees with the view of Henning and Jaeger.[19] The relative values of q and r/p adopted by the ICT[4] are based directly on the work of the Bureau of Standards[13,17].

Henning and Jaeger[19] consider that the variation from unity of either r/p or q is less than the experimental error, and under the circumatances think it more practical to assume $r/p = q = 1.0000$. It seems best, however, to accept the mean value of $r/p = 0.99995$, as determined in four different laboratories,[14] but to take the probable error as ± 0.00005. Assuming then no distinction between int. volt (v), and int. volt (a), we write equations (1) to (7)as

1 int. ohm $= p$ abs. ohm, $\quad p = 1.00051 \pm 0.00002$ (1′)
1 int. amp. $= q$ abs. amp., $\quad q = 0.99995 \pm 0.00005$ (2′)
1 int. coul. $= q$ abs. coul., (3′)
1 int. volt $= pq$ abs. volt. $\quad pq = 1.00046 \pm 0.00005$ (4′)
1 int. joule $= pq^2$ abs. joule, $\quad pq^2 = 1.00041 \pm 0.00010$ (5′)
1 int. henry $= p$ abs. henry, (6′)
1 int. gauss $= q$ abs. gauss, (7′)

Section F

THE ATOMIC WEIGHTS OF CERTAIN ELEMENTS (H, HE, N, AG, I, C, CA).

In evaluating some of the general constants, it is necessary to use the atomic weights of various elements. Since it is only relative magnitudes of

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[17] E. B. Rosa, G. W. Vinal, and A. S. McDaniel, Bur. Standards Bull. **10**, 475–536 (1914). See pp 477 and 486. See also reference 13, page 367.

[18] W. Jaeger and H. v. Steinwehr, Zeits. f. Instrumentenk. **28**, 327 and 353 (1908), and H. v. Steinwehr and A. Schulze, ibid, 42, 221 (1922).

[19] HP **2**, 501–2.

atomic weights which have physical significance, it follows that in the ultimate analysis, only ratios of atomic weights enter into our formulas for the general constants. All atomic weights are likewise derermined from ratios, but in general not directly from the particular ratios in which we are interested. Hence it is necessary to consider individual atomic weights, and for convenience, all those relevant to this paper are collected in the present section. The elements here discussed are hydrogen, helium, nitrogen, silver iodine, carbon, and calcium.

The present system of atomic weights is based on the purely arbitrary assumption that the weight of oxygen is 16 exactly. Any other assumed value for oxygen would be equally permissible, since as noted, atomic weights are entirely relative quantities. In choosing oxygen as a basis, it is tacitly assumed that this element has always the same atomic weight; in other words, that it has no isotopes. This however is seemingly not the case, for Giauque and Johnston[1] have very recently found an isotope of oxygen of atomic weight roughly 18, from an analysis of the atmospheric absorption bands of oxygen. The relative abundance of the two species of oxygen is not yet known, so that one cannot give accurately the new atomic weight of the"16" isotope, in terms of the standard mixture of the two isotopes taken as *exactly* 16. One practical result of this discovery is that the atomic weights determined recently by Aston,[2] from the mass spectrograph, need not be identical with those determined by chemical means, since Aston's atomic weights are based on the mass 16 isotope of oxygen considered as exactly 16, while the chemical atomic weights are based on the ordinary mixture of the two isotopes considered as exactly 16. As we shall see presently, Aston's values of the atomic weights of hydrogen, helium, nitrogen and iodine seem to agree with the chemical values within his *limit* of error(one part in ten thousand to one part in five thousand). If this is really the case, it means that, in ordinary oxygen, O_{16} is more than one thousand times as abundant as O_{18}.

The most detailed account of the very numerous investigations dealing with atomic weights, so far as I am aware, is that by F. W. Clarke.[3] This document of more than 400 large pages gives complete material on the subject, up to the date of publication. The method adopted by Clarke is to collect together all published determinations of a given atomic weight made by a given method, assign to each a probable error based entirely on the internal consistency of the data, and obtain a weighted average value with *its* probable error, using standard least squares formulas. No attention is given to the possibility of systematic errors, the author claiming that such errors will, in the end, more or less cancel out. His weighting of the results of different investigators is thus quite impersonal, being based wholly on internal consistency. He then collects together the determinations of a given atomic

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[1] W. F. Giauque and H. L. Johnston, Nature **123,** 318 (1929), J. Am. Chem. Soc. **51,** 1436 (1929), and private communications. For more recent information see section O.

[2] F. W. Aston, Proc. Roy. Soc. **115A,** 487 (1927).

[3] F. W. Clarke "A Redetermination of Atomic Weights," 4th edition (1920). Printed as Part 3, Vol 16 of Memoirs, Nat. Acad. Sci.

weight, made by the various methods previously considered, and obtains a final weighted average and probable error. As might be expected, the average results from different methods often differ from each other by many times the probable error of each, thus clearly showing the presence of systematic errors in some or all of the methods employed. Whether such systematic errors do in fact cancel out in the long run may seriously be debated. The atomic weight committees, to be mentioned presently, seem more inclined to base the final value on what appears to be the most accurate determination by the most reliable method; in other words, on the determination in which it is most probable that systematic errors have been eliminated. The atomic weight committees also try to use ratios connected most directly with oxygen while Clarke considers all possible ratios.

Since the world war, there has been one report on the atomic weights of the elements by a so-called international committee.[4] Since 1925 the annual report on atomic weights by G. P. Baxter, for the American Chemical Society, has contained a table of atomic weights, for which he alone is responsible. Since 1918, a German committee on atomic weights has published annual reports. These various sources do not agree in many cases on the best value, and—what is more important for our purpose—do not give the probable error. There does not seem to be even a consistent practise in regard to the number of stated figures. Sometimes the last quoted figure is definitely doubtful, while in the great majority of cases it is certain. In the case of hydrogen the first five significant figures are certain, while only four figures are given by Baxter[5] and by the German committee.[6] The writer however is informed that the 1929 reports will give hydrogen as 1.0078. It is therefore necessary to investigate the probable values and probable errors, for the elements previously mentioned.

HYDROGEN

Baxter[5] quotes Moles[7] as deciding, from a critical review of gas density measurements, that $H=1.0078$. This value, as given by Moles[7] is actually taken from a previous article[8] where full details are given. Moles lists nine results lying in the narrow range 1.00766 to 1.00783, with a mean value of 1.00777 ± 0.00002, or a rounded figure of 1.0078. These nine results are given in Landolt-Börnstein[9], while Moles' article[8] is discussed in the 6th German report.[10] The final average represents the result of 223 different measurements by five different investigators, using four different methods, and seems to be the most reliable now available. It is quite possible that

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[4] Second Report of the International Committee on Chemical Elements, J. Am. Chem. Soc. **47**, 597 (1925). This committee did not include German and Austrian scientists.

[5] G. P. Baxter, 34th annual report, J. Am. Chem. Soc. **50**, 603 (1928).

[6] German committee (8th report), Berichte, **61B**, 1–31 (1928).

[7] E. Moles, Gazz. chim. ital. **56**, 915 (1926).

[8] E. Moles, Zeits. f. physik. Chem. **115**, 61, and **117**, 157 (1925).

[9] Landolt-Börnstein, Physik. Chem. Tables, 5th ed. (E), p. 3 (1927).

[10] German committee (M. Bodenstein, etc.), Berichte, **59**, SII (A) (1926).

the probable error should be larger than that given by Moles, but even if it were many times larger, this would not affect any of the conclusions regarding the general constants. I accordingly adopt $H = 1.00777 \pm 0.00002$. Aston,[2] from positive ray analysis, obtains $H = 1.00778$ with a limit of error 0.00015. This last result is quoted by Baxter,[5] and is the only value for hydrogen mentioned in the latest German report.[6] As already pointed out, the recent discovery of an isotope of oxygen makes it now permissible to use Aston's value only as an indication of the relative abundance of O_{18} and O_{16}, and not as an atomic weight determination. It is in perfect agreement with the chemical value, which indicates a very low abundance of O_{18}.

HELIUM

In obtaining the best atomic weight of helium, one need consider only the most recent determinations. Baxter,[5] in his latest report, refers to an investigation by Baxter and Starkweather,[11] giving $L = 0.17846\ \mathrm{g \cdot l^{-1}}$ as the density of helium, for g_{45}. They also conclude that helium behaves like a perfect gas. Hence, using our adopted value[12] of R in the equation

$$R = M(1-\alpha)/L \tag{1}$$

we obtain

$$M = (22.4146 \pm 0.0008)(0.17846) = 4.00011 \tag{2}$$

as the atomic weight of helium. Baxter[5] then quotes unpublished work by R. B. Ellestad and himself, yielding $(1-\alpha) = 0.9995$. With this value, in Eq. (1), one obtains $M = 4.0021$. As far as chance variations are concerned, the probable error in L is one part in 20000 or less. The probable error in $(1-\alpha)$ is not stated,[12a] but it is presumably at least one part in 10000. Hence the final probable error in M is at least ± 0.0004. Aston[2] obtained 4.00216 with a limit of error 0.0004, in perfect agreement with the revised Baxter and Starkweather value. As in the case of hydrogen, this indicates a small prevalence of O_{18} and encourages one to use Aston's values for other elements as true determinations of atomic weight.

The German committee[6] criticises the original Baxter and Starkweather value of $(1-\alpha) = \text{unity}$, and points out that all previous work indicates that $(1-\alpha) = 0.9995$. It therefore also decides that $\mathrm{He} = 4.002$, in agreement with Baxter.[5] It is evident that the true atomic weight of helium must be close to the value found by Aston. The chemical value is at present slightly less accurate than Aston's, and I shall accordingly adopt his value but his assumed error as the *probable* error, although such a procedure may well be

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[11] G. P. Baxter and H. W. Starkweather, Proc. Nat. Acad. Sci. **12**, 20 (1926).

[12] See section D.

[12a] Just before this paper was to go to the printer, the writer received from Professor Baxter his individual values of $(1-\alpha)$ for He. There are four of these, averaging 0.99958, with a maximum variation of only 14 parts in 10^5. This new result would give $\mathrm{He} = 4.0018$, with a final probable error of possibly 0.0003 or less. Since the *limit* of error of Aston's value is 0.0004, the chemical value is still the less accurate and no change is required in the final adopted value.

open to criticism, in view of the present situation regarding the oxygen isotopes. I accordingly write

$$\mathrm{He}=4.0022\pm0.0004.$$

NITROGEN

The atomic weight of nitrogen is not used directly in any of the following sections, but the error in this atomic weight produces practically the entire error in the atomic weight of silver. Since the great majority of the accepted atomic weights are derived more directly from silver than from oxygen, the atomic weight of silver is of the highest importance. It is therefore necessary to consider carefully the best value of N and its probable error.

The atomic weight of nitrogen can be obtained by direct comparison with oxygen, and also from density measurements, using the adopted value of R in Eq. (1). According to Clarke,[3] the final average of these two methods gives $\mathrm{N}=14.0076$. The atomic weight can be obtained indirectly in many ways. The results of all methods, including the two just mentioned, are summarized by Clarke[13] and give $\mathrm{N}=14.0081$. This was presumably the best value in 1920. At the present time it is generally agreed that, just as in the case of helium, the atomic weight of nitrogen can be determined most accurately from its density and deviation from a perfect gas, by the use of Eq. (1).

The various results have recently been discussed in full by Moles.[14] As explained in section D, Moles derives $R=22.4148\pm0.0007\ \mathrm{g\cdot l^{-1}}$, from data on oxygen, while we have adopted $R=22.4146\pm0.0008$. For nitrogen, Moles quotes six values of L, varying from 1.25032 to 1.25071 $\mathrm{g\cdot l^{-1}}$, with a mean value 1.25048 ± 0.00006. Omitting the first two values, which he considers uncertain, the mean is 1.25046 ± 0.000045. This last value is adopted by Moles. Among the four values used in getting this average is 1.25049, obtained by Moles and Clavera.[15] This work represents probably the most accurate single investigation on nitrogen. The individual results are quoted in full by Baxter.[5] Moles and Clavera discuss carefully the various sources of systematic error, and conclude finally that $L=1.25049\pm0.00003$. The chief reason for not adopting this as the best value of L is the lower result obtained by Baxter and Starkweather.[16] Their result is 1.25036, and they give no estimate of probable error, but remark that the result is slightly lower than the results of previous observers. It is therefore difficult to decide on the relative weighting of these various values. Moles and Clavera[15] give 1.25048 as the weighted average, including their own result. Moles,[14] as noted, gives 1.25046 ± 0.000045, based on the same available data. It appears to the writer that this result is not unreasonable, and it is adopted here.

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[13] F. W. Clarke, reference 3, p. 118.

[14] E. Moles, Zeits. f. anorg. allgem. Chem. **167,** 40 (1927). See also section D.

[15] E. Moles and J. M. Clavera, Zeits. f. anorg. allgem. Chem. **167,** 49 (1927).

[16] G. P. Baxter and H. W. Starkweather, Proc. Nat. Acad. Sci. **12,** 703 (1926). See ibid **14,** 57(1928) for a better method of calculation which yields 1.25037.

The value of $(1-\alpha)$, or $(1+\lambda)$ as it is often denoted, is known with greater certainty for nitrogen than for oxygen.[12] The four best determinations, as quoted by Moles,[14] vary only from 1.00042 to 1.00046, with a mean of 1.000437 ± 0.000014. The value of $(1-\alpha)$ has been determined, indirectly, by Baxter and Starkweather,[16] as 1.00033. By another method of calculation, to which Moles[14] takes exception, they get 1.00051. Their recalculation[16] gives 1.00040. Moles himself concludes that, including also the Baxter and Starkweather value, $(1-\alpha)=1.000424 \pm 0.000014$. Moles and Clavera[15] give 1.00043 as a final best value. Baxter and Starkweather give 1.00047 as the average of all work, excluding their own, but do not decide on a final average. I shall adopt $(1-\alpha)=1.00043 \pm 0.00002$.

With the adopted values of R, L, and $(1-\alpha)$, we have in Eq. (1),

$$22.4146 \pm 0.0008 = \frac{M(1.00043 \pm 0.00002)}{1.25046 \pm 0.000045} \cdot \tag{3}$$

Hence $M=28.01651$, or $M/2=\mathrm{N}=14.0083 \pm 0.0008$, the value adopted here. The error is larger than that adopted by Moles.[14] His own estimates of error for the three factors lead to one part in 20000 for the final probable error in N. He however states that the probable error is one part in 30000 (± 0.00045). Then in his summary he writes $\mathrm{N}=14.0082 \pm 0.0002$, an error of only one part in 70000. This is presumably a misprint. Moles and Clavera,[15] with slightly different assumed values and errors for $(1-\alpha)$ and L, deduce[17] $\mathrm{N}=14.0082 \pm 0.0004$.

In reviewing the literature on this subject, one is struck by the fact that, in the cases of both oxygen and nitrogen, the values finally adopted for both $(1-\alpha)$ and L tend to rise and fall together, so that the final result (value of R for oxygen, and the atomic weight for nitrogen) remains almost constant. A cursory survey of various critical summaries would thus indicate that the probable error in R and in the atomic weight of nitrogen is much smaller than that assumed here. Since the evaluation of $(1-\alpha)$ is quite independent from that of L, this seeming connection must be purely accidental. It is however quite possible that the error in N assumed above is a little high. Aston[2] obtains $\mathrm{N}=14.008$, but his assumed accuracy in this case is only one part in 5000. It is to be recalled, in this connection, that Aston gives always the *limit* of error, and his *probable* error should be much smaller. Fortunately his values all agree beautifully with the chemical values, so that the decision as to his actual probable error may be left open.

SILVER

As has already been stated, the best atomic weight of silver is at the present time directly dependent on that of nitrogen. A good summary of the situation is given by Moles and Clavera.[15] Of the many methods for obtaining the value of Ag, it is now generally agreed that the most accurate

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[17] Written accidentally as ±0.00004, but stated correctly as 3 parts in 10^5, and given in their summary correctly as ±0.00042.

is that based on the reduction of $AgNO_3$ to Ag. Since $O=16.000$, by definition, the sole error in Ag is due to that in N. Moreover, the proportional error in Ag is due to that in NO_3, and is therefore only about one-fourth the probable proportional error in N. The ratio $AgNO_3/Ag$ can be determined with great precision. Clarke[18] gives all the values of this ratio, up to 1920, the weighted average being 1.57479. Of these the most accurate, by far, is that by Richards and Forbes,[19] yielding also 1.57479. Since then a very elaborate investigation has been carried out by Hönigschmidt, Zintl and Thile,[20] which gives again exactly the same ratio. This latter work is discussed by Baxter[5] and by the German committee,[6] and is specifically accepted by the latter. There are twenty published values of the ratio which show a maximum range of only two parts in 150,000, and the probable error in the final mean is only about one part in 10^6. Hence, except for unsuspected systematic errors, the error in this ratio is quite negligible. With our adopted value of N, and with the above value of the ratio $AgNO_3/Ag=r$, one has

$$Ag=\frac{NO_3}{r-1}=\frac{62.0083\pm0.0008}{0.57479}=107.8799\pm0.0014$$

If we had used the Moles[14] or the Moles and Clavera[15] value $N=14.0082$, we would have obtained $Ag=107.8797$. These results seem to indicate very strongly that the best value of $Ag=107.880$. Hönigschmidt, Zintl and Thile[20] use the rounded figure $N=14.008$, and therefore get $Ag=107.879$.

The atomic weight of silver can be obtained in many other ways. Clarke[21] lists 43 different methods, yielding a final weighted average of 107.8804. It seems however clear at the present time that only the $AgNO_3/Ag$ ratio results need be considered, with a final real error in Ag due merely to that in N. It has already been pointed out that the error assumed here for N is larger than that assumed by others. It therefore does not seem unreasonable[21a] to adopt

$$Ag=107.880\pm0.001$$

In order that the true result for Ag should be 0.001 more or less than this, N must be respectively 14.0089 or 14.0078.

IODINE

The atomic weight of iodine enters into the discussion of the value of the Faraday, in section J, and is therefore considered here. As in the case of silver, the atomic weight of iodine may be obtained in various different

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[18] F. W. Clarke, reference 3, page 98.

[19] T. W. Richards and G. S. Forbes, J. Am. Chem. Soc. **29,** 808 (1907) and Zeits. f. anorg. Chem. **55,** 34 (1907).

[20] Hönigschmid, E. Zintl and P. Thilo, Zeits. f. anorg. allgem. Chem **163,** 65 (1927).

[21] F. W. Clarke, reference 3, page 116.

[21a] O. Hönigschmid and R. Sachtleben, Zeits. f. anorg. allgem. Chem. **178,** 1 (1929), have very recently been able to get a precise determination of silver in terms of oxygen, from the reduction of $Ba(ClO_4)_2$ and subsequent titration with silver, yielding finally $Ag=107.880\pm0.001$.

ways. Clarke[13] lists eight methods, with a mean result of 126.926. This result will, however, bear closer scrutiny. Of the eight methods, the most accurate is the direct determination of the I/Ag ratio, assuming the atomic weight of silver as known. All of the experimental results for this ratio are listed by Clarke.[22] There seems to be no more recent work on the subject. Among the values of this ratio, that obtained by Baxter,[23] in 1910, is the most reliable. His value of I/Ag is 1.176603, with an apparent probable error (from 13 determinations) of only 3 parts in 10^6. Clarke, following his usual custom, lists all determinations of this ratio. Now the four earlier results are all approximately 1.1753, while the later results run much higher. It therefore seems evident that these earlier results are vitiated by some systematic error. They are however quite self consistent, and so by Clarke's method are given a high weighting. Clarke's final weighted average is 1.176454. With the four earlier results eliminated, I obtain a new weighted average of 1.176549, in much closer agreement with Baxter's 1910 result. This revised ratio, combined with Ag = 107.880, gives I = 126.926, while Baxter's result gives 126.932. Using the revised average value for the I/Ag ratio with Clarke's results[13] for the other seven methods, I obtain a final weighted average of I = 126.932, in place of Clarke's value 126.926 previously quoted, and in exact agreement with Baxter's result. The International atomic weight committee,[4] in 1925, adopted I = 126.932, based entirely on Ag = 107.880 and Baxter's 1910 value of the I/Ag ratio. The German committee still retains[23a] the older international value 126.92. The above results seem to indicate quite definitely that I = 126.932, with a proportional error only slightly in excess of that for Ag. Hence I conclude I = 126.932 ± 0.002.

