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INFLUENCE OF IMPURITY FERROMAGNETISM AND OF THE EXTERNAL MAGNETIC FIELD ON THE RESISTANCE OF A METAL WITH MAGNETIC IMPURITIES

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Abstract

The calculation performed in [*1*] *is extended to the case of impurity ferromagnetism and of an external magnetic field. It is shown that depending on the ratio between* T_{max} *(see* [*1*]*),* T_c *(Curie temperature) and* μH *various kinds of non-monotonic behaviour of the resistance versus temperature are possible. It has been found also that the resistance may decrease with magnetic field.*

In the previous work [1] we have examined the electric resistance of a non-magnetic metal with small admixtures of magnetic impurity atoms. We have found that the part of the resistance connected with the exchange interaction of electrons with the admixed atoms varies with temperature. If this interaction has the anti-ferromagnetic sign ($J < 0$), this part of the resistance will increase with decreasing temperature attaining a maximum at a certain temperature, after which it begins to fall. This temperature does not depend on the concentration of the impurity.

Experimentally (see, for example, [2]) curves with a maximum have been obtained, however, the location of the maximum essentially depended on the concentration of the impurity. In the present paper it is shown that this results from impurity ferromagnetism. The influence of the external magnetic field on the resistance is also examined.*

I. Impurity Ferromagnetism

For the description of the impurity ferromagnetism and of the external magnetic field we shall proceed in the spirit of communication [3]. From the diagrams for G and $\mathcal{G}$ functions (see [1]) we shall isolate the diagrams of the first order represented in Fig. 1. These diagrams equal zero in the absence of spin ordering and differ from zero in the cases considered. They can be regarded as the result of the action of some effective "field" in one case on the spin of the electron and in the other on the spin of the impurity. We shall combine this "field" with the real magnetic field and shall examine the impurity and the electron under the action of some combined field which we later will determine self-consistently. The "free" Green functions will have the form:

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* The influence of an external magnetic field on the resistance of a metal with magnetic impurities was examined by another method by Gurevich and Vassilevich. The Kondo effect was not taken into account.

$$G = \frac{1}{i\omega - \xi + \sigma_Z P} \quad (a)$$

$$\mathscr{G} = \frac{1}{i\omega - \lambda + S_Z Q} \quad (b) \qquad (1)$$

The symbols are the same as in [1], z is the direction of spin ordering.

Let us point out that the averaging with respect to the spins of the impurities must be done taking into account the "field" Q. Due to this the normalizing factor equals not $e^{\lambda/T}/(2S + 1)$ but

$$e^{\lambda/T} \Bigg/ \left(\sum_{-s}^{s} e^{-MQ}\right) = e^{\lambda/T} \frac{\sinh(Q/2T)}{\sinh[Q(S + ½)/T]}$$

From diagrams I (a) and (b) and formulas (1) we obtain self-consistent equations for P and Q (here and further on $\hbar = 1$):

$$Q = g\mu_0 H + \frac{J}{N} \frac{p_0 m}{2\pi^2} S_{p\sigma}\, T \sum_{\omega} \int d\xi \frac{\sigma_Z}{i\omega - \xi + \sigma_Z P}$$

$$P = \mu_0 H + \frac{N_i J}{N} \frac{e^{\lambda/T}\sinh(Q/2T)}{\sinh[Q(S + ½)/T]} S_{p_s} T \sum_{\omega} \frac{S_Z}{i\omega - \lambda + S_Z Q}$$

(N_i is the number of admixed atoms per cm^3, μ_0Bohr's magneton, $g \approx 2$ the gyromagnetic factor of the impurity). Taking the sums and integral and setting $\lambda >> T$ we obtain:

$$P = \mu_0 H + JcSB_S\left(\frac{SQ}{T}\right) \quad (a)$$

$$Q = g\mu_0 H + \frac{3zJ}{2\varepsilon_F} P \quad (b) \qquad (2)$$

($B_S(x) = \frac{2S + 1}{2S} \coth \frac{(2S + 1)x}{2S} - \frac{1}{2S} \coth \frac{x}{2S}$ is the Brillouin function, z is the number of electrons per atom, C the atomic concentration). By introducing (2b) into (2a) we obtain exactly condition (13) of reference [3]

$$\left(S = \frac{\chi_0}{2\mu_0^2} P, \quad \frac{\chi_0}{\mu_0^2} = \frac{p_0 m}{\pi^2}, \quad a = \frac{2J}{N}\right)$$

With the help of equation (2) we can find P and Q and consequently also the "free" Green's functions.

