Physics Vol. 1, No. 5, pp. 307-309, 1965. Physics Publishing Co. Printed in Great Britain

NOTE ON THE BAND THEORY OF MAGNETISM

A.W. OVERHAUSER

*Scientific Laboratory, Ford Motor Company,
Dearborn, Michigan*

(Received 2 March 1965)

Abstract

The conclusions given by Mattis in a recent study of an exactly solvable, one-dimensional model are criticized. A correct Hartree-Fock treatment of the model leads to an interaction independent spin susceptibility, in agreement with the exact result. Therefore here, at least, Hartree-Fock theory is vindicated rather than vitiated; and the paradox concerning correlations, posed by Mattis, is spurious. Finally, the model is shown to be not pertinent to the band theory of magnetism.

IN A RECENT paper [1] published in this journal Mattis has explored several consequences of a solvable Fermi gas model in one dimension. The model is an extension of one treated originally by Luttinger [2] and later by Lieb and Mattis [3]. The relevant generalization is the inclusion of an ordinary spin coordinate, which endows the model with magnetic properties – for example, a Pauli spin susceptibility. The remarkable feature is that exact solution of the quantum mechanical many-body problem, including rather arbitrary interaction between the Fermions, is feasible. The commutation rules which play such an important role in the solution were first derived in 1936 by Born and Nagendra Nath, [4] and have been emphasized by Pryce [5] and also by Case [6].

The model consists of two types of Fermi particles, which we shall call x electrons and y electrons. The x electrons have an energy spectrum, $E = +k$; and the k-states between $-\infty$ and $+k_F$ would be occupied for the non-interacting ground state. The y electrons have an energy spectrum, $E = -k$; and the corresponding occupied states would run from $-k_F$ to $+\infty$. It is important to realize that the Hilbert spaces of the x and y electrons are completely orthogonal. In other words, the distinction between x and y electrons is the analog of an isotopic-spin degree of freedom. Both species of electron have, of course, a complete Pauli-spin degree of freedom.

For the present discussion we need only consider space-coordinate interactions between the electrons. Between every pair of electrons, regardless of species, there will be an interaction potential $V(z)$, where z is the spatial separation of the two electrons. $V(q)$ will be the Fourier transform of this potential. The only property we need assume for the discussion that follows is that $V(q)$ is an integrable function.

The eigenstates of the model just described can be explicitly enumerated by the method given by Mattis [1]. It should be noted in passing that the quasi-particle Hamiltonian defined by Mattis, his equation (III.2), is merely a constant and can be disregarded [7]. The result on which we shall now focus attention is the invariance of the spin susceptibility under the action of spatial forces. This property, derived rigorously by Mattis, seems at first surprising since in all band theory models previously studied, exchange interactions modify the spin susceptibility, at least within the approximation schemes employed. We shall now show, however, that the present result has a trivial and intuitive explanation.

In order to understand the essential ingredient of a one-dimensional model appropriate to a band theory of magnetism, we will first consider the usual one-dimensional model with $E = k^2$, and with occupied k-states for the non-interacting ground state extending from $-k_F$ to $+k_F$. In order to be specific, let us take an interaction with a Fourier transform,

$$V(q) = \exp(-|q|).$$

We explicitly evaluate the total exchange energy, W_{ex}, of the Hartree-Fock state having a small fractional spin polarization, δ/k_F. That is,

$$k_F \text{ (up-spin)} = k_F + \delta,$$

$$k_F \text{ (down-spin)} = k_F - \delta.$$

For a one-dimensional gas of unit length, we obtain,

$$W_{ex} = -\left[2k_F - 1 + \exp(-2k_F)\cosh 2\delta\right]/2\pi^2.$$

The change in W_{ex} arising from the small fractional polarization is, accordingly,

$$\Delta W_{ex} \cong -\delta^2 \exp(-2k_F)/\pi^2.$$

The factor $\exp(-2k_F)$ in the coefficient of δ^2 shows that the exchange interactions which promote magnetic alignment are those between parallel spin states on opposite sides of the Fermi surface. The generality of this conclusion for a one-dimensional model can be easily established.

Returning now to the Mattis model, we observe that exchange interactions exist only between x electron pairs with parallel spin and between y electron pairs with parallel spin. There are no exchange interactions between x electrons and y electrons. The fundamental reason is, of course, that exchange interactions exist only between particles with equal spin and equal isotopic spin. (The required absence of x-y exchange interactions in the Mattis Hamiltonian can be readily verified.) A magnetic polarization in the Mattis model corresponds, say, to an increase in the number of up-spin x electrons near the right-hand Fermi surface together with an increase in the number of up-spin y electrons near the left-hand Fermi surface. Since there is no exchange interaction between these two groups of electrons, our previous remarks imply that no tendency toward magnetic order should be anticipated. In other words, the model is magnetically impotent.

The foregoing conclusion can be verified explicitly. Consider, say, an x electron at the Fermi surface. Its exchange potential,

$$A = \int_0^\infty V(q)\,dq/2\pi,$$

is a constant, independent of the location of the Fermi surface, and has the same value for the spin-up or spin-down electron of maximum energy. Consequently, if we were to magnetically polarize the x electrons, one electron at a time, the change in exchange potential of each electron as it is transferred from the uppermost spin-down state to the lowermost, empty spin-up state is zero, independent of the accumulated polarization. Therefore, the total Hartree-Fock exchange energy of the Mattis model is invariant under magnetic polarization. It follows that expression (V.7) of reference [1] does not depend on the interaction, contrary to the assertion made there. Hartree-Fock theory gives the correct spin susceptibility, and no paradox concerning correlation corrections exists.

We have shown that the exchange interactions, arising from spatial forces, which could give rise to a magnetically active, one-dimensional model are simply not present in the model considered by Mattis. The model is therefore not pertinent to the band theory of magnetism, be it paramagnetism, ferromagnetism, or spin-density-wave antiferromagnetism

Acknowledgement

The author is pleased to thank Drs. R.V. Penney and A.D. Brailsford for many helpful discussions.

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7. This can be seen readily by observing that the non-interacting ground state is also an eigenstate of H_{qp}. Noting further that the various ρ and S operators can be used to generate all of Hilbert space by operating on the non-interacting ground state, and that these operators commute with H_{qp}, one quickly establishes the vacuous nature of H_{qp}.