Among the experimental values of the ratio I/Ag, as listed by Clarke[22] is that by Vinal and Bates,[24] from electrolysis. Their final value of the ratio is[25] 1.176235, leading to I = 126.892. This appears to be a relatively inaccurate determination of I, and is mentioned again in section J, in connection with the value of the Faraday. In conclusion it is of interest to note that Aston[2] gets I = 126.932, in exact agreement with our adopted value.

CARBON

The atomic weight of carbon can be determined directly from oxygen. The final result of all such determinations, as obtained by Clarke,[26] is 12.0000 ± 0.00026. This result (written 12.000) was accepted in 1925 by the International atomic weight committee,[4] and has since been used by Baxter.[5] There are however a number of other methods for obtaining C.

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[22] F. W. Clarke, reference 3, p. 84.

[23] G. P. Baxter, J. Am. Chem. Soc. **32,** 1591 (1910).

[24] G. W. Vinal and S. J. Bates, Bur. Standards, Bull. **10,** 425 (1910).

[25] F. W. Clarke, reference 3, quotes 1.176260 (the reciprocal of 0.85015) while the true Vinal and Bates' value of Ag/I is 0.85017.

[23a] The 1929 report, Berichte **62,** 1 (1929), which has just appeared, gives 126.93. It also gives Ag = 107.880 in place of the older 107.88, and H = 1.0078, in place of the older 1.008. The other atomic weights of interest here are unchanged in value.

[26] F. W. Clarke, reference 3, p. 47.

Thirteen methods, including the above direct result, are listed by Clarke,[27] and give a weighted mean of 12.0025 ± 0.00019. Aston[2] finds $C = 12.0036$, with a limit of error 0.0012. It seems difficult to decide on the best chemical value. Clarke's printed probable errors as usual seem far smaller than the actual probable errors. We have now found that in the cases of H, He, N, and I, Aston's result agrees with the best chemical value well within his limit of accuracy, and this, it seems to the writer, furnishes sufficient justification for adopting a value of C which lies within Aston's stated limit of error. The mean of the final Clarke value (12.0025) and Aston's value (12.0036) is 12.003, and is adopted here. To be conservative, however, I adopt as the *probable* error essentially the value given by Aston as his *limit* of error, and write

$$C = 12.003 \pm 0.001 .$$

CALCIUM

The atomic weight of calcium, as well as that of carbon, is needed for the evaluation of the grating space of calcite. The value of Ca which has been universally accepted since 1911 is 40.07. When this value was readopted in 1925 by the International committee, reference was made to the work of Richards and Hönigschmid.[28] These investigators precipitate calcium chloride by a solution of silver, and determine the amount of silver chloride produced. They assume $Ag = 107.88$ and $Cl = 35.457$. These are still the accepted atomic weights, that for Cl having recently been verified by Hönigschmid, Chan and Birckenbach.[29] The final result is $Ca = 40.075$, based on four determinations ranging from 40.085 to 40.070. Clarke[30] gives as the final result from all methods, $Ca = 40.0902$, but in obtaining this result he uses values for Cl, Br, and S not accepted at the present time. It seems probable that the Richards and Hönigschmid value of 40.075 is the best and is adopted here. The probable error seems to be about 0.005, and this figure is adopted, although it is admittedly very uncertain.

The atomic weights adopted in this section are then as follows:

$H = 1.00777 \pm 0.00002$

$He = 4.0022 \pm 0.0004$

$N = 14.0083 \pm 0.0008$

$Ag = 107.880 \pm 0.001$

$I = 126.932 \pm 0.002$

$C = 12.003 \pm 0.001$

$Ca = 40.075 \pm 0.005$

Section G

THE NORMAL ATMOSPHERE (A_n)

The normal atmosphere is defined as the pressure exerted by a column of mercury 76 cm high, of normal density (i.e., at 0°C, A_n), under normal

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[27] F. W. Clarke, reference 3, p. 120.

[28] T. W. Richards and O. Hönigschmid, J. Am. Chem. Soc. **32,** 1577 (1910) and **33,** 28 (1911)

[29] O. Hönigschmid, S. B. Chan and L. Birckenbach, Zeits. f. anorg. allgem. Chem. **163,** 315 (1927).

[30] F. W. Clarke, reference 3, p. 144.

gravity. The very small change in the density of mercury, with change of pressure, permits an approximate value of A_n to be used, in determining the normal density. The normal atmosphere has been used in connection with certain preceding constants, but in such a way as not to involve its value in absolute units, as discussed in the present section.

The ICT[1] gives $A_n = 1.013250 \times 10^6$ dyne·cm^{-2}. This is based on the definition[2] of A_n as the pressure of a column of a liquid of density 13.5951 grams per cc, under normal gravity. The HP[3] gives $A_n = 1.01325_3 \times 10^6$, from the defining equation

$$A_n = H_n \cdot \rho_n(\mathrm{Hg}) \cdot \delta_m(\mathrm{H_2O}) \cdot g_n,$$

in which H_n = height of normal barometer = 76.000 cm,

ρ_n = normal specific gravity of Hg (at 0°C, A_n), referred to air-free water of maximum density,

$\delta_m(\mathrm{H_2O})$ = maximum density of water,

g_n = normal gravity[4] = 980.665 cm·sec^{-2}.

The ICT value of A_n follows directly from its definition, i.e.,

$A_n = 76 \times 13.5951 \times 980.665 = 1.013250 \times 10^6$ dyne · cm^{-2}

Henning and Jaeger,[5] using the density of mercury in the definition, investigate the most probable value of ρ_n, and give a table of values taken from Scheel and Blankenstein,[6] which leads to an average value 13.59546. This table includes their own result of 13.59549. Henning and Jaeger then *adopt* $\rho_n = 13.5955$. The value of $\delta_m(\mathrm{H_2O})$ is[7] 0.999973 g·cm^{-3}. The product $\rho_n(\mathrm{Hg}) \cdot \delta_m(\mathrm{H_2O}) = D_n = 13.5955 \times 0.999973 = 13.595133$ g · cm^{-3}, agreeing with the ICT value of D_n to the six significant figures given by the ICT, but, with the use of seven figures, leading to $A_n = 1013253$, as given by the HP.[2] Hence the discrepancy in the seventh significant figure between the ICT and HP values of A_n seems to be due to carrying that result to seven figures, while the factors entering into it are given only to six.

I shall adopt as the most probable value of ρ_n, the figure calculated by Scheel and Blankenstein,[6] viz. 13.59546. This should have a probable error of a few units in the last digit. Then $D_n = 13.59546 \times 0.999973 = 13.59509$ g·cm^{-3}, and $A_n = 13.59509 \times 76 \times 980.665 = 1.013249 \times 10^6$ dyne·cm^{-2}. This in turn should have a probable error of not more than two or three units in the last digit. I adopt ±0.000003.

The 45° atmosphere is obtained by the mere substitution of $g_{45}(980.616)$ [8] for g_n. Hence

$$A_{45} = 13.59509 \times 76 \times 980.616 = (1.013199 \pm 0.000003) \times 10^6 \text{ dyne} \cdot \text{cm}^{-2}.$$

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[1] ICT. **1,** 18.

[2] ICT. **1,** 34, definition of "Atmosphere."

[3] F. Henning and W. Jaeger, HP. **2,** 495.

[4] Adopted by the Fifth General Conference (1913), C. E. Guillaume, Trav. et Mem. du Bur. int. **16,** 114 (1913), See HP **2,** 490.

[5] HP. **2,** 490 and 494–496.

[6] K. Scheel and F. Blankenstein, Zeits. f. Physik **31,** 202 (1925). Given also in Landolt-Börnstein, 5th ed. (E) p. 12.

[7] See section C.

[8] See section B.

Note added in proof. It is evident from the preceding discussion that the definition of the normal atmosphere given by N. E. Dorsey in the ICT[2] is technically quite different from that given by Henning and Jaeger in the HP.[3] In particular, the ICT definition makes the normal atmosphere a conventional constant, with therefore no probable error. The writer had some correspondence on this matter with Dr. Dorsey, leading to the conclusion that the HP definition was correct. The adopted value is therefore based on this HP definition, as given in the first sentence of this section.

The writer unfortunately overlooked a recent article by G. K. Burgess[9] in which the "standard atmosphere" is defined as "the pressure due to a column of mercury 760 mm high, having a mass of 13.5951 $g \cdot cm^{-3}$, subject to a gravitational acceleration of 980.665 $cm \cdot sec^{-1}$, and is equal to 1,013,250 $dyne \cdot cm^{-2}$." It is thus a conventional constant, with no error. From direct correspondence with Dr. Burgess I now learn that this definition was adopted in 1927 by the International Commission of Weights and Measures. Fortunately this new definition makes no change in either the magnitude or error of any derived constant. It should be noted, in conclusion, that no temperature is specified in the definition and that the word "mercury" is technically superfluous. This seems to the writer very objectionable, since there is thus technically no simple method for reducing to standard atmospheres an actual barometer reading at an actual observed temperature. The HP definition, as used by the writer, therefore seems preferable, in spite of international agreement to the contrary.

Section H

THE ABSOLUTE TEMPERATURE OF THE ICE-POINT (T_0)

The generally accepted value of T_0 was, for many years, 273.09°K. This is based on Berthelot's analysis[1] of the data of Chappuis[2] giving $1/T_0 = \gamma = 366208 \times 10^{-8}$, and of Joule and Thomson for the porous plug experiment. The final average value was $\gamma = 36618 \times 10^{-7}$, or $T_0 = 273.09°$. The ICT[3] gives $T_0 = 273.1$ as one of its basic constants, a value apparently taken from Berthelot.

A most extensive series of observations on the volume and pressure coefficients (α and β) of certain gases has recently been carried out by Henning and Heuse,[4] at the Reichsanstalt. The value of γ was obtained by two different methods.[5] In the first it is assumed that the volume and pressure coefficients (α and β)[6] are linear functions of the pressure, so that their linear extrapolation to $p = 0$, gives γ ($= \alpha = \beta$ of an ideal gas $= 1/T_0$). In

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[9] G. K. Burgess, Bur. Standards, J. Research **1,** 635 (1928).

[1] D. Berthelot, Trav. et Mem. du Bur. inter. **13,** 12 (1907).

[2] P. Chappuis, ibid. vols **6** and **13.**

[3] ICT. **1, 17.**

[4] (a) F. Henning, Zeits. f. Physik **5,** 264 (1921). (b) F. Henning and W. Heuse, ibid. **5,** 285 (1921). (c) W. Heuse, ibid. **37,** 157 (1926).

[5] See F. Henning, HP. **9,** 527–529, for a description of these methods.

[6] $(v_t/v_0)_p = 1 + \alpha t_p$, $(p_t/p_0)_v = 1 + \beta t_v$.

the second method it is assumed that the isothermal pv is a linear function of p. Choosing one meter of Hg as the unit of pressure and making pv unity for unit pressure, Henning and Heuse[4] denote the slope of the resulting pv curve by κ_t[7]. It can then be shown[8] that

$$\gamma=\beta+3.73(\alpha-\beta)+1.367p_0\kappa_0\cdot 10^{-2}$$

where α and β are the coefficients for pressure p_0, taken from the data of the first method, and κ_0 refers to the 0°C isothermal.

From the first method they obtained for the gases He, H_2 and N_2, $\gamma\times 10^7=36600$, 36607, and 36606 respectively, or $T_0=273.224°$, 273.172° and 273.179°. The mean is $\gamma\times 10^8=366043$ or $T_0=273.190°\pm 0.015$.

From the second method they get for He (two determinations at slightly different p_0), H_2 and N_2, $\gamma\times 10^7=36598$, 36597, 36617, and 36604, respectively. The mean is 36604.0 or $T_0=273.194°$. They conclude that the best mean value of all the experiments is $\gamma\times 10^7=36604$. The reciprocal of this is $T_0=273.19°$. They write it, however, as 273.20°.

In the later article[4c] by Heuse, neon is used, and the above value of γ is confirmed. Because of a discrepancy between their value of κ_0, and that obtained by other observers, using high pressures,[9] a discrepancy which occurs in all the four gases used, Heuse writes the final probable error as ±0.03°, rather than the ±0.01° which the above data seem to indicate.

The only other determination of T_0 of comparable accuracy is that by Roebuck,[10] using the Joule-Thomson effect in air.[11] This method requires a knowledge of α, the volume coefficient, as well as of the Joule-Thomson coefficient μ. Roebuck measured μ, but for α used older data, mainly by Chappuis. Henning and Jaeger[12] make note of this fact, and as the most probable value of T_0 they adopt merely the Henning and Heuse value 273.20° (which as previously noted should be 273.19°). Roebuck obtained three results, 273.18°, 273.16° and 273.12°, with an average of 273.15°. He gives a list of all previous determinations of T_0, and as the most probable value chooses 273.17°, a value lying midway between his own result and that of Henning and Heuse. He gives ±0.02° as the probable error of the final average result. The writer feels that these two results (273.15° by Roebuck, and 273.19° by Henning and Heuse) are entitled to far more weight than any of the older work, but that the second result is probably the most accurate, being based on new determinations of α. Hence I adopt $T_0=273.18\pm 0.03°$K. The probable error is that given by Henning and Jaeger,[12] and is a very conservative estimate. Roebuck's ±0.02° may well be more reasonable.

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[7] See footnote 3, section D, for the relation of κ_t to α, and the meaning of α (or κ_t) in terms of an ideal gas.

[8] F. Henning, Temperaturmessung, Braunschweig 1915, p. 68. See also references 5 and 4(b).

[9] See reference 4b for references.

[10] J. R. Roebuck, Proc. Am. Acad. Arts Sci. **60**, 537 (1925).

[11] See HP. **9**, 526–527, for the theory of this method.

[12] HP. **2**, 496.

Section I

THE MECHANICAL EQUIVALENT OF HEAT (J) AND THE ELECTRICAL EQUIVALENT OF HEAT (J')

A detailed description of the methods which have been employed for the evaluation of J, and a critical discussion of the results, is given by Jaeger in the HP.[1] The value adopted by Henning and Jaeger in the HP[2] is one $\text{cal}_{15} = 4.184_2$ int. joule $= 4.186_3$ abs. joule. The ICT value[3] is one $\text{cal}_{15} = 4.185$ abs. joule. The cal_{15} is defined as the amount of thermal energy required to heat one gram of pure water from 14.5° to 15.5°C.

In the original work by Joule, mechanical energy was turned directly into thermal energy, and J was thus directly evaluated in abs. joules. In most of the modern work, however, electrical energy is turned directly into thermal energy, thus evaluating the *electrical* equivalent of heat (J', measured in int. joules). Since the relation between the int. joule and the abs. joule (10^7 ergs) is known with considerable precision,[4] the value of the mechanical equivalent of heat may be obtained from the observed value of the electrical equivalent.

The value of J adopted by the HP results from the work of Jaeger and Steinwehr.[5] These investigators determined J', for many different mean temperatures[6] lying between 4.75°C and 49.60°C. This is undoubtedly the most extensive and accurate work on J' now available. They list 67 results in all. These results, plotted against temperature, are represented as a parabolic function of t. The resulting least squares equation is

$$J'(\text{int. joule}) = 4.2047_7 - 1.768 \times 10^{-3} t + 2.644_7 \times 10^{-5} t^2, \qquad (1)$$

where t is measured from 0°C, on the thermodynamic scale. This equation gives 4.18420 int. joules for the 15° calorie. The HP writes this 4.184_2.

On examining their data, the writer finds that a parabola is not a sufficiently complex function to give a proper representation of the observations. Their residuals show pronounced trends, and unfortunately the largest trend is just in the vicinity of 15°C, where it is most important that the curve give a true most probable representation of the data. I have accordingly made a separate investigation of the best curve for the representation of their data. The detailed work will be presented in a separate paper, and only the final result is quoted here. This is in the form of a fourth degree polynomial

$$J' = 4.21040 - 2.78958 \times 10^{-3} t + 7.73723 \times 10^{-5} t^2 - 8.52567 \times 10^{-7} t^3 + 3.7540 \times 10^{-9} t^4 \qquad (2)$$

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[1] W. Jaeger, HP. **9**, 476–495.

[2] HP. **2**, 497.

[3] ICT. **1**, 18.

[4] See section E.

[5] W. Jaeger and H. v. Steinwehr, Ann. Physik **64**, 305 (1921). See also HP **9**, 490–493.

[6] The temperature change in the calorimeter was usually about 1.4°C, for the six minute run used.

This gives $J'_{15}=4.18327$ int. joules, and is, I believe, the true most probable value resulting from the work of Jaeger and Steinwehr. It differs by 9.3×10^{-4} joules from their own deduced value.

These investigators consider with great care all of the possible errors, both systematic and accidental. They conclude that the electrical standards (normal resistance and normal Weston cell) together involve a possible uncertainty of one or two parts in 10000, and that the calorimetric measurements have the same accuracy. Jaeger, in quoting this work[7] gives two parts in 10000 (i.e., 8×10^{-4} joules) as the probable error. I therefore write $J_{15}'=4.1833\pm0.0008$ int. joules as the revised Jaeger and Steinwehr result. Using the results of section E, we have one int. joule $=pq^2$ abs. joule, where $pq^2=1.00041\pm0.00010$. Hence there results

$$J_{15}=(4.1833\pm0.0008)(1.00041\pm0.00010)=4.1850\pm0.0009 \text{ abs. joules.}$$

The most accurate direct determination of the *mechanical* equivalent of heat J is that resulting from the work of Laby and Hercus,[8] which has appeared since the HP was compiled. These investigators use a continuous flow calorimeter and make 23 determinations, grouped about six different mean temperatures, the temperature change in the calorimeter being always about 5°C. These are then averaged to give six determinations at different temperatures, weighted arbitrarily in rough accordance with the self consistency of the individual results within each group. Their weighting is certainly more logical in the present case than the strict least squares weighting, and is accepted by the writer. They then obtain a *weighted* average of the six results, a corresponding weighted mean temperature, and the probable error of the weighted average, by the usual least squares formula. This result is $J=4.1841\pm0.0001$ abs. joules, at 16.67°C. This method of averaging would be legitimate if the actual $f(t)$ representing J could be replaced by a straight line, over the range of temperature used, and if the residuals of all the six points, for the true $f(t)$ curve, are such as to indicate only the usual accidental errors.

A more precise method of averaging is first to adopt a curve for the variation of the specific heat of water with temperature. Such a curve is given immediately by Eq. (2). If it is desired that the specific heat at 15°C be unity, Eq. (2) is to be divided by 4.18327. Laby and Hercus, later in their article, assume that Callendar's revision of Barnes' data for the specific heat of water[9] gives the most reliable data, in terms of the thermodynamic scale. This matter will be discussed by the writer in the separate paper to which reference has been made, with the conclusion that the Jaeger and Steinwehr results, because of the later date of the work, are the more reliable. Over the limited temperature range now being discussed, the two specific heat curves differ by almost negligible amounts.

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[7] W. Jaeger, HP. **9,** 493.

[8] T. H. Laby and E. O. Hercus, Phil. Trans. **A227,** 63 (1927). See also T. H. Laby, Proc. Phys. Soc. London **38,** 169 (1926) for a critical review of all previous determinations of the *electrical* equivalent of heat.

[9] See Landolt-Börnstein, 5th Ed. p. 1250, for all data on the specific heat of water. The Callendar reference is Phil. Trans. **A212,** 1 (1912).

Using Eq. (2) as a basis for a specific heat curve, I have reduced each Laby and Hercus result to the 15° calorie, the six temperatures involved being15.97, 15.88, 16.15, 16.74, 15.62, and 20.50, all in °C. It then appears that the reduced result for the 20.50°C temperature is quite inconsistent with the other five.[10] Thus the first five results yield a weighted average of 4.18526 abs. joules for the 15° cal., with a weighted mean residual of 3.0×10^{-4} joules, while the residual of the sixth determination is 22.1×10^{-4}. If the Callendar specific heat curve is used, the corresponding results are 4.18545, 3.1×10^{-4}, and 31.5×10^{-4}, respectively. An observation with a residual 7 to 10 times the mean residual should be rejected, according to any of the well known criteria of rejection. Hence I obtain from the Laby and Hercus data, $J_{15}=4.18526$ abs. joules.