For the following we shall need some approximations and limiting expressions. Let H equal 0. Assuming the argument in B_S to be small and developing in series we obtain the relation:

$$P^2 = \frac{10\, cS(S+1)\,\varepsilon_F}{3z(S^2 + S + ½)}\,(T_C - T) \tag{3}$$

where T_C is the Curie temperature, equal to

$$T_C = \frac{J^2 czS(S+1)}{2\varepsilon_F} \tag{4}$$

Formula (3) is correct in the vicinity of T_C. But if $T \ll T_C$ then

$$P = JcS. \tag{5}$$

As to Q, according to (2) it equals

$$Q = J\left[\frac{15}{2}\,\frac{cS(S+1)Z}{\varepsilon_F(S^2 + S + ½)}\,(T_C - T)\right]^{½}, \quad T_C - T \ll T_C$$

$$Q = \frac{3czJ^2S}{2\varepsilon_F}, \quad T \ll T_C \tag{6}$$

Therefore, for $T_C - T \ll T_C$, $QS \ll T_C$ and for $T \lesssim T_C$, $QS \sim T_C$.

Now let us examine what happens in the presence of a field. We are interested in the case of $\mu_0 H \gg T_C$ and $T \gg T_C$.


(a) (b)

FIG. 1.

Assuming again the argument in B_S to be small we obtain

$$P = \mu_0 H\left(1 + \frac{JcS(S+1)g}{3T}\right)$$

The condition of applicability of this formula is $g\mu_0 SH/T \ll 1$. In the reverse limiting case $P = \mu_0 H + JcS$.

From (2) we find that in both limiting cases

$$Q \approx g\mu_0 H \tag{7}$$

These formulae will be useful later on.

2. Self-Energy

Now let us consider the scattering. In the present case there are several types of diagrams which contribute to the self-energy part. One diagram corresponds to [1] (Fig. 2). The others

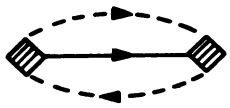

FIG. 2.

are represented in Fig. 3, where both interactions in each diagram refer to one atom (for the sake of simplicity we use Born's approximation for ordinary scattering). The diagrams on Fig.3 equal zero if the spins are not polarized.*

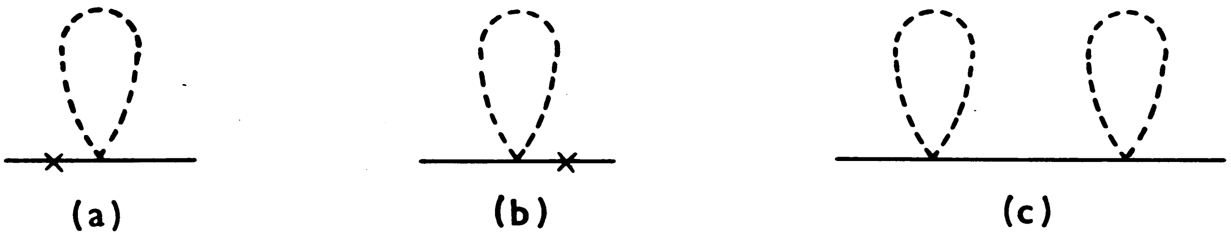

FIG. 3.

