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Viewpoint

Electrons in graphene: an interacting fluid par excellence

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Along with the quark gluon plasma and cold atom gasses, graphene is establishing its place as a perfect liquid.

Subject Areas: **Graphene**

A Viewpoint on:
Graphene: A Nearly Perfect Fluid
Markus Müller, Jörg Schmalian and Lars Fritz
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Ever since it was shown that graphene—a single layer of carbon atoms—could be isolated from graphite, it has occupied a center stage of condensed matter physics. The popularity of graphene is rooted in the unusual nature of its low-energy excitations: near the Fermi level, the electron energies scale linearly with their momenta. This means that the electrons can be described as "massless" fermions, though with a velocity of about 300 times less than the velocity of light. The linear dispersion relation also implies a vanishing density of single-particle states at the Fermi level, which should make the effects of the Coulomb interaction between electrons weak.

This usual mantra, however, may sometimes be quite misleading, as argued by Markus Müller at the ICTP in Trieste, Italy, Jörg Schmalian at Ames Lab and Iowa State University, US, and Lars Fritz at Harvard University, US, in a paper appearing in *Physical Review Letters*[1]. They show that a particularly suitable measure of how strongly the excitations in a given quantum fluid interact is given by the dimensionless ratio between the fluid's sheer viscosity and entropy density. They find that the value of this ratio in graphene is surprisingly close to its likely lower bound [2]. Such a low viscosity-to-entropy ratio, somewhat paradoxically, means that the electrons in graphene form a quantum liquid that is, in fact, strongly interacting. By this criterion graphene comes closer to being a "perfect fluid" than several other quantum systems that have often been labeled as strongly correlated.

Landau's notion of a Fermi liquid as a system of interacting fermions that, at low energies, effectively behave as noninteracting quasiparticles is the central paradigm of many-body physics. Our modern way of thinking about a Fermi liquid is to use the language of renormalization group theory—a theory that extracts the essential physics of many-body systems by zooming out from the microscopic details. In this framework, one would say that although the Coulomb interaction between electrons in a typical metal in an absolute sense is not weak, its effective strength depends on the energies at which the system is probed [3]. In a Fermi liquid, the effective interaction parameters decrease at lower temperatures and frequencies until they reach saturation. This idea receives its simplest realization precisely in graphene: The Fermi surface is shrunk to just two points ("Dirac" points) in momentum space, near which the energy depends linearly on the quasiparticle's momentum and the density of quasiparticle states also vanishes linearly. The scarcity of low-energy excitations renders all the short-range components of the Coulomb electron-electron interactions *irrelevant*[4]. Using a term from renormalization group theory, these interactions "flow" towards zero as fast as the first power of temperature. From this perspective, graphene appears to be a perfect example of a weakly interacting Fermi liquid. Or so it would seem.

What about the fact that the Coulomb interaction is a long-ranged force? In metals, this does not matter much, since the quasiparticles screen the interaction and make it effectively short ranged. But in graphene there are not enough low-energy quasiparticles to screen effectively, and the Coulomb interaction remains long ranged [5]. As a result, the Coulomb interaction does not change with the energy scale. Or, in the parlance of renormalization group theory one would say that the coupling g in the Coulomb interaction $V(r) = g/r$ represents an exactly marginal coupling, which does not flow at all with the change in energy scale. Its main effect, it turns out, is to produce a shift in the Fermi velocity that diverges at low temperatures, albeit only as a logarithm: $v(T) \sim g \log(T_0/T)$, where the high-energy scale $T_0 \sim 10^5$ K is set by the width of the conduction energy band [6]. If one defines the dimensionless strength of the Coulomb interaction $\alpha(T) = g/(\hbar v(T))$, this coupling constant would slowly approach zero as the system is probed at progressively lower temperatures (Fig. 1).

So theorists can paint a picture of graphene with only a few strokes: At temperature or frequency scales

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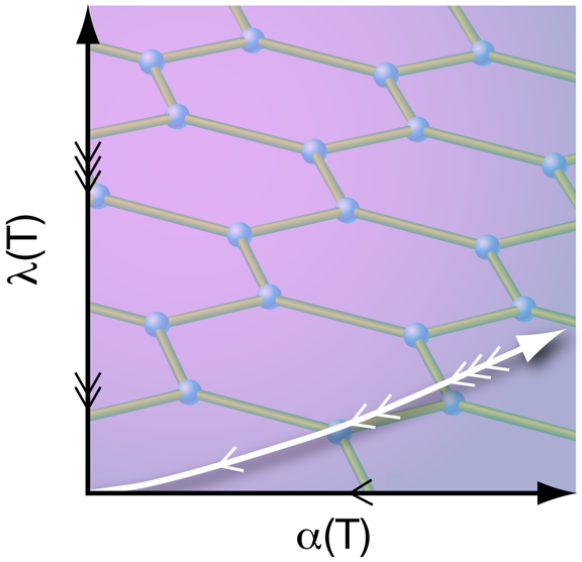

FIG. 1: The schematic "flow" of the coupling constants describing weak electron-electron interaction with decreasing temperature in graphene. α parameterizes the $\sim 1/r$ tail of the Coulomb interaction, and λ stands collectively for the electron-electron repulsion on the lattice scale. The density of the arrows increases with the speed of the flow. (Illustration: Alan Stonebraker)

much below T_0 any physical quantity is, to leading order, given by its value for the noninteracting system of quasirelativistic particles with an effective velocity $v(T)$. The leading corrections are small at low temperature and proportional to $\alpha(T) \sim 1/\log(T_0/T)$[7]. As long as the electron-electron interaction is not strong enough to turn graphene's ground state into an insulator [4], the effect of the interaction on the low-energy properties is fairly small. In this respect graphene resembles some of our most cherished physical theories: quantum electrodynamics (at low energies) and quantum chromodynamics (at high energies).