Laby and Hercus discuss the accuracy of the various elements entering into the determination of their result, but make no estimate of the probable error in the final result, due to the various possible sources of error. The only error given (±0.0001) is, as noted, merely a least squares error, due to accidental variations of the individual results. Jaeger and Steinwehr, on the other hand, make no *specific* allowance for such an error, and their stated error (±0.0008) seems to be their estimate of the various possible errors, constant and otherwise, affecting the result. Both these investigations have been carried out during the past decade, with every possible experimental refinement, and it is natural to assume that the final probable error in the two cases cannot be greatly different. Of the 67 individual determinations by Jaeger and Steinwehr, six show very large residuals, and have been rejected by the writer, in deriving Eq. (2). The remaining 61 have a mean residual, in terms of Eq. (2), of 11.7×10^{-4} joules. In the case of Laby and Hercus, the four individual determinations made at the relatively high mean temperature, 20.50°C, have been rejected as a group, because of their very large mean residual. The remaining 19 determinations forming the other five groups have an average residual of 8.8×10^{-4} joules, assuming the weighted result 4.18526 abs. joules for the 15° calorie, and the specific heat curve given by Eq. (2). The purely accidental errors, as anticipated, are thus roughly the same for the two investigations, and in lieu of more definite evidence it seems best to assume that the *total* probable error in both cases is the same as that given by Jaeger and Steinwehr for their own work. Hence I take as the final Laby and Hercus result, $J_{15}=4.1853\pm0.0008$ abs. joules.

This is to be averaged with the reduced value 4.1850 ± 0.0009 abs. joules, resulting from the work of Jaeger and Steinwehr. Because of the greater accuracy claimed by Laby and Hercus, I adopt for the mean value 4.1852, rather than 4.1851. The probable error of the mean can however be taken slightly less than that of either determination. My final assumed result is

$$\text{one } 15^\circ \text{ calorie } (J_{15}) = 4.1852 \pm 0.0006 \text{ abs. joules.}$$

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[10] This is most probably due to some constant error at the 20.5° temperature. All other observations, it should be noted, are close to 16°C mean temperature.

Using now the assumed value $pq^2 = 1.00041 \pm 0.00010$, we obtain

$$\text{one } 15^\circ \text{ calorie } (J_{15}') = 4.1835 \pm 0.0007 \text{ int. joules,}$$

as a *slightly* more accurate value than the direct determination by Jaeger and Steinwehr of 4.1833 ± 0.0008 int. joules.

Finally, using Eq. (2) one obtains $J_{20}/J_{15} = 0.999058$. Hence one 20° calorie

$$J_{20} = 4.1813 \pm 0.0006 \text{ abs. joules}$$

$$J_{20}' = 4.1796 \pm 0.0007 \text{ int. joules.}$$

In conclusion it should be noted that the revised Jaeger and Steinwehr result, $J_{15}' = 4.1833 \pm 0.0008$ int. joules, when taken with the revised Laby and Hercus result, $J_{15} = 4.1853 \pm 0.0008$ abs. joules, gives directly one int. joule $= 1.00048 \pm 0.00027$ abs. joules. This is an entirely independent method for evaluating pq^2, but the probable error is about three times as large as that assumed for the more direct method discussed in section E. The good agreement of the two values, 1.00041 and 1.00048, is therefore probably more or less accidental. Attention should be directed also to an investigation by W. Groth[11] of the electrical and mechanical equivalents of heat. He found that within the limits of error of his work (one part in two thousand) the two quantities were identical. This agrees with our adopted values, the difference between J and J' being slightly less than one part in two thousand.

Section J

THE FARADAY (F)

The Faraday is defined as the quantity of electricity carried in electrolysis by one gram equivalent of any element. It is believed to be a general constant of nature, and its true value is accordingly independent of experimental conditions. According to modern ideas, each univalent ion carries a charge numerically equal to the electronic charge e. The Avogadro number N_0 gives the number of atoms (or molecules) in one gram equivalent. Hence one may define the Faraday (F) more precisely as equal to the product $N_0 \cdot e$. The fact that, as a matter of experimental practise, F can be most accurately evaluated from electrolysis, and N_0 is then evaluated from F and e, does not affect the validity of the definition.

The electrochemical equivalent of a substance is the mass associated with unit electric charge. Like the Faraday, its true value is independent of experimental conditions, and depends only on the adopted unit of charge. On the other hand, as noted in section E, we can measure only the amount of a substance deposited or released in an electrolytic cell, per unit current per second. This amount is affected by experimental conditions, and may or may not equal the electrochemical equivalent. The Faraday is then, by definition, the ratio of the gram equivalent of a substance to its electro-

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[11] W. Groth, Ann. Physik **82,** 1156 (1927).

chemical equivalent. It seems to be almost universal practise to ignore the distinction between mass deposited per unit charge, and electrochemical equivalent. This practise has resulted in considerable confusion in the literature regarding the best value of certain electrochemical equivalents, and the resulting best value of the Faraday.

In spite of the above remarks, it is convenient to *assume*, for the moment, that the amount of silver deposited per unit charge in a silver voltameter, operating under the conditions laid down in defining the international ampere, is in fact the electrochemical equivalent of silver (E_{Ag}). *With this assumption*, the value of Faraday follows immediately from constants already adopted. The gram equivalent of silver, or of any univalent substance, is numerically equal to its atomic weight in grams (Ag). The amount of silver deposited in electrolysis by one international coulomb is, by definition,[1] 0.00111800 grams. Hence

$$F=\frac{\text{Ag}}{0.00111800}\text{ int. coul.} \tag{1}$$

Using our adopted value[2] of Ag, we have

$$F=\frac{107.880\pm0.001}{0.00111800}=96494\pm1\text{ int. coul.} \tag{2}$$

With our adopted value[1] of q, there results

$$F=\frac{(107.880\pm0.001)(0.99995\pm0.0005)}{0.00111800}=96489\pm5\text{ abs. coul.} \tag{3}$$

If one uses $q=1$, as adopted by the HP[3], one gets $F=96494$ int. coul. *or* abs. coul. This is the actual value of the Faraday adopted by Henning and Jaeger[3] If one uses $q=0.99993$, as adopted by the ICT,[4] there results

$$F=\frac{(107.880)(0.99993)}{0.00111800}=96487\text{ abs. coul.} \tag{4}$$

The ICT, however, adopts[5] $F=96500\pm10$ abs. coul., which with its adopted value of q, leads to[6] $F=96507$ int. coul. This last value, if substituted in Eq. (1) obviously requires Ag$=107.893$, in direct contradiction to the known facts. The value $F=96500\pm10$ abs. coul. is evidently taken from the work of Vinal and Bates,[7] and in order to understand the seeming discrepancy, it is necessary to examine in some detail this last quoted investigation, and to employ the distinction just mentioned between mass carried in electrolysis and mass deposited.

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[1] See section E.

[2] See section F.

[3] Henning and Jaeger, HP **2** 502.

[4] ICT. **1**, 18.

[5] ICT. **1**, 17.

[6] ICT. **1**, 26.

[7] G. W. Vinal and S. J. Bates, Bur. Standards, Bull. **10**, 425 (1914). See page 447.

Vinal and Bates were investigating the iodine voltameter, and connected such a voltameter in series with a silver voltameter. They then measured the ratio of the mass of silver to the mass of iodine, deposited (or liberated) by the same current. *If* these masses truly represent the amounts of silver and of iodine required to carry a specified quantity of electricity, their ratio should, by Faraday's law, equal the ratio of the atomic weights of silver and iodine. The final measured value of this ratio (Ag/I) was 0.85017, with an apparent probable error of a few parts in 10^5. This result has been mentioned in section F. Combined with Ag = 107.880, it gives I = 126.892, while as discussed in section F, the best chemical value of the ratio leads to I = 126.932 ± 0.002. The Vinal and Bates value is thus in disagreement with the chemical value by about 32 parts per 10^5, and this discrepancy must be due to the fact that in the silver voltameter, or in the iodine voltameter, or in both, the measured amount of deposit does not represent the true mass carried by a given electric charge.

This question has been investigated in great detail, in the case of the silver voltameter. In particular, the possibility of inclusions in the silver deposit has been studied by various persons.[8] The final conclusion is that there is likely to be about 0.004 percent of inclusions in the silver deposit of a silver voltameter, operated under the specifications used in defining the int. amp. Applying this correction to the Vinal and Bates results, one gets Ag/I = 0.850136, accounting for only 3.4 parts per 10^5, out of the total discrepancy of 32 parts. It would therefore appear that, barring unsuspected sources of error in the silver voltameter, such as secondary chemical reactions, etc., the main cause of the above discrepancy lies in the iodine voltameter.

Vinal and Bates, working before the above facts as to inclusions were known, did not draw this conclusion. Their actual procedure was to use Eq. (1) to get $F = 96494$ int. coul. from the silver voltameter, and then to use I = 126.92 (the accepted value at that time), and a calculated electrochemical equivalent of iodine (E_{I}), to get a new value $F = 96514$ int. coul. from the iodine voltameter. Their value of E_{I} (0.00131502) was obtained by dividing 0.00111800 by their measured ratio 0.85017. They then take the arithmetic average of 96494 and 96514, or 96504, as the best value of the Faraday in int. coul. They state, finally, that the value of q (as we have defined it in section E) is uncertain, but that 96500 ± 10 abs. coul. can be considered the best working value of F. This assumed value in abs. coul. evidently corresponds to $q = 0.99996$, in close agreement with our accepted value.

Their adoption of 96504 int. coul. as the mean value of the Faraday necessarily implies the following conclusions. From Eq. (1), with $F = 96504$ int. coul., and Ag = 107.880, we obtain $E_{\text{Ag}} = 0.00111790$ grams. Hence one must conclude that although 0.00111800 grams of silver are deposited apparently by one int. coul., and serve as its definition, the true electrochemical equivalent is only 0.00111790 grams per int. coul. Using the Vinal

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[8] E. B. Rosa, G. W. Vinal, and A. S. McDaniel, Bur. Standards, Bull. **10**, 475 (1914); G. W. Vinal and W. M. Bouvard, ibid **13**, 147 (1916).

and Bates value of I, we have $96504 = 126.92/E_I$. Hence $E_I = 0.00131518$, as compared to 0.00131502 deduced by them, from the amount of iodine liberated. We must thus conclude that in this case, the amount of the substance (iodine) liberated is *less* than that required to carry the charge. This may very well be the case, due to the loss of liberated iodine, and the matter is discussed by Vinal and Bates. There does not, however, seem at the present time to be experimental justification for dividing up the discrepancy practically equally between the silver and iodine voltameters. The only known fact appears to be the observed 0.004 percent inclusion in the silver deposits, and the proper correction for this would give 0.00111796 grams as the true E_{Ag} per int. coul., or, with our adopted value of q, 0.00111801 g as the true E_{Ag} per abs. coul. These values in turn lead to $F = 96498$ int. coul., or 96493 abs. coul.

In a later paper Vinal and Bouvard[8] do in fact discard entirely the iodine value, and by this same method deduce $F = 96494$ abs. coul., based on $q = 0.99996$. They however again recommend 96500 abs. coul. as the best rounded value. On the other hand, Henning and Jaeger[3] make no distinction between mass carried and mass deposited, writing $E_{Ag} = 0.00111800$ g per int. coul., with *no* indicated error. It seems evident from the work of Vinal and Bouvard that there *are* inclusions in the silver deposit, tending to make E_{Ag} too large by 4×10^{-8} g, and F too small by 4 coulombs. On the other hand, there may well be small parasitic chemical reactions in the silver voltameter, tending to decrease the value of E_{Ag} and hence to increase the value of F. Under the circumstances it seems to the writer best to *adopt* the values of F given in Eqs. (2) and (3), but to assign to E_{Ag} a probable error of 5×10^{-8}g, i.e., an error slightly greater than the measured effect of the inclusions. Then

$$F = \frac{107.880 \pm 0.001}{(1.11800 \pm 0.00005) \times 10^{-3}} = 96494 \pm 5 \text{ int. coul.} \tag{5}$$

and

$$F = \frac{(107.880 \pm 0.001)(0.99995 \pm 0.00005)}{(1.11800 \pm 0.00005) \times 10^{-3}} = 96489 \pm 7 \text{ abs. coul.} \tag{6}$$

Eq. (5) thus assumes $E_{Ag} = (1.11800 \pm 0.00005) \times 10^{-3}$ g per int. coul., while Eq. (6) assumes $E_{Ag} = (1.11805 \pm 0.00007) \times 10^{-3}$ g per abs. coul. Finally from Eq. (6),

$$\begin{aligned} F &= 9648.9 \pm 0.7 \text{ abs. } em \text{ units} \\ Fc &= (9648.9 \pm 0.7)(2.99796 \pm 0.00004) \times 10^{10} \\ &= (2.89270 \pm 0.00021) \times 10^{14} \text{ abs. } es \text{ units.} \end{aligned}$$

Section K

THE ELECTRONIC CHARGE (e)

A relatively large number of important constants have values which depend directly on the value assigned to the electronic charge e, and in most cases the final probable error in these values is due mainly to the error in e.

From this standpoint the electronic charge may be considered the most important of the general physical constants, and it is therefore highly desirable that its value should be determined in many different ways, and by many different persons. Until very recently the situation has been the reverse of this. Only one really precision method for the evaluation of e was known, and the precision work, by this method, had been carried out by a single individual. Under such circumstances it is very fortunate that the investigations referred to represent a masterpiece of experimental technique, in which every possible cause of error, systematic or accidental, was studied in minute detail, and after many years of work reduced to a final minimum. Millikan's investigations on this subject, by the oil-drop method, extend over more than a decade, the latest value of e having been published[1] in 1917. Because of the great importance of this resulting value of e, and because of higher values which have recently been obtained, the writer has investigated the matter in more than usual detail.

Millikan found that if the viscosity of air is taken as constant, in Stokes' law of fall, the apparent value of e is a function of the radius of the drop and of the pressure of the air. The true value of e can be found by assuming a modification of Stokes' law. If the pressure p or the radius of the drop a is sufficiently large, the final equation for determining the true e is

$$e_1^{2/3}=e^{2/3}(1+b/pa) \qquad (1)$$

where e_1 is the apparent value, e the true value, and b an undetermined constant. This equation can be written

$$y=a_0+a_1x \qquad (2)$$

when

$$x=1/pa, \quad y=e_1^{2/3}, \quad a_0=e^{2/3}, \quad a_1=e^{2/3}\cdot b.$$

Hence if $e_1^{2/3}$ is plotted against $1/pa$, the resulting graph should be linear, and its intercept on the y axis gives $e^{2/3}$, leading to the desired quantity.

Millikan found that for values of $1/pa$ less than about 700 (p in cm Hg, a in cm), the resulting graph was definitely linear. For larger values of $1/pa$, the slope of the graph increases to a new larger value, and then remains constant. The complete law for the fall of such drops in air was discussed[2] in 1923. Only that part of the curve corresponding to $1/pa$ less than 700 was used in precise determinations of e. The 1917 value of e was deduced from data on 25 oil-drops, each drop giving one point on the graph. The 25 observations form a beautifully consistent set of data,[3] lying on a linear graph. The least squares solution, as calculated by the writer, gives for the intercept $a_0=(61.111\pm0.032)\times10^{-8}$. The plotted data are however

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[1] R. A. Millikan, Phil. Mag. (6) **34,** 1 (1917). For previous work by Millikan see Phys. Rev. **29,** 60 (1909), Phil. Mag. **19,** 209 (1910), Phys. Rev. **32,** 349 (1911), Phys. Rev. **2**; 109 (1913), and Physik. Zeits. **14,** 796 (1913). See also R. A. Millikan "The Electron," Chap. V.

[2] R. A. Millikan, Phys. Rev. **22,** 1 (1923).

[3] See R. A. Millikan, Phil. Mag (6) **34,** 12 (1917), or "The Electron" page 121 (first edition), for the plot of these data.

based on the 1913 value (0.0001824) for the viscosity of air. The value of $a_0(=e^{2/3})$ is directly proportional to the assumed viscosity. With the improved 1917 value of the viscosity (0.00018227), a_0 becomes 61.0676×10^{-8}. Hence $e=a_0^{3/2}=(4.7721\pm 0.0038)\times 10^{-10}$ *es* units.

Millikan stars 18 of the points, these having been taken when the conditions of observation were considered as perfect as possible. These 18 drops give $a_0=61.121\pm 0.038$ (or $e=4.7733\pm 0.0045$, based on the 1917 viscosity). As a matter of fact, these 18 drops deviate from the best straight line more than do the other 7. Thus the standard deviation (mean error of a single observation) of the 25 drops is 0.121×10^{-8}, while for the 18 drops it is 0.123×10^{-8}, each referred to its own least squares solution. The drops of smaller radius fall more slowly, and therefore can be more accurately timed. This might make them appear more reliable. Actually they are less reliable, due presumably to other errors. Thus 13 smaller drops whose radii lie between 12.17 and 15.92×10^{-5} cm, have a standard deviation of 0.134, when considered as part of the 25 drops. This is definitely larger than the 0.121 average of the entire 25. A least squares solution, based on these 13 drops alone gives $a_0=61.143\pm 0.050$, and a standard deviation of 0.132. This standard deviation is so close to 0.134 that we can conclude that the 13 drops fit on the graph of the entire 25 essentially as well as on a graph designed to fit them alone. On the other hand, the 12 larger drops (for which $a=16.57$ to 23.40×10^{-5} cm) give for the least squares solution, $a_0=61.078\pm 0.045$, and a standard deviation 0.117. They are thus definitely more reliable than the smaller drops, as judged by their mutual consistency. The resulting value of e, reduced to the 1917 viscosity, is 4.7759 ± 0.0058 for the 13 smaller drops, and 4.7683 ± 0.0053 for the larger drops. The weighted mean is 4.7718, in essential agreement with the value (4.7721) obtained from all 25 drops. This of course is just what we should expect.

The average deviation from the average of the two results (for small and large drops respectively) is 0.0038, and this is much less than the probable error of either. This is an analytic proof of the contention made by Millikan in 1913 and again in 1917, that the true value of e (intercept of the graph) is not in any way a function of the radius of the drop. The above results also indicate that the larger drops are, if anything, more reliable than the smaller. If the larger are given a higher weight, the resulting value of e would lie between 4.772 and 4.768, according to the weighting. It has been noted already that the 18 starred drops are slightly less consistent than the others. The final conclusion is then that there is no particular reason for giving different weights to the different drops, and that any such weighting, if made, would slightly lower e. I therefore take 4.772×10^{-10} *es* units as the best result of the 1917 work.

In 1913 data on 58 drops were obtained. These show practically the same consistency as the 25 drops just considered, the standard deviation being 0.135. Millikan actually used, in evaluating e, the 23 drops (out of 58) of smallest $1/pa$. These 23 are more consistent, having a standard deviation of only 0.092. They lead to $e=4.7665\pm0.0058$, based on the 1917 viscosity, while the entire 58 drops give 4.7703 ± 0.0022. This last figure might appear to be a more reliable value of e than that obtained in 1917, but such a conclusion ignores the errors due to other factors. In 1913 Millikan estimated that there were four such factors, each with a maximum uncertainty of 0.1 percent. In 1917 he estimated that there remained only two such factors of any consequence, each with a maximum of uncertainty of only 0.05 percent. His final estimate, in 1917, for the maximum of uncertainty in e is 0.1 percent, based mainly on these two factors. The above calculations show, however, that there is a *probable* error of 0.08 percent (±0.0038) in the 1917 value of e, due merely to accidental errors. The final maximum uncertainty is therefore several times as large. I estimate that the final *probable error* is about 0.1 percent, and write $e=(4.772\pm0.005)\times10^{-10}$ *es* units.

This value is now subject to two further corrections. The equation for e (or e_1) can be put in the form

$$e=k/F \tag{3}$$

where F is the electric field intensity, and k represents a number of factors, all mechanical in nature. F is measured directly in int. volts per cm, using a calibrated Weston cell. In reducing the result to *es* units per cm, Millikan used $c=2.999\times10^{10}$ cm·sec^{-1}, and made no distinction between international and absolute electrical units. Since the time of his work it has been shown definitely that the int. volt differs from the abs. volt by an appreciable amount, as discussed in section E. We have also now the new value, $c=2.99796$. Both the resulting corrections tend to lower e. That due to the change in c is of course obvious. It lowers e in just the proportion 2.99796/2.999, and so changes e from 4.772 to 4.770. The other change, due to the electrical units, seems to have been overlooked by everyone. Thus Henning and Jaeger, in the HP[4], state explicitly that Millikan used international volts, and that the resulting e is therefore in int. coulombs. The difference between int. coul. and abs. coul. being negligible, they therefore make no correction. This conclusion is however incorrect. Because the electrical potential forces the charged drops against the viscosity of air, instead of against *electrical* resistance, one has *only* electric voltage coming into the calculations, as shown by Eq. (3). It has been shown in section E that one int. volt $=1.00046\pm0.00005$ abs. volts. Hence the true value of F, in abs. volts, is larger and the true value of e, in *abs. es* units is smaller, by just this ratio. Hence, finally, the value of e is reduced[5] from 4.770 to 4.768. Since the error in each of these corrections is negligible, the final result is

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[4] HP **2**, 503.