Their contribution is easily calculable. The result is (in the time of Green's function)

$$\sum\nolimits^{(1)}_{\alpha\alpha'} = -\,i\pi \;\mathrm{sign}\;\omega\,\frac{p_0 m}{2\pi^2}\left[-\,2f_0(P-\mu_0 H)\sigma_Z+\frac{1}{N_i}(P-\mu_0 H)^2\right]_{\alpha\alpha'},$$

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* In the letter to the editor of JEThF presenting very briefly the results of the present paper, these terms were not considered. The author expresses his gratitude to A. Rusinow who drew his attention to them.

where f_0 is the amplitude of usual scattering (which for simplicity reasons is assumed to be isotropic), and P is defined in the proceeding section. As for the diagram on Fig. 2 it is calculated by analogy to the procedure used in [1]:

$$\sum\nolimits^{(2)}_{\alpha\alpha'} = i \ \mathrm{sign}\ \omega \frac{e^{\lambda/T}\sinh(Q/2T)}{\sinh[(S+½)Q/T]} N_i \frac{p_0 m}{2\pi^2}$$

$$\times \int \Gamma_{\alpha\beta,\ \alpha_1\beta_1}(\omega,\ \omega_1;\ \omega+\omega_1-\omega_2,\ \omega_2)\Gamma_{\alpha_2\beta_2,\ \alpha'\beta_3}(\omega+\omega_1-\omega_2,\ \omega_2;\ \omega_1\omega_1)$$

$$\times\ Im\ \mathcal{G}^R{}_{\beta_3\beta_1}(\omega_1)\ Im\ \mathcal{G}^R{}_{\beta_1\beta_2}(\omega_2)\ Im\ G^R{}_{\alpha_1\alpha_2}(\omega+\omega_1-\omega_2)$$

$$\times\left(\tanh\frac{\omega_2}{2T} - \coth\frac{\omega_2-\omega}{2T}\right)\left(\tanh\frac{\omega_1}{2T} - \tanh\frac{\omega+\omega_1-\omega_2}{2T}\right)\frac{d\omega_1}{2\pi}\ \frac{d\omega_2}{2\pi} \tag{8}$$

Substituting

$$Im\ \mathcal{G}^R_{\beta\beta_1} = -\pi\delta_{\beta\beta_1}(\omega - \lambda + S_{Z\beta}Q)$$

$$Im\ G^R_{\alpha\alpha_1} = -\pi\delta_{\alpha\alpha_1}(\omega - \xi + \sigma_{Z\alpha}P)$$

taking the integrals over ω_1, ω and ξ and because $\lambda >> T$, we obtain

$$\sum\nolimits^{(2)}_{\alpha\alpha'} = -i\ \mathrm{sign}\ \omega N_i \frac{p_0 m}{2\pi}\ \frac{\sinh(Q/2T)}{\sinh[Q(2S+1)/2T]}\ \frac{e^{\omega/T+1}}{e^{\omega - QS_{Z\beta}/T} - e^{-QS_{Z\beta_1}/T}}$$

$$\times\ \Gamma_{\alpha\beta,\alpha_1\beta_1}\left[\omega,\ \lambda - QS_{Z\beta};\ \omega + Q\left(S_{Z\beta_1} - S_{Z\beta}\right),\ \lambda - QS_{Z\beta_1}\right]$$

$$\times\ \Gamma_{\alpha_1\beta_1,\alpha'\beta}\left[\omega + Q\left(S_{Z\beta_1} - S_{Z\beta}\right),\ \lambda - QS_{Z\beta_1};\ \omega, \lambda - QS_{Z\beta}\right] \tag{10}$$

Restricting ourselves to logarithmic accuracy we can, as in [1], calculate Γ with the help of the time technique for $T = 0$, and let $\lambda = 0$ everywhere. In [1] the lower limit of the logarithmic integral was the quantity ω and upon calculation of the conductivity $\omega \sim T$ proved to be essential. In the present case the situation is changed. Let us consider the simplest diagram in Fig. 4. In analogy to [1] (formula 10*a*) we obtain (11)

$$-i\left(\frac{J}{N}\right)^2\left(\vec{\sigma}\ \vec{S}\right)_{\alpha\beta,\alpha_1\beta_1}\left(\vec{\sigma}\ \vec{S}\right)_{\alpha_1\beta_1,\alpha'\beta'}\frac{p_0 m}{2\pi^2}\int\frac{d\omega_1}{2\pi}\int d\xi_1\ \times$$