There are, however, quantities that do diverge in the noninteracting regime; namely, some response functions, which measure how quickly the system restores the thermodynamic equilibrium. In fact, in the complete absence of interactions, the relaxation time would be infinite. In graphene the relaxation time τ is inversely proportional to temperature and α^2. This suggests that the viscosity of the electronic fluid in graphene—which, like in any fluid, is a measure of resistance to a shear force and for graphene is given by $\eta \sim n\langle\varepsilon\rangle\tau$, where $n \sim (k_B T/\hbar v)^2$ is the density of thermally excited quasiparticles and $\langle\varepsilon\rangle \sim k_B T$ is their average energy—can to leading order be written as $\eta = (C_\eta/\hbar)(k_B T/v\alpha)^2$, where C_η is a numerical constant of proportionality. Normalizing this result by graphene's entropy density then yields

$$\eta/s = (C_\eta \pi/9\zeta(3))(1/\alpha(T))(\hbar/k_B), \qquad (1)$$

with all the numerical constants fully displayed. The last expression for the viscosity-to-entropy ratio shows it is a dimensionless number in the units of nature's constants $\hbar/k_B$. If the low-temperature limit of the flowing coupling constant $\alpha(T)$ were finite, this number would be a characteristic of a "fixed point" of the renormalization group flow, akin to other universal numbers that characterize fixed points, critical exponents and amplitude ratios as prime examples. (Fixed points are the special points in the coupling constant space where the flow stops.) Since $\alpha(T)$, however, in graphene approaches zero at low temperatures, the ratio η/s ultimately diverges, but only very slowly. Building on their previous calculation of graphene's dc conductivity [8], in a technical tour de force, Müller *et al.* determined the constant of proportionality C_η to ultimately find $\eta/s \sim 0.00815(\log(T_0/T))^2\ \hbar/k_B$. At room temperatures, this number is only $\sim 0.3\hbar/k_B$.

To appreciate the above result one obviously needs a useful point of reference. Let us ask what the same ratio would be, this time in a truly strongly interacting system. Kovtun, Son, and Starinets [2] studied a number of strongly interacting field theories, some of which were dual to those describing black holes, and found that the ratio is finite, universal, and in fact not much lower than the result in graphene: $\eta/s = (\pi/4)(\hbar/k_B)$. One expects that this number may provide a natural lower bound, and indeed the result for graphene conforms to this conjecture. The dimensionless viscosity of graphene, however, is significantly lower than in several other systems that would undoubtedly deserve to be characterized as strongly coupled. For example, in cold atoms with a diverging scattering length, $\eta/s \sim 0.5(\hbar/k_B)$, whereas for helium at the superfluid critical point, $\eta/s \sim 0.7(\hbar/k_B)$[9]. It should be noted, though, that if the chemical potential μ of graphene is tuned to lie away from the two Dirac points, it will behave as a conventional metal with a Fermi surface. In this case, the viscosity increases at low temperatures at a much faster rate as $\eta/s \sim (|\mu|/T)^3$.

The fact that the viscosity-to-entropy ratio in graphene is so low and almost temperature independent is another example that this material is what would be called a "quantum critical" system: there is no length scale other than the ones provided externally by the temperature, frequency of the measurement probe, or finite size of the sample. One of the main preoccupations of condensed matter physicists for many years has been understanding quantum critical points in various systems. Graphene appears to be a ready-made, particularly gentle example of fermionic quantum criticality, with comparatively few gapless fermions appearing only near special points in momentum space. One may expect many lessons about the nature of quantum transport, response to magnetic field, or the effects of disorder in critical systems to be learned from this deceptively simple-looking system.

The main lesson of the work of Müller, Schmalian,

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and Fritz, however, may be that graphene is, in a certain well-defined sense, rather far from being weakly interacting. In fact, with a possible exception of the ultrarelativistic quark-gluon plasma [9], from temperatures as low as 50 K to temperatures as high as 300 K graphene may be closer to the notion of a perfect strongly interacting fluid than any other quantum system we currently know. One cannot help but wonder what other surprises this fascinating material has in store.

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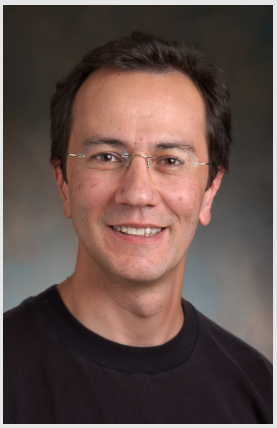

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