[5] These two corrections to e, due to the change in c, and to the electrical units, were first noted by the writer in Sept. 1928, and transmitted immediately to Professor Millikan who agreed with them. They were later presented to the Am. Phys. Soc., at the Pasadena meeting in December 1928 (Phys. Rev. **33**, 265, 1929).

$e=(4.768\pm0.005)\times10^{-10}$ *abs. es* units. This, the writer believes, is the most reliable value that can be deduced from Millikan's oil-drop work.

Very recently there has been devised an entirely different method for measuring e, which bids fair to become more accurate than the oil-drop method. The two results by this method which have already been published are, however, apparently less reliable than the oil-drop value. This new method measures directly the Avogadro number N_0, and from this and the known value of the Faraday, e immediately follows.[6] The method utilizes the absolute wave-lengths of x-ray lines, determined with an ordinary ruled grating at grazing incidence, as compared with the wave-lengths determined with a crystal grating. The latter values are given by

$$\lambda=2d\cdot\sin\theta \tag{4}$$

where d is the grating space. It has been pointed out by Siegbahn,[7] and by Compton, Beets and DeFoe,[8] that in order to obtain the *true* x-ray wave-length λ, it is necessary to use an *effective* grating space d in Eq. (4). Such an effective grating space automatically corrects for the actual refraction of the x-rays at the crystal surface. For first order spectra, and for the high frequencies of ordinary x-rays, the *true* grating space d' is connected with the effective space d by the relation

$$d=d'(1-0.000135)\,. \tag{5}$$

In evaluating λ, Siegbahn uses for the calcite grating space $d=3.02904\times10^{-8}$ cm, at 18°C. This is a more or less arbitrary value, based on the assumption that d for rock-salt, at 18°C, is 2.81400×10^{-8} cm. We shall denote by d''_{18} this 3.02904 value, and by λ'' the resulting wave-length. Hence

$$\lambda''=2d_{18}''\sin\theta\,. \tag{6}$$

From Eqs. (4) and (6)

$$\lambda/\lambda''=d_{18}/d_{18}'' \tag{7}$$

in which λ is the *true* wave-length of an x-ray, as determined with a ruled grating, d_{18} the effective grating space of calcite at 18°C, λ'' the supposed true wave-length resulting from measurements with a calcite crystal with d''_{18} as an assumed effective grating space at 18°C. Thus knowing λ'', d''_{18}, and λ, one calculates d_{18}. Then from Eq. (5) we obtain d'_{18}, the true grating space of calcite. The temperature coefficient of calcite[9] is 1.04×10^{-5} and the value of d'_{20} is accordingly 2.08×10^{-5} larger. This 20° value of d' is given theoretically by the following formula,

$$d_{20}'=\left\{\frac{nM}{\rho N_0\phi(\beta)}\right\}^{1/3} \tag{8}$$

in which $n=1/2$, M=molecular weight of calcite ($CaCO_3$), ρ=density of calcite at 20° C, $\phi(\beta)$=a geometrical constant depending on the crystal

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[6] See section J.

[7] M. Siegbahn, "Spectroscopy of X-rays" p. 26.

[8] A. H. Compton, H. N. Beets, and O. K. DeFoe, Phys. Rev. **25**, 625 (1925).

[9] M. Siegbahn, reference 7, p. 85.

structure of calcite, N_0 = Avogadro's number $= F/e$ where F = Faraday. Thus knowing d'_{20} we can calculate N_0 and finally e.

The best value of M follows directly from the atomic weights adopted in section F. It is

$$M=3\times 16.000+(12.003\pm 0.001)+(40.075\pm 0.005)=100.078\pm 0.005 \quad (9)$$

The best value of ρ is 2.7102 ± 0.0004 g·cm^{-3}, at 20°C, as determined by DeFoe and Compton.[10] The best value of $\phi(\beta)$ is 1.09630 ± 0.00007 at 20°C, as determined by Beets.[11] The adopted value of the Farady[12] is $(2.89270\pm 0.00020)\times 10^{14}$ abs. *es* units. Solving Eq. (8) for e, and making the above substitutions, we have

$$e=\frac{\rho F\phi(\beta)(d')^3}{nM}=(1.7176\pm 0.0003)\times 10^{13}\cdot(d_{20}')^3. \quad (10)$$

The two published determinations of d_{18}, based on absolute x-ray wave-lengths, are by Bäcklin,[13] and Wadlund.[14] Using Eq. (7), Wadlund obtains 1.5373 ± 0.0008A for the $K\alpha_1$ line of Cu, and this, combined with Siegbahn's values of d''_{18} and λ'', gives $d_{18}=(3.0290\pm 0.0016)\times 10^{-8}$ cm. The corresponding value of d_{20} is then 3.02906, and the true grating space d'_{20} is by Eq. (5), $(3.0295\pm 0.0016)\times 10^{-8}$ cm. This value[15] is to be substituted in Eq. (10). It gives $e=(1.7176\pm 0.0003)\times 10^{13}(3.0295\pm 0.0016)^3\times 10^{-24}=(4.7757\pm 0.0076)\times 10^{-10}$ abs. *es* units. Wadlund, using other values of the auxiliary constants, obtained 4.774 ± 0.007. This is not as accurate as the oil-drop value, and Professor A. H. Compton writes me that later work indicates that Wadlund's published probable error may be a slight underestimate.

It is very difficult to appraise the work of Bäcklin,[13] especially as regards its accuracy. He gets 8.333 ± 0.008A for the absolute wave-length of the Al $K\alpha$ line. Comparing this with an unpublished result by A. Larsson (8.3229 ± 0.0008A), obtained with a crystal, Bäcklin obtains $d_{18}=3.033\pm 0.003$A. This gives $d'_{20}=3.03347$A, and by Eq. (10), $e=(4.794\pm 0.015)\times 10^{-10}$ abs. *es* units. This value is 0.55 percent higher than the oil-drop result. His own result, with different values of the auxiliary constants, is 4.793 ± 0.015.

Before discussing further the absolute value, a word is necessary as to the errors. Throughout his paper, Bäcklin calculates the *mean error* of a *single* observation (i.e., the standard deviation) and seems to consider

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[10] O. K. DeFoe and A. H. Compton, Phys. Rev. **25,** 618 (1925).

[11] H. N. Beets, Phys. Rev. **25,** 621 (1925).

[12] See section J.

[13] E. Bäcklin "Absolute Wellenlängenbestimmungen der Röntgenstrahlen," Uppsala Dissertation, 1928.

[14] A. P. R. Wadlund, Proc. Nat. Acad. Sci. **14,** 588 (1928), and Phys. Rev. **32,** 841 (1928).

[15] Both Wadlund and Bäcklin omit these corrections and thus use their observed d_{18} as the value of d'_{20}.

that this is the *limit of error* of the resulting *average* value. For the 15 to 25 observations he is normally handling, such a standard deviation is several times the probable error of the average. It therefore *does* represent more nearly a reasonable *limit* of error than a *probable* error. On the other hand, in getting the final error in λ, he takes the square root of the sum of the squares of the individual limits of error. As stated in the Introduction one should in this case take the straight sum, to get the limit of error of the result. This sum is 0.17 percent for the error in λ, instead of his 0.10 percent, and so 0.51 percent in e. This gives $e=4.794\pm0.024$, where the *limit of error* is indicated. The probable error is then not over ±0.006, so that the apparent accuracy is practically that of Millikan's work. In fact Bäcklin compares the average residual of his observations with that of Millikan, and finds his definitely smaller, and also finds his residuals better distributed on the Gaussian error curve. In terms of the resulting value of e, his average residual is 0.12 percent, with 29 observations. The residuals for the oil-drop method, used by Bäcklin, are not based on a least squares solution of the data, but the average residual which he gives, 0.25 percent, is essentially the same as that given by the correct least squares solution of the 25 drops used by Millikan in 1917. His final error is however slightly larger than that for the oil-drop work, due to the fact that his other sources of error are larger. He rightly concludes that as far as internal consistency is concerned, the x-ray method is more accurate, and the problem thus becomes that of eliminating the various sources of systematic error.

In the preliminary work on this paper, I calculated each general constant quite independent of the relations between the various constants. The values so obtained were found to be unexpectedly satisfactory, when certain known or proposed relations were tested. In the subsequent extensive revisions, these relations have been kept in mind, and a deliberate attempt has been made to retain a *system* of constants which should be as satisfactory as possible. The most important of the relations between general constants is that given by the Rydberg constant, which according to Bohr is a simple function of e, e/m, h, and c. This relation is discussed in section O, in connection with the finally adopted values of these four constants. In anticipation of this discussion it may be said that any value of e as high as that of Bäcklin could hardly be reconciled with the observed values of h and e/m.

The investigation by Bäcklin constitutes a pioneer piece of work, and it is quite likely, as such, to contain various unsuspected sources of systematic error. If the three values of e (4.768 from Millikan's oil-drop work, 4.776 by Wadlund, and 4.794 by Bäcklin) are weighted according to the apparent probable error of each, the weighted average will still be suspiciously high. The very thorough examination made by the writer of the actual value of e and its probable error, as deduced from oil-drop work, was carried out mainly because of the inconsistency between that value and the result published by Bäcklin. In view of all the above circumstances, the writer has finally decided to reject the Bäcklin value, and to use the weighted mean of the remaining two values, viz. 4.768 ± 0.005 and 4.776 ± 0.008. This mean is 4.770, and as usual I adopt as its probable error the smaller of the two individual errors, rather than that given by least squares, since the latter is rather meaningless when only two observations are concerned. The finally adopted value is then

$$e = (4.770 \pm 0.005) \times 10^{-10} \text{ abs. } es \text{ units.}$$

In conclusion attention should be called to the fact that a more accurate determination of e, from x-ray measurements, is now under way, and it is to be hoped that this work will clear up the present somewhat uncertain situation. This is discussed further in Section O.

Section L

THE SPECIFIC CHARGE OF THE ELECTRON (e/m)

A very complete and critical account of all work on the measurement of e/m, up to 1919, has been given by Bestelmeyer.[1] His final conclusion is that $e/m = (1.76 \pm 0.02) \times 10^7$ *em* units. A more recent discussion of all experimental results is that by Gerlach,[2] who concludes that $e/m = 1.766 \times 10^7$ *em* units. The question is discussed very briefly by Henning and Jaeger,[3] who however adopt Gerlach's value. The ICT[4] adopts 1.769 ± 0.003.

There is an enormous amount of experimental material on this subject. The latest work, however, so greatly exceeds in accuracy all the preceding that it seems legitimate to confine the discussion to these new results. The value of e/m has been obtained with considerable accuracy by three distinct methods, (*a*) deflection of electrons in electric and magnetic fields, (*b*) Zeeman effect, (*c*) fine structure and relative wave-lengths of H and He$^+$ spectral lines. It may be obtained also from Bohr's theoretical expression for the Rydberg constant, R_∞, provided one assumes the value of e and of h. This last method is, however, not as accurate as the preceding. A fifth, and very interesting method for obtaining e/m involves the Compton shift. This also is as yet a relatively inaccurate method. The last two methods are discussed in sections O and M respectively.

The latest and most accurate work with method (*a*) is that by Wolf.[5] In this work a new method, first suggested and employed by Busch,[6] is carried out with every possible experimental refinement. The essential point in the method is the employment of a longitudinal instead of a transverse magnetic field. The electron velocity is calculated from the potential fall. Wolf makes an exhaustive study of the possible errors, and finally

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[1] Marx, Handbuch d. Radiologie V, pp. 1–82 (1919).

[2] HP. **22,** 41–82 (1926).

[3] HP. **2,** 504–506.

[4] ICT. **1,** 17.

[5] F. Wolf, Ann. Physik **83,** 849 (1927).

[6] H. Busch, Physik. Zeits. **23,** 438 (1922).

concludes that $e/m = (1.7679 \pm 0.0018) \times 10^7$ *em* units. So far as the writer can judge, the assumed probable error is entirely justified.

The actual magnitude, 1.7679, should however be corrected for the difference between the int. and abs. electrical units (section E). The equation used by Wolf is

$$\frac{e}{m} = \frac{8\pi^2 V}{H^2 l^2} \cos^2 \alpha \tag{1}$$

where V is the potential drop and H the magnetic field. Wolf does not state whether he corrected from int. to abs. units, but presumably, in conformity with current practise, he did not. If V' and H' denote the measured quantities in int. units, then

$$\frac{e}{m} = \frac{8\pi^2 (V'pq)}{(H'q)^2 l^2} \cos^2 \alpha \tag{2}$$

Hence his value of e/m should be multiplied by p/q and this, by section E, is 1.00056. Therefore his true result is $(1.7689 \pm 0.0018) \times 10^7$ abs. *em* units.

In the determination of e/m by means of the Zeeman effect, the basic equation is

$$\frac{e}{m} = \frac{4\pi c \Delta\lambda}{nH'q\lambda^2} \tag{3}$$

where e/m is in *em* units,

c = velocity of light in vacuum, in cm·sec^{-1},

n = index of refraction of the medium (air) in which $\Delta\lambda$ and λ are measured,

H' = intensity of the magnetic field in int. gauss,

$H'q$ = intensity of the magnetic field in abs. gauss,

$\Delta\lambda$ = separation of either component of a normal triplet of wavelength λ from the undisturbed position of the line, $\Delta\lambda$ and λ being measured in cm.

By section E, $q = 0.99995 \pm 0.00005$, and can be taken as unity without appreciable error.

The most recent and accurate work, using this method, is that by Babcock.[7] In his investigation a large number of spectral lines (116 in all) were employed. Nearly all of them showed a complex Zeeman effect pattern. For determining e/m it was necessary to assume the Runge denominator of each line. In cases where this is small, it was known with certainty. In some cases it was large and rather uncertain. Babcock states,[8] "It will be observed, however, that these large denominators play a minor part in the present investigation." In spite of this remark, his work has been

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[7] H. D. Babcock, Astrophys. J. **58,** 149 (1923).

[8] H. D. Babcock, reference 7, p. 155.

criticised,[9] and Gerlach, in his final table[10] of values of e/m, omits Babcock's result. It appears to the writer that the criticism is unjustified, and at his suggestion, Babcock has recently recalculated his data, omitting all Zeeman patterns which are in any way doubtful.

His previously published result, assuming $c=2.9986\times10^{10}$, is $e/m=(1.761\pm0.001)\times10^{7}$ *em* units, in which the error quoted is merely the observational error, as determined by least squares. Babcock estimates his probable uncertainty, when all sources of error are considered, as not more than two or three parts in 1800. This it seems to the writer is a very conservative estimate. The new result,[11] based on 48 lines for which the Zeeman pattern is definitely established, and using $c=2.99796\times10^{10}$, is 1.7606 ± 0.0012, where again the error is purely observational. The difference between the two values is just that produced by the change in the value of c. Hence the omission of the lines of doubtful Zeeman pattern makes no change in the result. Babcock considers that his latest value is correct to about one part in a thousand. I therefore write $e/m=(1.761\pm0.002)\times10^{7}$ abs. *em* units as the best result from Zeeman effect.

The latest and most accurate work on e/m, using method (c), is that by Houston.[12] This method is based on the Bohr-Sommerfeld model of an atom consisting of a positive nucleus and *one* encircling electron (moving in elliptic or circular orbits). As examples of such an atom we have hydrogen (H) and singly ionized helium (He^+). In order to determine e/m, we must evaluate the so-called Rydberg constant for hydrogen (R_H) and for ionized helium (R_{He}). As will appear presently, practically the entire error in e/m is merely the error in the *difference* $R_{He}-R_H$.

The pioneer work on e/m, by this method, is that performed by Paschen.[13] He obtained $R_H=109677.69\pm0.06$ cm^{-1}, $R_{He}=109722.14\pm0.04$ cm^{-1}. This gives $e/m=1.768\pm0.003$, using his own values and assumed errors for R_H and R_{He}, but the present accepted values and errors for H, He, and F. The writer,[14] using better data and more recent theoretical developments, later calculated $R_H=109677.7\pm0.2$. The argument in this last quoted paper shows that Paschen's estimate of error in the case of R_H is much too small, and the entire question, in its relation to the value of e/m was later discussed.[15] In this last article it was concluded that the method here considered gives $e/m=1.758\pm0.009$. The recent investigation by Houston, which assumes the latest theoretical developments, is so much more accurate than the work just mentioned that it alone will be considered in detail.

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[9] Back-Lande "Zeemaneffekt" p. 140 (1925).

[10] HP. **22,** 81.

[11] H. D. Babcock, Phys Rev. **33,** 268A (1929) and Astrophys. J. **69,** 43 (1929).

[12] W. V. Houston, Phys. Rev. **30,** 608 (1927). For his earlier work see Astrophys. J. **64,** 81 (1926), and Phys. Rev. **29,** 749 (1927) (abstract 7).

[13] F. Paschen, Ann. Physik (4) **50,** 901 (1916).

[14] R. T. Birge, Phys. Rev. **17,** 589 (1921).

[15] R. T. Birge, Nature **111,** 287 (1923).

From Bohr's theory we have

$$R_H = \frac{R_\infty}{1+m/m_H} \quad (4) \qquad R_{He} = \frac{R_\infty}{1+m/m_{He}} \quad (5)$$

where R_∞ = Rydberg constant corresponding to a nucleus of infinite mass,
m = mass of electron,
m_H = mass of *nucleus* of hydrogen atom
m_{He} = mass of *nucleus* of helium atom.

For convenience, m, m_H, and m_{He} are here measured in atomic weight units. From Eqs. (4) and (5) we obtain

$$\frac{m}{m_H} = \frac{(R_{He} - R_H)(m_{He}+m)}{R_H(m_{He} - m_H)} \cdot \qquad (6)$$

Houston[12] writes and uses $(m_{He}+m)$ incorrectly as $(m_{He}-m)$. In order to use directly the atomic weights, H and He, we write $m_H = H - m$, $m_{He} = He - 2m$, thus obtaining

$$m = \frac{(R_{He} - R_H)(He-m)(H-m)}{R_H(He-H-m)} \qquad (7)$$

The value of m, as used on the right side of the equation, need not be known with great accuracy. The equation then gives an *accurate* value of m, where now m *must* be expressed numerically as the "atomic weight" of an electron.

From the definition of the Faraday we have[16]

$$\frac{e}{m} = \frac{F}{m} = \frac{FR_H(He-H-m)}{(R_{He} - R_H)(He-m)(H-m)} \qquad (8)$$

Houston's new experimental results are

$$R_{He} = 109722.403 \pm 0.004 \text{ cm}^{-1}, \qquad R_H = 109677.759 \pm 0.008 \text{ cm}^{-1} \cdot$$

The stated errors are purely least squares probable errors. Houston remarks that he believes the *relative* values of R_{He} and R_H are correct to 0.02, although the *absolute* error in each may be about 0.05,[17] and this seems to the writer a reasonable estimate of error.

The actual value of R_H is obtained from the strong member of the long wave-length component of $H\alpha$, assuming in accordance with the new wave-mechanics that this component consists of one strong and two weak

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[16] In Eq. (8), $e/m = F/m$, the two m's are not numerically the same. e/m is merely a ratio, but if e = electronic charge, m = electronic mass in grams. In F/m, m is necessarily the atomic weight of an electron, as used in Eq. (7).

[17] The absolute values of R_H and R_{He}, depend on the absolute wave-lengths of the various spectral lines involved. These in turn depend on the wave-length of the red Cd line measured in cm. The wave-length of this line at 15°C, A_n, is 6438.4696 I.A., *defining* the International Angstrom unit (I.A.). An I.A. unit is intended to be 10^{-8} cm, and the probable error is of the order of one part in several million.

members.[18] Houston's microphotometric curve of the long wave-length component shows no asymmetry. The calculated correction to be made to the observed center of gravity in order to obtain the strong member is only 0.0056 cm^{-1}. This amounts to 0.04 cm^{-1} in R_H. In a similar way the correction in the case of $H\beta$ corresponds to 0.013 cm^{-1} in R_H. For R_{He} there is no such correction, since the desired lines are measured directly. Houston's values of R_H, resulting from $H\alpha$ and $H\beta$, differ by only 0.021 cm^{-1}. This tends to confirm the applied corrections, and it seems to the writer that these corrections must be essentially right, and that the resulting $(R_{He}-R_H)$ can scarcely contain unknown errors of more than 0.01 or 0.02 cm^{-1} I therefore assume

$$R_{He}-R_H=44.644\pm 0.020 \qquad (9)$$

where the second doubtful figure is carried mainly in order to get a more exact comparison of the various possible results.