$$\times \frac{1}{\omega_1 + QS_{Z\beta_1} + i\delta}\;\frac{1}{\omega - \omega_1 - \xi_1 + P\sigma_{Z\alpha_1} + i\delta\,\mathrm{sign}\,(\xi_1 - P\sigma_{Z\alpha_1})} =$$

$$= \left(\frac{J}{N}\right)^2 \left(\vec{\sigma}\,\vec{S}\right)_{\alpha\beta,\,\alpha_1\beta_1} \left(\vec{\sigma}\,\vec{S}\right)_{\alpha_1\beta_1,\,\alpha'\beta'} \frac{p_0 m}{2\pi^2} \int_0^{\varepsilon_F} \frac{d\xi_1}{\xi_1 - QS_{Z\beta_1} - \omega - i\delta}$$

(Here we made the substitution $\xi_1 - P\sigma_{Z\alpha} \to \xi_1$). From this it is obvious that the quantity P does not act at all in the logarithmic integral. The lower limit of the integral turns out to be the largest of the quantities Q and ω, i.e., in final analysis $\max(Q,\,T)$. Since the logarithmic integral comes from the region $\xi \gg \max(Q,\,T)$ therefore expression (2) preserves the same spinor form as in the absence of ordered spins. The same applies to all subsequent orders for Γ.

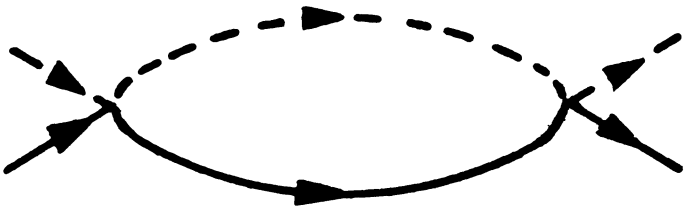

FIG. 4.

Thus, in expression (10) for $\Gamma_{\alpha\beta,\,\alpha_1\beta_1} \times \Gamma_{\alpha_1\beta_1,\,\alpha'\beta}$ according to [1] we can substitute

$$\frac{\left(\vec{\sigma}\,\vec{S}\right)_{\alpha\beta,\,\alpha_1\beta_1}\left(\vec{\sigma}\,\vec{S}\right)_{\alpha_1\beta_1,\,\alpha'\beta}}{\left[1 + \frac{3JZ}{2\varepsilon_F}\ln\frac{\varepsilon_F}{\max(Q,\,|\omega|)}\right]^2} \tag{12}$$

Setting $|\omega| \sim T$, we obtain, according to the preceding section, that if the magnetic field is absent or if $g\mu_0 HS \ll T_0$, then for $T \gg T_C$ in (12) one has $\ln \varepsilon_F/T$ and for $T \lesssim T_C$, $\ln \varepsilon_F/T_C$. But if $g\mu_0 HS \gg T_C$ then for $T \gg g\mu_0 HS$ we have $\ln \varepsilon_F/T$, and for $T \ll g\mu_0 HS$ we have $\ln \varepsilon_F/g\mu_0 HS$. However in the present case the temperature dependence derives not only from Γ but also from the preceding factor in (10). Using the spinor formula of Γ, (12) we obtain

$$\sum\nolimits_{\alpha\alpha'}^{(2)} = -\, i \text{ sign } \omega N_i \frac{p_0 m}{2\pi} \frac{\sinh(Q/2T)}{\sinh[Q(2S+1)/2T]} \frac{1}{\left[1 + \frac{3J_Z}{2\varepsilon_F} \ln \frac{\varepsilon_F}{\max(Q,, |\omega|)}\right]^2} \times$$

$$\times \sum_{M=-S}^{S} e^{QM/T} \left[M^2 + \frac{1}{2}(S+M)(S-M+1)(1+\sigma_Z)_{\alpha\alpha'} \frac{e^{\omega/T}}{e^{\omega/T} + e^{Q/T}} + \right.$$

$$\left. + \frac{1}{2}(S+M+1)(S-M)(1+\sigma_Z)_{\alpha\alpha'} \frac{e^{\omega/T}}{e^{\omega/T} + e^{-Q/T}} \right] \tag{13}$$