Houston used $m=5.4\times 10^{-4}$, $He=4.0001$, $H=1.0077$, $F=96470$ abs. coulombs, and obtained $e/m=(1.7606\pm 0.0010)\times 10^7$ *em* units. Using his constants and the correct formula (Eq. 8) the result is 1.7603. The error in his formula is therefore almost negligible. The values of the necessary auxiliary constants adopted in this paper are

$H=1.00777\pm 0.00002$ $\qquad$ $He=4.0022\pm 0.0004$

$F=96489\pm 7$ abs. coul. $=9648.9\pm 0.7$ abs. *em* units.

m is given by Eq. (8) as

$$m=\frac{F}{e/m}=\frac{9648.9}{1.761\times 10^7}=5.48\times 10^{-4}. \qquad (10)$$

The entire probable error in e/m, due to errors in all factors of Eq. (8), aside from $(R_{He}-R_H)$, is less than 0.01 percent and so is entirely negligible compared to the error in $(R_{He}-R_H)$.[19]

Using Houston's value of R_H, and of $R_{He}-R_H$ (Eq. 9), together with the values of H, He, etc., just quoted, we obtain from Eq. (8) $e/m=(1.7608\pm 0.0008)\times 10^7$ abs. *em* units. This value of e/m thus agrees with that obtained by Babcock. Summarizing the results we find

$e/m=1.769\pm 0.002$ from deflection experiments,
$=1.761\pm 0.002$ from Zeeman effect,
$=1.761\pm 0.001$ from H and He spectra.

The discrepancy between the first result and the last two is four times the probable error of the first, and according to the Gaussian error curve, there is only one chance in 143 of this occurring. Hence the discrepancy seems to be real.

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[18] See W. V. Houston, reference 12.

[19] It may be noted that in Eq. (8), He enters in such a manner that the combined proportional error in $(He-H-m)/(He-m)$ is only one-third the proportional error in He itself.

Now the last two results constitute measurements of e/m for electrons *inside of an atom,* based upon the quantum theory of atomic structure. The first result is the measurement of e/m for electrons *in free space.* The figures thus point to the startling conclusion that the e/m of an electron is less when it is *inside* an atom than when it is *outside.* If this conclusion seems unacceptable, then it would appear that there is some general error in the equations of the quantum theory of atomic structure. The final alternative is that there is some unknown general error in all the deflection experiments. No matter what may be the cause of the discrepancy, the very fact of its existence appears to the writer to be of profound significance. Under the circumstances, it seems to be necessary to assume two different values of e/m, one to be used in all cases involving atomic structure, and the other involving free electrons. These may be called the spectroscopic and the deflection values of e/m. Hence

e/m (spectroscopic) $= (1.761 \pm 0.001) \times 10^7$ abs. *em* units per gram,
e/m (deflection) $= (1.769 \pm 0.002) \times 10^7$ abs. *em* units per gram.

Using the adopted value of c,

$(e/m)c$ (spectroscopic) $= (5.279 \pm 0.003) \times 10^{17}$ abs. *es* units per gram,
$(e/m)c$ (deflection) $= (5.303 \pm 0.006) \times 10^{17}$ abs. *es* units per gram.

Section M

THE PLANCK CONSTANT h.

The Planck constant h has been evaluated in a number of different ways. There is however some difference of opinion as to the relative accuracy of the different results, and some of the results are more or less incompatible. Hence a satisfactory evaluation of this fundamental constant is rather difficult.

The first attempt to obtain a most probable value of h, from the results of all seven methods by which it may be evaluated, was made by the writer[1] in 1919. The value found was $(6.5543 \pm 0.0025) \times 10^{-27}$ erg·sec, the stated error being merely the least squares probable error, deduced by the usual formula from the seven separate values, each weighted according to its probable error. This error has been criticized by Ladenburg[2] as far too small. It is however *not* the final error since, as clearly stated in my article, one must add to it an error somewhat greater than the proportional error in e. This latter quantity occurs with some positive power (unity to two) in every known method for obtaining h. This makes the total probable error more nearly ±0.01. The writer's 1919 evaluation of h has been adopted by the ICT, but the probable error is accidentally given as ±0.001, instead of ±0.01.

In 1920 Ladenburg[2] wrote a detailed article on the evaluation of h, in which several of the writer's conclusions were criticized. His own result in that article was 6.54 ± 0.01. In 1925 Ladenburg wrote another article on this subject, for the HP.[3] He then concludes that $h = 6.547$, which value he rounds off

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[1] R. T. Birge, Phys. Rev. **14,** 361 (1919).

[2] R. Ladenburg, Jahr. Radioakt. und Elektronik **17,** 93 (1920).

[3] HP **23,** 279–306.

to 6.55 ± 0.01. Henning and Jaeger[4] also briefly discuss the most probable value of h, and adopt 6.55, although no reference is made to Ladenburg's article in another volume. Both of Ladenburg's articles give rather full details of the theory and the experimental facts concerning the various methods for determining h.

Let us now consider briefly the present evidence on this subject. In obtaining the average value I shall exclude the Lewis and Adam's theory of ultimate rational units, and will discuss that separately in section O. This leaves six methods for evaluating h.

(a) *Bohr's Formula for the Rydberg Constant.* Bohr's theory of the hydrogen atom leads to the equation

$$R_\infty = \frac{2\pi^2 e^5}{h^3 c^2 e/m} \tag{1}$$

in which R_∞ is the Rydberg constant for infinite mass, expressed as usual in cm^{-1} units, while e, the electronic charge, is as usual in abs. *es* units, and e/m in abs. *em* units. R_∞ is derived[5] from the observed R_H by the equation

$$R_\infty = R_H\left(1+\frac{m}{m_H}\right) = R_H\left(1+\frac{F}{(e/m)(H-m)}\right). \tag{2}$$

From section L we have

$$\begin{aligned} R_\infty &= (109677.759)\left(1+\frac{9648.9}{1.761\times 10^7(1.00722)}\right) \\ &= 109677.759+59.665 = 109737.424 \text{ cm}^{-1}. \end{aligned} \tag{3}$$

The probable error[6] in R_∞ is about 0.06 cm^{-1}. In absolute units we have $R_\infty \cdot c = (3.28988 \pm 0.00004)\times 10^{15}$sec^{-1}. Substituting in Eq. (1) the spectroscopic value of e/m, since we are here dealing with spectroscopic data, and the values found for e, c, and R_∞, we obtain

$$\begin{aligned} h &= \left[\frac{2\pi^2(4.770\times 10^{-10})^5}{109737.42(2.99796\times 10^{10})^2(1.761\times 10^7)}\right]^{1/3} \\ &= 6.54713\times 10^{-27} \text{ erg}\cdot\text{sec}. \end{aligned} \tag{4}$$

With the adopted probable errors of R_∞, e, c, and e/m, the resulting probable error in h is 0.011. It should be noticed that the errors in c, R_∞ and e/m, as affecting h, are negligible in comparison with the error in e. The result of this method is then

$$h = (6.547 \pm 0.011)\times 10^{-27} \text{ erg}\cdot\text{sec}. \tag{5}$$

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[4] HP **2**, 510–511.

[5] Using Eqs. (4) and (8) of section L.

[6] From section L the probable error in the absolute value of R_H is 0.05 cm^{-1} and that in the second term of Eq. (3) is 0.03 cm^{-1}.

It may be noted, in passing, that after adopting a weighted mean value of h, Eq. (1) becomes a method for calculating, indirectly, the value of R_∞. On the other hand, using the directly determined value of R_∞, Eq. (1) becomes a means for calculating e/m. Both these calculations are given in Table b of section N.

(b) *Ionization Potentials.* In 1919 the author had available thirteen values of ionization and resonance potentials, each of which could be used to calculate h. Since then many more such potentials have been obtained. The probable error in each is rather large, because of a lack of rigidly controlled conditions. We now, however, have one really accurate determination of an ionization potential, obtained with electrons of carefully controlled velocity. This is Lawrence' determination[7] of the ionization potential of Hg. His final conclusion is that this value equals 10.40 ± 0.02 int. volts. The value 10.399 ± 0.007 used by the writer[8] in a brief discussion of h and other constants, was a preliminary result,[9] in which the stated error was merely that deduced from internal consistency of the data.

The equation for obtaining h is

$$h\nu = eV \tag{6}$$

in which all quantities are in absolute units. The observed potential (V') is is always in int. volts. The potential in abs. *es* units is then

$$V = pq\ V'\ 10^8/c \tag{7}$$

p and q having been evaluated in section E. The spectral frequency ν (in sec^{-1}) is obtained always from the wave-length λ, in cm. Hence $\nu'(\text{cm}^{-1}) = 1/\lambda$, and $\nu = c/\lambda$. Eq. (6) may then be written

$$\frac{h}{e} = \frac{pqV'10^8}{c^2\nu'} = \frac{pqV'\lambda 10^8}{c^2}\,. \tag{8}$$

It seems to be quite customary to assume that $V_{es} = V'_{\text{volts}}/300$, and to write Eq. (8) as[10]

$$h/e = V'\lambda/300c \tag{9}$$

This is equivalent to assuming[11] $c = 3 \times 10^{10}$ cm·sec^{-1}, and introduces an error of 0.07 percent. An error of this size is by no means negligible, in the case of the work discussed in this section, and the writer should like to emphasize, this fact. Scarcely any one uses $c = 3 \times 10^{10}$ when reducing λ to ν, and thus in the *same* equation (8), it is quite customary to use two *different* values of c.

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[7] E. O. Lawrence, Phys. Rev. **28,** 947 (1926).

[8] R. T. Birge, Science **64,** 180 (1926).

[9] E. O. Lawrence, Phys. Rev. **27,** 809 (1926), abstract 49.

[10] See for instance Ladenburg, HP. **23,** 290. On the other hand, Henning and Jaeger (HP. **2,** 511) give the correct reduction from volts to abs. *es* units.

[11] More accurately, it assumes c' (the ratio of the *es* to *em* units) $= 3 \times 10^{10}$. As noted in section A, the best value of $c'(=2.9979)$ agrees with the slightly more accurate value of c $(=2.99796)$, and one can therefore safely assume $c' = c$ in all calculations.

The "term" of Hg corresponding to the ordinary ionization potential is[12] 84178.5 cm^{-1}. Hence in Eq. (8) we have

$$h=\frac{(1.00046)(10.40)10^8(4.770\times10^{-10})}{(2.99796\times10^{10})^2\,84178.5}=(6.560\pm0.015)\times10^{-27} \tag{10}$$

in which the probable error in V' is 0.2 percent, as already stated, and in e, 0.1 percent. The errors of the other factors are negligibly small.

(c) *X-ray Continuous Spectrum.* This method, like the preceding, uses Eq. (8), λ being measured by means of a calcite crystal, i.e.,

$$\lambda=2d\sin\theta \tag{11}$$

where d is the grating space of calcite, and θ the angle at which the given wave-length shows constructive interference. Substituting Eq. (11) in Eq. (8), we obtain

$$h/e=pq\,2d(V'\sin\theta)10^8/c^2 \tag{12}$$

In 1919 the most accurate work by this method was that of Blake and Duane.[13] Since then Duane, Palmer and Yeh[14] have carried out a still more accurate investigation, and their result was used by the writer[8] in his 1926 article. The resulting value of h, as given by these investigators, is $(6.556\pm0.009)\times10^{-27}$. Another result for which equal accuracy is claimed, is that by Wagner.[15] The full details of this latter investigation have not been published, but Ladenburg[16] gives a complete list of Wagner's experimental results. Wagner's own result, based on $c=3\times10^{10}$, is 6.526 ± 0.01. Ladenburg, using Eq. (9), with $c=2.9985\times10^{10}$, changes this to 6.529 ± 0.01.

Duane, Palmer and Yeh used a definitely known potential (in int. volts) and measured the angle θ of their ionization chamber at which the ionization suddenly started (or stopped). This gives the critical ionization frequency (or wave-length) corresponding to the given voltage. On the other hand, Wagner used a known wave-length (or rather a series of known wave-lengths) and varied the voltage for a given wave-length, until ionization suddenly began (or ceased). As shown by Eq. (12), both methods involve a knowledge of the calcite grating space d. This important constant has been discussed in section K. In that section the absolute wave-lengths of rays were used to evaluate d', the true grating space, and d' was then used with other known constants to evaluate the electronic charge e. In the present section we use the finally adopted value of e, $(4.770\pm0.005)\times10^{-10}$ abs. *es* units, with these same constants, to evaluate d'. We therefore write Eq. (10) of section K as

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[12] A. Fowler, Report on Line Series, p. 149.

[13] F. C. Blake and W. Duane, Phys. Rev. **10,** 624 (1917).

[14] W. Duane, H. H. Palmer, and Chi-Sun Yeh, Proc. Nat. Acad. Sci. **7,** 237 (1921).

[15] E. Wagner, Physik. Zeits. **21,** 621 (1920).

[16] HP **23,** 296.

$$d_{20}'=\left[\frac{4.770\times10^{-10}}{1.7176\times10^{13}}\right]^{1/3}=(3.0283\pm0.0010)\times10^{-8}\ \text{cm}. \tag{13}$$

It should be noted that this value of d'_{20} *includes* the result of the x-ray work, since the value of e used in Eq. (13) is the weighted average obtained from both oil-drop and x-ray work. Thus we might have used $e=4.768$ to get a value of d' based on oil-drop work. A second value of d' might then be obtained from absolute x-ray measurements. The weighted average of these two values of d' would however be just the value given in Eq. (13), provided we use the data and probable errors indicated in section K.

Using Eq. (5) of section K, we obtain for the effective grating space of calcite at 20°C, for the first order spectrum,

$$d_{20}=(3.0279\pm0.0010)\times10^{-8}\text{cm}. \tag{14}$$

This value is now to be substituted in Eq. (12), together with the direct experimental value of $V'\sin\theta$. For the latter quantity Duane, Palmer and Yeh[14] found 2039.9 ± 1 int. volts, the work being performed at a mean temperature of about 20°C. Using the values of p, q, e, and c already adopted, we have

$$\begin{aligned}h&=\frac{(1.00046)2(3.0279\times10^{-8})(2039.9)(4.770\times10^{-10})10^{8}}{(2.99796\times10^{10})^{2}}\\&=6.5591\times10^{-27}\ \text{erg}\cdot\text{sec}.\end{aligned}\tag{15}$$

with the assumed probable errors in the various factors, the final result is

$$h=(6.559\pm0.008)\times10^{-27}\ \text{erg}\cdot\text{sec}. \tag{16}$$

This error is due almost entirely to that in e. If the only method for determining d were by means of the density of calcite combined with the oil-drop value of e, the error in $e^{4/3}$ should be used in connection with h, the error in d due to $e^{1/3}$ being withdrawn. This has been pointed out by the writer.[1] The error in h then becomes ±0.009. With d determined in the two different ways, the error in h lies somewhere between 0.008 and 0.009.

The result found by Wagner[15] is now to be similarly revised. Using in Eq. (12) 2.99796 for c, in place of 3.00, $pq=1.00046$ in place of unity, and $e=4.770$ in place of 4.774, we obtain 6.532 ± 0.010, in place of 6.526 ± 0.010. It is difficult to judge what revision is required in the values of λ used by Wagner, but any such change is probably rather small. We thus have, as the two best values of h, from x-ray data, 6.559 ± 0.008 (or 0.009) and 6.532 ± 0.010. The writer does not feel competent to discuss the probable causes for this discrepancy, and these have not been discussed by others. The work of Wagner has not yet been published in sufficient detail to allow an examination of possible sources of error. For this reason in adopting a weighted average of the two results, I shall give only one-half as much weight to Wagner. Since the two results differ by much more than the probable error of either, I use the regular least squares probable error of the resulting weighted average. Hence, from x-ray data,[16a]

$$h=(6.550\pm0.009)\times10^{-27}\text{ erg}\cdot\text{sec.}\tag{17}$$

(d) *Photo-electric Effect.* The most accurate evaluation of h, from photoelectric work, is that by Lukirsky and Prilezaev.[17] These investigators use a somewhat different experimental technique from that employed earlier by Millikan,[18] and obtain a simple empirical relation for the ionization current as a function of voltage. By means of this, the actual curve may be transposed into a linear graph, making the extrapolation to zero current more certain. They also carry the readings very close to this zero point.

The equation for evaluating h is the same as that used in the previous method, except that now the energy (P) to pull an electron out of the metal is no longer negligible compared to $h\nu$. Hence we write

$$Ve=h\nu-P.\tag{18}$$

In order to eliminate P, it is customary to use light of varying frequencies, measuring for each the critical voltage V at which ionization starts. V is then plotted against ν, and the resulting curves should be linear, with a slope

$$dV/d\nu=h/e.\tag{19}$$

With V measured as V' int. volts, and ν as ν' cm^{-1}, we have

$$\frac{dV}{d\nu}=\frac{pa10^8dV'}{c^2d\nu'}=\frac{h}{e}\cdot\tag{20}$$

Lukirsky and Prilezaev obtain a considerable number of such curves, using the metals Al, Zn, Sn, Ni, Cd, Cu, and Pt. Six curves, three with Zn, two with Al, and one with Ni, gave the most reliable results, and the six resulting values of h are tabulated. Unfortunately these investigators give no detailed data, and give no indication of the actual equation used for evaluating h. They show a small-sized plot of certain sample graphs. Their final value of h is 6.543×10^{-27} erg·sec, the individual results being 6.539, 6.542, 6.540, 6.556,6.536, and 6.546. These give a probable error of only 0.035 percent, based on mutual consistency. Because of the lack of information as to the source of these values, and especially because of the surprisingly small value of the resulting error, it seems safer to assume about 0.1 percent as the true probable error of the result, exclusive of the error in the auxiliary factors.

[16a] H. Feder, Ann. d. Physik (5) **1**, 494 (1929), working under E. Wagner, has just obtained by this same method, $h=6.547\pm0.003$, exclusive of the error in e. He explains Wagner's low result as directly due to the large size of the slit width used. His own value, 6.547, is identical with the value of h finally adopted in this section, and is lowered only one part in 10^4 by the use of my own values of all auxiliary constants.

[17] P. Lukirsky and S. Prilezaev, Zeits. f. Physik **49**, 236 (1928).

[18] R. A. Millikan, Phys. Rev. **7**, 355 (1916).

As just noted, there is no statement as to what values were used in Eq. (20). It seems quite certain that they used $e=4.774$, in place of our 4.770. Because of this, their final value of h should be lowered 0.0055×10^{-27}. It is very probable that they used $pq=1$, instead of our 1.00046. Their h should therefore be raised 0.0030×10^{-27}. According to the value of c used in each factor of c^2 in Eq. (20), there are other possible increases in h. My final conclusion is that the *probable* correction to their value of h lies between -0.0025 and $+0.0020$, and the *possible* correction between -0.0025 and $+0.0064$, all in 10^{-27} erg·sec units. Under the circumstances I adopt their original value, but add a probable error of 0.003×10^{-27} ($=0.05$ percent) to cover this uncertainty. The only other appreciable error is 0.1 percent due to e. The final probable error $(0.1^2+0.05^2+0.1^2)^{1/2}$ is then 0.15 percent. Hence

$$h=(6.543\pm 0.010)\times 10^{-27}\ \text{erg}\cdot\text{sec}. \tag{21}$$

These investigators estimate their final error in h as 0.1 to 0.2 percent, in agreement with my own assumption.

(e) *Wien's Displacement Law and the Planck Equation.* h may be determined from radiation constants in two different ways.[19] The first is by means of the constant c_2, occuring in the Wien displacement law. This law is

$$\lambda_{\max}T=c_2/\beta=A. \tag{22}$$

where $\beta=4.9651$ (root of the equation $e^{-\beta}+\beta/5-1=0$). The radiation constant c_2 occurs also in Planck's black body radiation law in the form

$$c_2=hc/k. \tag{23}$$

where $c=$velocity of light, k(Boltzmann constant) $=R_0/N_0$, R_0 (gas constant per mole) $=\nu_nA_n/T_0$, and N_0 (Avogadro's number) $=Fc/e$. Substituting the values of ν_n, A_n, T_0, F and e previously adopted,

$$N_0=(6.0644\pm 0.0061)\times 10^{23}\ \text{mole}^{-1}. \tag{24}$$

$$R_0=(8.3136\pm 0.0010)\times 10^{7}\ \text{erg}\cdot\text{deg}^{-1}\cdot\text{mole}^{-1}. \tag{25}$$

$$k\ =(1.3709\pm 0.0014)\times 10^{-16}\ \text{erg}\cdot\text{deg}^{-1}. \tag{26}$$

In 1919 the writer asked Coblentz, an expert in this field, what in his opinion was then the best value of c_2. Dr. Coblentz recommended 1.433 cm·deg. and this value was adopted. In a long critical review of the radiation constants, published three years later, Coblentz[20] gives 1.432 as the most probable value. No probable error is given but the four results, obtained by four different investigators, for which 1.432 is the weighted average, are 1.436, 1.430, 1.430, and 1.4318, the last being the final result of

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[19] A good description of methods for measuring the radiation constants is given by W. W. Coblentz in Glazebrook's "Dictionary of Applied Physics" IV, 541–565.

[20] W. W. Coblentz, Bur. Standards, Bull. **17,** 7 (1922). Much of the contents of this article is given also by Coblentz in J. Optical Soc. Am. **5,** 131 (1921). For a still later brief report see Coblentz, ibid **8,** 11 (1924).

Coblentz' own work in this field. Ladenburg,[21] in his HP article on h, gives 1.432 ± 0.006 as the most probable value. The separate results which he used in obtaining this average are 1.425 to 1.441, 1.4295 ± 0.007, 1.435, 1.4318, and 1.430. The chief error in all these results arises from the various corrections which must be applied to the directly observed values. Thus Coblentz' original value[22] of 1.4456 has become 1.4318 in his latest article,[20] as a result of various additional corrections. Coblentz originally considered his accuracy quite high, but in his later articles no definite estimate of the probable error is given. The writer can however see no justification for Ladenburg's assumption of 0.006 as the probable error of the weighted mean of all observations now available. Even if one grants that each separate investigation may be in error by this amount, the average surely has a smaller probable error, unless there are large, as yet unknown and unsuspected sources of *constant* error. I believe 0.003 is a much more reasonable estimate of error. Both Coblentz and Ladenburg agree on the absolute value. Hence I adopt

$$c_2 = 1.432 \pm 0.003 \text{ cm} \cdot \text{deg}. \tag{27}$$

Substituting in Eq. (23), one obtains

$$h = (6.548 \pm 0.015) \times 10^{-27} \text{ erg} \cdot \text{sec}. \tag{28}$$

It may be noted, in passing, that the radiation constant c_2 occurs in the general Boltzmann factor $e^{-\epsilon/kT}$, (ϵ = energy, T = absolute temperature) in the form $e^{-c_2\nu/T} = e^{-c_2/\lambda T}$, where ν in cm^{-1}, or λ in cm, is the quantum equivalent of ϵ ergs.

(f) *The Stefan-Boltzmann Law and the Planck Equation.* The second method for determining h by means of radiation constants is through the Stefan-Boltzmann law

$$E = \sigma T^4 = acT^4/4 \tag{29}$$

h is connected with σ, using Planck's law, by the relation

$$h = (2\pi^5 k^4/15c^2\sigma)^{1/3}, \tag{30}$$

k and c having already been defined. Just as in the case of c_2, there seems to be a difference of opinion among experts in this field concerning the accuracy with which σ may be measured. The best value, in 1919, according to the published literature, was that obtained by Coblentz,[23] namely $(5.722 \pm 0.012) \times 10^{-5}$ erg $\cdot$ cm^{-2} $\cdot$ deg^{-4} $\cdot$ sec^{-1}. In his more recent critical discussion of this question, Coblentz[20] gives a table of all available data, and concludes that the most probable value lies between 5.72 and 5.73.

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[21] HP **23,** 303.

[22] W. W. Coblentz, Bur. Standards, Bull **10,** 1–77 (1914).

[23] W. W. Coblentz, Proc. Nat. Acad. Sci. **3,** 504 (1917). Full details were published later in Bur. Standards, Bull. **15,** 529 (1920). His earlier work is given in Bur. Standards, Bull. **12,** 503 (1916).

Since this 1922 article by Coblentz, there have been two new determinations of σ, one by Hoffman,[24] using the method of Westphal,[25] giving $\sigma = 5.764 \pm 0.052$, and the other by Kussman,[26] using the modified Angstrom pyrheliometer. This latter method was used also by Coblentz[23] giving 5.722 as stated, by Gerlach[27] giving 5.80, and by Kahanowicz[28] giving 5.69 to 5.73 as corrected by Coblentz.[20] Kussman obtained $\sigma = 5.795 \pm$ one percent. Ladenburg,[29] in his HP article on h, quotes merely the four results by Gerlach,[27] Hoffman,[24] Coblentz[23] and Kussman.[26] He adopts the *unweighted* average of these four results, namely 5.77, and ± 0.06 as the probable error. Here again Ladenburg assumes for the probable error of an average, a value which he himself estimates to be the probable error of each separate determination. He agrees with Gerlach[27] in thinking that Coblentz' true error is more nearly 0.06 than 0.012. It is interesting to note that the actual experimental results of Kussman[26] and Coblentz[23] are in almost perfect agreement, as shown by a plot made by Kussman. The discrepancy in their final results is due entirely to the difference in the correction applied by these two investigators for the lack of complete absorption of the receiver. Michel and Kussman[30] claim to prove that the correction Coblentz applied is too small, thus accounting for his lower value of σ.

In deciding upon the most probable value of σ there is a certain psychological factor involved. We know from the preceding discussion what is at least approximately the most probable value of h. Now the values of σ found by Kussman and by Hoffman, as well as Gerlach's earlier value of 5.80, correspond to impossibly low values of h (6.528 to 6.514), while Coblentz' own result gives an h in essential agreement with that obtained by more accurate methods.[31] This fact tends, in itself, to indicate the correctness of Coblentz' correction for incomplete absorption of the receiver, as opposed to the correction used by Kussman.

It appears to the writer that Coblentz' estimated error for his own work (5.722 ± 0.012) is too small, but that his final average of the work of all investigators up to 1922 (5.72 to 5.73) should be more trustworthy than any single determination. I shall therefore choose 5.725 as this average, and 0.02 for its probable error. This result is then to be averaged with the more recent work by Hoffman and by Kussman, using the usual rule for weighting. The final weighted average is

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[24] K. Hoffman, Zeits. f. Physik **14,** 301 (1923).

[25] W. H. Westphal, Verhandl. deut. physik. Ges. **14,** 987 (1912) and **15,** 897 (1913).

[26] A. Kussman, Zeits. f. Physik **25,** 58 (1924).

[27] W. Gerlach, Ann. Physik **50,** 259 (1916) and Zeits. f. Physik **2,** 76 (1920).

[28] M. Kahanowicz, Nuovo Cimento (6) **13,** 142 (1917).

[29] HP **23,** 304.

[30] G. Michel and A. Kussman, Zeits. f. Physik **18,** 263 (1923).

[31] L. Strum, Zeits. f. Physik **51,** 287 (1928), suggests that the difference between Ladenburg's assumed experimental $\sigma = 5.77$, and the value (5.72) given by the Planck equation with the value of h assumed as known, is real and is due to the fact that the Planck equation is not strictly correct.

$$\sigma=(5.735\pm0.011)\times10^{-5}\ \mathrm{erg\cdot cm^{-2}\cdot deg^{-4}\cdot sec^{-1}}. \tag{31}$$

Hence, in Eq. (29), $a=4\sigma/c=(7.652\pm0.015)\times10^{-15}\ \mathrm{erg\cdot cm^{-3}\cdot deg^{-4}}$

Since this weighted average value of σ was calculated and adopted, there has appeared a further determination of this quantity, by Hoare.[32] He used a Callendar radio balance, and the advantage of the method is that both the source and the receiver are essentially "black bodies," so that no corrections are necessary. This is very important for, as already noted, the chief difference of opinion regarding the value of σ has hinged on the correction to be applied for incomplete absorption. Hoare obtains $\sigma=5.735$, agreeing exactly with the value already adopted. He gives no estimate of probable error, but lists 38 separate results, showing an average deviation from the average of only 0.016. The inclusion of this new result obviously leaves the average value unchanged, and I shall leave the probable error of Eq. (31) unchanged. Objection might have been made to this adopted error as too small, but such an objection can hardly hold in the face of Hoare's work, which seems to be the most accurate now available. This new work also speaks against Strum's assumption of an inadequacy of Planck's formula.

Using Eq. (31) in Eq (30) we obtain

$$h=(6.539\pm0.010)\times10^{-27}\ \mathrm{erg\cdot sec}. \tag{32}$$

(g) *Summary* We have now obtained six determinations of h, as listed in the table.

Table of values of h

Method	h	Power of e involved
Rydberg constant	6.547 ± 0.011	5/3
Ionization potentials	6.560 ± 0.015	3/3
X-rays	6.550 ± 0.009	4/3
Photo-electric	6.543 ± 0.010	3/3
c_2	6.548 ± 0.015	3/3
σ	6.539 ± 0.010	4/3

The weighted average of these six results is $h=6.5466\times10^{-27}$ erg·sec The least squares probable error of the average, determined by the usual formula, is ±0.0017, an error definitely smaller than that deduced in 1919, indicating an increased consistency of the data. It may seem surprising that the probable error of each of the six results is so nearly the same. This is due to the fact that in each case, the error listed is due chiefly to the error in e. Each result involves e to the power given in the last column of the table. The *weighted* average power is 1.26. Hence the weighted average value of h can be assumed to involve e to the power 1.26. The error in h due to this is $1.26\times10^{-3}\times6.55=0.00825$. Combining this with the above least squares error in h, we have a total final error of 0.00842, essentially no greater than the error due to e alone. Hence our final result is

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[32] F. E. Hoare, Phil. Mag. **6**, 828 (1928).

$$h = (6.547 \pm 0.008) \times 10^{-27} \text{ erg} \cdot \text{sec.} \tag{33}$$

It is of interest to note that this value of h is identical with Ladenburg's most recent estimate.[3] This identity is however spurious, since Ladenburg assumes $e = 4.774 \times 10^{-10}$. If this older value of e had been used in the present work, we should have obtained $h = 6.5535$, in practically exact agreement with the writer's 1919 value (6.5543).

Another potentially accurate method for evaluating h is given by the Compton shift of x-ray lines. The theoretical equation for this is

$$\Delta\lambda = (h/mc)(1 - \cos\phi) \tag{34}$$

where m is the mass of an electron, as deduced from the values of e and e/m. Since, however, h varies in value with e, Eq. (34) can better be used to evaluate e/m, as mentioned incidentally in section L. We can in fact write

$$\Delta\lambda = (h/e)(e/m)(1 - \cos\phi) \tag{35}$$

in which e as usual is in *es* units, and e/m in *em* units. Then

$$\frac{e}{m} = \frac{\Delta\lambda}{(h/e)(1 - \cos\phi)}$$

The most accurate work on this subject has been done by Sharp,[33] who obtains $\Delta\lambda = (0.04825 \pm 0.00017) \times 10^{-8}$ cm, for $(1 - \cos\phi) = (1.984 \pm 0.001)$. With the adopted values of h and e, we have[34] $h/e = (1.3725 \pm 0.0005) \times 10^{-17}$ erg·sec·es^{-1}. Substituting in Eq. (36) one finds $e/m = (1.772 \pm 0.006) \times 10^7$ abs. *em* units, the final error being due almost entirely to the error in $\Delta\lambda$. It seems to the writer possibly significant that this value agrees better with the deflection than with the spectroscopic value of e/m, for the theory used in the derivation of Eq. (35) is essentially the collision theory of classical dynamics for free electrons. Greater accuracy is however needed before any real conclusion can be drawn.[35]

Section N

TABLES

The various important constants and ratios which have been evaluated in sections A to M are collected in Table a of this section. In the process of evaluating these constants, it has been necessary to calculate certain auxiliary constants, and also to use certain conventional quantities, such as g_{45} and g_n. All such quantities are listed in Table b, in the order in which they appear in the preceding sections. The section in which each quantity

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[33] H. M. Sharp, Phys. Rev. **26,** 691 (1925).

[34] See footnote 2 of section N.

[35] B. Davis and D. P. Mitchell, Phys. Rev. **32,** 331 (1928), have recently found, for the "undisplaced" line, a fine structure corresponding directly to the *Raman effect* observed with ordinary light. The observed displacement of the lines is considerably less than the Compton shift, and so the accuracy of Sharp's work is presumably unaffected by this new phenomenon

is discussed is indicated on the left side of the table. For convenience the indirect calculation of R_∞, $R_\infty \cdot c$, e/m, c_2, σ and a are also included in this table. All of these indirect calculations depend on the finally adopted value of h, and so could not be obtained when the more directly observed value was under consideration. It should be noted that in the case of R_∞, $R_\infty \cdot c$ and e/m, the direct evaluation is the more accurate, while in the case of c_2, σ and a the indirect evaluation is the more accurate.

Table a. Principal Constants and Ratios

Section		
A	Velocity of light	$c = (2.99796 \pm 0.00004) \times 10^{10}$ cm·sec^{-1}
B	Gravitation constant	$G = (6.664 \pm 0.002) \times 10^{-8}$ dyne·cm^2·g^{-2}
C	Liter	$l = 1000.027 \pm 0.001$ cm^3
D	Volume of perfect gas (0°C, A_n)	$v_n = (22.4141 \pm 0.0008) \times 10^3$ cm^3·mole^{-1} $R_n = 22.4135 \pm 0.0008$ liter·mole^{-1}
D	Volume of perfect gas (0°C, A_{45})	$R = 22.4146 \pm 0.0008$ liter·mole^{-1}
E	International ohm ($= p$ abs-ohm)	$p = 1.00051 \pm 0.00002$
E	International ampere ($= q$ abs-amp)	$q = 0.99995 \pm 0.00005$
F	Atomic weights	
	O (oxygen)	$= 16.0000$ (definition)
	H (hydrogen)	$= 1.00777 \pm 0.00002$
	He (helium)	$= 4.0022 \pm 0.0004$
	N (nitrogen)	$= 14.0083 \pm 0.0008$
	Ag (silver)	$= 107.880 \pm 0.001$
	I (iodine)	$= 126.932 \pm 0.002$
	C (carbon)	$= 12.003 \pm 0.001$
	Ca (calcium)	$= 40.075 \pm 0.005$
G	Normal atmosphere	$A_n = (1.013249 \pm 0.000003) \times 10^6$ dyne·cm^{-2}
G	45° atmosphere	$A_{45} = (1.013199 \pm 0.000003) \times 10^6$ dyne·cm^{-2}
H	Ice point (absolute scale)	$T_0 = 273.18 \pm 0.03$°K
I	Mechanical equivalent of heat[1]	$J_{15} = 4.1852 \pm 0.0006$ abs-joule·cal$_{15}^{-1}$ $J_{20} = 4.1813 \pm 0.0006$ abs-joule·cal$_{20}^{-1}$
I	Electrical equivalent of heat[1]	$J'_{15} = 4.1835 \pm 0.0007$ int-joule·cal$_{15}^{-1}$ $J'_{20} = 4.1796 \pm 0.0007$ int-joule·cal$_{20}^{-1}$
J	Faraday constant[1]	$F = 96494 \pm 5$ int-coul·g-equiv^{-1} $= 96489 \pm 7$ abs-coul·g-equiv^{-1} $= 9648.9 \pm 0.7$ abs-*em*-unit·g-equiv^{-1} $Fc = (2.8927_0 \pm 0.0002) \times 10^{14}$ abs-*es*-unit·g-equiv^{-1}
K	Electronic charge[1]	$e = (4.770 \pm 0.005) \times 10^{-10}$ abs-*es*-units $e/c = (1.5910_8 \pm 0.0016) \times 10^{-20}$ abs-*em*-units
L	Specific electronic charge (spectroscopic)[1]	$e/m = (1.761 \pm 0.001) \times 10^7$ abs-*em*-unit·g^{-1} $(e/m)c = (5.279_{41} \pm 0.003) \times 10^{17}$ abs-*es*-unit·g^{-1}
L	Specific electronic charge (deflection)[1]	$e/m = (1.769 \pm 0.002) \times 10^7$ abs-*em*-unit·g^{-1} $(e/m)c = (5.303_{39} \pm 0.006) \times 10^{17}$ abs-*es*-unit·g^{-1}
M	Planck constant	$h = (6.547 \pm 0.008) \times 10^{-27}$ erg·sec

In addition to certain constants listed in Table b, there are many others which are functions of constants given in Tables a and b. A number of these derived constants are collected in Table c. This table could be extended almost indefinitely, but an attempt has been made to include the more important or more frequently used constants. The process for obtaining the correct probable error for many of the constants of Tables b and c is sometimes involved. The general principle underlying it is stated and illustrated in footnote 2 of this section. The possible theoretical significance of certain relations appearing in these tables is considered in section O.

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[1] In the expressions for all derived quantities in these tables, e is to be understood as in abs-*es*-units, (e/m) as in abs-*em*-units, R_H, R_{He} and R_∞ as in cm^{-1} units, and J as in joules per cal. F is in abs.-*em*-units except as noted in one constant of Table c.

Table b. Additional Quantities Evaluated or Used in Connection with Table a.

Section		
A	Ratio of *es* to *em* units (direct)	$c' = (2.9971 \pm 0.0001) \times 10^{10}$ cm$^{1/2}\cdot$sec$^{-1/2}\cdot$int $\Omega^{-1/2}$ $= (2.9979 \pm 0.0001) \times 10^{10}$ cm$\cdot$sec^{-1}
A	Ratio of *es* to *em* units (indirect)	$c' = c = (2.99796 \pm 0.00004) \times 10^{10}$ cm$\cdot$sec^{-1}
B	Acceleration of gravity (45°)	$g_{45} = 980.616$ cm$\cdot$sec^{-2}
B	Mean density of earth	$\delta = 5.522 \pm 0.002$ g$\cdot$cm^{-3}
C	Maximum density of water	$\delta_m = 0.999973 \pm 0.000001$ g$\cdot$cm^{-3}
D	Acceleration of gravity (normal)	$g_n = 980.665$ cm$\cdot$sec^{-2}
D	Density of oxygen gas (0°C, A_{45})	$L = 1.428965 \pm 0.000030$ g$\cdot$liter^{-1}
D	Factor converting oxygen (0°C, A_{45}) to ideal gas	$1 - \alpha = 1.000927 \pm 0.000030$
E	International coulomb ($= q$ abs-coul)	$q = 0.99995 \pm 0.00005$
E	International henry ($= p$ abs-henry)	$p = 1.00051 \pm 0.00002$
E	International volt ($= pq$ abs-volt)	$pq = 1.00046 \pm 0.00005$
E	International joule ($= pq^2$ abs-joule)	$pq^2 = 1.00041 \pm 0.00010$
E	International gauss ($= q$ abs-gauss)	
F	Density of nitrogen (0°C, A_{45})	$L = 1.25046 \pm 0.000045$ g$\cdot$liter^{-1}
F	Factor converting nitrogen (0°C, A_{45}) to ideal gas	$1 - \alpha = 1.00043 \pm 0.00002$
G	Specific gravity of Hg (0°C, A_n) referred to air-free water at maximum density	$\rho_n = 13.59546 \pm 0.00003$
G	Density of Hg (0°C, A_n)	$D_n = 13.59509 \pm 0.00003$ g$\cdot$cm^{-3}
J	Electrochemical equivalent of silver	$E_{\text{Ag}} = (1.11800 \pm 0.00005) \times 10^{-3}$ g$\cdot$int-coul^{-1} $= (1.11805 \pm 0.00007) \times 10^{-3}$ g$\cdot$abs-coul^{-1}
K	Arbitrary calcite grating space (18°C)	$d_{18}'' = 3.02904 \times 10^{-8}$ cm
K	Density of calcite (20°C)	$\rho = 2.7102 \pm 0.0004$ g$\cdot$cm^{-3}
K	Structural constant of calcite (20°C)	$\phi(\beta) = 1.09630 \pm 0.00007$
L	Rydberg constant for hydrogen[1]	$R_{\text{H}} = 109677.759 \pm 0.05$ cm^{-1}
L	Rydberg constant for ionized helium[1]	$R_{\text{He}} = 109722.403 \pm 0.05$ cm^{-1}
L	Wave-length of red Cd line (15°C, A_n)	$\lambda(\text{Cd}) = 6438.4696$ I.A. (definition of I.A. unit)
M	Rydberg constant for infinite mass[1]	$R_\infty = 109737.42 \pm 0.06$ cm^{-1} $R_\infty c = (3.28988 \pm 0.00004) \times 10^{15}$ sec^{-1}
	R_∞(indirect)$^{1,2} = 2\pi^2 e^5/h^3c^2(e/m)_{sp}$	$= 10974_4 \pm 170$ cm^{-1}
	$R_\infty c$ (indirect)^{1,2}	$= (3.290_{03} \pm 0.005) \times 10^{15}$ sec^{-1}
M	e/m (spectroscopic, indirect)^{1,2} $e/m = 2\pi^2 e^5/h^3c^2R_\infty$	$= (1.7611_2 \pm 0.0025) \times 10^7$ abs-*em*-unit$\cdot$g^{-1}
M	True grating space of calcite (20°C)	$d'_{20} = (3.0283 \pm 0.0010) \times 10^{-8}$ cm

[2] The error is derived by the process used in section M, in getting the final probable error in h itself. This process is used in these tables for every constant which involves e in more than one factor. It may be illustrated as follows. Let $y(=10^{-3})$ denote the proportional error in e, and $z(=0.0017/6.547=0.26\times10^{-3})$ the proportional error in h excluding the error in e. Since h varies as $e^{1.26}$, the final proportional error in h is $[(1.26y)^2+z^2]^{1/2}=1.287\times10^{-3}$, corresponding to the absolute error 0.00842×10^{-27} erg$\cdot$sec, already given in section M. Similarly, since h/e varies as $e^{0.26}$, the proportional error in h/e is given by $[(0.26y)^2+z^2]^{1/2}=0.368\times10^{-3}$. As a third example consider e^5/h^3, as it occurs in the indirect evaluation of R_∞ or e/m. This ratio varies as $e^{1.22}$. The proportional error in e^5/h^3 is then $[(1.22y)^2+(3z)^2]^{1/2}=1.45\times10^{-3}$. A fourth example is the indirect calculation of $c_2(=hc/k)$. Since k varies directly as e, h/k varies as $e^{0.26}$. Let u be the proportional error in k, exclusive of the error due to e, $(u=1.27\times10^{-4})$. Then the error in h/k is $[(0.26y)^2+z^2+u^2]^{1/2}=3.89\times10^{-4}$, the error in c being negligible. As a final example consider the indirect calculation of σ, in Table b. The error here is $[(0.22y)^2+(4u)^2+(3z)^2]^{1/2}=9.6\times10^{-4}$ In the case of some of the derived constants, notably the chemical constants S_0 and i_0 of Table c, the calculation of the error is much more involved. This matter is mentioned again in Section O.