Taking the sum over M, we find

$$\sum\nolimits_{\pm}^{(2)} = -\, i \text{ sign } \omega N_i \frac{p_0 m}{2\pi} \frac{1}{\left[1 + \frac{3J_Z}{2\varepsilon_F} \ln \frac{\varepsilon_F}{\max(Q, |\omega|)}\right]^2} \times \tag{14}$$

$$\times \left\{ S(S+1) + S B_S\left(\frac{SQ}{T}\right) \left[\frac{e^{\omega/T} + 1}{e^{\omega/T} + e^{\mp Q/T}} \left(\coth \frac{Q}{2T} \mp 1\right) - \coth \frac{Q}{2T} \right] \right\}$$

where the plus and minus signs denote different orientations of electron spins. The coefficient in $\Sigma^{(1)} + \Sigma^{(2)}$ by $-\, i$ sign ω we shall write as $1/2\, \tau_\pm$. The ordinary scattering gives, as before (see [1]), an additional term in Σ of a considerably larger size (except in the vicinity of "resonance").

In analogy to [1] we obtain for the conductivity

$$\sigma = \frac{ZNe^2}{2m} \int_0^\infty \frac{d\omega}{2T \cosh \frac{2\omega}{2T}} \left[\tau'_+(\omega) + \tau'_-(\omega)\right]$$

here $\tau'_\pm(\omega) = [1/\tau_{ord} + 1/\tau_\pm(\omega)]^{-1}$. We took advantage of the fact that the quantity $\tau'_+(\omega) + \tau'_-(\omega)$ is symmetrical with respect to the change of the sign of ω. Taking $1/\tau_{ord} \gg 1/\tau_\pm(\omega)$ we obtain $\rho = \rho_{ord} + \rho_{ex}$ (ρ_{ex} includes also the interference part; the corresponding term in $\Sigma^{(1)}$ contributes in the second order of J/f_0)

$$\rho_{ex} = \rho_{ex,0} \left\{ \left[1 - \frac{1}{S+1} B_S\left(\frac{SQ}{T}\right) \frac{\sinh \frac{Q}{T} - \frac{Q}{T}}{\coth \frac{Q}{T} - 1} \right] \left(1 + \frac{3JZ}{2\varepsilon_F} \ln \frac{\varepsilon_F}{\max(Q, T)}\right)^{-2} - \right.$$

$$\left. - 3 \frac{S}{S+1} B_S{}^2 \left(\frac{SQ}{T}\right) \right\} \tag{15}$$

where $\rho_{ex,0} = \dfrac{3\pi m J^2 S(S+1)C}{2N\varepsilon_F\ e^2 h}$ (in the usual units), C is the atomic concentration.

The asymptotic values equal:

$$\rho_{ex} = \rho_{ex,\ 0}\left\{\left[1-\left(\frac{Q}{3T}\right)^2\right]\left(1+\frac{3Jz}{2\varepsilon_F}\ln\frac{\varepsilon_F}{T}\right)^{-2} - \frac{S(S+1)Q^2}{3T^2}\right\},\quad Q \ll T \tag{16}$$

$$\rho_{ex} = \rho_{ex,\ 0}\left\{\frac{S}{S+1}\left(1+\frac{3Jz}{2\varepsilon_F}\ln\frac{\varepsilon_F}{Q}\right)^{-2} - \frac{3S}{S+1} + \right.$$

$$\left. + \frac{1}{S+1}\left[\left(\frac{1}{S}-2+\frac{2Q}{T}\right)\left(1+\frac{3Jz}{2\varepsilon_F}\ln\frac{\varepsilon_F}{Q}\right)^{-2} + 6\right]e^{-Q/T}\right\},\quad e^{-Q/T} \ll 1$$

Thus the ordering of spins terminates the logarithmic increase of the resistance and, furthermore, results in a decrease of resistance due to the substitution of $S(S+1)$ in the logarithmic factor by S^2 and the appearance of a supplemental negative term coming basically from an interference with ordinary scattering. Naturally there is also the effect of increasing resistance in the presence of magnetic field, connected to the usual curvatures of electron trajectories. We shall speak about this later.