M Effective grating space of calcite (20°C).................... $d_{20}=(3.0279\pm0.0010)\times10^{-8}$ cm
M Avogadro's number[1]...... $N_0=Fc/e=(6.064_{36}\pm0.006)\times10^{23}$ mole^{-1}
M Gas constant per mole.. $R_0=\nu_n A_n/T_0=(8.3136_0\pm0.0010)\times10^7$ erg·deg^{-1}·mole^{-1}
$R_0'=R_0/(J_{15}\times10^7)=1.9864_3\pm0.0004$ cal$_{15}$·deg^{-1}·mole^{-1}
$R_0''=R_0/(A_n\times1000.027)=(8.2046_4\pm0.0009)\times10^{-2}$ liter·atmos·deg^{-1}·mole^{-1}
$R_0'''=R_0/A_n=\nu_n/T_0=82.048_8\pm0.009$ cm^3·atmos·deg^{-1}·mole^{-1}
M Boltzmann constant...... $k=R_0/N_0=(1.3708_9\pm0.0014)\times10^{-16}$ erg·deg^{-1}
M Second radiation constant (experimental)................. $c_2=1.432\pm0.003$ cm·deg
M Second radiation constant (indirect)[2].............. $c_2=hc/k=1.4317_4\pm0.0006$ cm·deg
M Stefan-Boltzmann constant (experimental)................. $\sigma=(5.735\pm0.011)\times10^{-5}$ erg·cm^{-2}·deg^{-4}·sec^{-1}
M Stefan-Boltzmann constant (indirect)[2]........ $\sigma=2\pi^5k^4/15c^2h^3=(5.713_9\pm0.006)\times10^{-5}$ erg·cm^{-2}·deg^{-4}·sec^{-1}
M Radiation density constant (experimental)........... $a=4\sigma/c=(7.651_8\pm0.015)\times10^{-15}$ erg·cm^{-3}·deg^{-4}
M Radiation density constant (indirect)[2]........ $a=8\pi^5k^4/15c^3h^3=(7.623_7\pm0.007)\times10^{-15}$ erg·cm^{-3}·deg^{-4}

As in the case of the basic constants listed on page 17 of Vol. 1 of the ICT, the constants of the present Table a are, for purposes of computation, to be taken as exactly correct. That is, all additional digits in each constant are to be assumed as zero. The real probable error in each constant is of course that indicated in the table, and each such constant has an accepted value carried only to the number of significant figures required by the adopted probable error. On the other hand, the various *derived* constants of Table c (and the occasional derived constant appearing in Tables a and b) are given with one and often two more digits than is required by the probable error. Such additional digits are printed below the line, and have been added in order that calculations of derived constants, when made in different ways (often in terms of other derived constants) shall not introduce any appreciable error. The ICT accomplishes the same result by printing the logarithm, to seven digits, of each constant. This does not help in calculations of derived constants, made with a calculating machine. If such a machine is not available, the writer has found Crelle's multiplication tables more convenient than logarithms.

A brief explanation of certain points of nomenclature seems desirable.

(a) The nomenclature adopted in this paper is intended to conform with current practise. On the other hand, an attempt has been made to avoid duplication of the same symbol, especially in the same general field. Some duplication seems unavoidable, examples of this being the use of H for atomic weight of hydrogen and for the strength of a magnetic field, σ for the Stefan-Boltzmann radiation constant and also in the expression for the rotational specific heat constant in Table c.

(b) In certain cases the writer has arbitrarily chosen a symbol for a certain constant (such as $\nu_0=e\cdot10^8/hc^2$, in Table c), when this constant is important and is needed in other relations. Thus the ionization potential of hydrogen equals R_H/ν_0, in Table c.

(c) *e/m* always indicates merely the *ratio* of charge to mass for an electron, in *em* units, this being the simplest and most commonly used

Table c. Miscellaneous Derived Constants

$c^2=(8.98776\pm0.00024)\times10^{20}$ cm$^2\cdot$sec^{-2}	$h/c=(2.183_{81}\pm0.003)\times10^{-37}$ g$\cdot$cm
$c^3=(26.9449\pm0.0011)\times10^{30}$ cm$^3\cdot$sec^{-3}	$e^2=(22.752_9\pm0.045)\times10^{-20}$ *es*-units^2
$c^4=(80.7798\pm0.0043)\times10^{40}$ cm$^4\cdot$sec^{-4}	$e^3=(108.53_{13}\pm0.33)\times10^{-30}$ *es*-units^3
$h^2=(42.86_{42}\pm0.10)\times10^{-54}$ erg$^2\cdot$sec^2	$e^4=(517.6_{94}\pm2.1)\times10^{-40}$ *es*-units^4
$h^3=(280.6_{25}\pm1.0)\times10^{-81}$ erg$^3\cdot$sec^3	$e^5=(2469._{40}\pm12)\times10^{-50}$ *es*-units^5

Mass of electron (spectroscopic)	$m_0=\dfrac{e}{c(e/m)_{sp}}=(9.035_{10}\pm0.010)\times10^{-28}$ g
Mass of electron (deflection)	$m_0=\dfrac{e}{c(e/m)_{defl}}=(8.994_{25}\pm0.014)\times10^{-28}$ g
Atomic weight of electron (spectroscopic)	$m=F/(e/m)_{sp}=(5.479_{22}\pm0.003)\times10^{-4}$
Atomic weight of electron (deflection)	$m=F/(e/m)_{defl}=(5.454_{44}\pm0.006)\times10^{-4}$
Mass of atom of unit atomic weight	$M_0=1/N_0=(1.6489_8\pm0.0016)\times10^{-24}$ g
Mass of hydrogen atom	$M_{\mathrm{H}}=\mathrm{H}/N_0=(1.6617_9\pm0.0017)\times10^{-24}$ g
Atoms per gram of hydrogen	$1/M_{\mathrm{H}}=(6.017_{61}\pm0.006)\times10^{23}$ g^{-1}
Mass of proton	$M_P=(\mathrm{H}-m)/N_0=(1.6608_9\pm0.0017)\times10^{-24}$ g
Mass of α-particle	$M_\alpha=(\mathrm{He}-2m)/N_0=(6.597_{74}\pm0.007)\times10^{-24}$ g
Charge in electrolysis of 1 g hydrogen	$e/M_{\mathrm{H}}=F/\mathrm{H}=9574.5_1\pm0.7$ abs-*em*-units$\cdot$g^{-1}
Specific charge of proton	$e/M_P=F/(\mathrm{H}-m)=9579.7_3\pm0.7$ abs-*em*-units$\cdot$g^{-1}
Specific charge of α-particle	$2e/M_\alpha=2F/(\mathrm{He}-2m)=4823.1_1\pm0.6$ abs-*em*-units$\cdot$g^{-1}
Ratio mass H atom to mass electron (spectroscopic)	$\dfrac{(e/m)_{sp}}{e/M_{\mathrm{H}}}=1839._{26}\pm1$
Ratio mass H atom to mass electron (deflection)	$\dfrac{(e/m)_{defl}}{e/M_{\mathrm{H}}}=1847._{61}\pm2$
Ratio mass proton to mass electron (spectroscopic)	$=1839.26-1=1838._{26}\pm1$
Ratio mass proton to mass electron (deflection)	$=1847.61-1=1846._{61}\pm2$
Energy associated with unit wave-number	$\epsilon/\nu'=hc=(1.9627_{64}\pm0.0025)\times10^{-16}$ erg$\cdot$cm
Potential associated with unit frequency[2]	$V/\nu=h/e=(1.3725_4\pm0.0005)\times10^{-17}$ erg$\cdot$sec$\cdot$*es*-unit^{-1}
Frequency associated with unit potential	$\nu/V=e/h=(7.2857_7\pm0.0027)\times10^{16}$ *es*-unit$\cdot$erg$^{-1}\cdot$sec^{-1}
	$\nu/V''=e\cdot10^8/hc=(2.4302_5\pm0.0009)\times10^{14}$ sec$^{-1}\cdot$abs-volt^{-1}
Wave-number associated with 1 abs-volt	$\nu_0=\nu'/V''=e\cdot10^8/hc^2=8106._{31}\pm3$ cm$^{-1}\cdot$abs-volt^{-1}
Wave-length associated with 1 abs-volt	$\lambda_0=\lambda V''=hc^2/e=(12336._1\pm5)\times10^{-8}$ cm$\cdot$abs-volt
Energy of 1-abs-volt-electron	$h\nu/V''=e\cdot10^8/c=(1.5910_3\pm0.0016)\times10^{-12}$ erg
Speed of 1-abs-volt-electron (spectroscopic)	$v_e=[2\cdot10^8(e/m)_{sp}]^{1/2}=(5.9346_4\pm0.0017)\times10^7$ cm$\cdot$sec^{-1}
Speed of 1-abs-volt-electron (deflection)	$v_e=[2\cdot10^8(e/m)_{defl}]^{1/2}=(5.9481_1\pm0.0034)\times10^7$ cm$\cdot$sec^{-1}

TABLE C. *Miscellaneous Derived Constants—(Continued)*

Fine structure constant	$\alpha=2\pi e^2/hc$	$=(7.283_{64}\pm 0.006)\times 10^{-3}$
Reciprocal of fine structure constant	$1/\alpha$	$=137.29_4\pm 0.11$
Square of fine structure constant	α^2	$=(5.305_{12}\pm 0.008)\times 10^{-5}$
Bohr unit of angular momentum	$h/2\pi$	$=(1.0419_9\pm 0.0013)\times 10^{-27}$ erg·sec
	$e^2/c=\alpha h/2\pi$	$=(7.589_{45}\pm 0.015)\times 10^{-30}$ *es*-unit²·sec·cm⁻¹
Magnetic moment of 1 Bohr magneton (spectroscopic)	$\mu_1=(h/4\pi)(e/m)_{sp}$	$=(0.9174_{70}\pm 0.0013)\times 10^{-20}$ erg·gauss⁻¹
Magnetic moment of 1 Bohr magneton (deflection)	$\mu_1=(h/4\pi)(e/m)_{defl}$	$=(0.9216_{38}\pm 0.0016)\times 10^{-20}$ erg·gauss⁻¹
Ratio Bohr magneton to Bohr mechanical unit (spectroscopic)	$\dfrac{\mu_1}{h/2\pi}=\dfrac{1}{2}\left(\dfrac{e}{m}\right)_{sp}$	$=0.8805\pm 0.0005)\times 10^{7}$ gauss⁻¹·sec⁻¹
Ratio Bohr magneton to Bohr mechanical unit (deflection)	$\dfrac{\mu_1}{h/2\pi}=\dfrac{1}{2}\left(\dfrac{e}{m}\right)_{defl}$	$=(0.8845\pm 0.0010)\times 10^{7}$ gauss⁻¹·sec⁻¹
Magnetic moment per mole for one Bohr magneton per molecule (spectroscopic)	$\mu_1 N_0$	$=5563._{87}\pm 10$ erg·gauss⁻¹·mole⁻¹
Same (deflection)	$\mu_1 N_0$	$=5589._{14}\pm 11$ erg·gauss⁻¹·mole⁻¹
Zeeman displacement per gauss	$\dfrac{\Delta\nu'}{H}=\dfrac{(e/m)_{sp}}{4\pi c}$	$=(4.674_{38}\pm 0.003)\times 10^{-5}$ cm⁻¹·gauss⁻¹
Band spectra constant connecting wave-number and moment of inertia, I	$h/8\pi^2 c$	$=(27.65_{83}\pm 0.04)\times 10^{-40}$ g·cm
Same, connecting frequency and I	$h/8\pi^2$	$=(8.291_{87}\pm 0.011)\times 10^{-29}$ erg·sec
Same, connecting ergs and I	$h^2/8\pi^2$	$=(5.428_{69}\pm 0.014)\times 10^{-55}$ erg²·sec²
Atomic specific heat constant	$c_2/c=h/k$	$=(4.7757_3\pm 0.0019)\times 10^{-11}$ sec·deg
Rotational specific heat constant	$\sigma IT=h^2/8\pi^2 k$	$=(39.59_{97}\pm 0.06)\times 10^{-40}$ g·cm²·deg
Reduced mass of H atom	$\mu_{\mathrm{H}}=R_{\mathrm{H}}(m_0)_{sp}/R_\infty$	$=(9.030_{19}\pm 0.010)\times 10^{-28}$ g
Schroedinger constant for H atom	$8\pi^2\mu_{\mathrm{H}}/h^2$	$=(1.663_{42}\pm 0.003)\times 10^{27}$ g·erg⁻²·sec⁻²
Schroedinger constant for electron	$8\pi^2(m_0)_{sp}/h^2$	$=(1.664_{32}\pm 0.003)\times 10^{27}$ g·erg⁻²·sec⁻²
Ionization potential for H atom	R_{H}/ν_0	$=13.529_9\pm 0.005$ abs-volt
Ionization potential for He⁺	$4R_{\mathrm{He}}/\nu_0$	$=54.141_7\pm 0.020$ abs-volt
Separation of nucleus and electron in normal H atom, using experimental value of R_{H}	$a_0=\dfrac{\alpha(1-\alpha^2)^{1/2}}{4\pi R_{\mathrm{H}}}$	$=(0.5284_{56}\pm 0.0004)\times 10^{-8}$ cm
Same, using calculated value of R_{H}	$a_0=\dfrac{h^2(1-\alpha^2)^{1/2}}{4\pi^2 e^2\mu_{\mathrm{H}}}$	$=(0.5284_{21}\pm 0.0005)\times 10^{-8}$ cm
Radius of Bohr orbit in normal H, referred to center of mass, using experimental R_∞	$a_0'=\dfrac{\alpha(1-\alpha^2)^{1/2}}{4\pi R_\infty}$	$=(0.5284_{69}\pm 0.0004)\times 10^{-8}$ cm

TABLE C. *Miscellaneous Derived Constants*—(*Continued*)

Same, using calculated R_∞	$a_0' = \frac{h^2c(e/m)_{sp}(1-\alpha^2)^{1/2}}{4\pi^2e^3}$	$=(0.5281_{34}\pm0.0005)\times10^{-8}$ cm
Speed of electron in normal H orbit, referred to nucleus	$v_0'=\alpha c$	$=(2.1836_1\pm0.0017)\times10^8$ cm·sec^{-1}
Same, referred to center of mass, using experimental R_H and R_∞	$v_0=\alpha cR_H/R_\infty$	$=(2.1824_2\pm0.0017)\times10^8$ cm·sec^{-1}
General spectroscopic doublet constant	$R_\infty\alpha^2$	$=5.821_{71}\pm0.009$ cm^{-1}
Hydrogen doublet constant	$\Delta\nu_H=R_H\alpha^2/16$	$=0.3636_{59}\pm0.0006$ cm^{-1}
Dispersion constant	$e^2/2\pi m_0=ec(e/m)_{sp}/2\pi$	$=(4.007_{97}\pm0.005)\times10^7$ cm^3·sec^{-2}
X-ray scattering coefficient	$\frac{8\pi e^4}{3m_0^2c^4}=\frac{8\pi e^2(e/m)^2_{sp}}{3c^2}$	$=(6.576_{91}\pm0.015)\times10^{-25}$ cm^2
Compton shift at 90° (spectroscopic)	$\frac{h}{m_0c}=\frac{h}{e}\left(\frac{e}{m}\right)_{sp}$	$=(0.024170_4\pm0.000016)\times10^{-8}$ cm
Compton shift at 90° (deflection)	$\frac{h}{m_0c}=\frac{h}{e}\left(\frac{e}{m}\right)_{defl}$	$=(0.02428_{02}\pm0.00003)\times10^{-8}$ cm
Wave mechanics λ of 1-abs-volt-electron (spectroscopic)	$\frac{h}{m_0(v_e)_{sp}}$	$=(12.210_0\pm0.006)\times10^{-8}$ cm
Loschmidt number (0°C, A_n)	$n_0=N_0/\nu_n$	$=(2.705_{60}\pm0.003)\times10^{19}$ cm^{-3}
Wien's displacement constant (indirect)	$A=c_2/4.9651$	$=0.28836_1\pm0.00011$ cm·deg
First radiation constant[3]	$c_1=8\pi hc$	$=(4.932_{96}\pm0.006)\times10^{-15}$ erg·cm
	or hc^2	$=(0.5884_{29}\pm0.0008)\times10^{-5}$ erg·cm^2·sec^{-1}
	or $2\pi hc^2$	$=(3.697_{20}\pm0.005)\times10^{-5}$ erg·cm^2·sec^{-1}
Energy per mole for 1-abs-volt-electron per molecule	$\frac{F\text{ (abs. coul per gram equiv).}}{J_{15}\text{ (abs. joules per cal.).}}$	$=23054._8\pm4$ cal$_{15}$·mole^{-1}
Translational energy of molecule at 0°C	$E_0=3kT_0/2$	$=(5.617_{49}\pm0.006)\times10^{-14}$ erg
Translational energy per degree	$e_0=E_0/T_0=3k/2$	$=(2.0563_3\pm0.0021)\times10^{-16}$ erg·deg^{-1}

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[3] Planck's radiation law is

$$E_\lambda=\frac{c_1}{\lambda^5}\left(e^{-c_2/\lambda T}-1\right)^{-1}.$$

E_λ may be defined in various ways, and c_1 varies accordingly. If $E_\lambda d\lambda$ denotes the energy density of unpolarized radiation in range $d\lambda$, $c_1=8\pi hc$ (used by Coblentz and others). If $E_\lambda d\lambda$ denotes the intensity of emission of linearly polarized radiation in range $d\lambda$, perpendicular to a surface, per unit of surface, per unit solid angle, $c_1=hc^2$ (Planck's own nomenclature, used generally in German texts). If $E_\lambda d\lambda$ denotes the emission of unpolarized radiation in range $d\lambda$, per unit surface, in all directions (2π solid angle), $c_1=2\pi hc^2$ (used in the ICT and elsewhere)

TABLE C. *Miscellaneous Derived Constants*—(*Continued*)

Sackur-Tetrode constant[4] (ϵ, base of ln $=2.71828$)	$S_0=R_0' \ln \frac{(2\pi k)^{3/2}\epsilon^{5/2}}{h^3 N_0^{5/2}}=-11.0533\pm0.0026$ $\text{cal}_{15}\cdot\text{deg}^{-1}\cdot\text{mole}^{-1}$, or entropy units
Sackur-Tetrode constant, alternative form	$S_0/R_0'=-5.5643_9\pm0.0009$
	$S_0'(=S_0+R_0'\ln R_0''')=-2.2985_2\pm0.0018$ $\text{cal}_{15}\cdot\text{deg}^{-1}\cdot\text{mole}^{-1}$
Chemical constant (unit at. wt., pressure in dynes·cm^{-2})	$i_0=\frac{3}{2}\log\frac{2\pi k^{5/3}}{N_0 h^2}=+4.4174_6\pm0.0004$
Same (pressure in atmospheres)	$i_0'=i_0-\log A_n=-1.5882_5\pm0.0004$
Debye-Hückel constant for electrolytes[5]	$(\pi e^6/k^3)^{1/2}=(2.512_{51}\pm0.004)\times10^5$ *es*-unit$^3\cdot$erg$^{-3/2}\cdot$deg$^{3/2}$
Multiplier of (Curie constant)$^{1/2}$ to give magnetic moment per molecule	$(3k/N_0)^{1/2}=(2.604_{17}\pm0.003)\times10^{-20}$ erg$^{1/2}\cdot$deg$^{-1/2}\cdot$mole$^{1/2}$
Same, to give Bohr magnetons per molecule	$\frac{(3k/N_0)^{1/2}}{\mu_1}=2.8384_2\pm0.0019$ erg$^{-1/2}\cdot$gauss$\cdot$deg$^{-1/2}\cdot$mole$^{1/2}$

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[4] S_0 is to be used in the formula for the entropy of a perfect monatomic gas $S=(3/2)\ R_0'\ \ln\ M+(3/2)\ R_0'\ \ln\ T+R_0'\ \ln\ V+S_0$ where $M=$ molecular weight in grams, $T=$ °C abs., $V=$ vol. in cm^3. S_0' is to be used in $S=(3/2)\ R_0'\ \ln\ M+(5/2)\ R_0'\ \ln\ T-R_0'\ \ln\ P+S_0'$ where $P=$ pressure in normal atmospheres (A_n).