Let us consider now how the curve $\rho(T)$ should behave in various cases. If $J > 0$, ρ_{ex} will decrease with the temperature. This will last until T becomes of the order T_0, the temperature at which the spins become ordered. According to the preceding this happens at $T \sim QS$, i.e., at $T \sim \max(T_C,\ g\mu_0 SH)$. Thereafter $\rho \to$ const.

However, if $J < 0$ various cases are possible. For $Q = 0$ on the complete curve $\rho(T)$ in the general case there must be a minimum deriving from the summation of the ordinary component of resistance which rises with temperature and the decreasing ρ_{ex}. Let us assume that it is located at temperature T_{min}. At a lower temperature T_{max} ρ has a maximum deriving from the pole in (15). If $T_0 \ll T_{max}$ then the ordering will manifest itself as follows on curve $\rho(T)$. T_{min} and T_{max} are preserved. Temperature T_{max} is independent of the concentrations of the admixtures. At $T \lesssim T_0$ the resistance $\rho(T) \to$ const.

If $T_{max} \ll T_0 \ll T_{min}$ the situation arises which probably is most frequently observed in experiments [2]. The minimum $\rho(T)$ is preserved. The old maximum disappears, however a new one arises. On approaching T_0 from above the rate of increase of $\rho(T)$ slows down and reverts to a decrease, after which $\rho(T)$ becomes constant. In the absence of a magnetic field or if it is weak, $T_0 \sim T_C$ so that the temperature of the maximum is approximately proportional to the concentration of magnetic impurities. But if $g\mu_0 HS \gg T_C$, then the temperature of the maximum is approximately proportional to H and does not depend on the concentration.

Finally in case $T_0 \gg T_{min}$ even for $J < 0$ the minimum is not present on the $\rho(T)$ curve.

In all cases ordering results in a decrease of ρ_{ex}. Still, as pointed out above, ρ_{ord} can increase at the same time under the influence of the external magnetic field. If the time between collisions $\tau \ll 1/\Omega$, $\Omega = eB/mC$ (B being the average field in the specimen) then the correction to the resistance is of the order $\rho_{ord}(\Omega\tau)^2$. This magnitude has to be compared with the change of exchange resistance, which for $Q/T \ll 1$ is of the order $\rho_{ex}(Q/T)^2$, and for $Q/T \gtrsim 1$ is of the same order as ρ_{ex}.

First the question arises whether in the absence of an external field the increase of ρ_{ord} in a ferromagnetic transition can outweigh the decrease of ρ_{ex}. According to [3], as the result of a ferromagnetic transition there appears an internal field $B \sim 4\pi g\mu_0 SN_C$. By determining the upper limit $4\pi g\mu_0 SN$, from the value B_{sat} of the saturation induction of usual ferromagnetics it follows that $B < 10^4C$. The change of ρ_{ord} will not appear for $\tau \ll \sqrt{(\rho_{ex}/\rho_{ord})}\ 1/\Omega$. Setting $\rho_{ex}/\rho_{ord} \gtrsim 10^{-2}$ we obtain $\tau \ll 10^{-12}/C$, or the mean free path $l \ll 10^{-4}/C$. If one evaluates l by formula $l \sim 1/N_i\sigma \sim I/N\sigma C$ and sets $N \sim 10^{22}\mathrm{cm}^{-3}$, $\sigma \sim 10^{-16}\mathrm{cm}^2$ then $l \sim 10^{-6}/C$, so that our requirement is fulfilled even in the case that the metal does not contain foreign non-magnetic impurities.

Let us now consider the influence of the external magnetic field. According to the above estimation the magnetic field will cause a drop of the total resistance, if

$$\max(T,\ \mu_0 H) \ll \frac{\hbar}{\tau}\sqrt{\frac{\rho_{ex}}{\rho_{ord}}} \tag{17}$$

For $l \sim 10^{-3}$ on the right side one has approximately 0.1°K. From there it follows that the total resistance can either increase or drop upon introduction of the field. The increase of the usual resistance can be kept down by non-magnetic impurity addition. However, in all cases the increase of the ordinary resistance with the field is temperature independent, whereas the change of the exchange component is highly temperature dependent.

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