[5] See page 328 of Eucken, Jette and Lamer "Physical Chemistry."

nomenclature. On the other hand e, standing alone, indicates electronic charge in *es* units, and m_0 indicates electronic mass while m equals the atomic weight of an electron. Hence a more logical but less convenient nomenclature would have been (e/m_0) *es* units, and possibly (e'/m_0) *em* units.

(d) In the fundamental quantum relation connecting spectral frequency and energy, $\epsilon = h\nu = eV$, each side of the equation represents energy in ergs, provided all quantities are in abs. c.g.s. units. ν/V $(=e/h)$ then measures the frequency in $\sec^{-1}$ associated with one abs. *es* unit of potential. It is usually convenient to substitute the wave number (ν') or the wave-length (λ) in place of ν, and to substitute the number of abs. volts (V'') in place of V. $(V' =$ int. volts, throughout this paper). The values of the various ratios, such as ν'/V'' etc., are given in Table c.

(e) An electron which has fallen through one abs. volt of potential is termed an abs. volt-electron, and its energy in ergs and speed in $\mathrm{cm} \cdot \sec^{-1}$ are given in Table c. Corresponding to any *ionization potential* of an atom or molecule in volts (V''), there is an *energy of ionization* (eV'') which can conveniently be measured in units equal to the energy of a volt-electron, and is so designated. Thus an ionization potential of 10 volts corresponds to an energy of ionization of 10 volt-electrons. Similarly it is convenient, in the case of molecules, to speak of a *dissociation potential* of, let us say, 10 volts, and a corresponding energy of dissociation (heat of dissociation) of 10 volt-electrons per molecule. The factor by which this last quantity must be multiplied to give the heat of dissociation in calories per mole is given in Table c. Unfortunately there has arisen the practise, to which the writer also must plead guilty, of designating the *heat* of dissociation as 10 volts, instead of stating, more correctly, that the equivalent dissociation *potential* is 10 volts, or that the heat of dissociation per molecule is 10 volt-electrons.

(f) The name of the units given in the tables conforms as far as possible with current practise. Certain difficulties arise in the case especially of magnetic and electrical quantities. These difficulties are connected with the unknown dimensions of magnetic permeability μ, and specific inductive capacity ϵ. It is customary in the ICT, and in most other tables and texts, to indicate these unknown dimensions by the symbols μ and ϵ. A given unit, such as the gauss, is applied only to quantities of a given set of dimensions, *including* μ and ϵ. In the present paper we are concerned only with numerical magnitudes and no particular attention has accordingly been paid to this matter of dimensions. Thus in section E, the statement that the absolute *em* unit of resistence is one $\mathrm{cm} \cdot \sec^{-1}$ involves the assumption not only of unit permeability, but also of dimensionless permeability. In a number of the equations given in Table c, the two sides of the equation do *not* check dimensionally unless one assumes μ and ϵ to be dimensionless. It follows from this that the name of the unit stated in the table applies strictly only to one side of such an equation. In such cases the unit applies to the left side of the equation, since this is the quantity being evaluated.

The right side gives merely the most direct derivation of the numerical magnitude, in terms of quantities already evaluated. Since this ambiguity does not affect the numerical magnitude, it is inconsequential in the present discussion. As examples of this situation we cite the fine structure constant α, which is dimensionless. To satisfy this condition one should write $\alpha = 2\pi e^2/\epsilon_0 hc$ where ϵ_0 is numerically unity, and represents merely the dimensions of ϵ. Similarly the ratio of the Bohr magneton μ_1 to the Bohr unit of angular momentum $(h/2\pi)$ is strictly $\mu_0 \frac{1}{2}(e/m)$, where μ_0 is numerically unity, and represents merely the dimensions of permeability.

(g) The mole is a (variable) unit of mass, equal to the molecular weight in grams. The gram equivalent is a similar (variable) unit of mass, equal to the atomic or molecular weight in grams, divided by the valence.

The various quantities appearing in Tables a and b have been discussed in the preceding sections. No general explanation will be given of the meaning or use of the quantities appearing in Table c, since any adequate explanation would constitute almost a text-book of modern physics and physical chemistry. In the case of the more specialized constants, no explanation is needed by investigators actually working with each constant, and it is to such persons that the data given here will be most useful.

I wish, in conclusion, to direct attention merely to two constants for which the formula used here differs from that normally given. It is customary to use for the speed of the electron in the normal orbit of hydrogen, as given by Bohr's original theory, a value which refers to the nucleus considered as the center of coordinates. This I call $v_0(=\alpha c)$ in Table c. It would seem more logical to give the speed referred to the center of mass, the quantity denoted v_0' in Table c. There is a similar discrepancy in the case of the radius of this orbit. The electron, according to Bohr, moves about the center of mass in a circle of radius a_0', as it is denoted in Table c. This is not the same as the constant separation of the nucleus and electron, which is here denoted a_0. In the literature these two quantities, a_0, and a_0', are sometimes confused. The expressions for v_0, v_0', a_0, and a_0' given in Table c include also the factor $(1-\alpha^2)^{1/2}$, arising from the variation of mass with speed.

Section O

CONCLUSIONS

This final section is devoted to a brief discussion of the status of the general physical constants, as it appears to the writer at the present time (April 1929). As noted in the Introduction, the values of all constants adopted in sections A to M have been based on material available on January 1, 1929. In the present section one can scarcely refrain from mentioning more recent material, especially if it has any direct bearing on the conclusions already reached. Some of this new material has been given in footnotes in the preceding sections. Each section, from A to M, will be considered in order.

(A) It seems unlikely that a more accurate value of the velocity of light, c, will be required for some years to come. In the important relations to be considered in this concluding section, the probable error in c is far less than that in any of the other constants. Hence, as far as such relations are concerned, the present value of c is entirely satisfactory, and can be considered as more or less permanently established.

(B) The constant of gravitation, G, although of great intrinsic importance, has not been used in later sections of the present paper. The value obtained by Heyl is so much more accurate than any previously available that it also is likely to remain unchanged for some time in the future.

(C) The relation of the liter to the cubic decimeter has not been investigated for many years. The value adopted in section C seems to be universally used, and can be considered as essentially permanent.

(D) The normal mole volume of an ideal gas, v_n, has been actively investigated in recent years. The value adopted is based on 1928 calculations, and it is quite possible that additional work will, at any time, change the most probable value. Although the Baxter and Starkweather investigations, on which the adopted value is founded, are generally considered more trustworthy than any previous work, their results differ somewhat from the previous values and hence a future change in v_n as great as, or even greater than, the adopted probable error would not be surprising.

(E) As has been pointed out in section E, the exact evaluation of the relation between the international and absolute electrical units is a matter of great experimental difficulty, especially since there is no one official standard. Each national laboratory has its own international standards and these may change with time. Progress is being made toward the adoption of the *absolute* electrical units as standards, but for the purpose of correcting work done in terms of the present international units, it is necessary to know the relation between the two systems. The int. ampere differs from the abs. ampere by so small an amount, if at all, that it makes practically no difference whether the ratio q is taken to be unity or not. The int. ohm does, however, definitely differ from the abs. ohm by about one part in 2000 (as shown by the value of p). Because experimental physicists have usually ignored the distinction, many slight changes in published results have been necessary in the later sections of this paper. Due to the variability of the international standards, it is possible that the proper value of p to be used in each specific correction may differ more or less from the adopted value, but the entire possible variation is not more than 2 or 3 parts in 10^5, and this is small compared to the 50 parts in 10^5 by which p differs from unity. The corrections made in this paper are therefore believed to be essentially correct and are not likely to be contradicted by future work.

(F) Of the atomic weights considered in section F, only that of calcium is at all uncertain. This atomic weight has been used in connection with the grating space of calcite and also in the evaluation of the electronic charge e by x-ray measurements. The adopted probable error (0.005) represents only 5 parts in 10^5 in the molecular weight of calcite and thus would have remained negligibly small even if a value several times as large had been adopted.

In this same section evidence was presented to show the agreement of Aston's atomic weights with the chemical values, to at least one part in 10,000, thus indicating that the new oxygen isotope of mass 18 had an abundance of less than one part in 1000 (more accurately, one in 1300) compared to oxygen 16. I have now learned from H. D. Babcock that experiments he has performed on absorption coefficients in the atmospheric bands indicate that O_{18} has an abundance* of only one part in 1250, the probable error being some 25 percent. Hence Aston's atomic weights should be greater than the chemical values by about one part in 10,000. Babcock's determination of relative abundance, however, involves the *assumption* that the absorption coefficient is the same, per molecule, for each species of molecule ($O_{16}-O_{16}$ and $O_{16}-O_{18}$), and this may not be true.

As indicated in footnote 21a of section F, a very recent accurate determination has been made of the atomic weight of silver, directly in terms of oxygen, with a resulting value and error identical with that adopted. The atomic weight of nitrogen therefore becomes unnecessary, for the purposes of this paper.

(G) The normal atmosphere, A_n, is the most accurately known of all the constants considered here. In one sense it is a conventional constant, rather than a general physical constant, since its value involves the *conventional* constant of normal gravity. In experimental work one must measure the actual value of gravity at the position of the apparatus, and this involves its own error. The value of A_n also involves the density of mercury, and since mercury has several isotopes, there are possibilities of uncertainty here, not apparent on the face of the results. Attention is directed also to the "Note added in proof," at the end of section G.

(H) The absolute temperature of the ice-point has an error which is not at all negligible in certain expressions in which it occurs. In fact this accounts for nearly the entire error in the gas constant R_0. As noted in section H, the work of Henning and Heuse indicates an apparent probable error less than the 0.03°C adopted by them and also by the writer, but their result differs from that of Roebuck by 0.04°C. It is quite possible that further work may slightly modify the adopted value and its probable error. There is, however, no question but that the present value of 273.18° is much more trustworthy than the value of 273.09° used until very recently.

(I) The recent precision measurements of the mechanical and of the electrical equivalent of heat are in beautiful agreement and this agreement forms strong evidence that each is substantially correct. It should be remembered, however, that the writer has recalculated the published values of both investigations, and the very fact that the final result depends so much on the method of computation, lends a certain element of uncertainty to the situation. Regardless of this, the present adopted values of J and J' are certainly much more trustworthy than the values based on earlier work.

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* This is a corrected figure in which allowance has been made for the fact that the $O_{16}\cdot O_{18}$ molecule has twice as many possible levels as the $O_{16}\cdot O_{16}$ molecule.

(J) As has been pointed out in section J, the value of the Faraday, *F*, and its adopted error, depend somewhat on the definition of this constant, and there has been some confusion on this point. There are many possibilities of small systematic error in electrolytic work, and these directly affect the value of the Faraday. The writer is aware, from personal correspondence, that his adopted value of *F* in abs. coulombs (96489 ± 7) is not approved by certain workers in this field. He believes, however, that no valid arguments have been presented to show that it is not the most probable value according to present evidence, and he feels that the true value is likely to differ from the adopted value by less than 7 coulombs rather than by more.

(K) (L) (M). The constants e, e/m, and h, considered in these three sections, together with the velocity of light, appear in many important derived constants. Due to the fact that h depends for its value on e, this latter quantity appears implicitly, if not explicitly, in every quantum relation. The detailed results considered in section M have shown that the value of h would be known with considerable certainty, were it not for the probable error in e. The most outstanding discrepancy in section M was that between the work of Wagner and of Duane and co-workers, on the measurement of h from the x-ray continuous spectrum. As stated in footnote 16a, of that section, the very recent work of Feder, using this same method, gives a value of h in exact agreement with that adopted, and also explains Wagner's very low value. The writer now feels that the value of h/e listed in Table *c* can be assumed with some confidence. The real problem concerns the values of e and of e/m.

The need of adopting two different values of e/m, one resulting from quantum theory experiments, and one from deflection experiments, is very annoying, and is fundamentally unsatisfactory. The same situation seems to be arising in regard to e. Millikan's oil-drop value has been accepted for many years, since it was the only one available. The new work on x-rays has now opened up another possibility. The value of Bäcklin, considered in some detail in section K, is one-half percent higher than my adopted value. Within the past few days I have been informed by Professor A. H. Compton of the final result of the work of Dr. J. A. Bearden, to which anonymous reference was made in the last paragraph of section K. Dr. Bearden obtains for the absolute wave-length of the (unresolved) Cu $K\alpha$ line, 1.5439 ± 0.0002A, and for the Cu $K\beta$ line, 1.3940 ± 0.0002A. These results are obtained from many different plates, each containing several orders of spectrum, and with three different gratings. The first is 0.345 percent higher than Siegbahn's value, the second 0.336 percent higher. The *relative* wave-lengths are thus in entire agreement with Siegbahn, but the *absolute* wave-lengths lead to an average value for calcite of $d'_{20} = 3.0398$A, and from Eq. (10), section K, $e = 4.825 \times 10^{-10}$ abs. *es* units. This is 1.15 percent above my adopted value of e. Before expressing any opinion as to this result, it is desirable to consider the various relations that have been suggested as holding between the constants now under consideration.

The most famous of these relations is that connecting e, e/m, h, and c, in Bohr's formula for the Rydberg constant, as stated in Eq. (1) of section M. This relation was used to evaluate h, and the value so obtained (6.54713) is identical to the first four digits with that finally adopted. Hence, in Table b, the indirectly calculated value of e/m is also practically identical with that adopted. In other words, the *adopted* values of e, e/m, h and c form a self-consistent system, as judged by the Bohr formula for R_∞. The writer[1] attempted in 1926, to obtain a similarly consistent set of constants, and at that time, in order to accomplish this, altered arbitrarily the value of e by a small amount. This however, was *not* a legitimate procedure, since the necessarily resulting change in h was not made. Fortunately the difficulty has now vanished, assuming the adopted values of the present paper to be correct.

Lewis and Adams,[2] from their theory of ultimate rational units, have obtained, with the aid of Planck's black body radiation law, the following relation between h, c and e,

$$\frac{hc}{2\pi e^2}=8\pi\left(\frac{8\pi^5}{15}\right)^{1/3}. \tag{1}$$

The right side of this equation equals 137.348, while the left side, with the constants here adopted equals $137.29_4 \pm 0.11$. The left side, it should be noted, equals the reciprocal of the fine structure constant α, and the value just quoted is taken directly from Table c. The numerical agreement of the two sides of Eq. (1) is very striking. This equation was used by the writer,[3] in 1919, as one method for calculating h. The present agreement shows that this method now yields a value of h almost identical (6.5496, to be precise) with that adopted.

The fact that α is a dimensionless constant involving fundamental general constants has often been noted, although it should be remembered, as stated in section N, that to make α strictly dimensionless, we must include with the factor hc the unknown dimensions of specific inductive capacity. H. S. Allen[4] has considered quite fully the various proposed relations of α to other quantities.

Recently Perles[5] has pointed out that the ratio of the mass of the proton to that of the electron (M_p/m_0) is another dimensionless constant which should have some significance, and has found that

$$\frac{hc}{e^2}\left(=\frac{2\pi}{\alpha}\right)=\frac{M_p/m_0}{\pi-1}\,. \tag{2}$$

With the values adopted here, the left side of Eq (2) equals 862.64 ± 0.68, while the right side equals 858.36 ± 0.49 or 862.26 ± 0.99 depending on whe-

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[1] R. T. Birge, Science **64,** 180 (1926). See footnote 8, section M.
[2] G. N. Lewis and E. Q. Adams, Phys. Rev. (2) **3,** 92 (1914).
[3] R. T Birge, Phys. Rev. **14,** 361 (1919).
[4] H. S. Allen "The Quantum and its Interpretation" (1928), pp. 171–183.
[5] J. Perles, Naturwiss. **16,** 1094 (1928).

ther one uses the spectroscopic or the deflection value of e/m. The agreement is evidently very good for the deflection value, but poor for the spectroscopic value. Whether it has any theoretical significance remains to be seen.

A far more important piece of work is Eddington's recent calculation[6] that $1/\alpha=136$ exactly, from considerations of the new quantum mechanics. This is to be contrasted with our value of 137.29 ± 0.11. The writer[7] has already pointed out briefly this conflict between Eddington's value of α and that deduced from the assumed values of h, c and e. It is desirable now to consider the question more specifically. The equation for $1/\alpha$ can be written

$$\frac{1}{\alpha}=\left(\frac{h}{e}\right)\left(\frac{c}{2\pi e}\right). \tag{3}$$

As has been noted more than once, h/e is known much more accurately than e. Assuming $1/\alpha=136$, and assuming the adopted value of h/e, we obtain $e=4.815\times 10^{-10}$*es* units. This is equivalent to assuming that h is to be varied directly as e, and , with the exception of the Rydberg constant method, most of the other methods *do* involve such a direct proportionality. On the other hand, if one assumes my average variation of h and e (the 1.26 power), then $1/\alpha=136$ requires that $e=4.8308\times 10^{-10}$ (and that $h=6.6520\times 10^{-27}$). The value of e obtained by Bearden, from x-rays, lies, strangely enough, between the two values just given, and so agrees as well as can be expected with Eddington's value of $1/\alpha$. Hence, so far as Eq. (3) is concerned, we must conclude that if the oil-drop value of e is correct within the assumed probable error, or even within several times that amount, Eddington's value of α, and Bearden's value of e, cannot possibly be correct. On the other hand, if Bearden's value of e, is correct, then Eddington's value of α is also correct, and the oil-drop value of e is wrong.

Direct evidence on these two alternatives is furnished by the Rydberg constant. Eq. (1) of section *M* can be written as

$$R_\infty=\frac{\alpha^2}{2}\left(\frac{e}{h}\right)\frac{1}{(e/m)}\cdot \tag{4}$$

Using our adopted value of e/h and its adopted error, together with Eddington's value of α, one obtains $e/m=(1.7948\pm 0.0007)\times 10^7$ abs. *em* units, as contrasted with our adopted value of (1.761 ± 0.001), obtained from similar quantum relations. It has already been pointed out that the adopted value of e/m, together with the adopted values of α and e/h, exactly satisfies Eq. (4). The figures just quoted indicate the impossiblity of satisfying Eq. (4)with Eddington's values of α. As already suggested, the discrepancy of 0.5 percent between the spectroscopic and deflection values of e/m may be due to some general error in quantum theory formulas. The above discrepancy of nearly two percent is evidently far more serious In his derivation of the value of $1/\alpha$, Eddington assumes the truth of Eq. (4). Hence the discrepancy

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[6] A. S. Eddington, Proc. Roy. Soc. **A122,** 358 (1929).

[7] R. T. Birge, Nature **123,** 318 (1929)

cannot be attributed to a possible theoretical incorrectness of this formula In fact, if Eddington's value of $1/\alpha$ (and by implication Bearden's value of e) is right, one can only conclude that all measurements of e/m, by any method, are too low by 1.5 to 2 percent, and in addition that our adopted value of e is too low by more than one percent. It seems to the writer—an opinion apparently shared by others—that it is more probable that the x-ray values of e are incorrect due to a difference in the structure of a crystal (density, etc.) near the surface from that in the interior.

It must be admitted that this is a very unsatisfactory way to leave the situation in the case of the most important constants known to science. It is to be hoped that before another year has passed, some if not all of these difficulties will have been removed.